On the enumeration of plane bipolar posets and transversal structures

Éric Fusy, Erkan Narmanli, and Gilles Schaeffer

Summary

Maps and decorated maps

1. Bijection with quadrant tandem walks

a. The KMSW bijection
b. Plane bipolar posets
c. Plane bipolar posets by vertices
d. Transversal structures

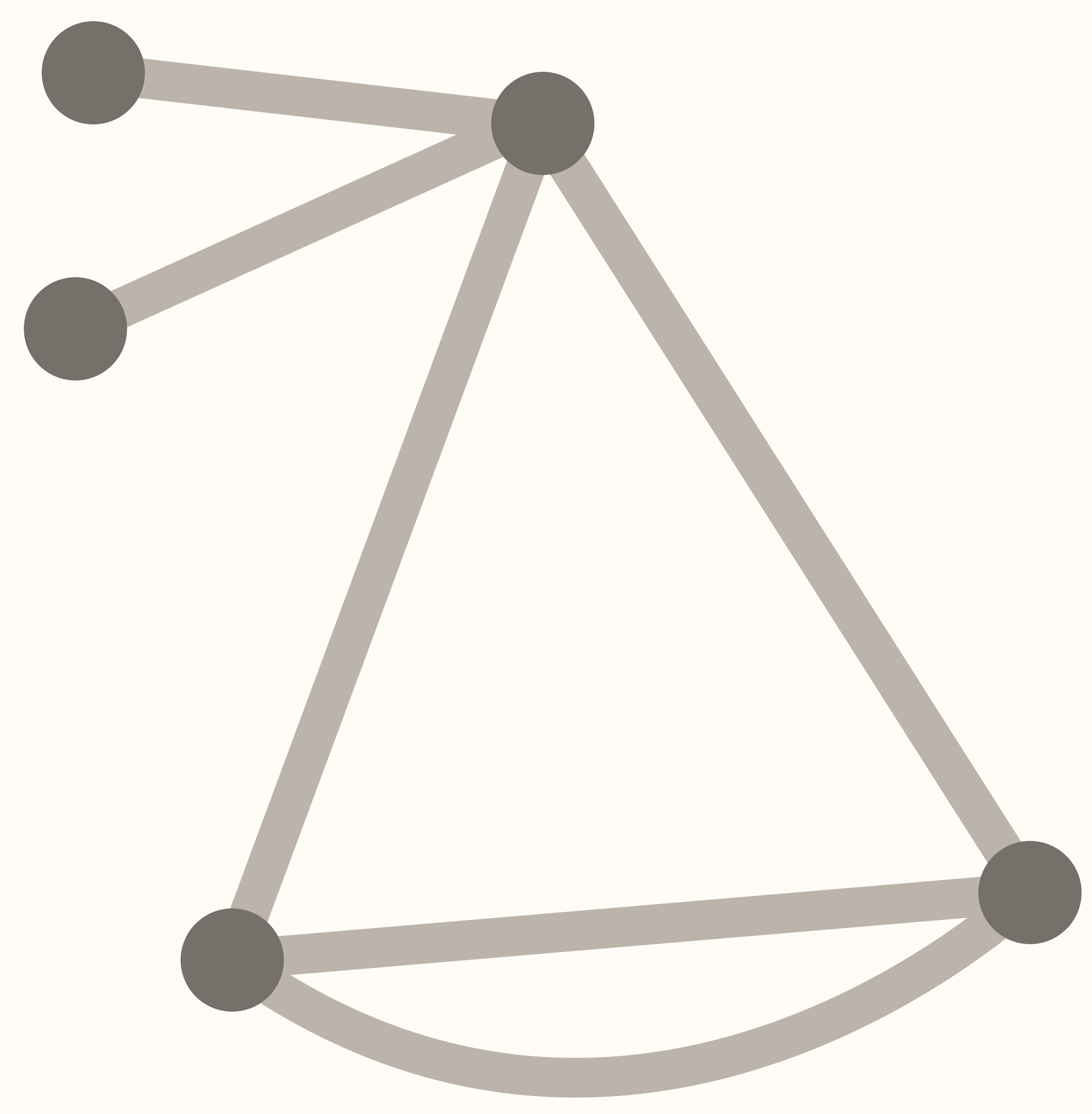
2. Asymptotic enumeration

(Digression on plane permutations)

3. Generic transversal structures

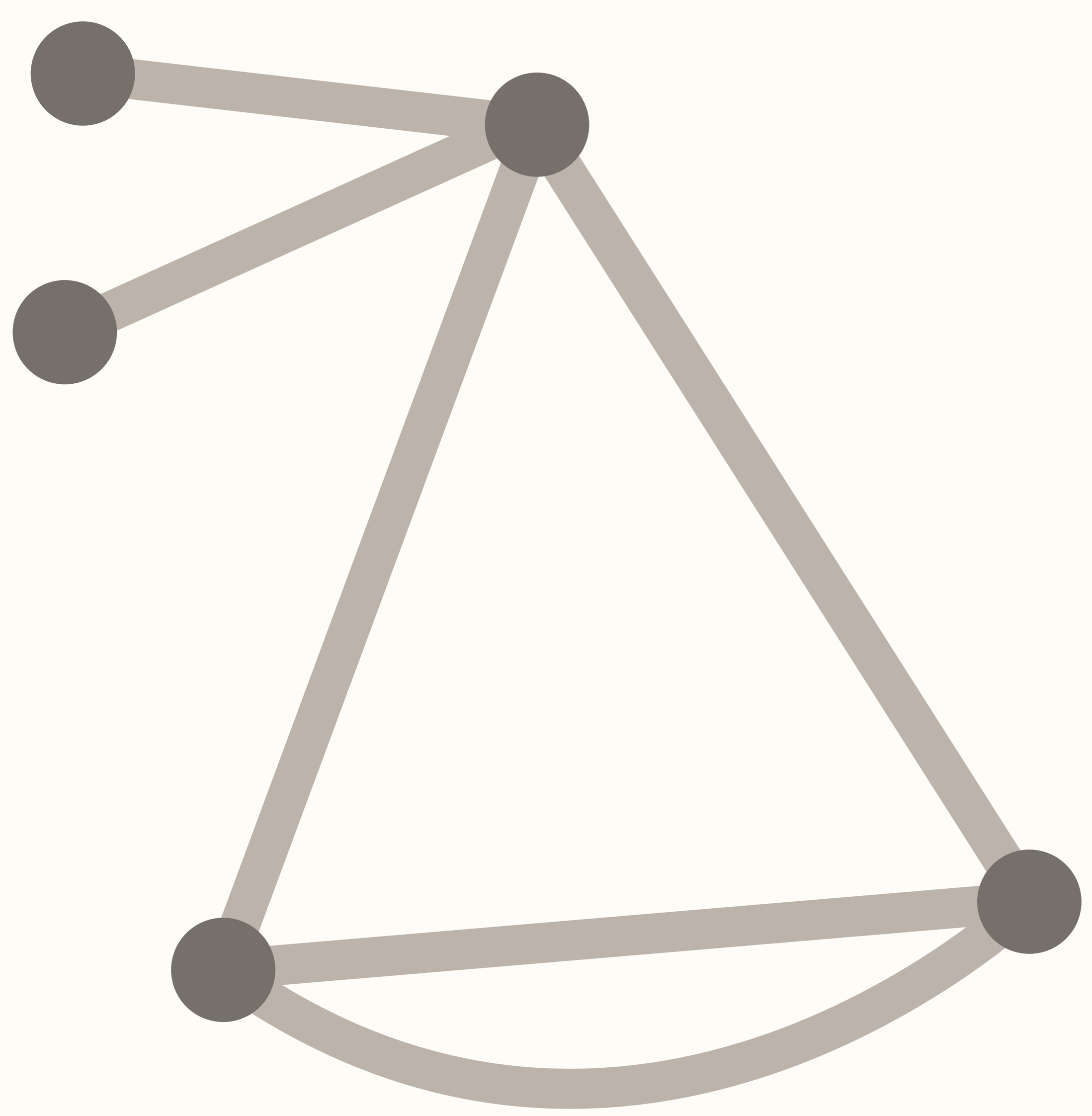
Decorated maps

Graphs



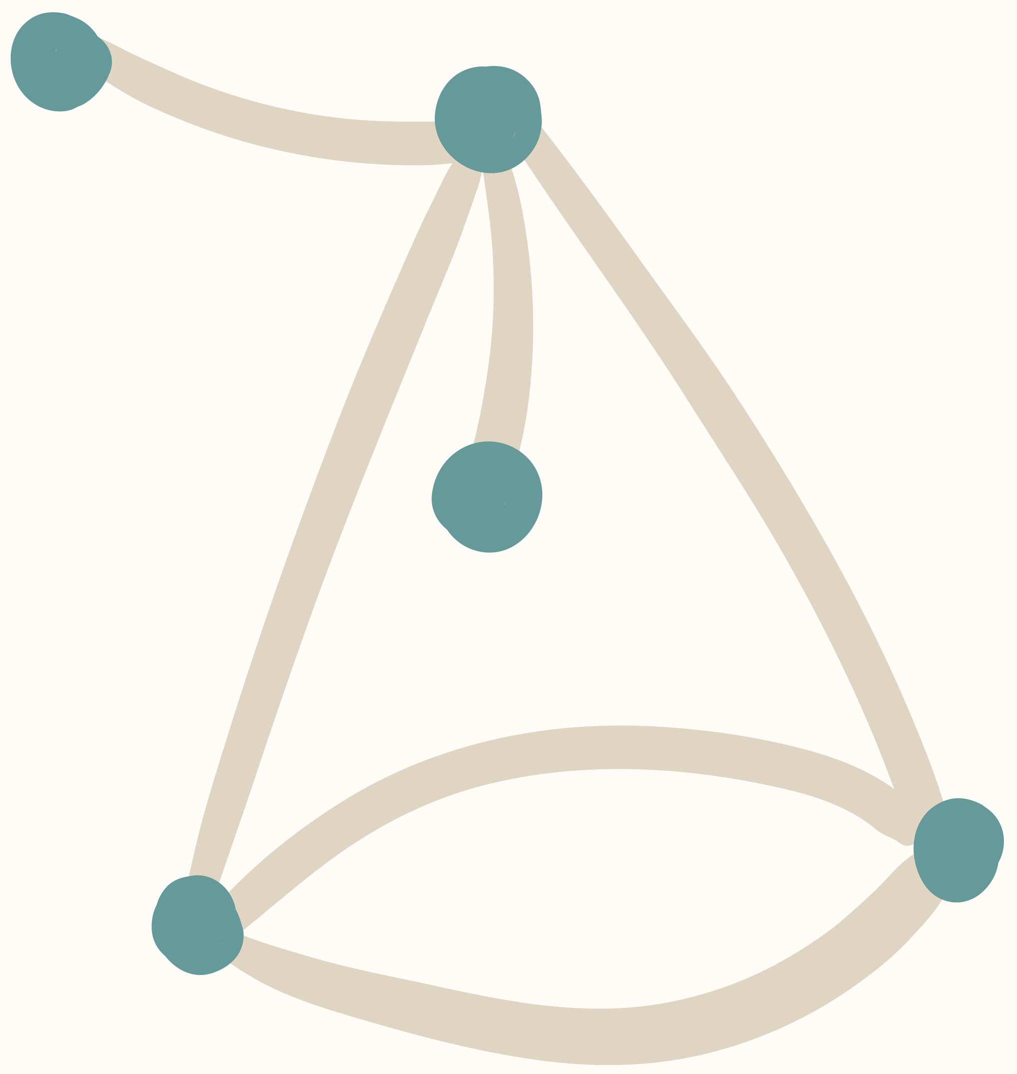
Decorated maps

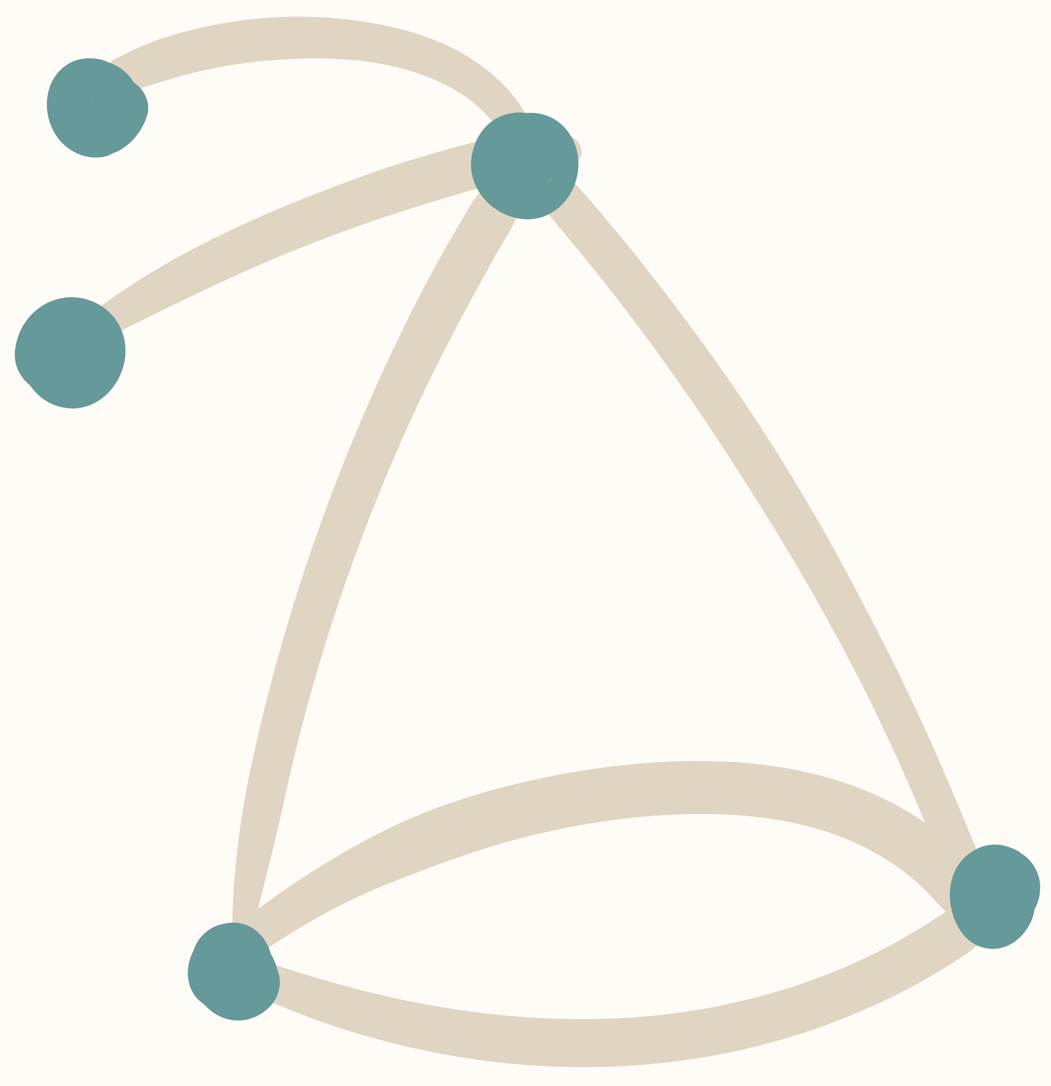
Graphs



Maps

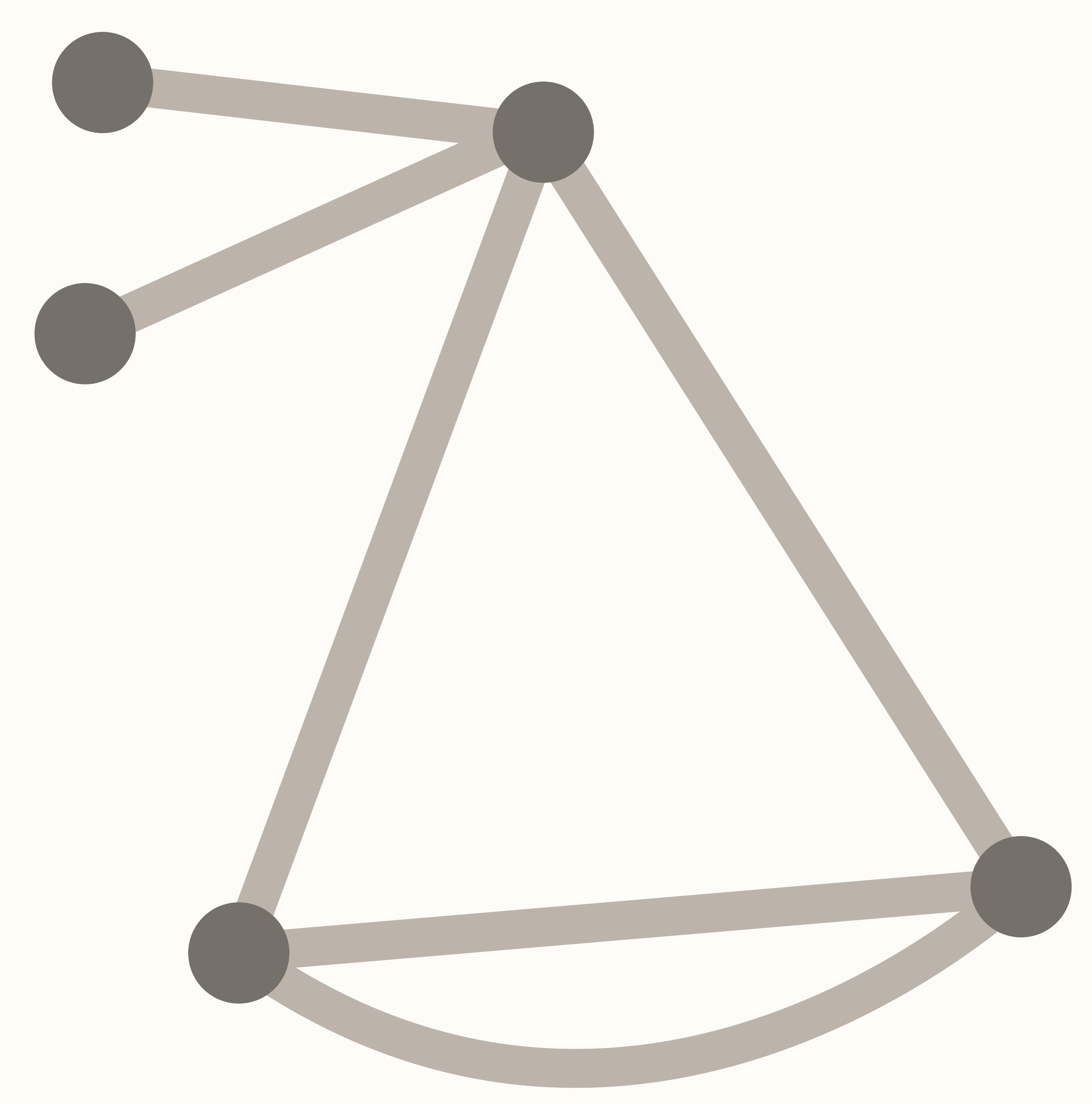
embedding on the plane





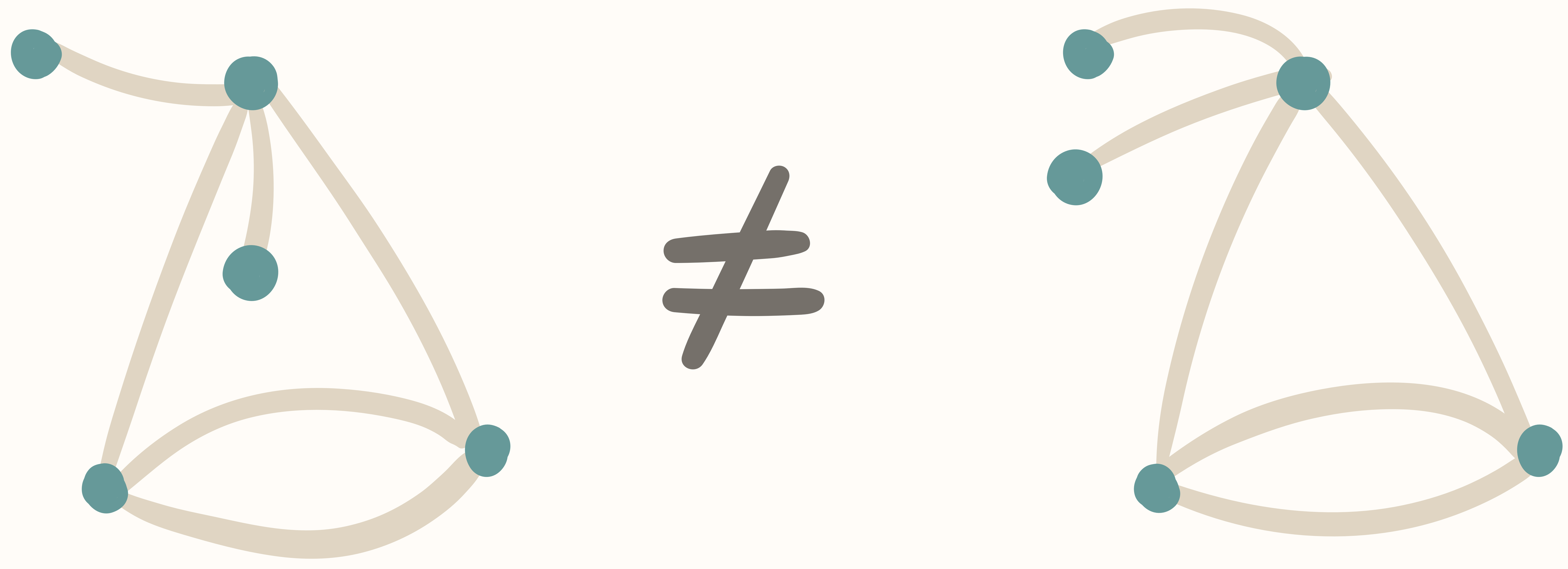
Decorated maps

Graphs



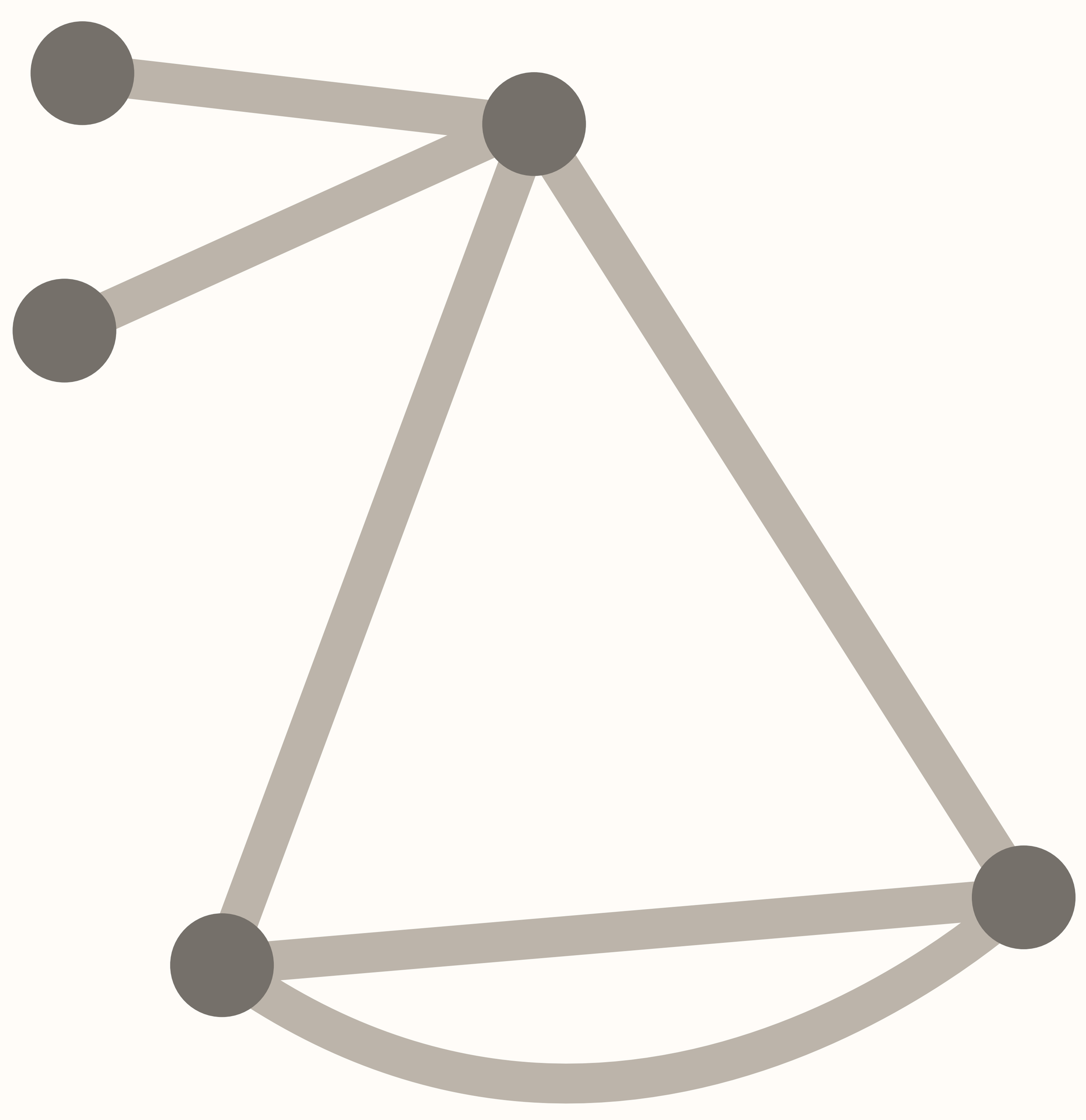
Maps

embedding on the plane



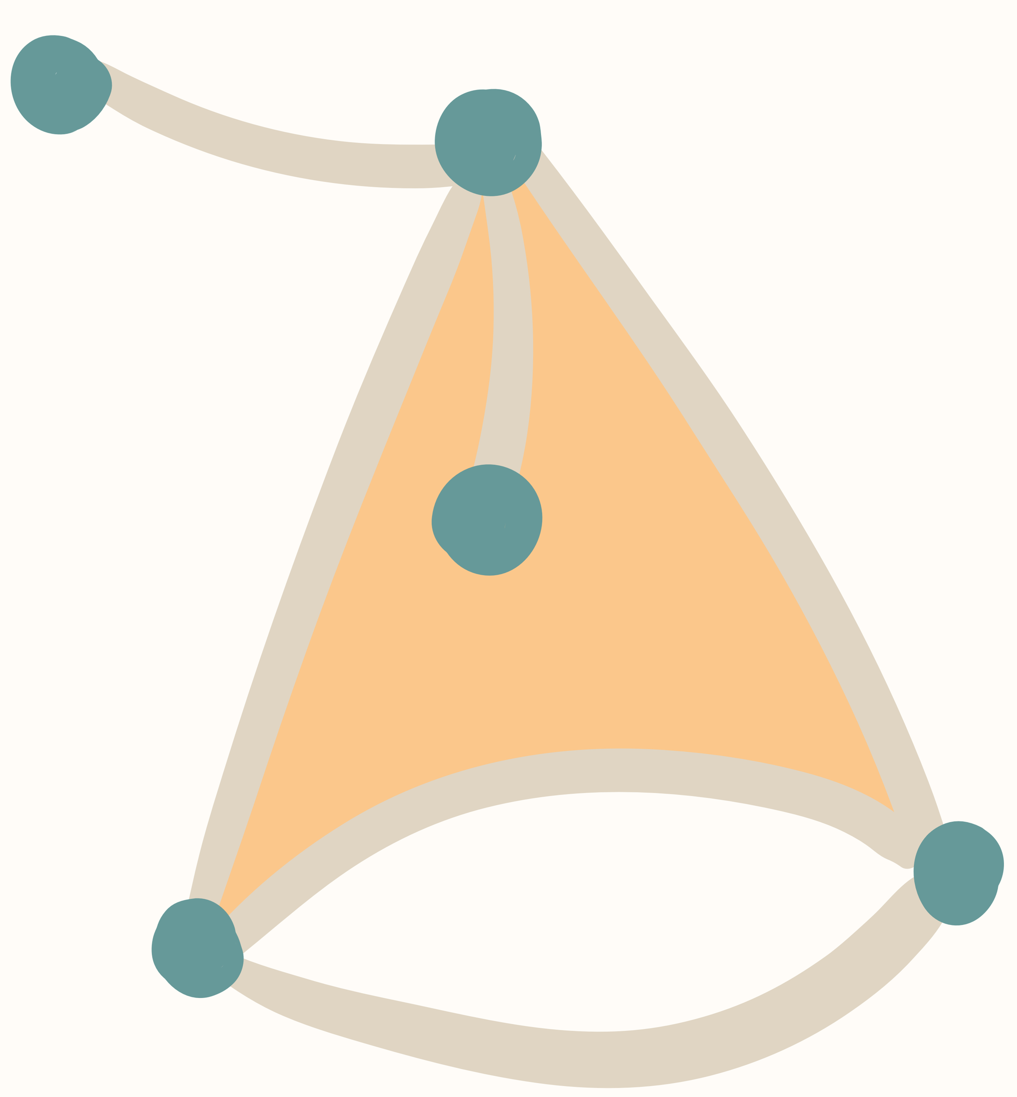
Decorated maps

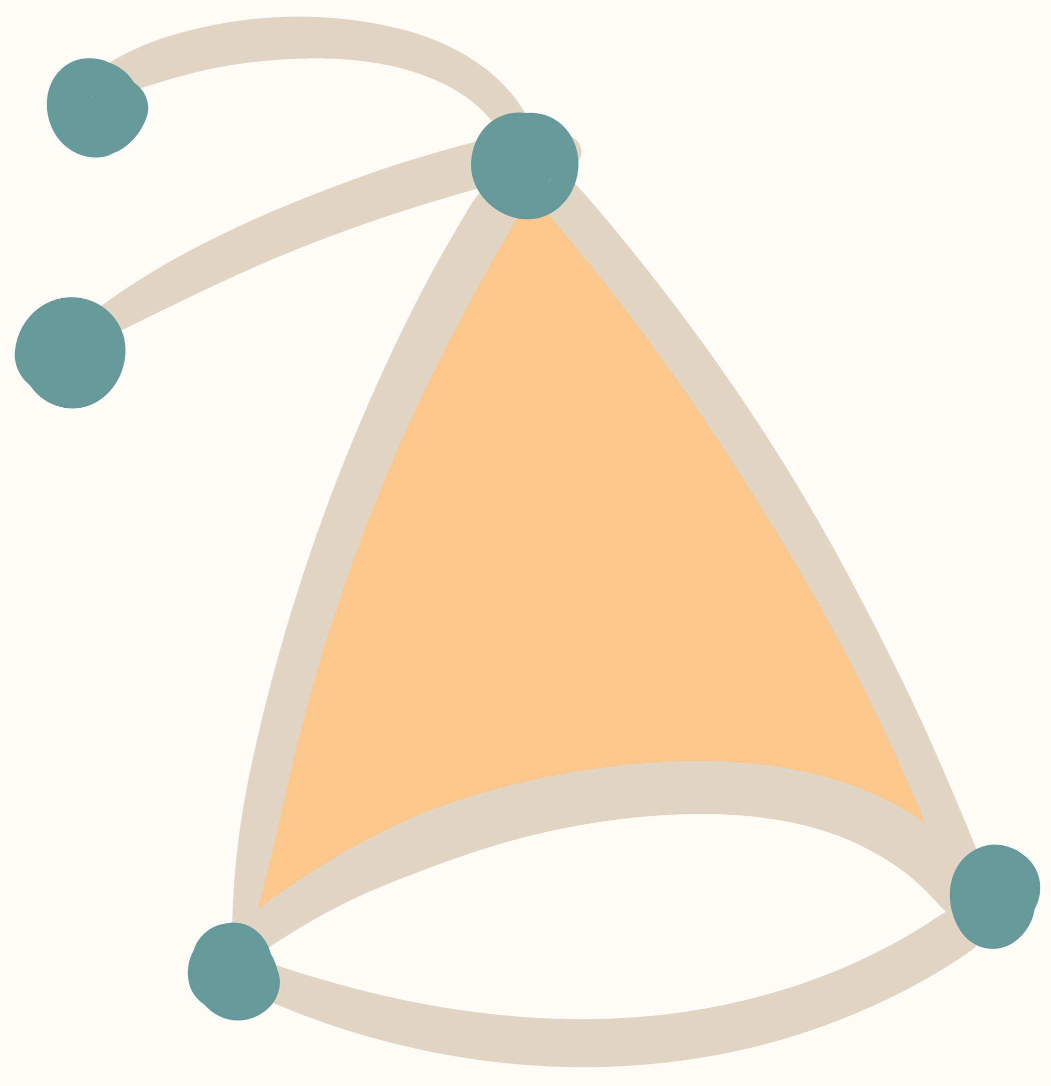
Graphs



Maps

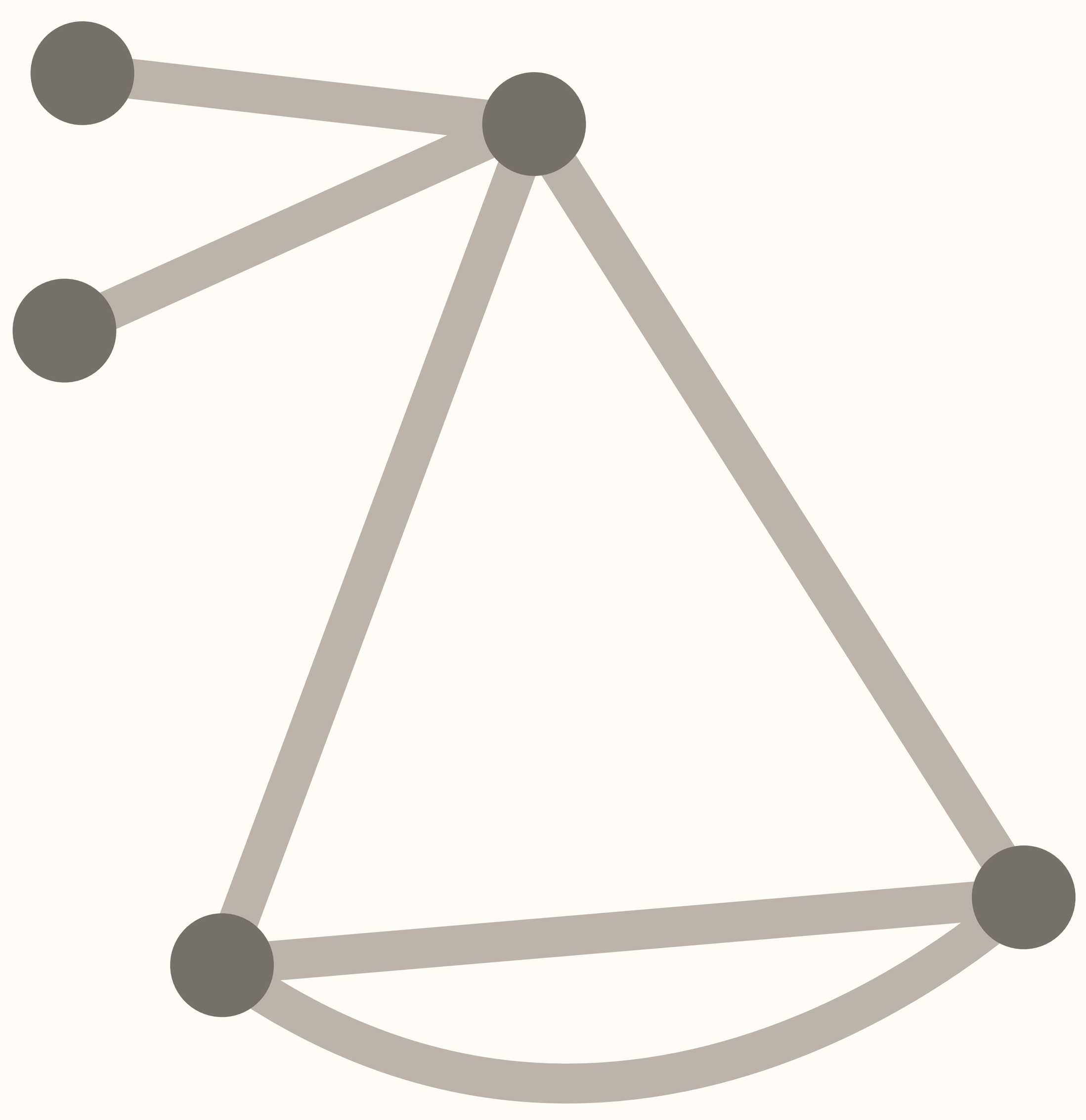
embedding on the plane





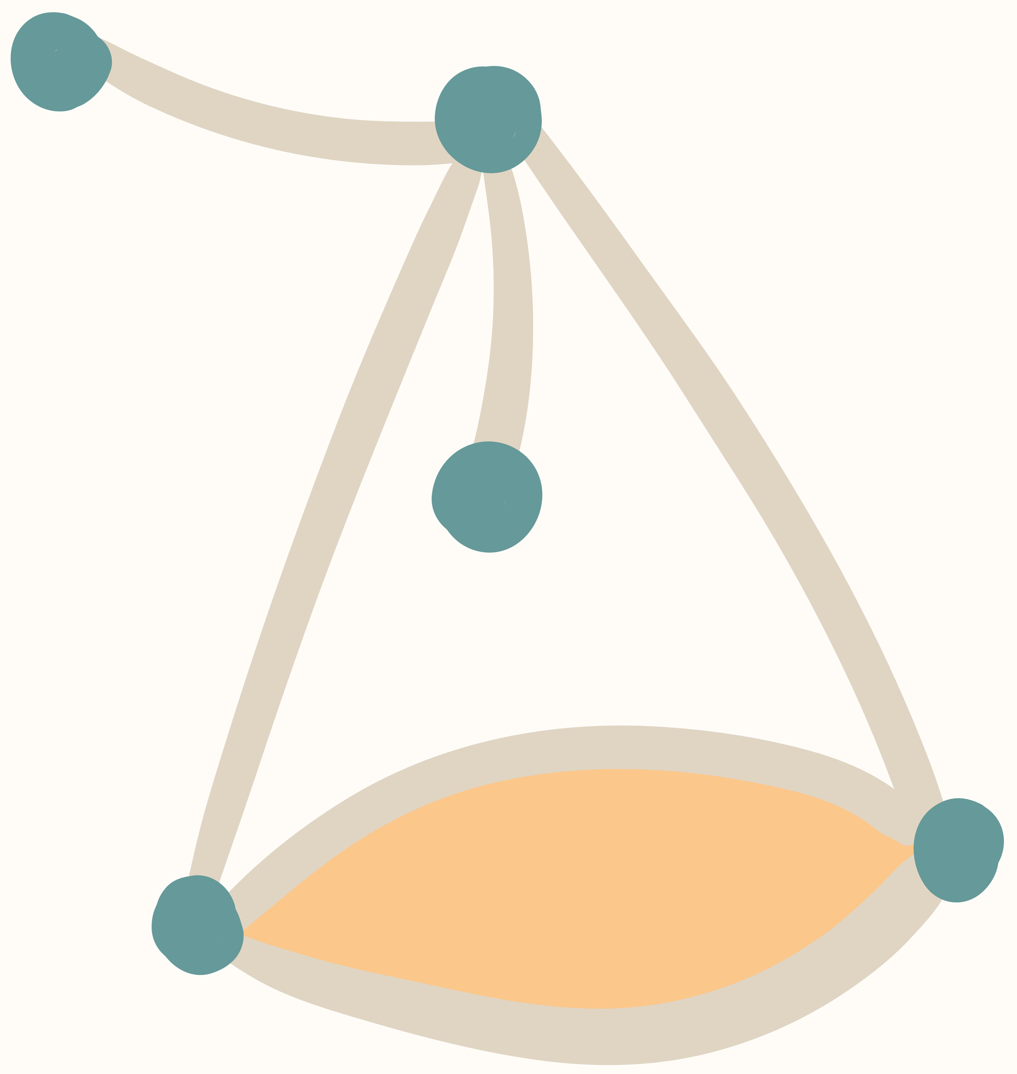
Decorated maps

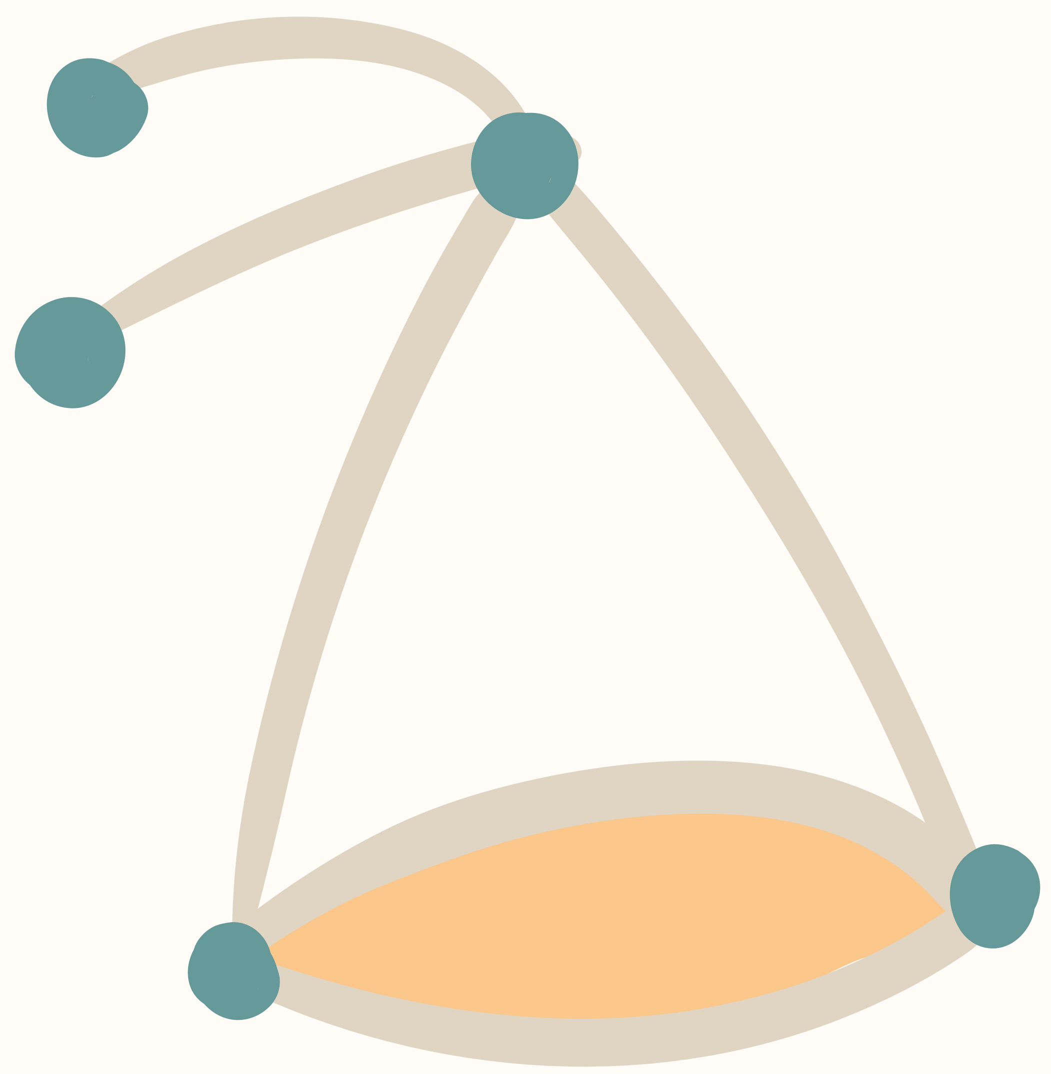
Graphs



Maps

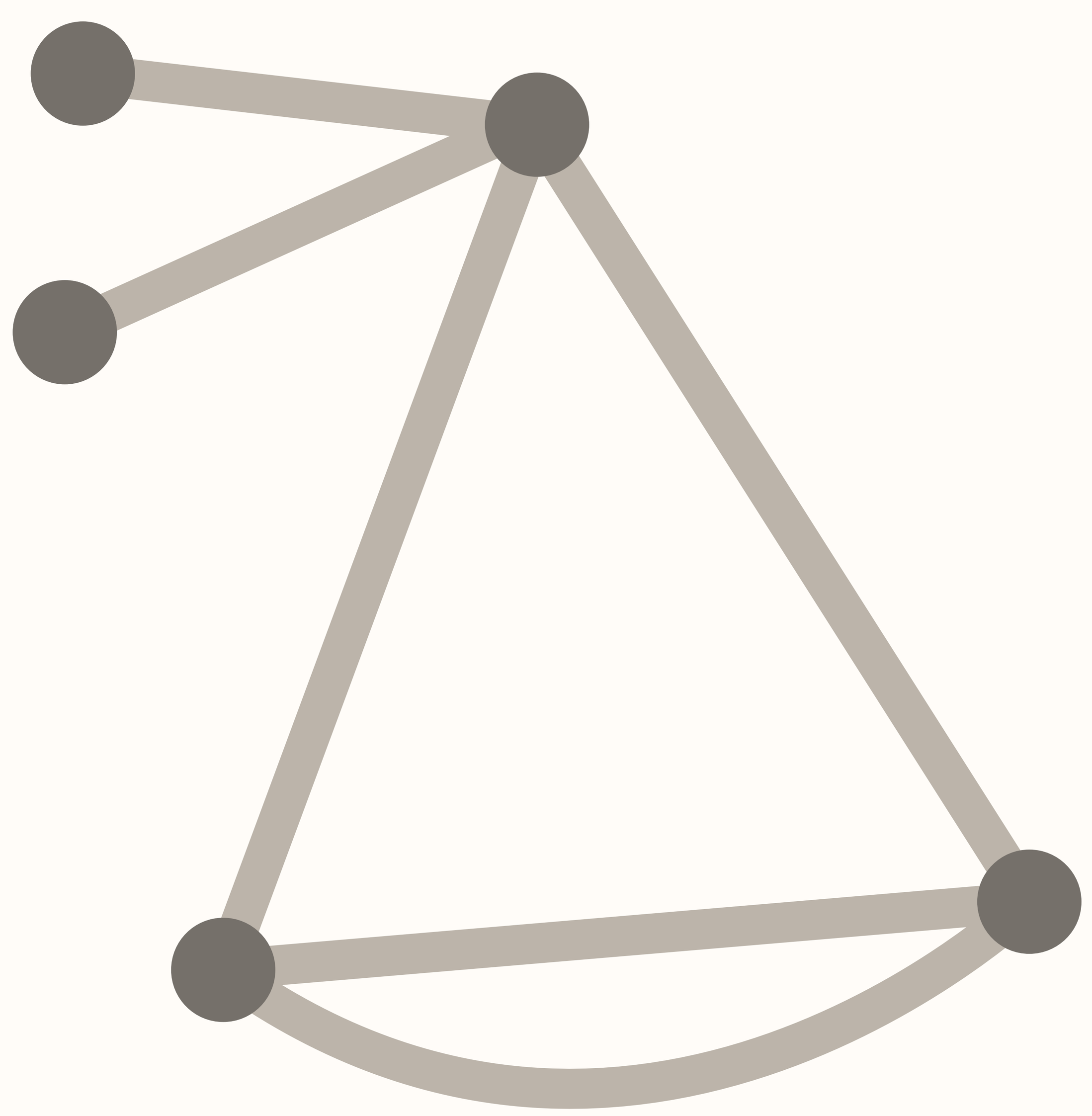
embedding on the plane





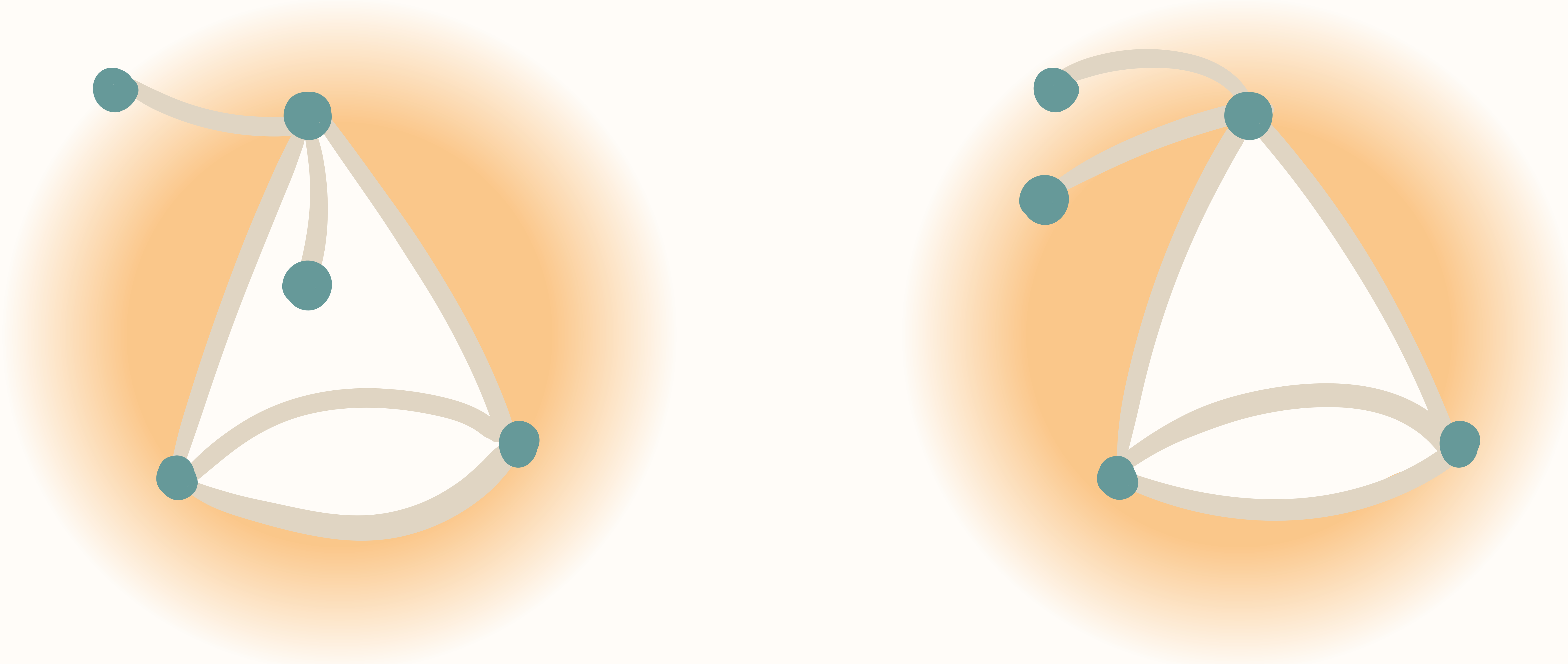
Decorated maps

Graphs



Maps

embedding on the plane



Decorated maps

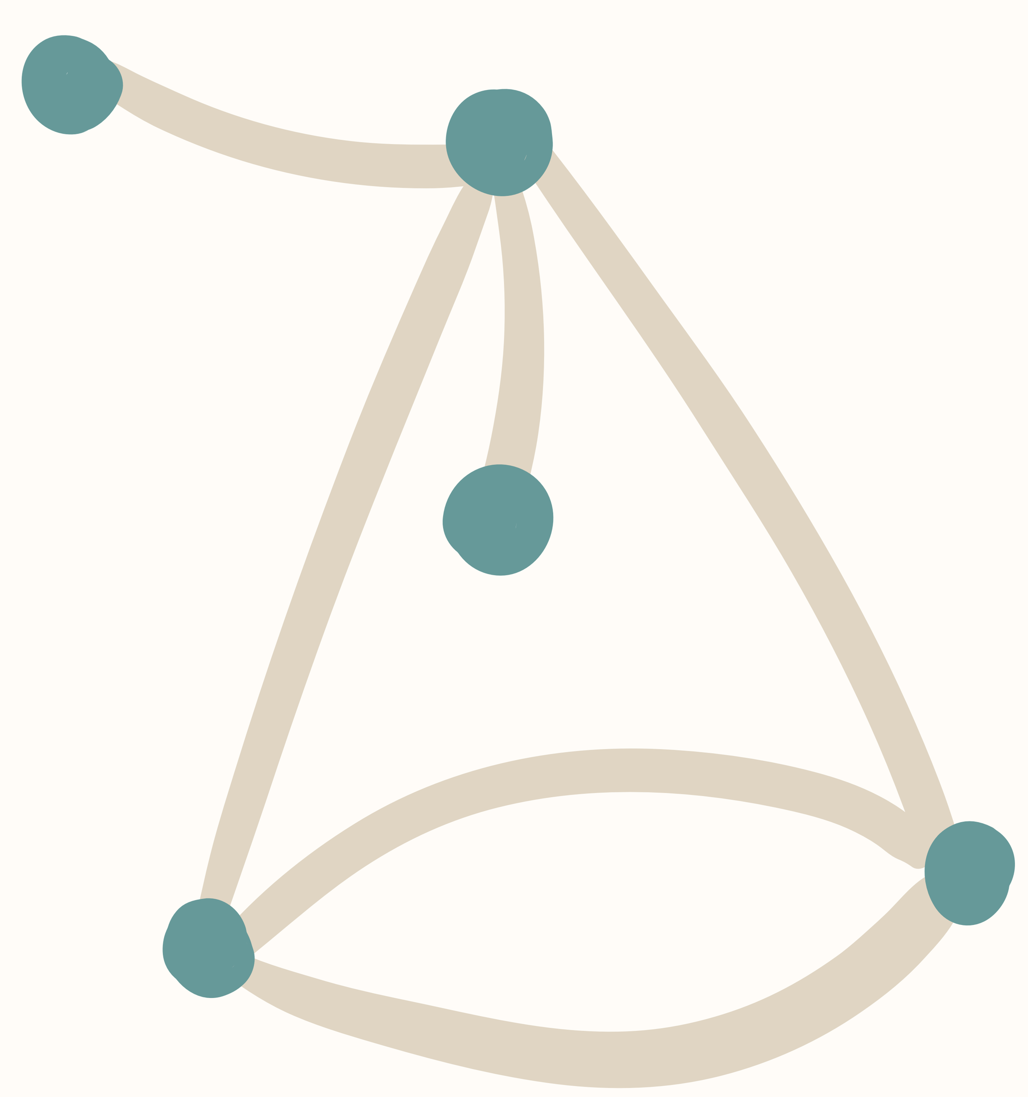
→ *exact enumeration formulas*

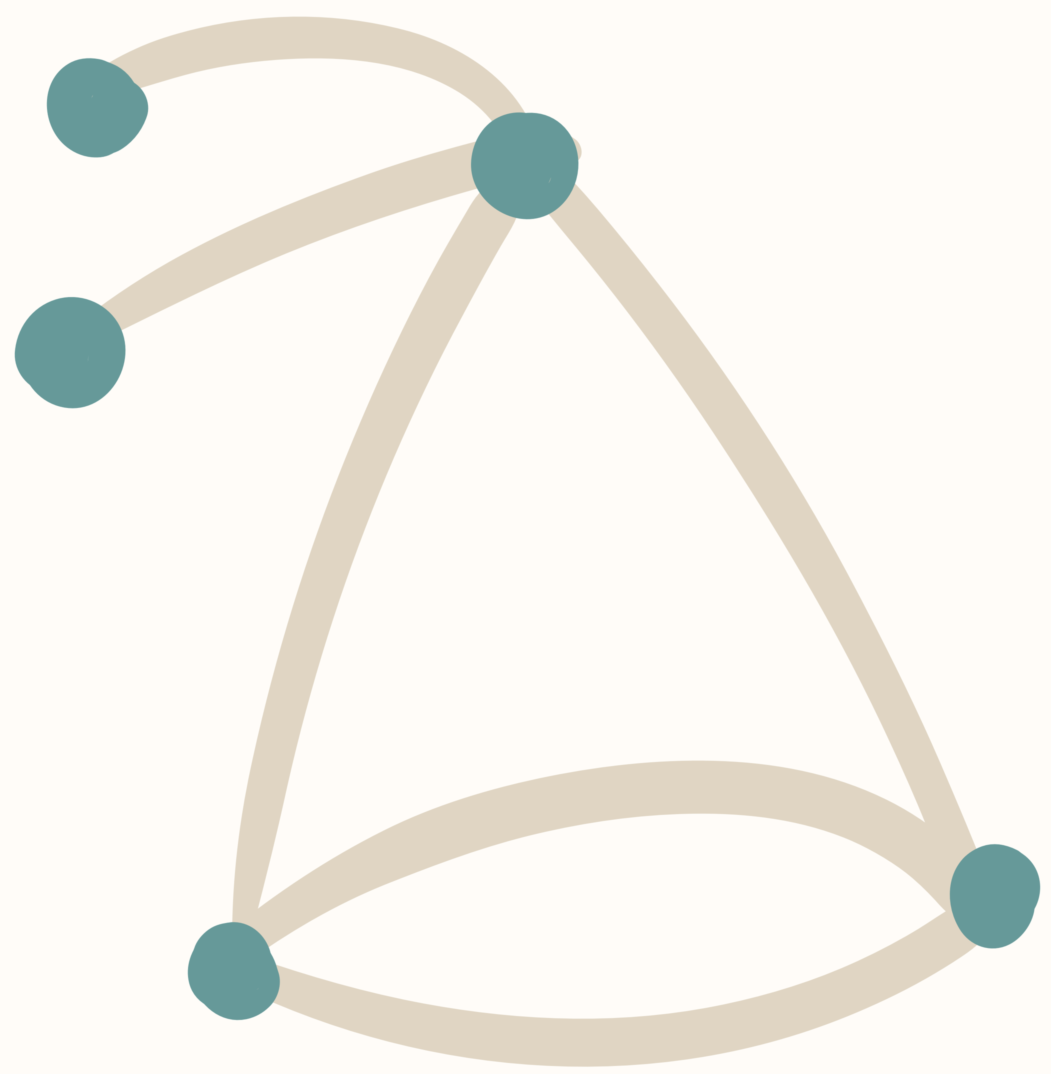
→ *universal critical exponent*

 # maps with n edges : $\varkappa \cdot \gamma^n n^{-5/2}$

Maps

embedding on the plane





Decorated maps

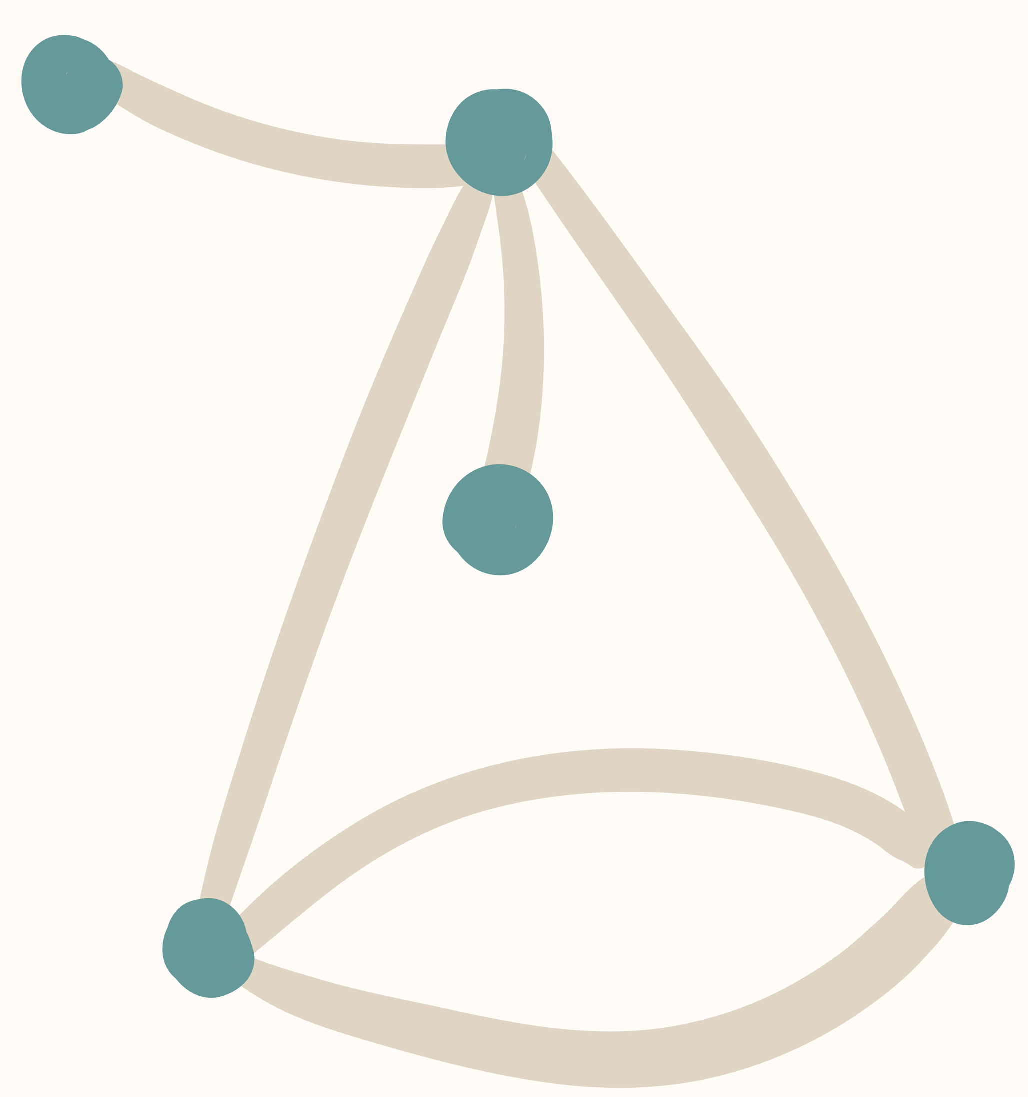
→ *exact enumeration formulas*

→ *universal critical exponent*

\# maps with n edges : $\varkappa \cdot \gamma^n n^{-5/2}$

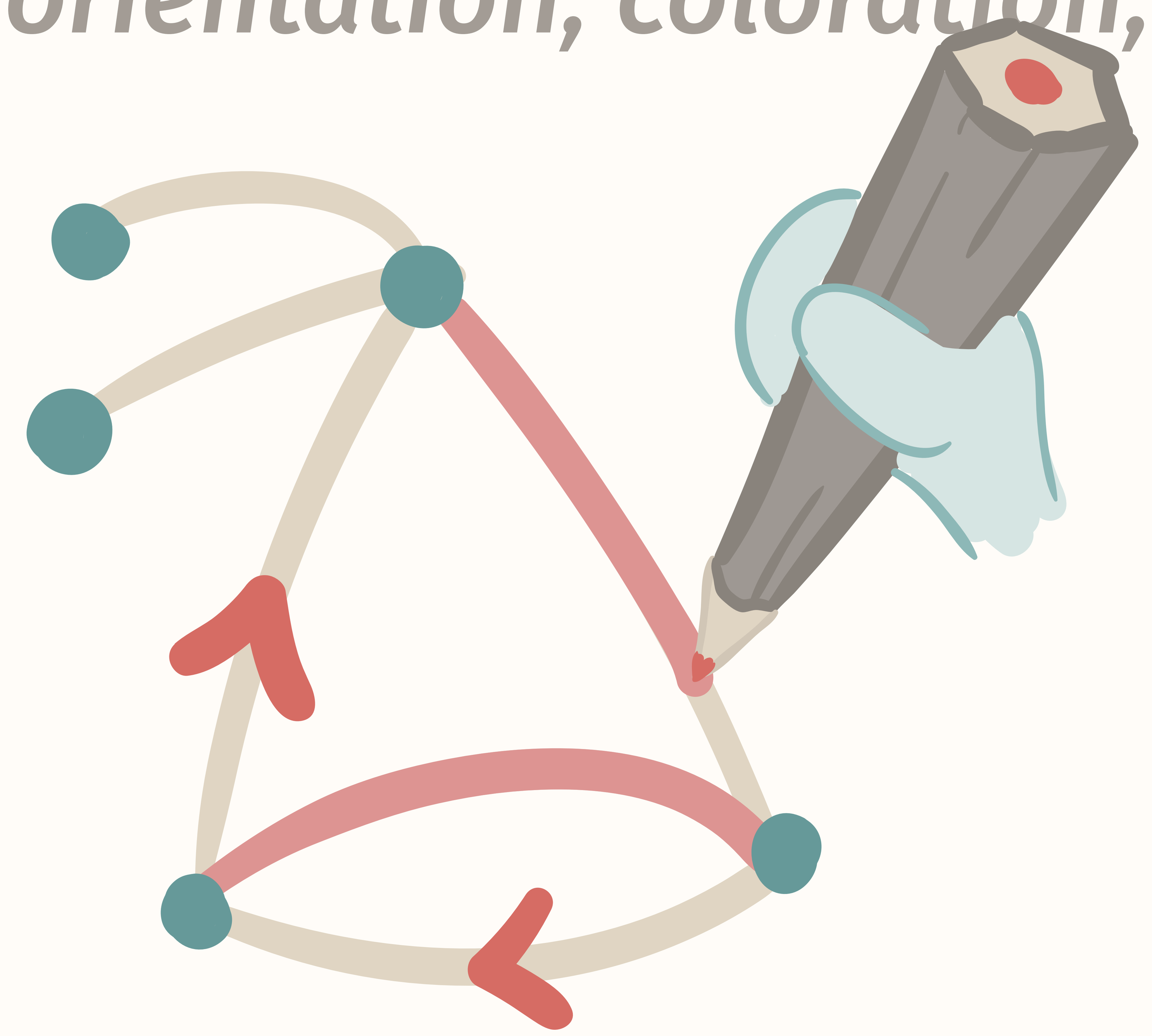
Maps

embedding on the plane



Decorated maps

orientation, coloration, etc.



Decorated maps

→ *exact enumeration formulas*

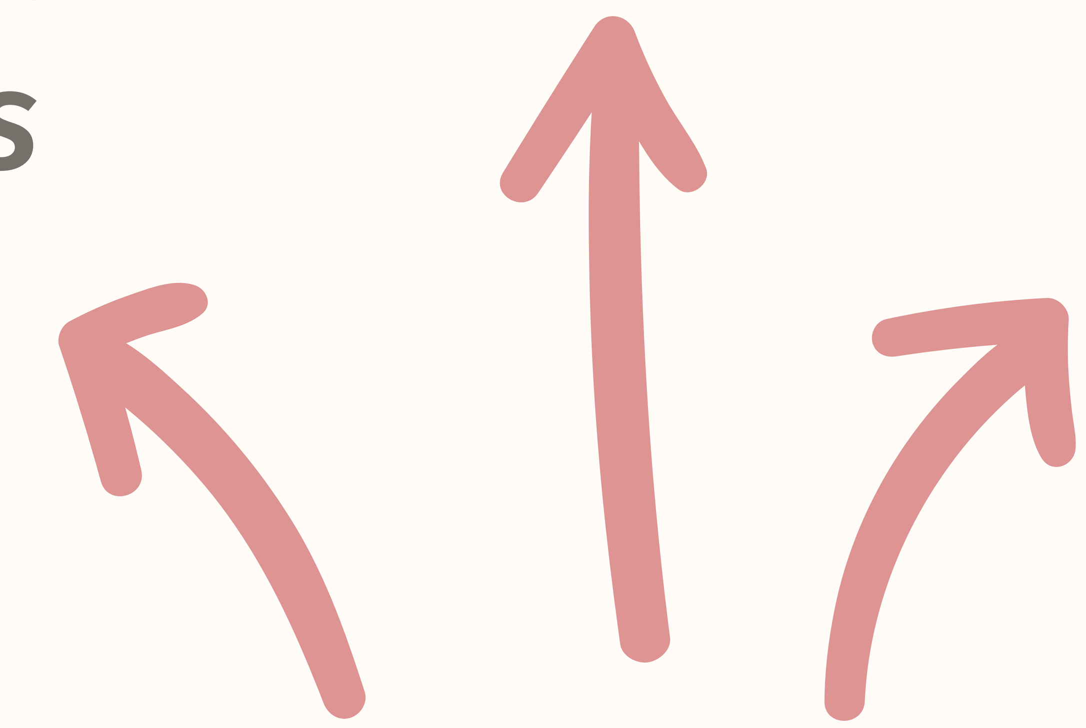
→ *universal critical exponent*

maps with n edges $: \varkappa \cdot \gamma^n n^{-5/2}$

transversal structures

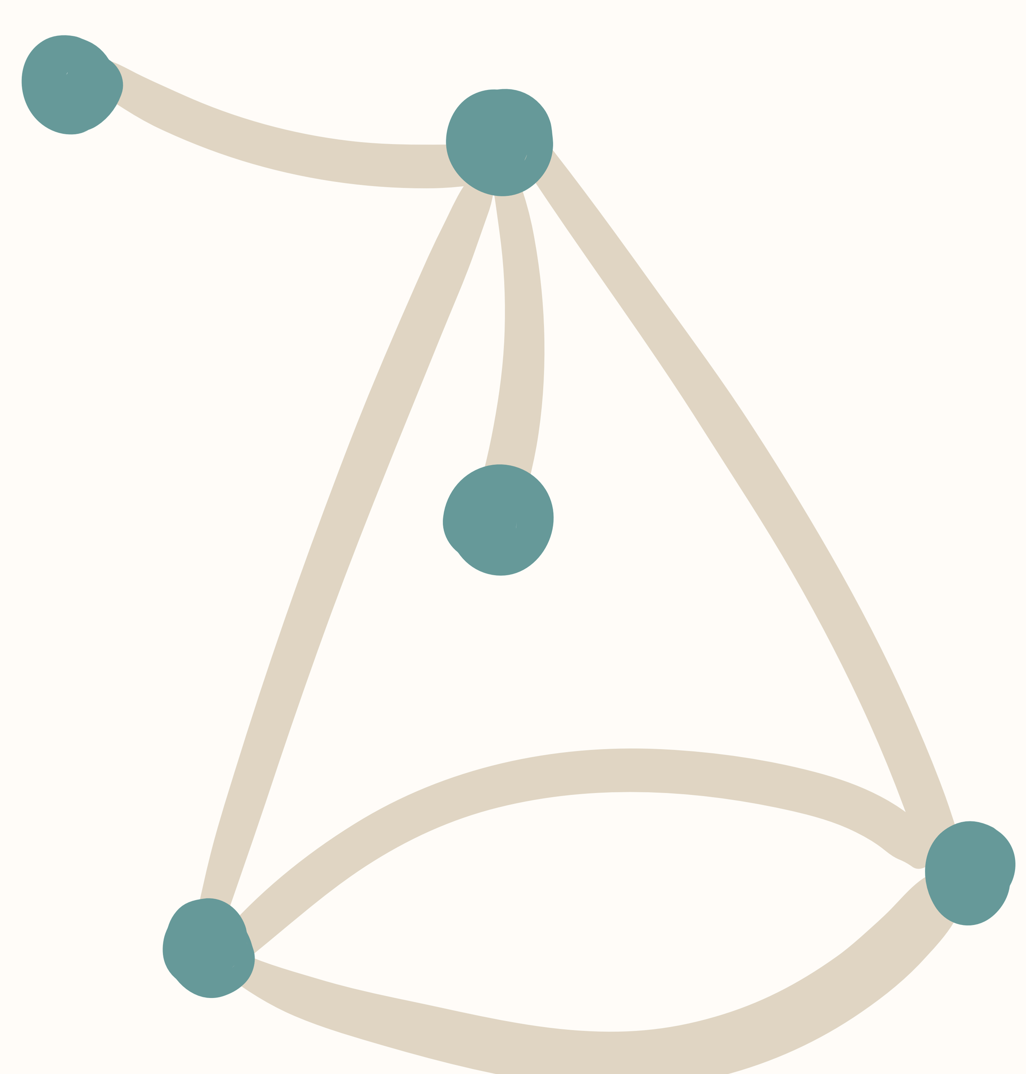
Schnyder woods

plane bipolar posets



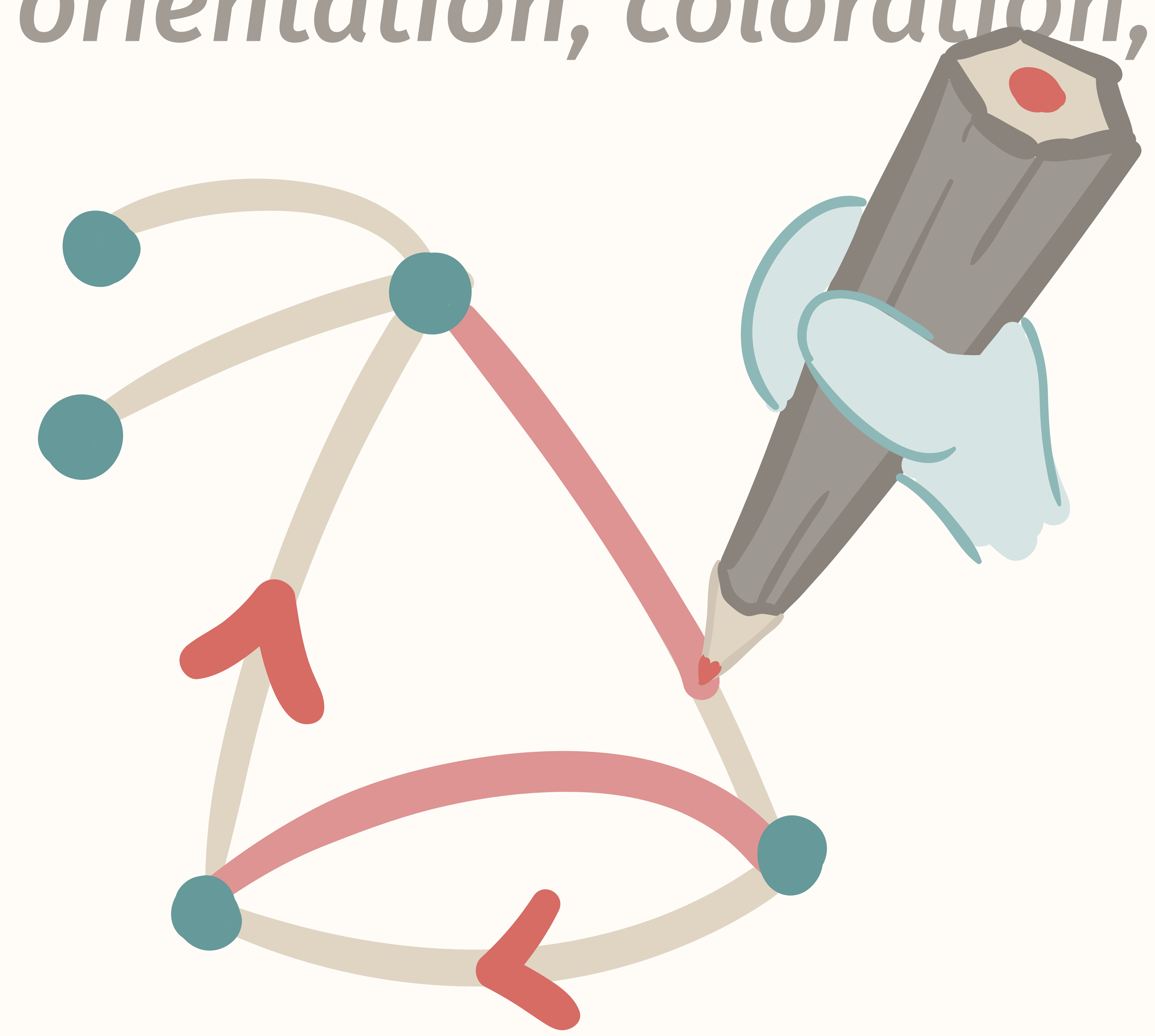
Maps

embedding on the plane

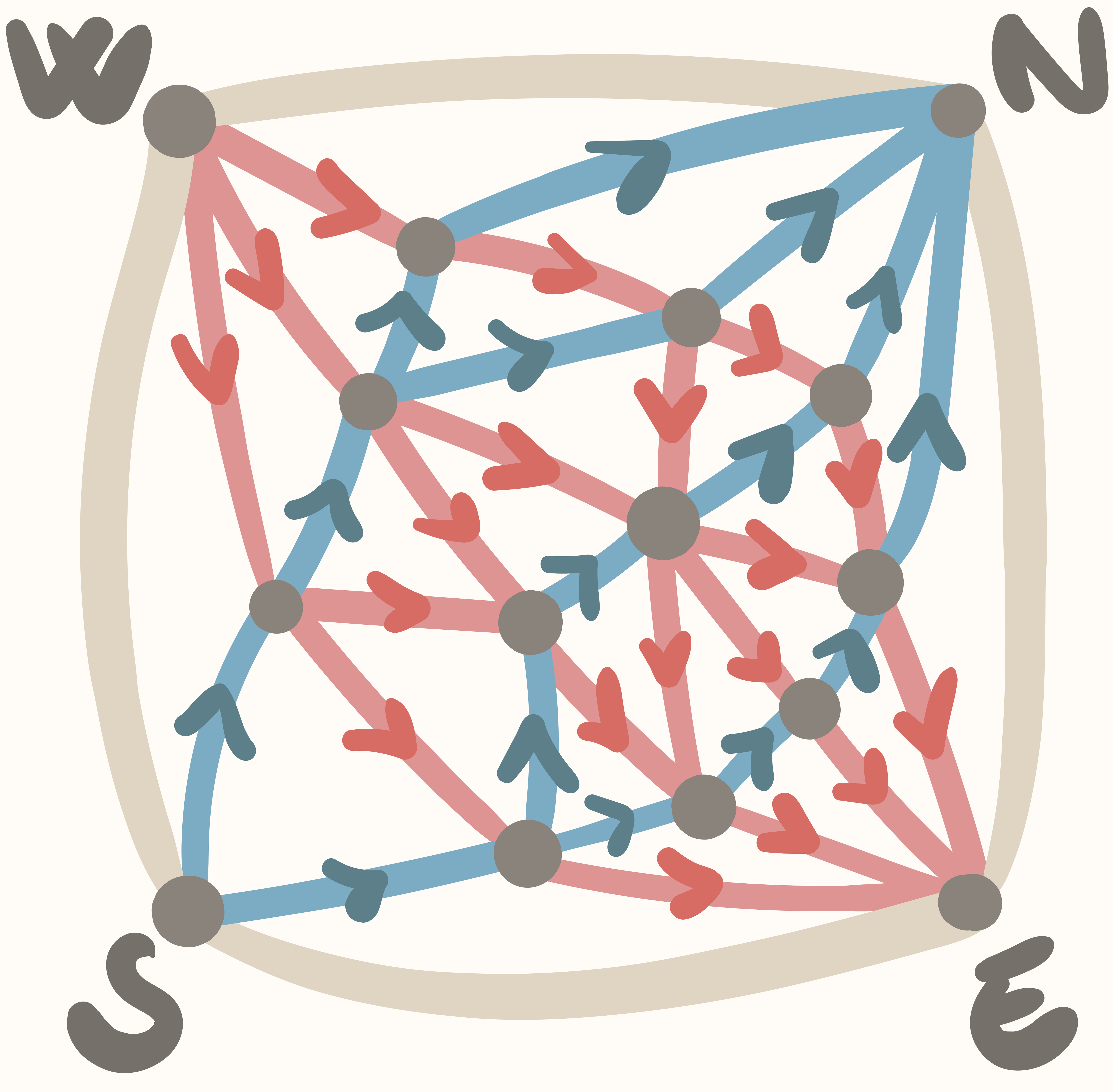


Decorated maps

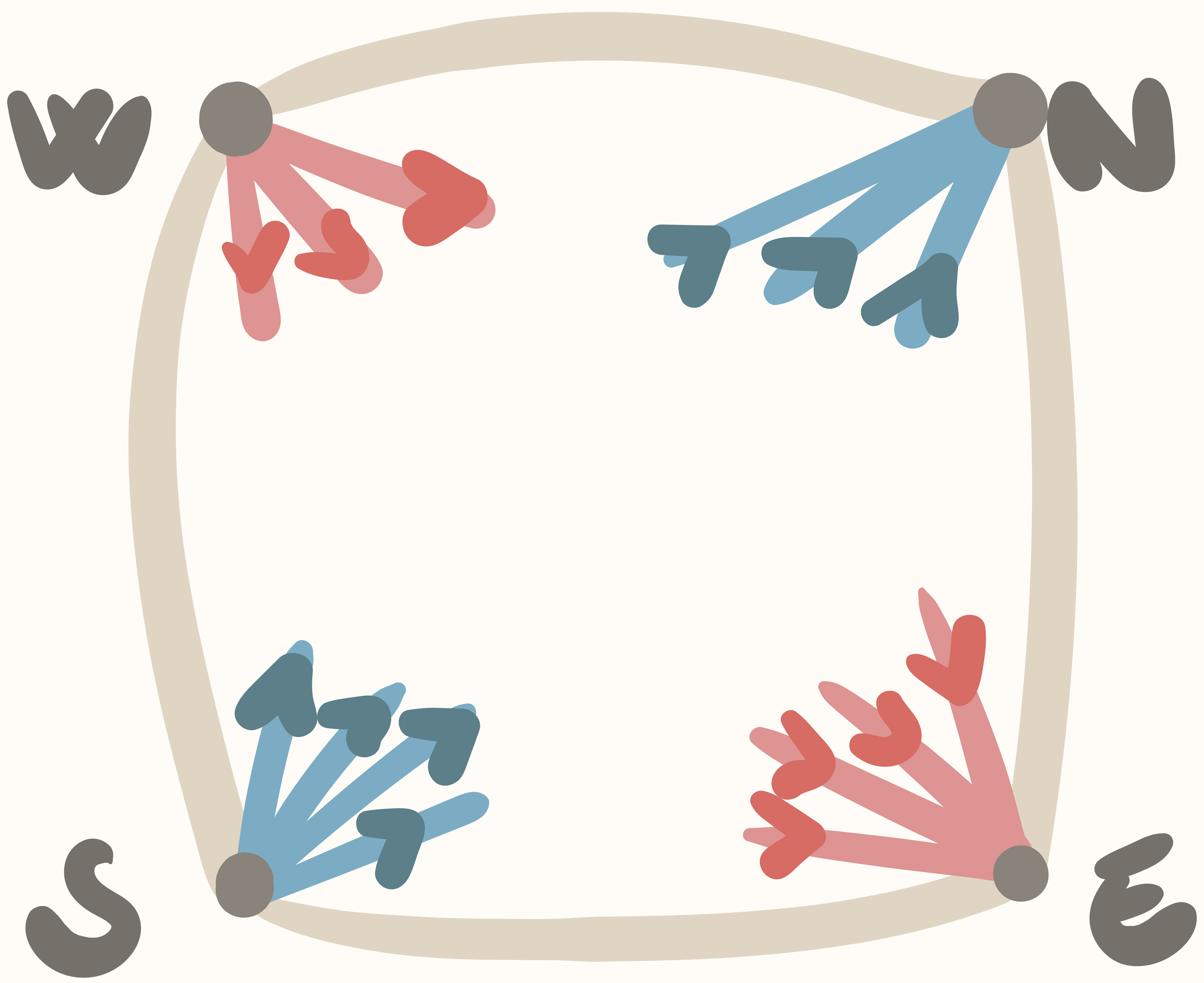
orientation, coloration, etc.

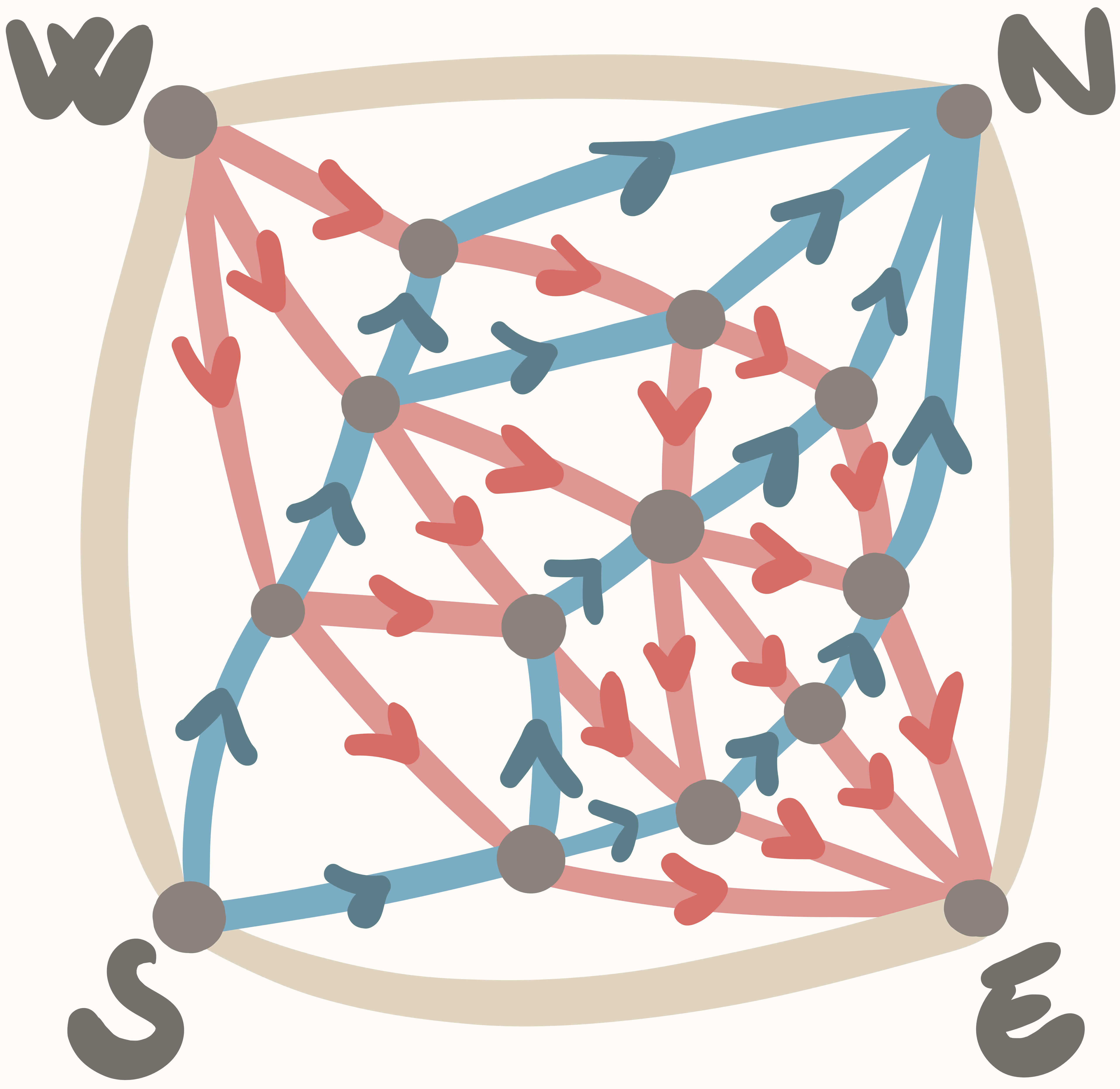


Transversal structures

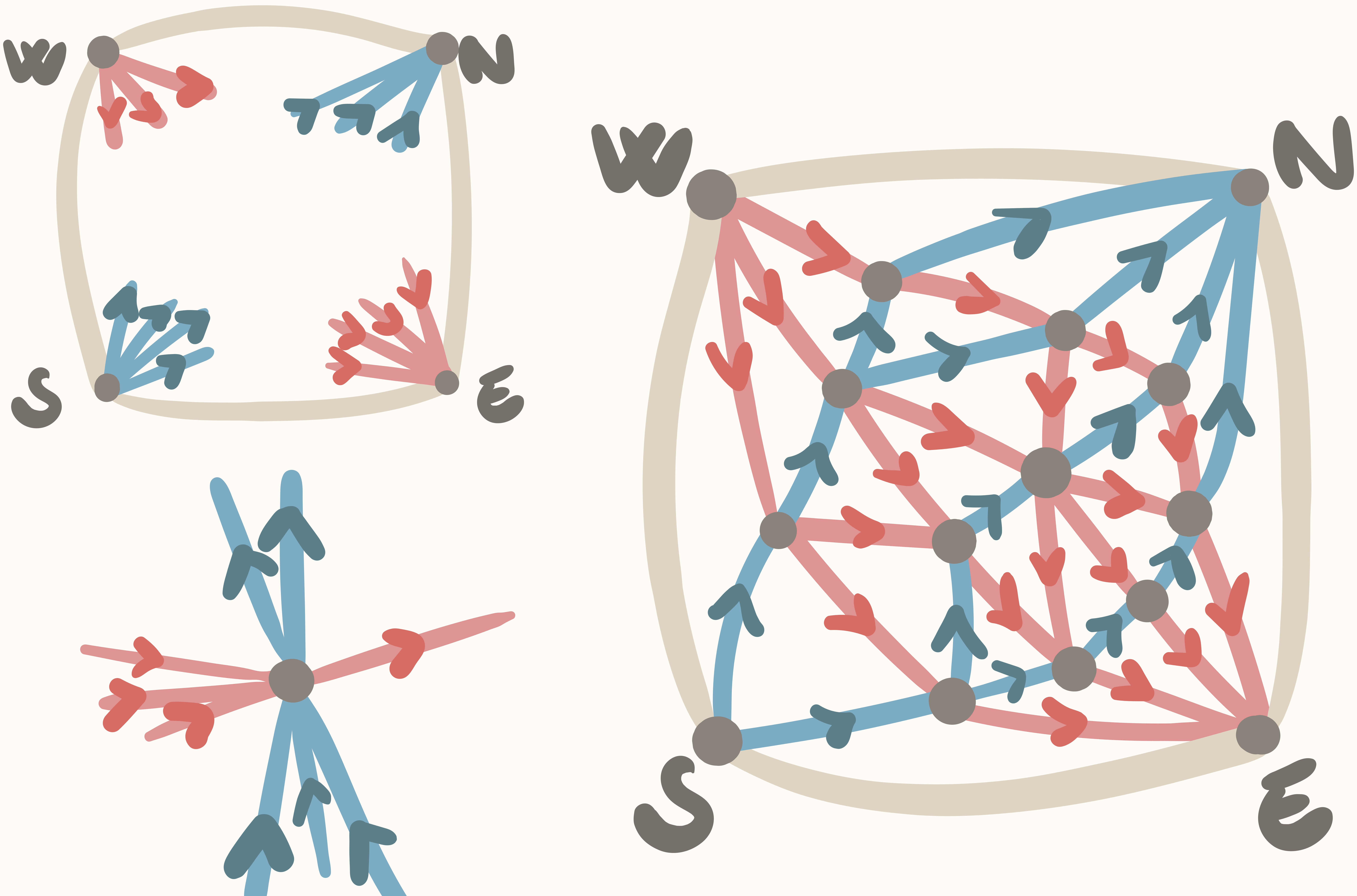


Transversal structures

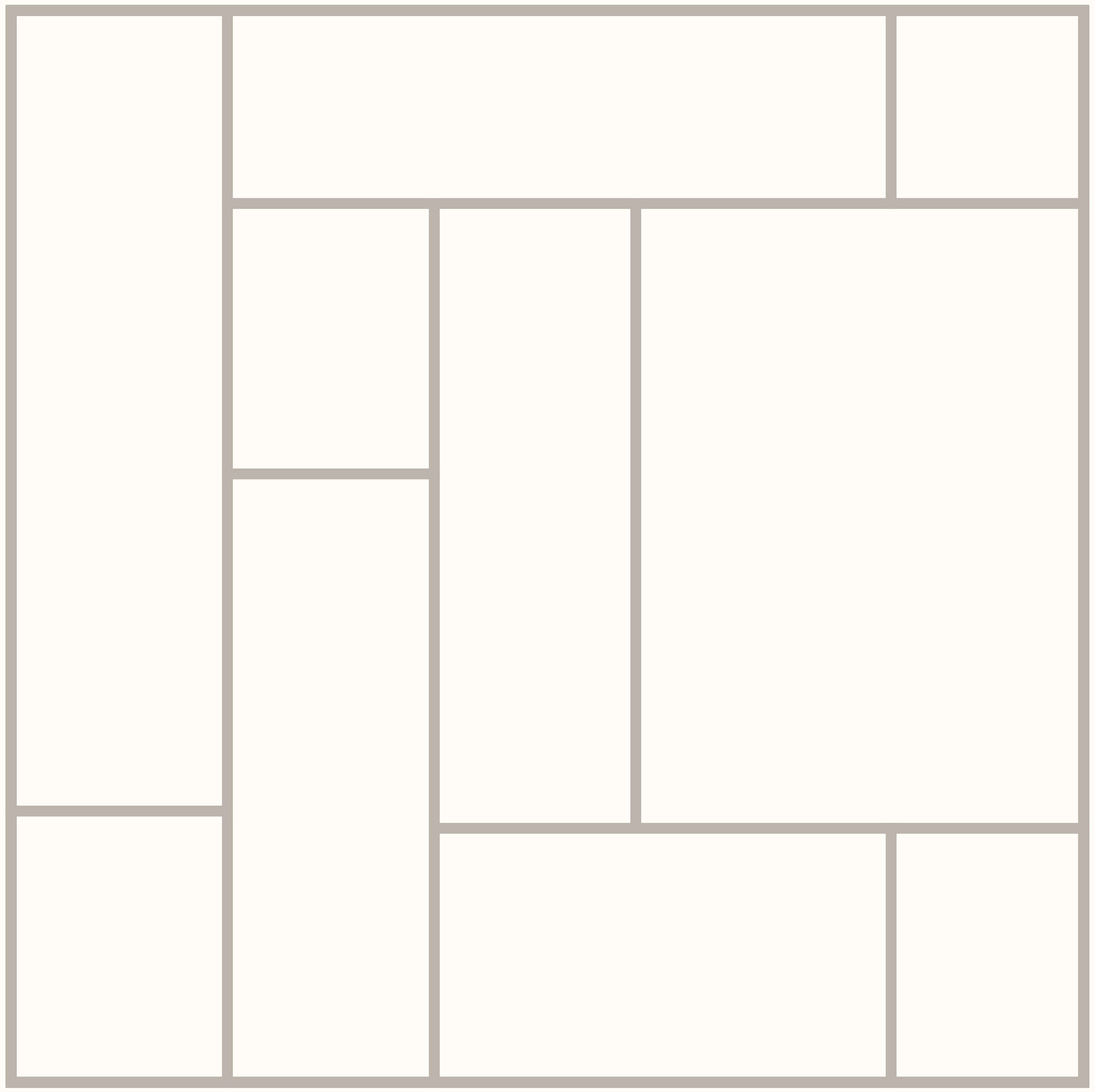




Transversal structures

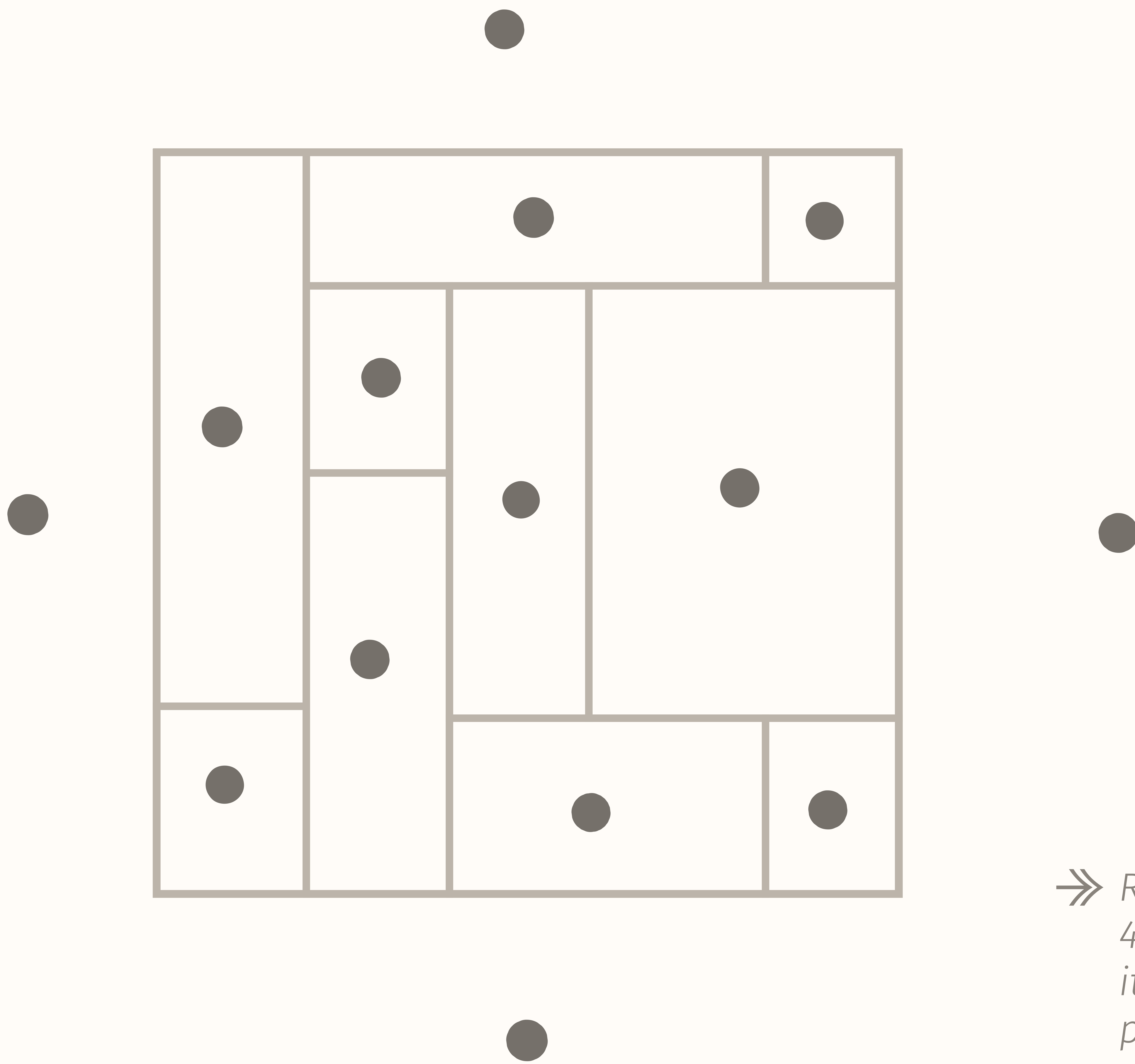


Link with rectangular tilings



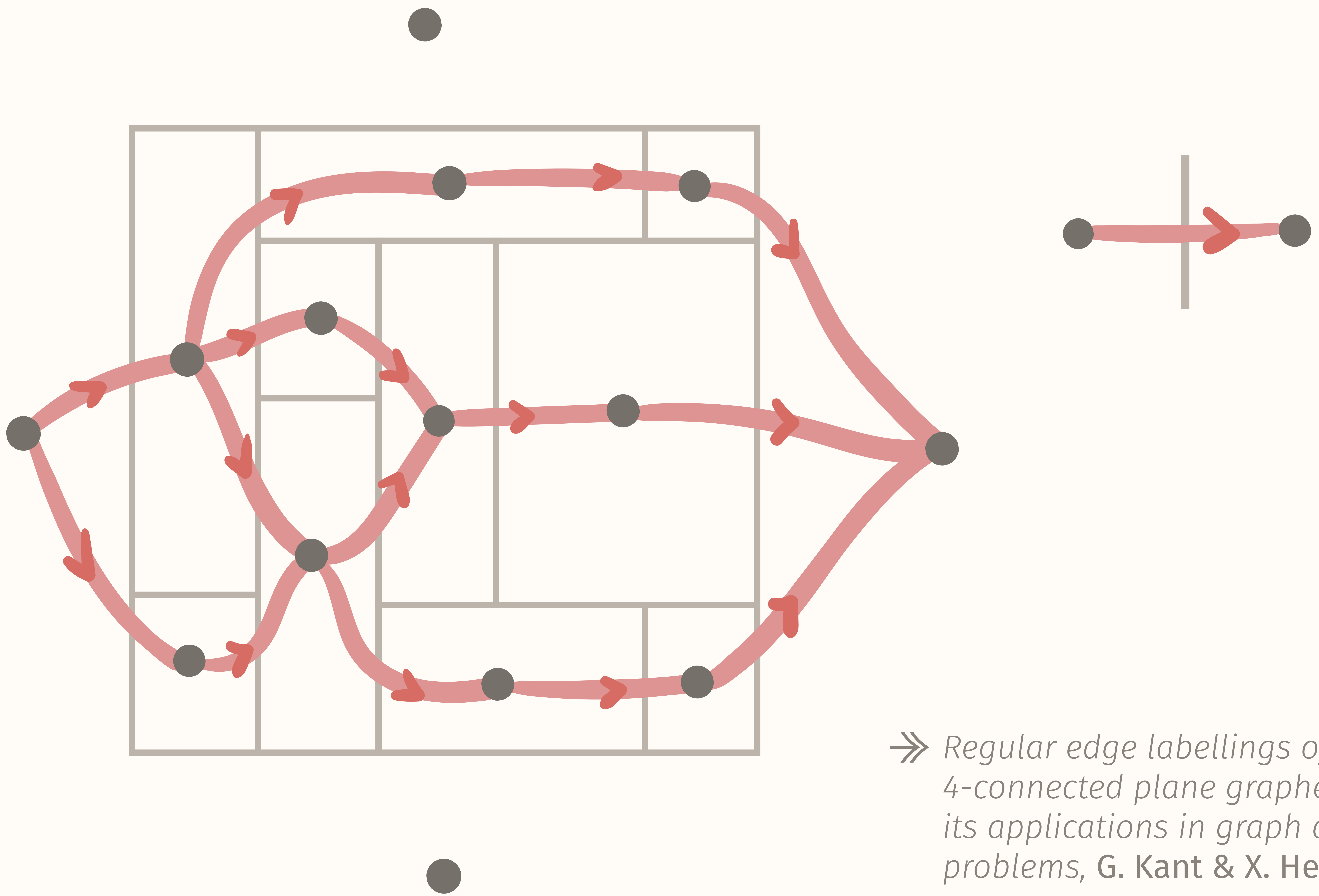
→ *Regular edge labellings of 4-connected plane graphes and its applications in graph drawing problems*, G. Kant & X. He (1997)

Link with rectangular tilings



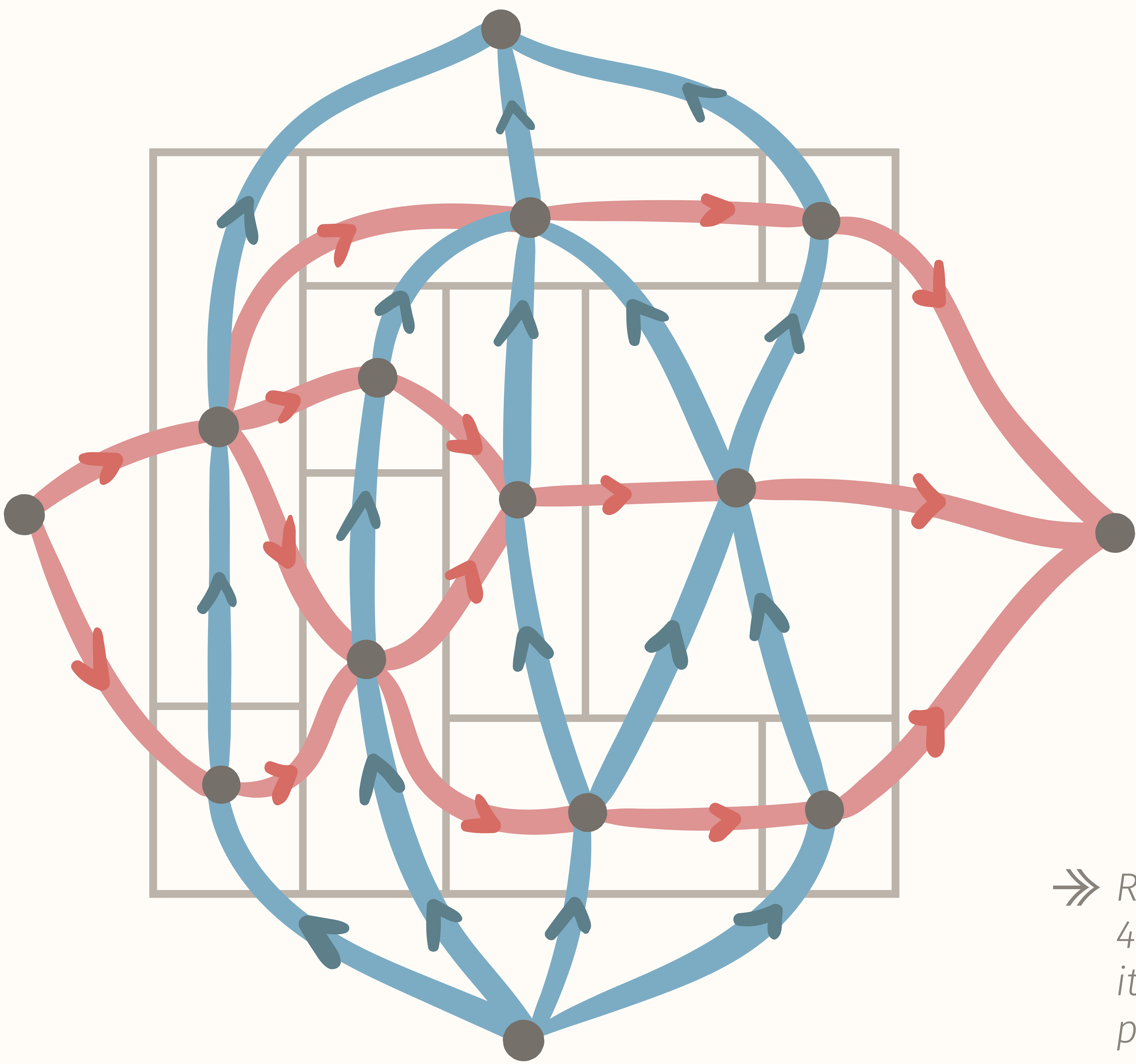
→ *Regular edge labellings of 4-connected plane graphes and its applications in graph drawing problems,* G. Kant & X. He (1997)

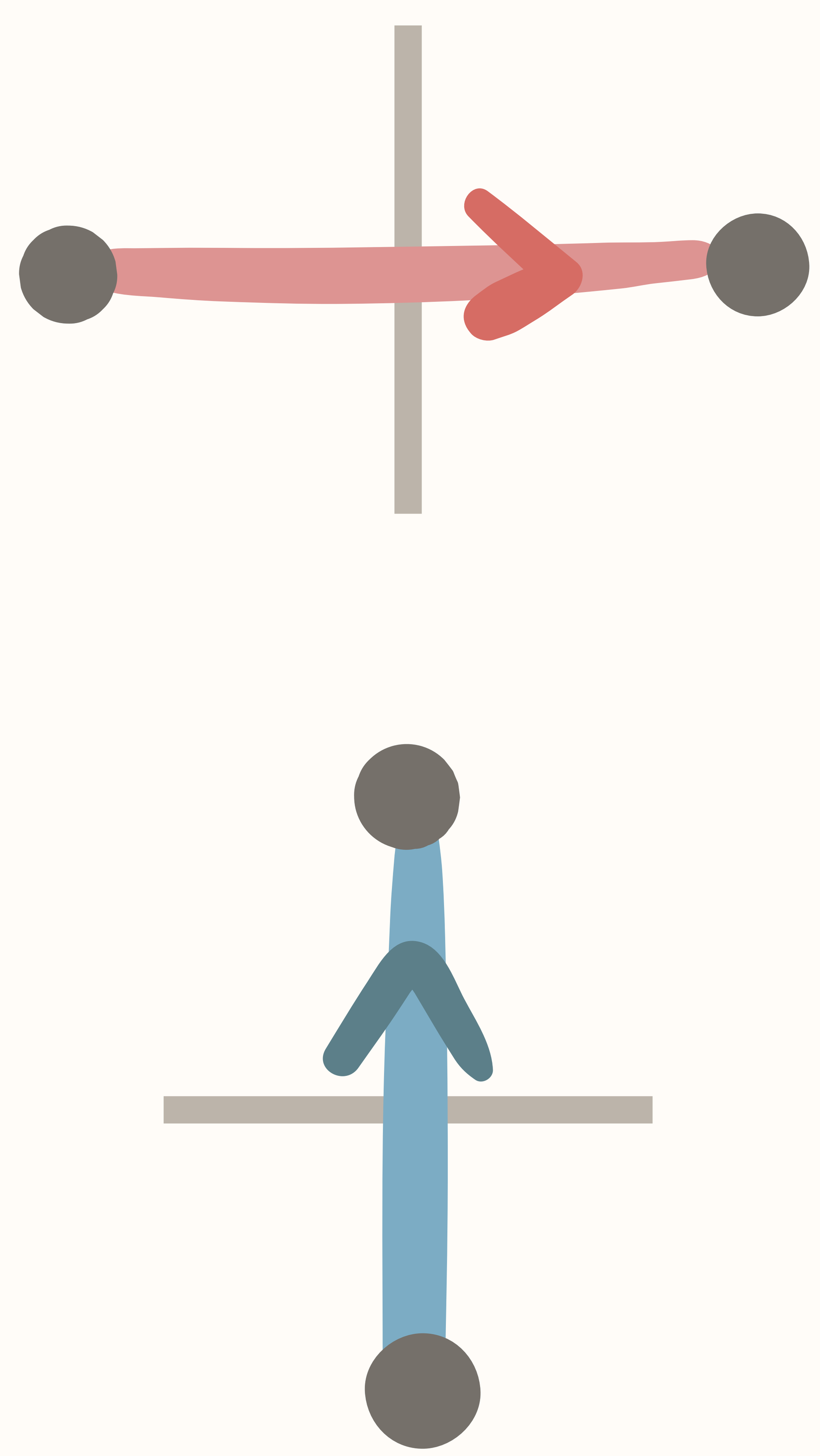
Link with rectangular tilings



⇢ *Regular edge labellings of 4-connected plane graphes and its applications in graph drawing problems,* G. Kant & X. He (1997)

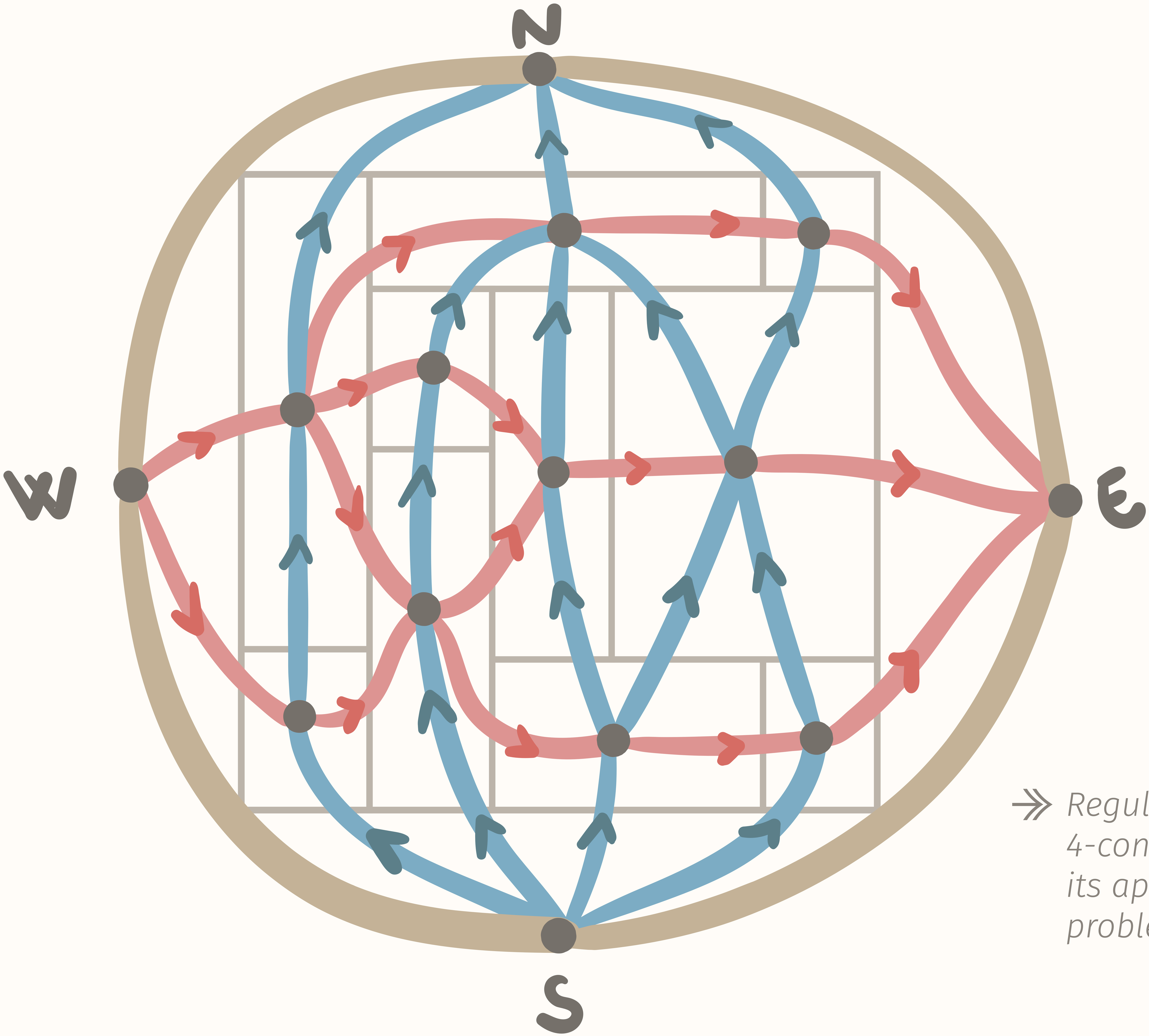
Link with rectangular tilings

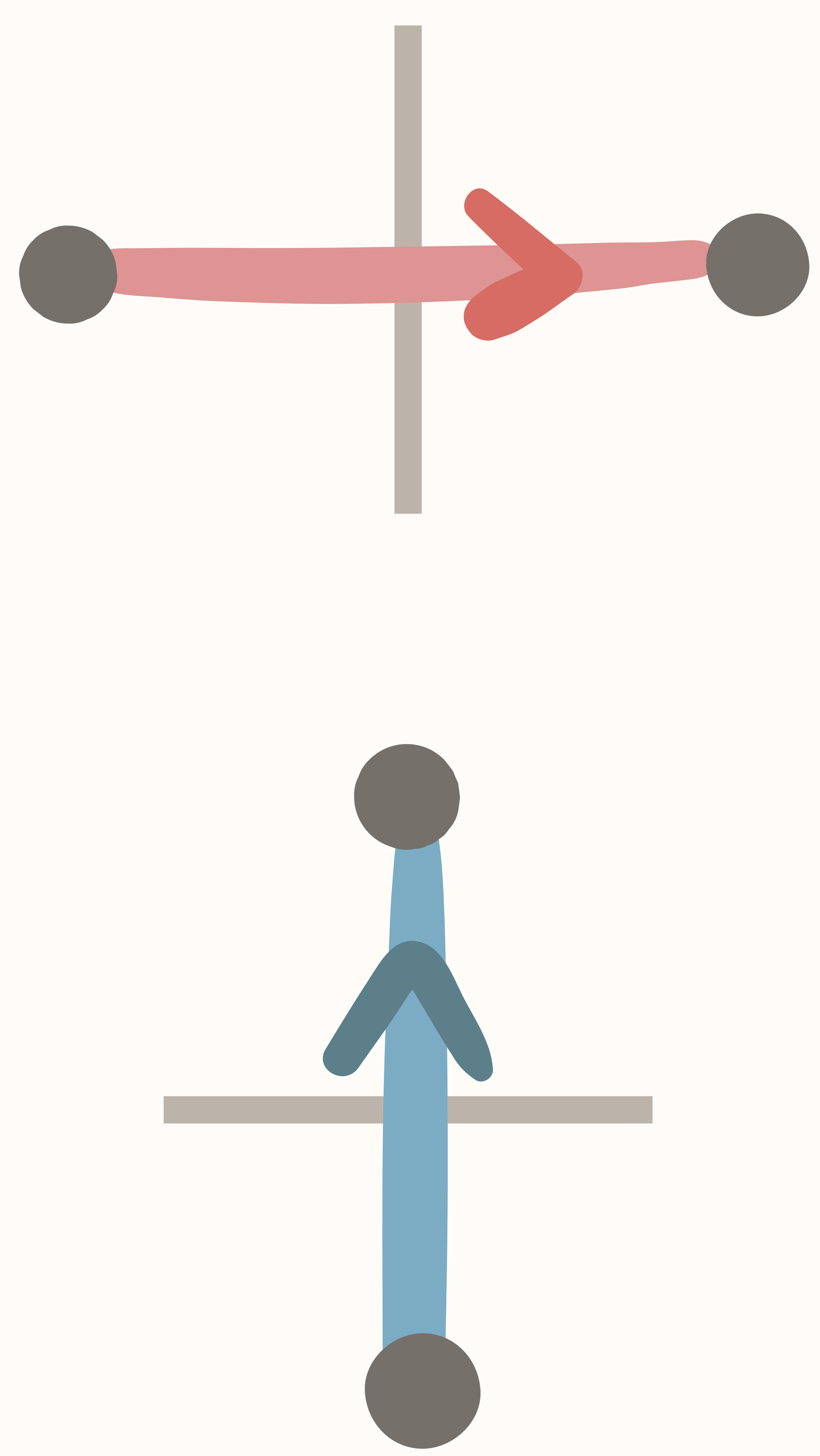




⇛ *Regular edge labellings of 4-connected plane graphes and its applications in graph drawing problems,* G. Kant & X. He (1997)

Link with rectangular tilings





→ *Regular edge labellings of 4-connected plane graphes and its applications in graph drawing problems*, G. Kant & X. He (1997)

Summary

Maps and decorated maps

1. Bijection with quadrant tandem walks

a. The KMSW bijection

b. Plane bipolar posets

c. Plane bipolar posets by vertices

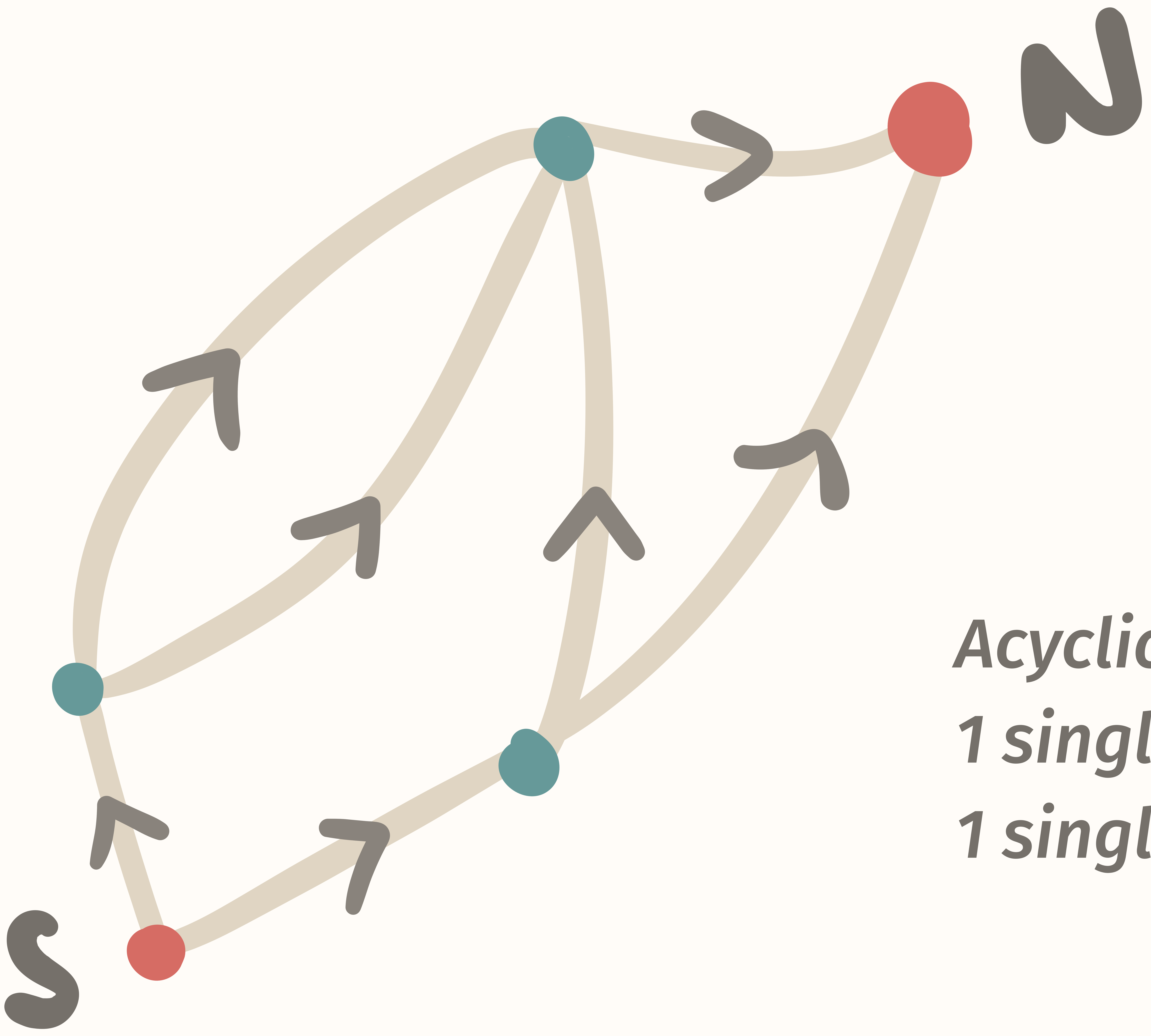
d. Transversal structures

2. Asymptotic enumeration

(Digression on plane permutations)

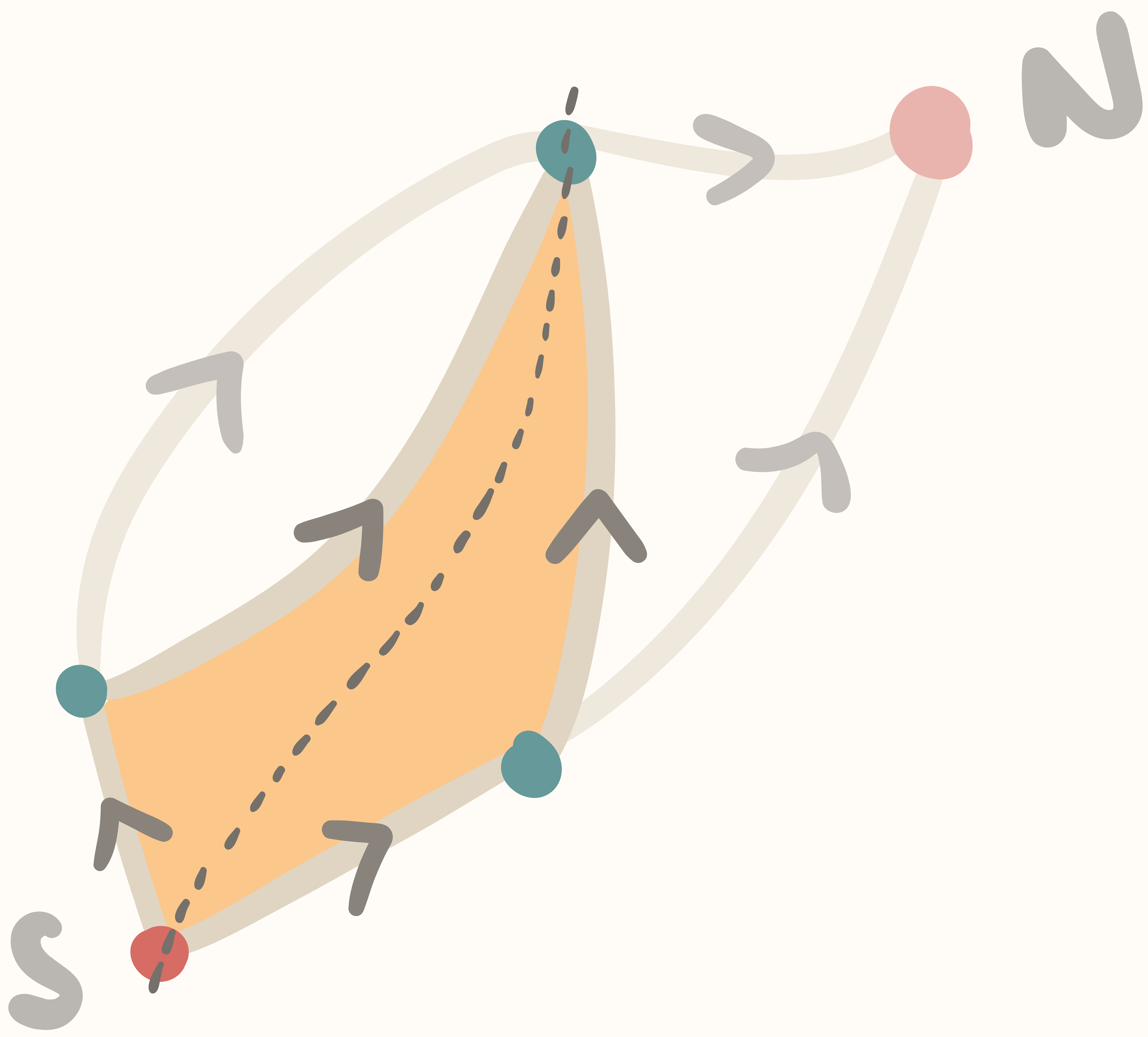
3. Generic transversal structures

Plane bipolar orientation

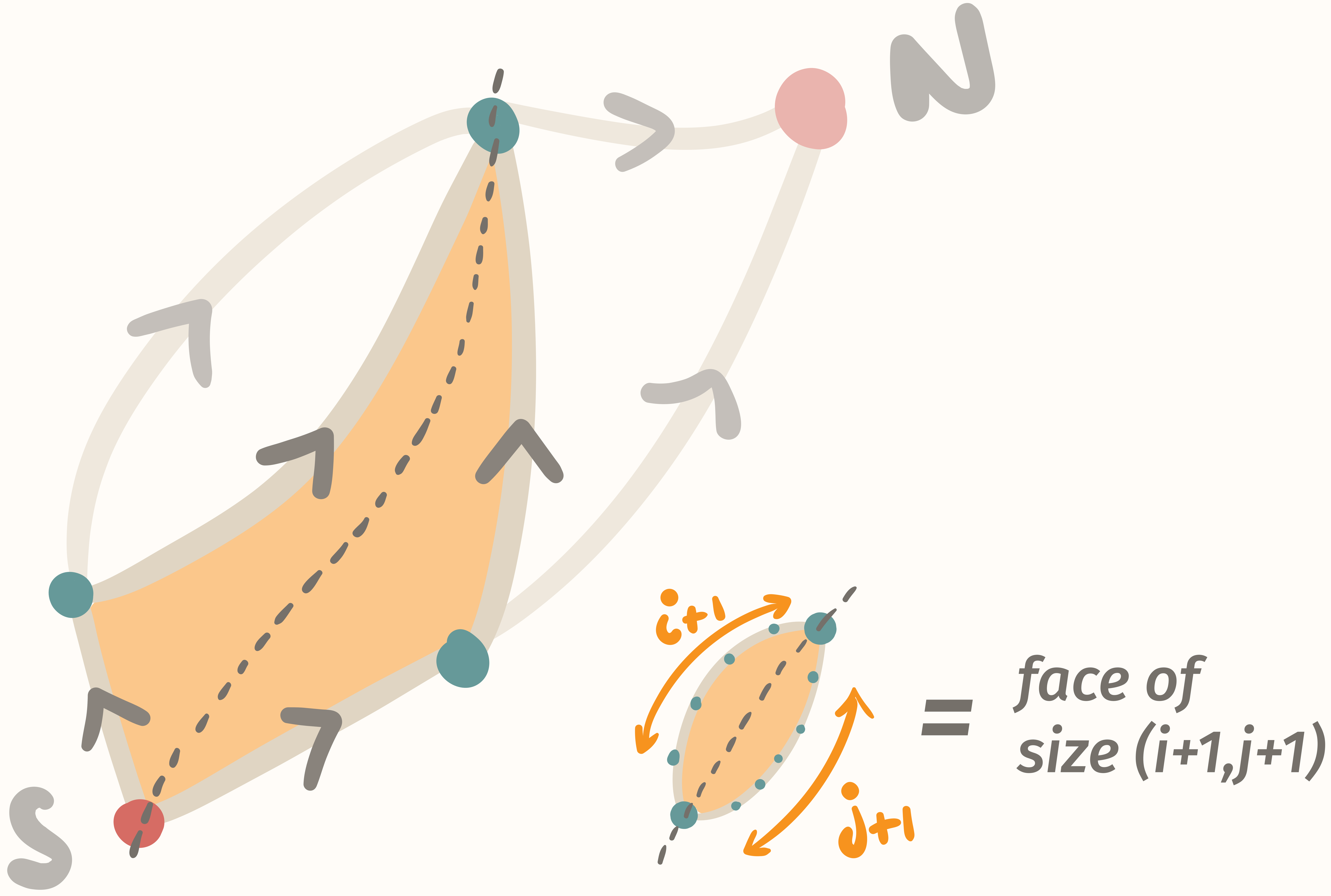


Acyclic
1 single source S
1 single sink N

Plane bipolar orientation



Plane bipolar orientation

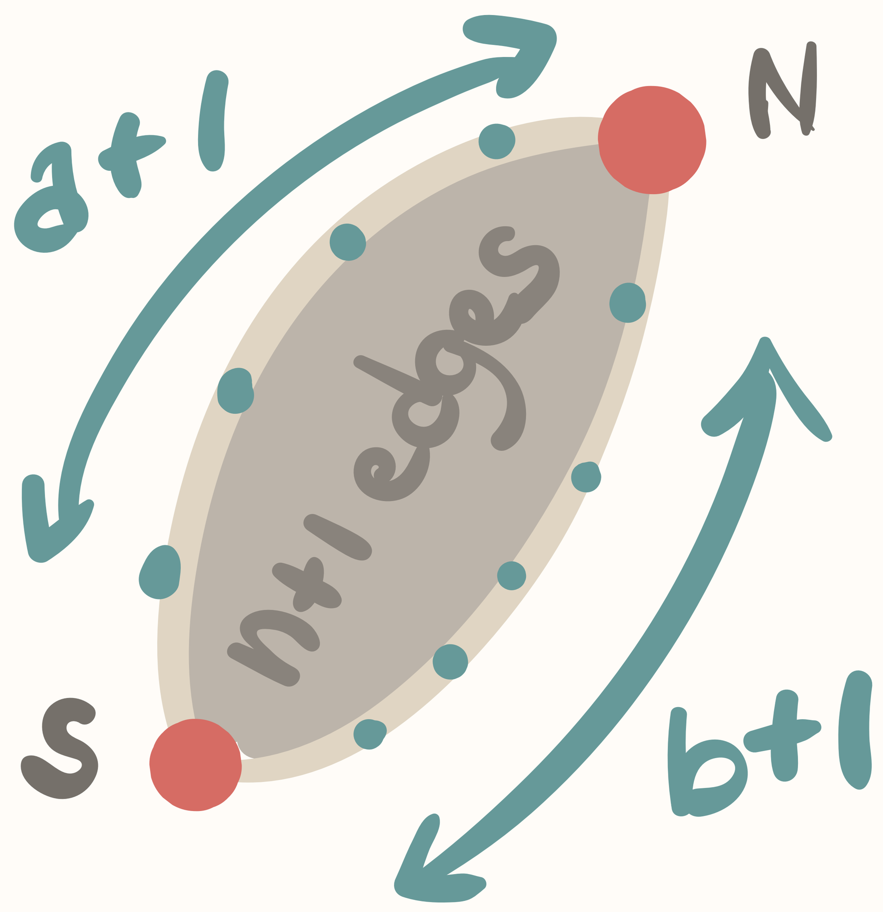


The KMSW bijection

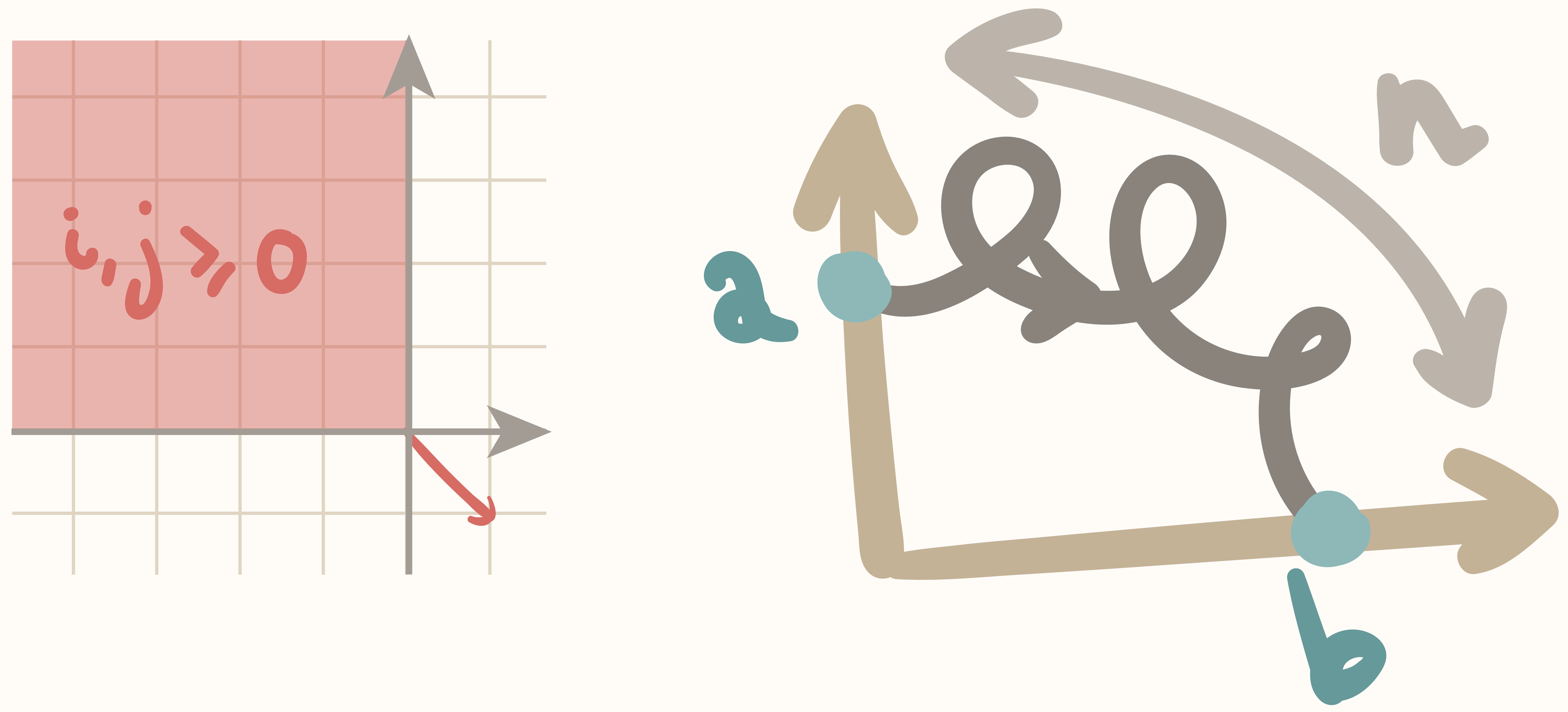
Plane bipolar orientations ⟷ *tandems walks in the quarter plane*

The KMSW bijection

Plane bipolar orientations ⟷ *tandems walks in the quarter plane*

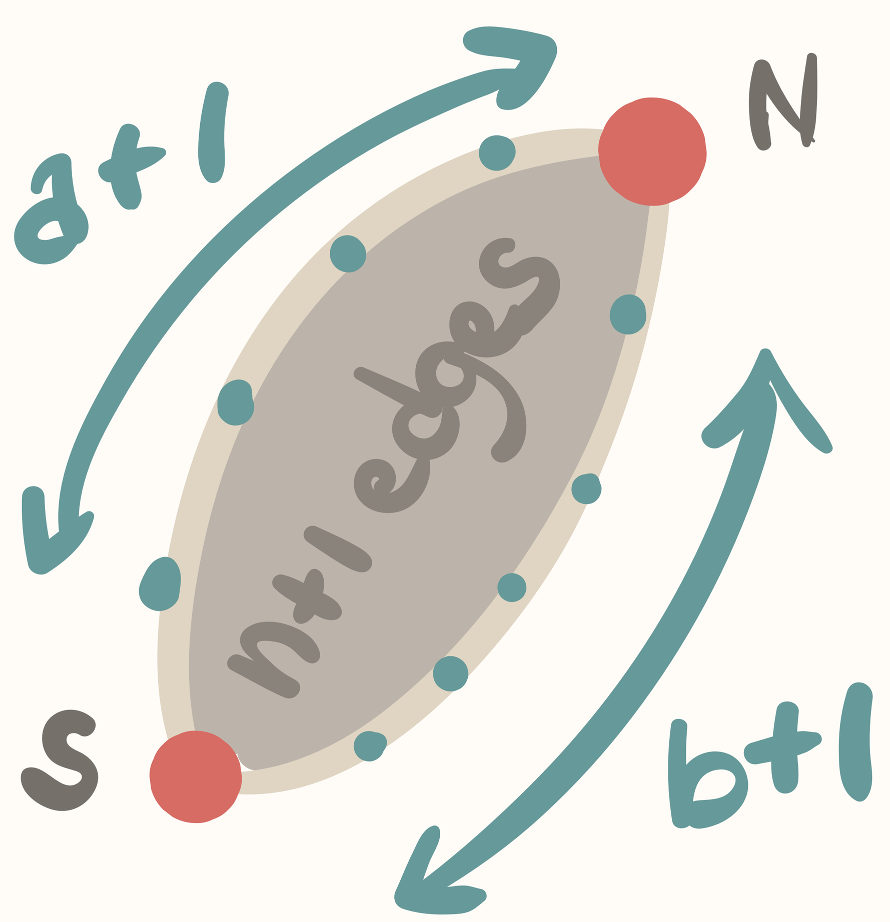


⟷

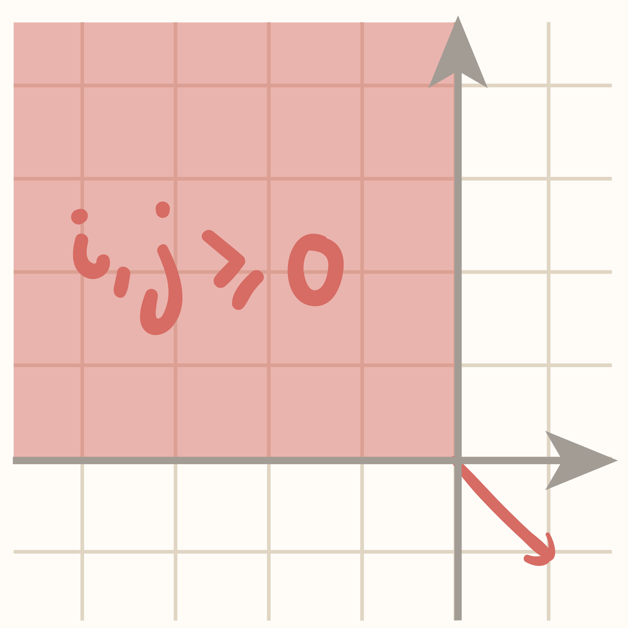


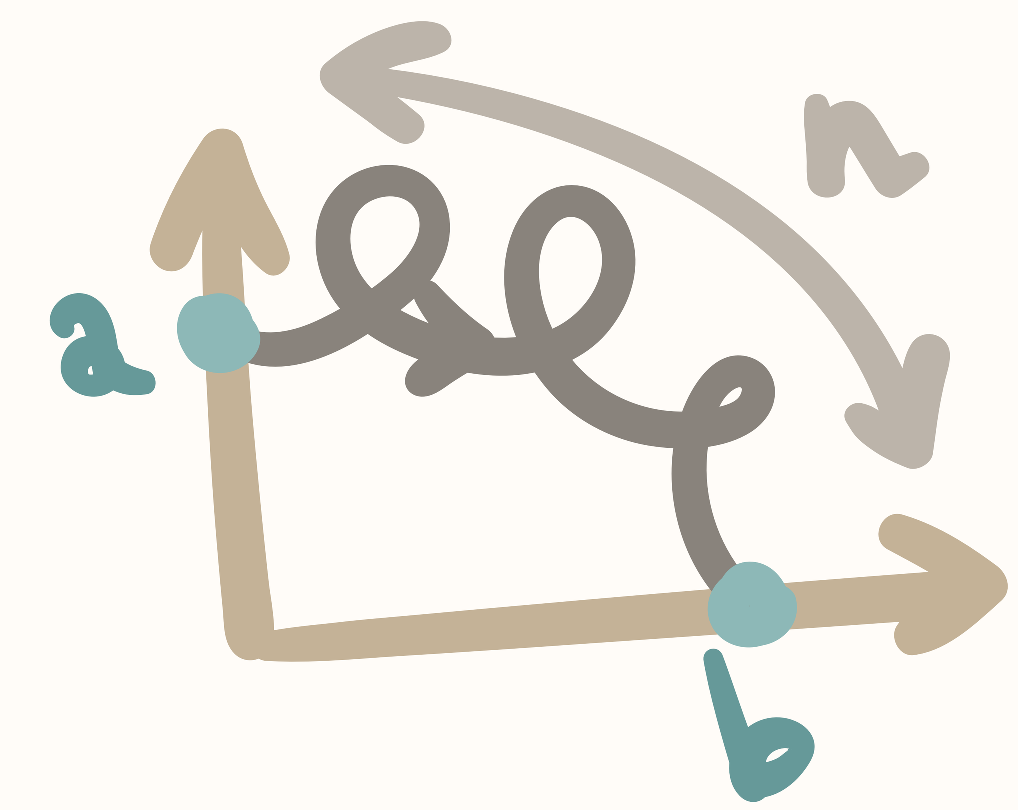
The KMSW bijection

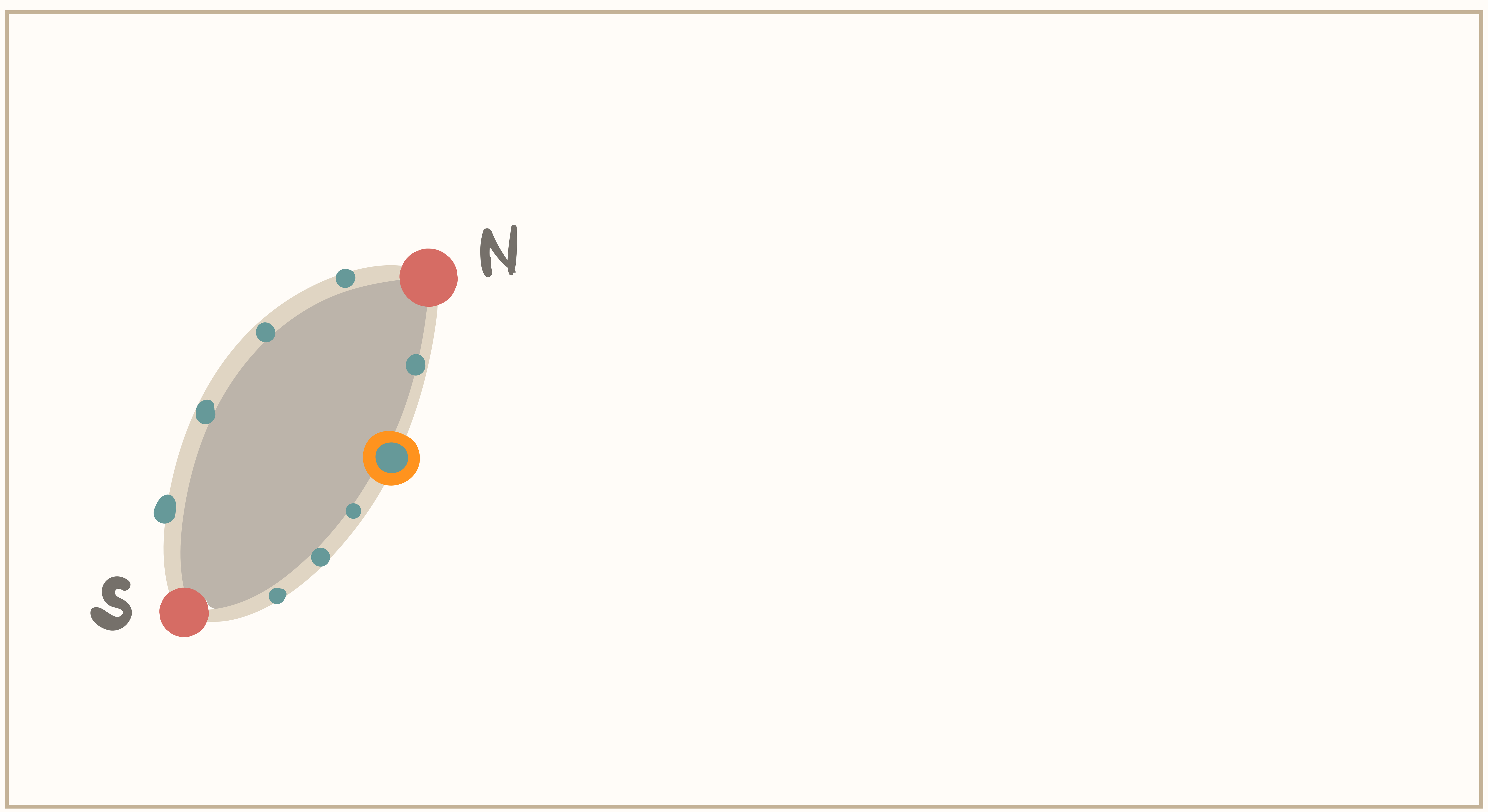
Plane bipolar orientations ⟷ *tandems walks in the quarter plane*



⟷

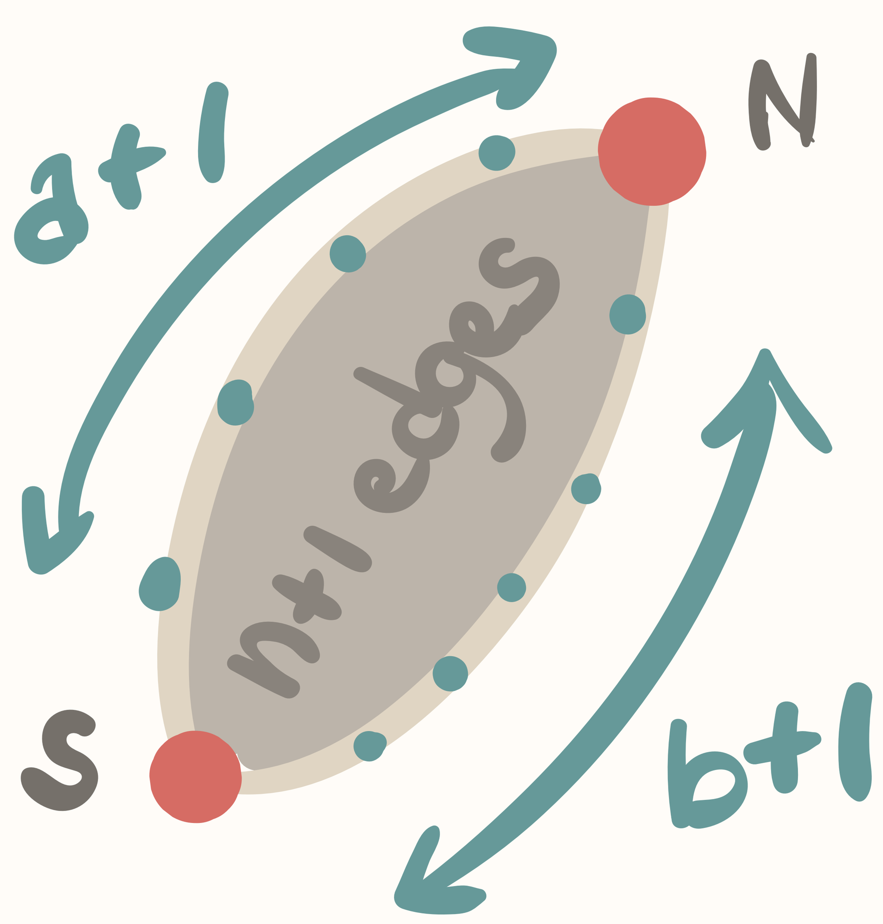


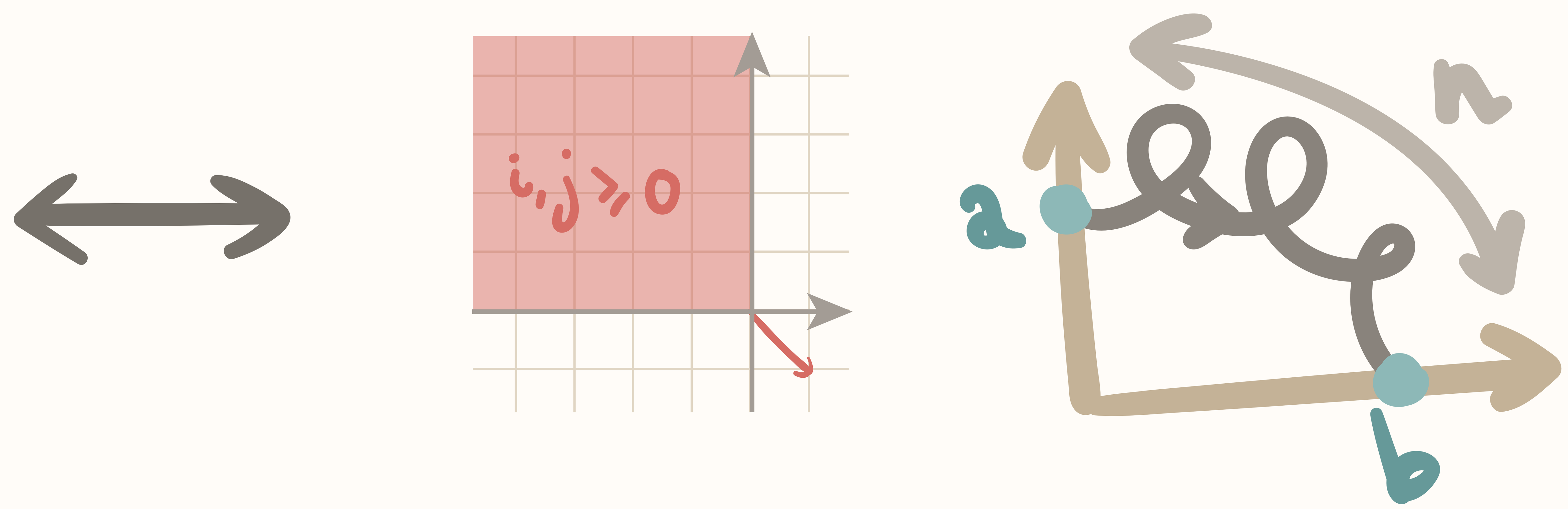


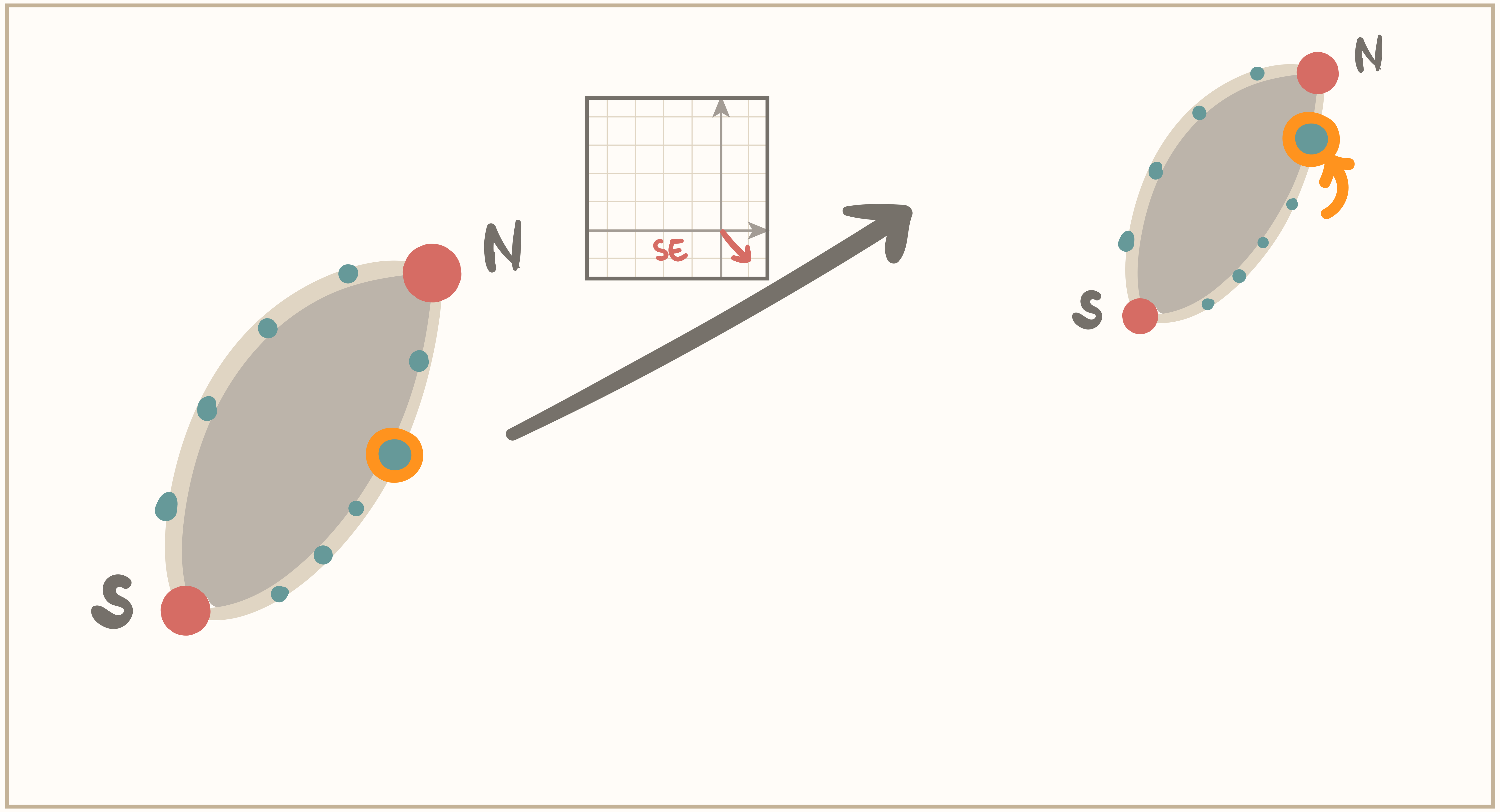


The KMSW bijection

Plane bipolar orientations ⟷ *tandems walks in the quarter plane*

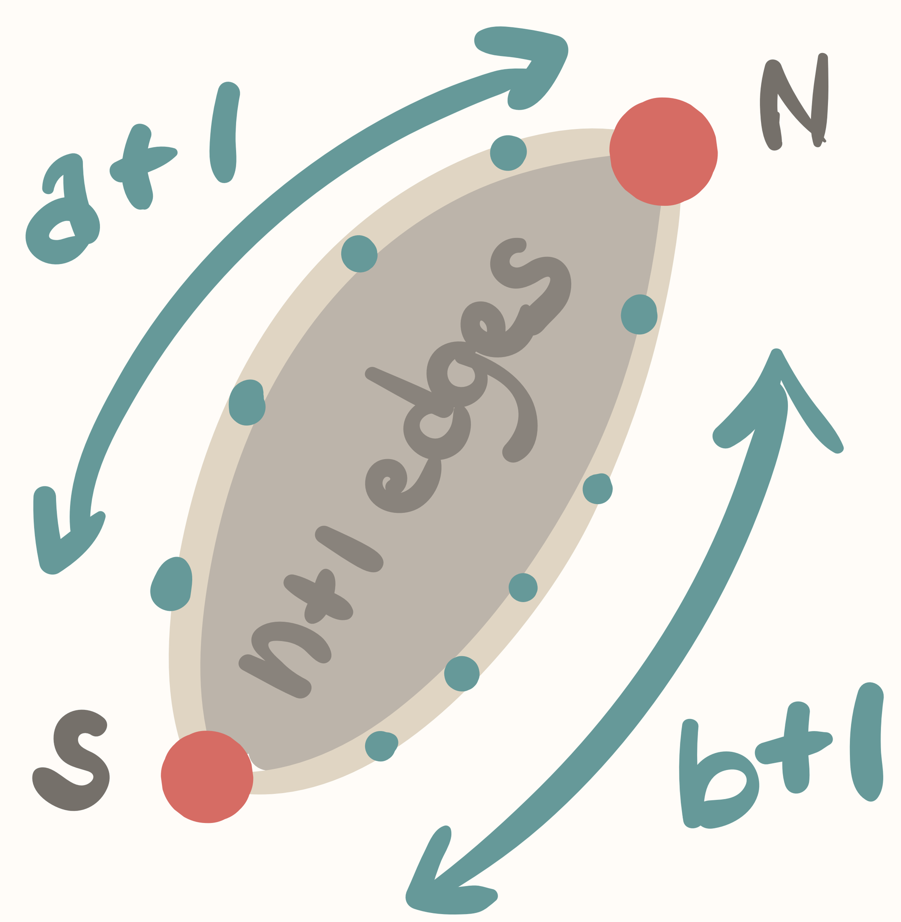


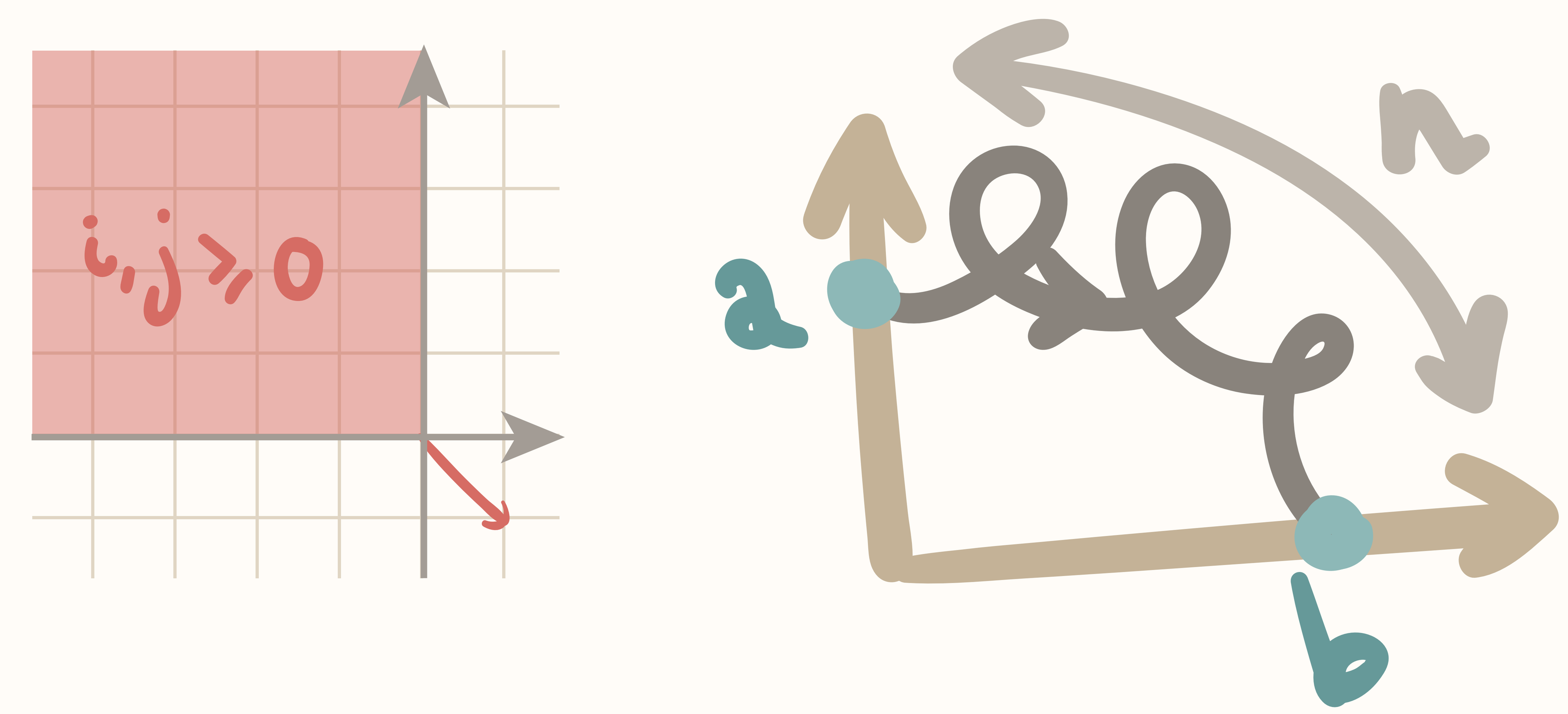


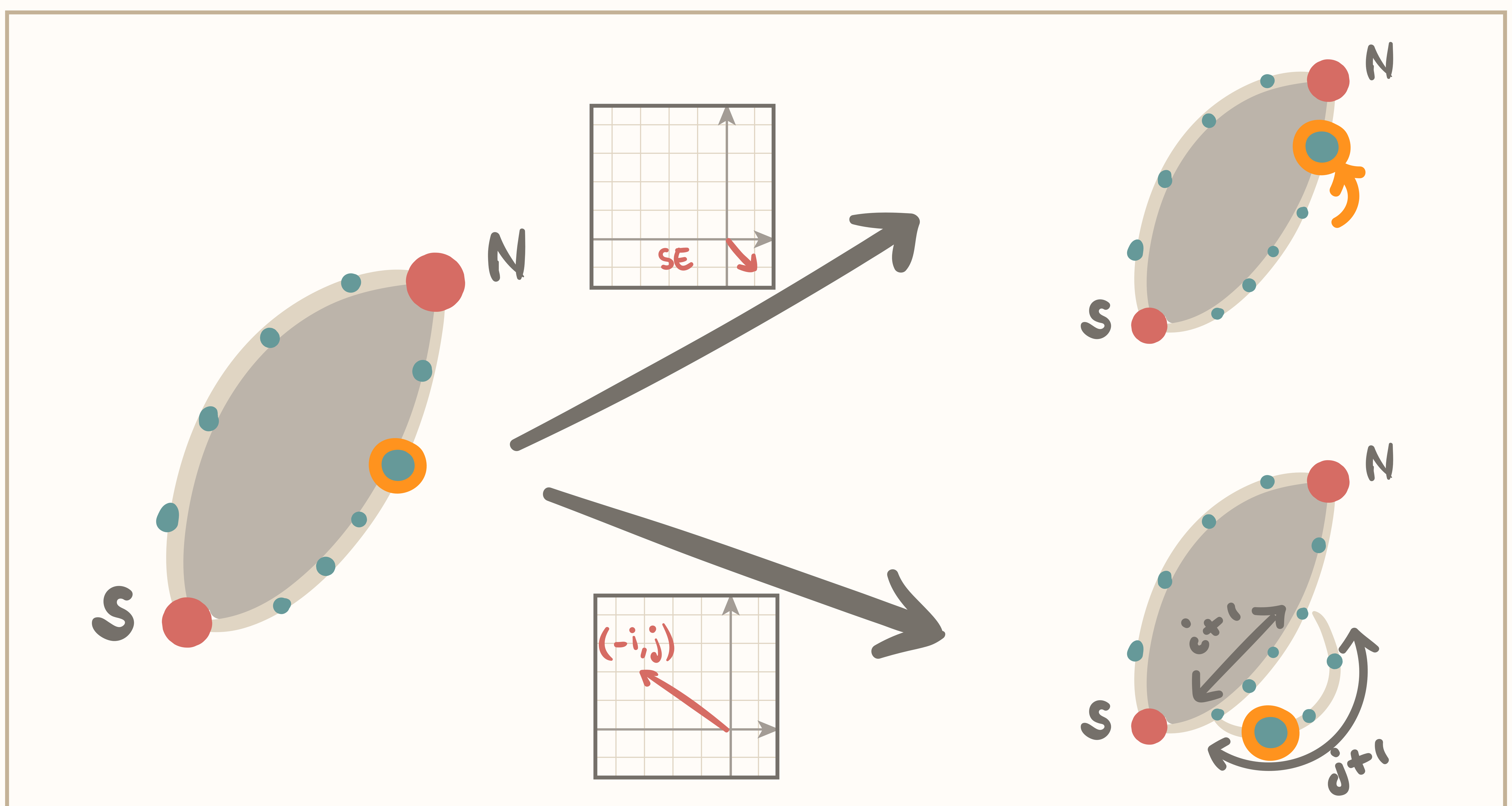


The KMSW bijection

Plane bipolar orientations ⟷ *tandems walks in the quarter plane*

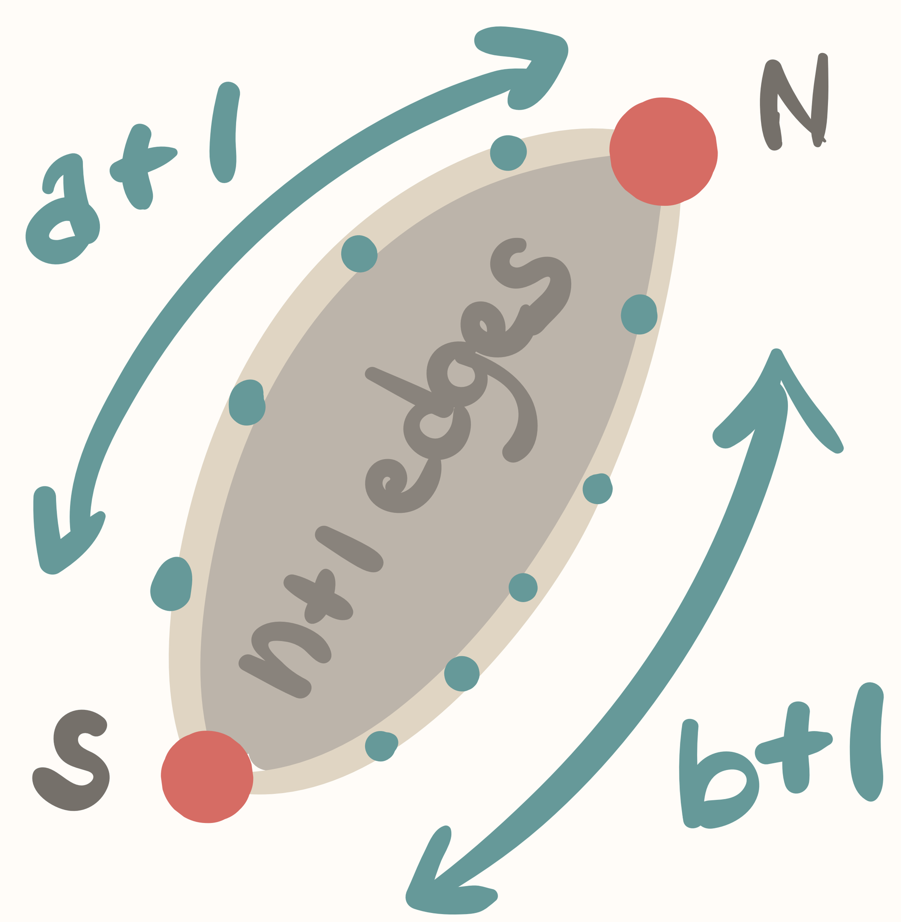


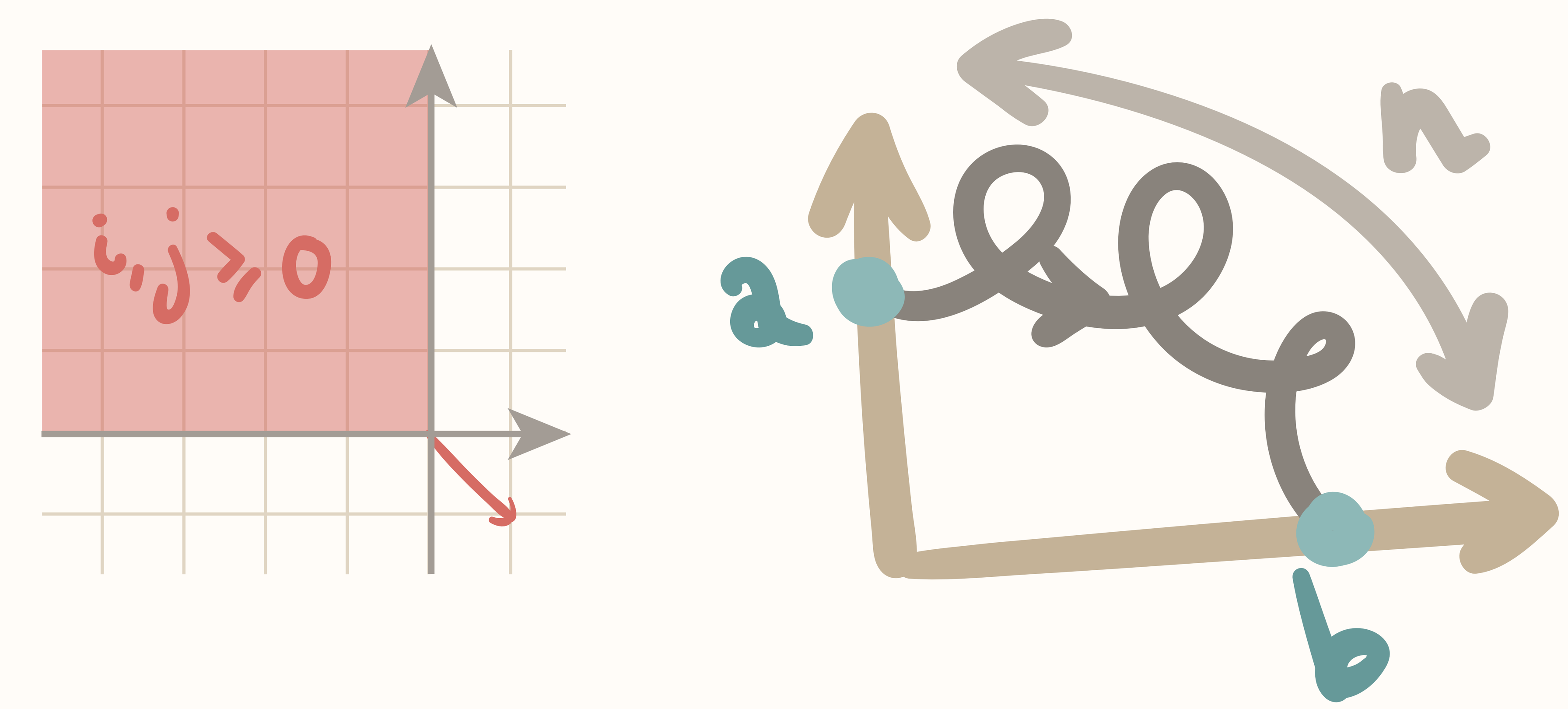


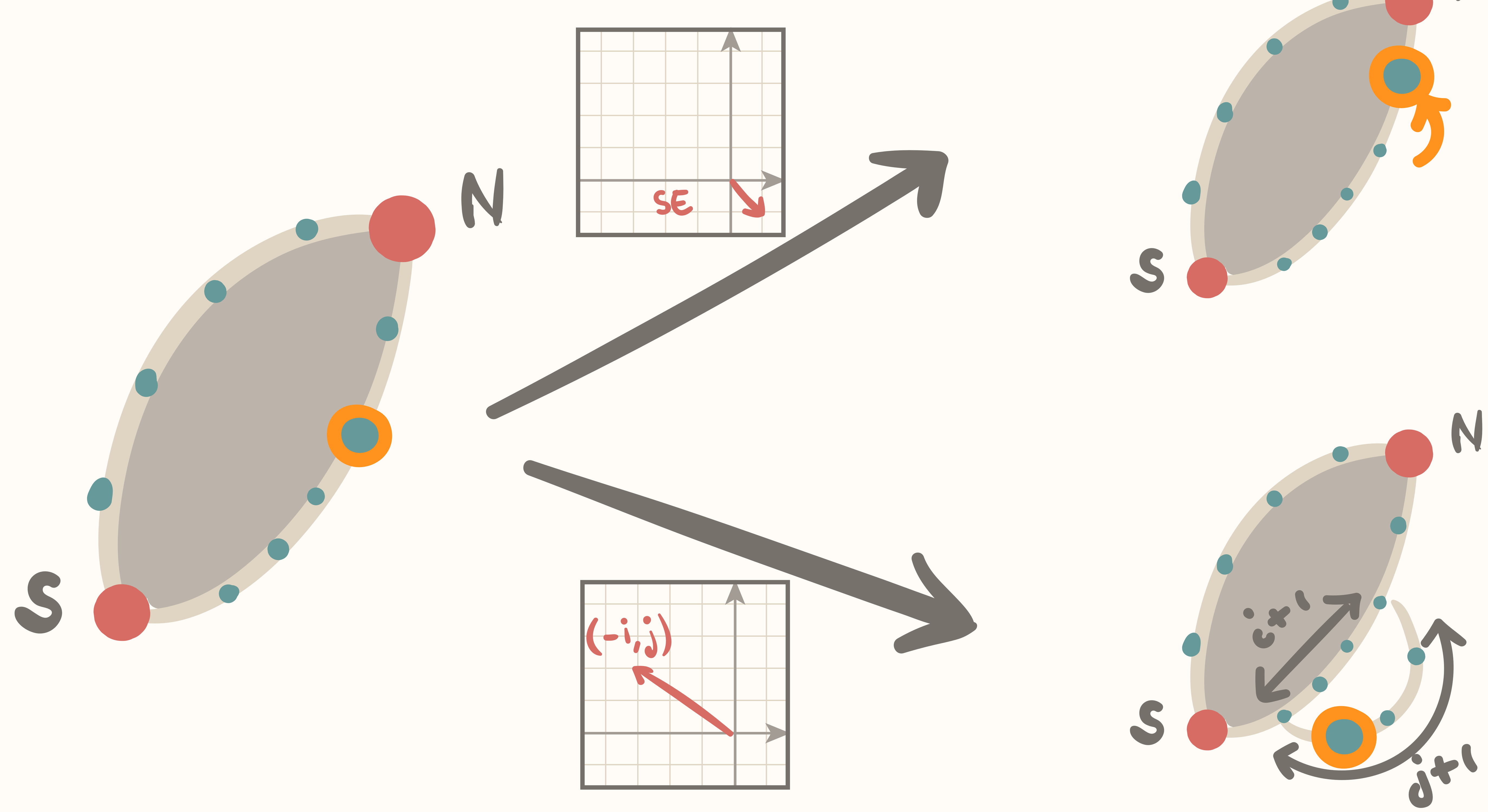


The KMSW bijection

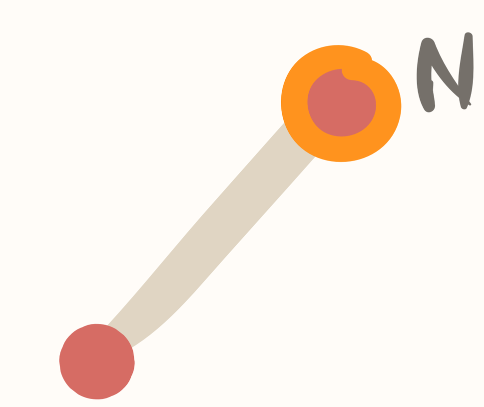
Plane bipolar orientations ⟷ *tandems walks in the quarter plane*





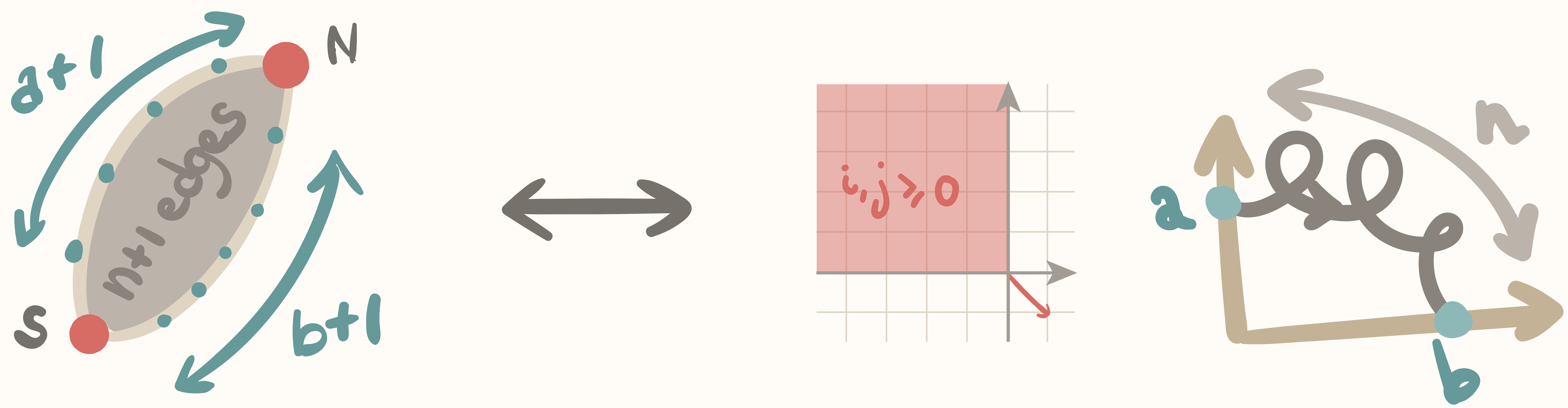


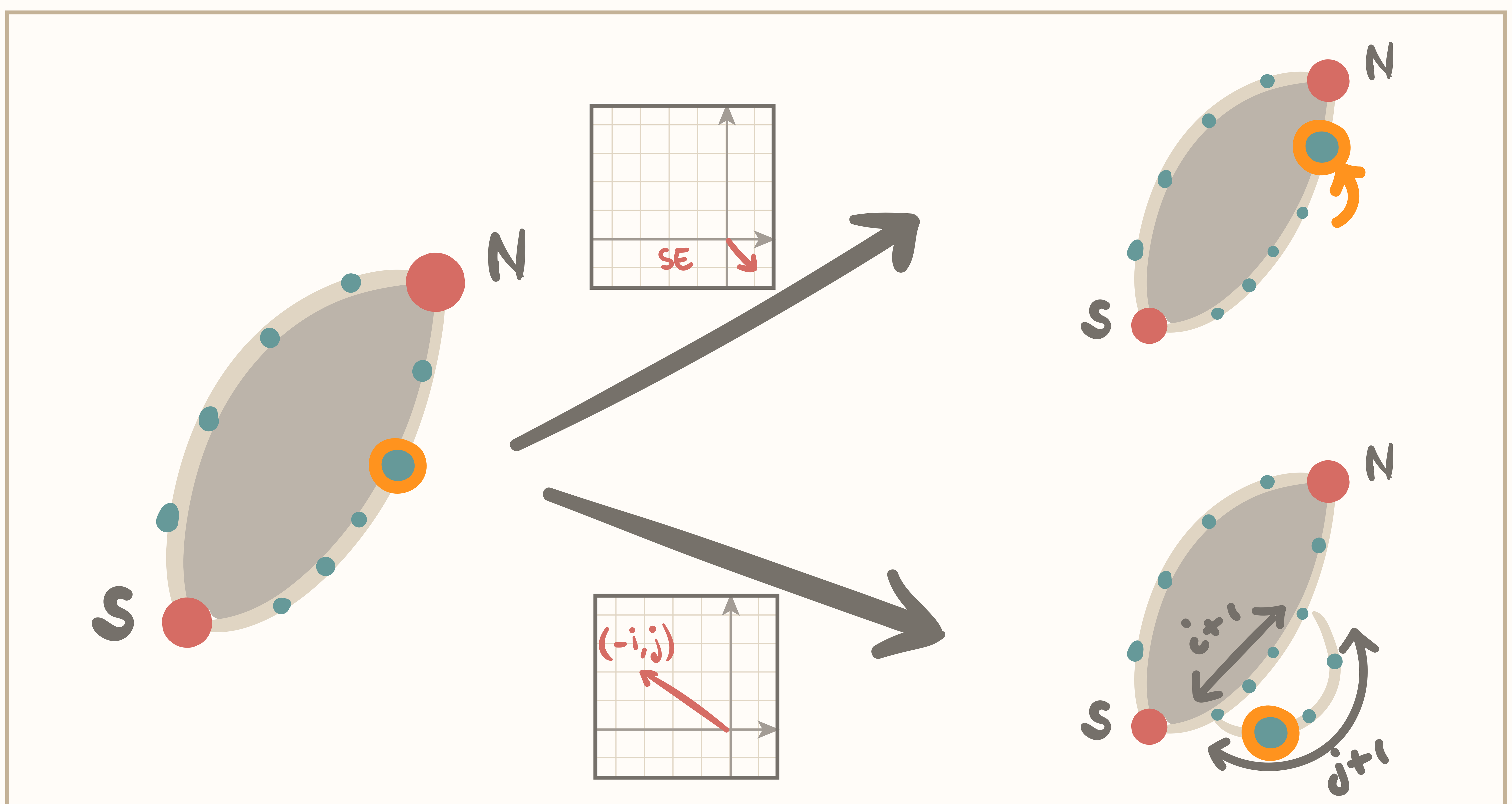
start :



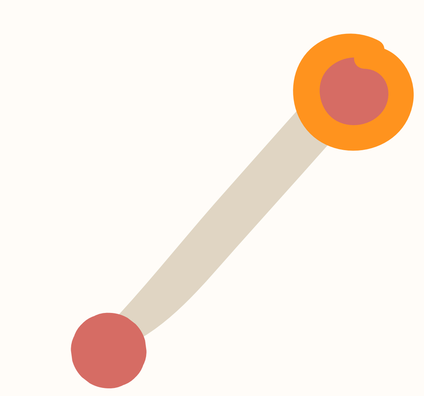
The KMSW bijection

Plane bipolar orientations ⟷ *tandems walks in the quarter plane*

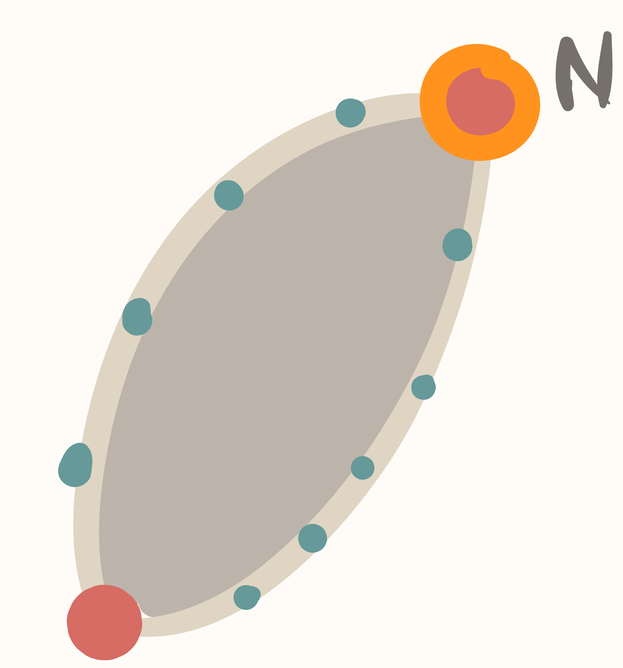




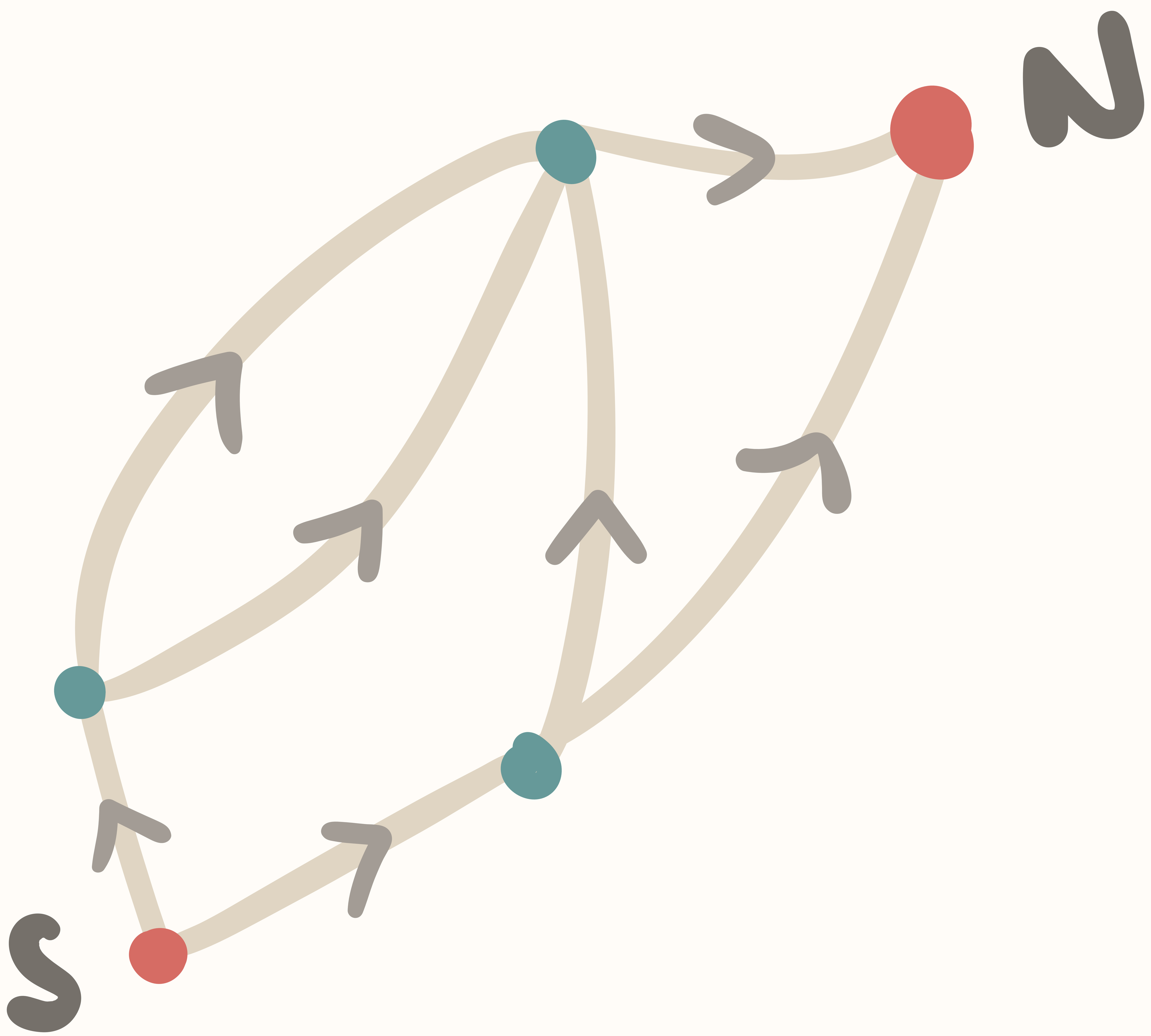
start :

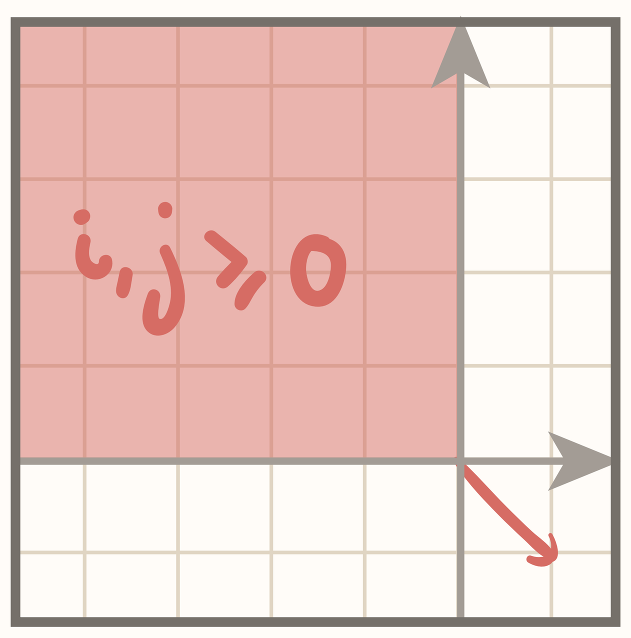


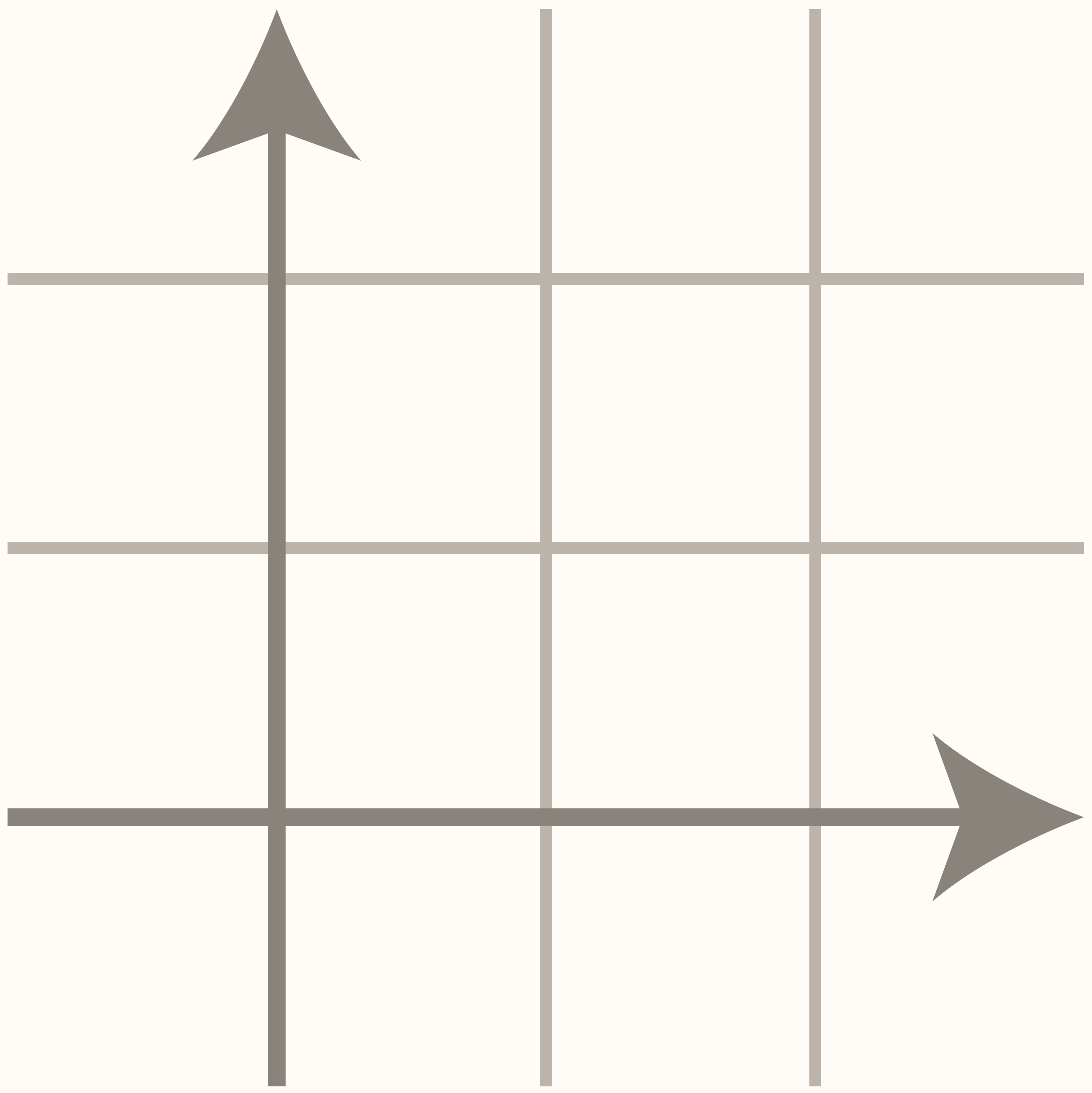
end :



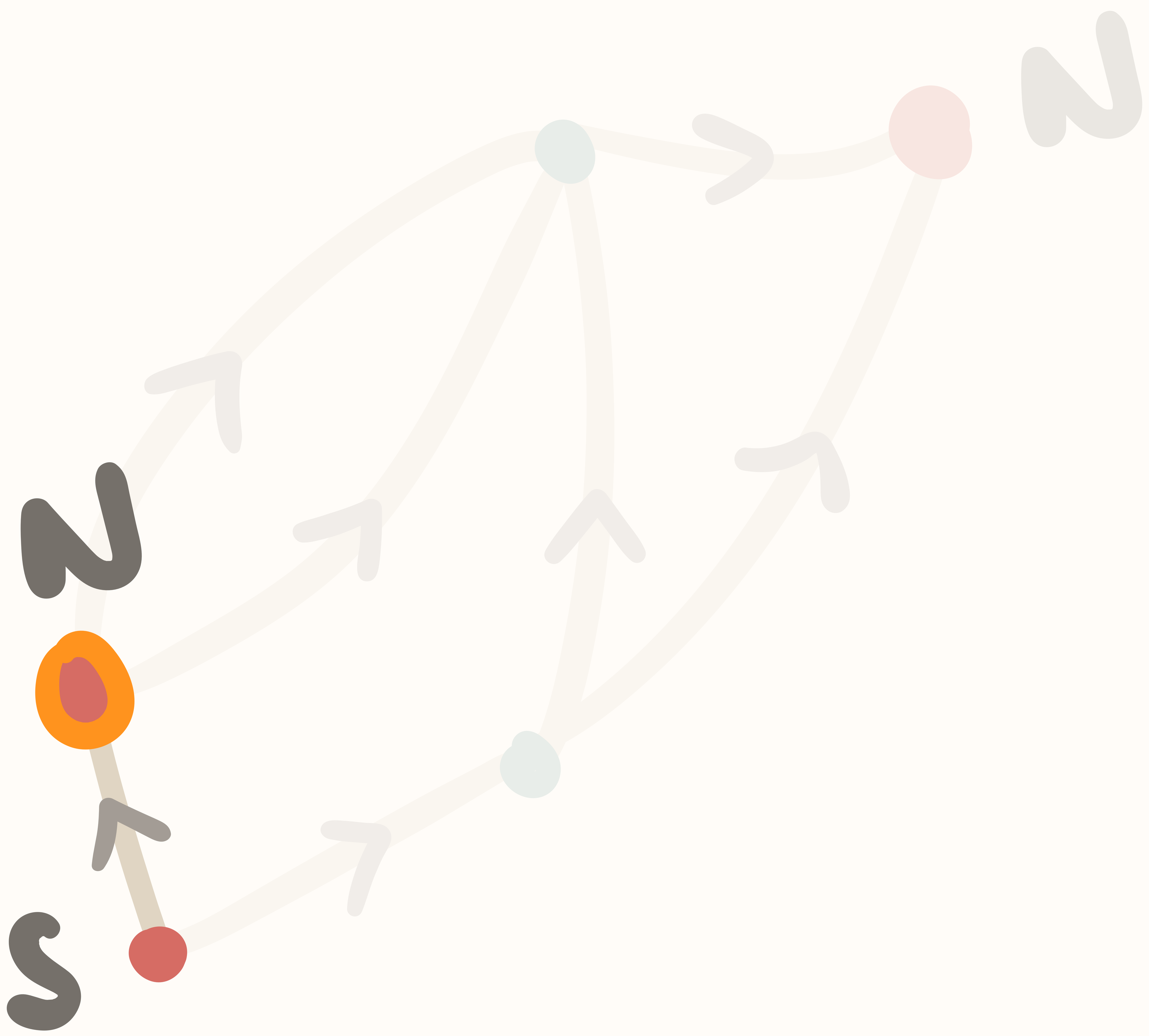
KMSW bijection example

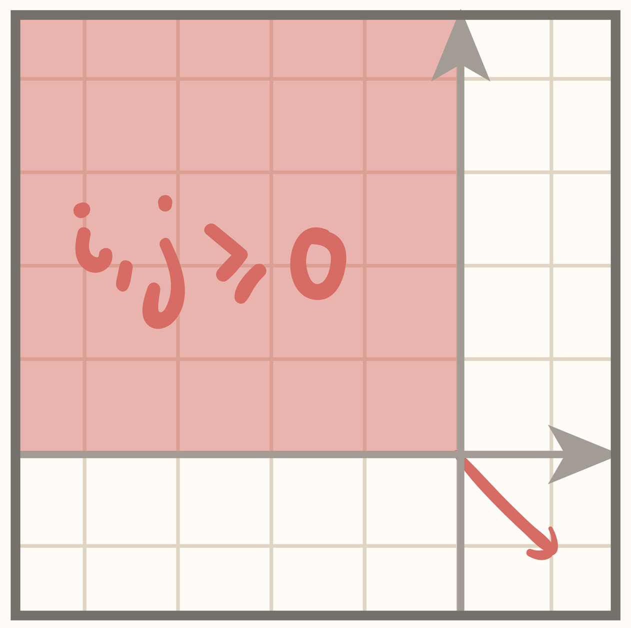


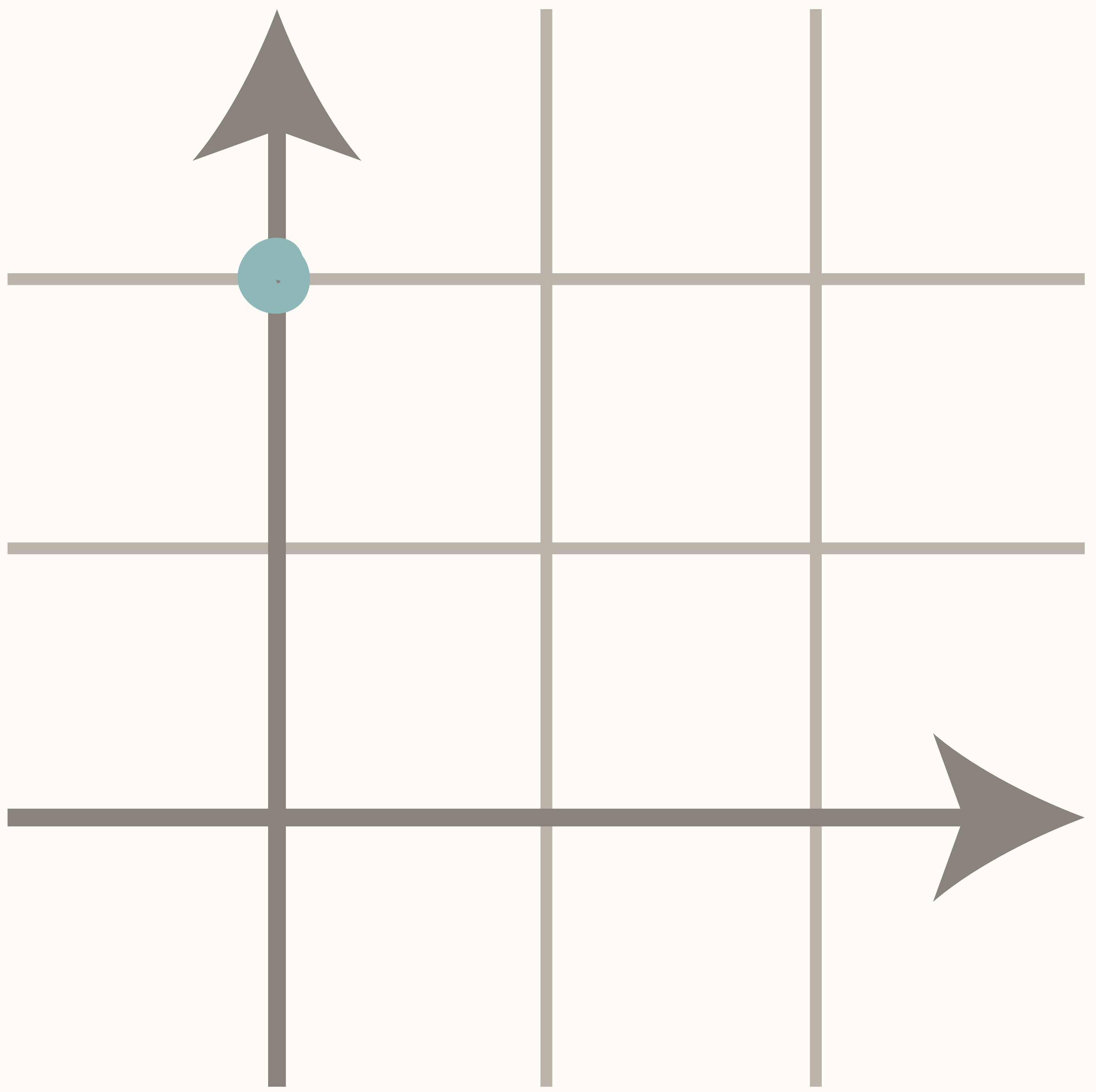




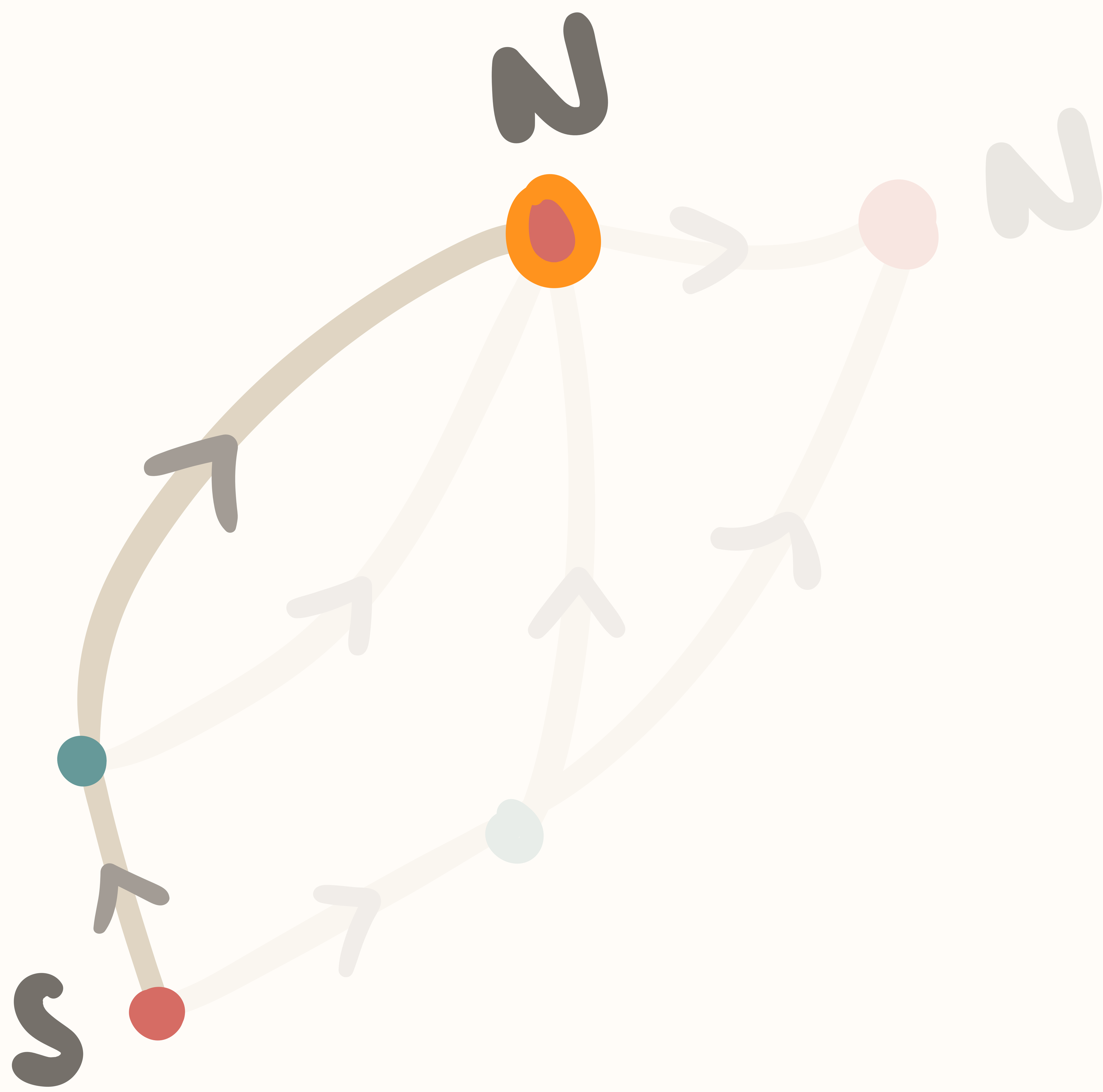
KMSW bijection example

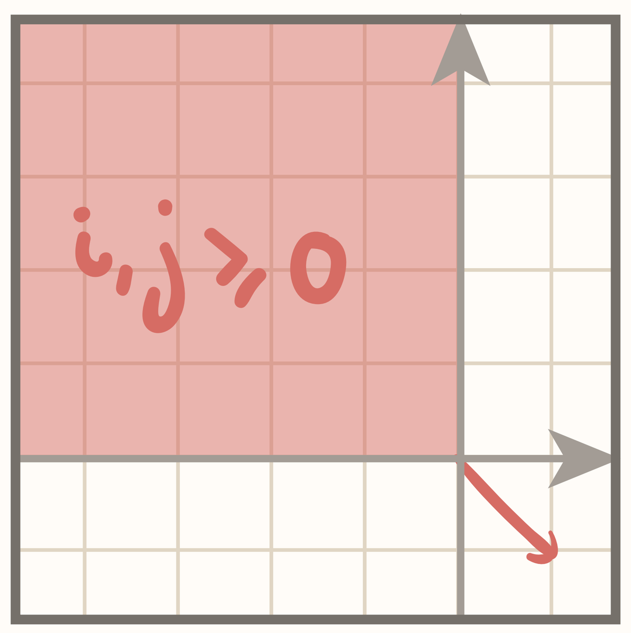


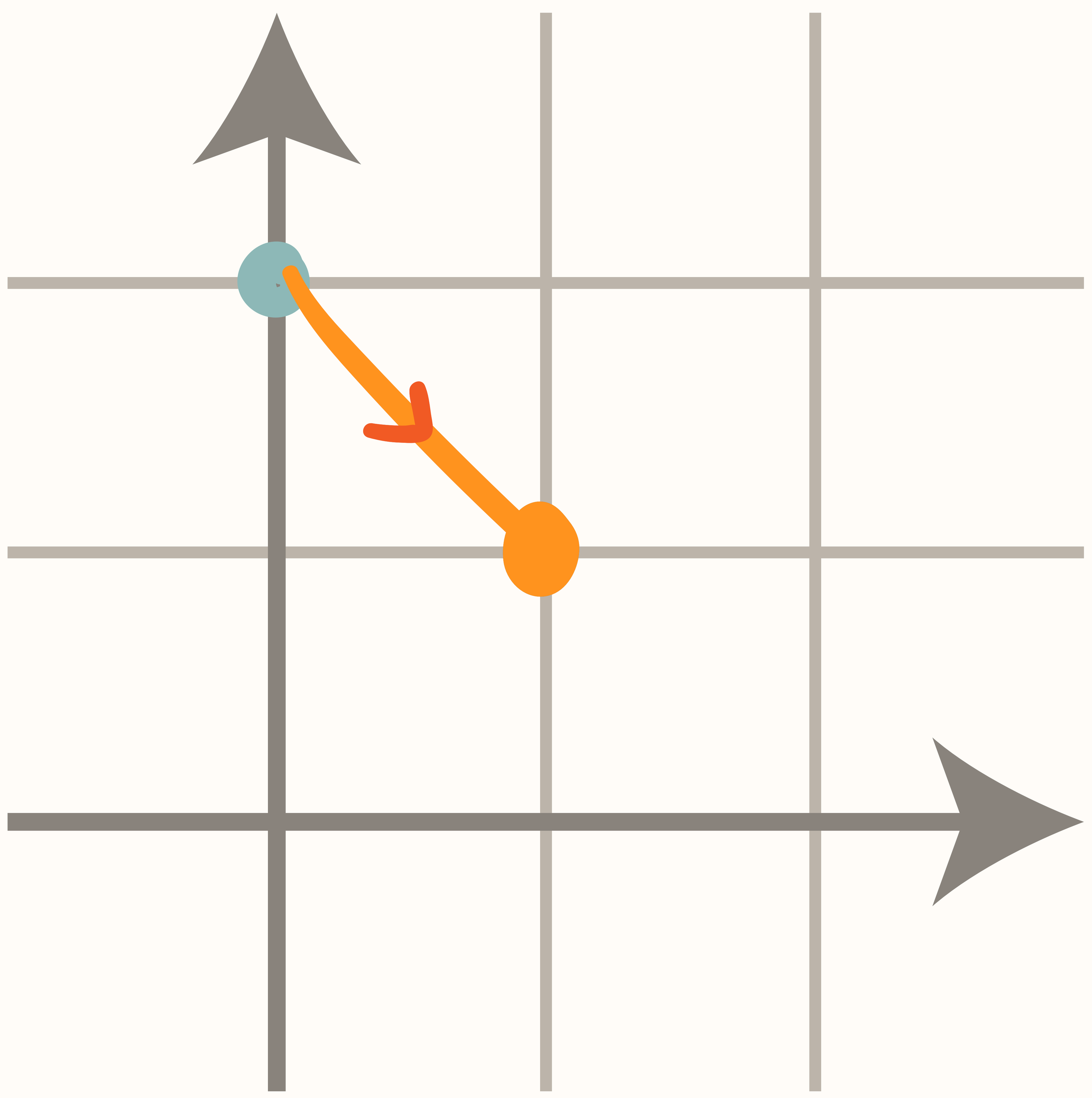




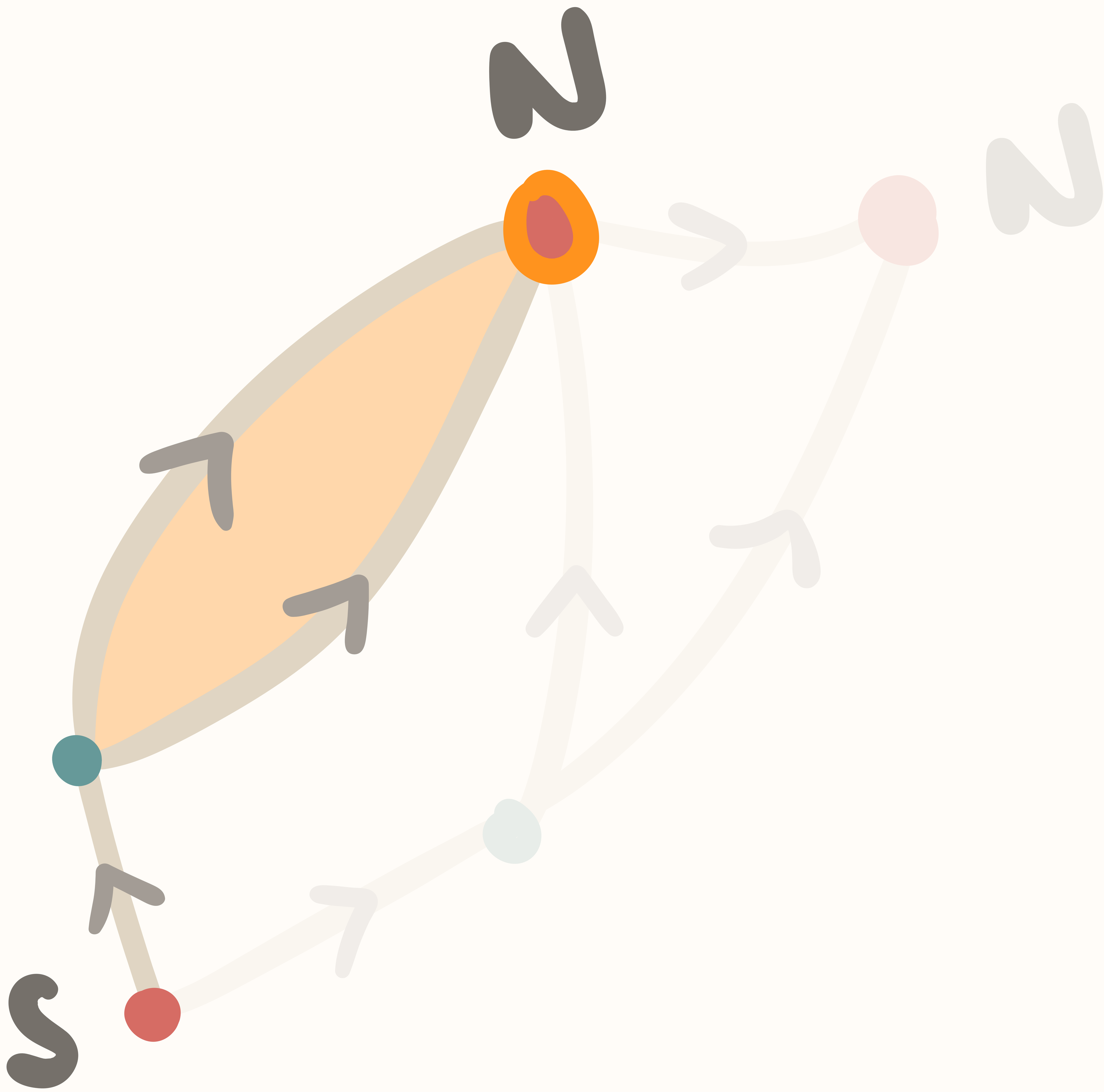
KMSW bijection example

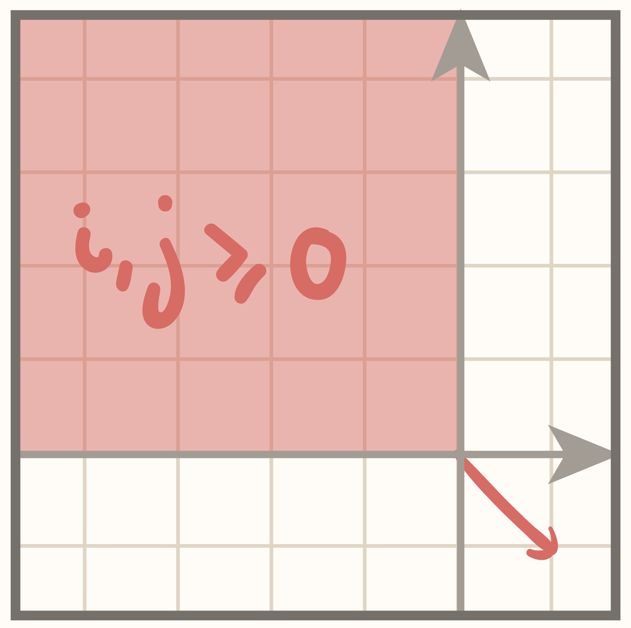


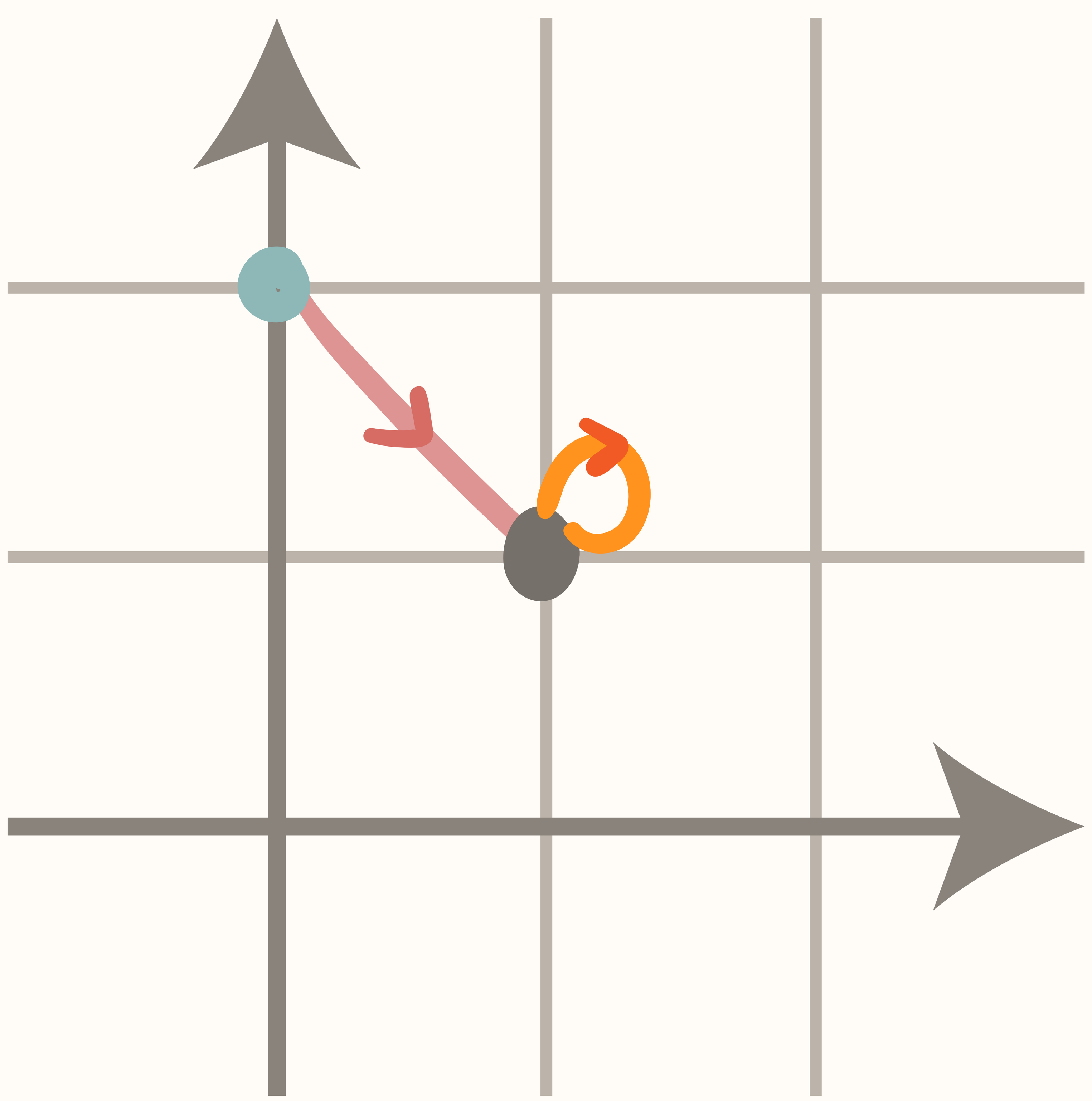




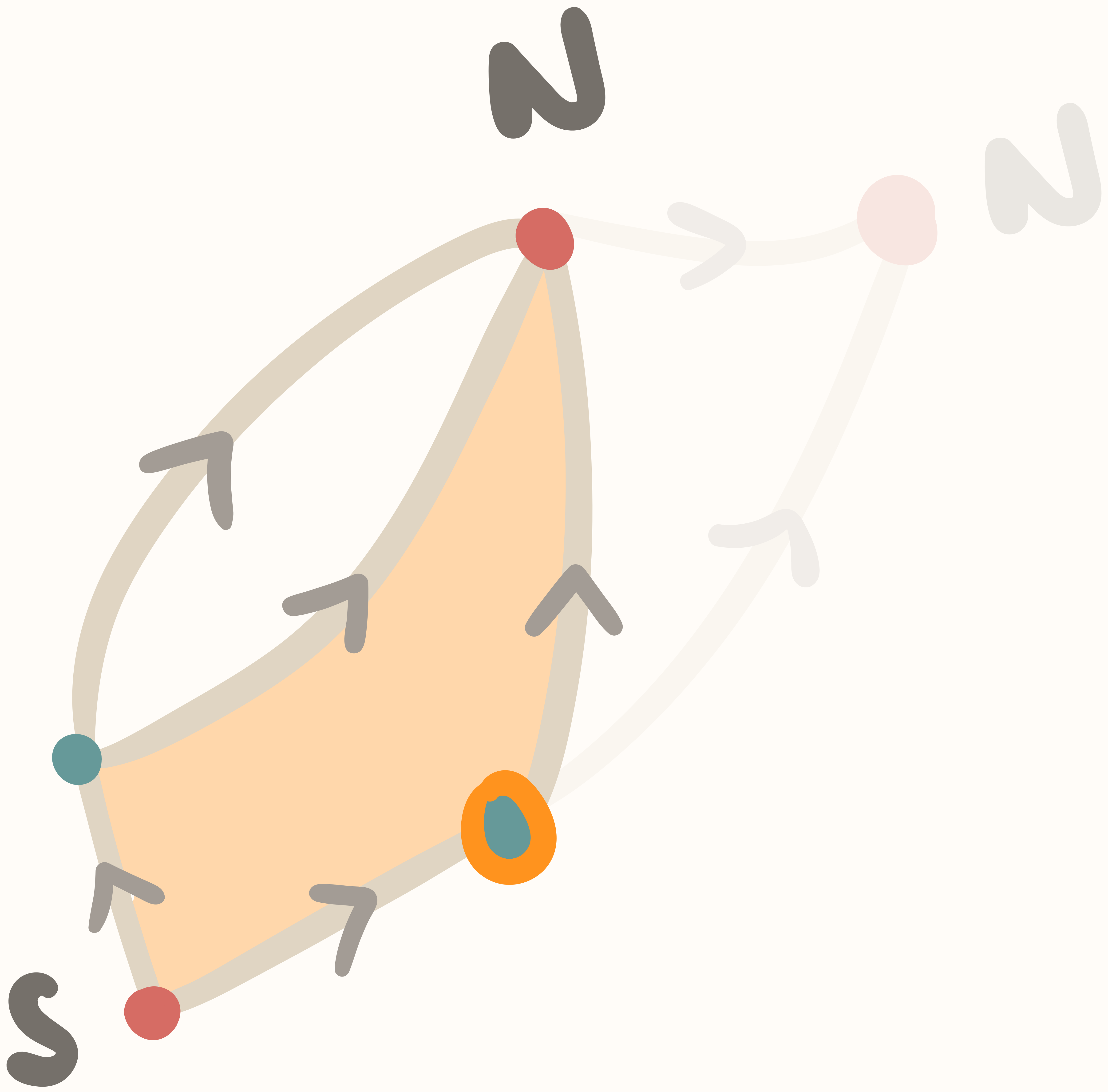
KMSW bijection example

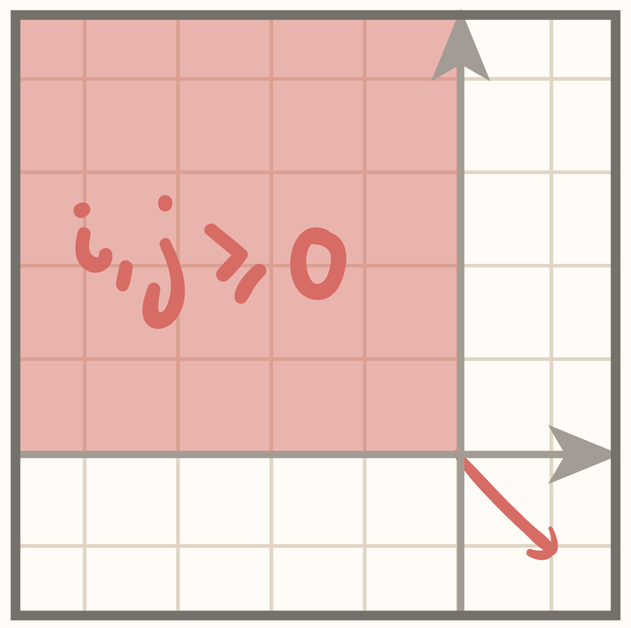


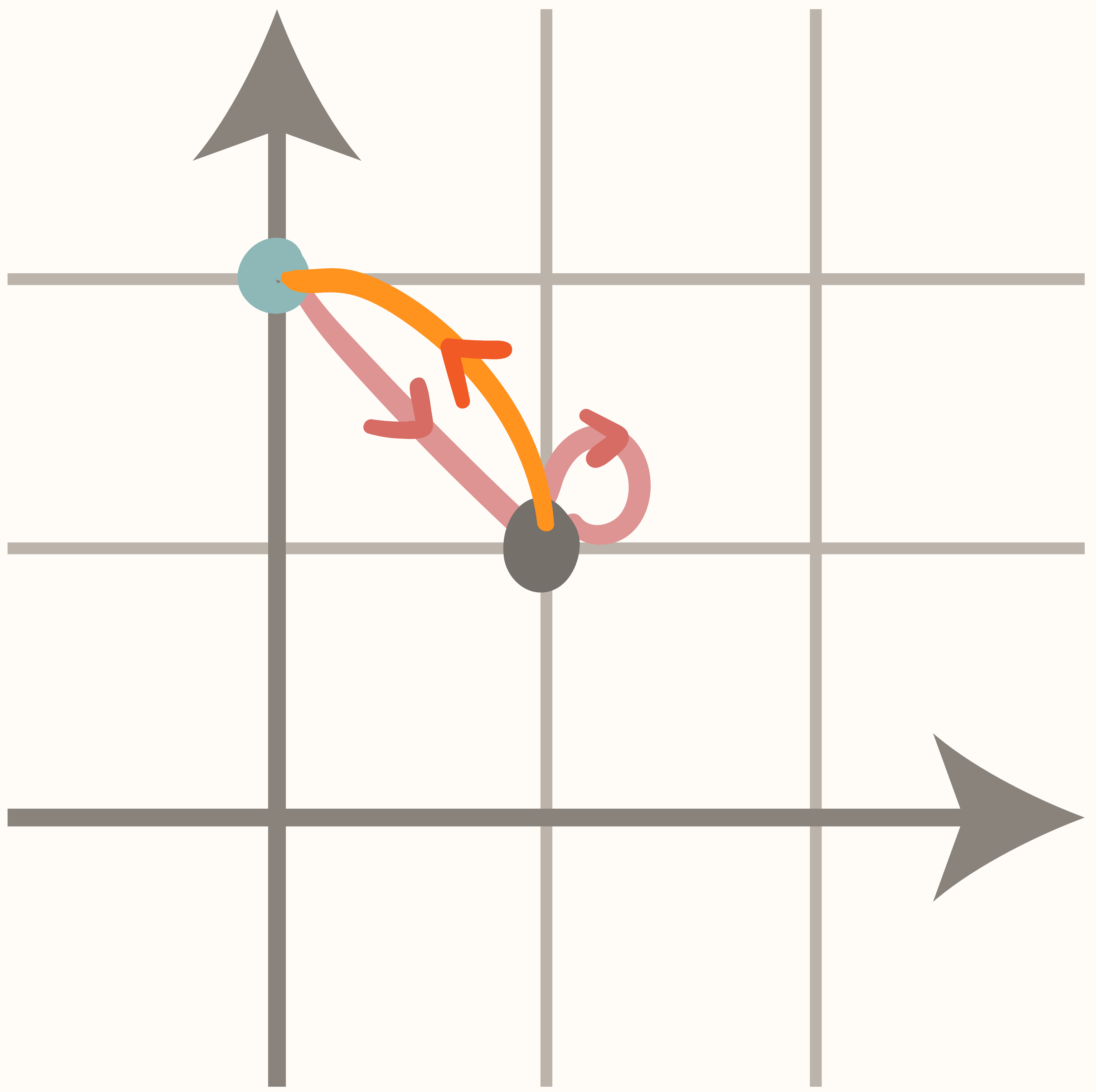




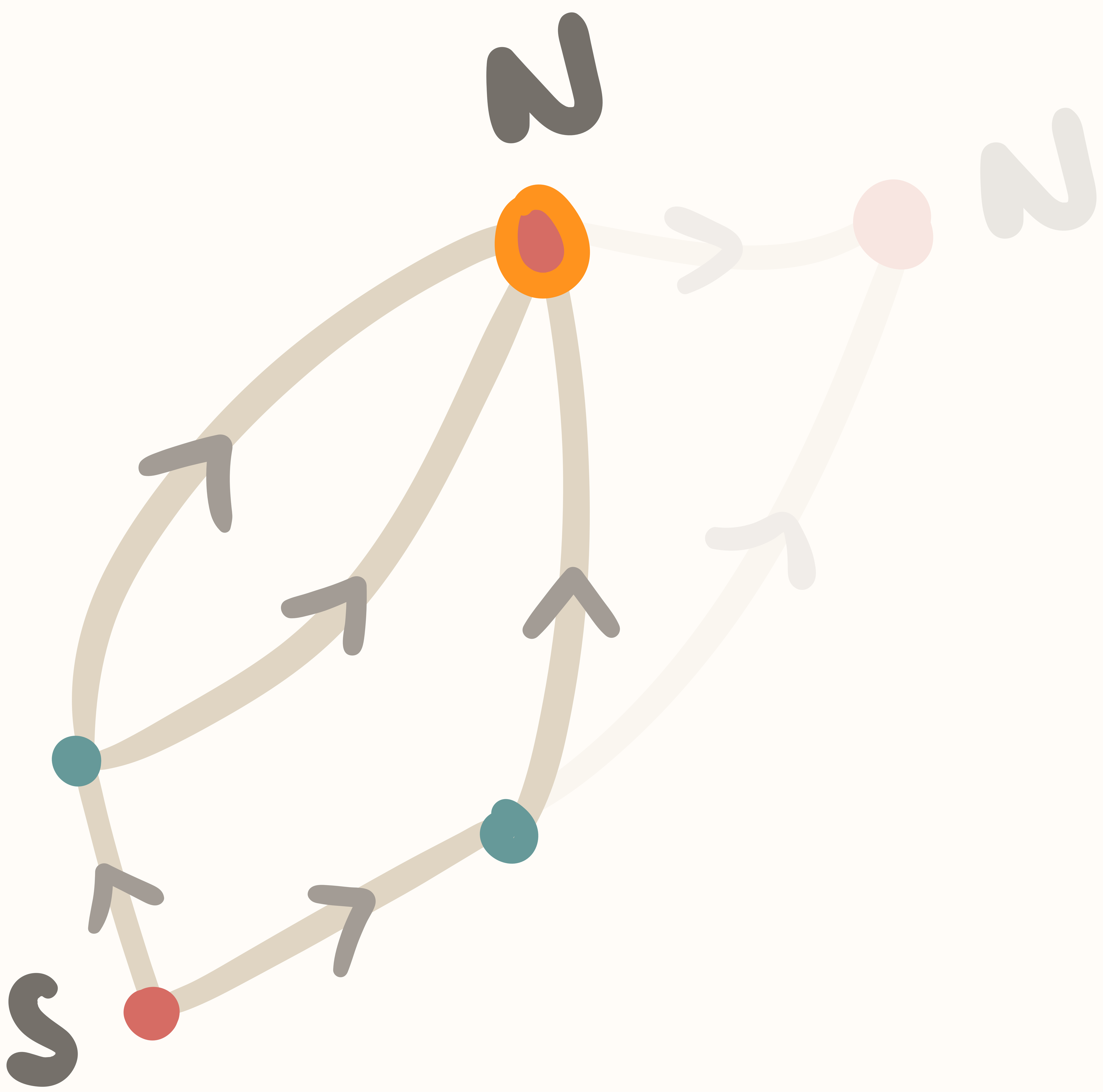
KMSW bijection example

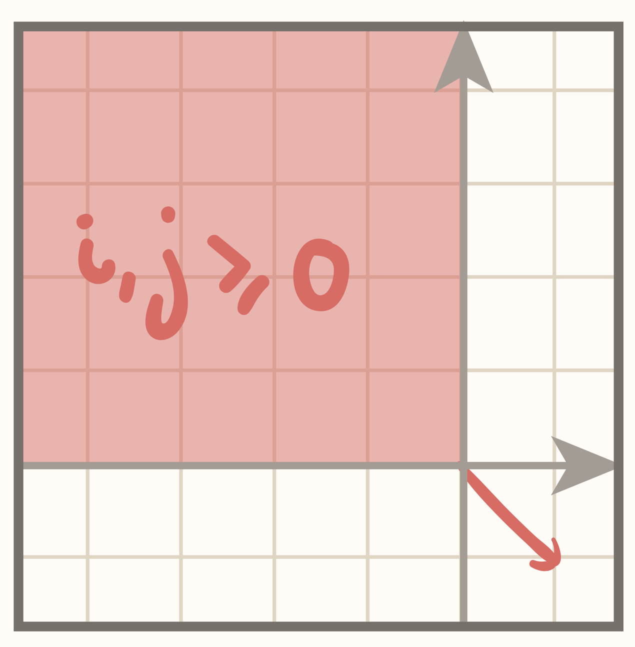


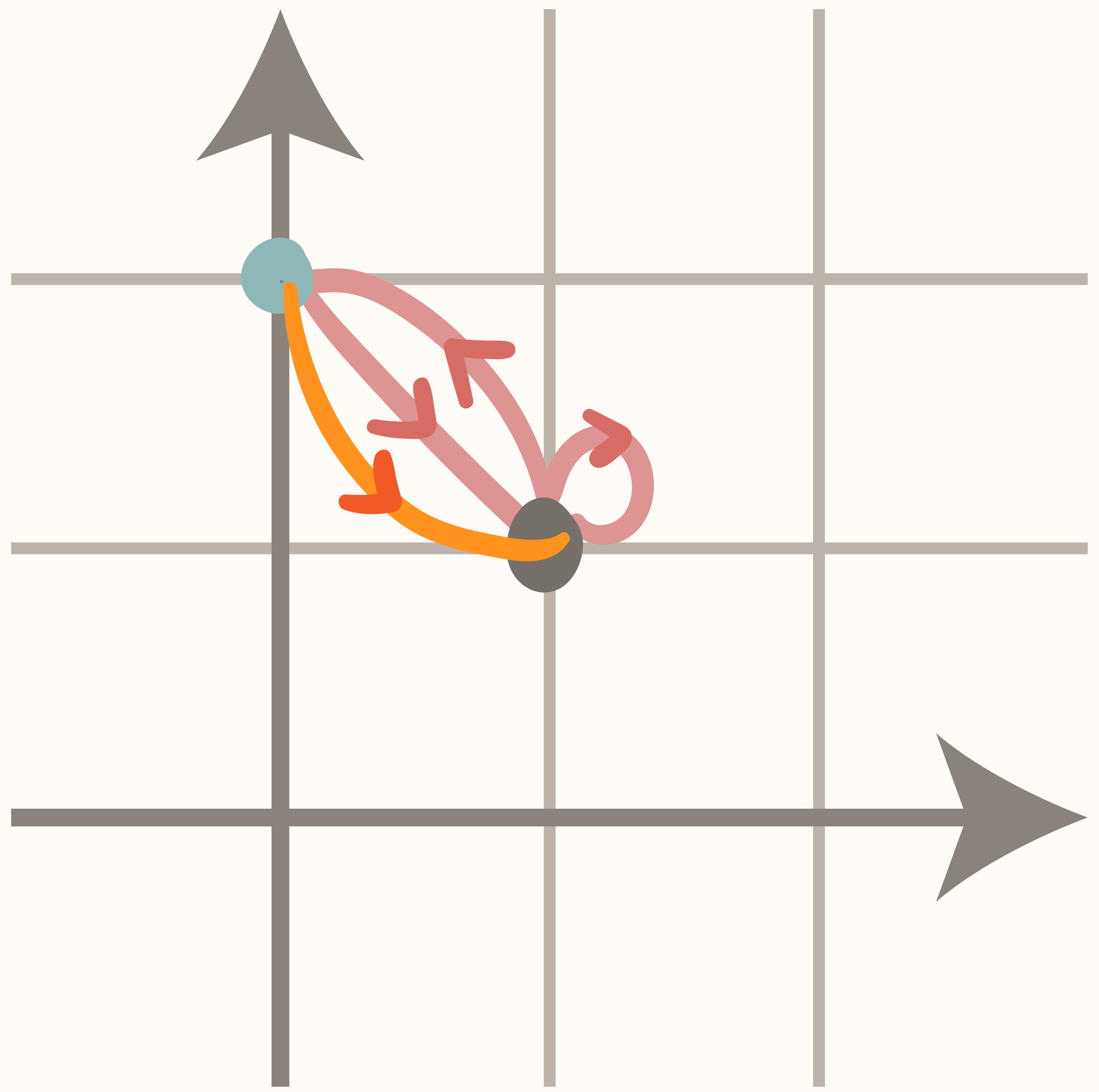




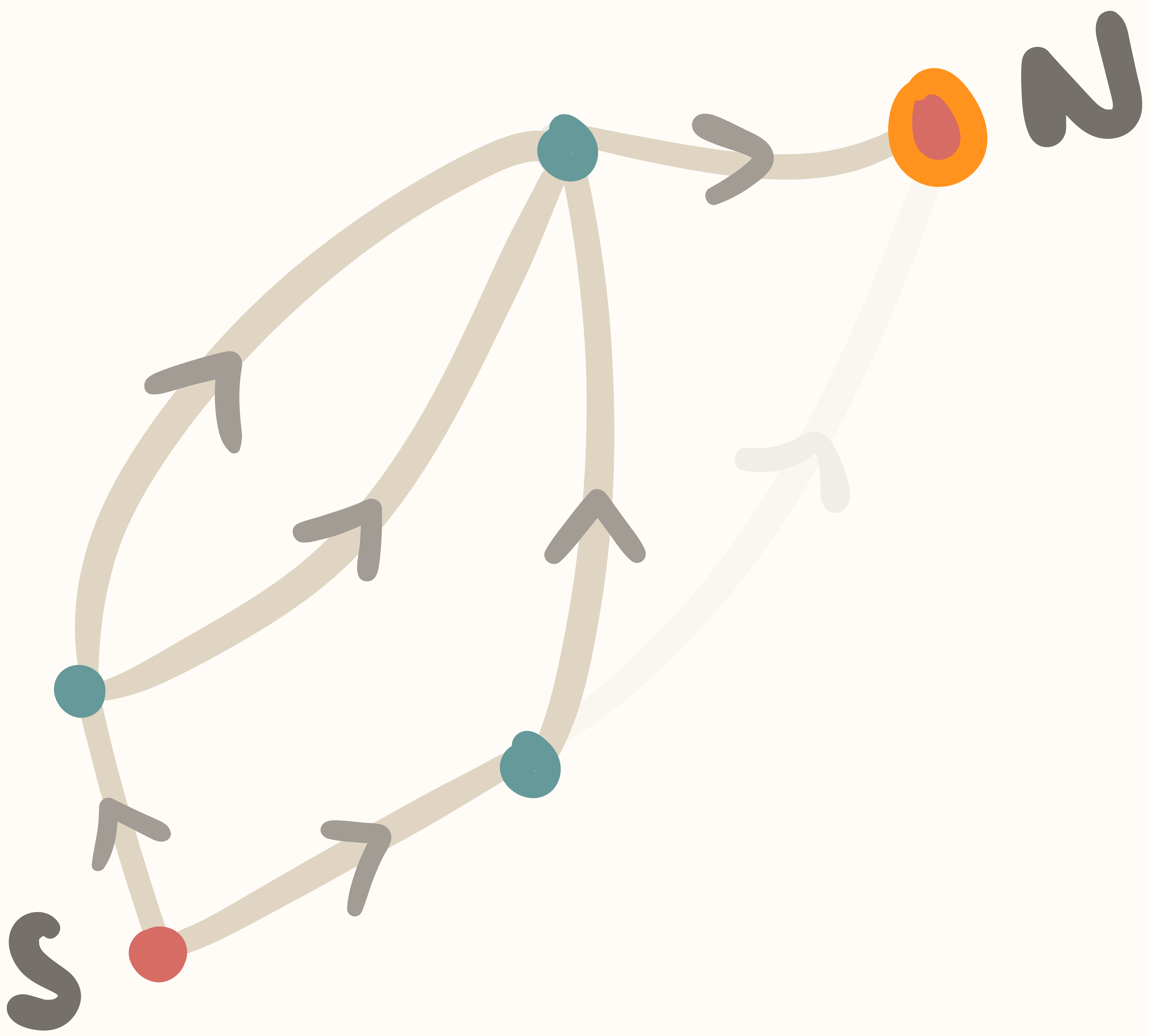
KMSW bijection example

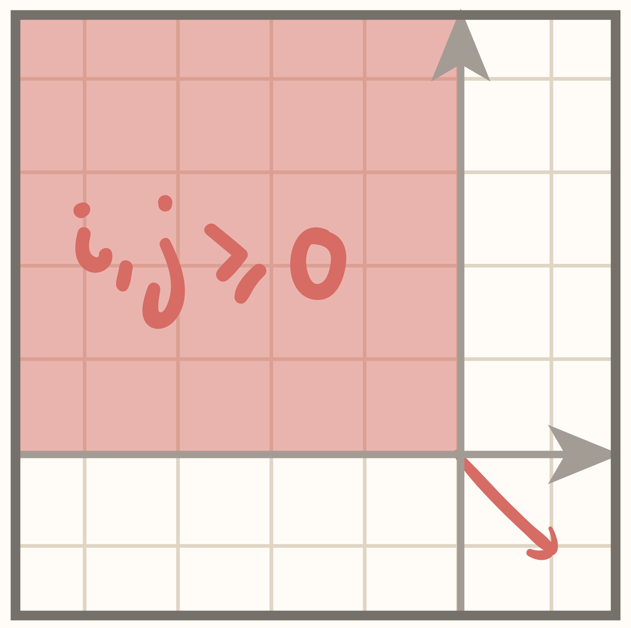


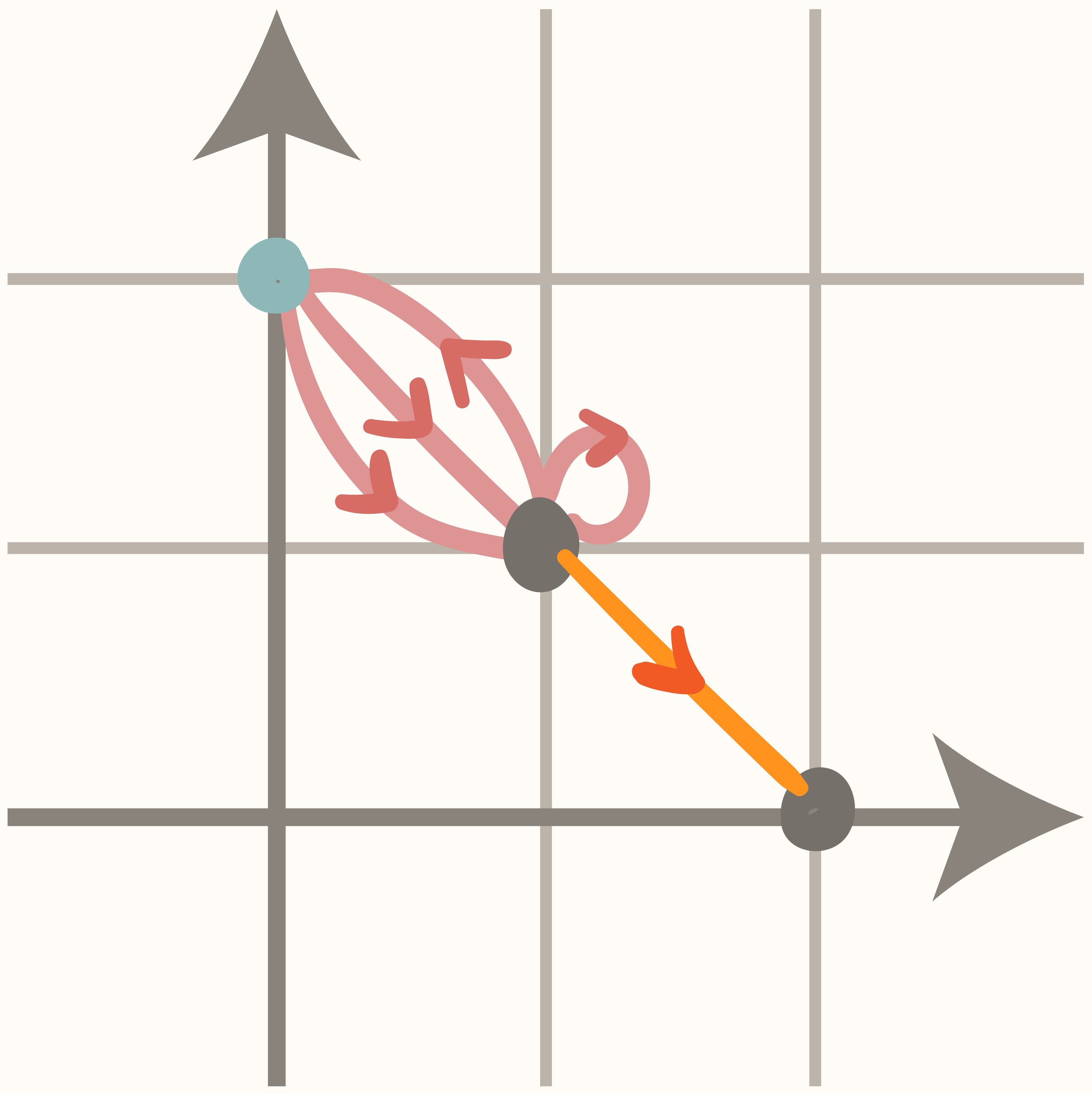




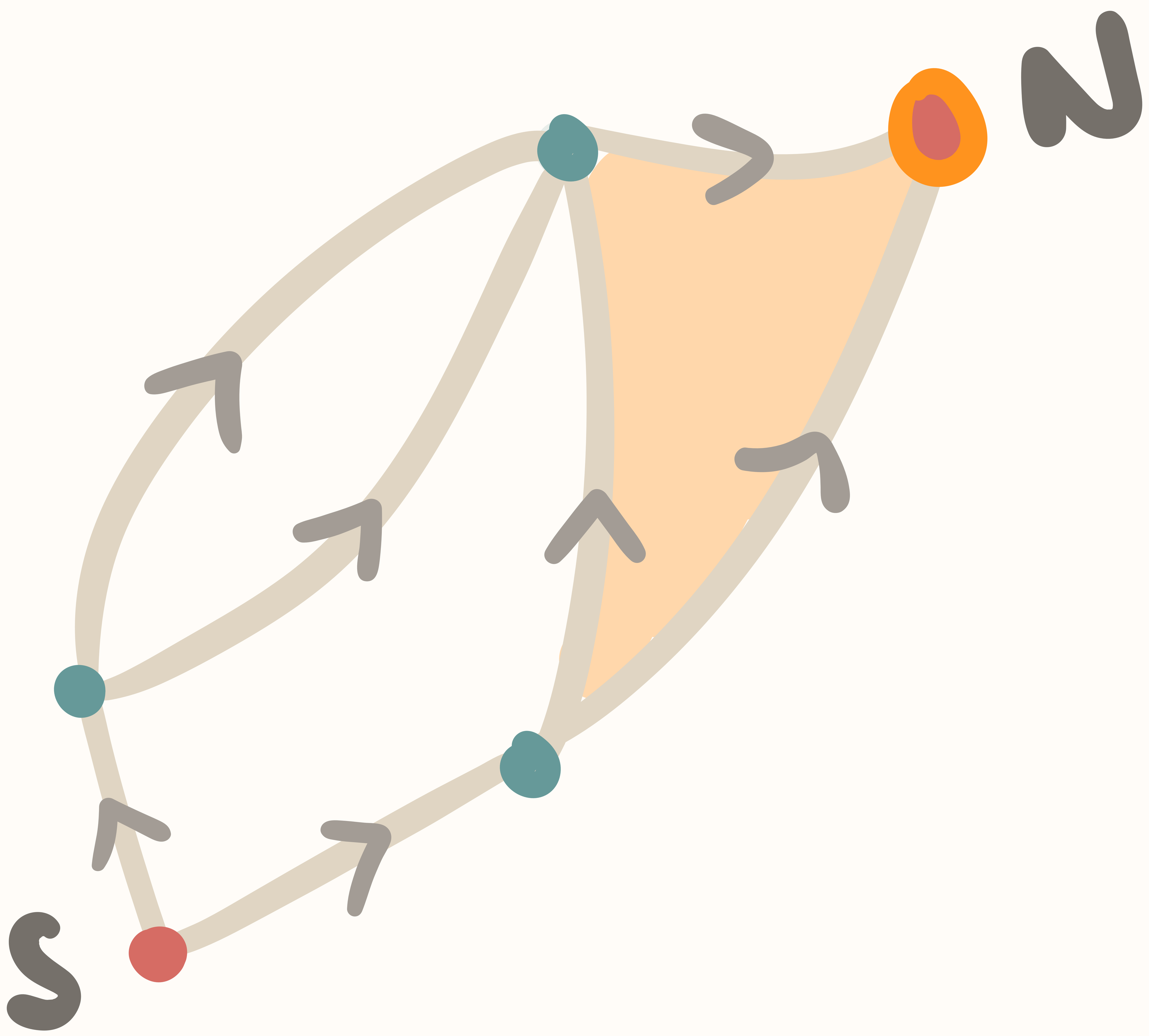
KMSW bijection example

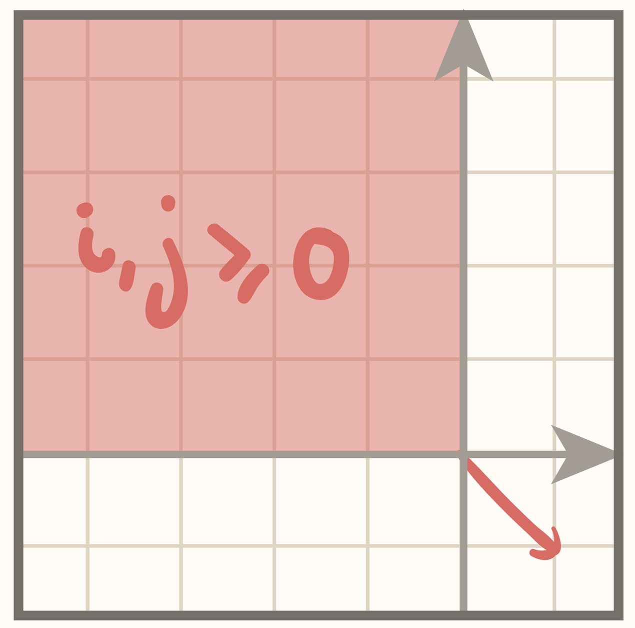


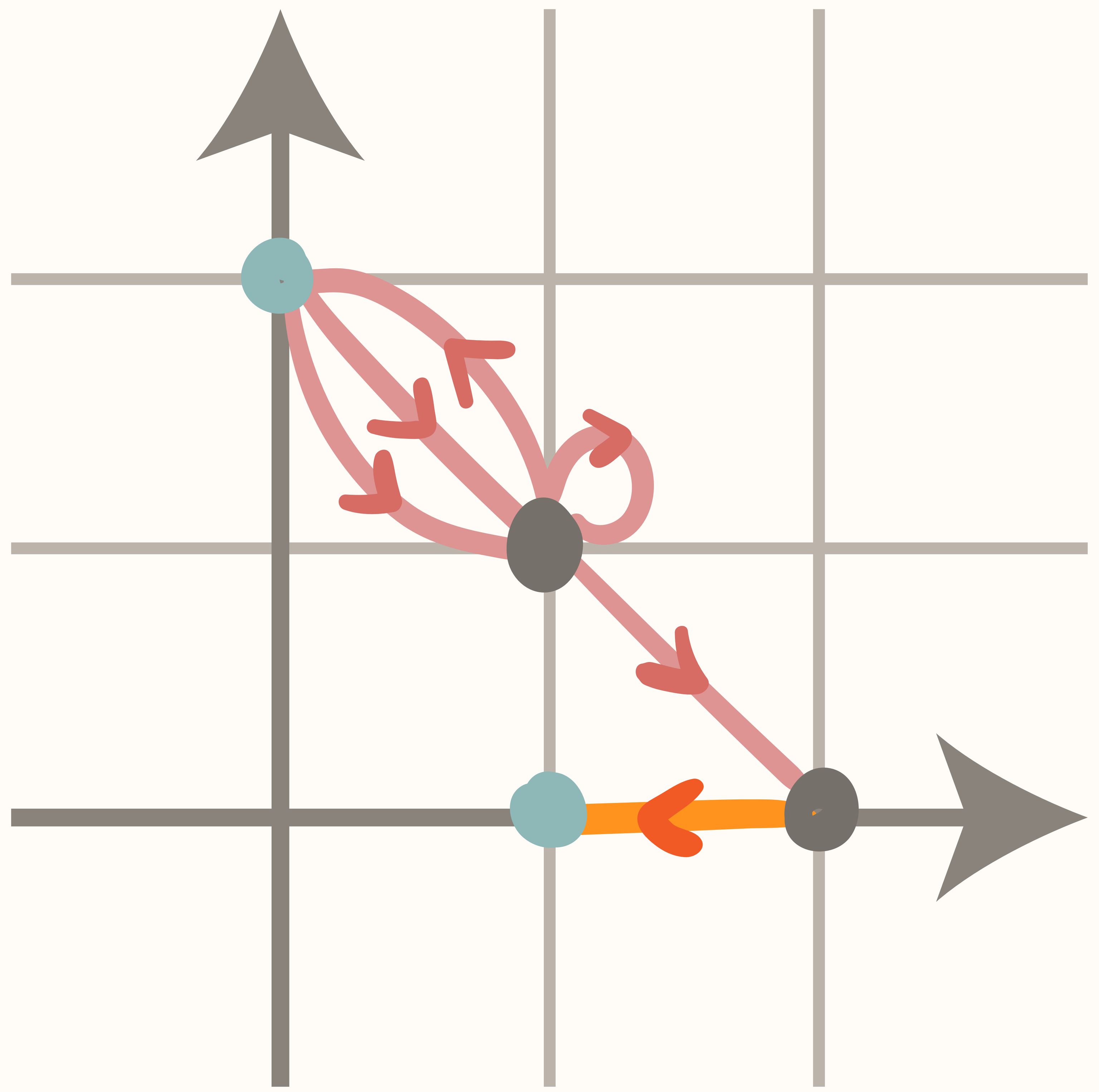




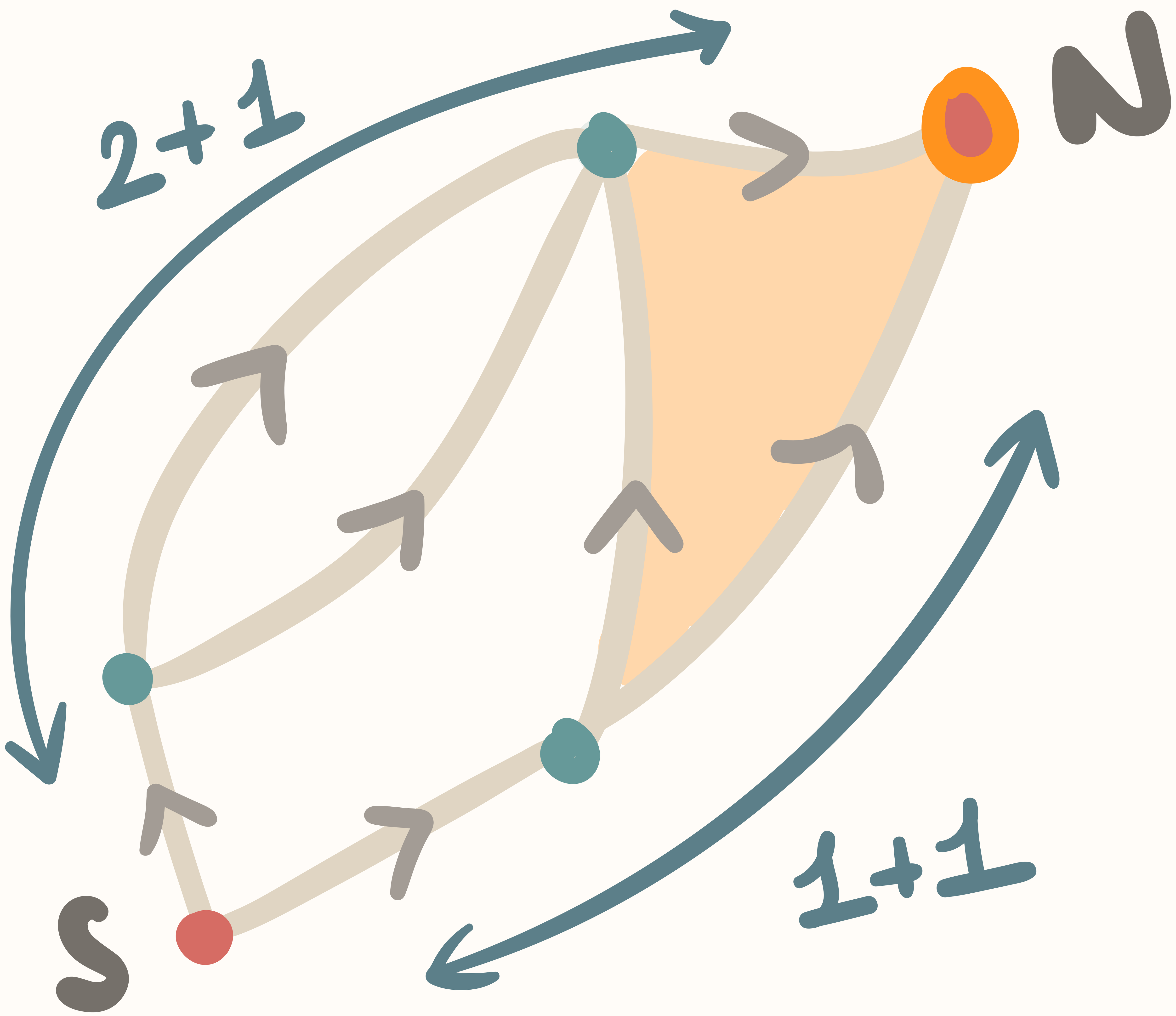
KMSW bijection example

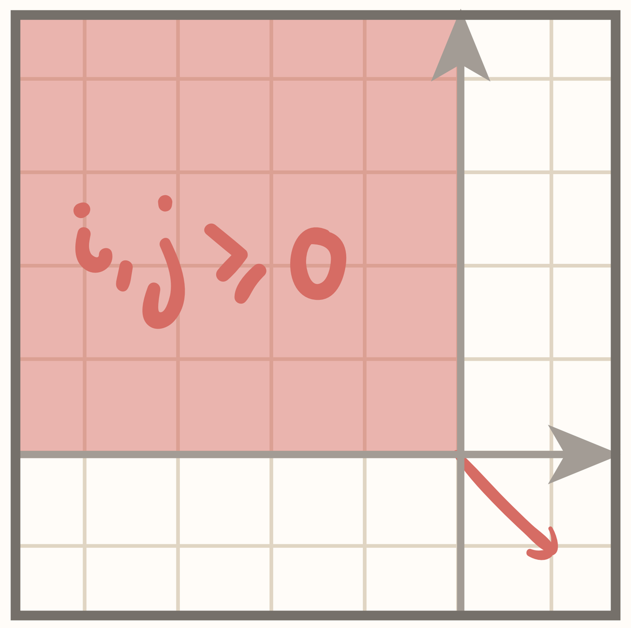


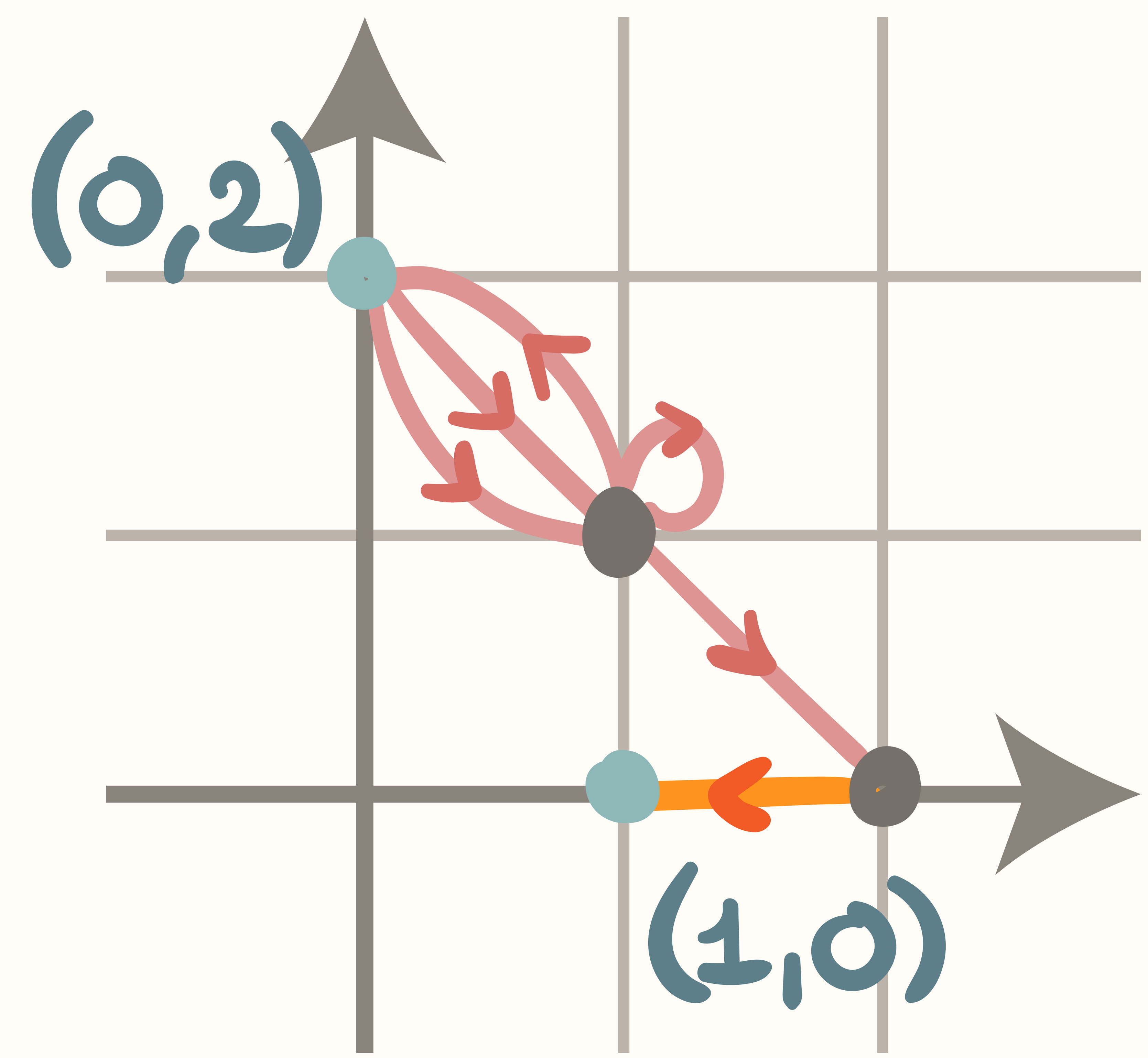




KMSW bijection example

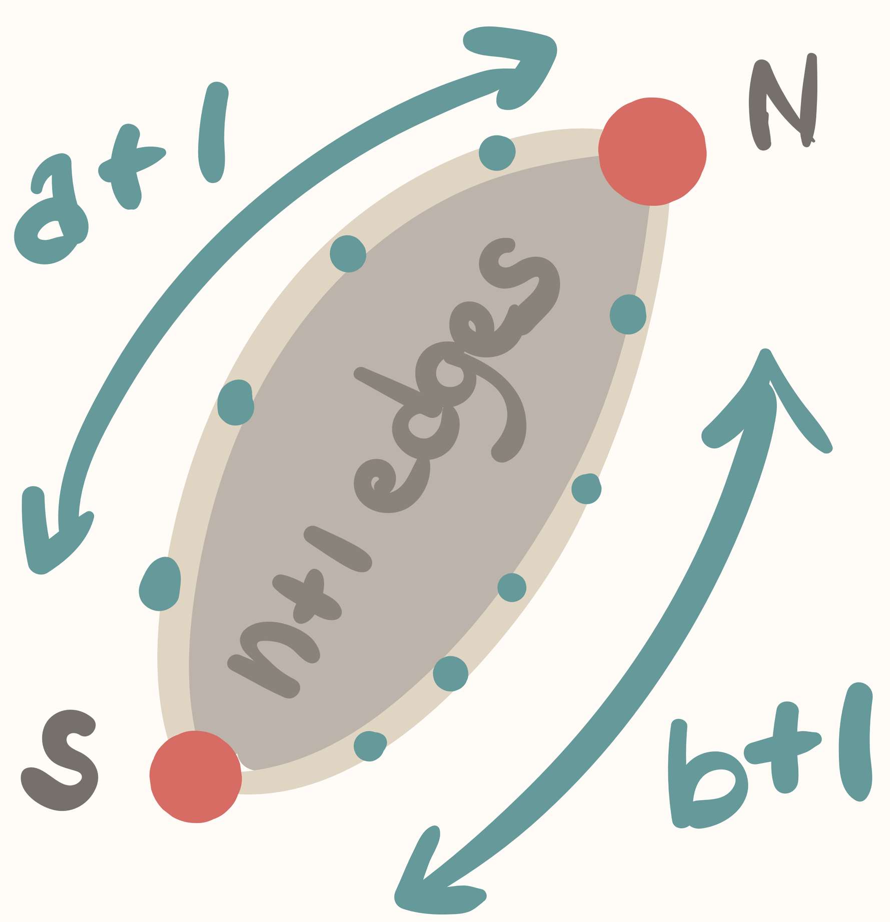




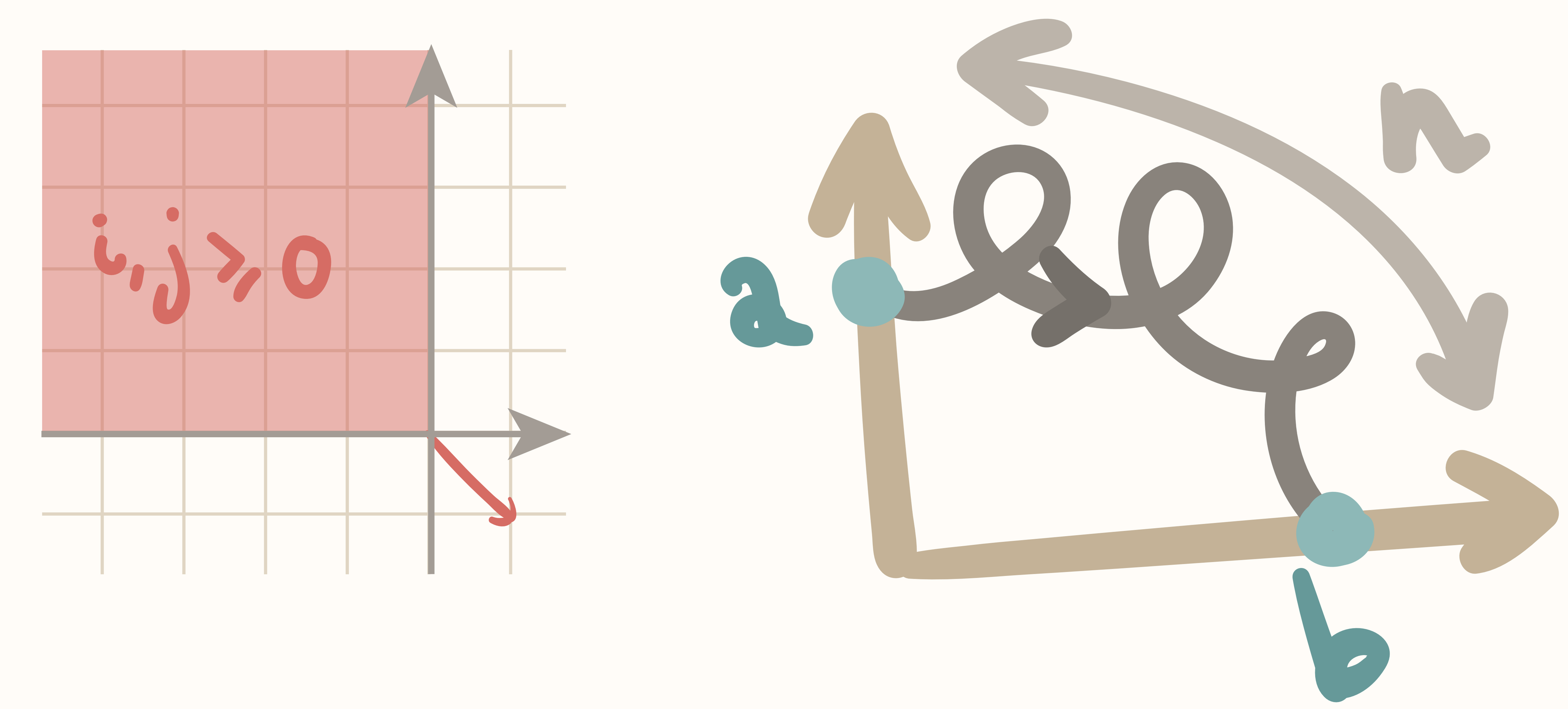


La bijection KMSW

bipolar orientations ⟷ ***tandem walks in the quarter plane***

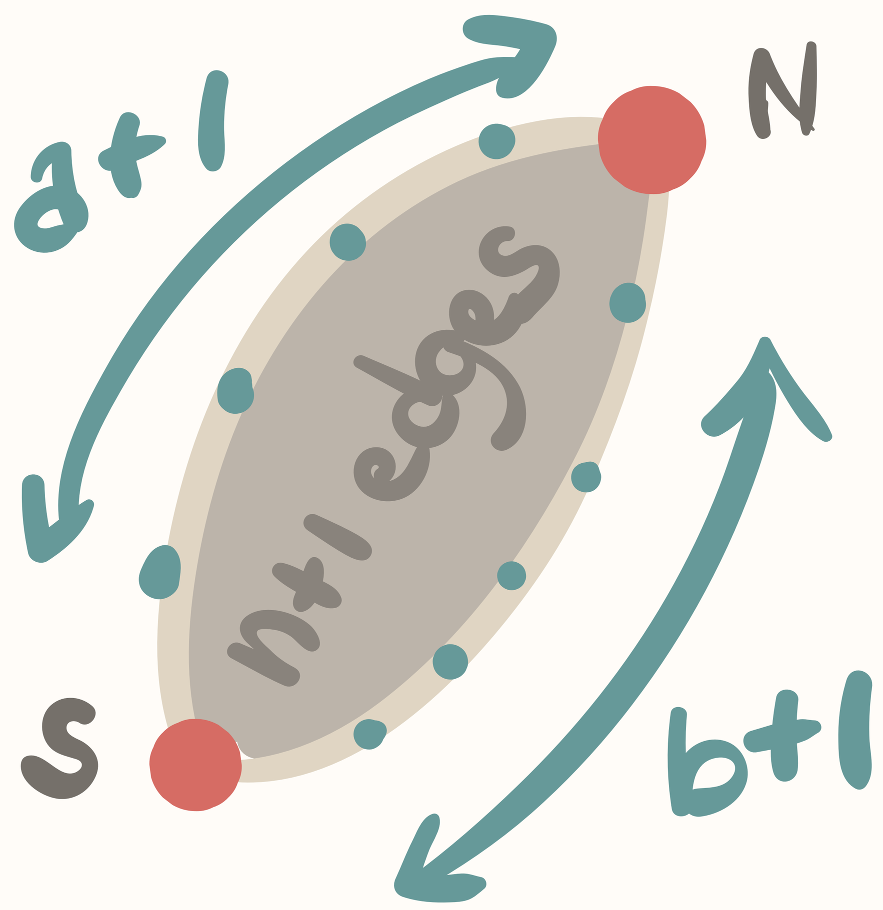


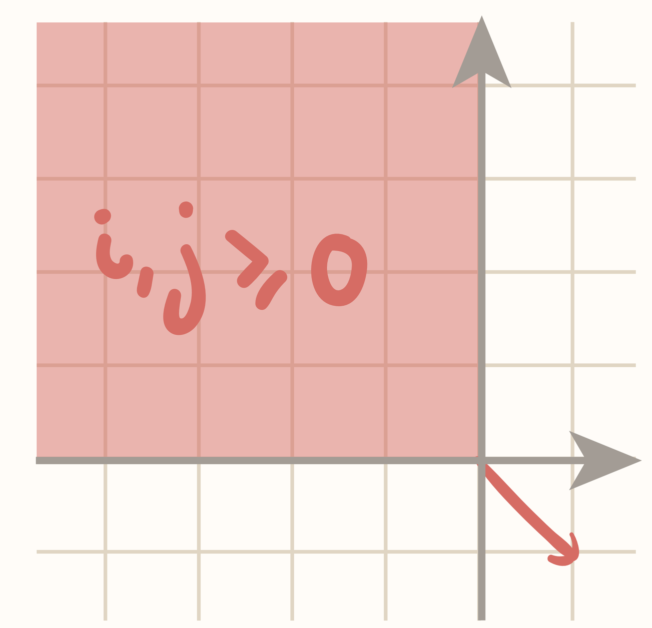
⟷

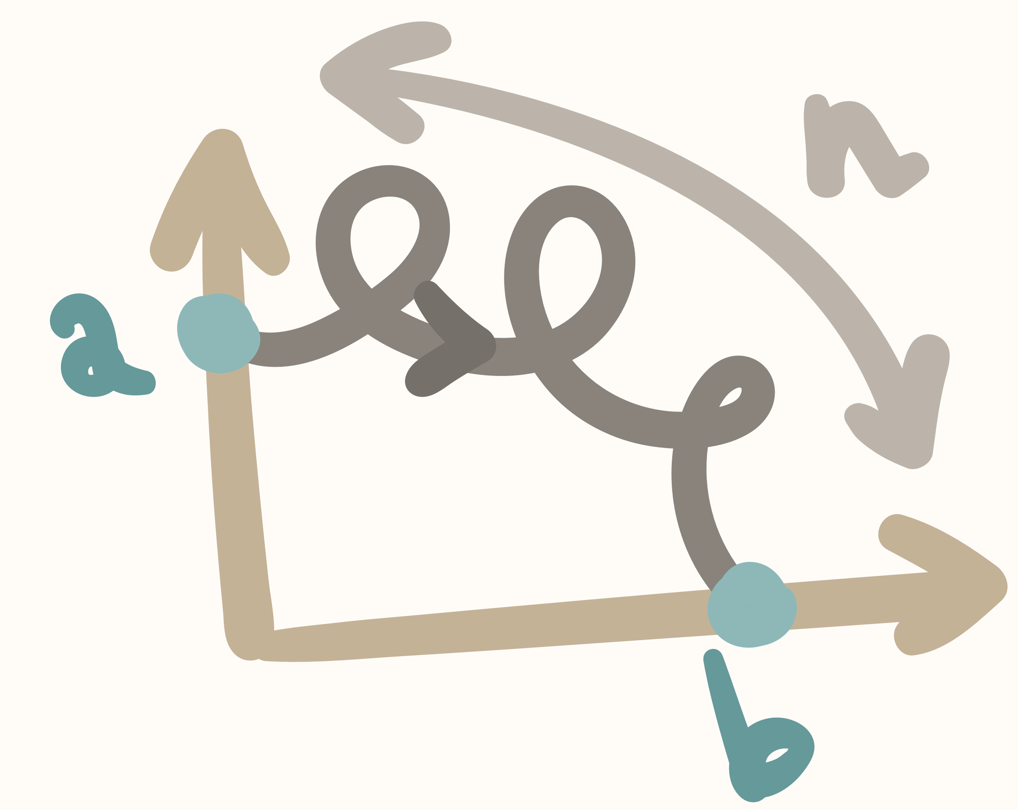


La bijection KMSW

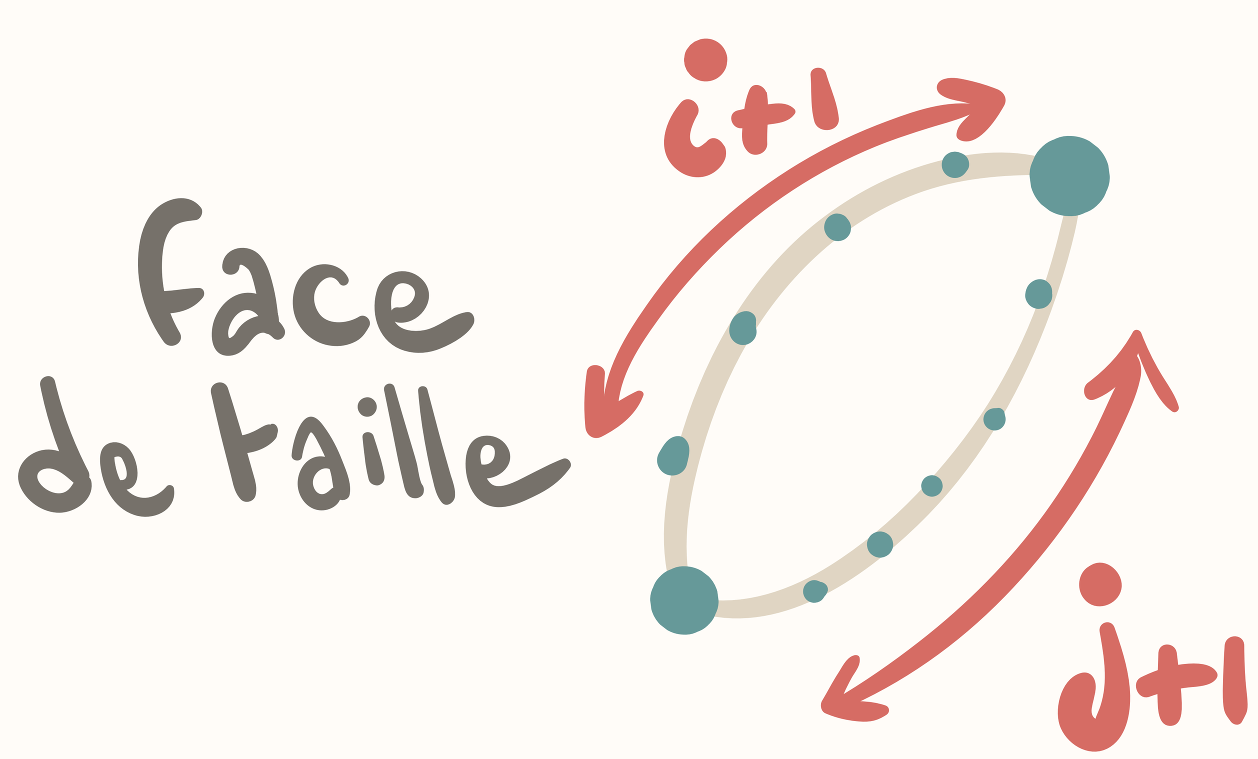
bipolar orientations ⟷ ***tandem walks in the quarter plane***







↠ *Bipolar orientations on planar maps and* SLE_{12}, R. Kenyon, J. Miller, S. Sheffield and D. Wilson (2015)



Summary

Maps and decorated maps

1. Bijection with quadrant tandem walks

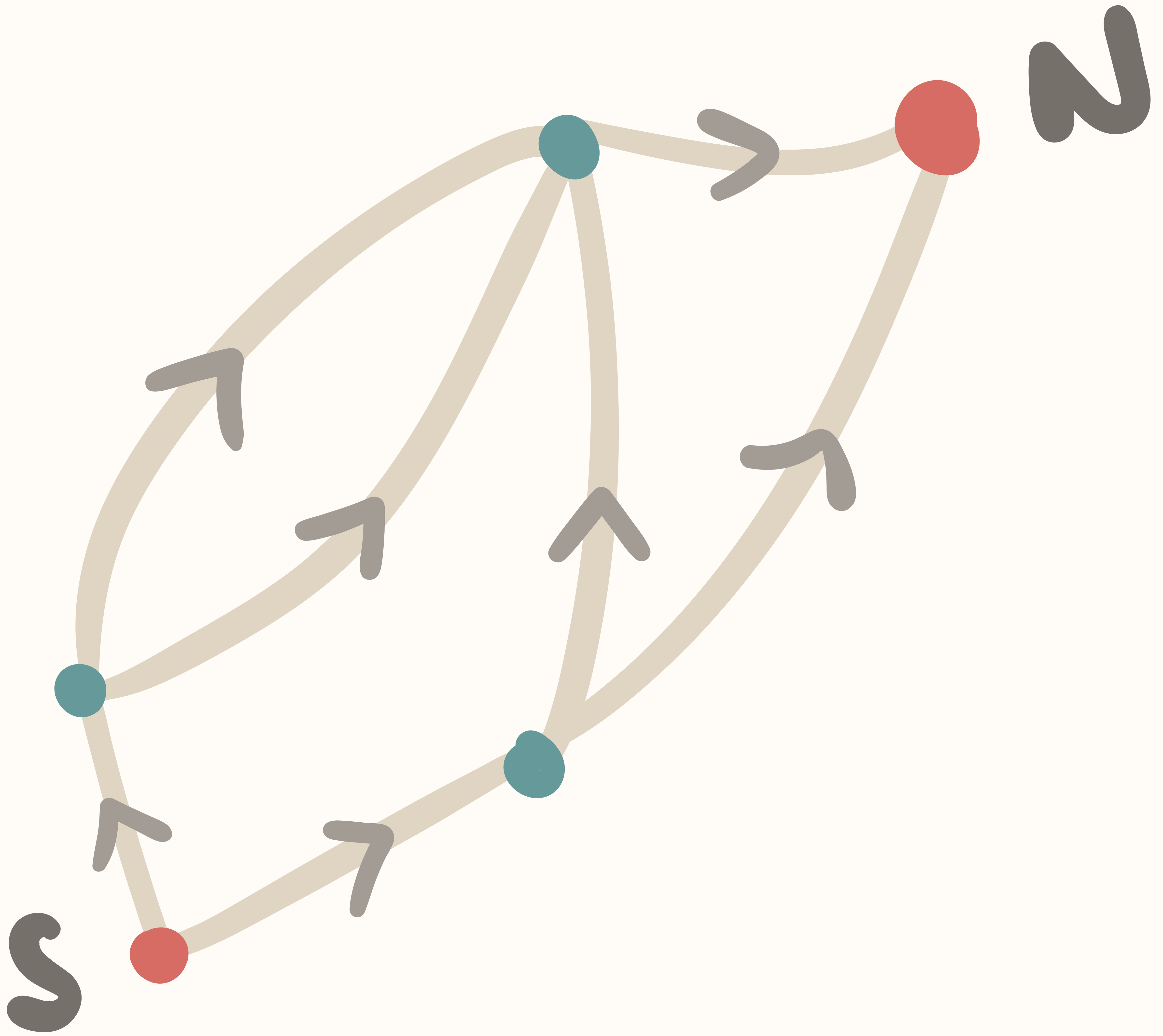
a. The KMSW bijection
b. Plane bipolar posets
c. Plane bipolar posets by vertices
d. Transversal structures

2. Asymptotic enumeration

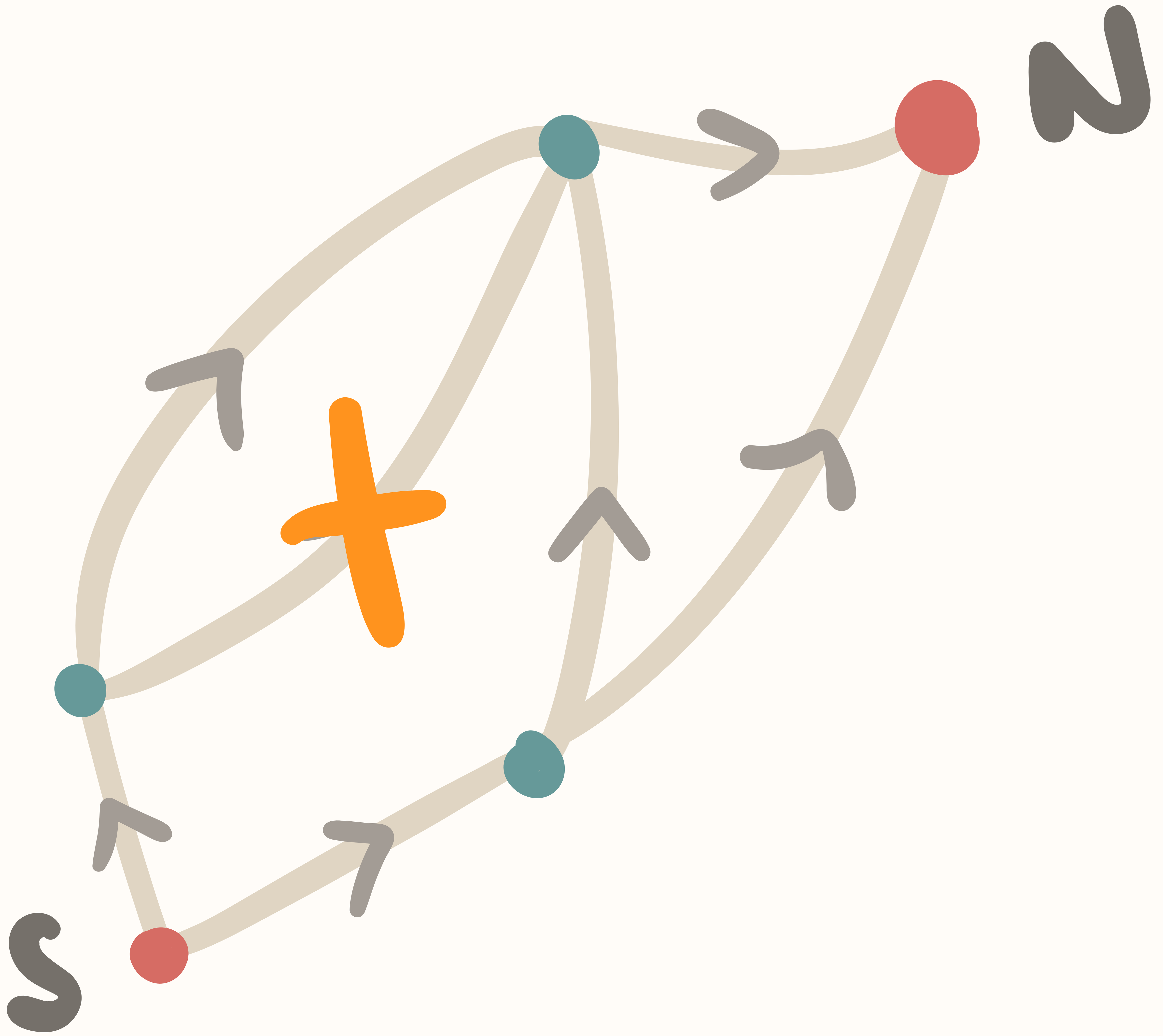
(Digression on plane permutations)

3. Generic transversal structures

Plane bipolar poset



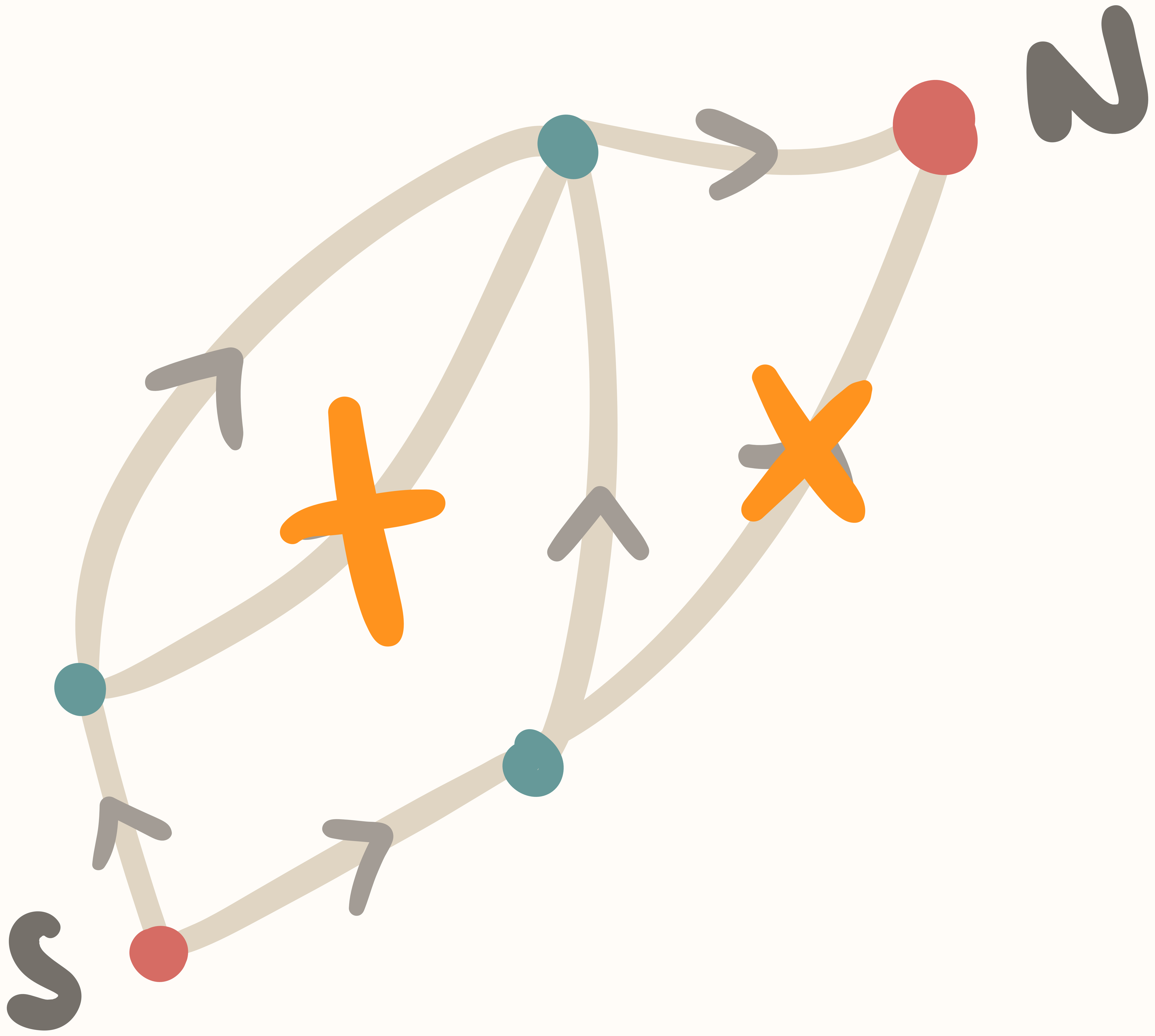
Plane bipolar poset



Poset
(plane bipolar poset)

= ***Bipolar orientation***
No multiple edge

Plane bipolar poset

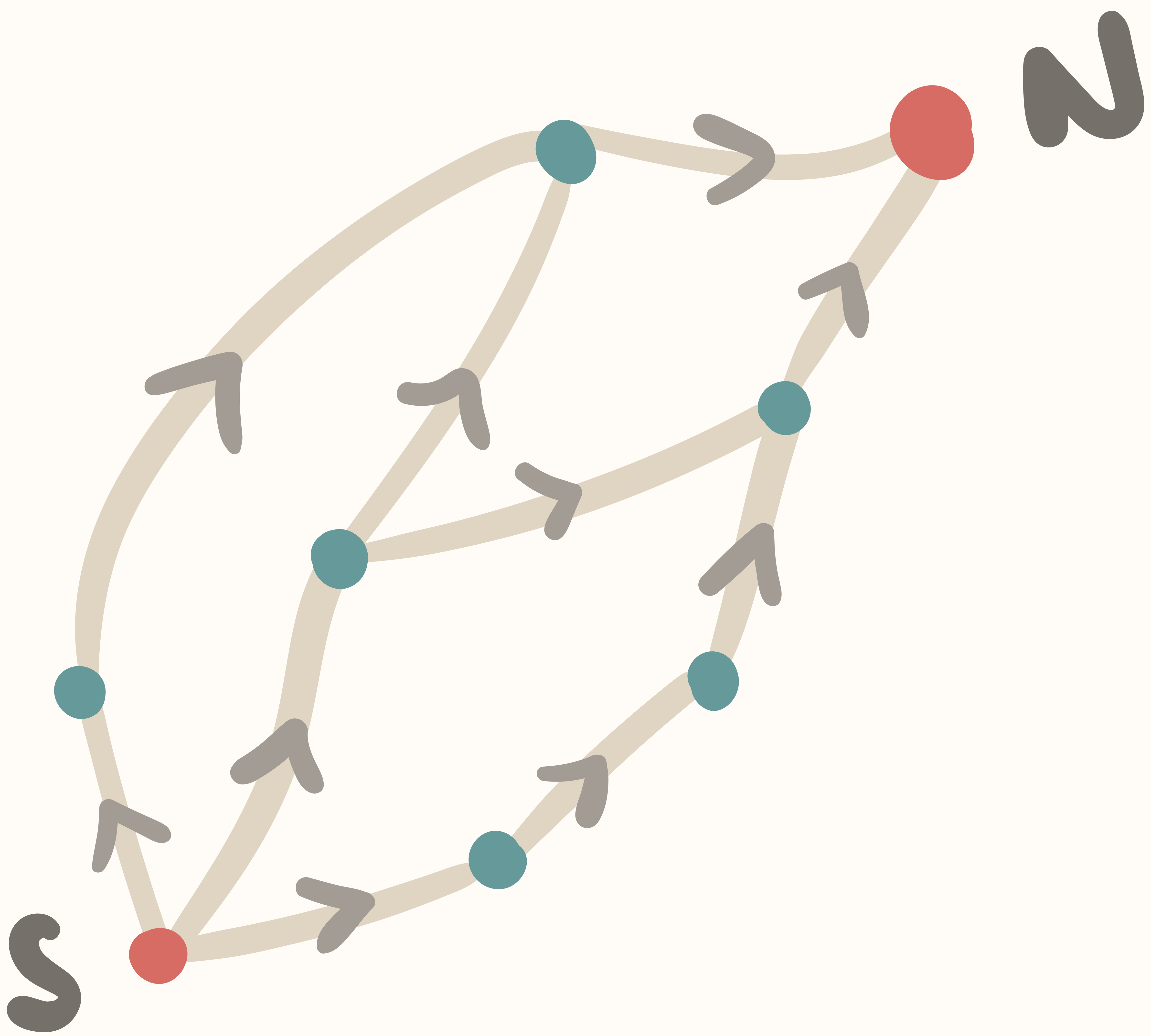


Poset
(plane bipolar poset)

= ***Bipolar orientation***

No multiple edge
No transitive edge

Plane bipolar poset

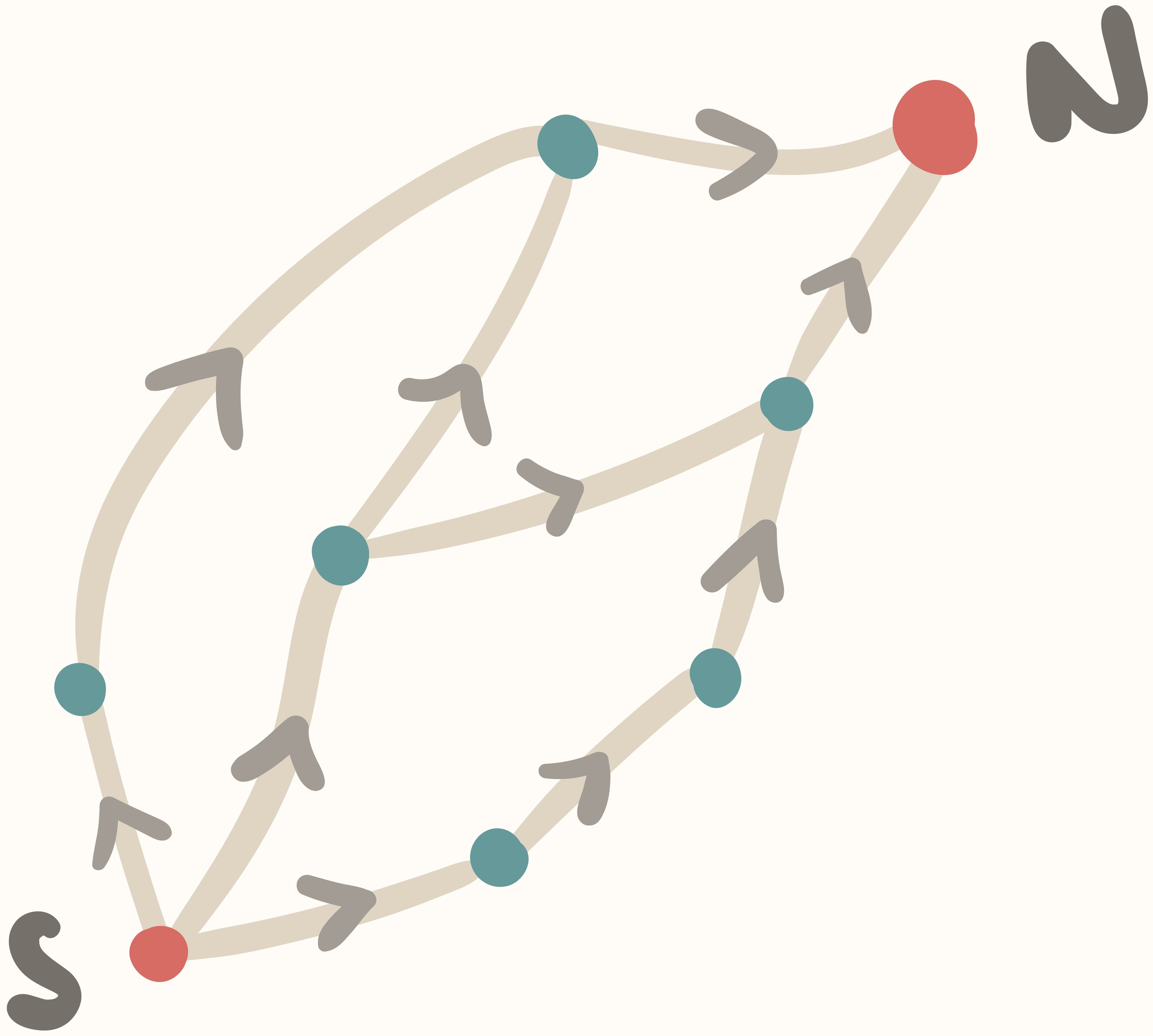


Poset
(plane bipolar poset)

= **Bipolar orientation**

No multiple edge
No transitive edge

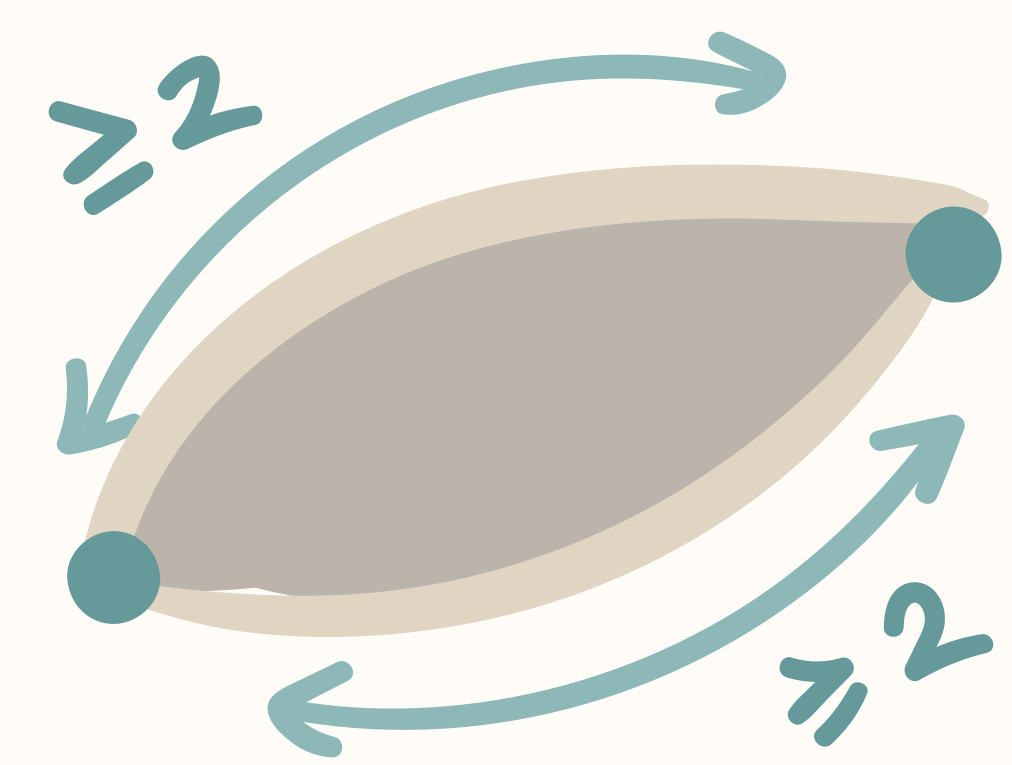
Plane bipolar poset



Poset
(plane bipolar poset)

= ***Bipolar orientation***
No multiple edge
No transitive edge

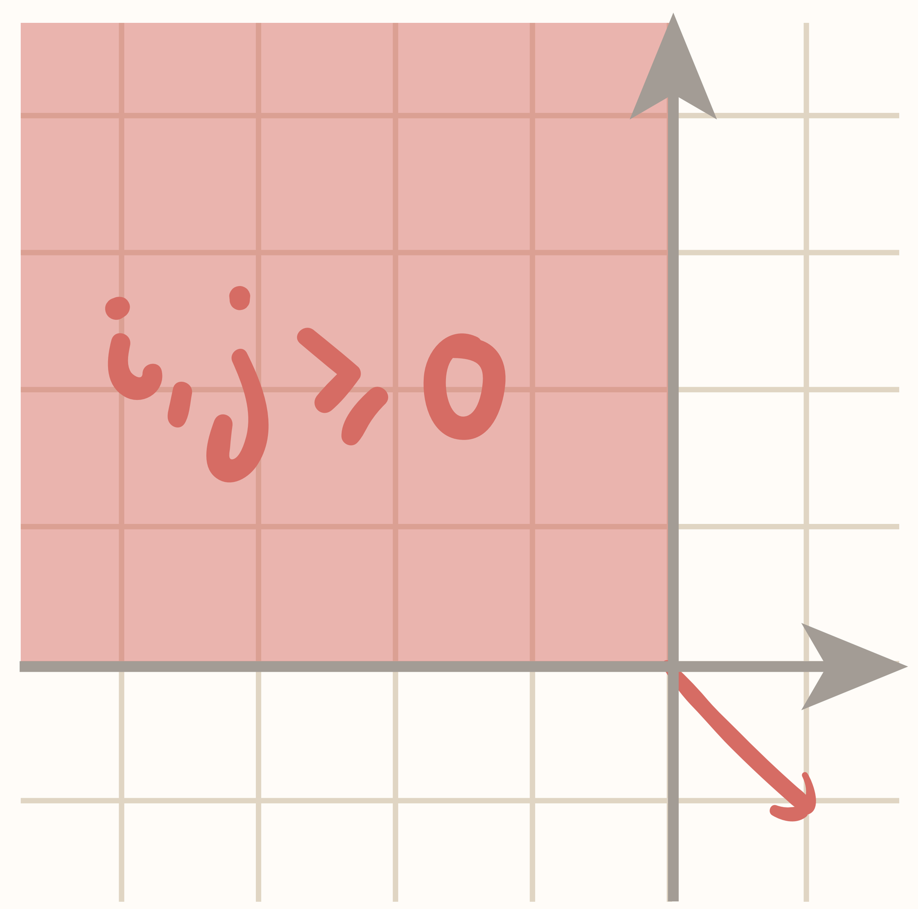
= ***Bipolar orientation***



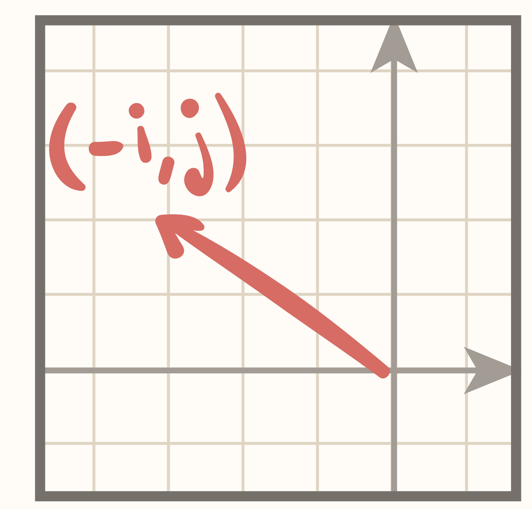
Specialization to Posets

Specialization to Posets

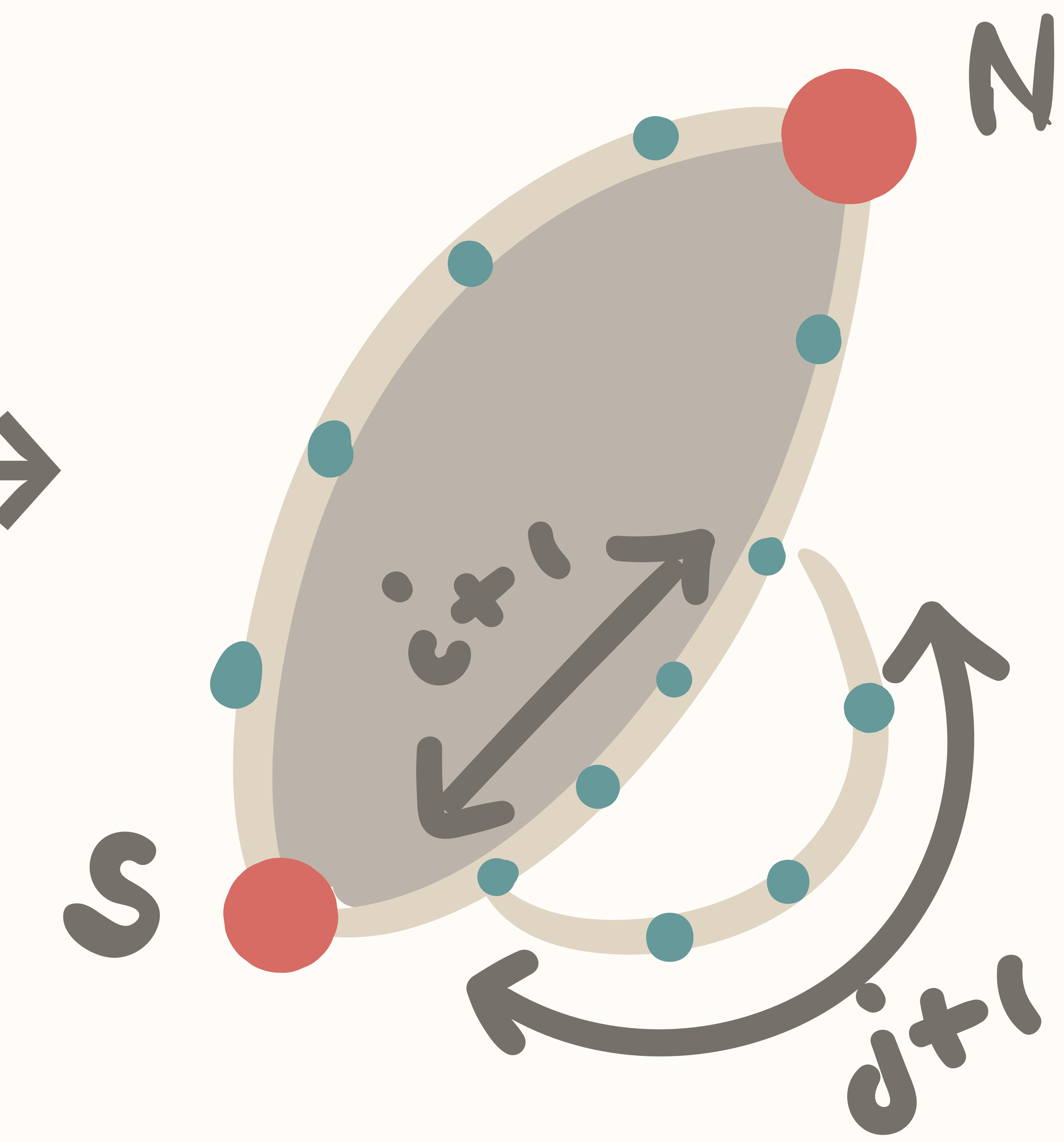
Bipolar orientation =



where

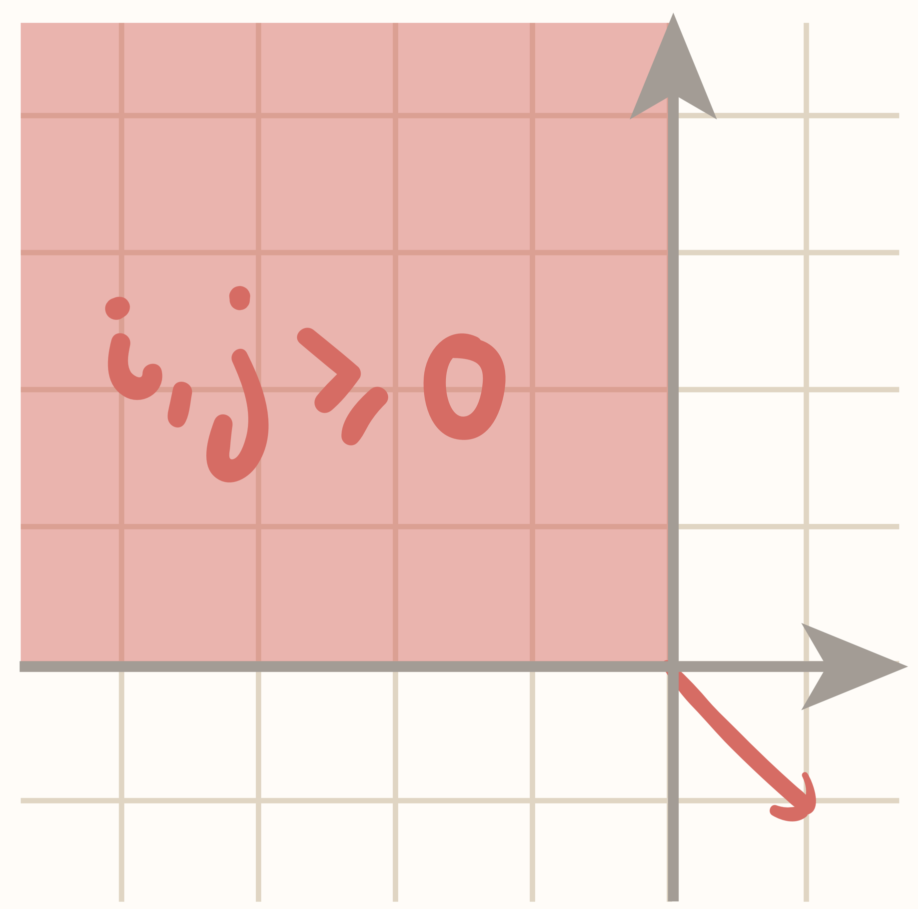


→

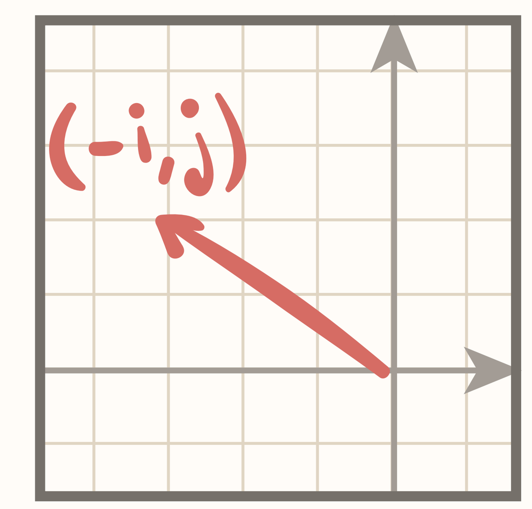


Specialization to Posets

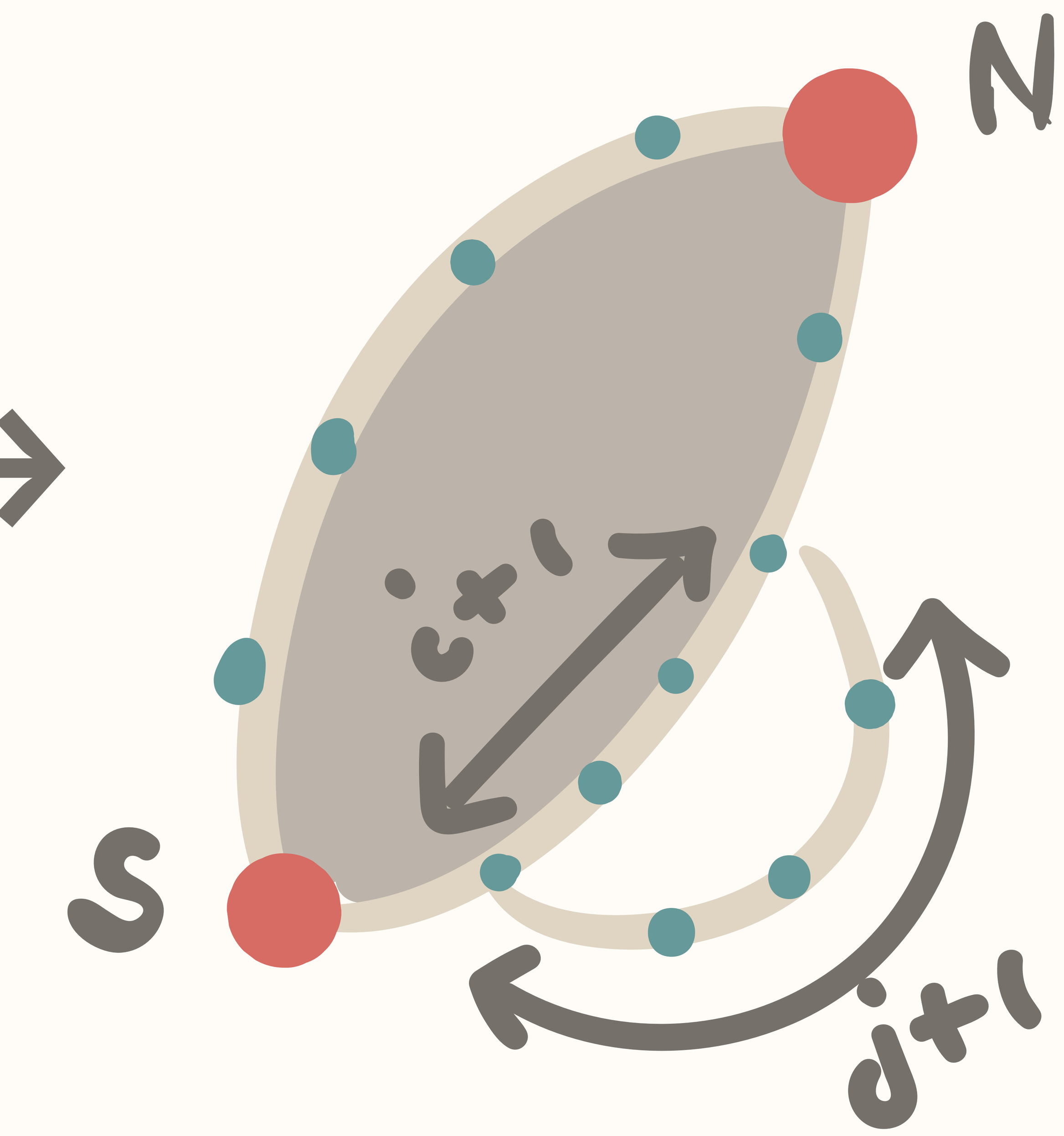
Bipolar orientation =



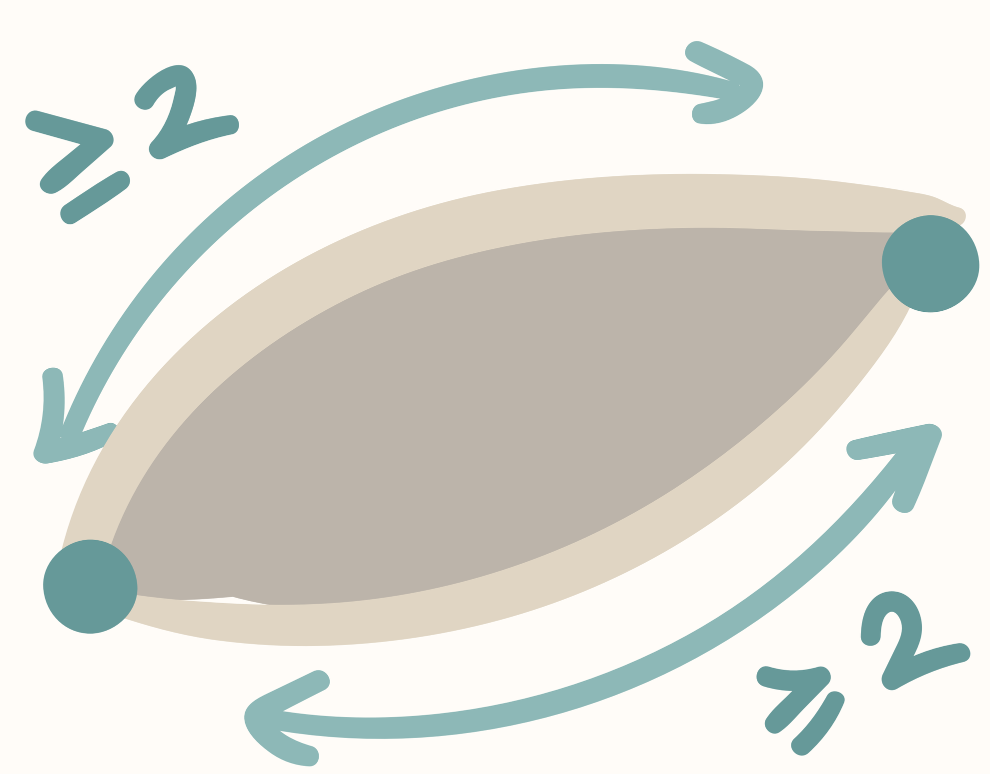
where



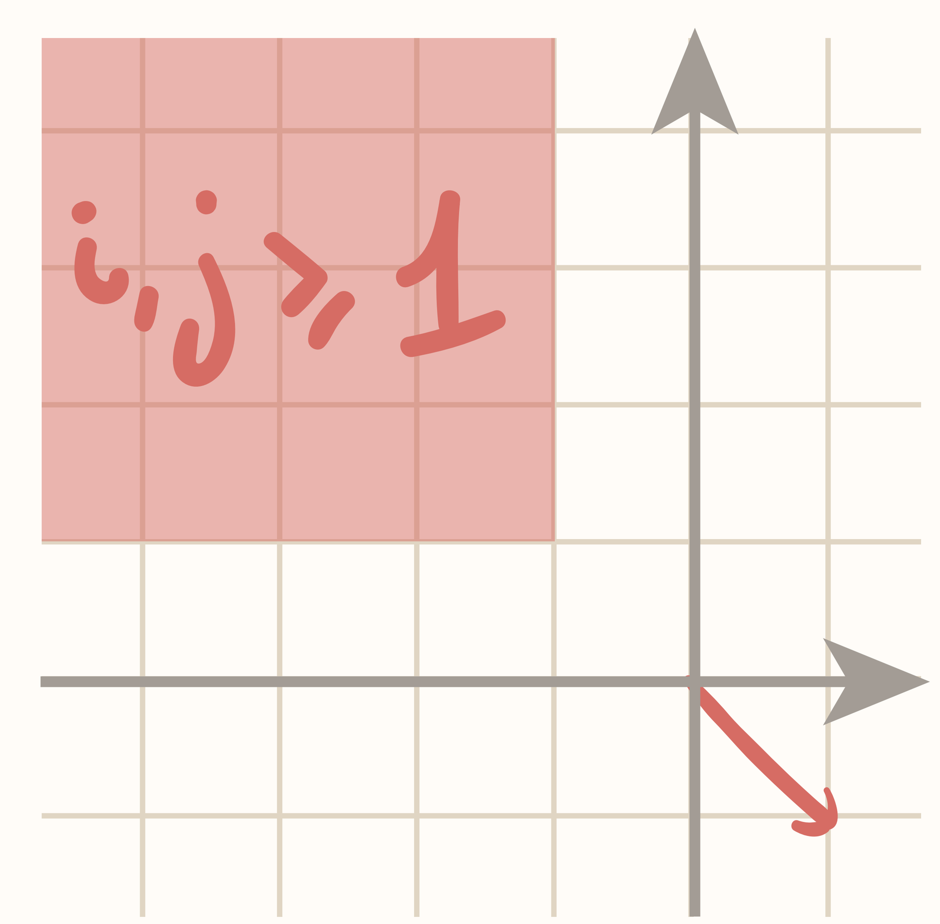
→



Poset =



=



Summary

Maps and decorated maps

1. Bijection with quadrant tandem walks

a. The KMSW bijection
b. Plane bipolar posets
c. Plane bipolar posets by vertices
d. Transversal structures

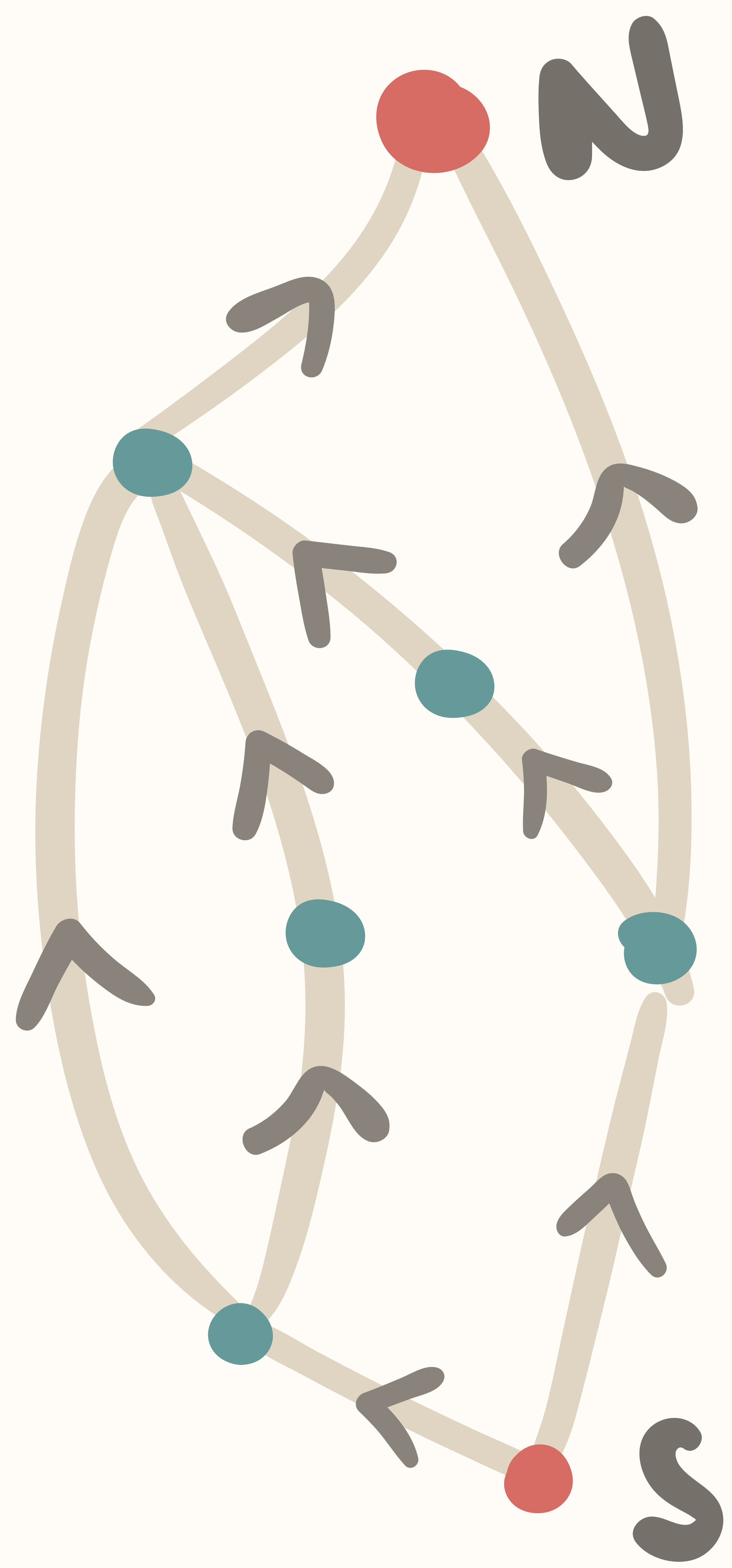
2. Asymptotic enumeration

(Digression on plane permutations)

3. Generic transversal structures

Specialization to Posets by vertices

Bipolar orientation

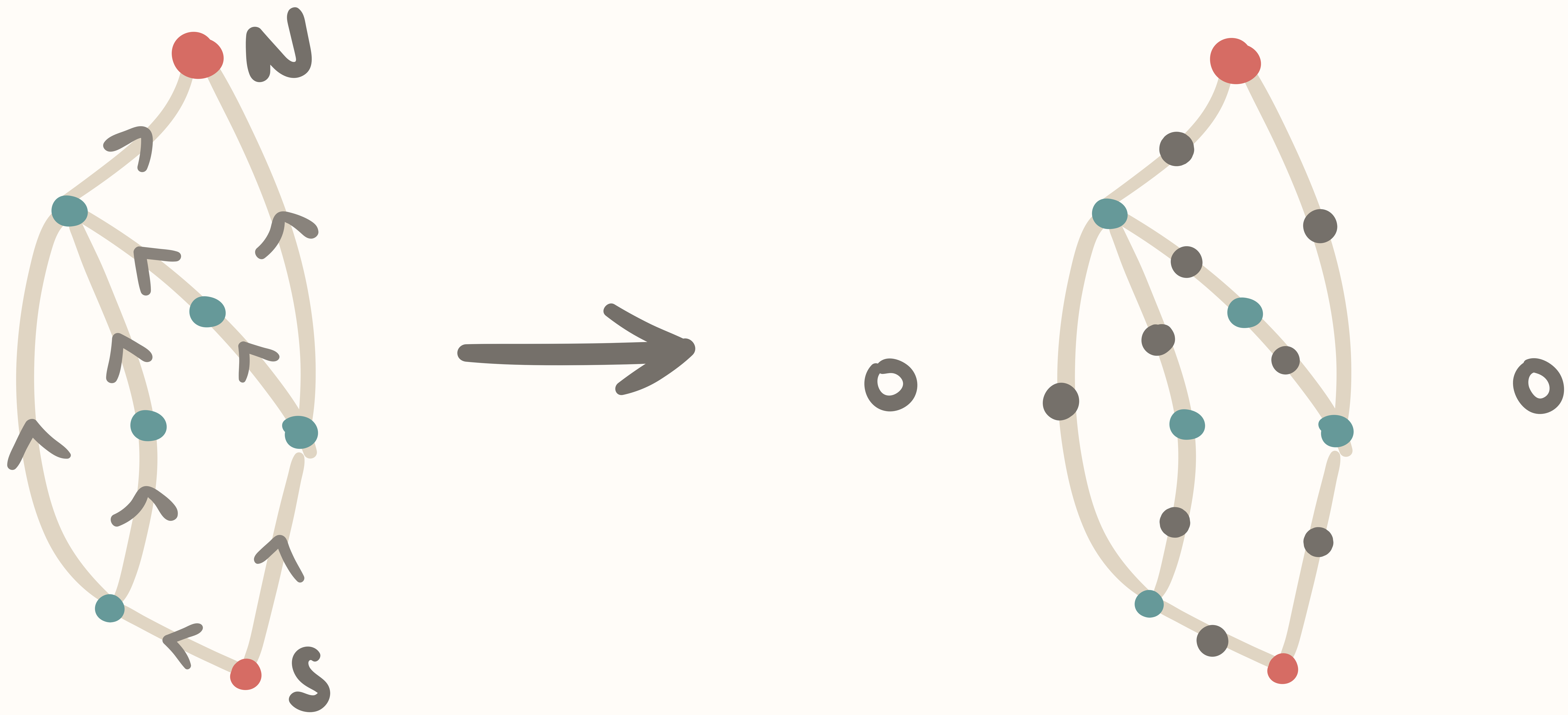


Specialization to Posets by vertices

Bipolar orientation

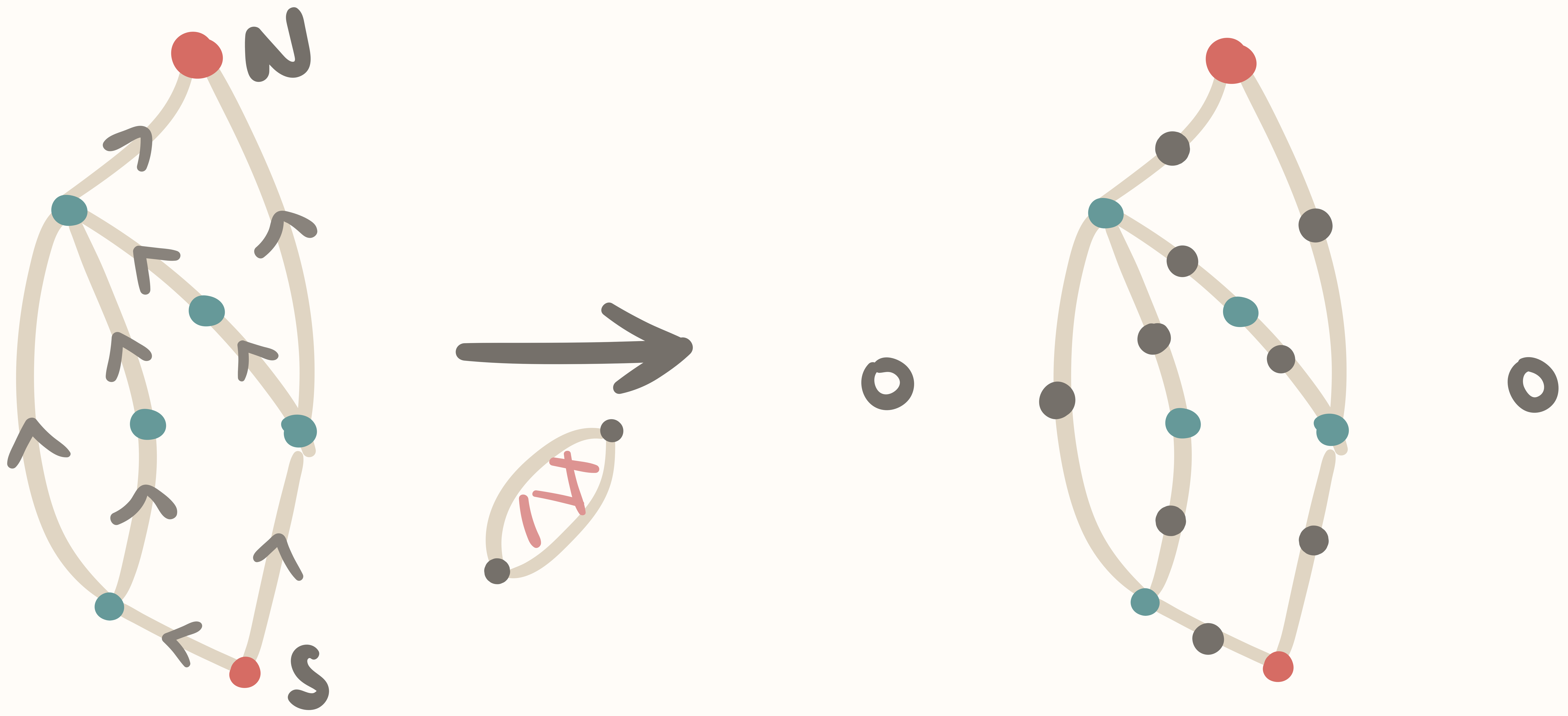
Specialization to Posets by vertices

Bipolar orientation



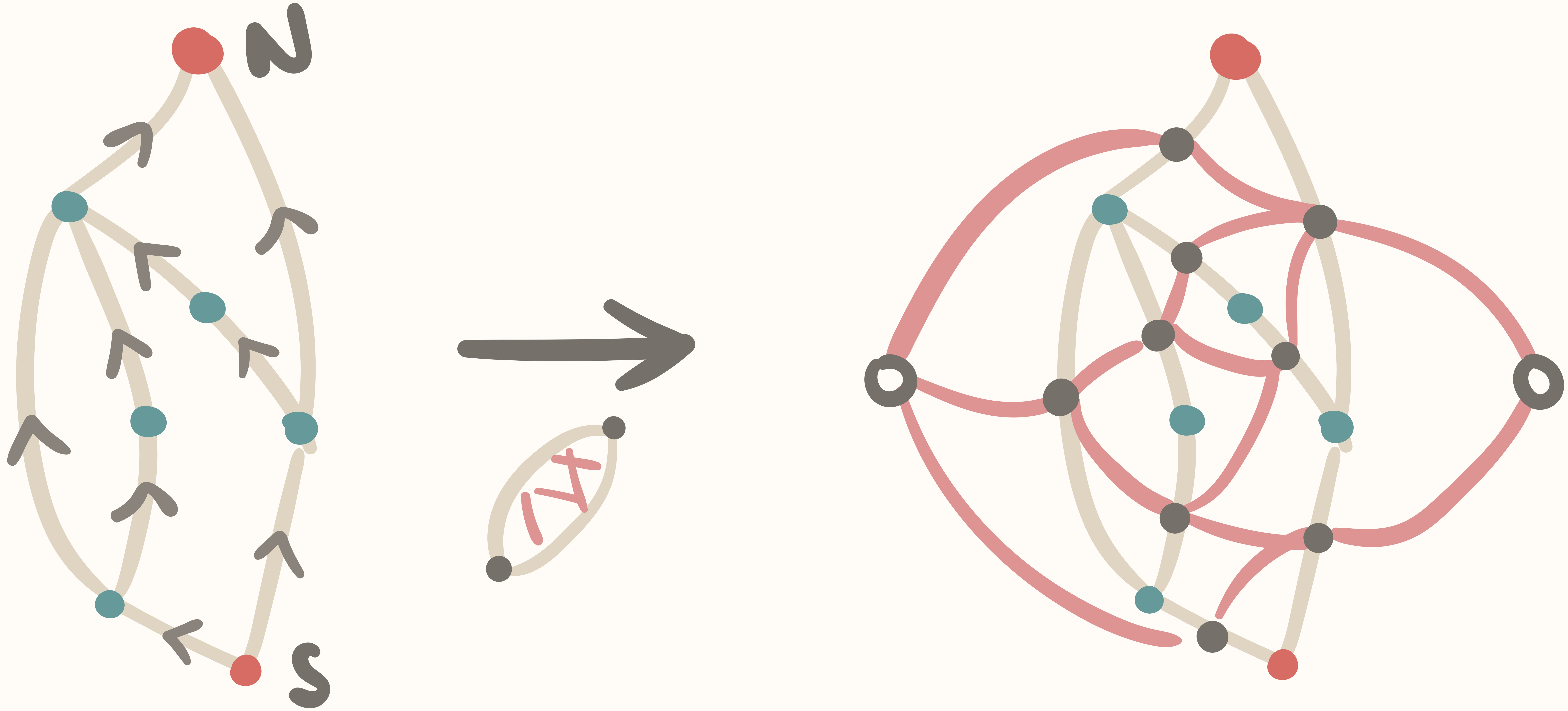
Specialization to Posets by vertices

Bipolar orientation

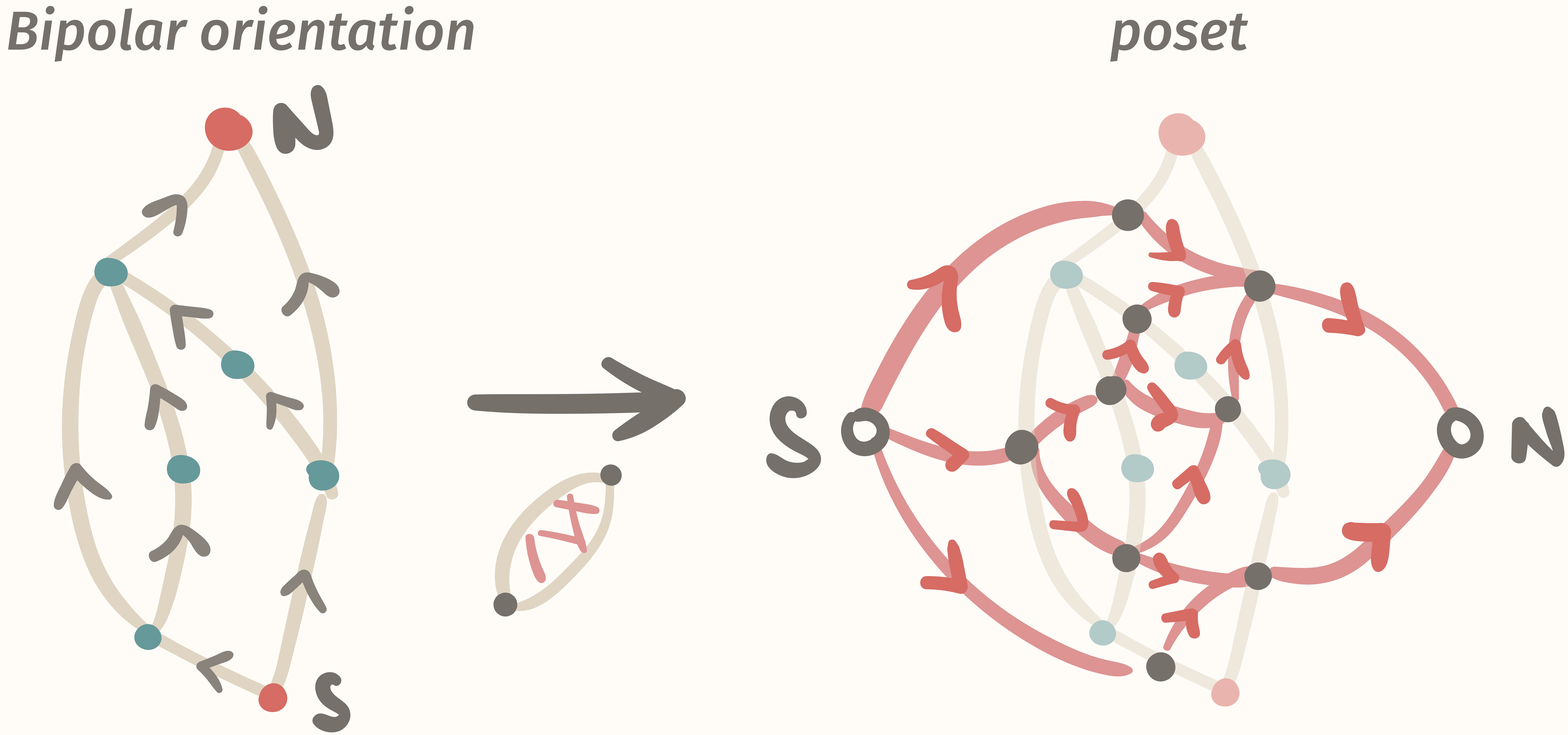


Specialization to Posets by vertices

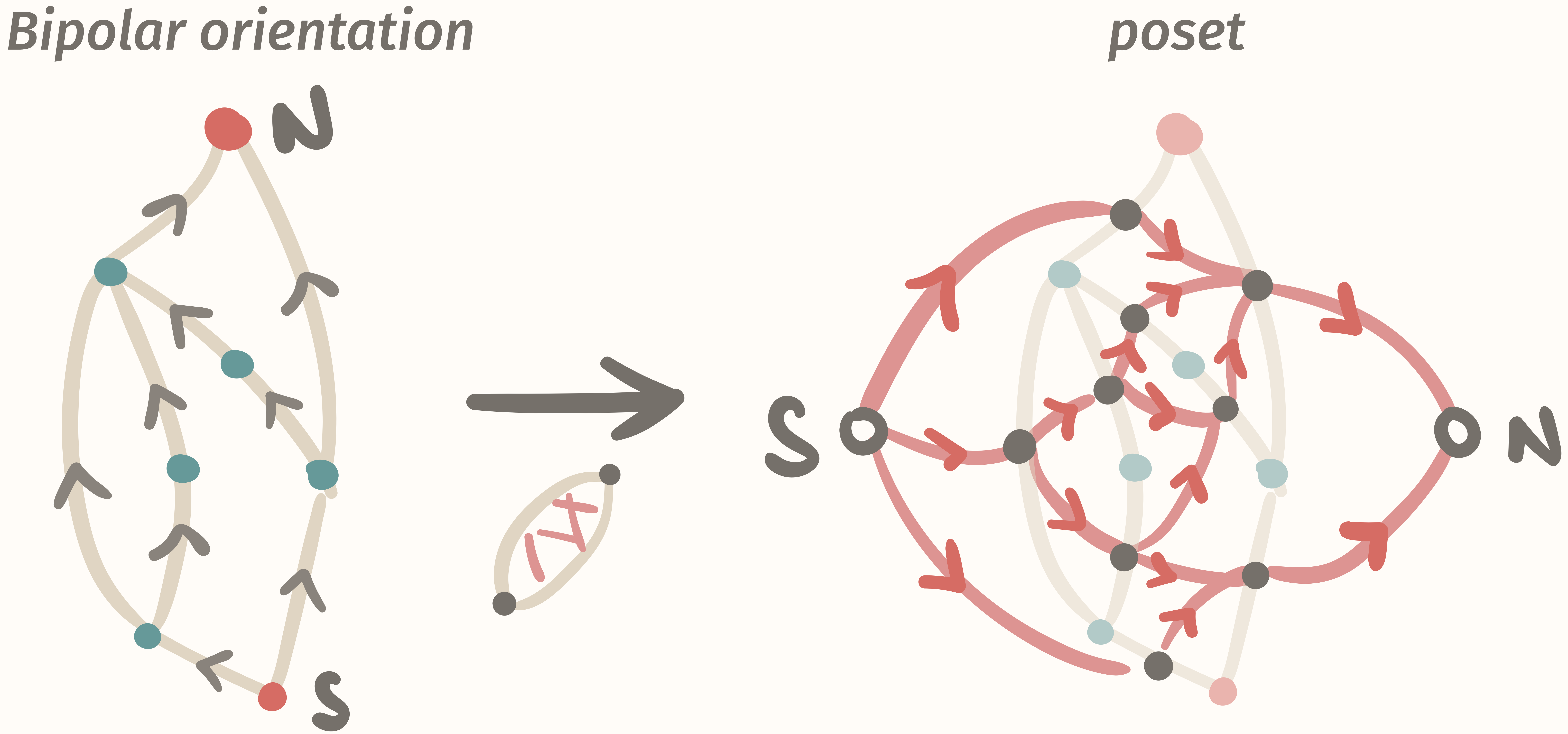
Bipolar orientation

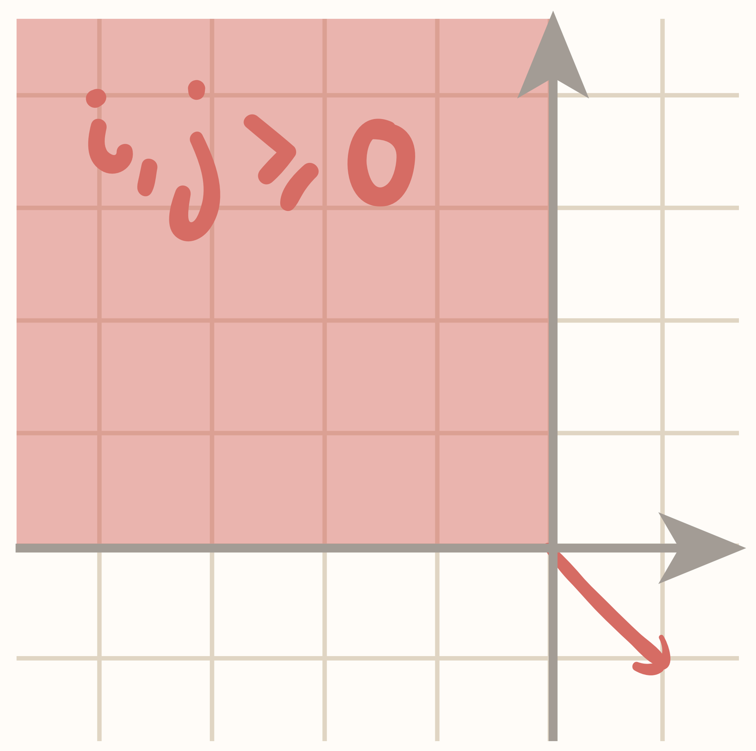


Specialization to Posets by vertices



Specialization to Posets by vertices

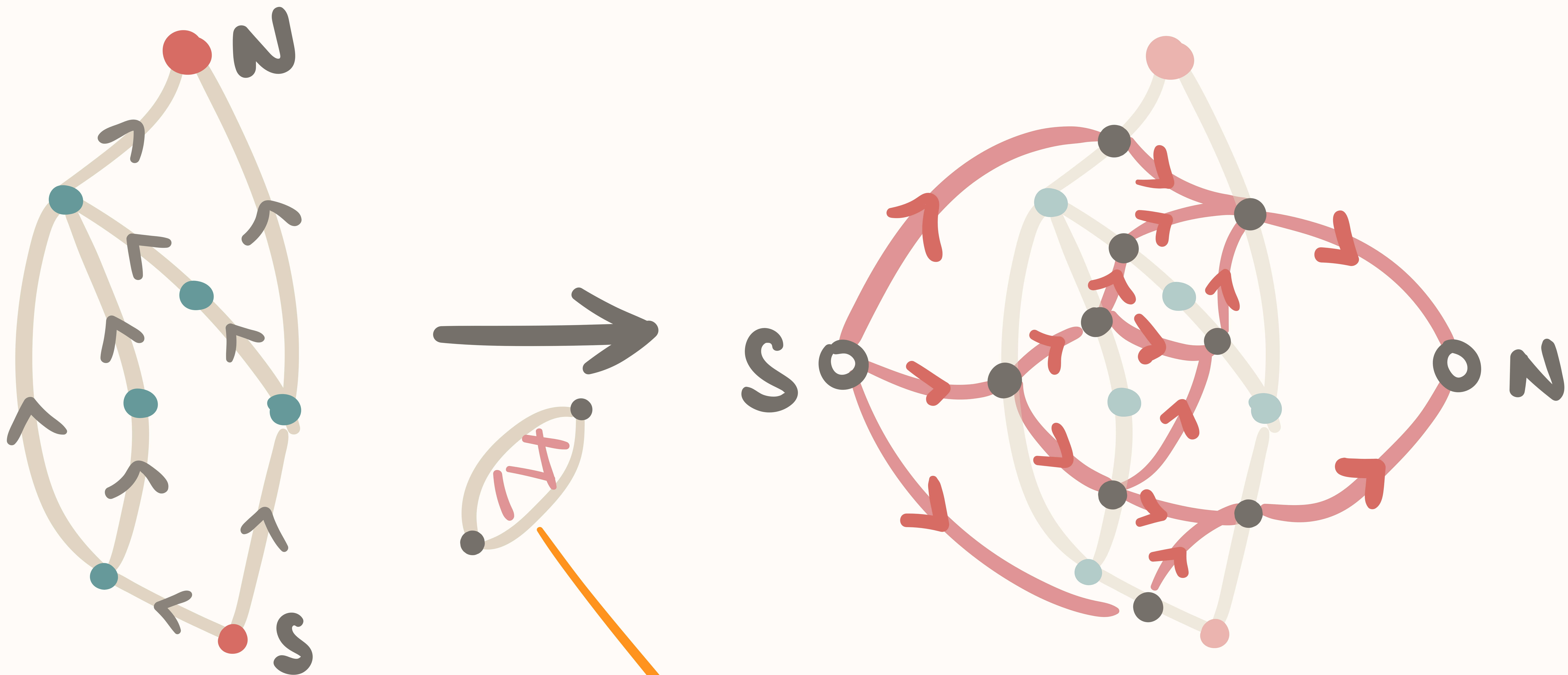


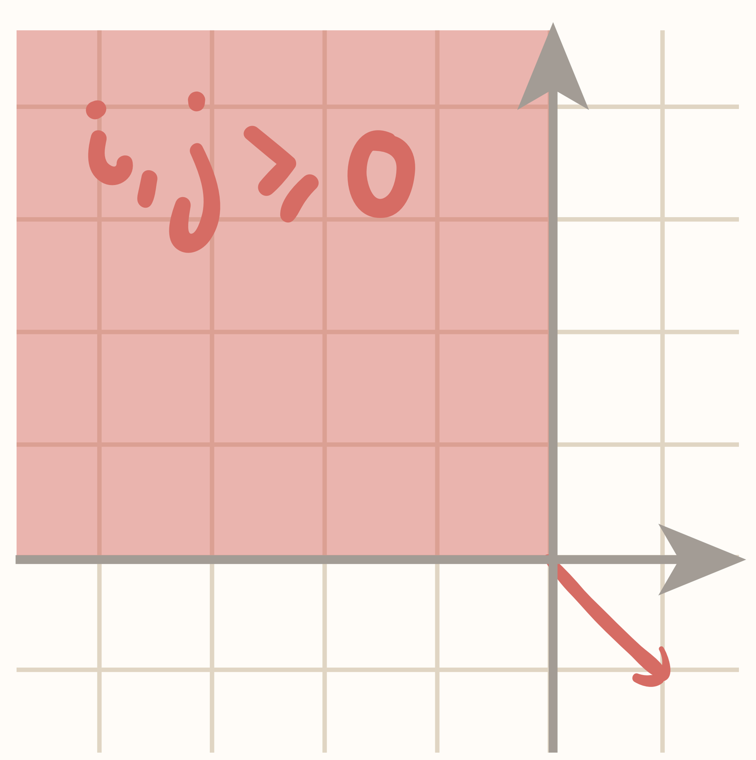


Specialization to Posets by vertices

Bipolar orientation

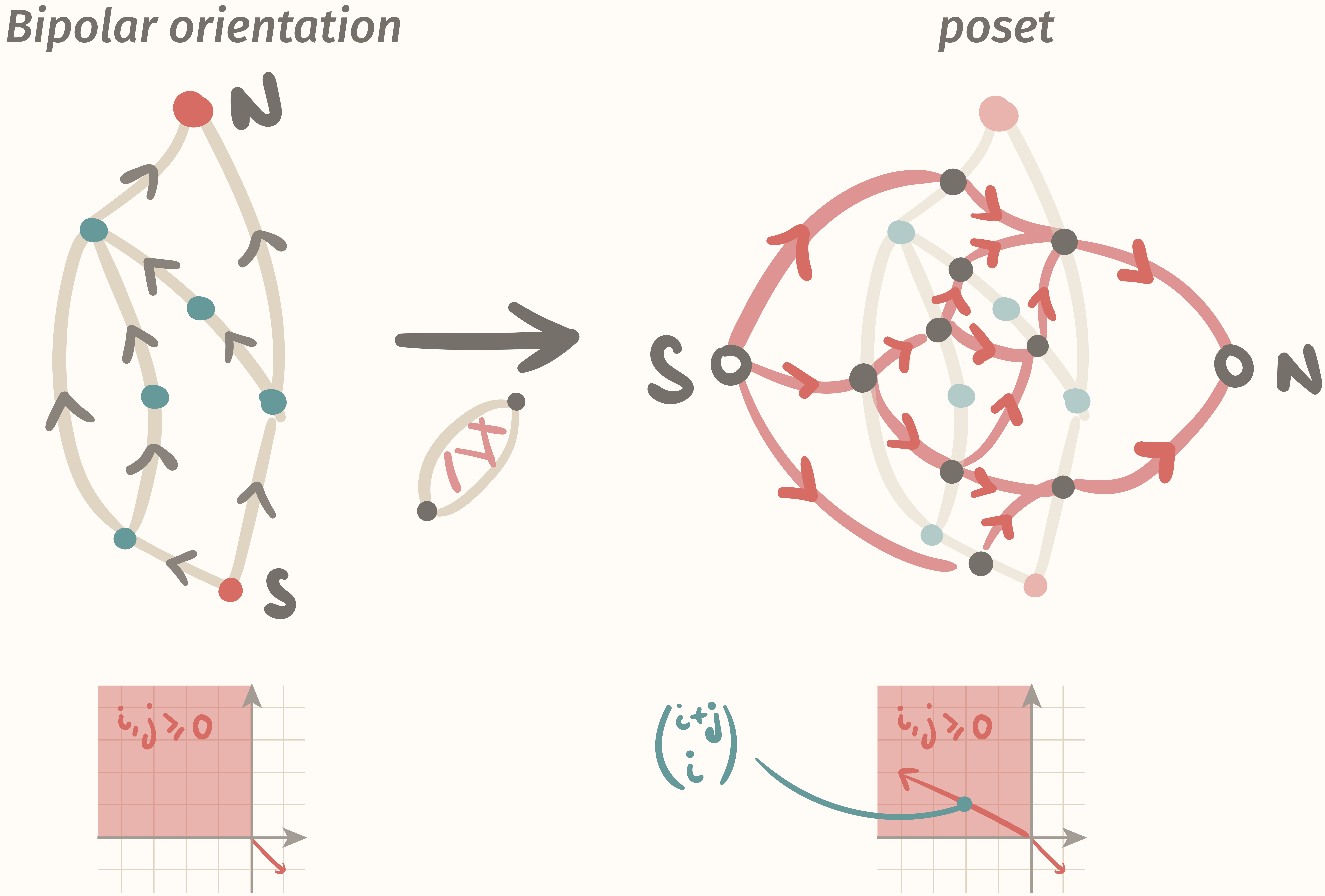
poset





$\binom{i+j}{i}$

Specialization to Posets by vertices



Summary

Maps and decorated maps

1. Bijection with quadrant tandem walks

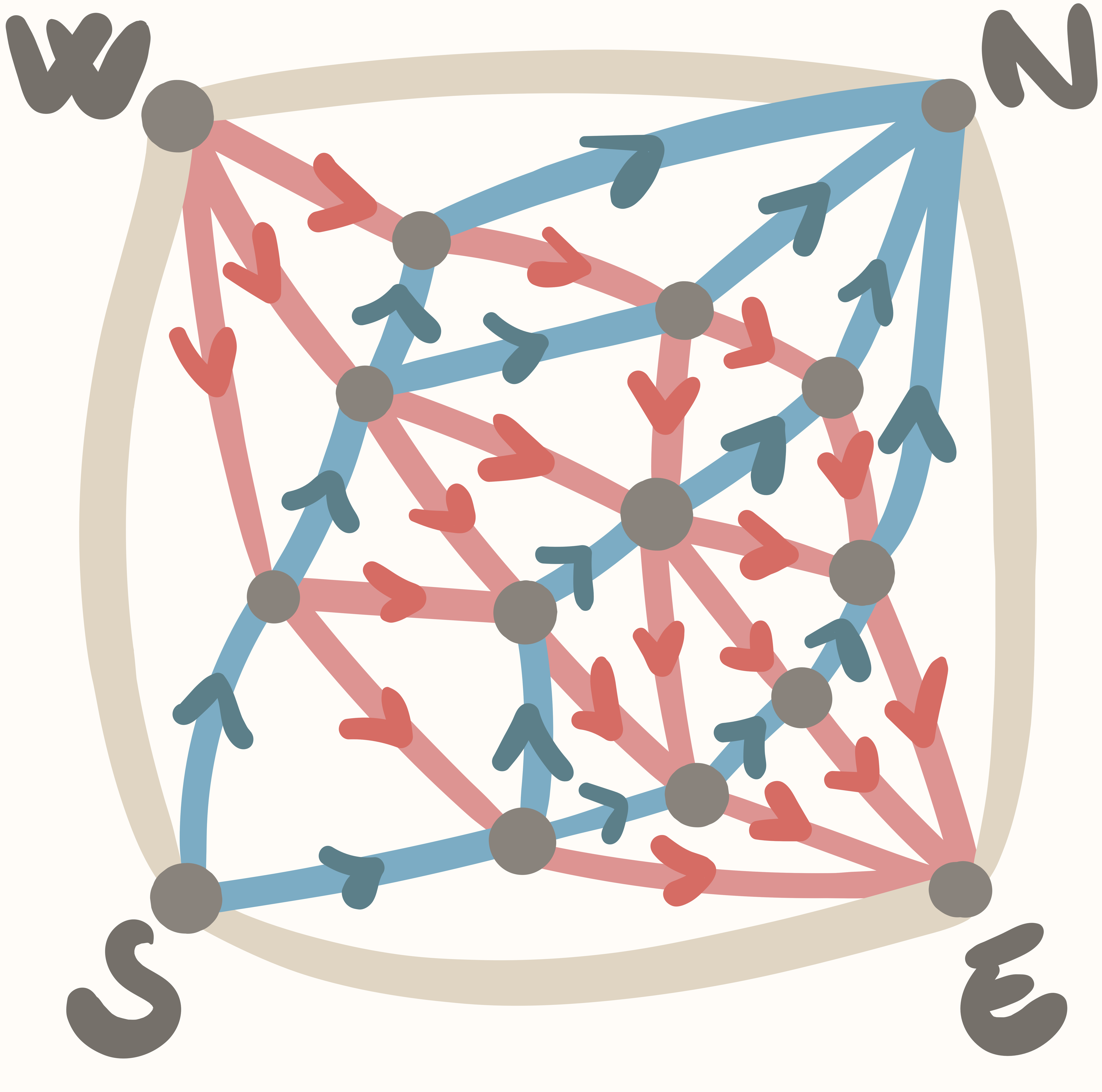
a. The KMSW bijection
b. Plane bipolar posets
c. Plane bipolar posets by vertices
d. Transversal structures

2. Asymptotic enumeration

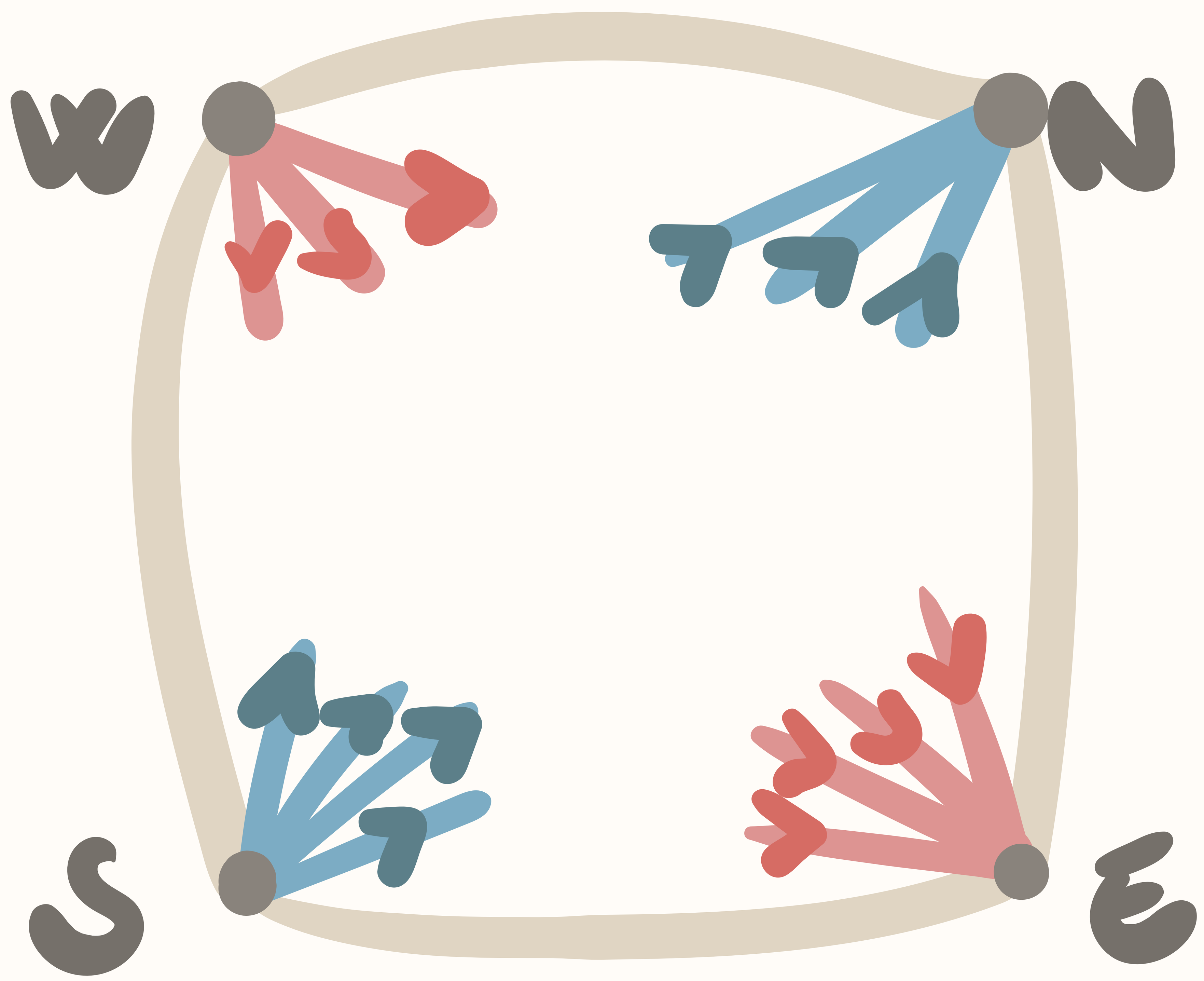
(Digression on plane permutations)

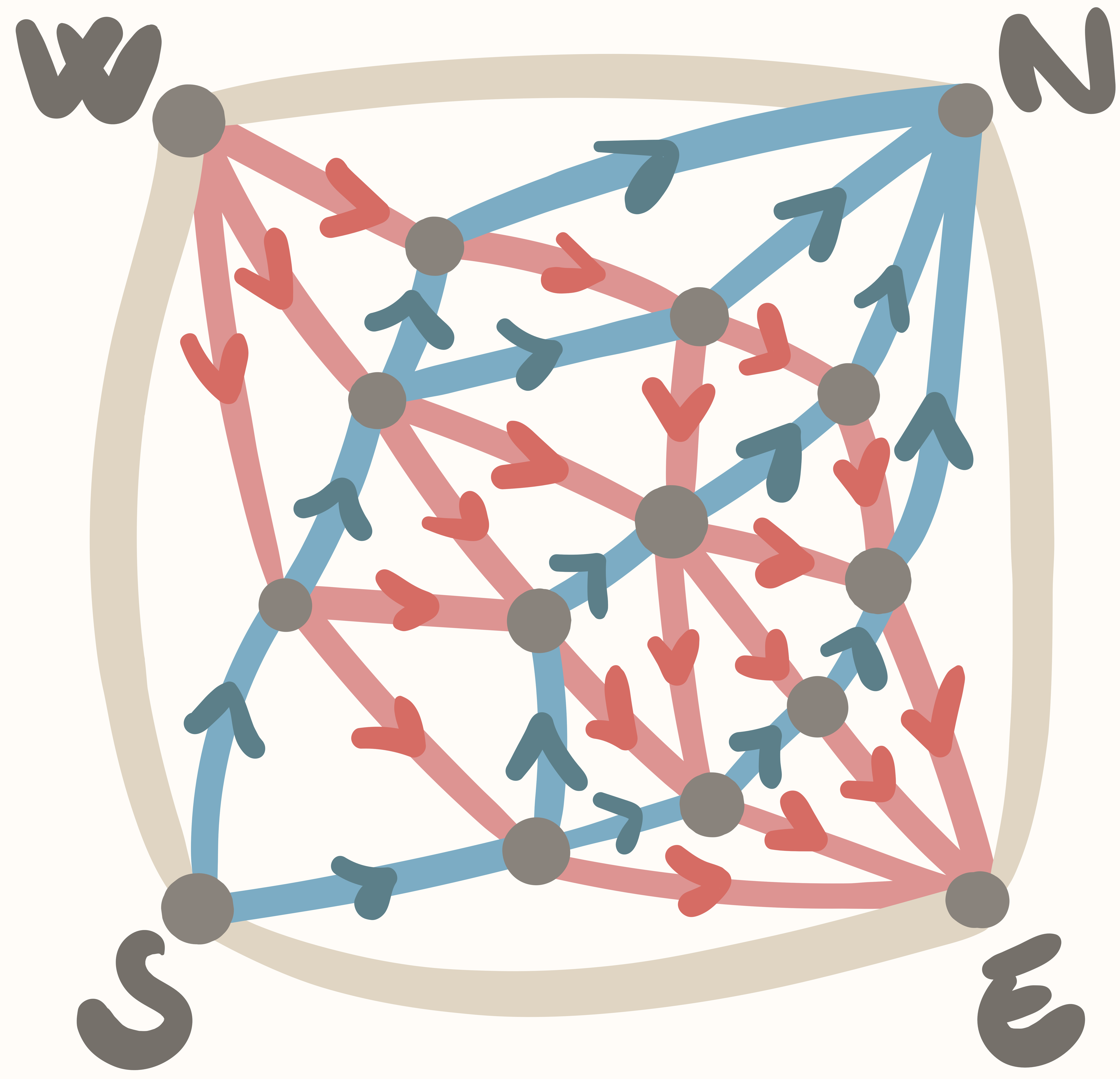
3. Generic transversal structures

Transversal structures

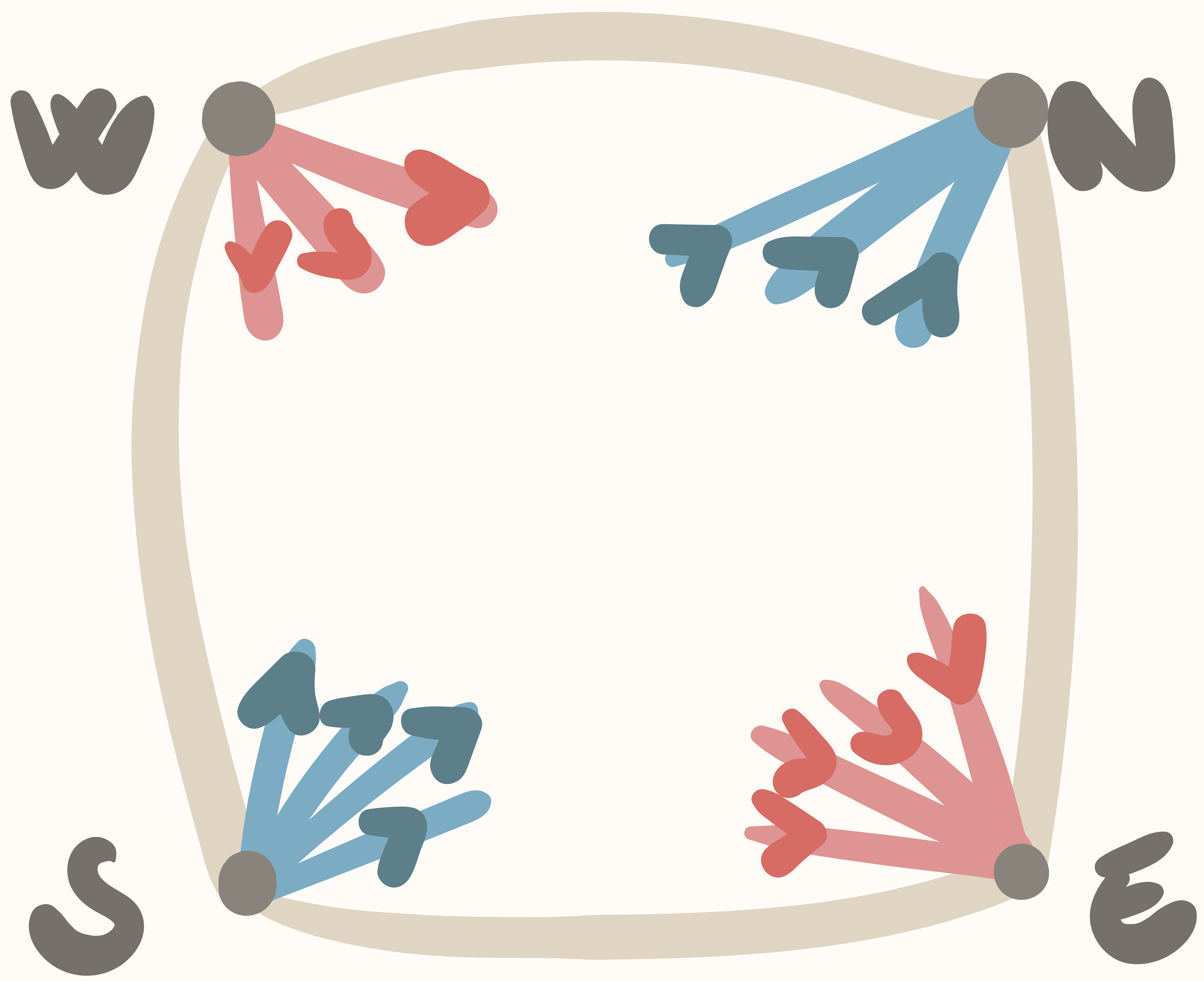


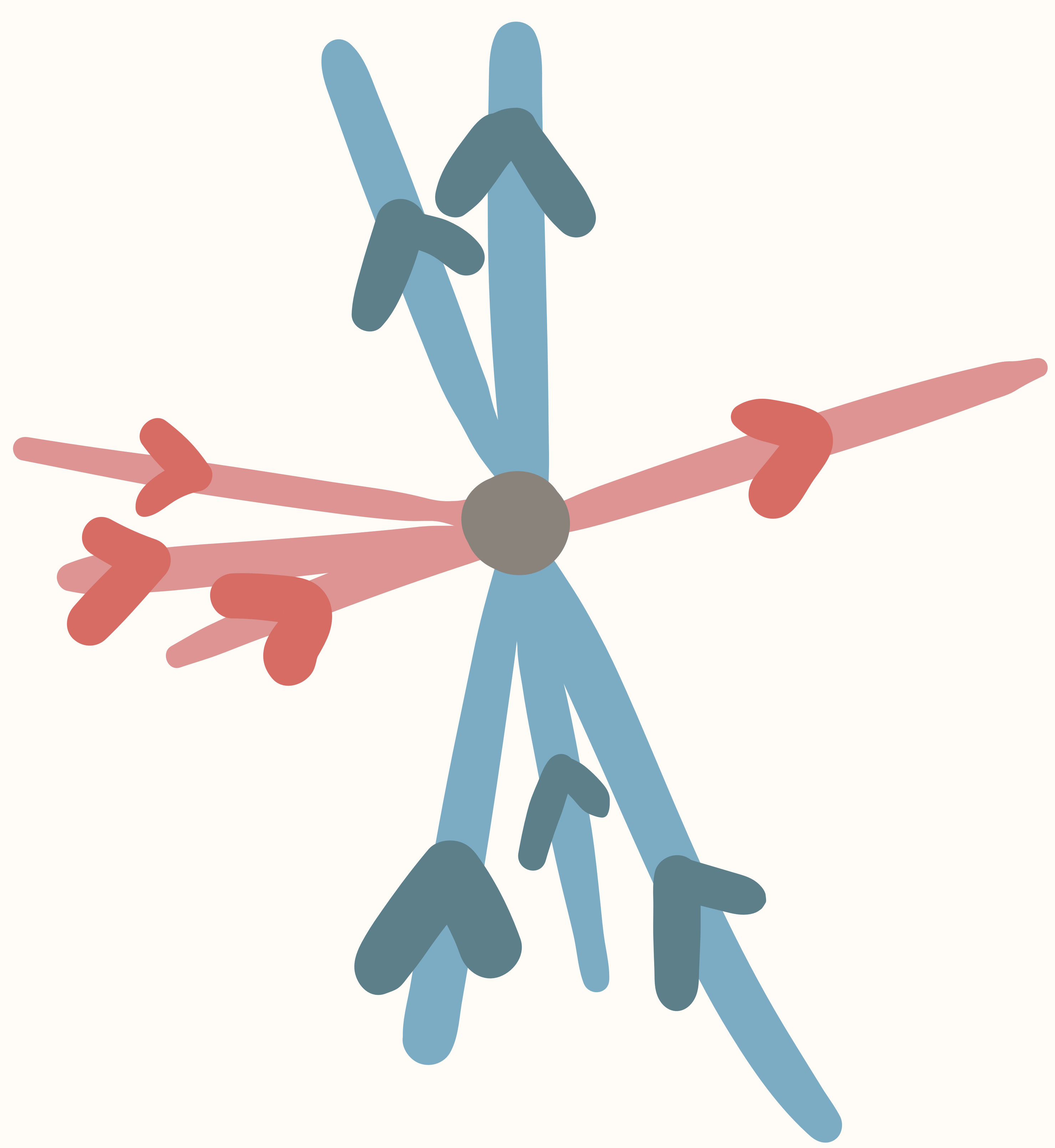
Transversal structures

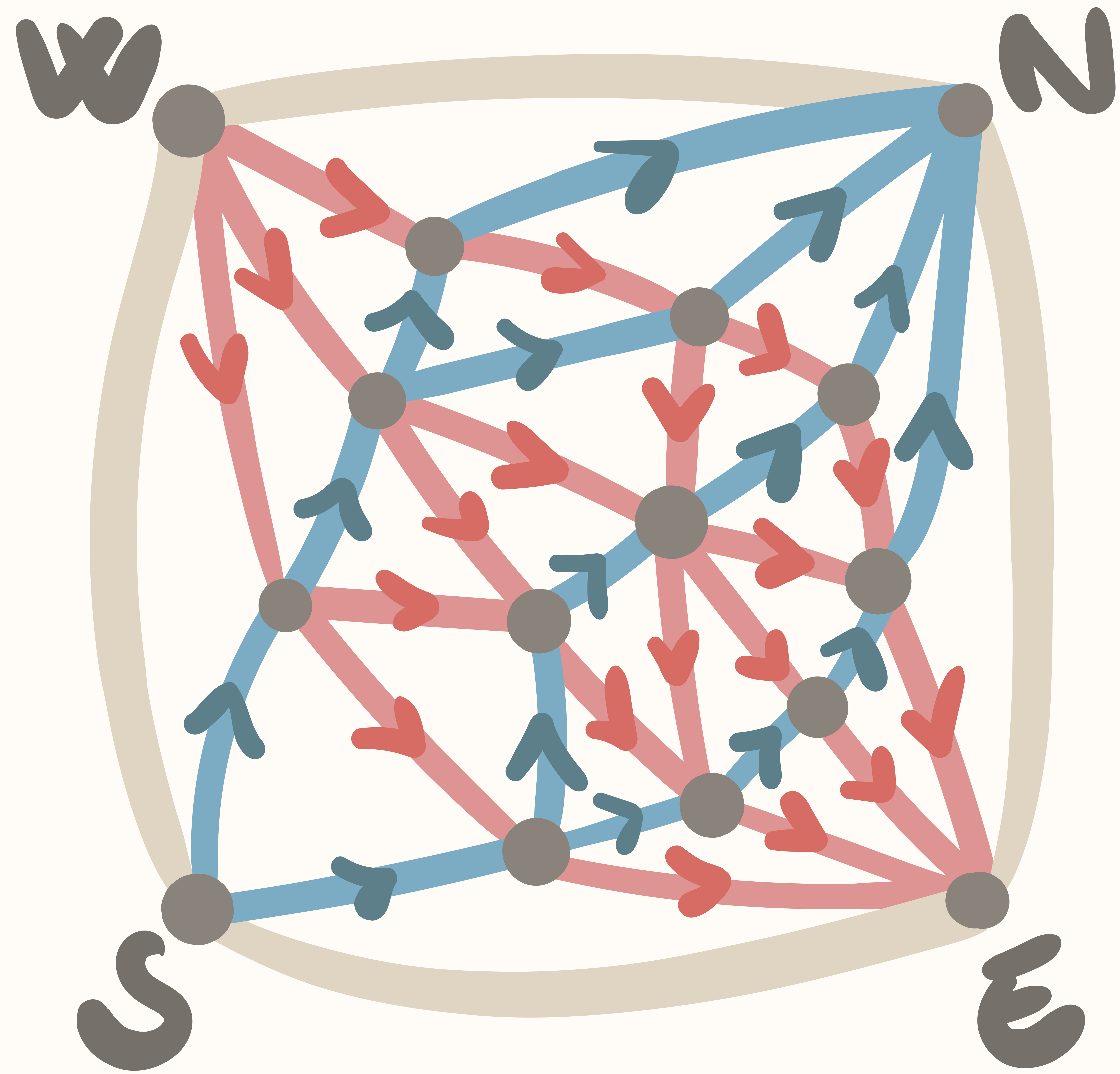




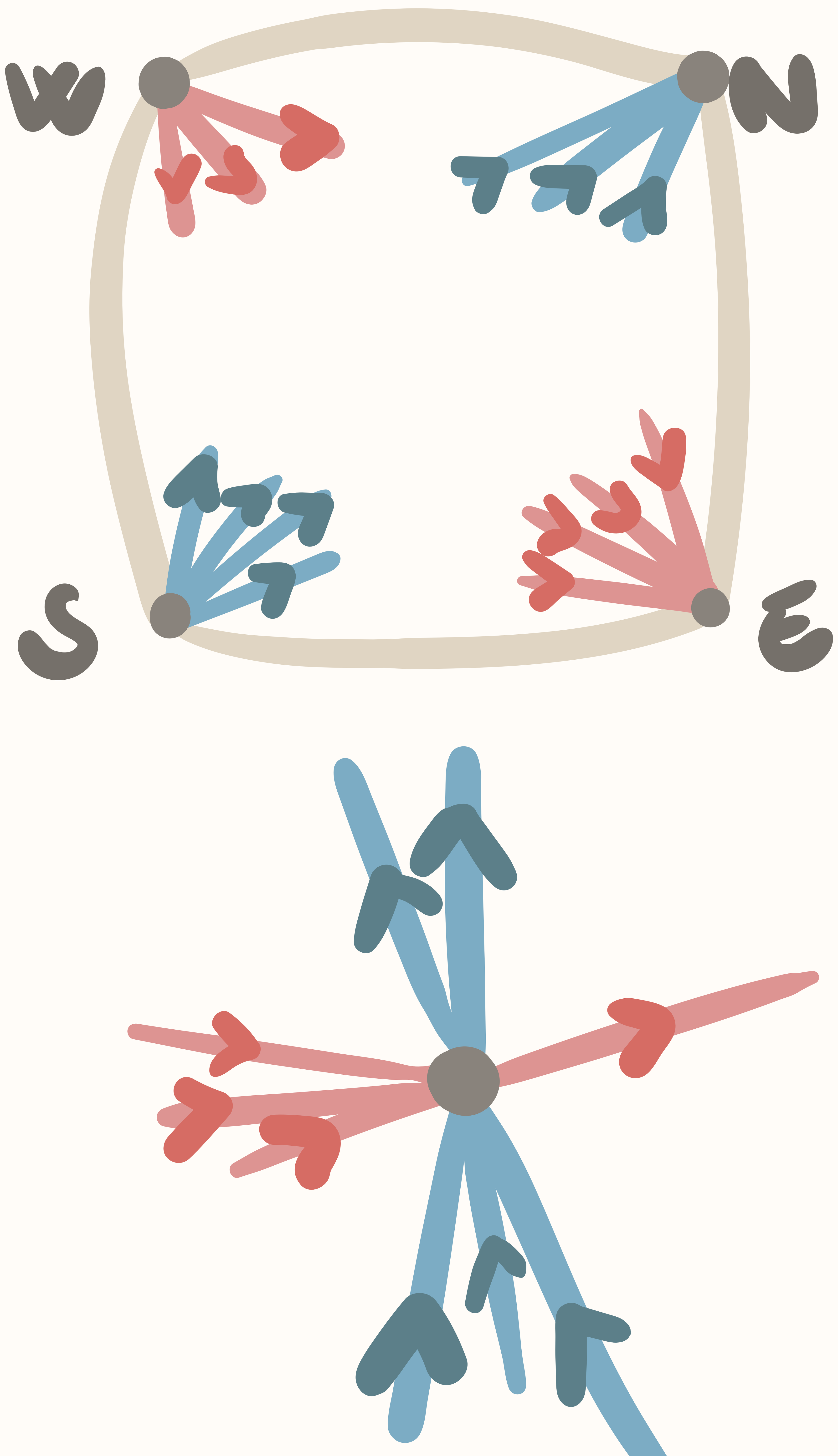
Transversal structures

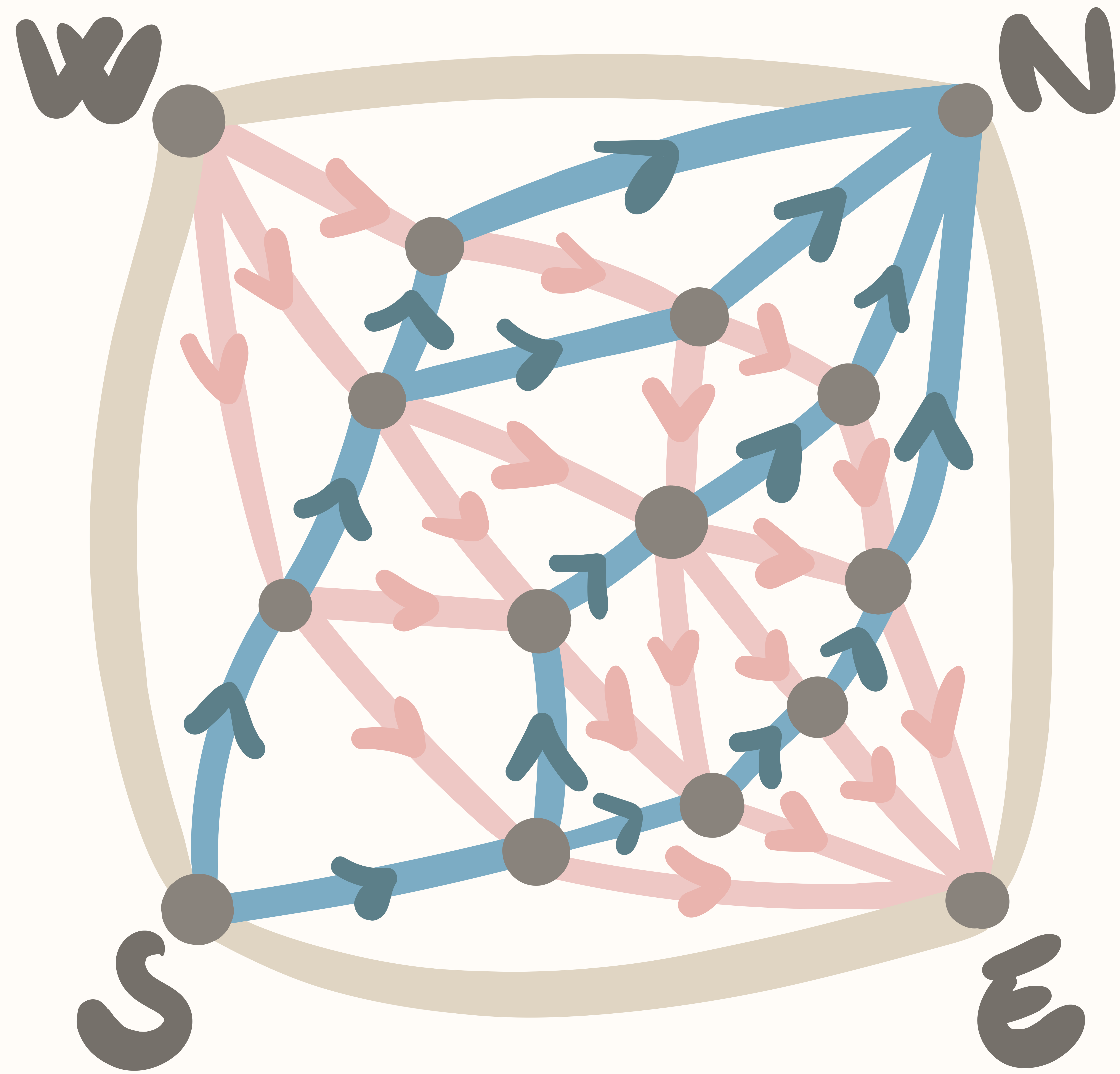




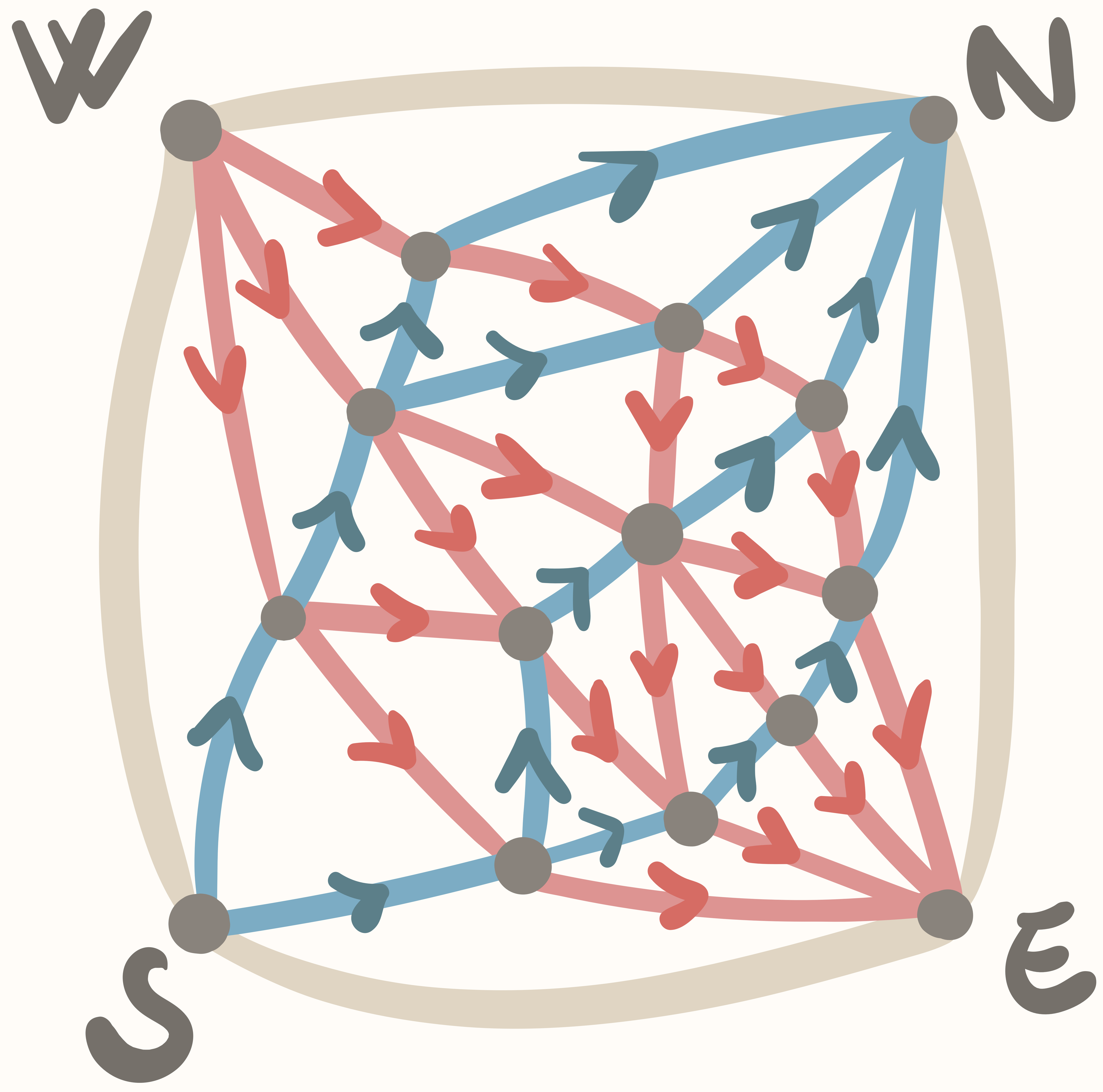


Transversal structures

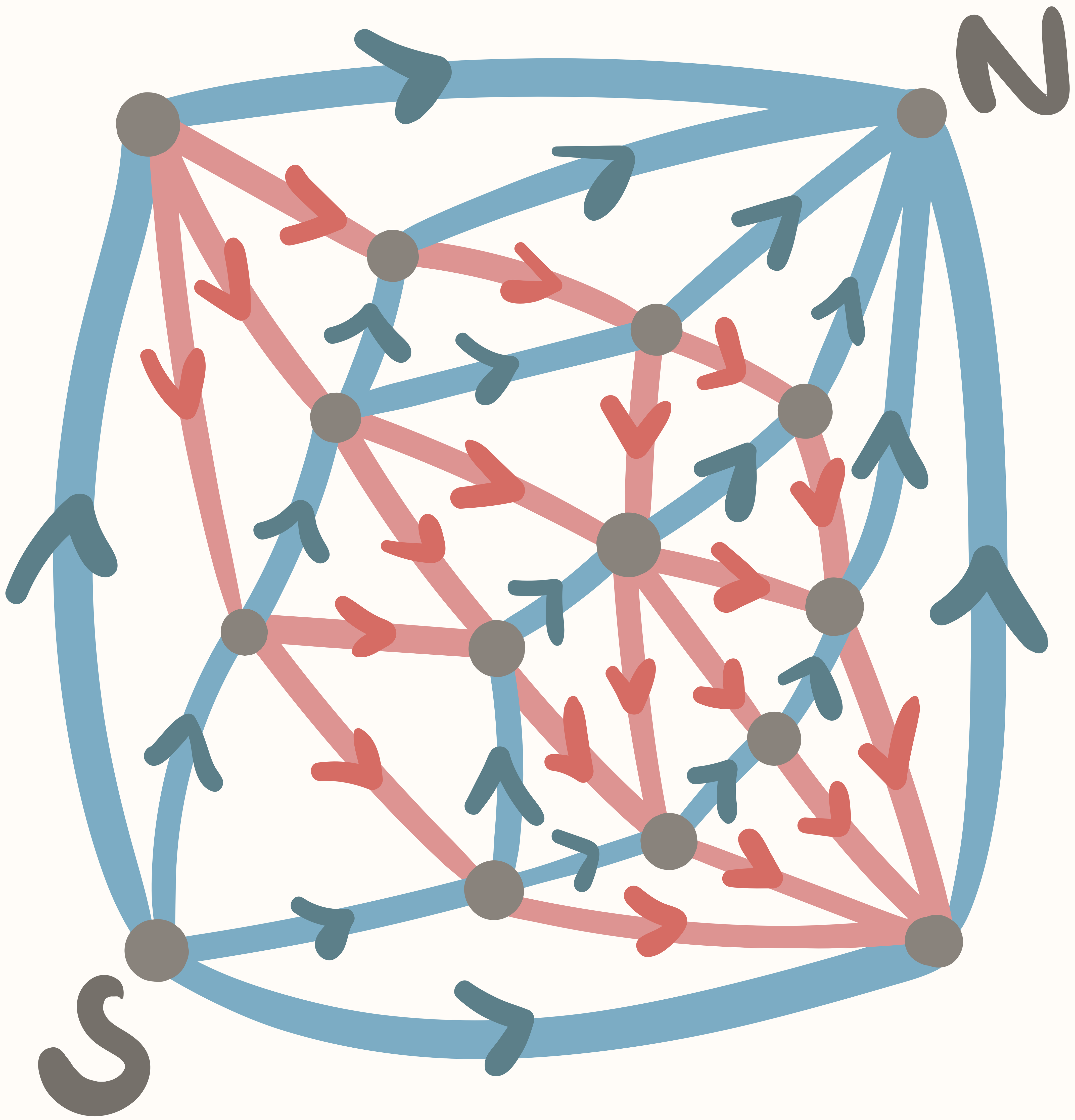




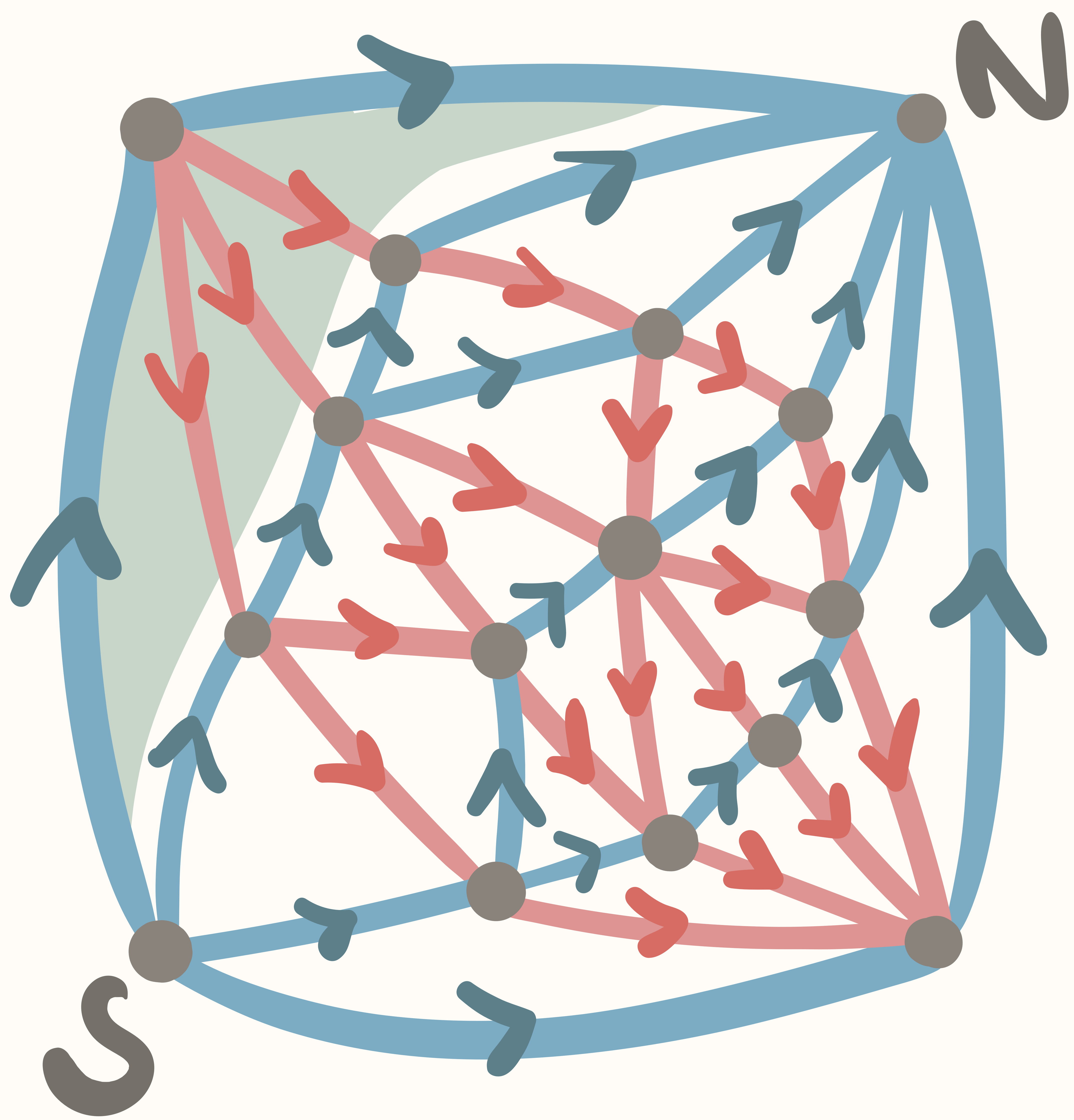
Specialization to transversal structures



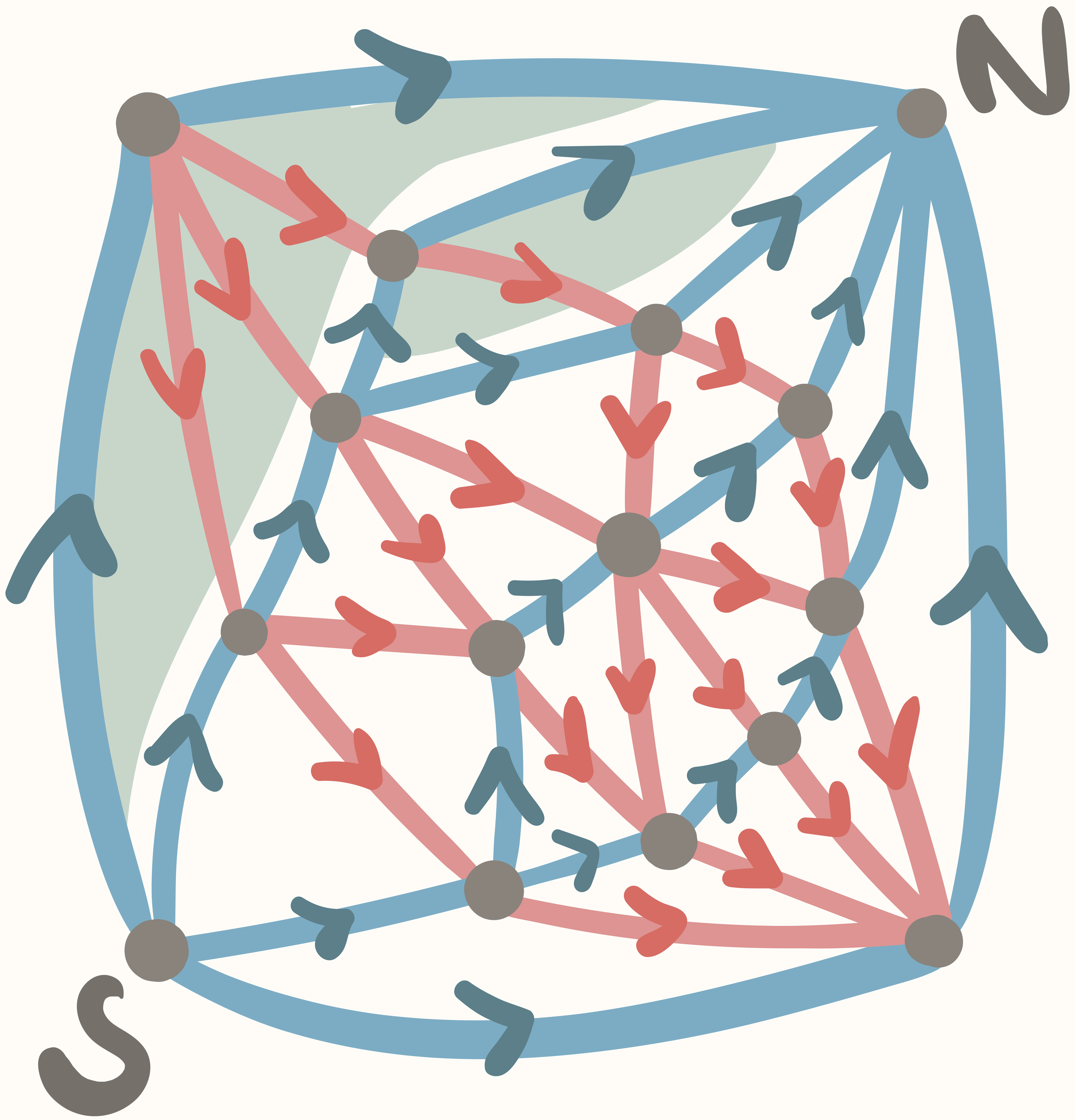
Specialization to transversal structures



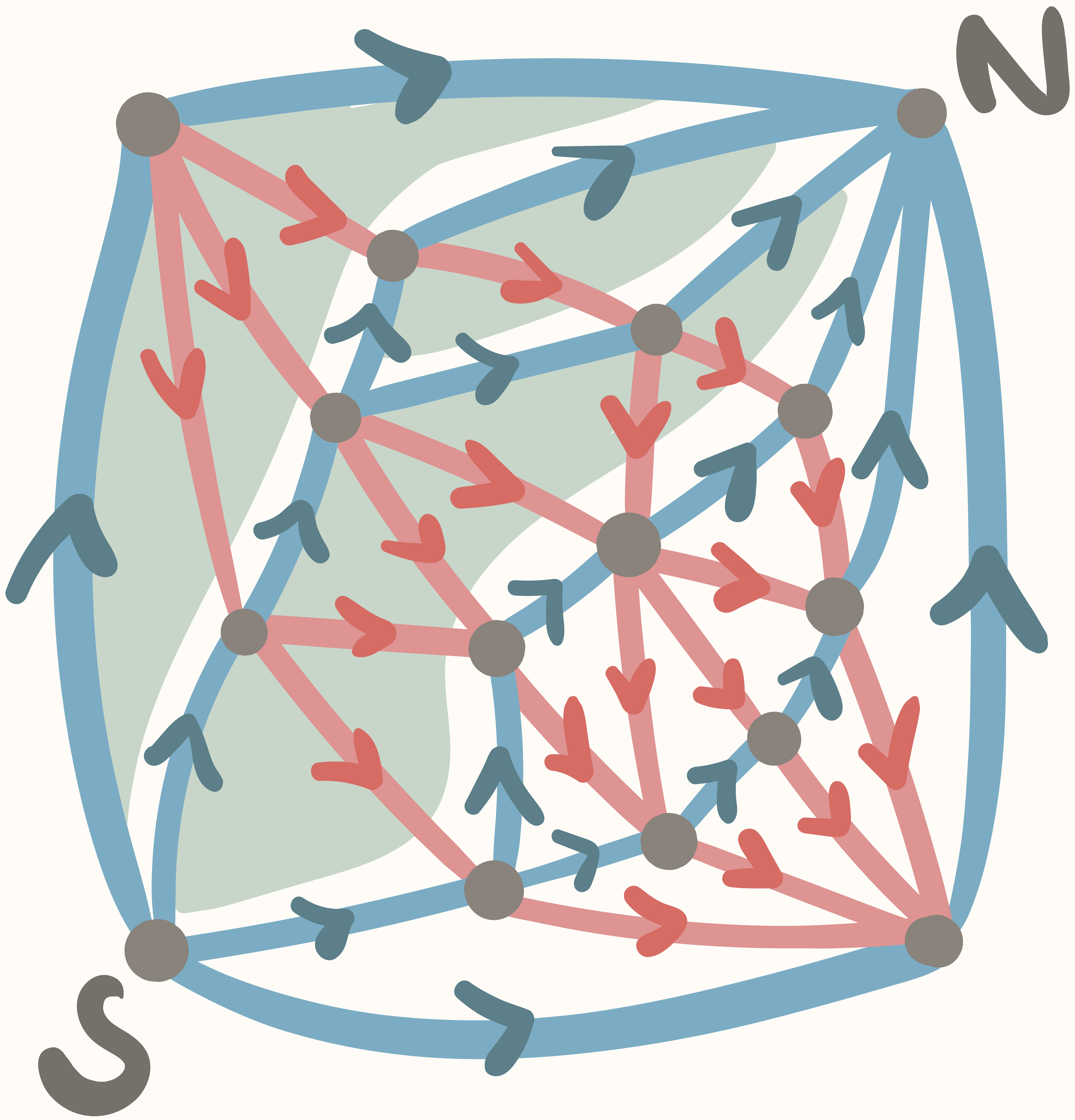
Specialization to transversal structures



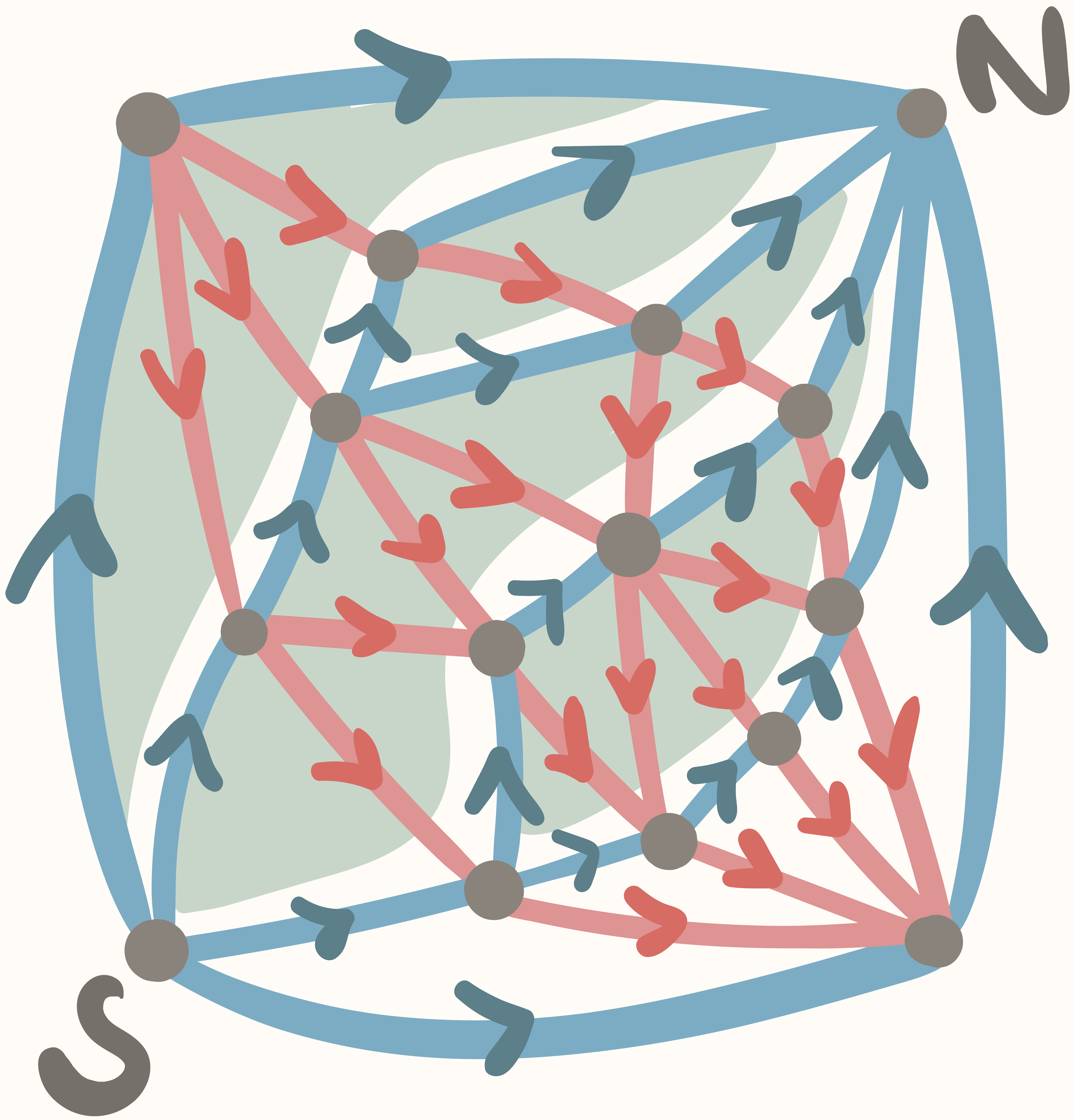
Specialization to transversal structures



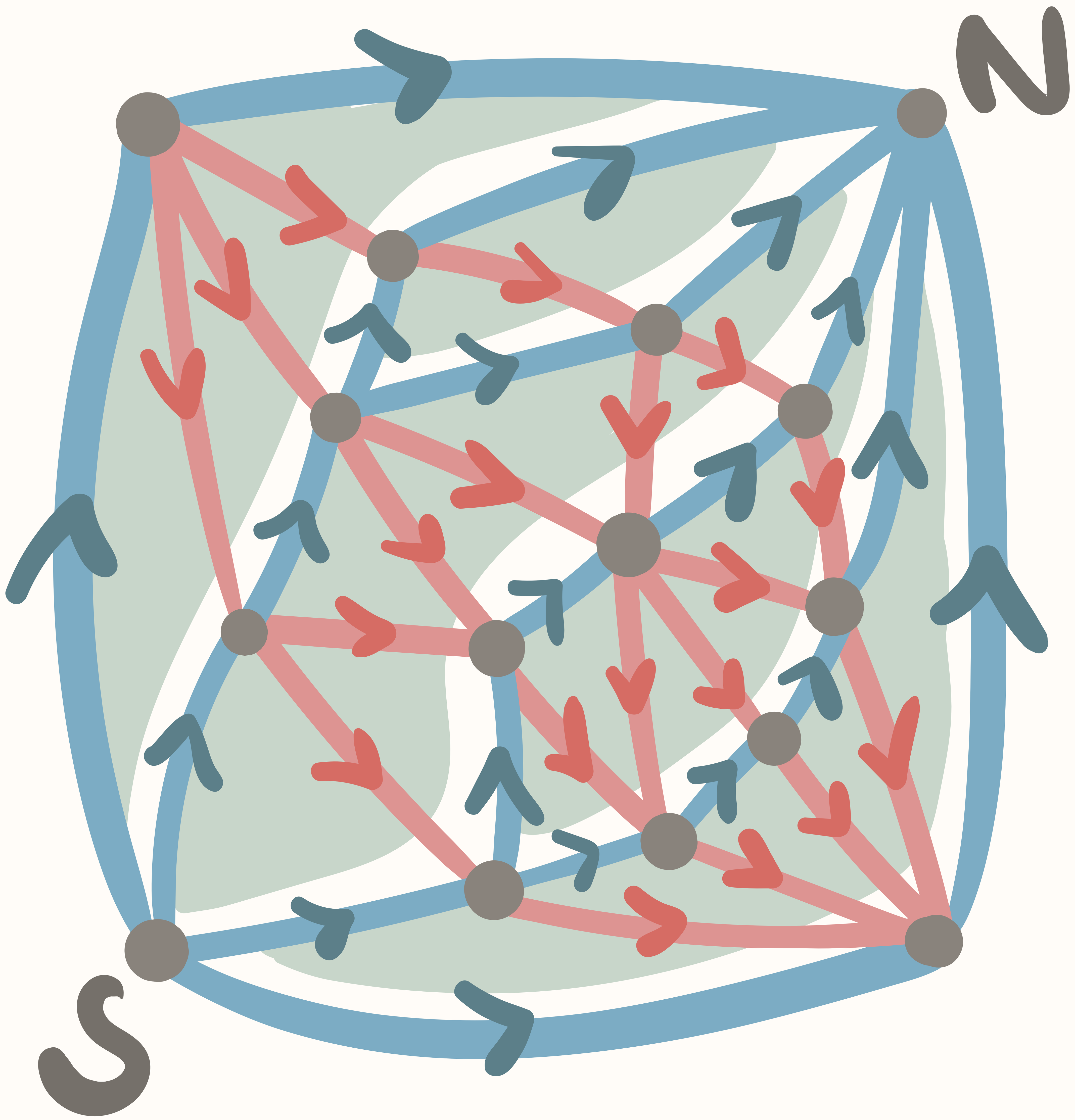
Specialization to transversal structures



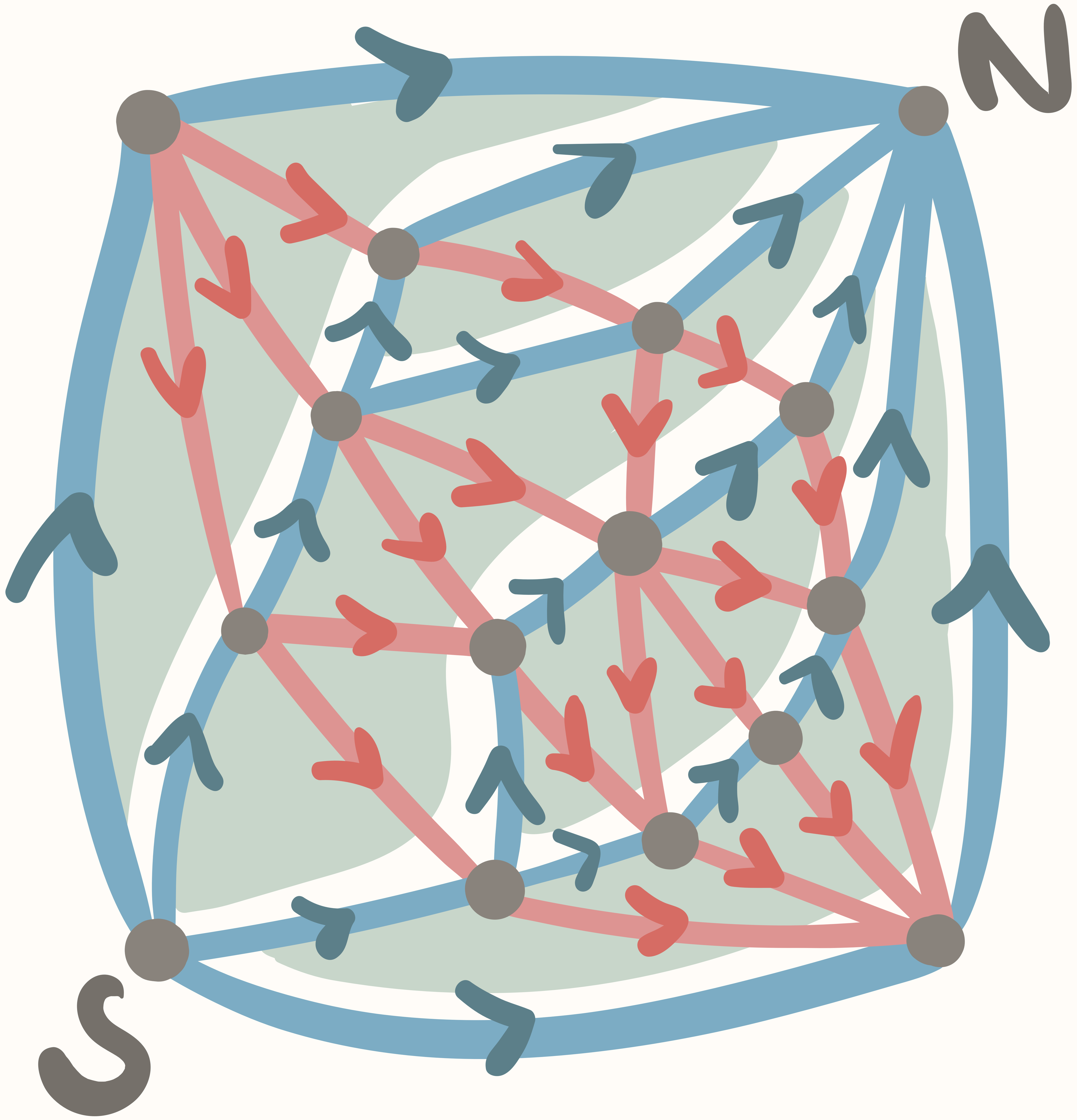
Specialization to transversal structures

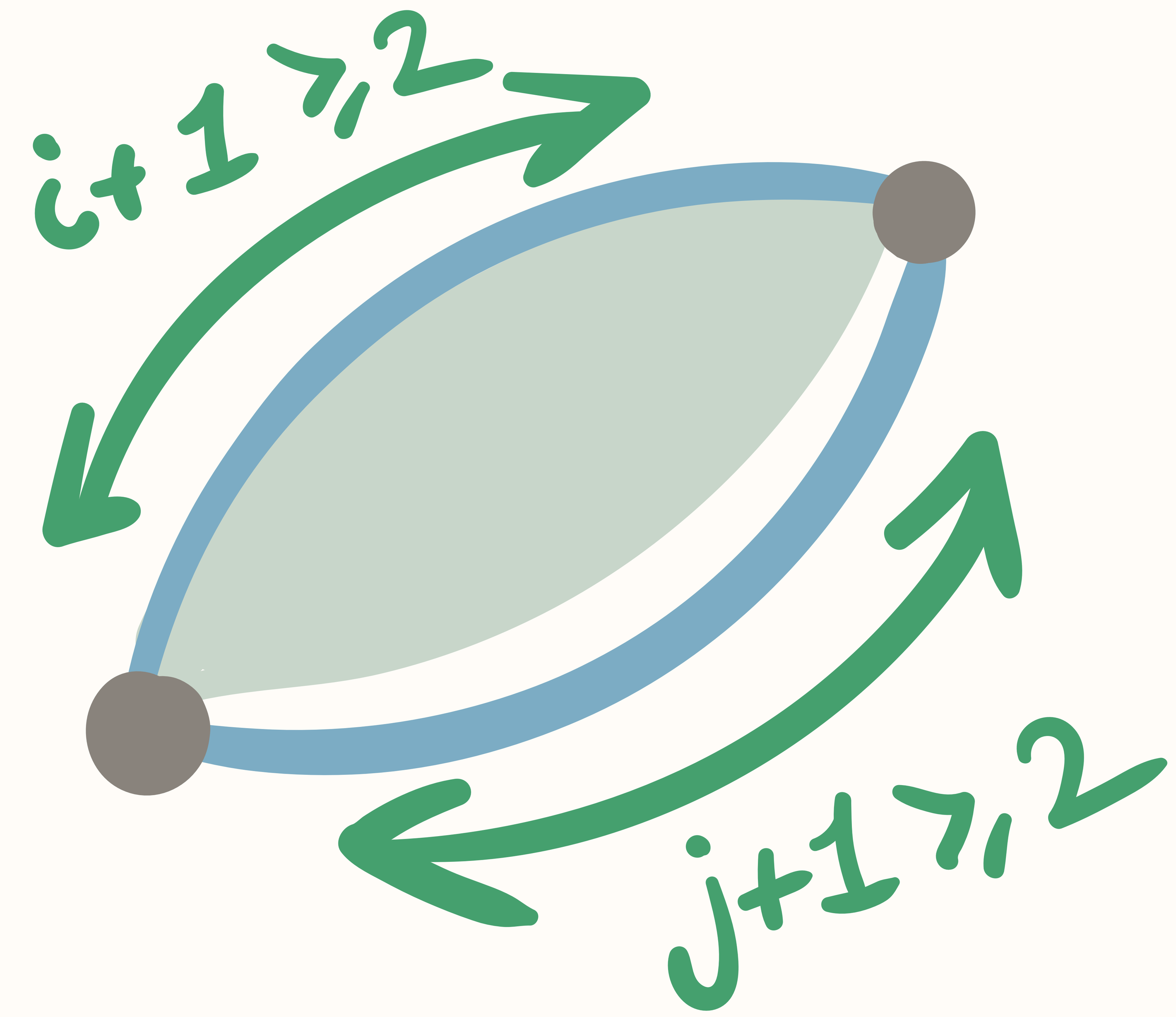


Specialization to transversal structures

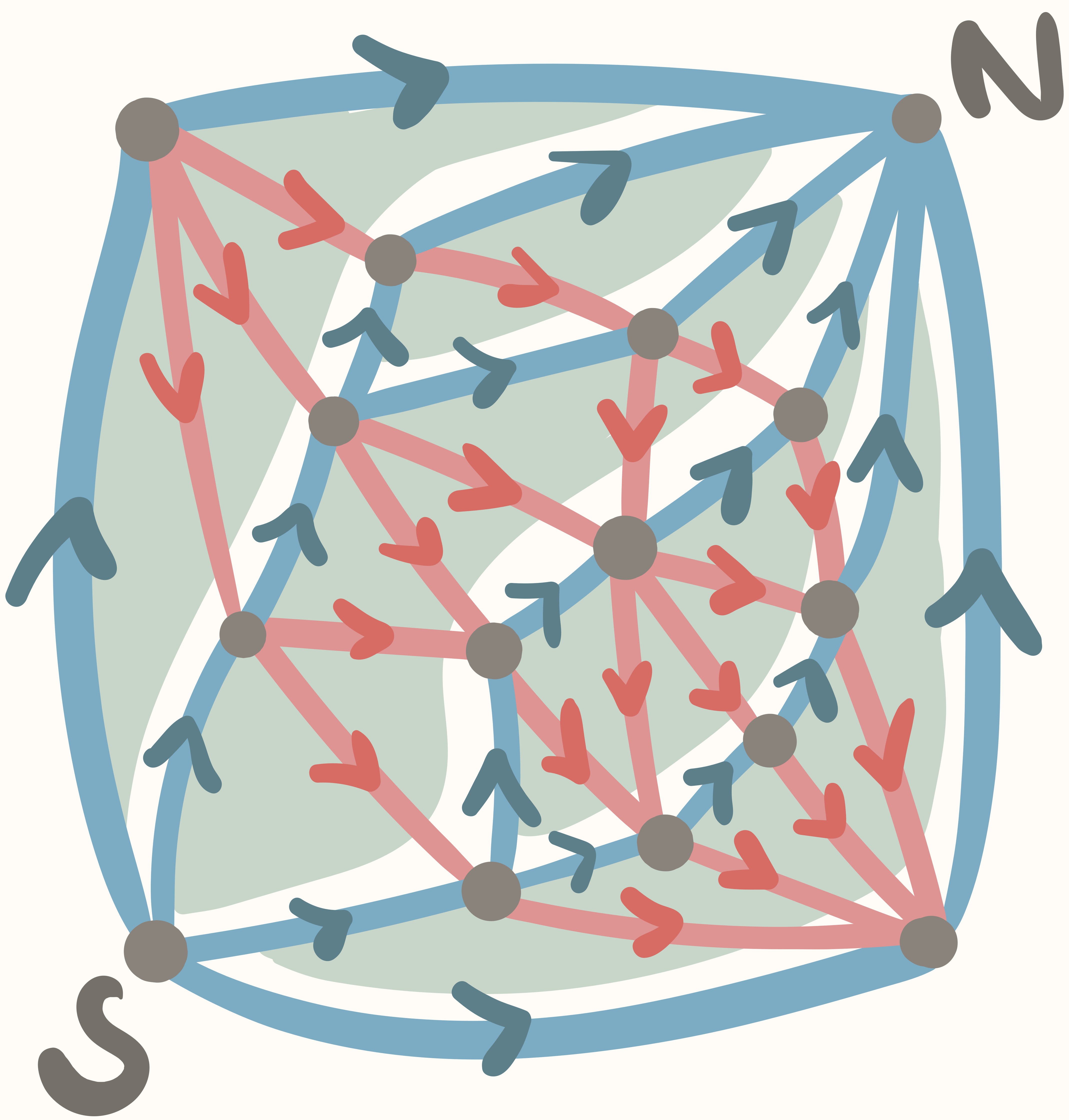


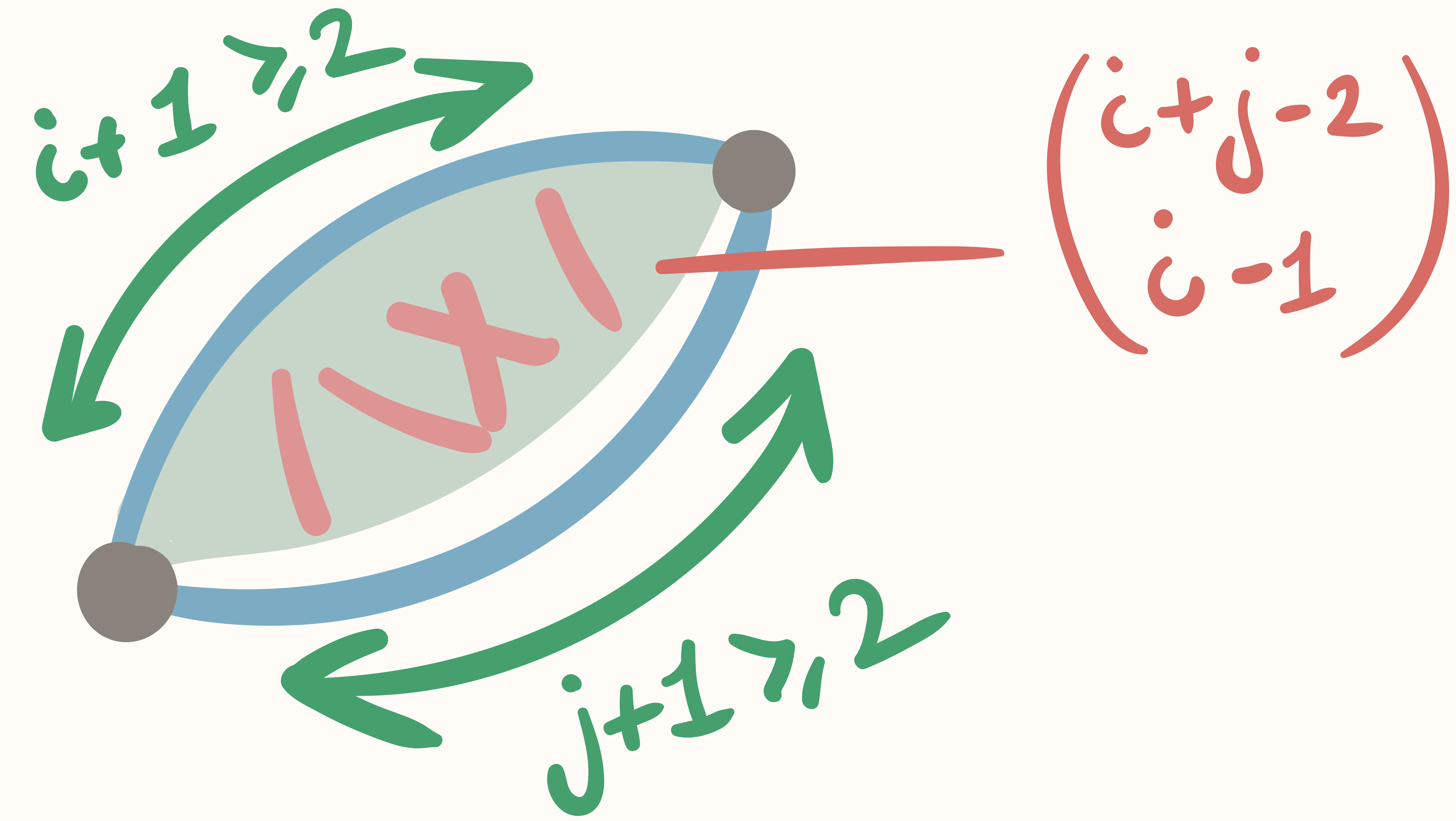
Specialization to transversal structures



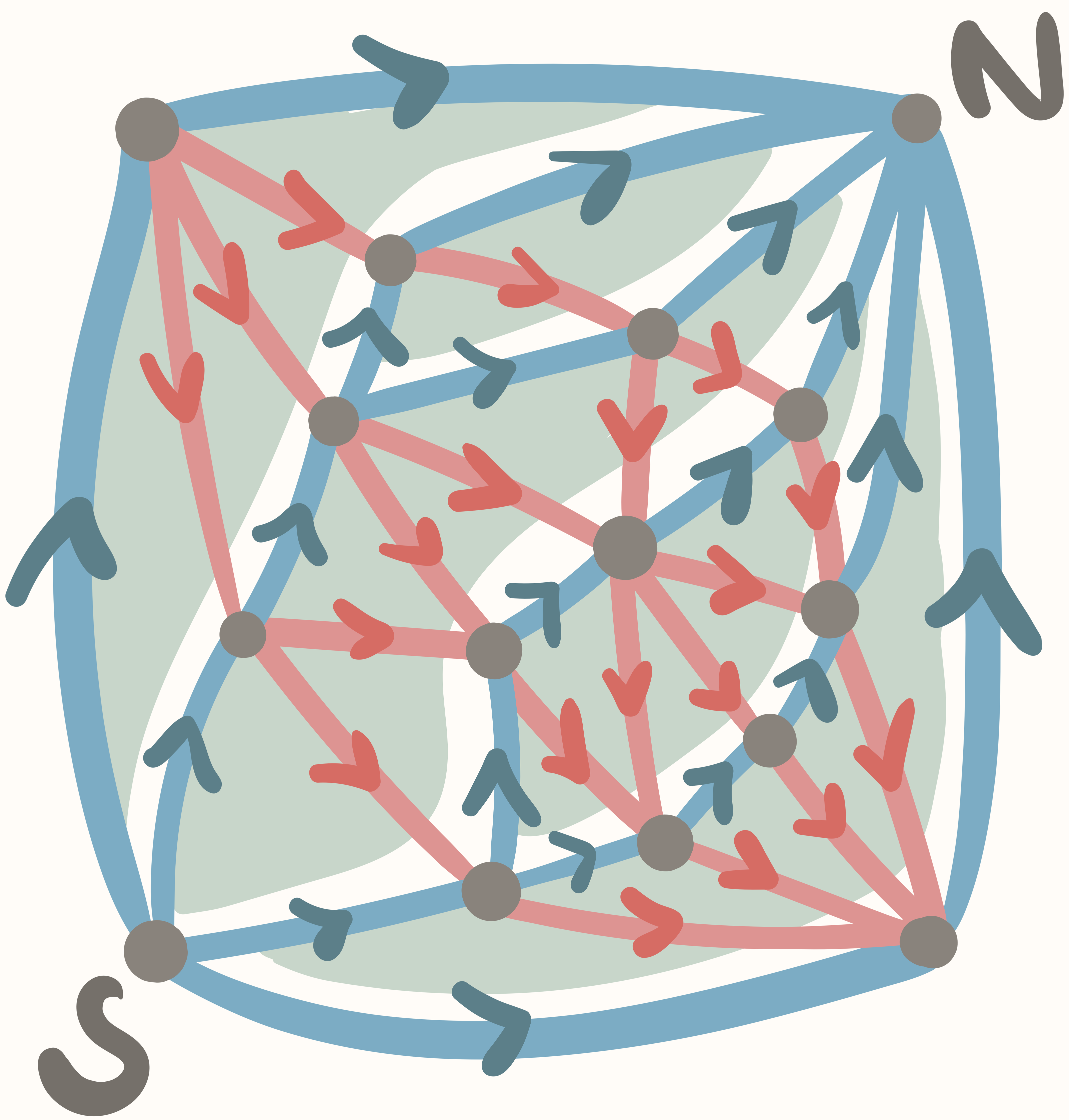


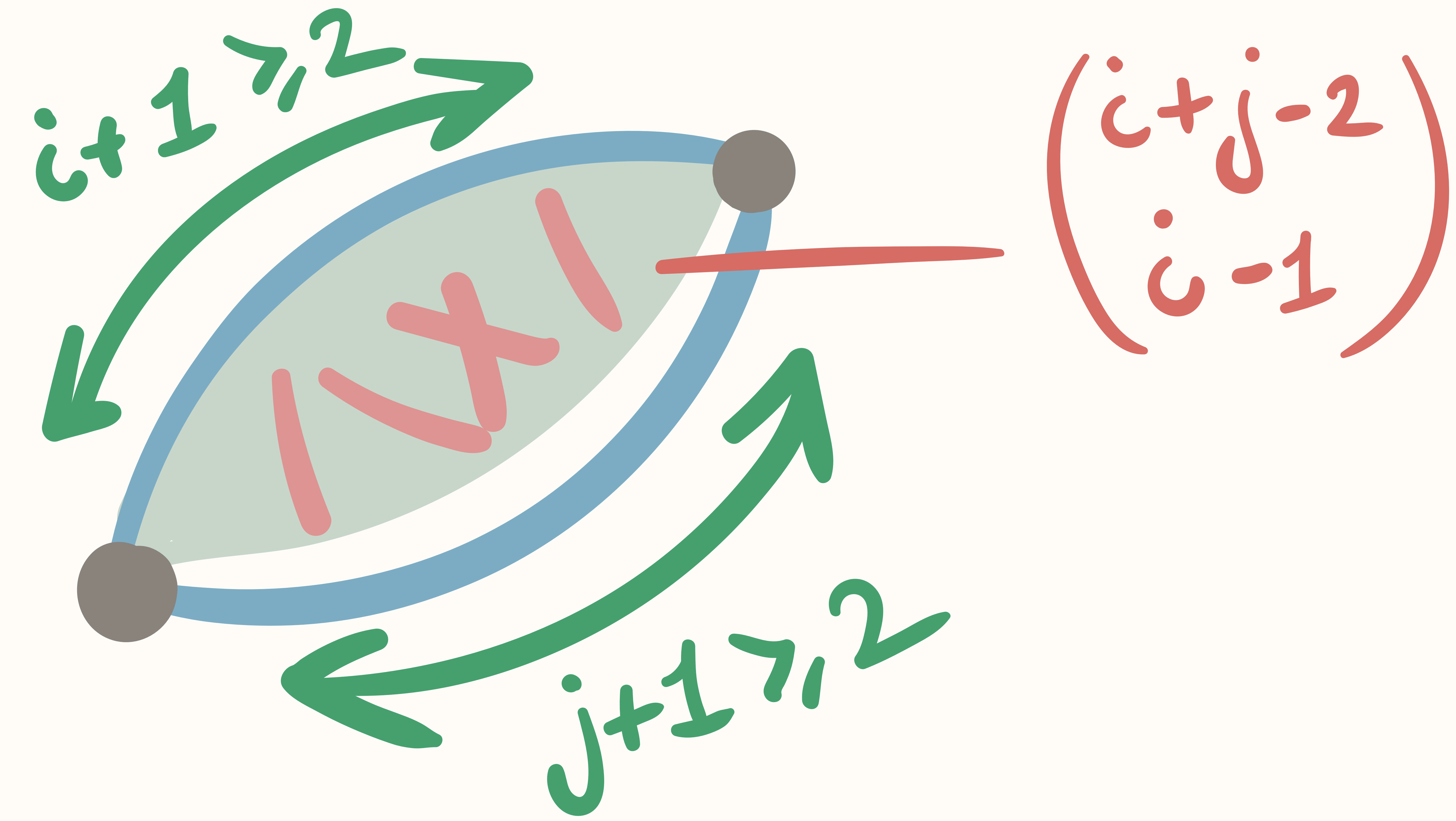
Specialization to transversal structures

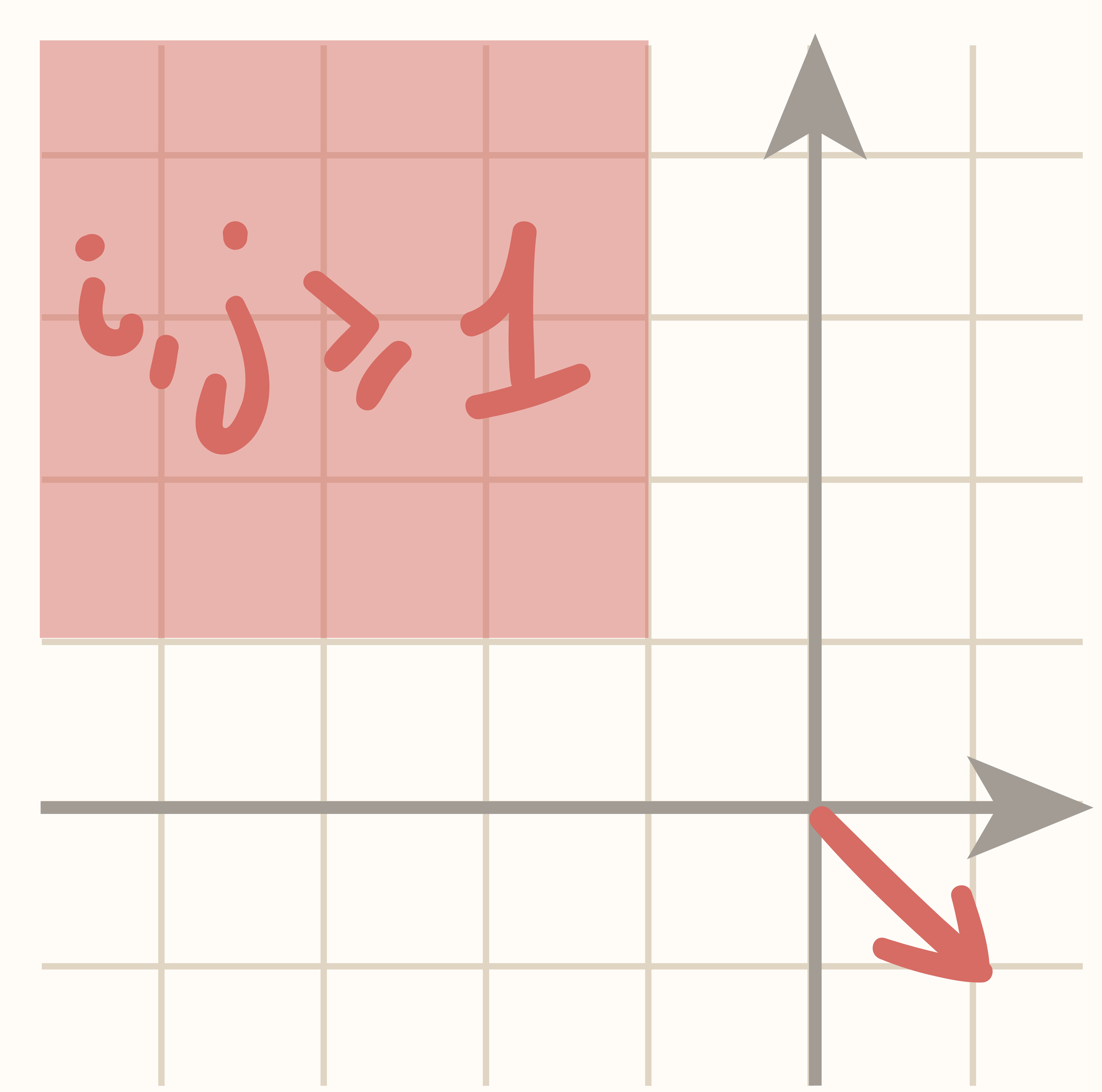




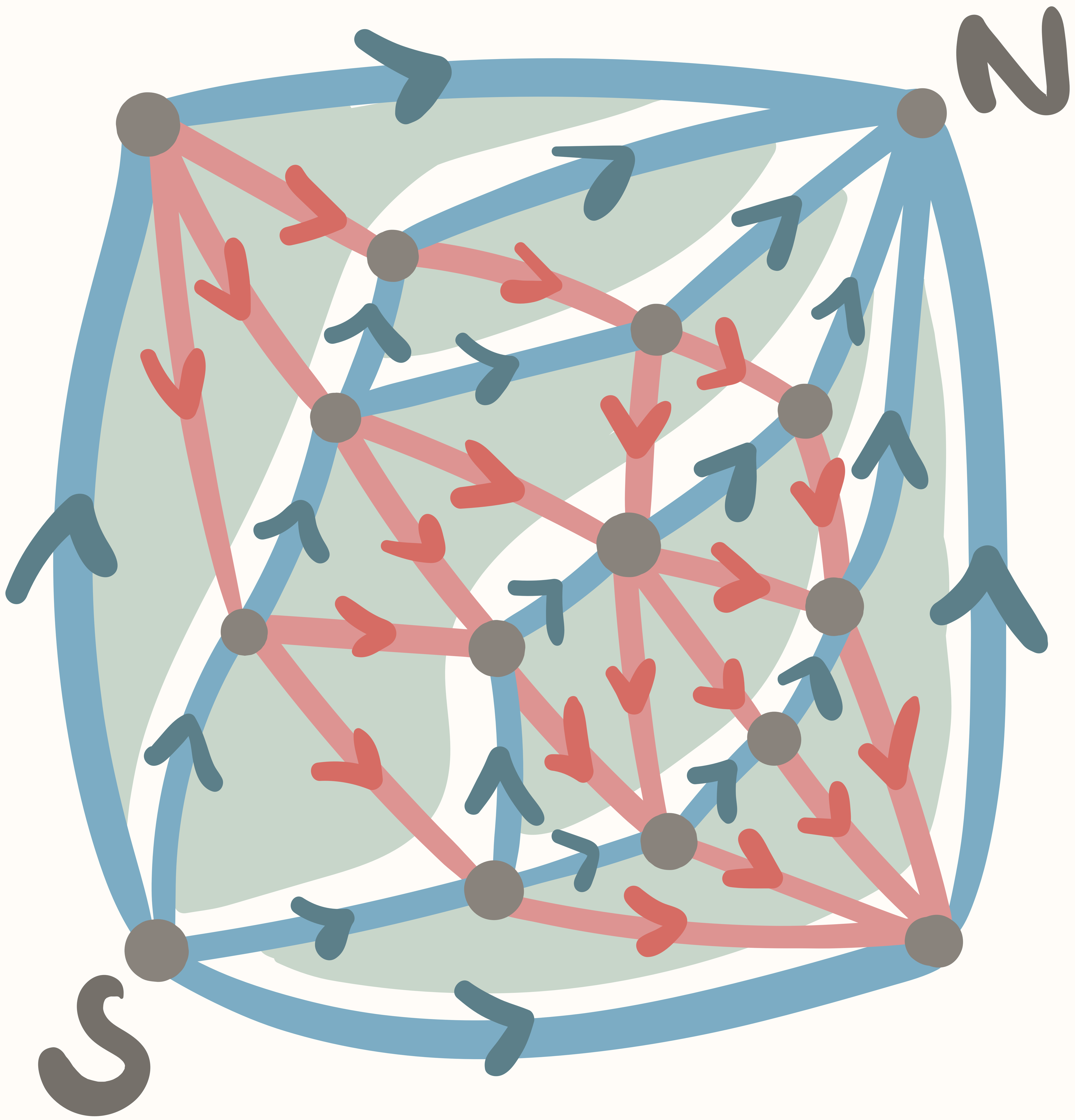
Specialization to transversal structures

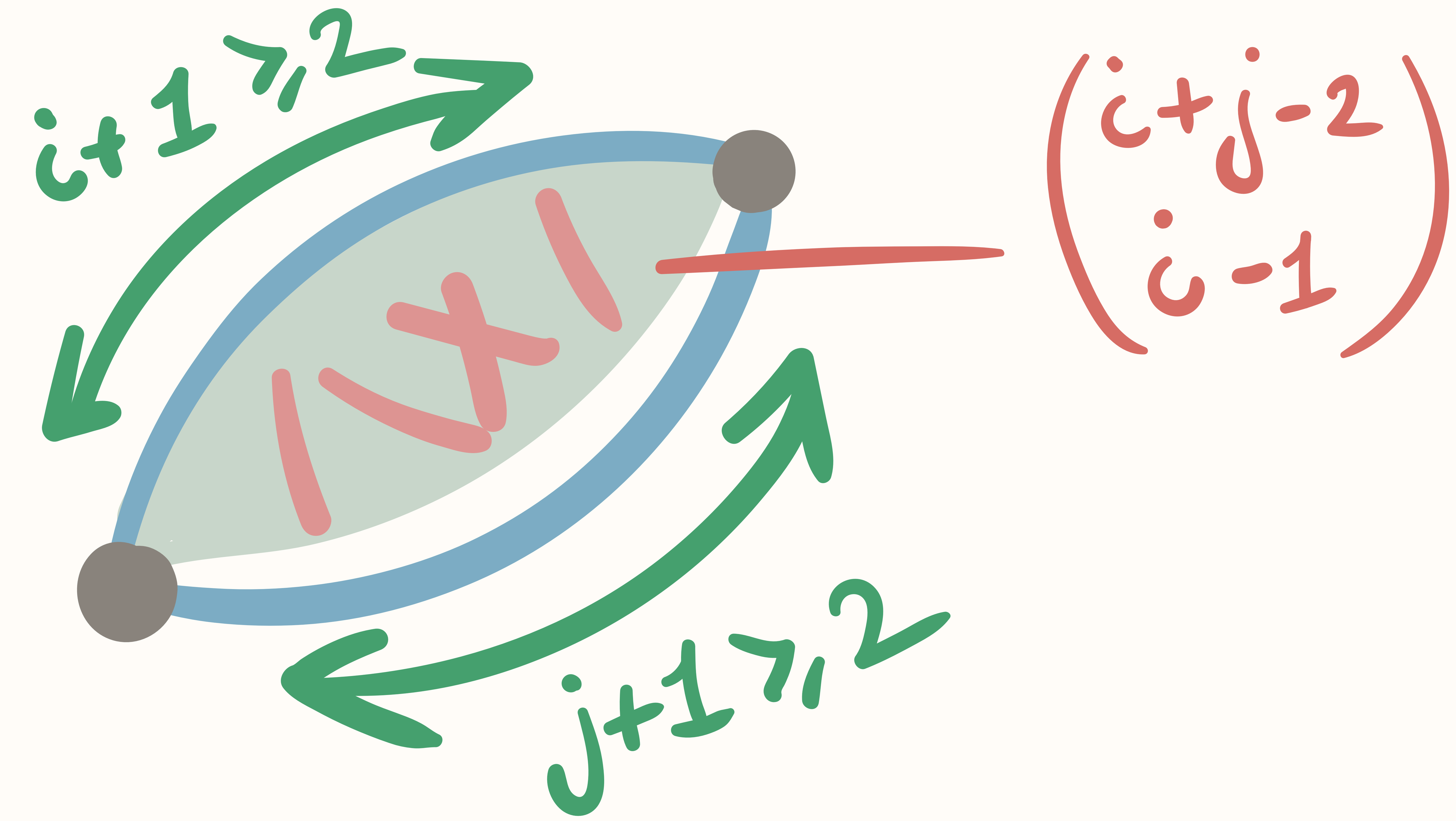


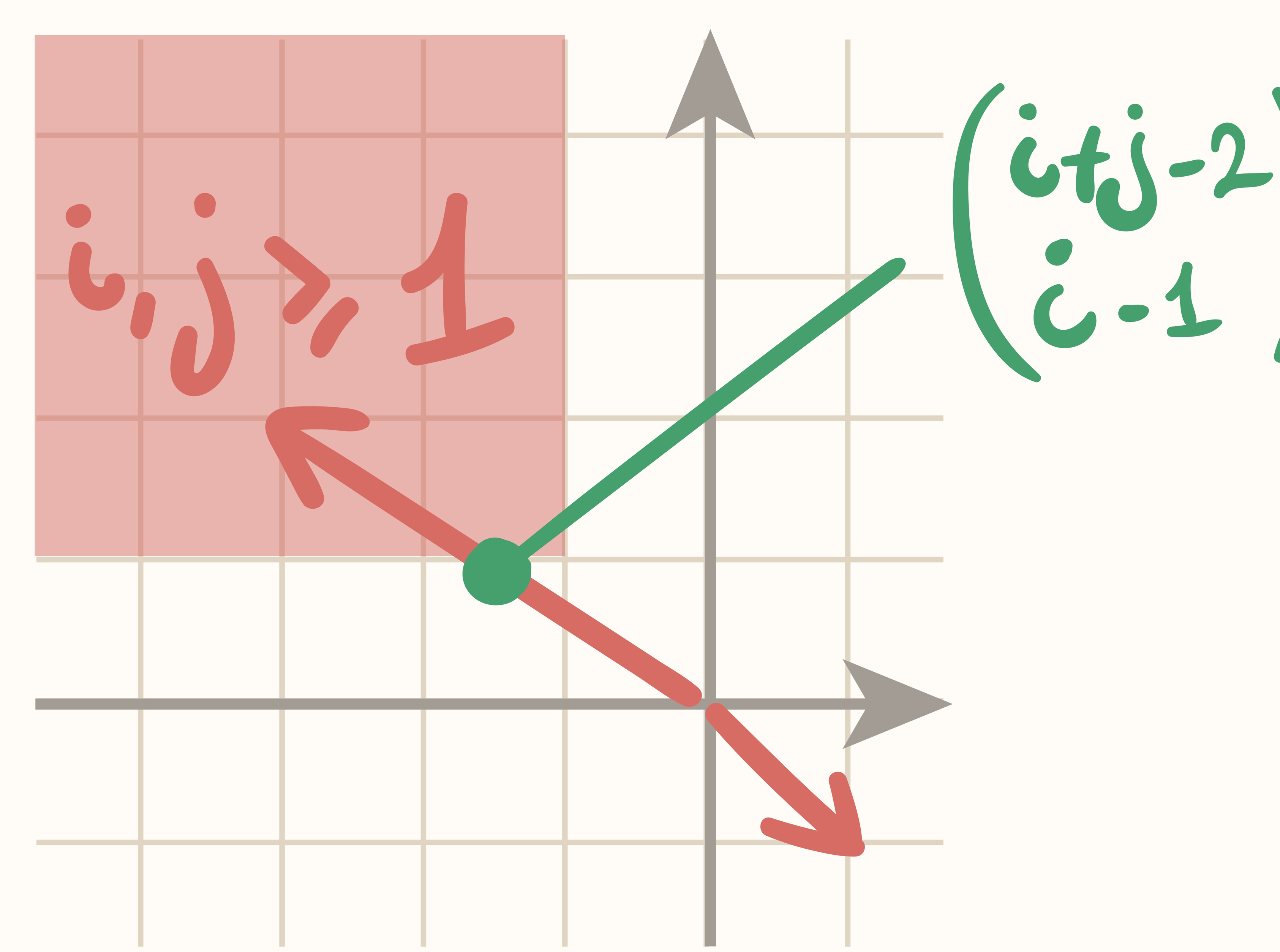




Specialization to transversal structures





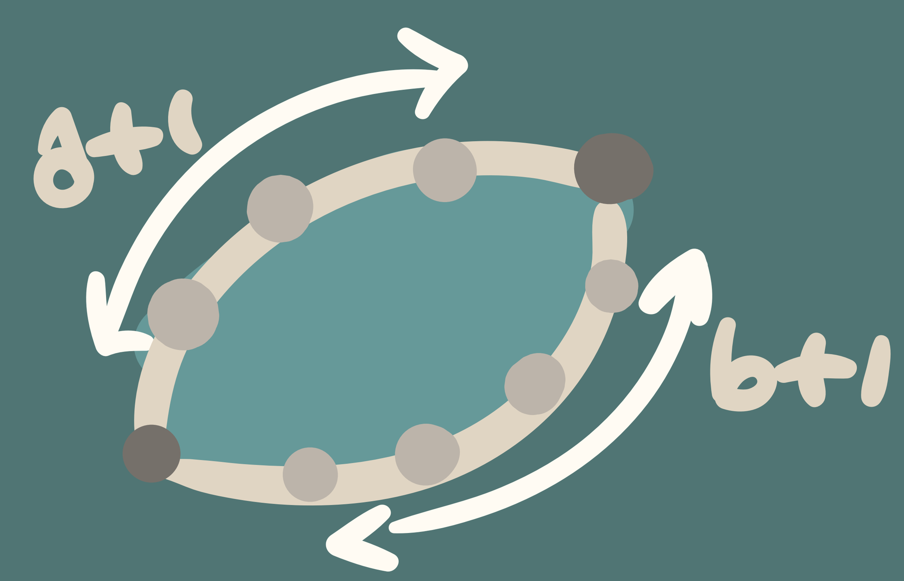


Specializations summary

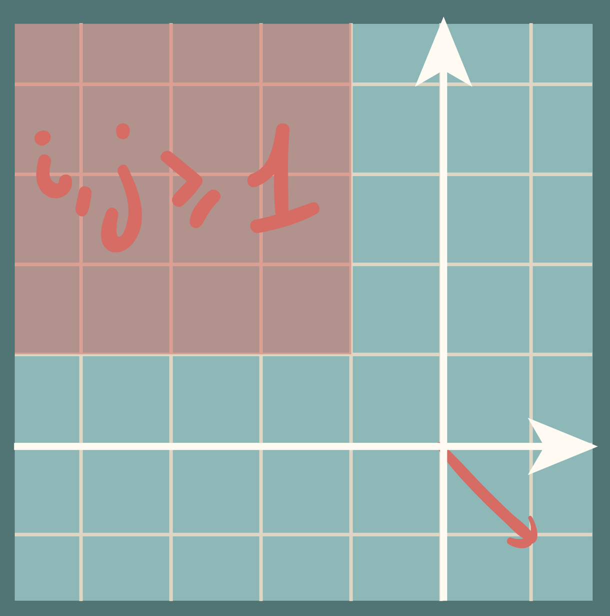
Model	*Tandem walk*

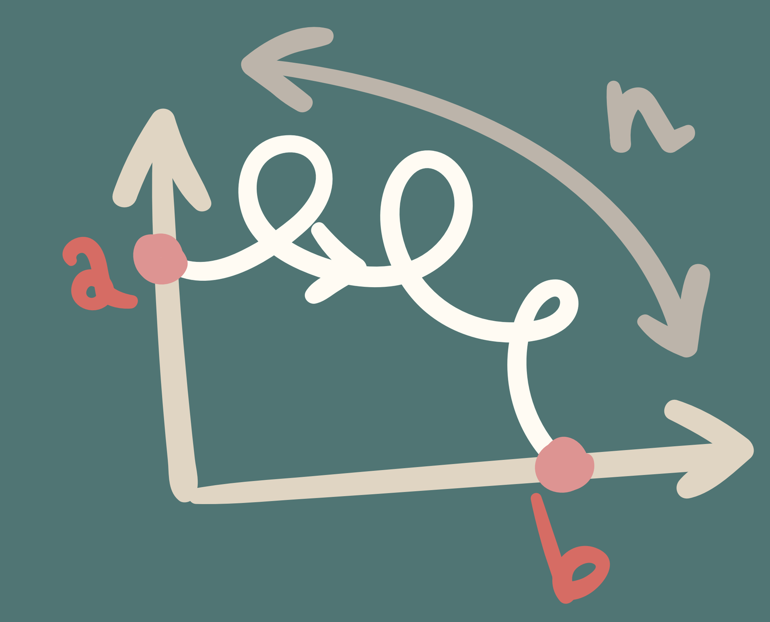
Specializations summary

Model	*Tandem walk*
Posets *n+2 edges*	



←*bijection*→

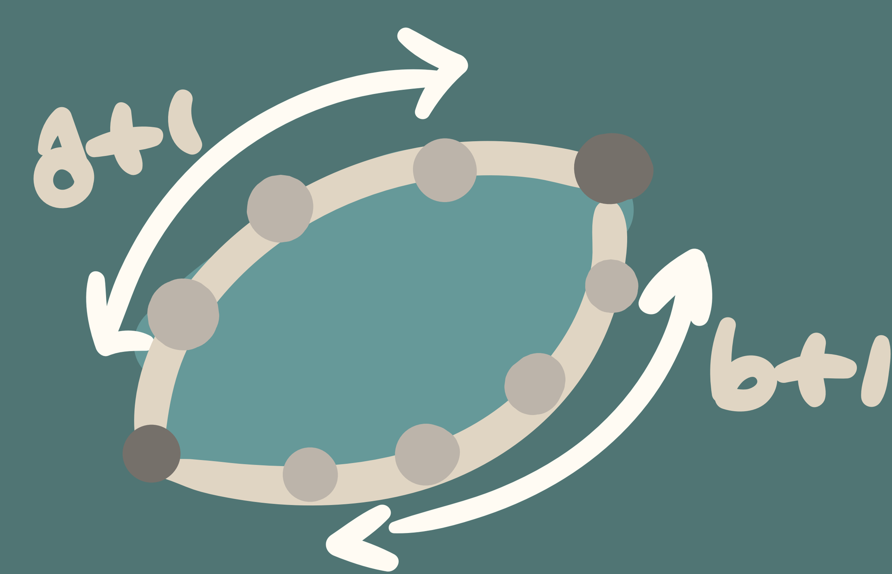




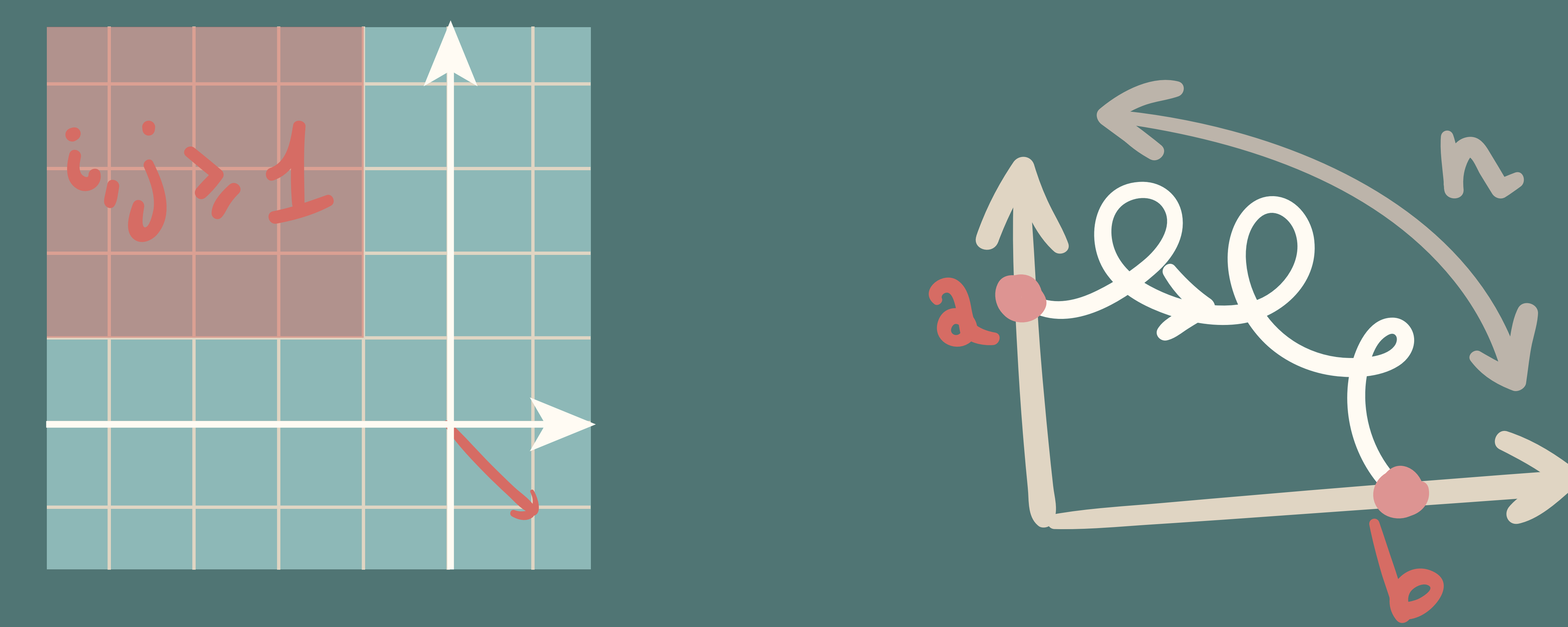
Specializations summary

Model	Tandem walk

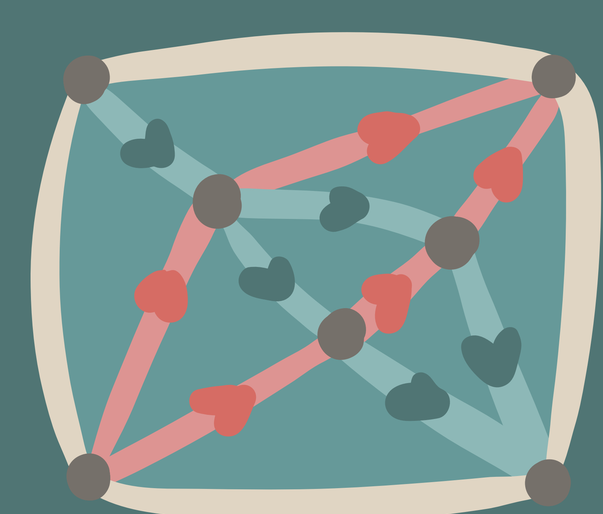
Posets
n+2 edges



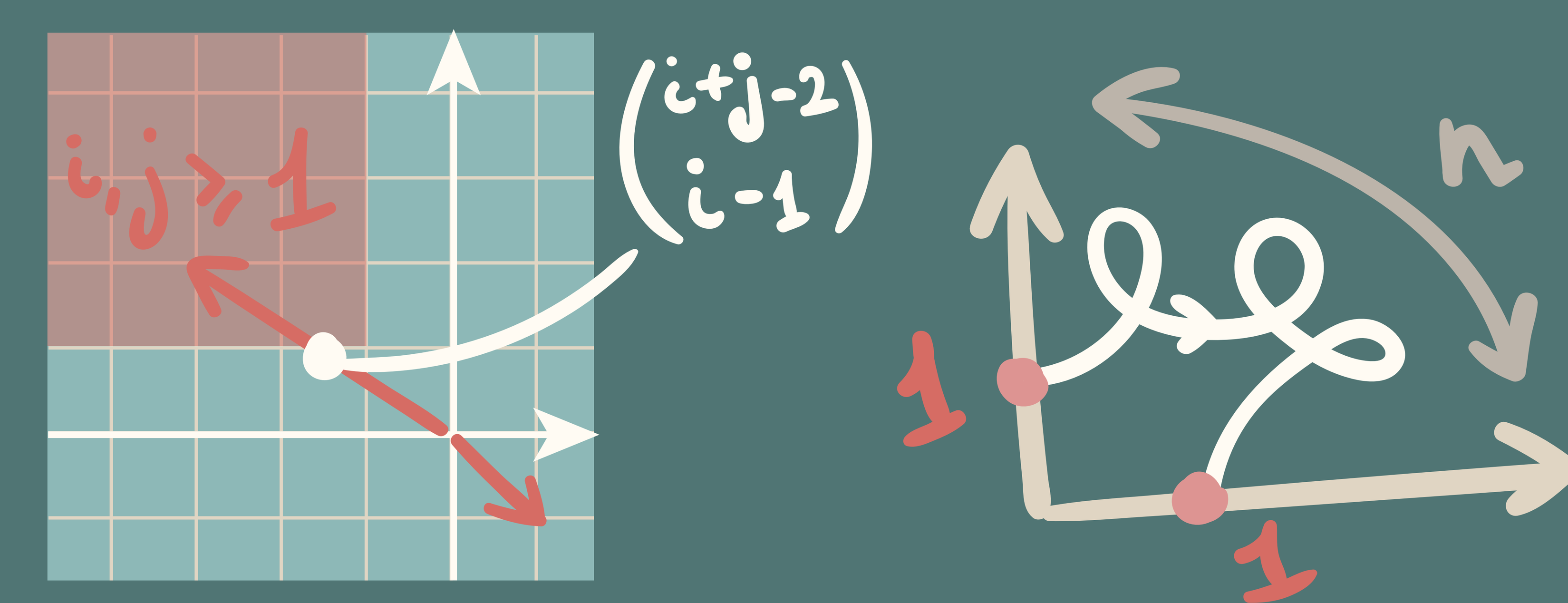
←bijection→



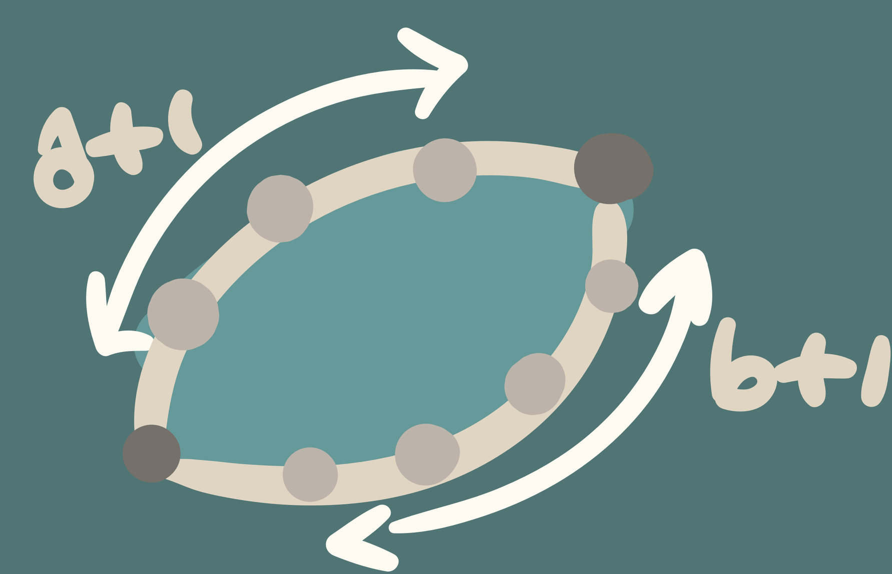
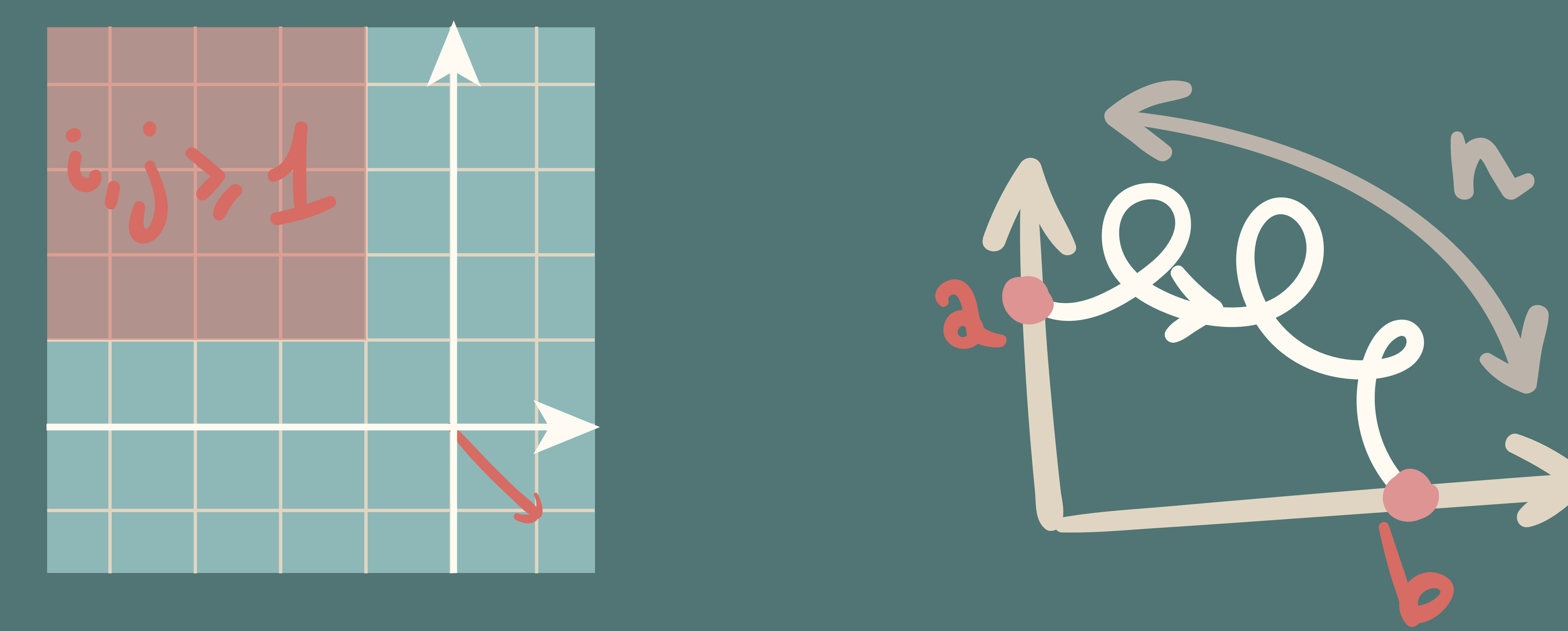
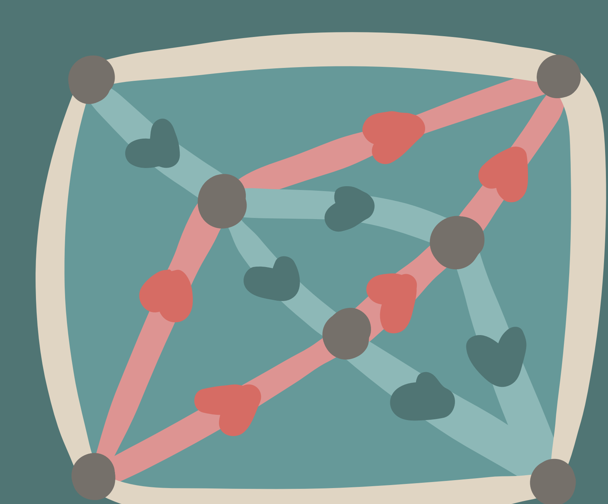
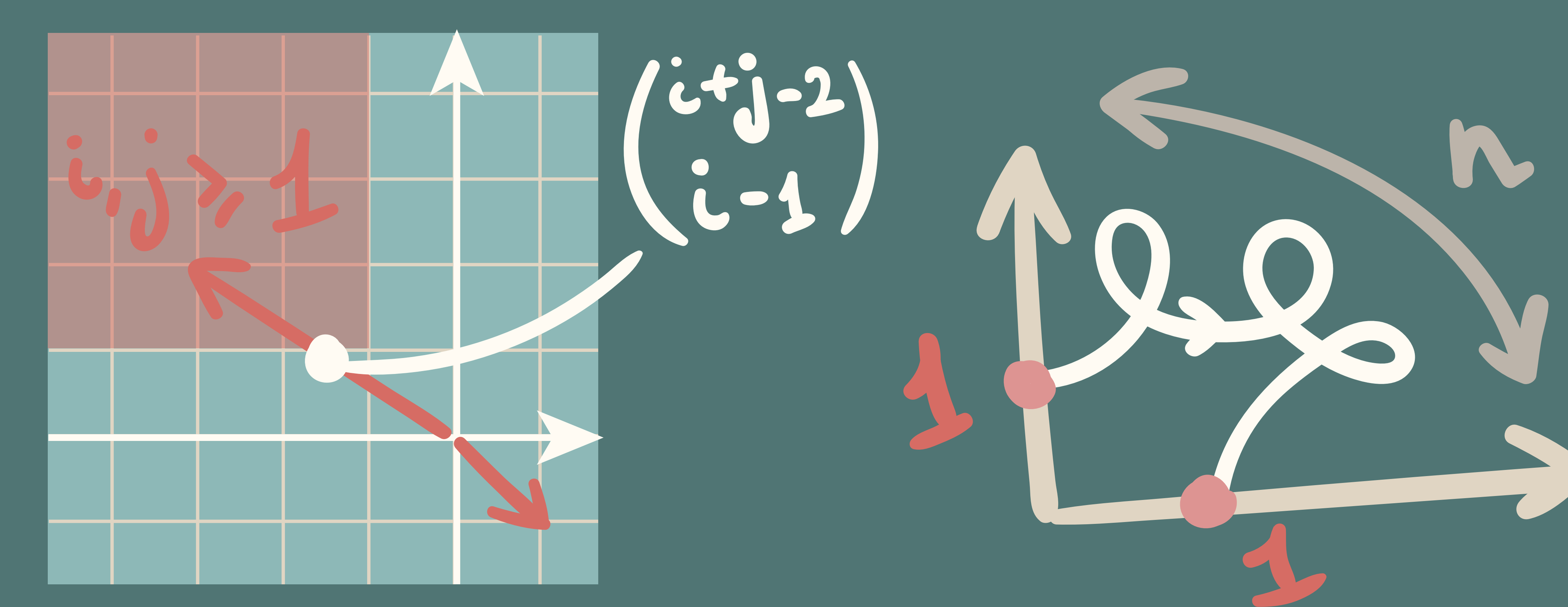
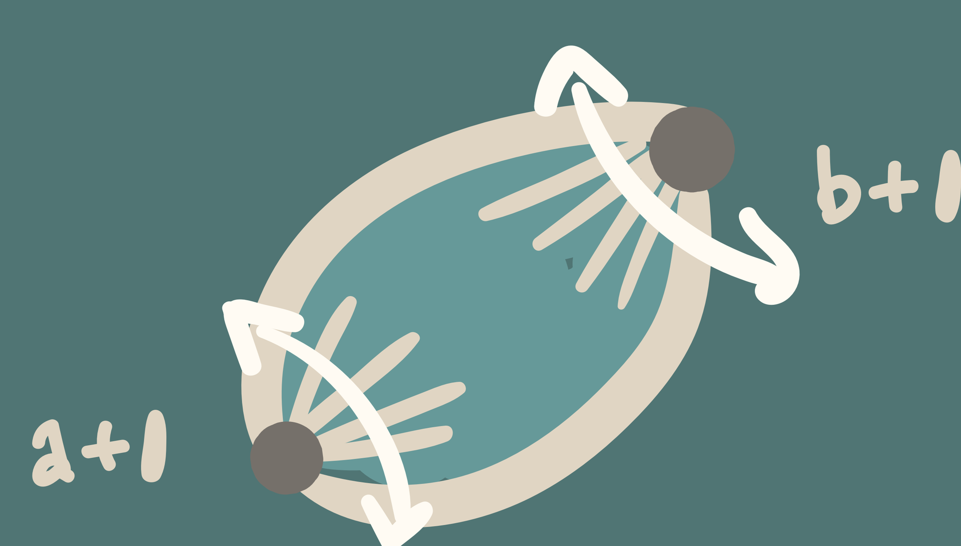
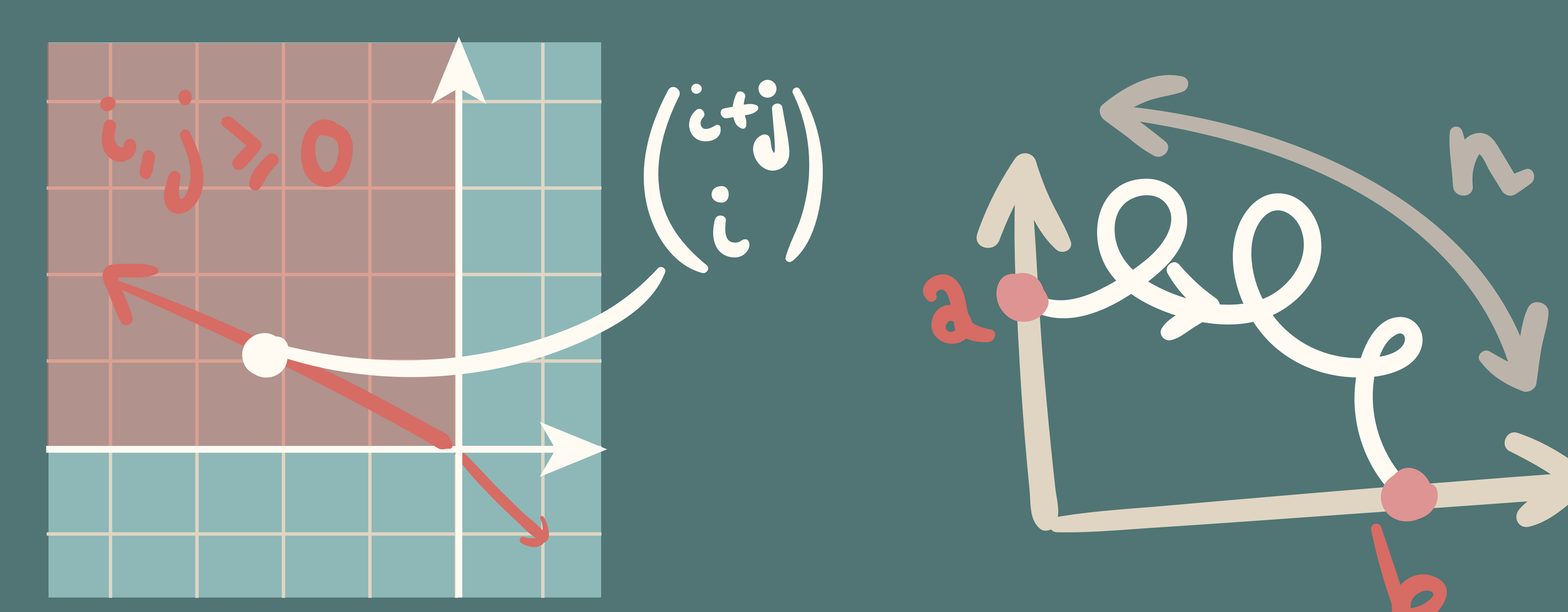
Transversal structures
n blue edges



←bijection→



Specializations summary

Model		Tandem walk
Posets *n+2 edges* 	←bijection→	
Transversal structures *n blue edges* 	←bijection→	
Posets *n vertices* 	←bijection→	

Summary

Maps and decorated maps

1. Bijection with quadrant tandem walks

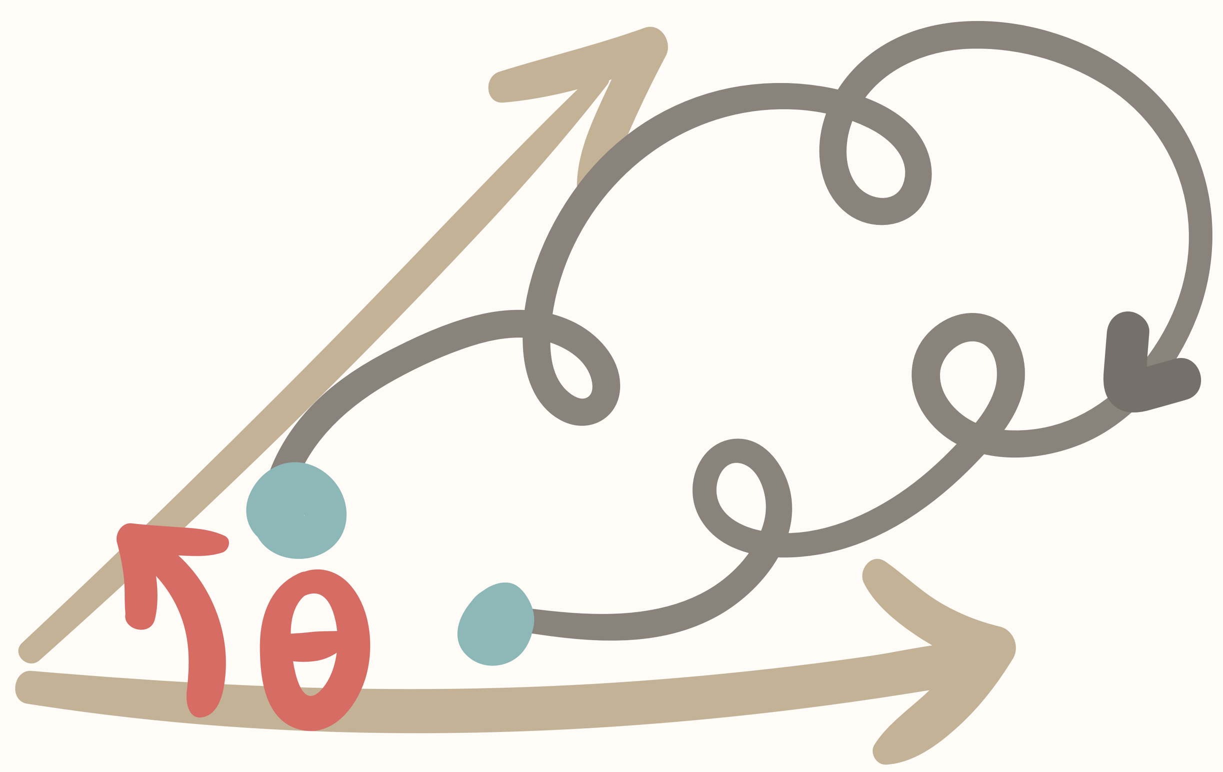
a. The KMSW bijection
b. Plane bipolar posets
c. Plane bipolar posets by vertices
d. Transversal structures

2. Asymptotic enumeration

(Digression on plane permutations)

3. Generic transversal structures

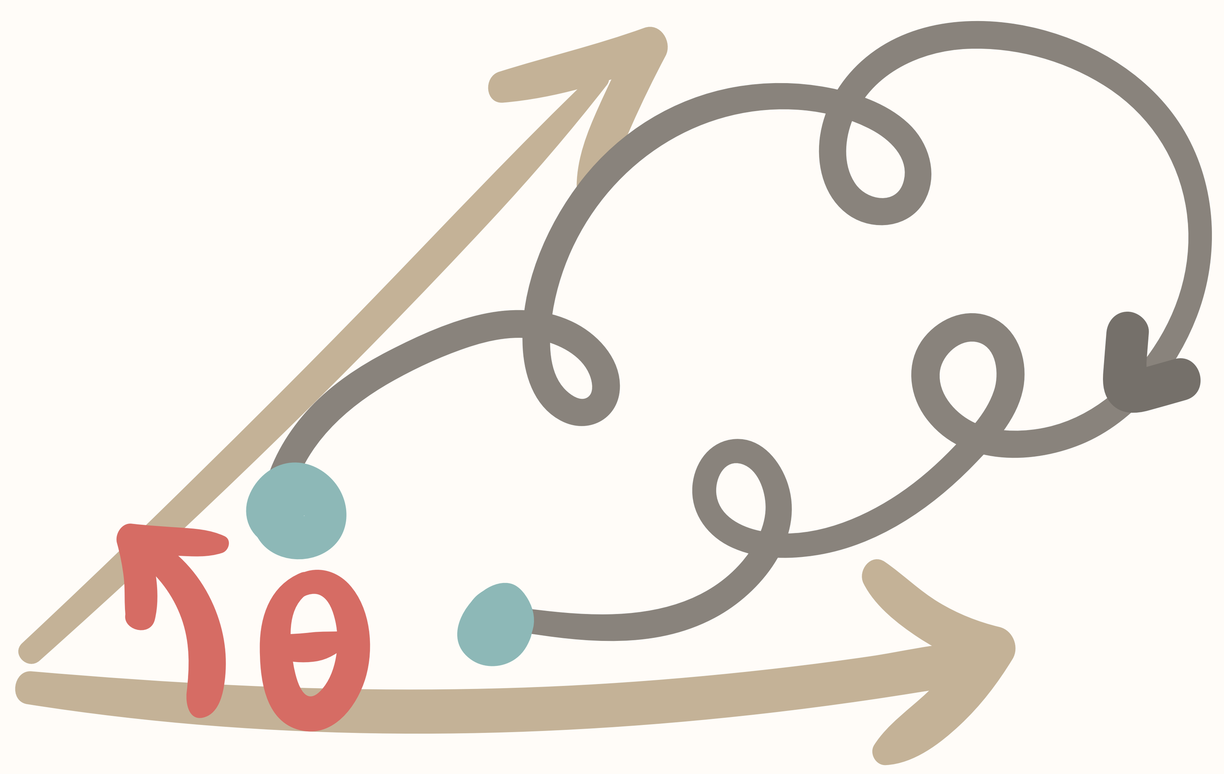
Asymptotic counting results



- *Random walks in cone,* D. Denisov & V. Wachtel (2015)
- *Non-D-finite excursions in the quarter plane,* D. Bostan, K. Raschel, B. Salvy (2012)

$$a_n \sim \varkappa \cdot \gamma^n n^{-1-\pi/\theta}$$

Asymptotic counting results



- *Random walks in cone,* D. Denisov & V. Wachtel (2015)
- *Non-D-finite excursions in the quarter plane,* D. Bostan, K. Raschel, B. Salvy (2012)

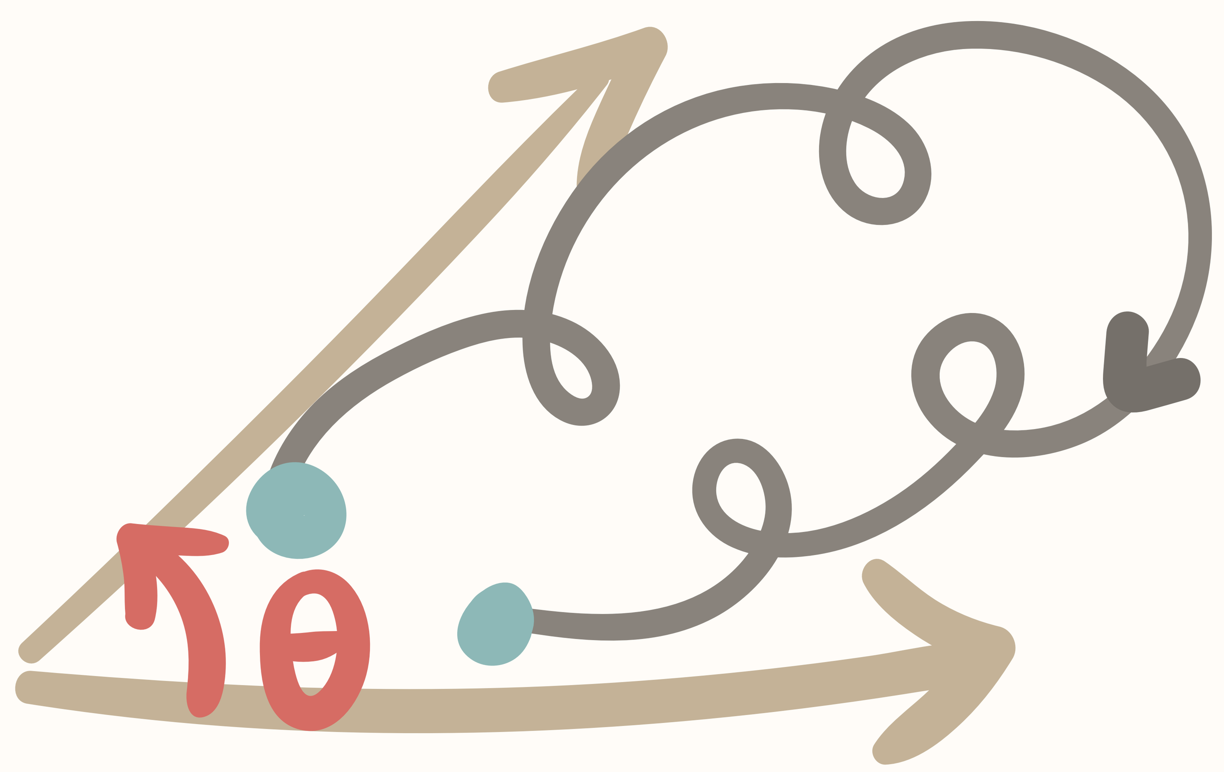
$$a_n \sim \varkappa \cdot \gamma^n n^{-1-\pi/\theta}$$

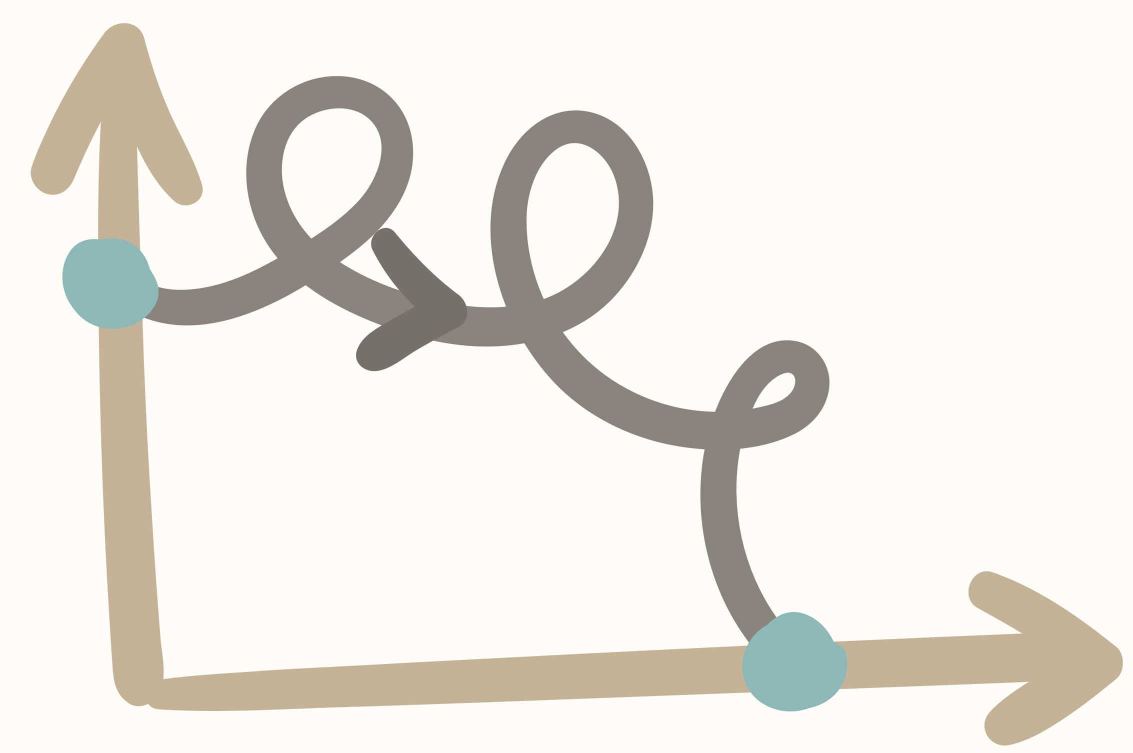
If the drift is zero, *i.e.* :

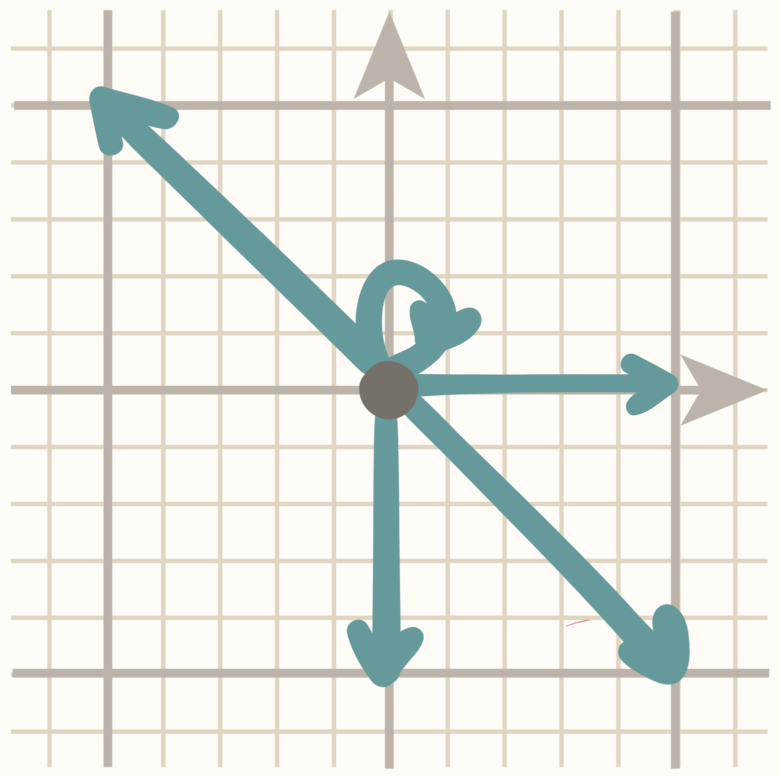
$$\mathbf{E}[X] = \mathbf{E}[Y] = 0$$

And the covariance matrix is identity.

Asymptotic counting results







- *Random walks in cone,* D. Denisov & V. Wachtel (2015)
- *Non-D-finite excursions in the quarter plane,* D. Bostan, K. Raschel, B. Salvy (2012)

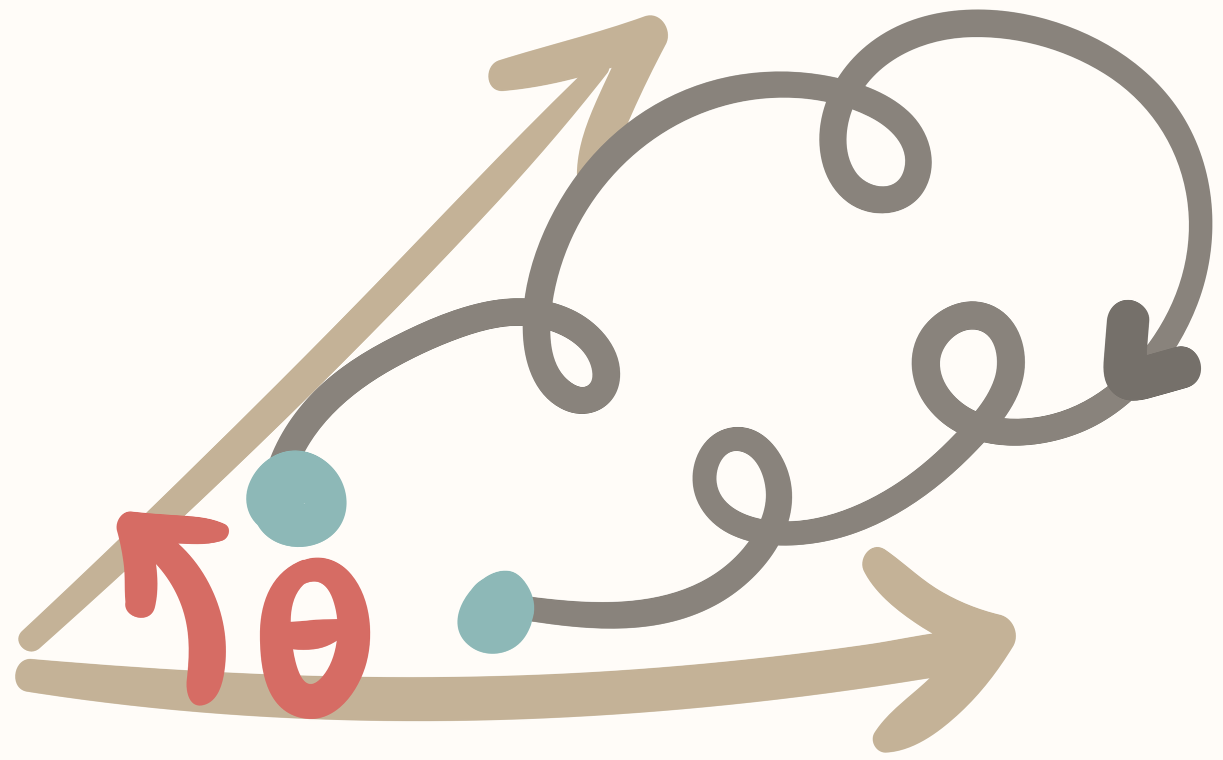
$$a_n \sim \varkappa \cdot \gamma^n n^{-1-\pi/\theta}$$

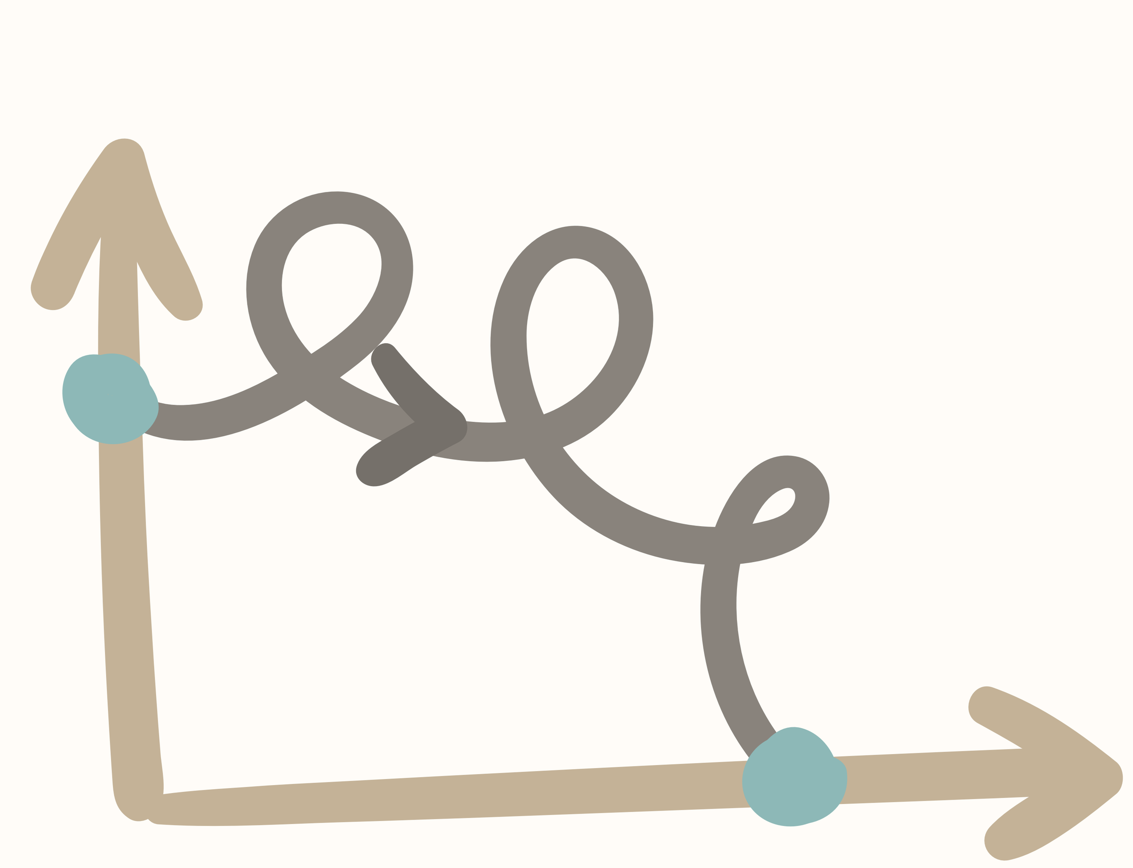
If the drift is zero, *i.e.* :

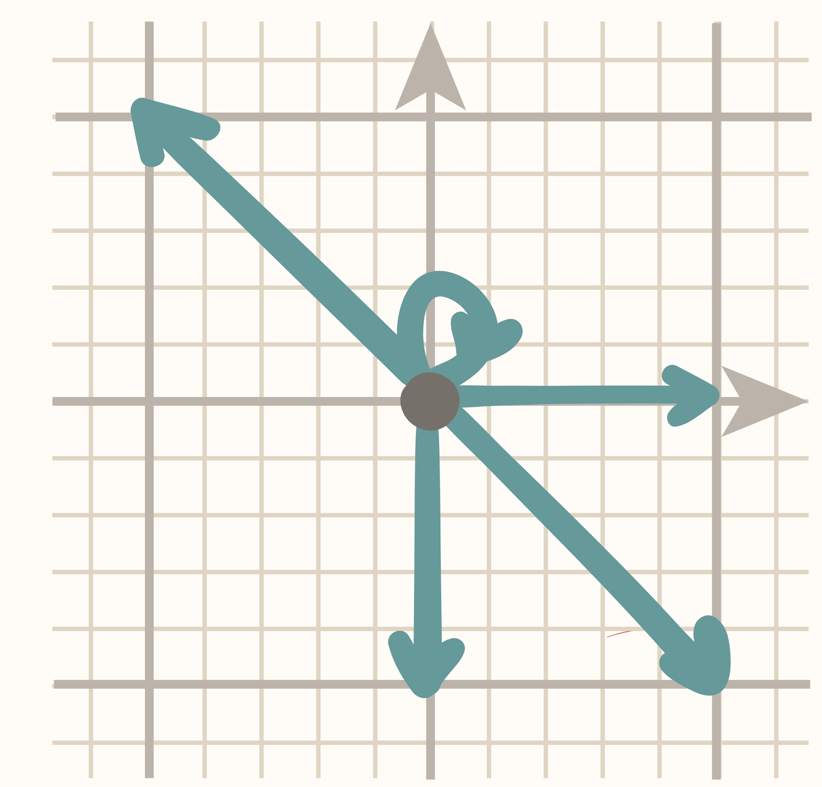
$$\mathbf{E}[X] = \mathbf{E}[Y] = 0$$

And the covariance matrix is identity.

Asymptotic counting results

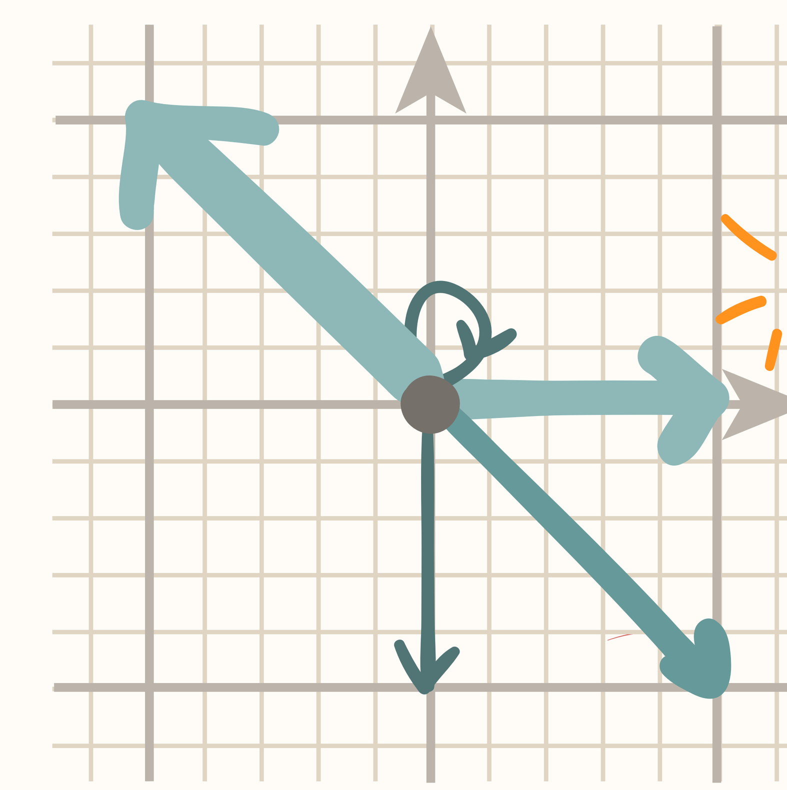






- *Random walks in cone,* D. Denisov & V. Wachtel (2015)
- *Non-D-finite excursions in the quarter plane,* D. Bostan, K. Raschel, B. Salvy (2012)

$$a_n \sim \varkappa \cdot \gamma^n n^{-1-\pi/\theta}$$



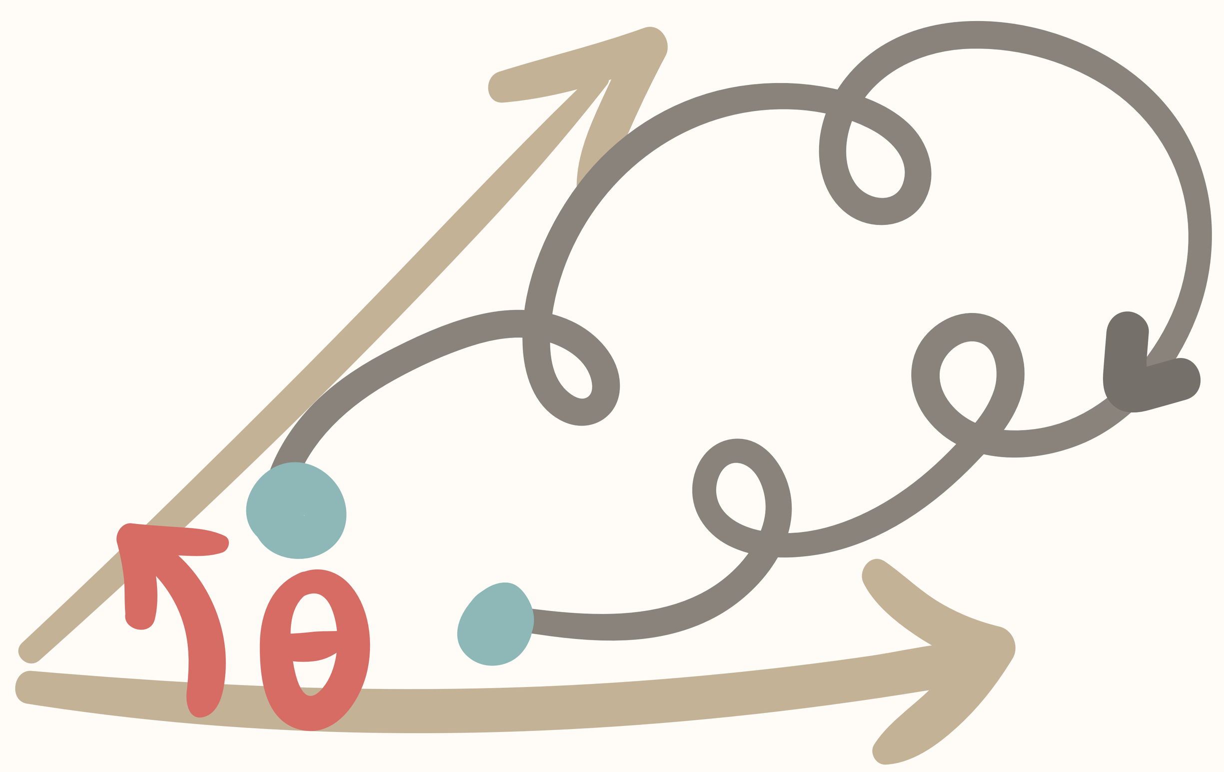
Weighted steps

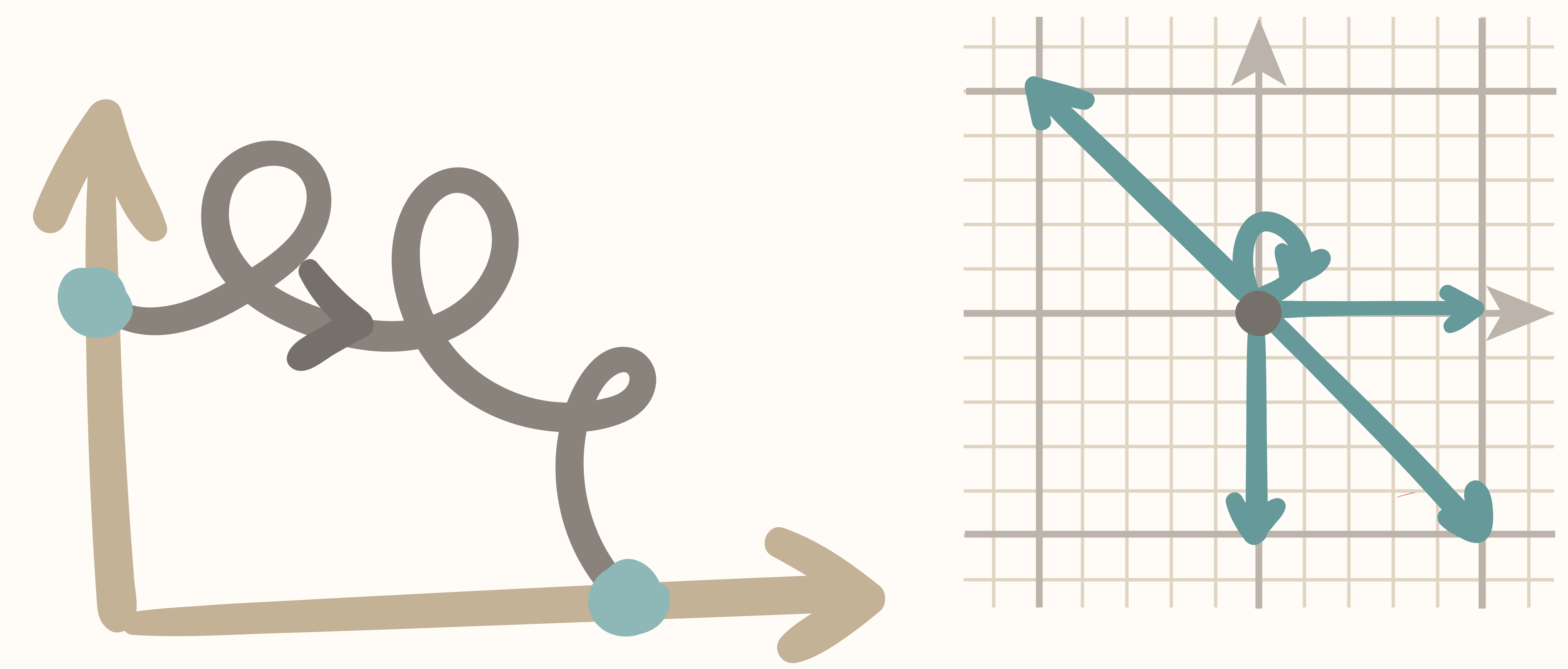
If the drift is zero, *i.e.* :

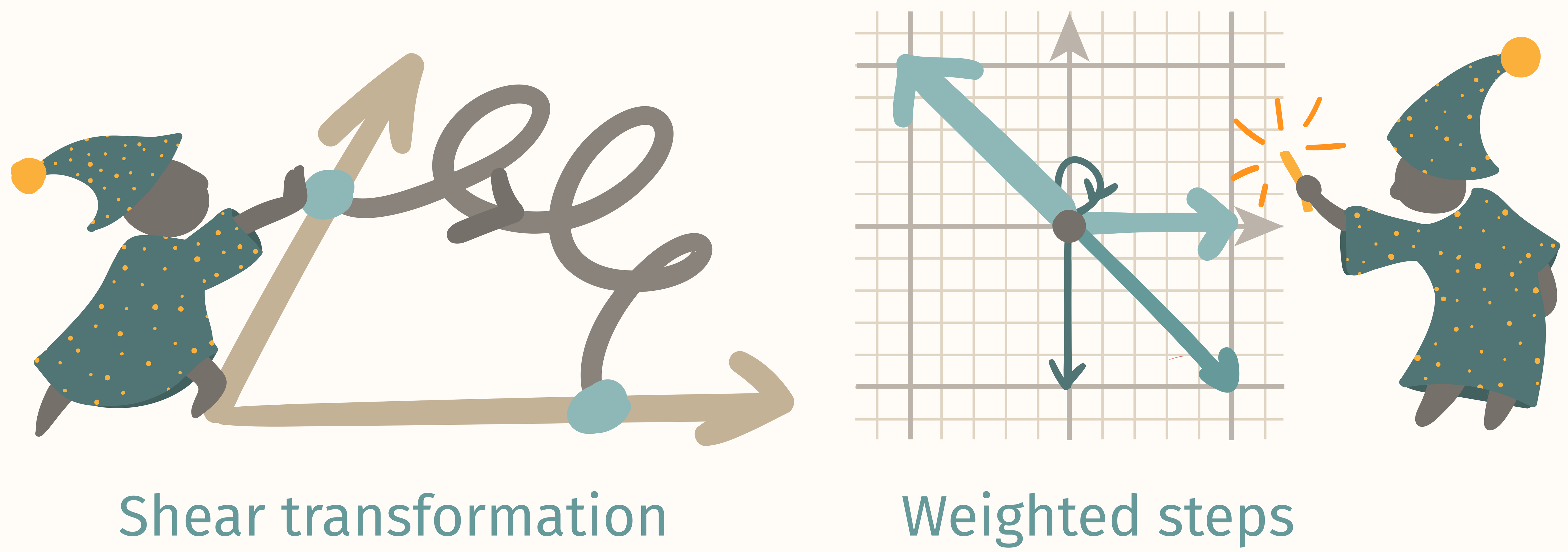
$$\mathbf{E}[X] = \mathbf{E}[Y] = 0$$

And the covariance matrix is identity.

Asymptotic counting results







- *Random walks in cone*, D. Denisov & V. Wachtel (2015)
- *Non-D-finite excursions in the quarter plane*, D. Bostan, K. Raschel, B. Salvy (2012)

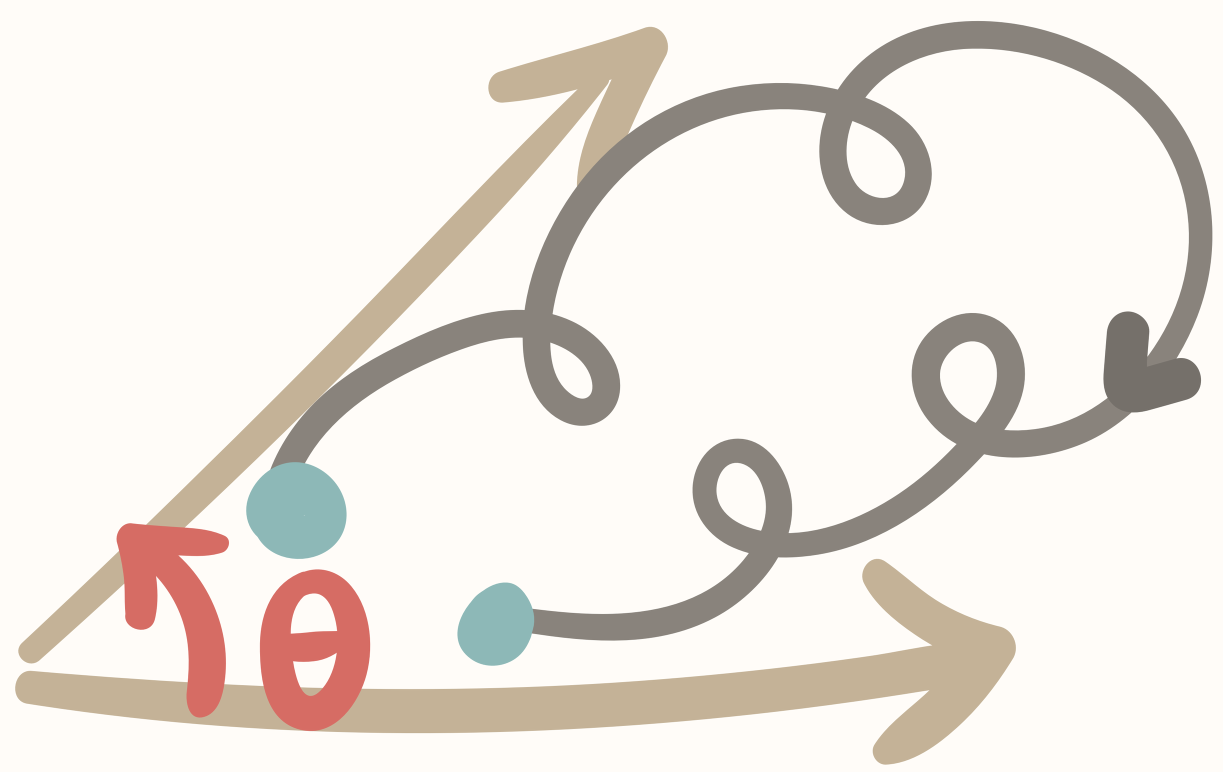
$$a_n \sim \varkappa \cdot \gamma^n n^{-1-\pi/\theta}$$

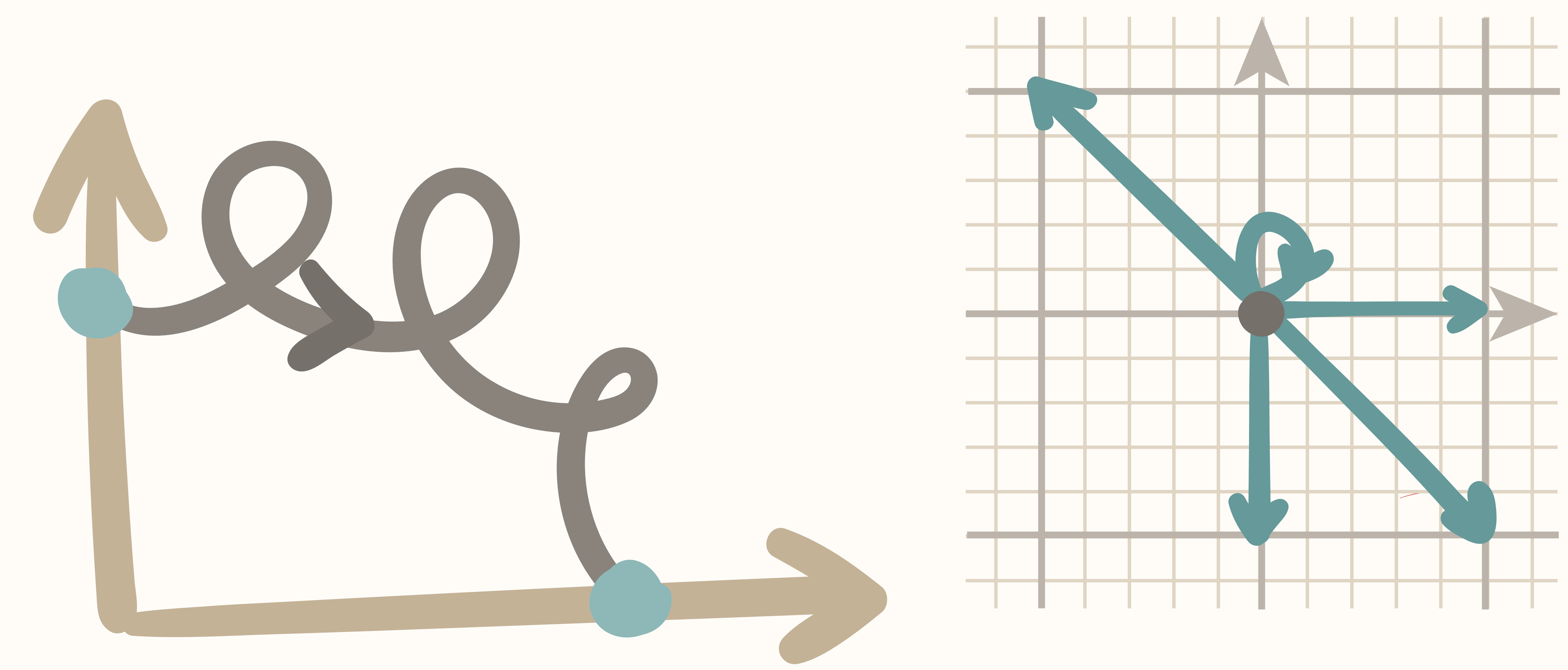
If the drift is zero, *i.e.* :

$$\mathbf{E}[X] = \mathbf{E}[Y] = 0$$

And the covariance matrix is identity.

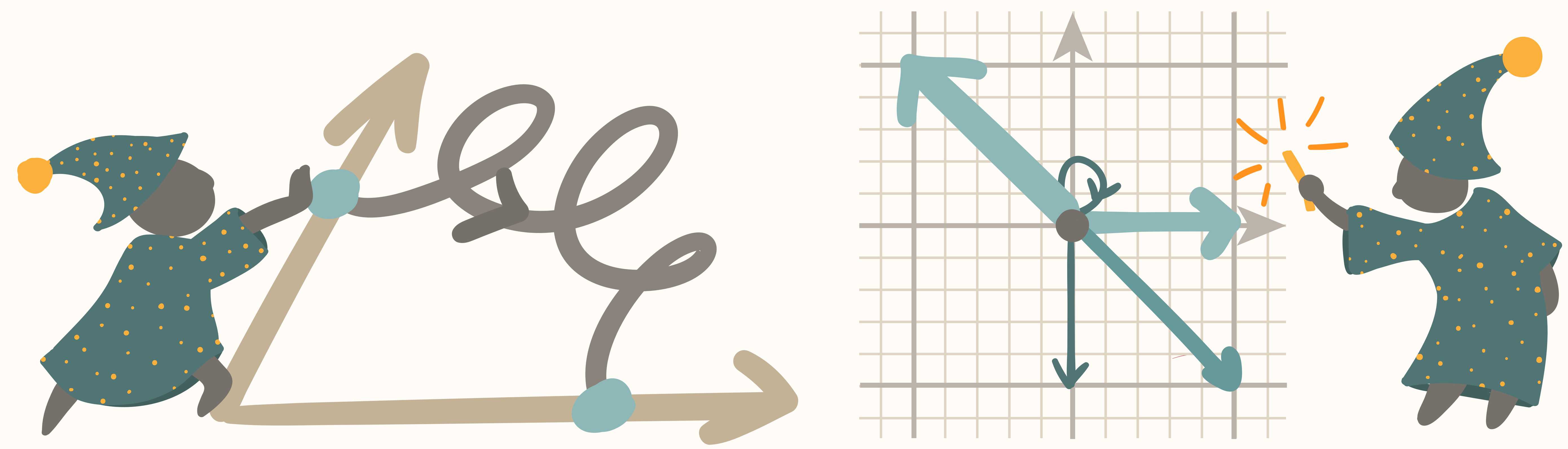
Asymptotic counting results





- *Random walks in cone,* D. Denisov & V. Wachtel (2015)
- *Non-D-finite excursions in the quarter plane,* D. Bostan, K. Raschel, B. Salvy (2012)

$$a_n \sim \varkappa \cdot \gamma^n n^{-1-\pi/\theta}$$



Shear transformation

Weighted steps

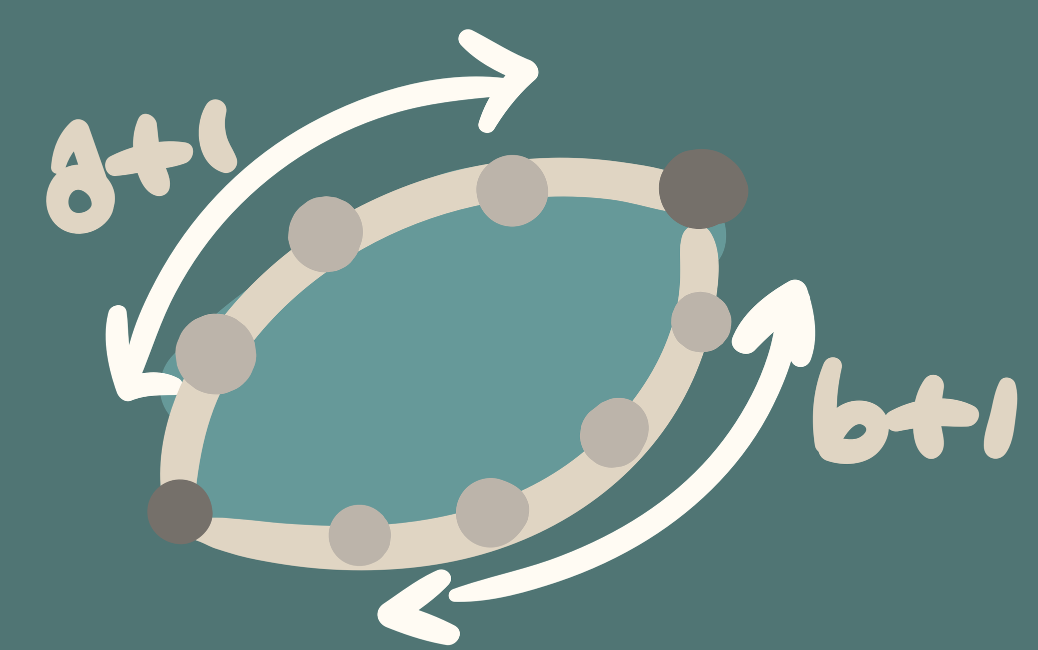
If the drift is zero, *i.e.* :

$$\mathbf{E}[X] = \mathbf{E}[Y] = 0$$

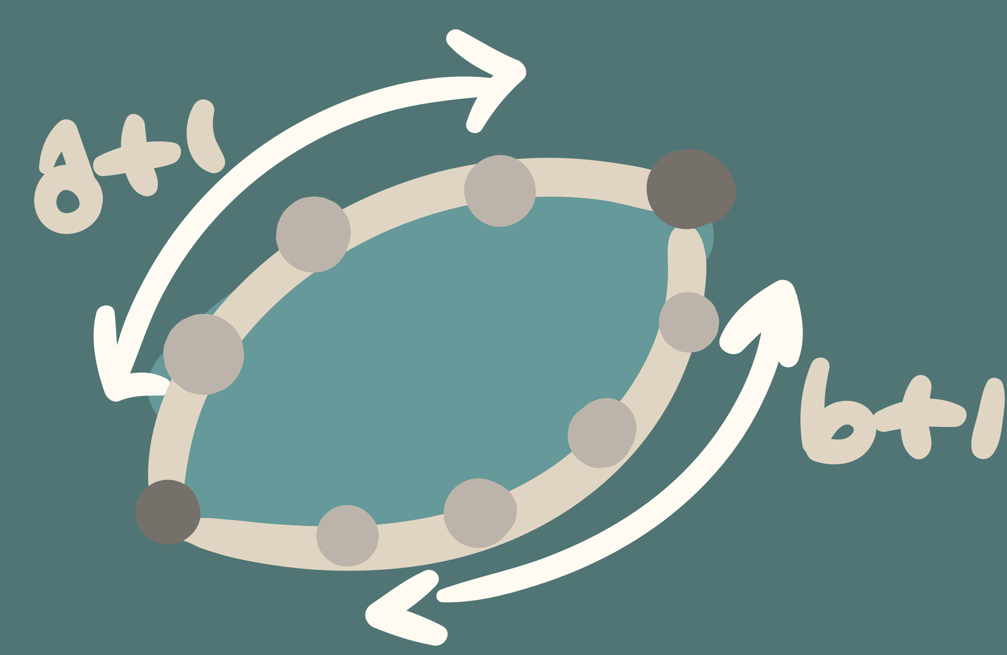
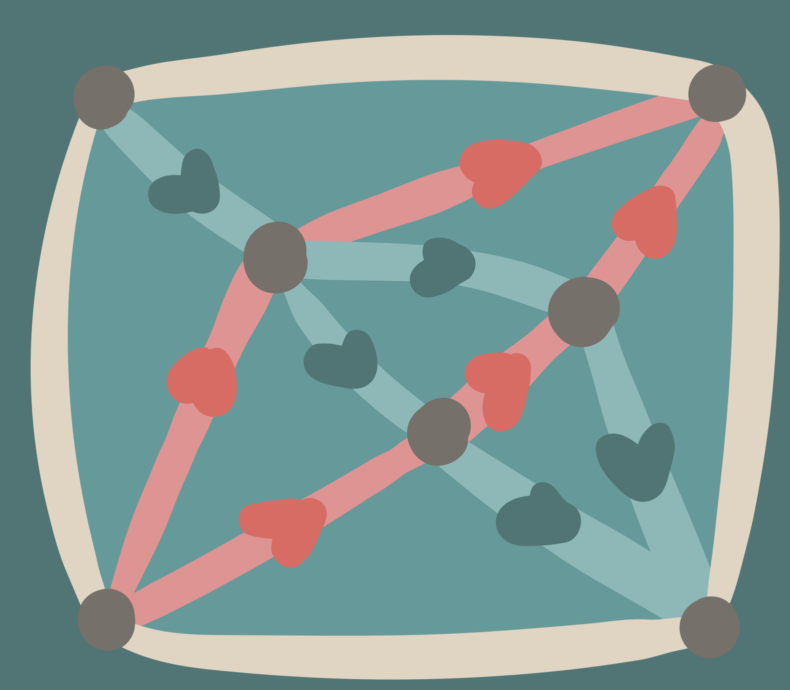
And the covariance matrix is identity.

Asymptotics

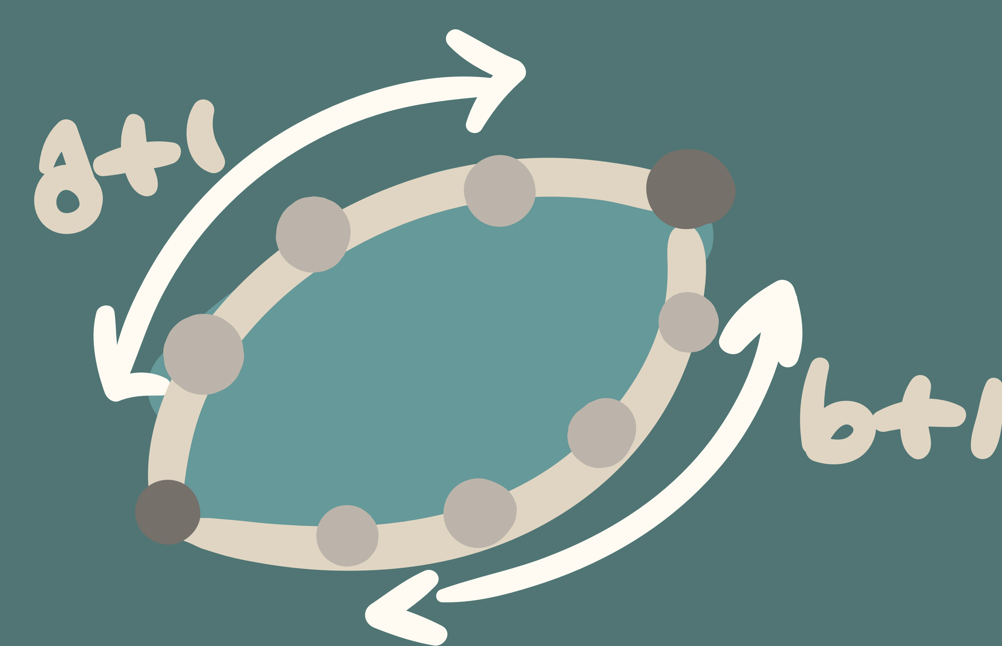
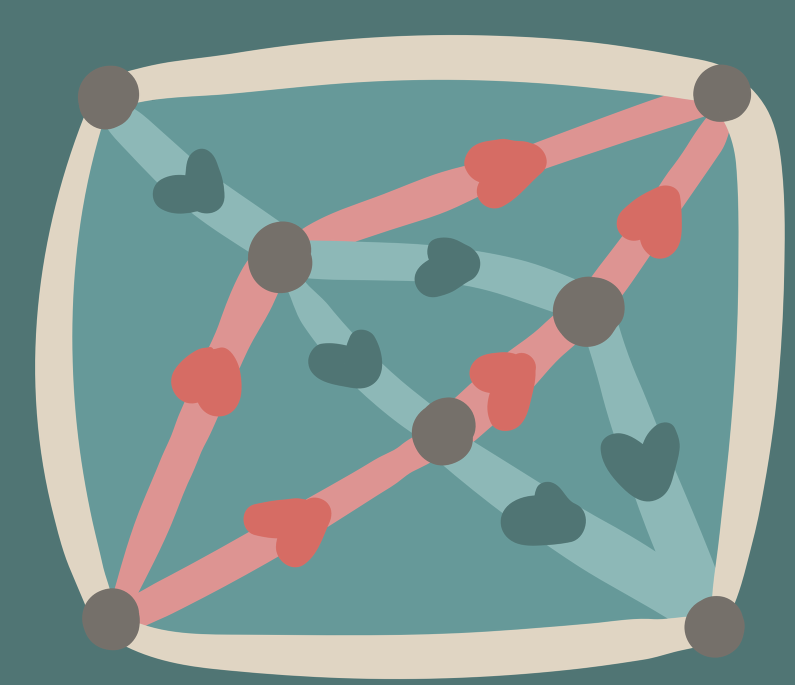
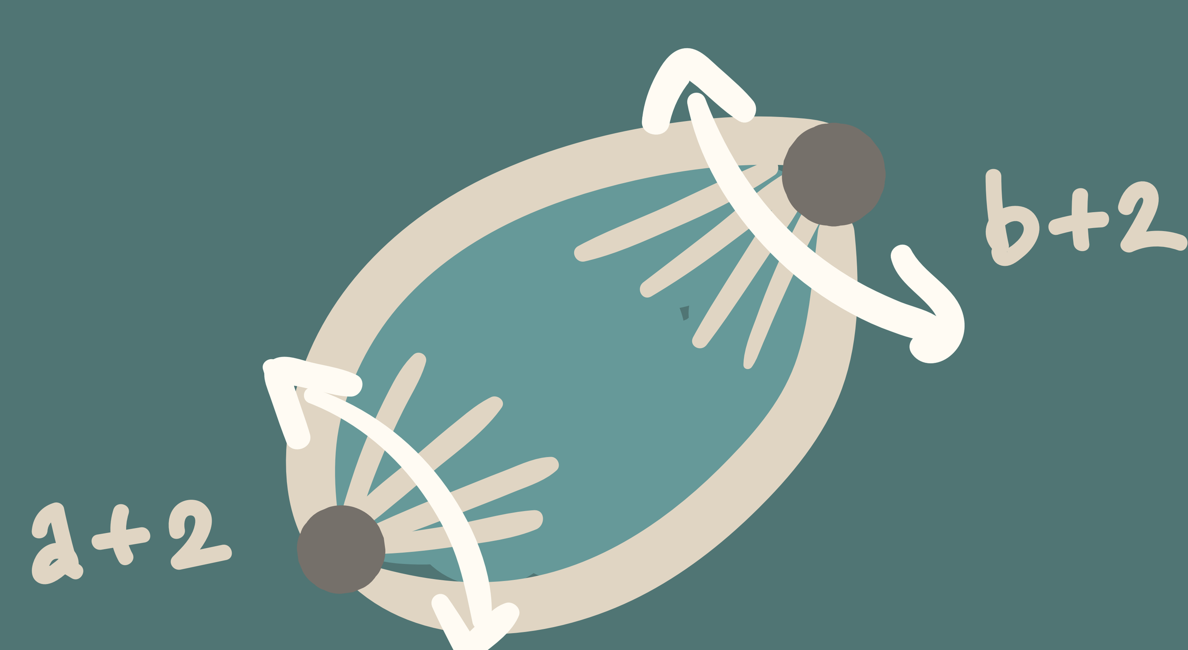
Asymptotic counting results

Model	*Asymptotics*
Posets *n+2 edges*  	$e_n \sim \varkappa\, \gamma^n n^{-\alpha}$ $\quad \alpha = -1 - \frac{\pi}{\arccos(\xi)}$ γ *and* ξ *are algebraic* $\gamma \approx 4.80\ldots$ $\alpha \approx 5.14\ldots$

Asymptotic counting results

Model		*Asymptotics*
Posets *n+2 edges*		$e_n \sim \varkappa\, \gamma^n n^{-\alpha}$ $\quad \alpha = -1 - \frac{\pi}{\arccos(\xi)}$ γ *and* ξ *are algebraic* $\gamma \approx 4.80\ldots \quad \alpha \approx 5.14\ldots$
Transversal structures *n vertices*		$t_n \sim \varkappa \left(\frac{27}{2}\right)^n n^{-1-\frac{\pi}{\arccos(7/8)}}$ ↠ *Counting rectangular drawings,* Y. Inoue, T. Takahashi & R. Fujimaki (2009)

Asymptotic counting results

Model		***Asymptotics***
Posets *n+2 edges*		$e_n \sim \varkappa\, \gamma^n n^{-\alpha}$ $\quad \alpha = -1 - \frac{\pi}{\arccos(\xi)}$ γ *and* ξ *are algebraic* $\gamma \approx 4.80\ldots$ $\alpha \approx 5.14\ldots$
Transversal structures *n vertices*		$t_n \sim \varkappa \left(\frac{27}{2}\right)^n n^{-1-\frac{\pi}{\arccos(7/8)}}$ ↠ *Counting rectangular drawings,* Y. Inoue, T. Takahashi & R. Fujimaki (2009)
Posets *n vertices*		$b_n \sim \varkappa \left(\frac{11+\sqrt{5}}{2}\right)^n n^{-6}$

Asymptotic counti

Mod

$\frac{\pi}{\cos(\xi)}$

Pose

n ver

POSETS PER VERTEX

$$b_n \sim \varkappa \left(\frac{11+\sqrt{5}}{2}\right)^n \cdot n^{-6}$$

Asymptotic counti

Mod
Pose   *n ver*

POSETS PER VERTEX

$$b_n \sim \varkappa \left(\frac{11+\sqrt{5}}{2}\right)^n \cdot n^{-6}$$

PLANE PERMUTATIONS

$$p_n \sim \varkappa \left(\frac{11+\sqrt{5}}{2}\right)^n \cdot n^{-6}$$

≫ *Semi-Baxter and strong-Baxter: two relatives of Baxter Sequences,* **M. Bouvel, V. Guerrini, A. Rechnitzer & S. Rinaldi (2018)**

Asymptotic counti

Mod

Pose

n ver

POSETS PER VERTEX

$$b_n \sim \varkappa \left(\frac{11+\sqrt{5}}{2}\right)^n \cdot n^{-6}$$

1, 1, 2, 6, 23, 104, 530, 2958, 17734, 112657, …

PLANE PERMUTATIONS

$$p_n \sim \varkappa \left(\frac{11+\sqrt{5}}{2}\right)^n \cdot n^{-6}$$

↠ Semi-Baxter and strong-Baxter : two relatives of Baxter Sequences,
M. Bouvel, V. Guerrini, A. Rechnitzer & S. Rinaldi (2018)

Asymptotic counti

Mod

Pose
n ver

Asymptotic counti

Mod

Pose
n ver

Summary

Maps and decorated maps

1. Bijection with quadrant tandem walks

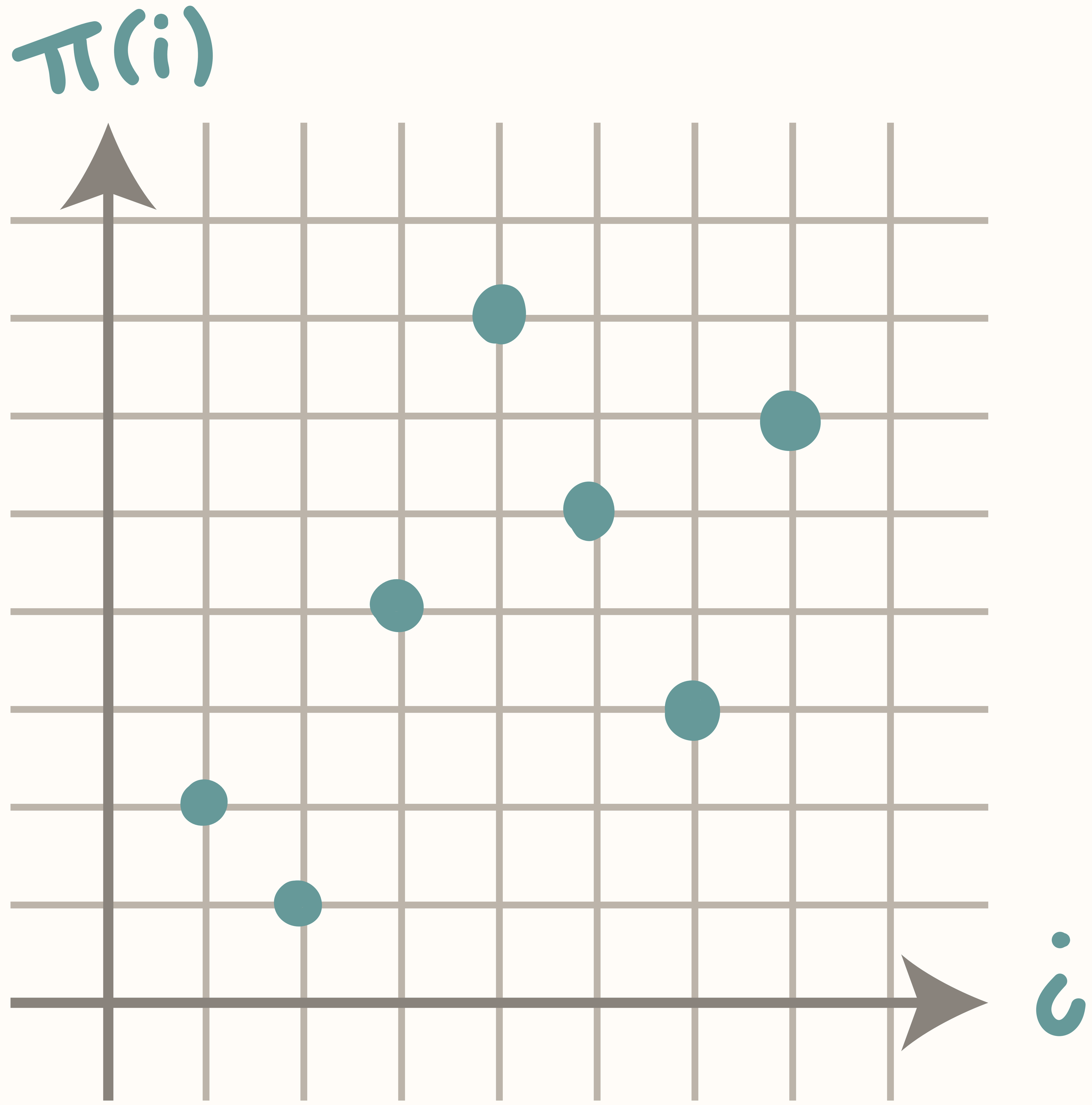
a. The KMSW bijection
b. Plane bipolar posets
c. Plane bipolar posets by vertices
d. Transversal structures

2. Asymptotic enumeration

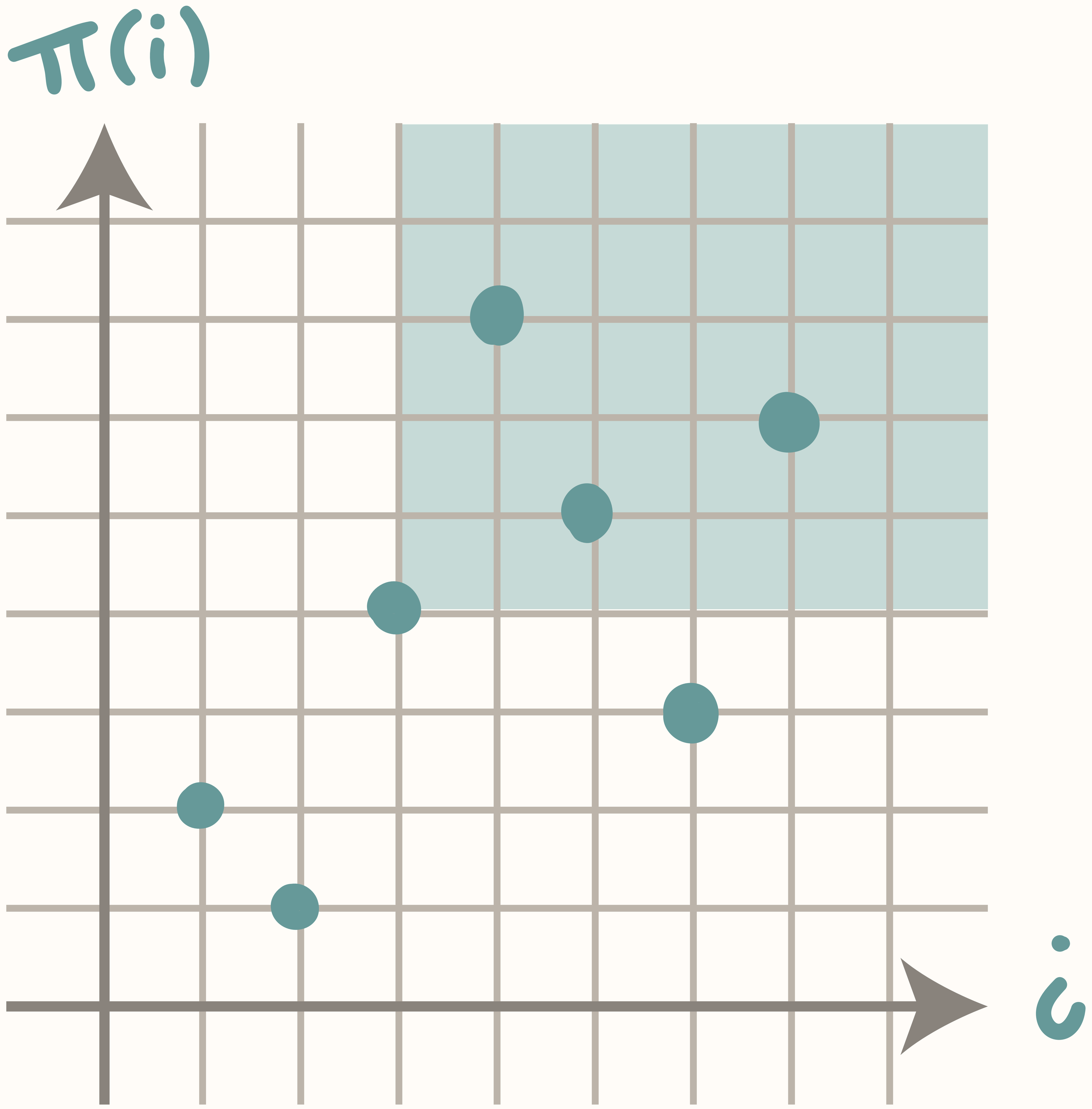
(Digression on plane permutations)

3. Generic transversal structures

Plane permutations

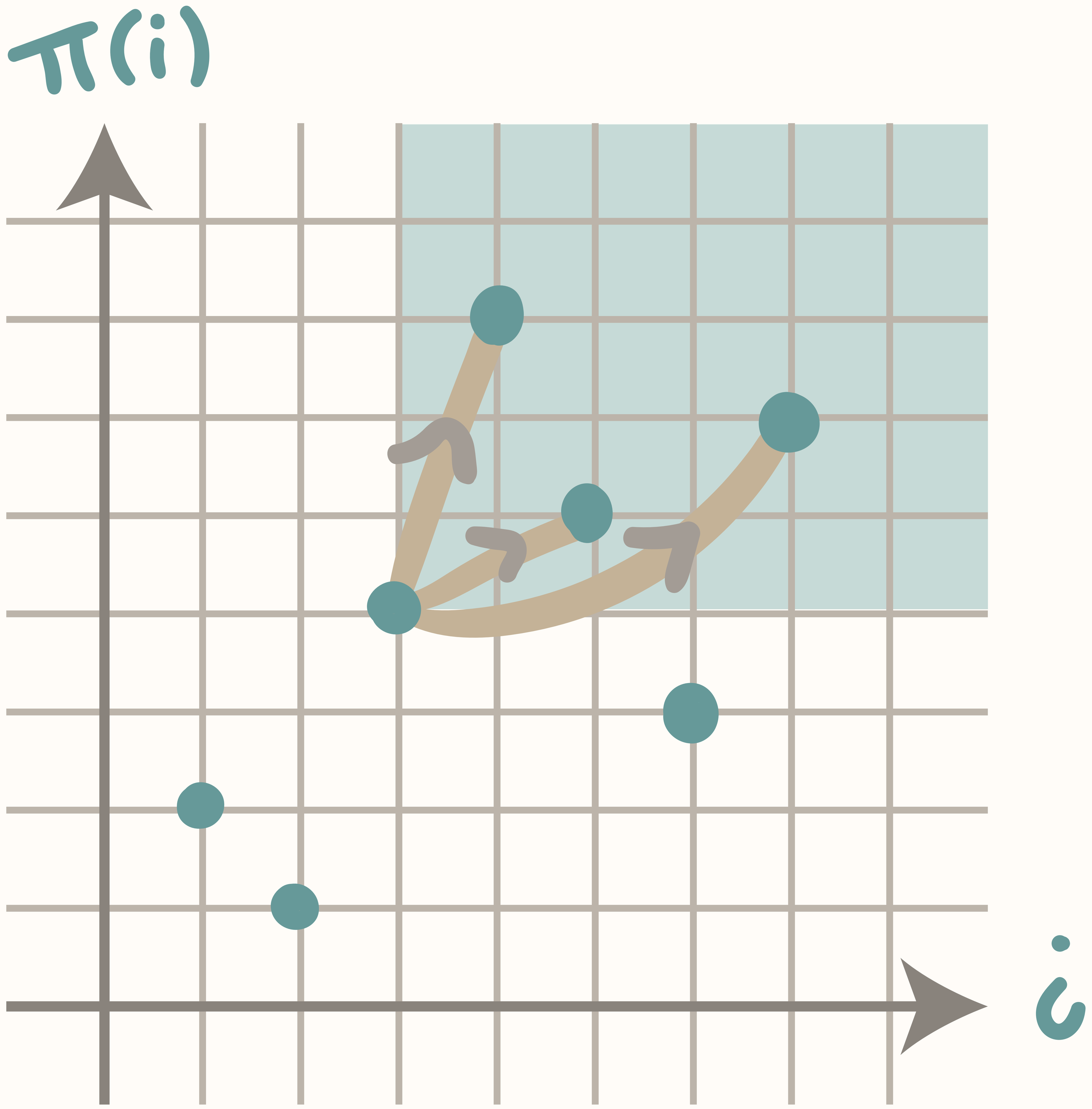


Plane permutations



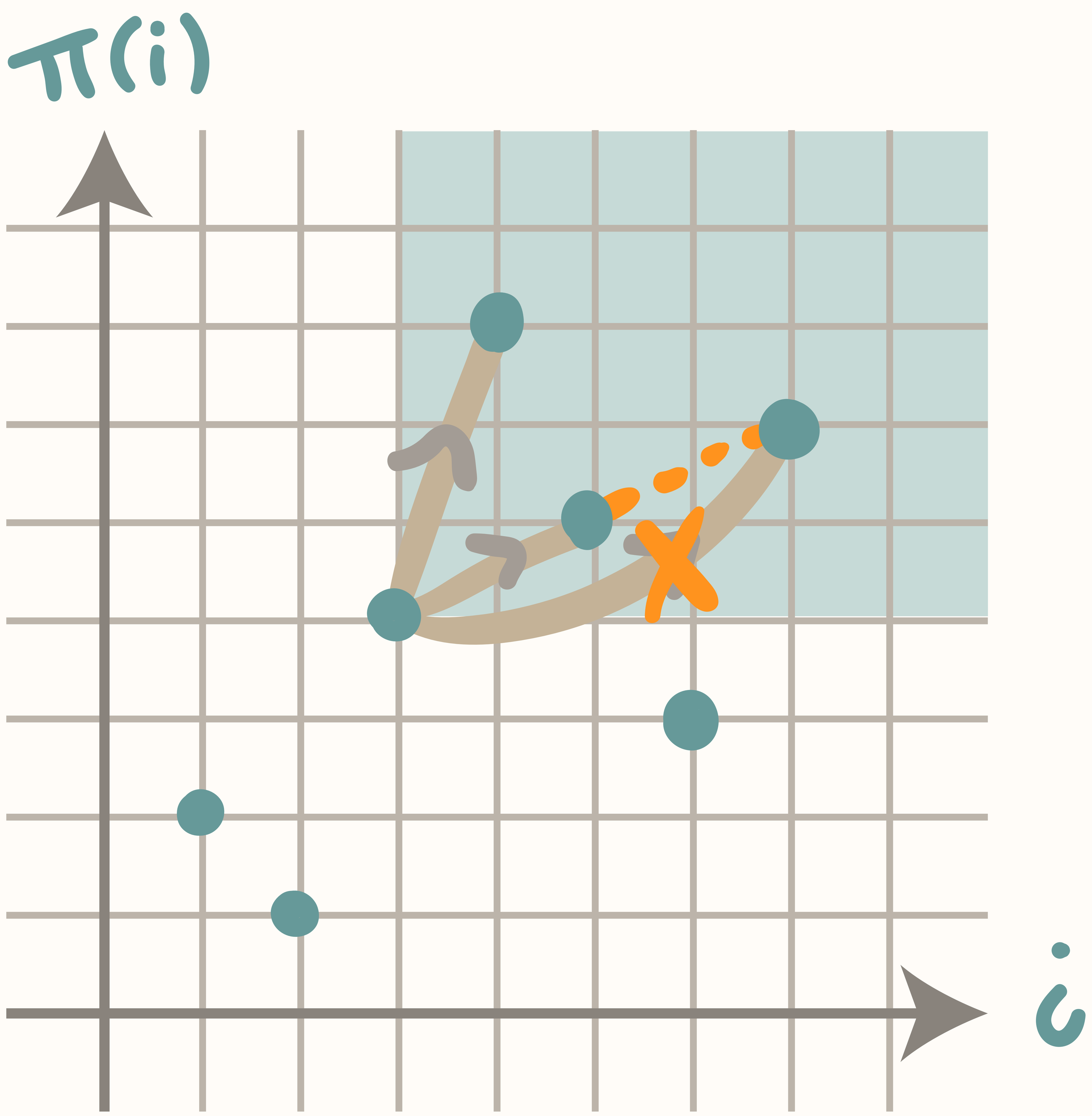
Dominance relation

Plane permutations



Dominance relation

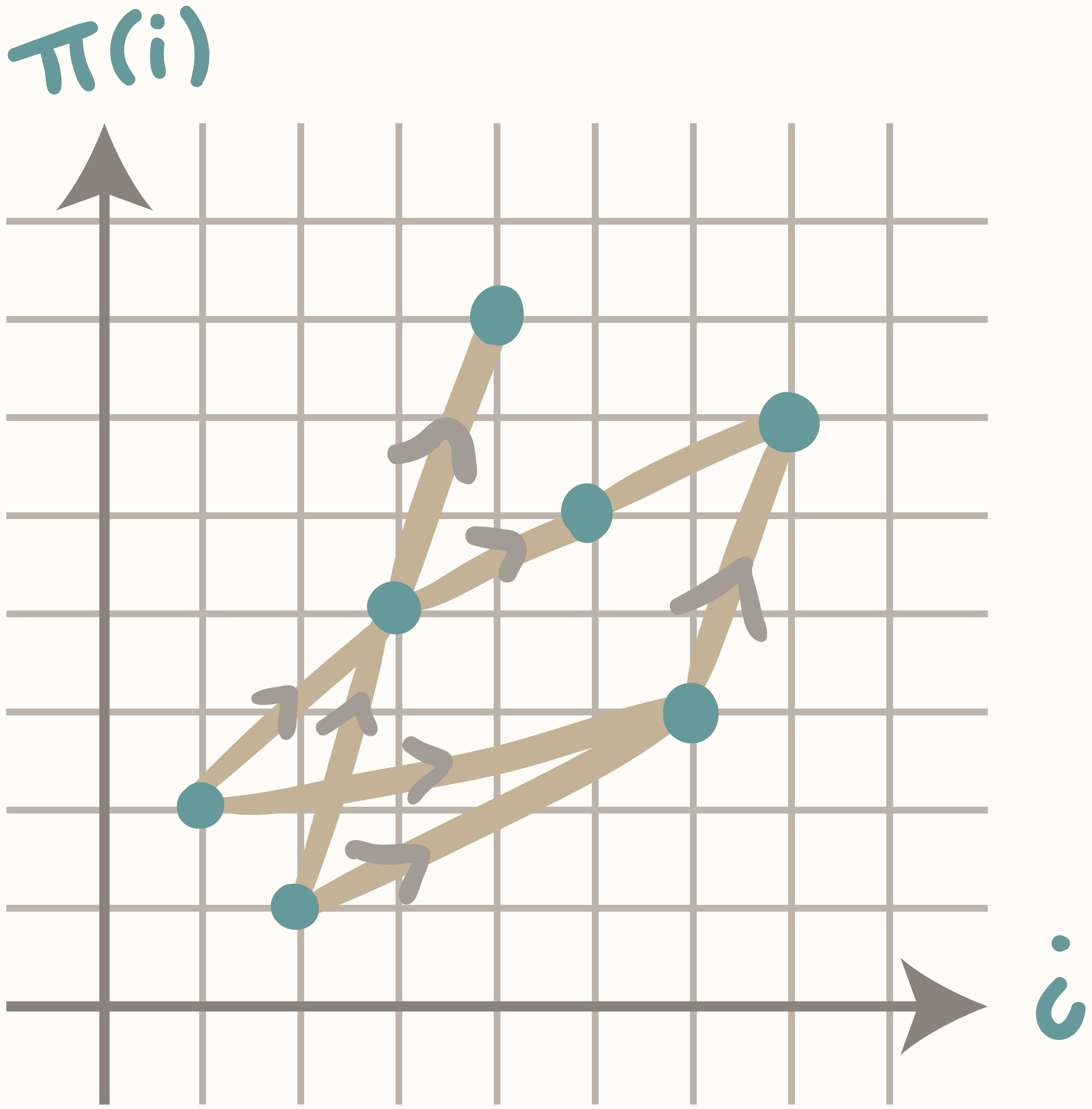
Plane permutations



Dominance diagram

= *Dominance relation with no transitive edges*

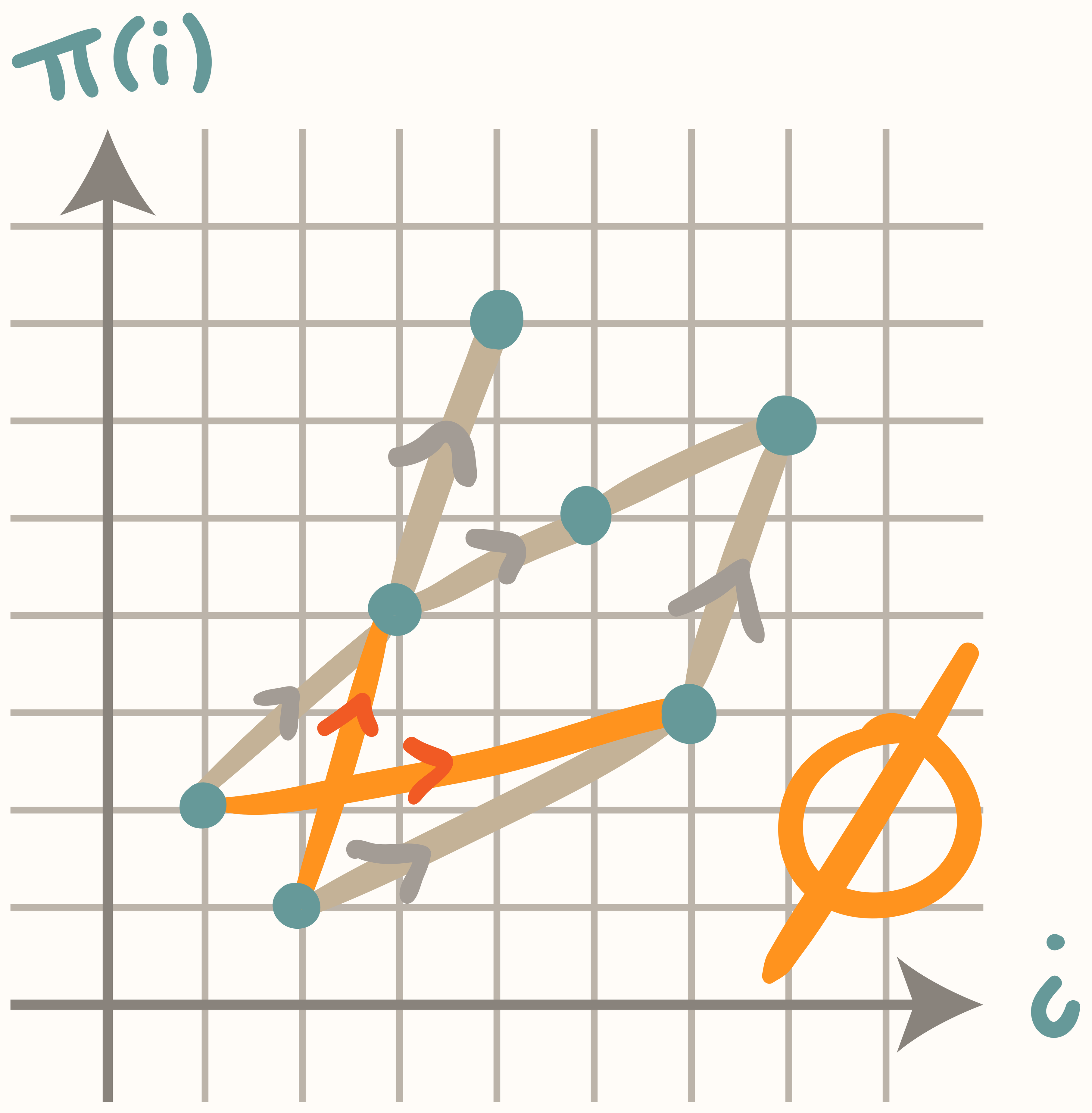
Plane permutations



Dominance diagram

= *Dominance relation*
with no transitive edges

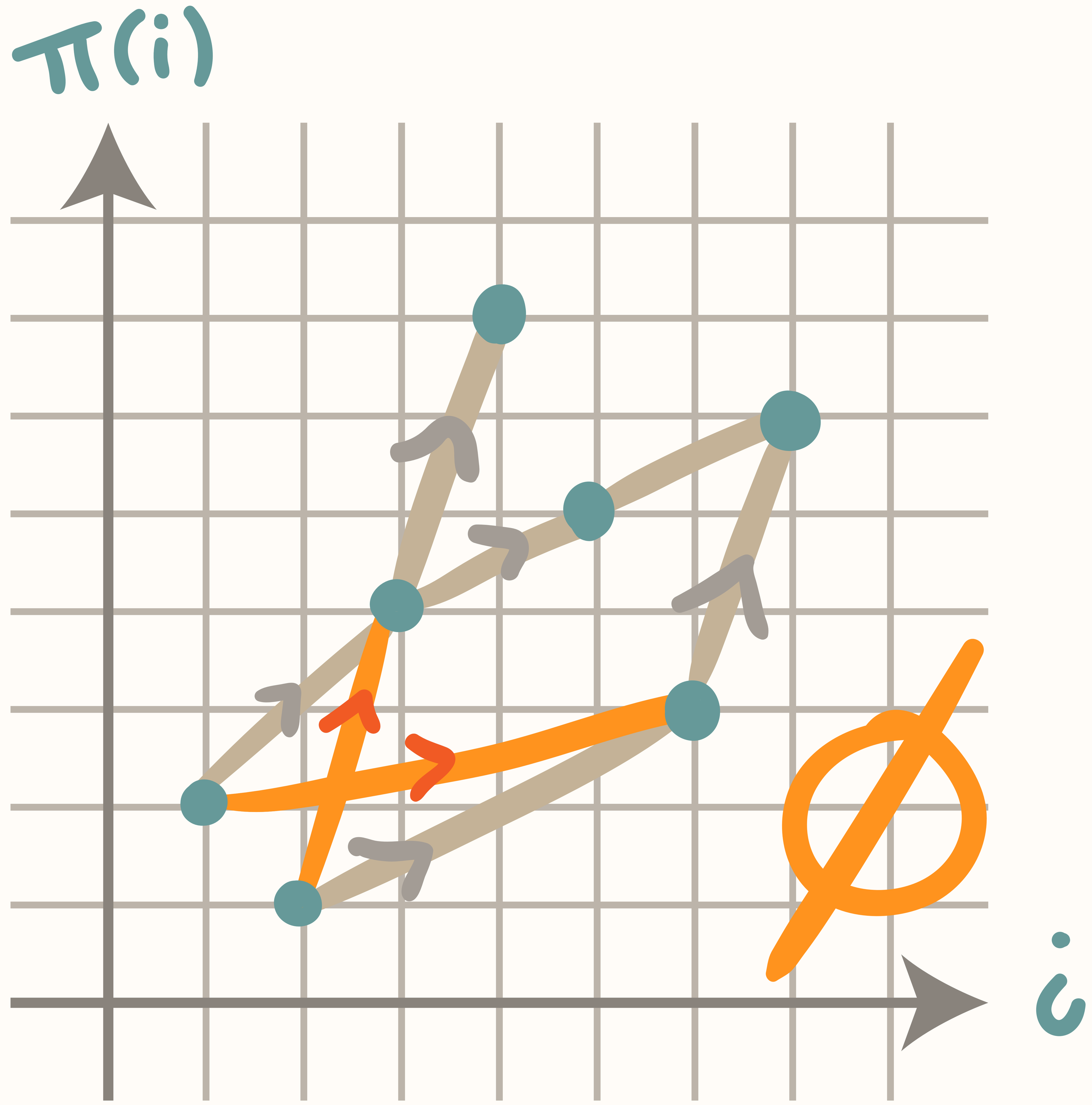
Plane permutations



Plane permutation

= *No edge crossing in the dominance diagram*

Plane permutations

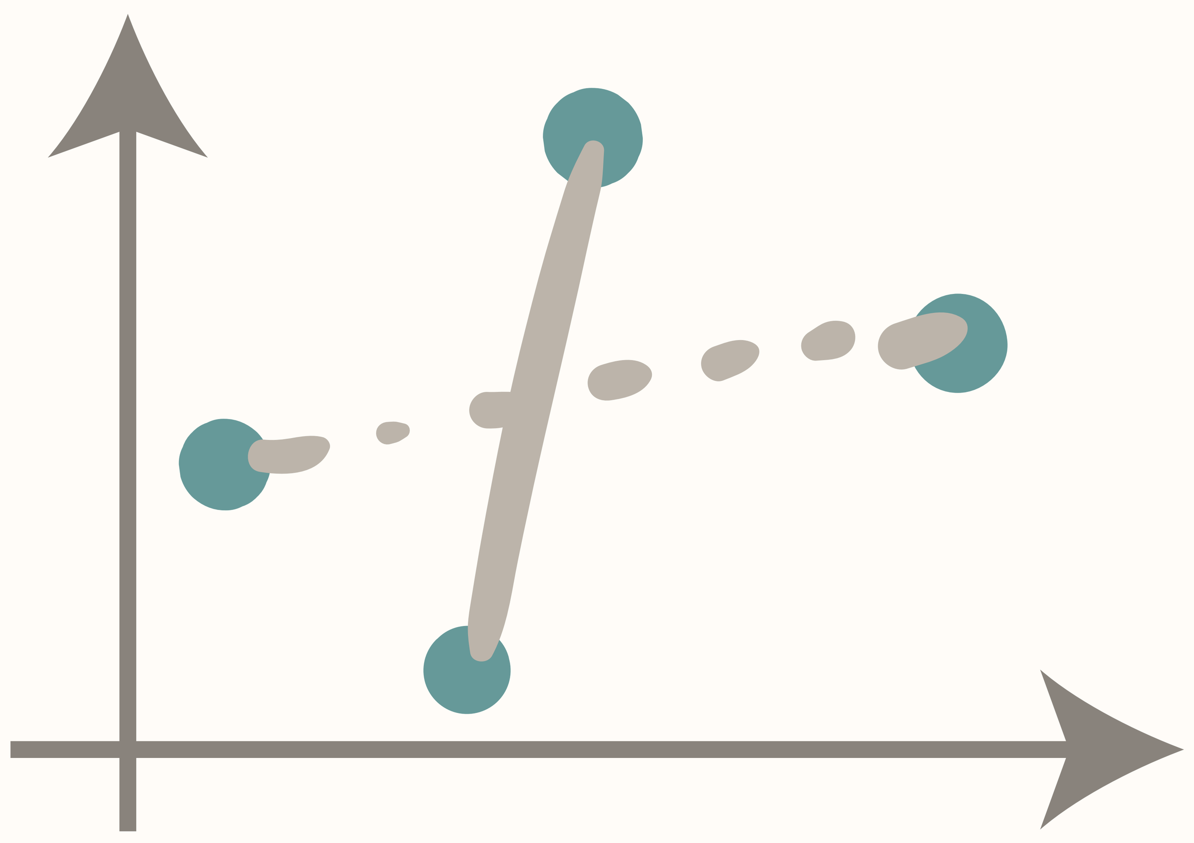


Plane permutation

= *No edge crossing in the dominance diagram*

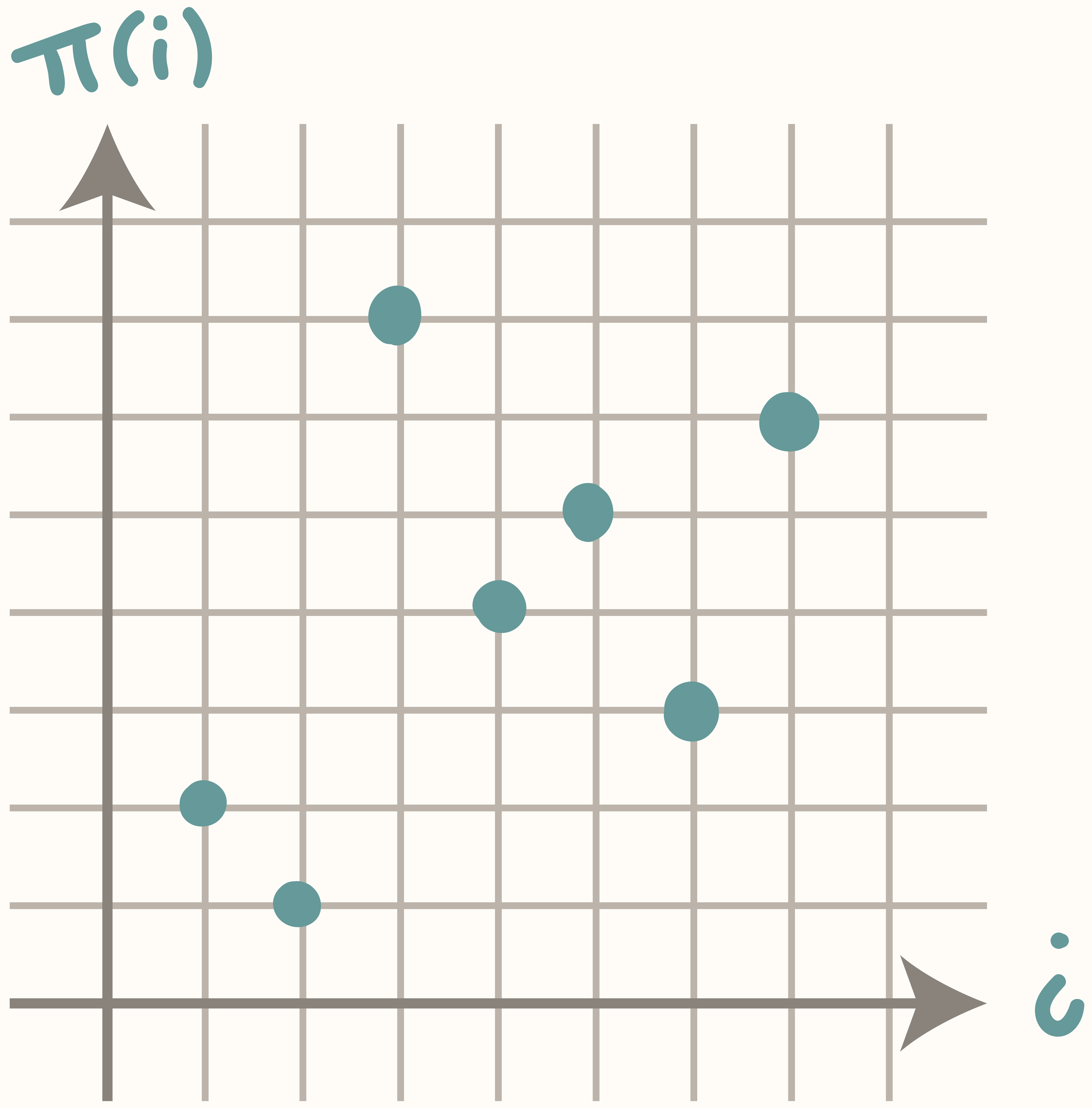
= *Avoid the vincular pattern :*

2 <u>14</u> 3



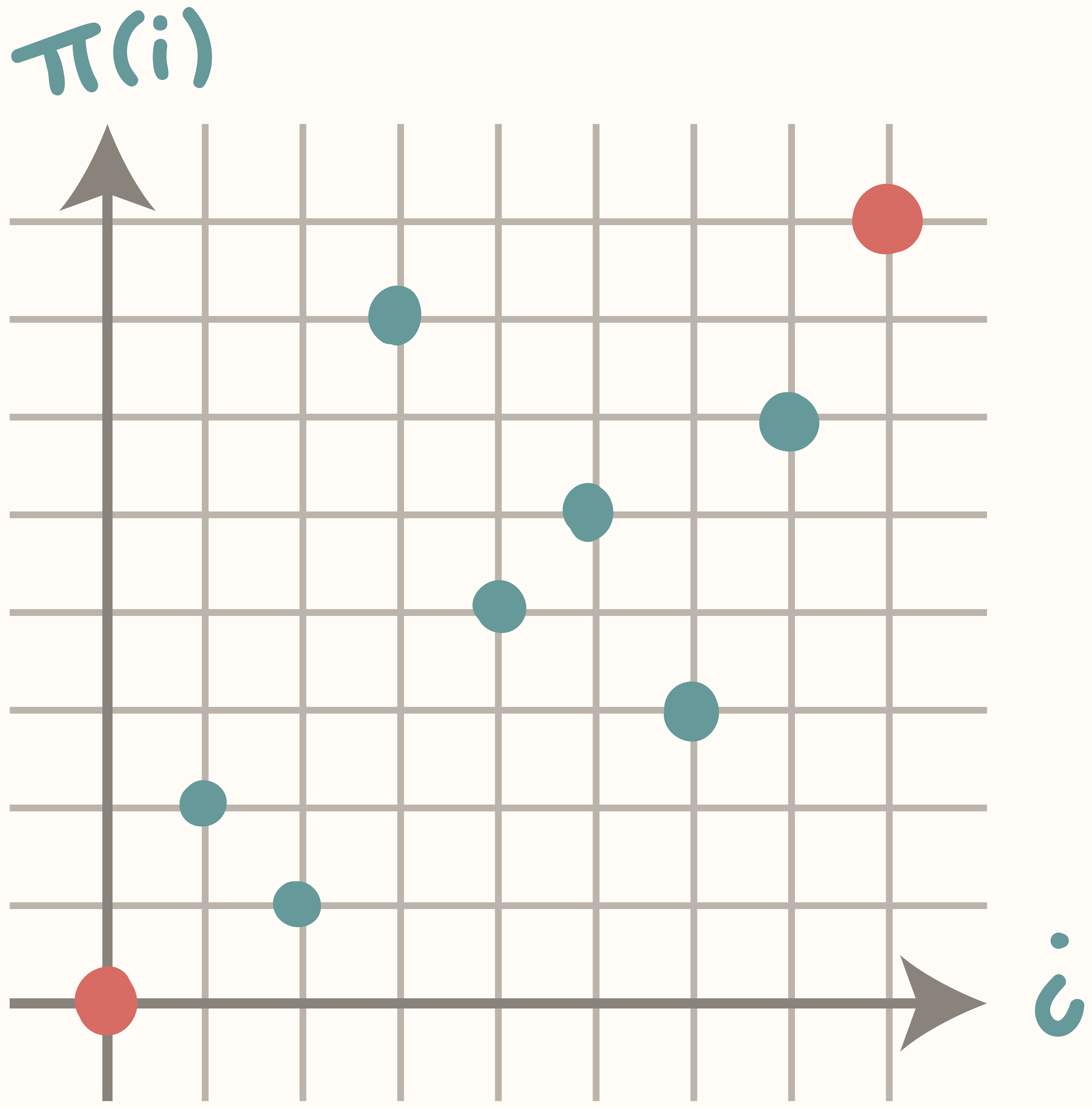
Link with plane permutations

Plane permutation ⟶ *Poset*



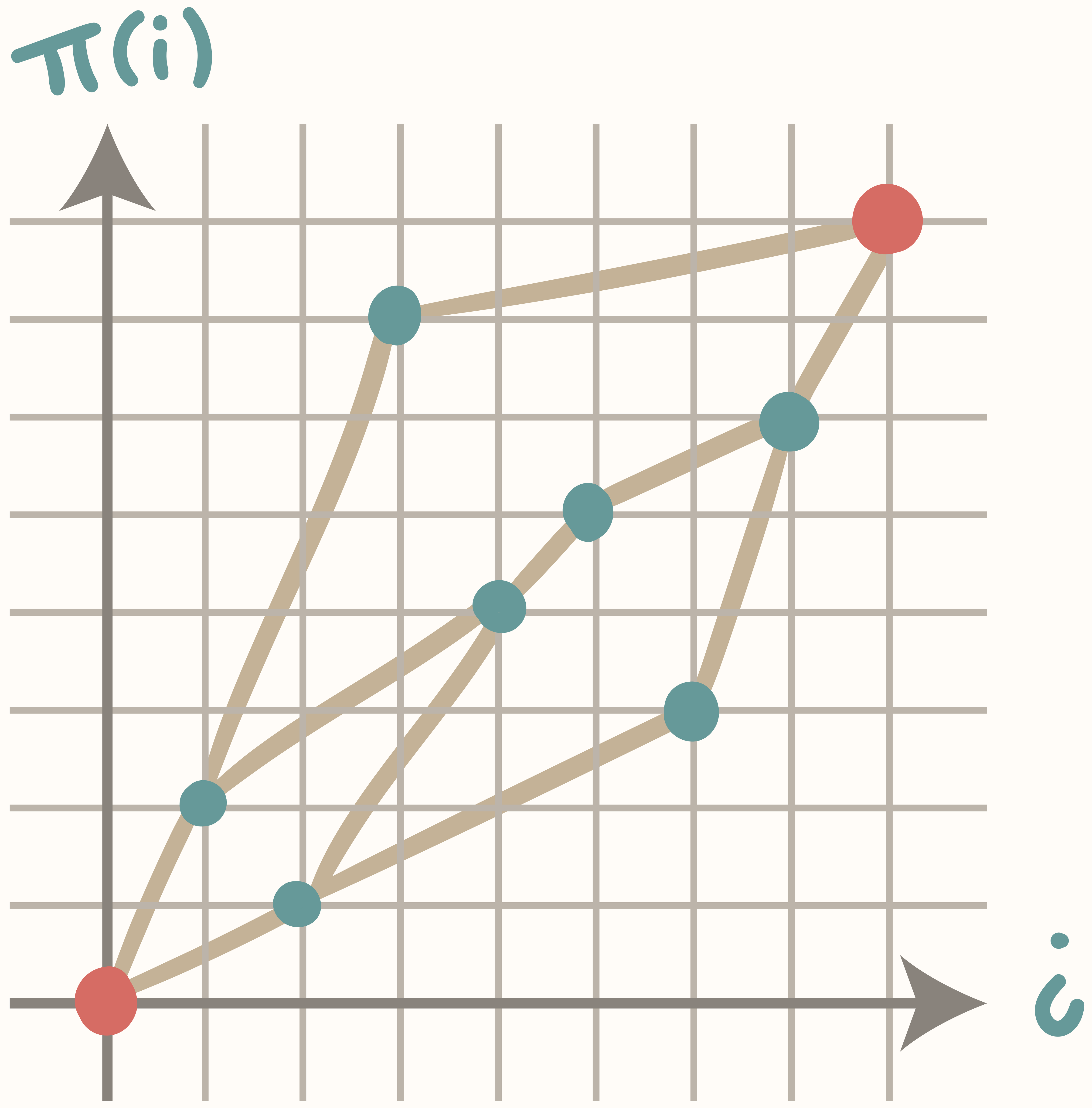
Link with plane permutations

Plane permutation → *Poset*



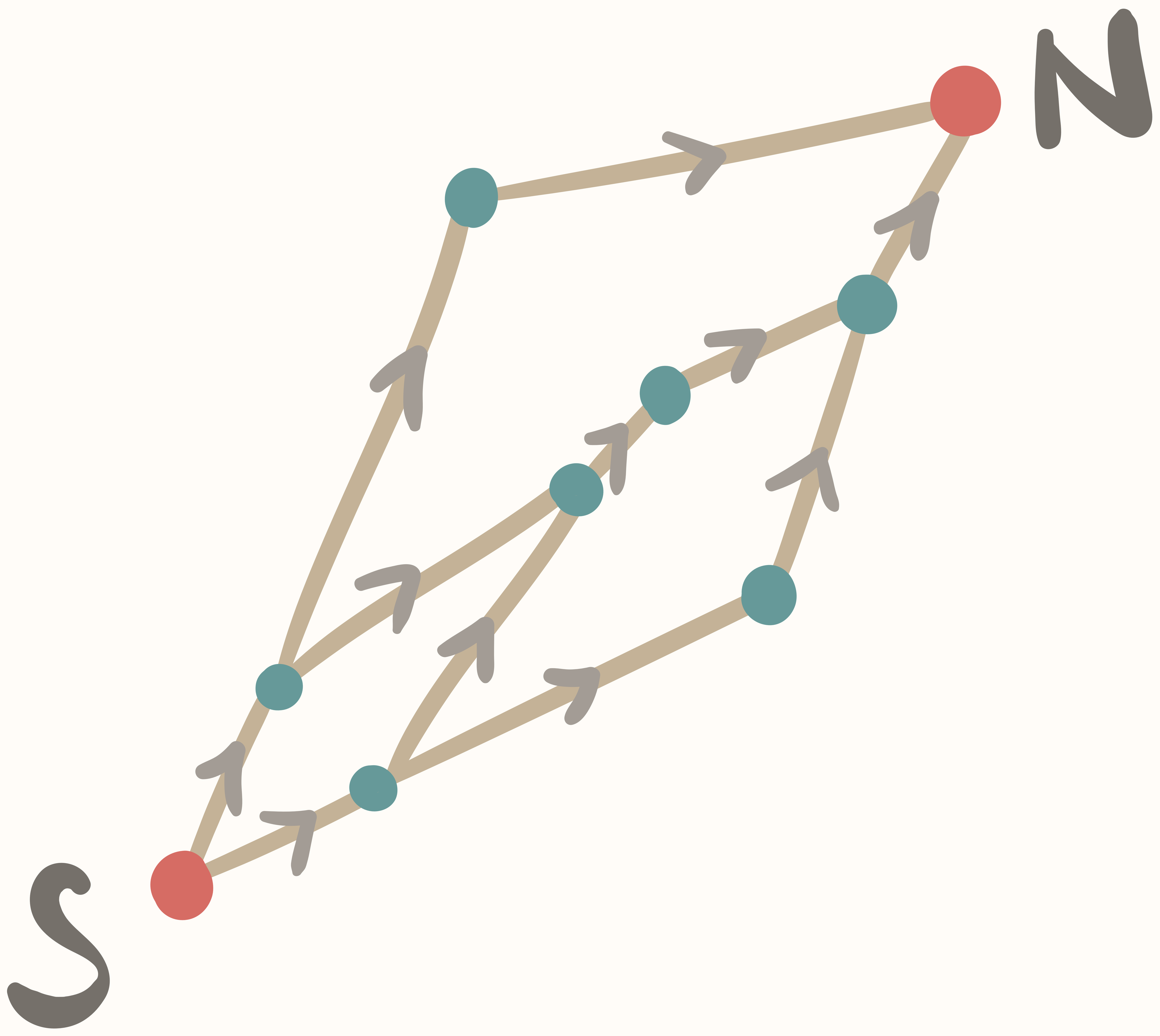
Link with plane permutations

Plane permutation ⟶ *Poset*



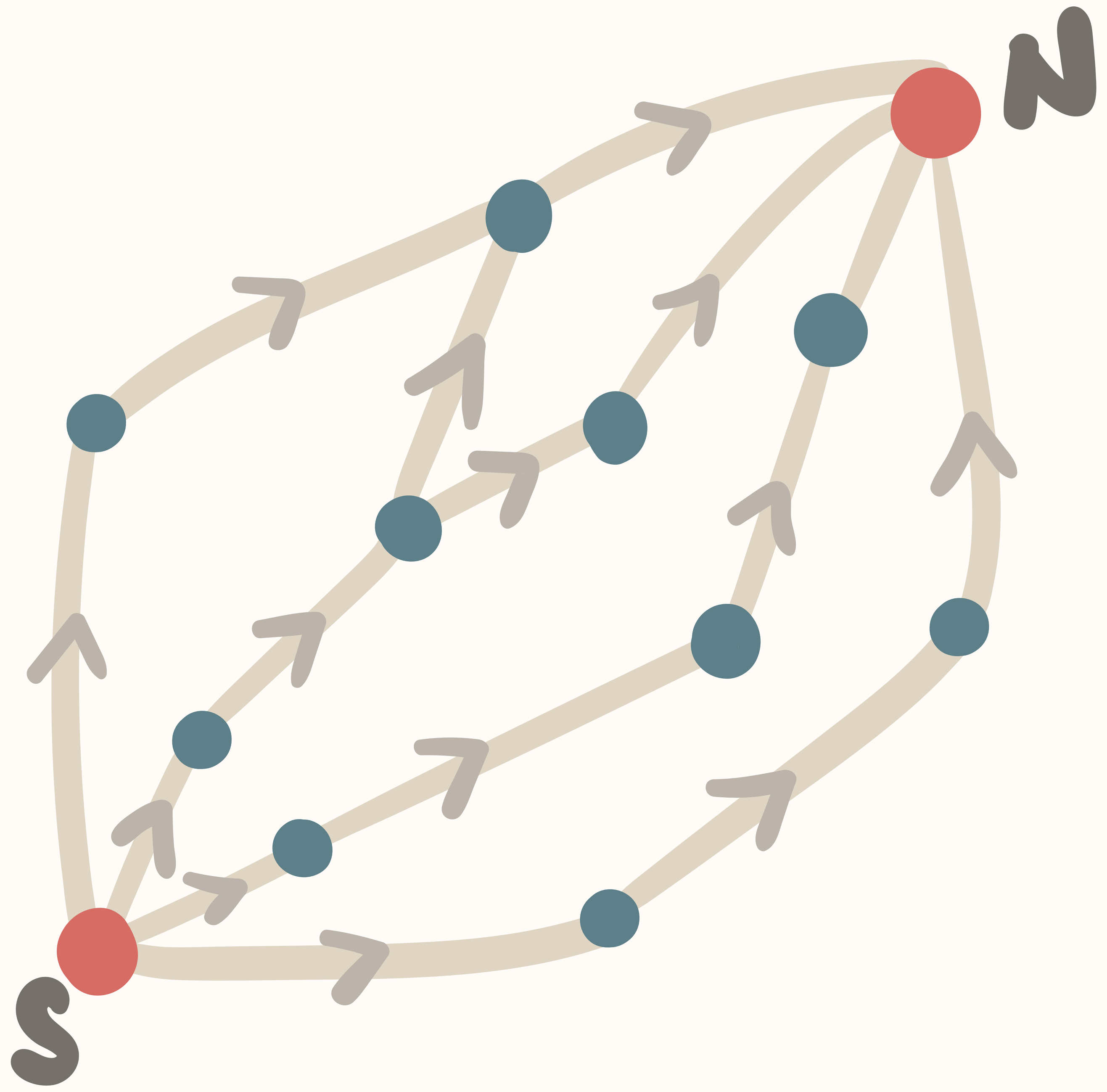
Link with plane permutations

Plane permutation → *Poset*



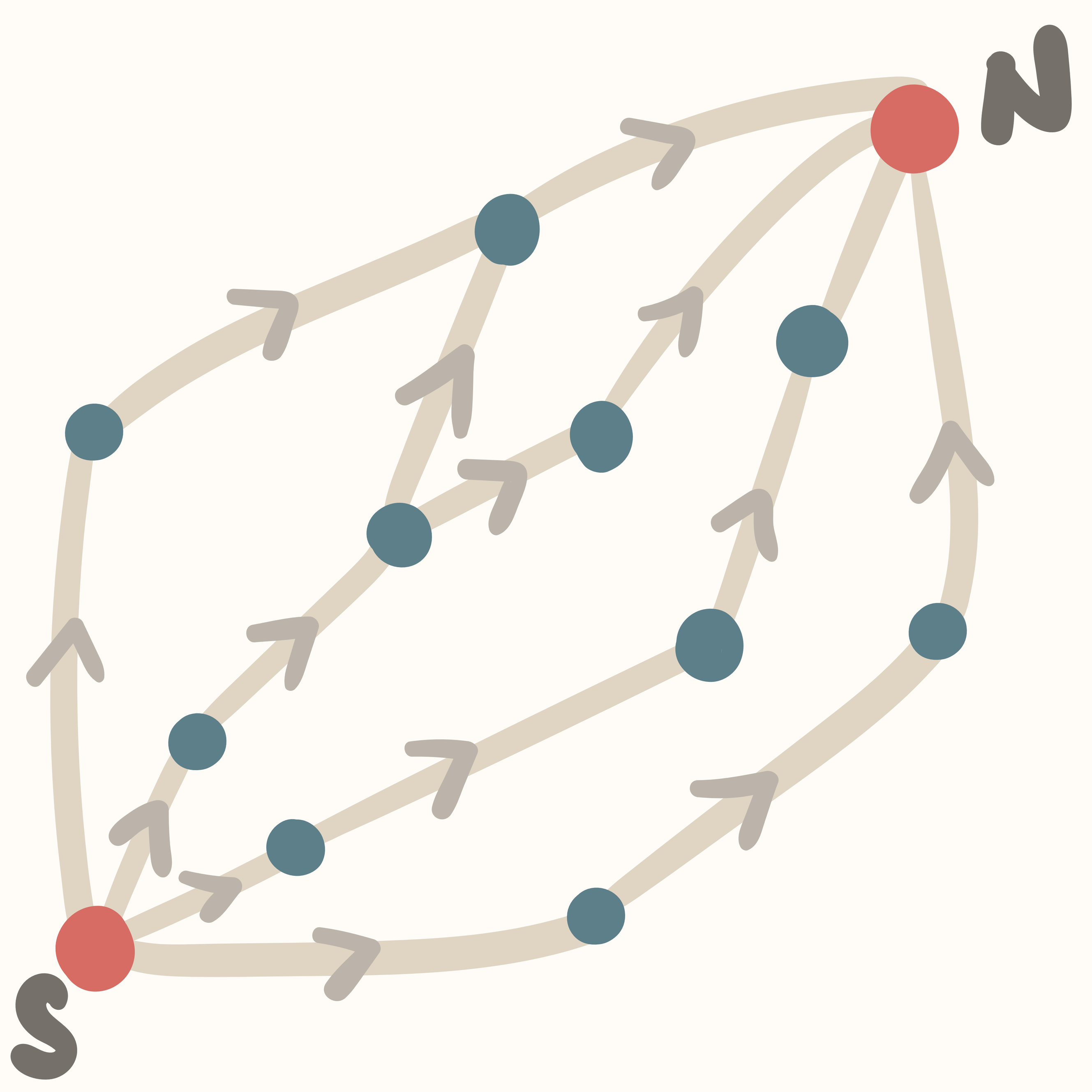
Link with plane permutations

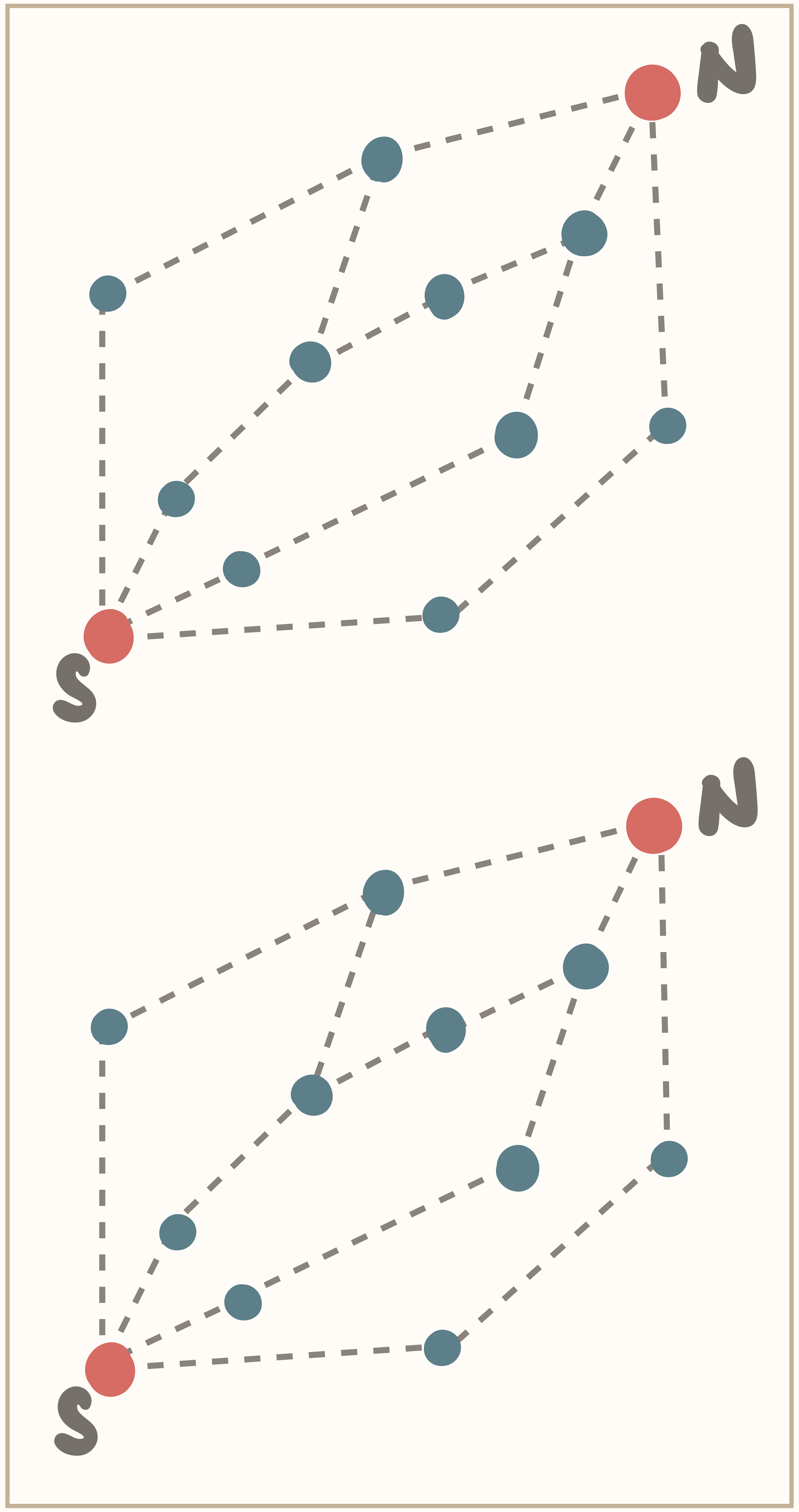
Poset ⟶ *Plane permutation*



Link with plane permutations

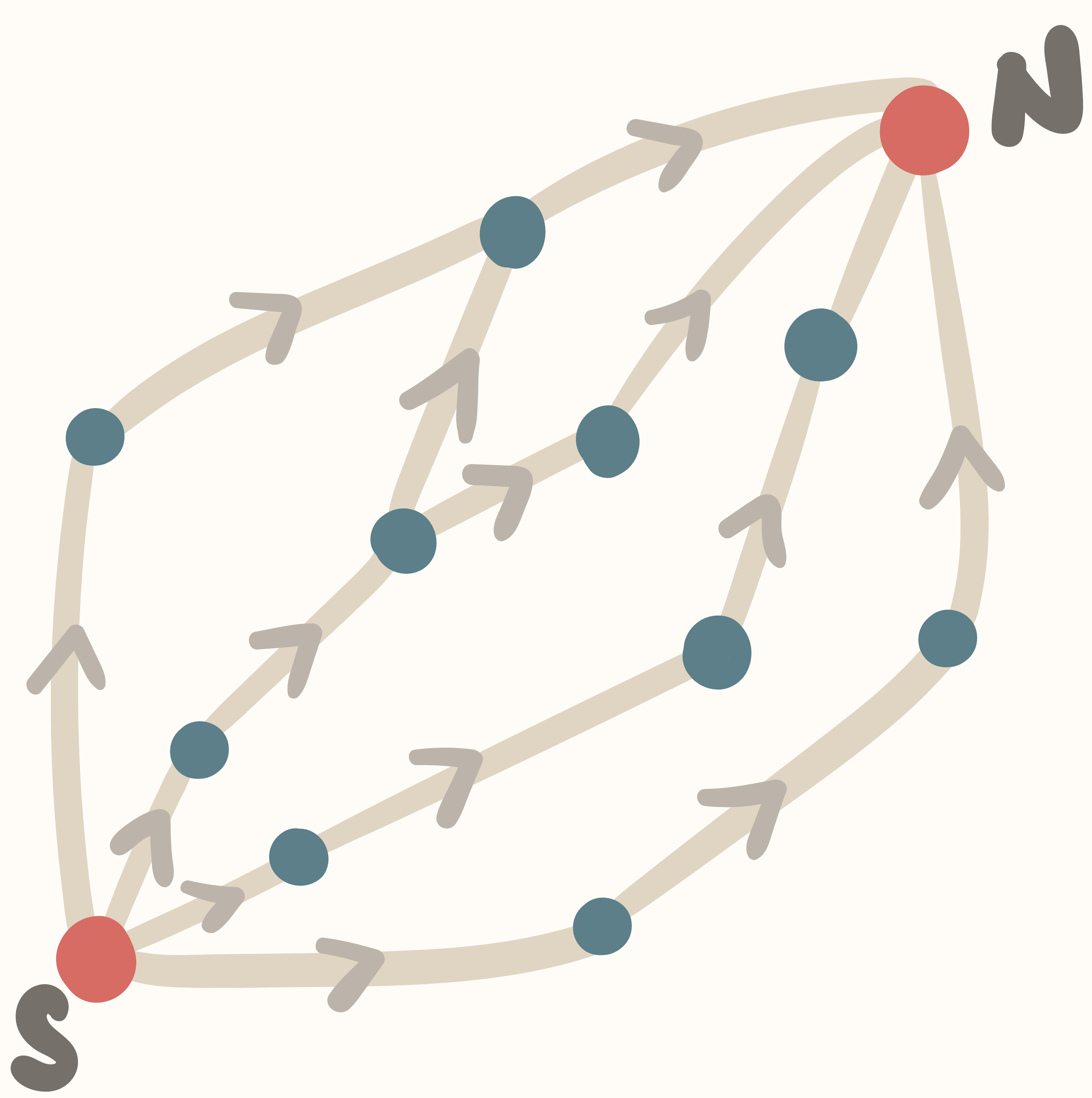
Poset ⟶ *Plane permutation*

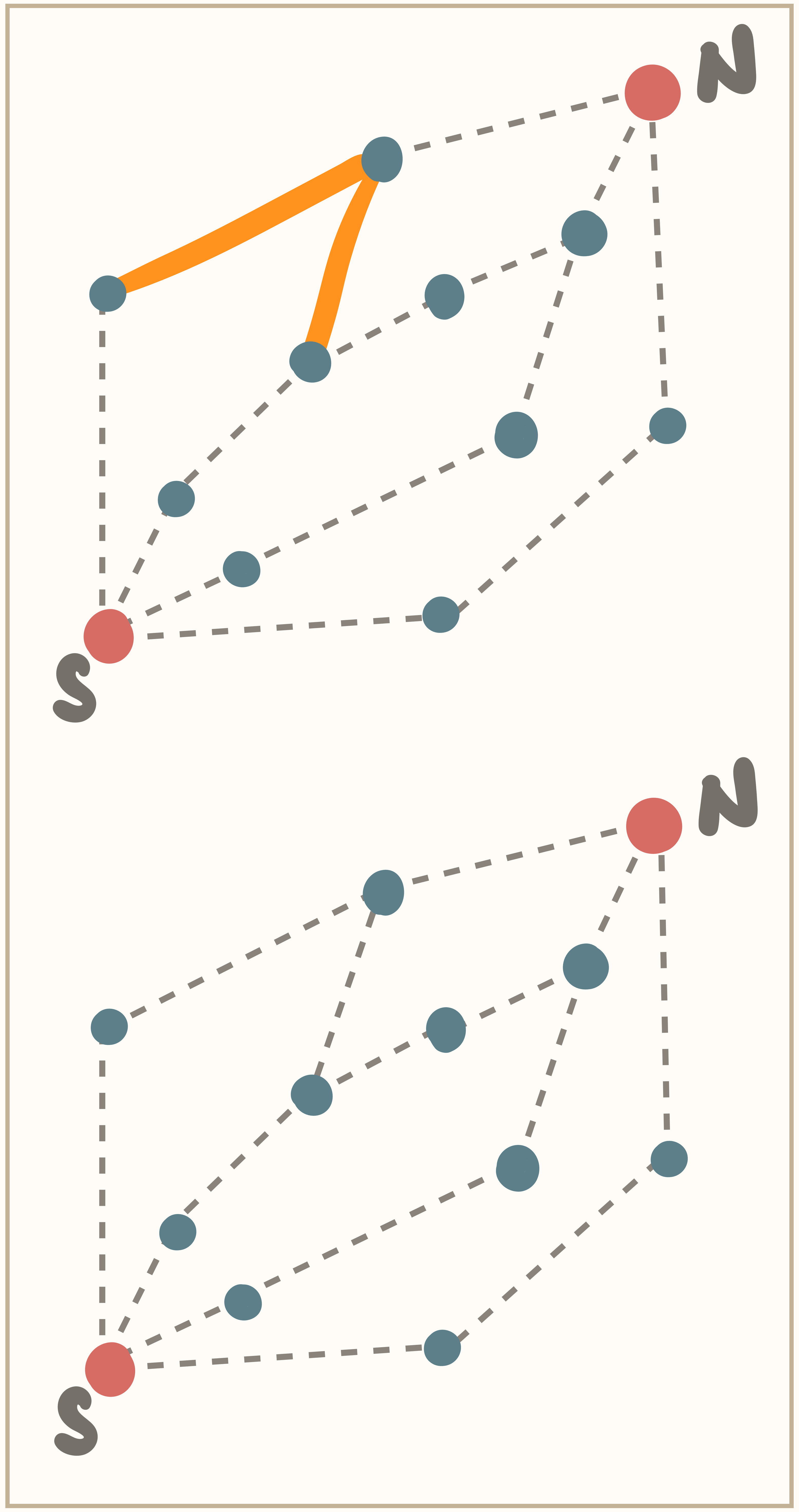




Link with plane permutations

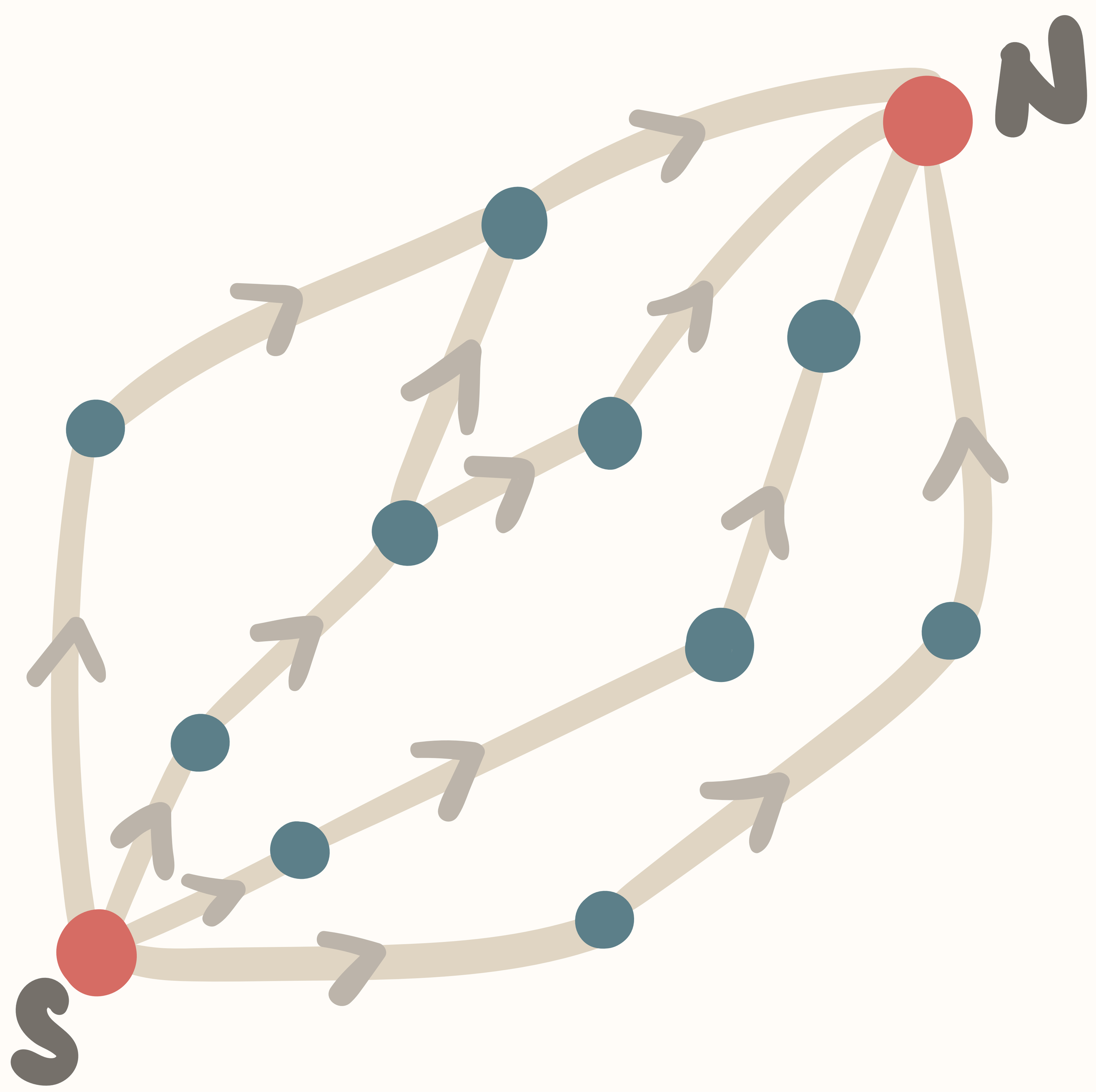
Poset ⟶ *Plane permutation*

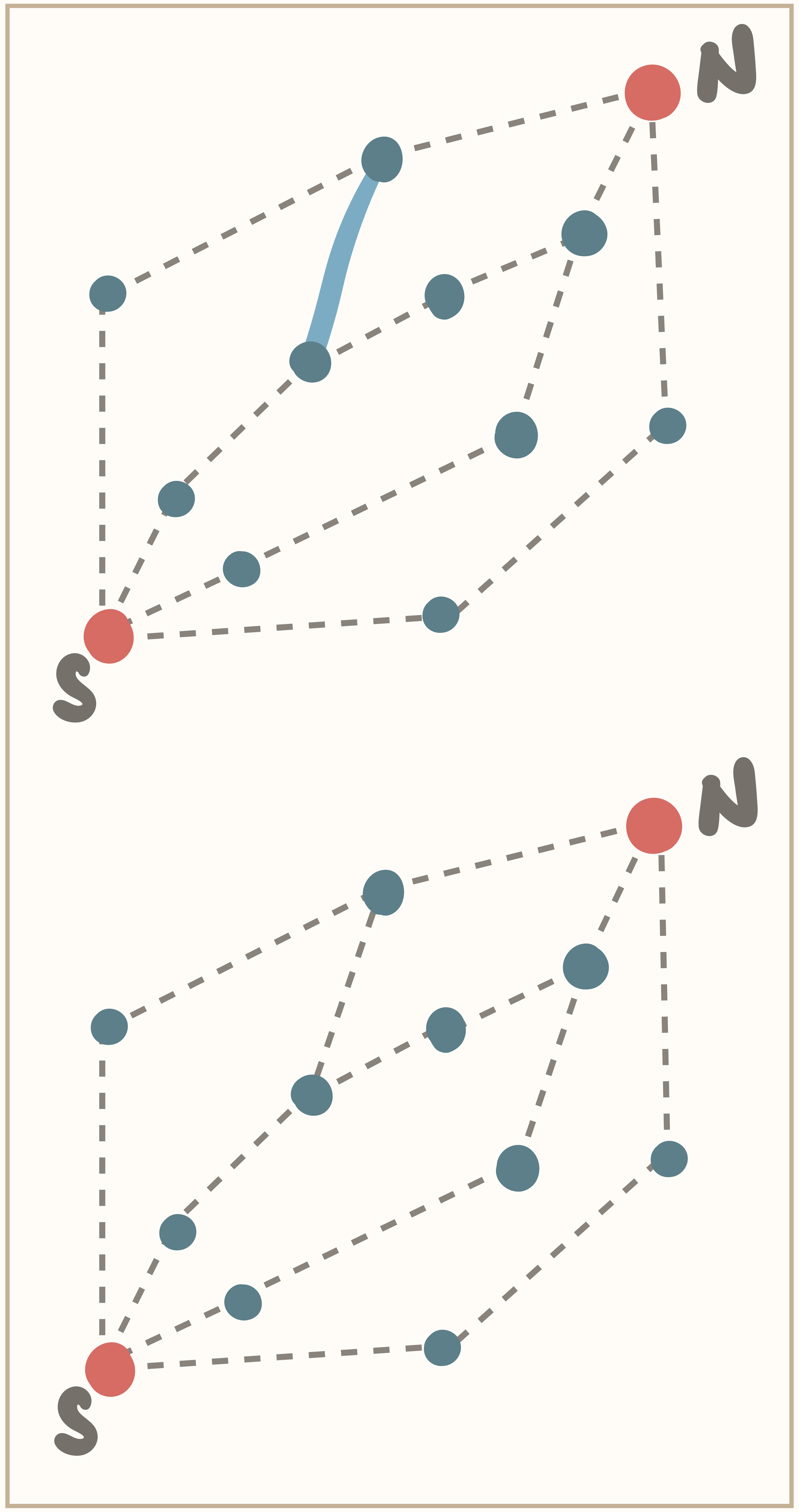




Link with plane permutations

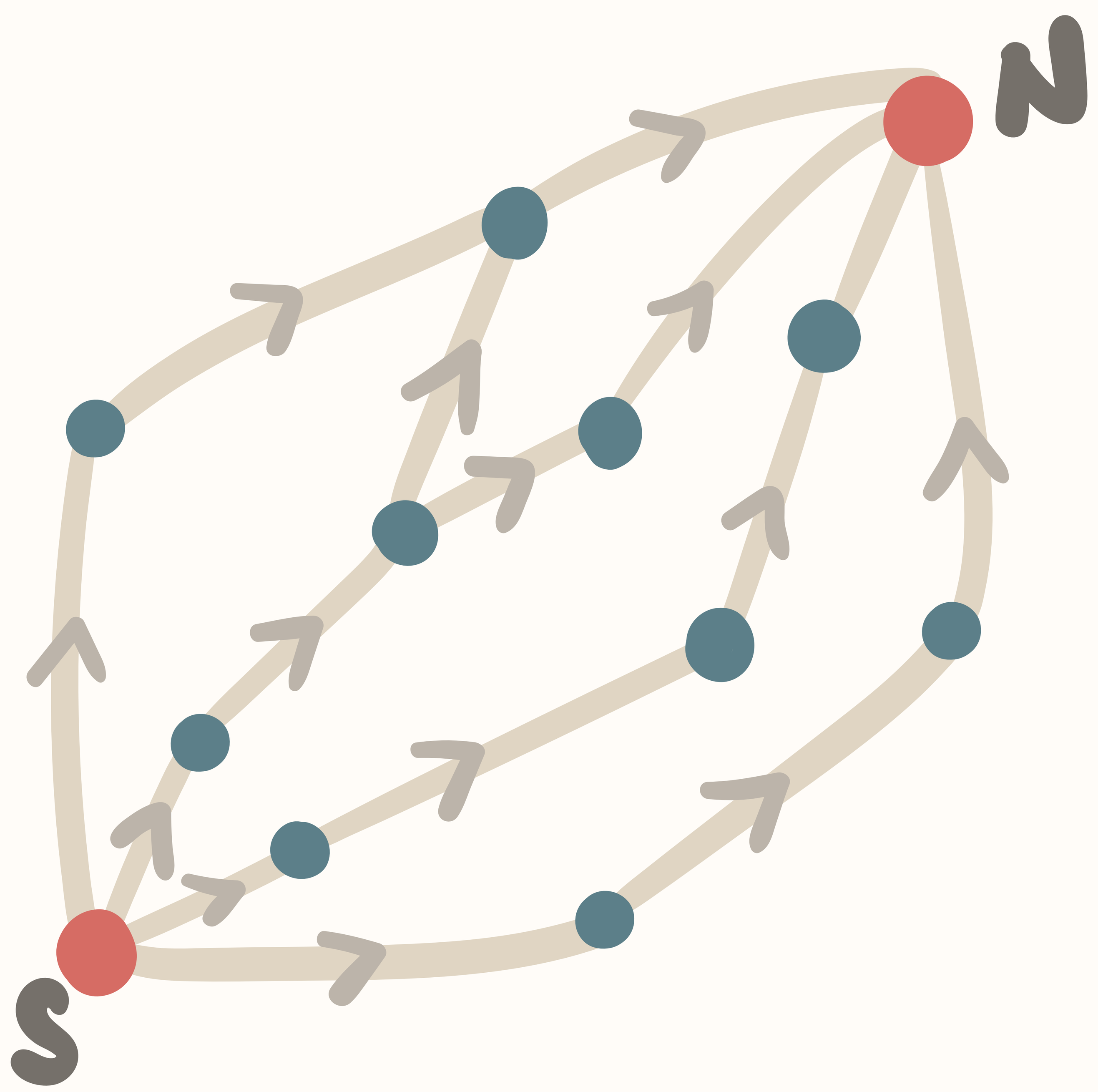
Poset ⟶ *Plane permutation*

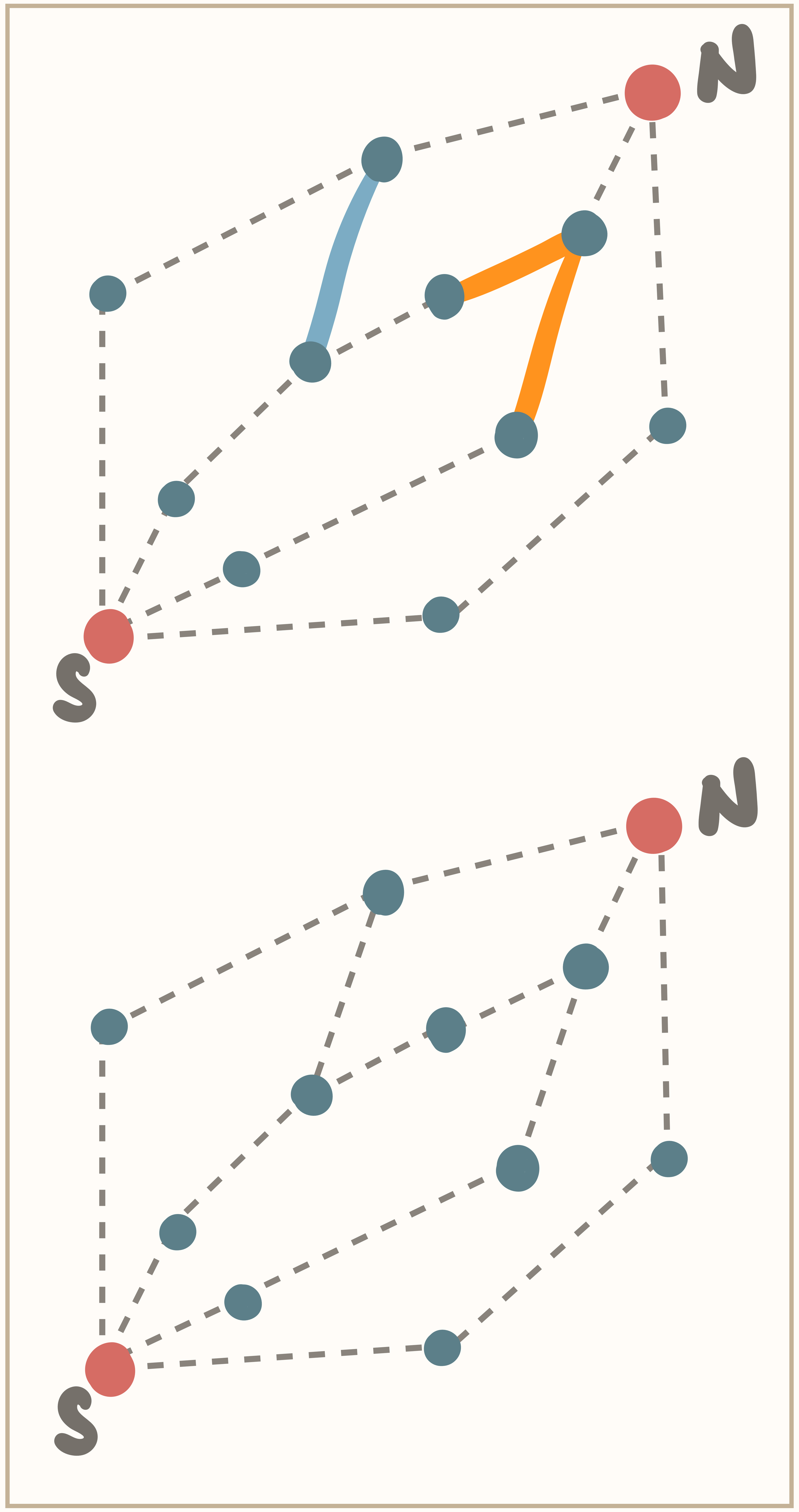




Link with plane permutations

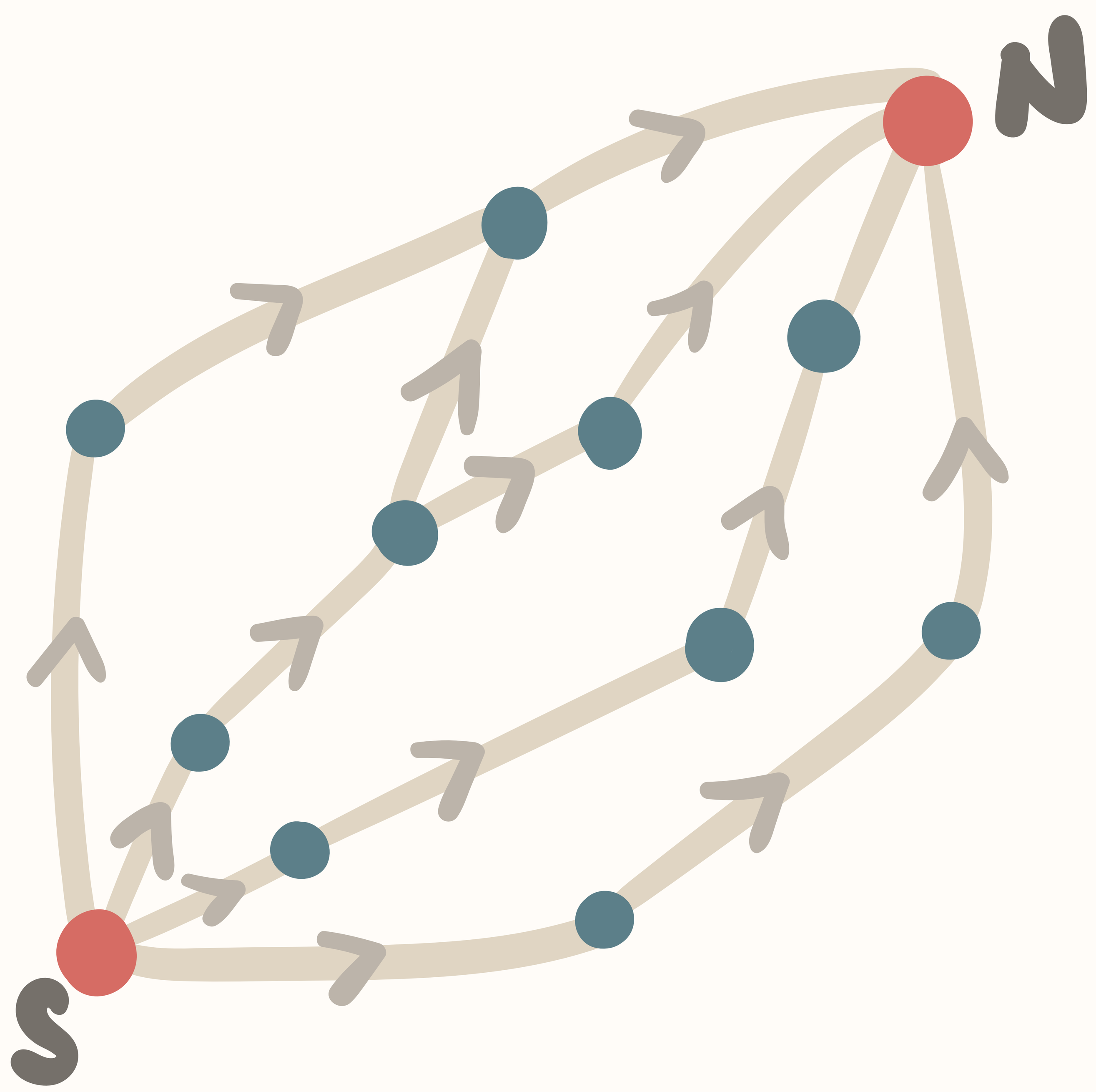
Poset ⟶ *Plane permutation*

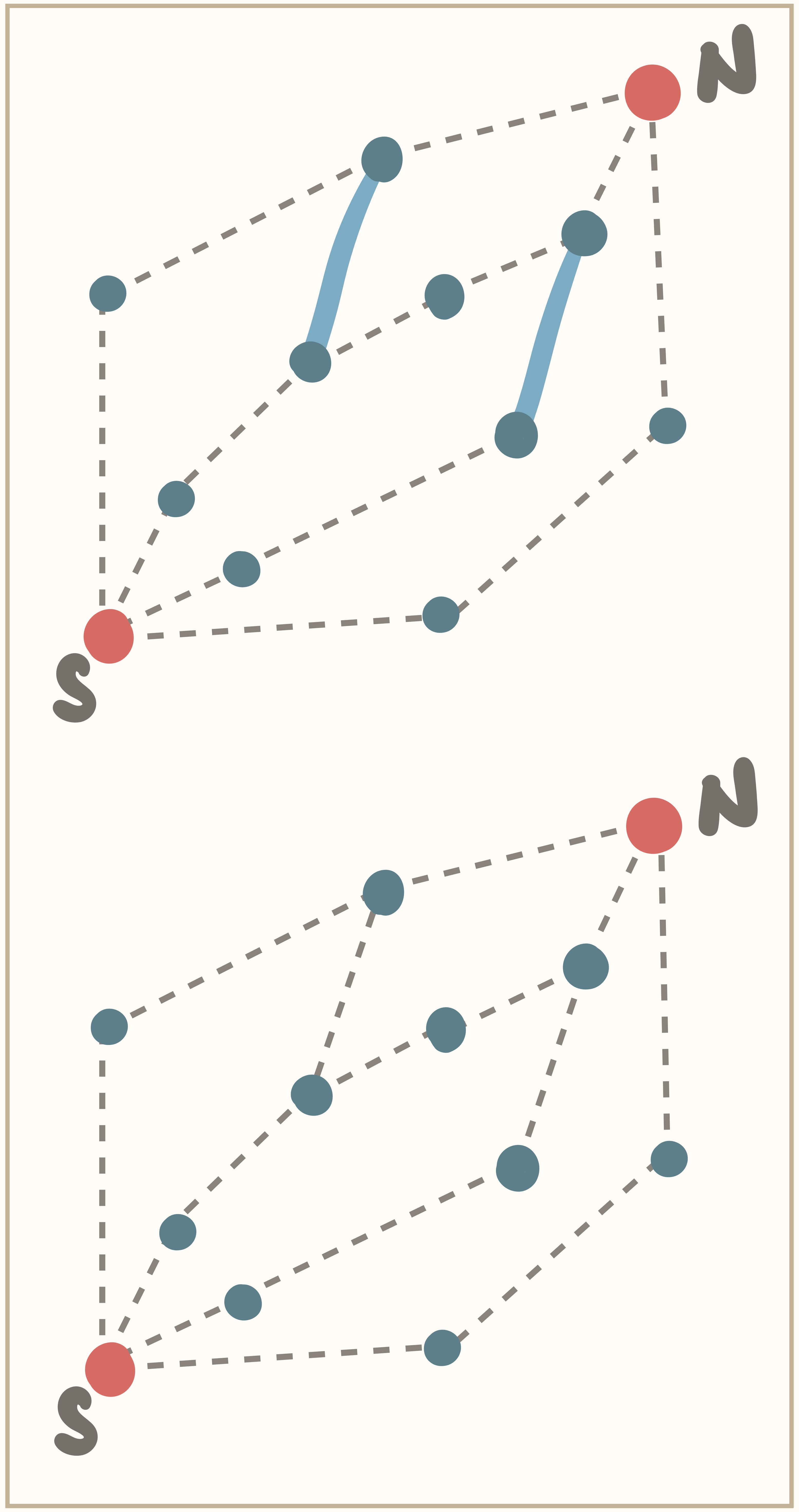




Link with plane permutations

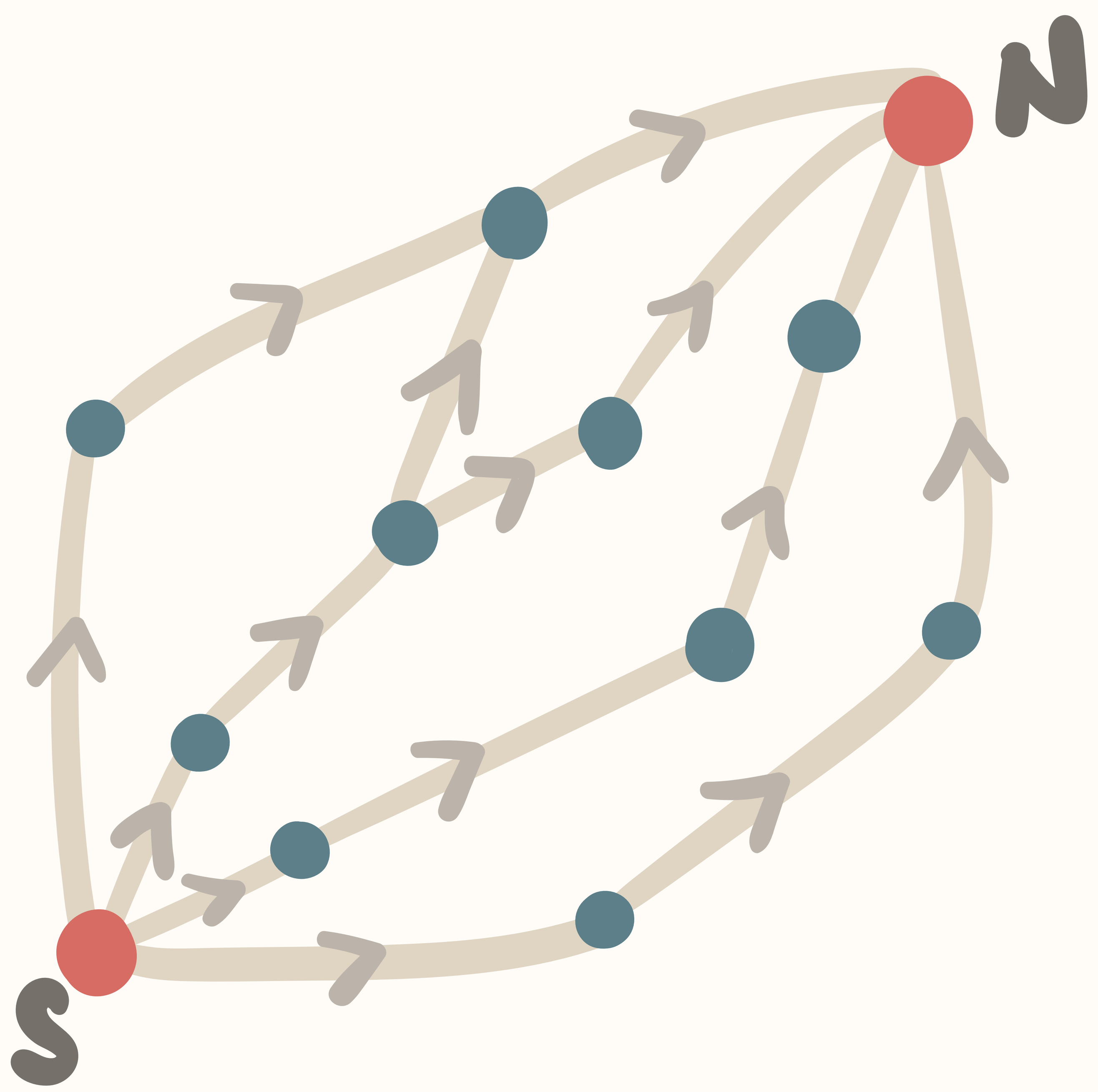
Poset ⟶ *Plane permutation*

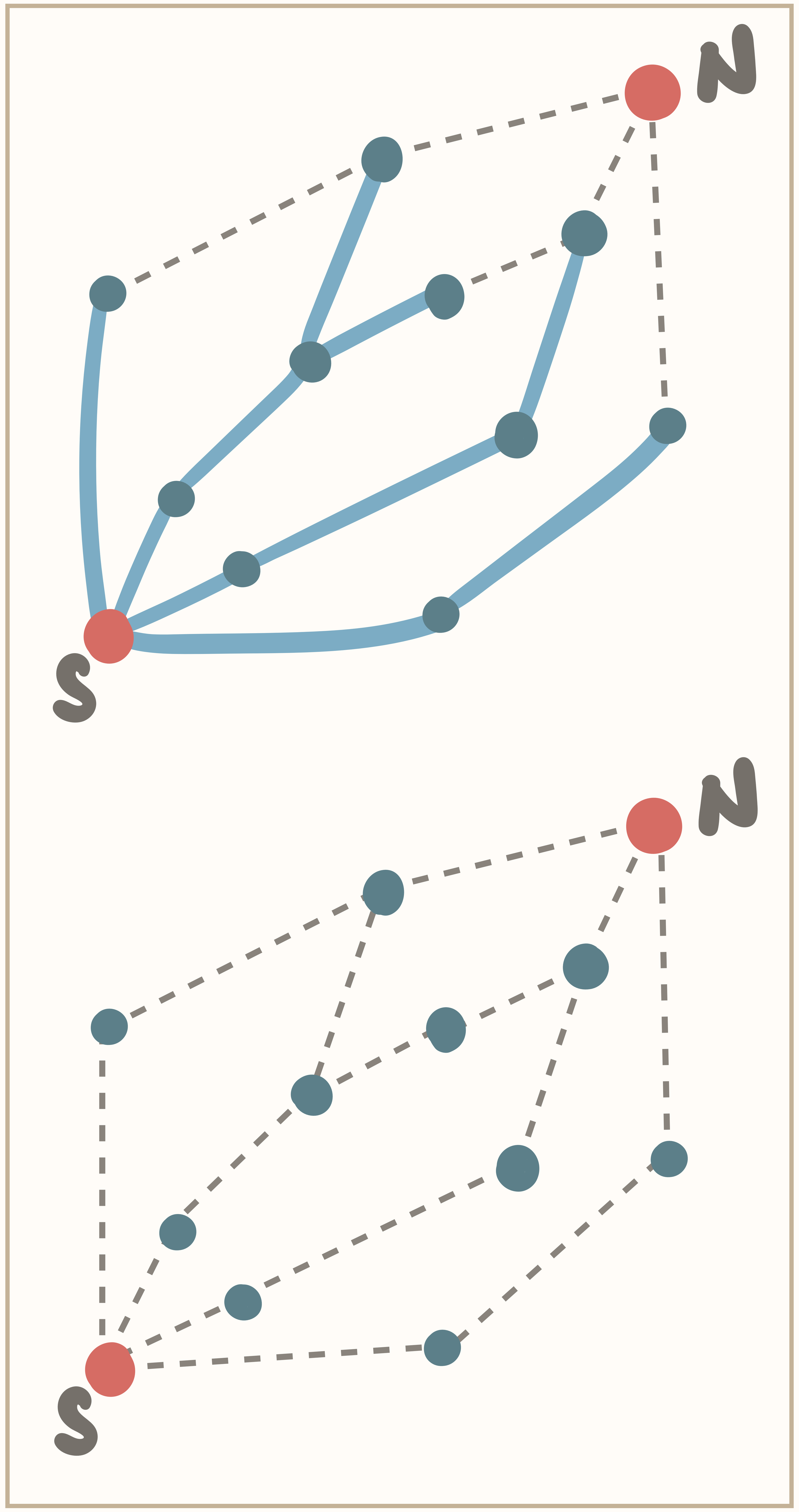




Link with plane permutations

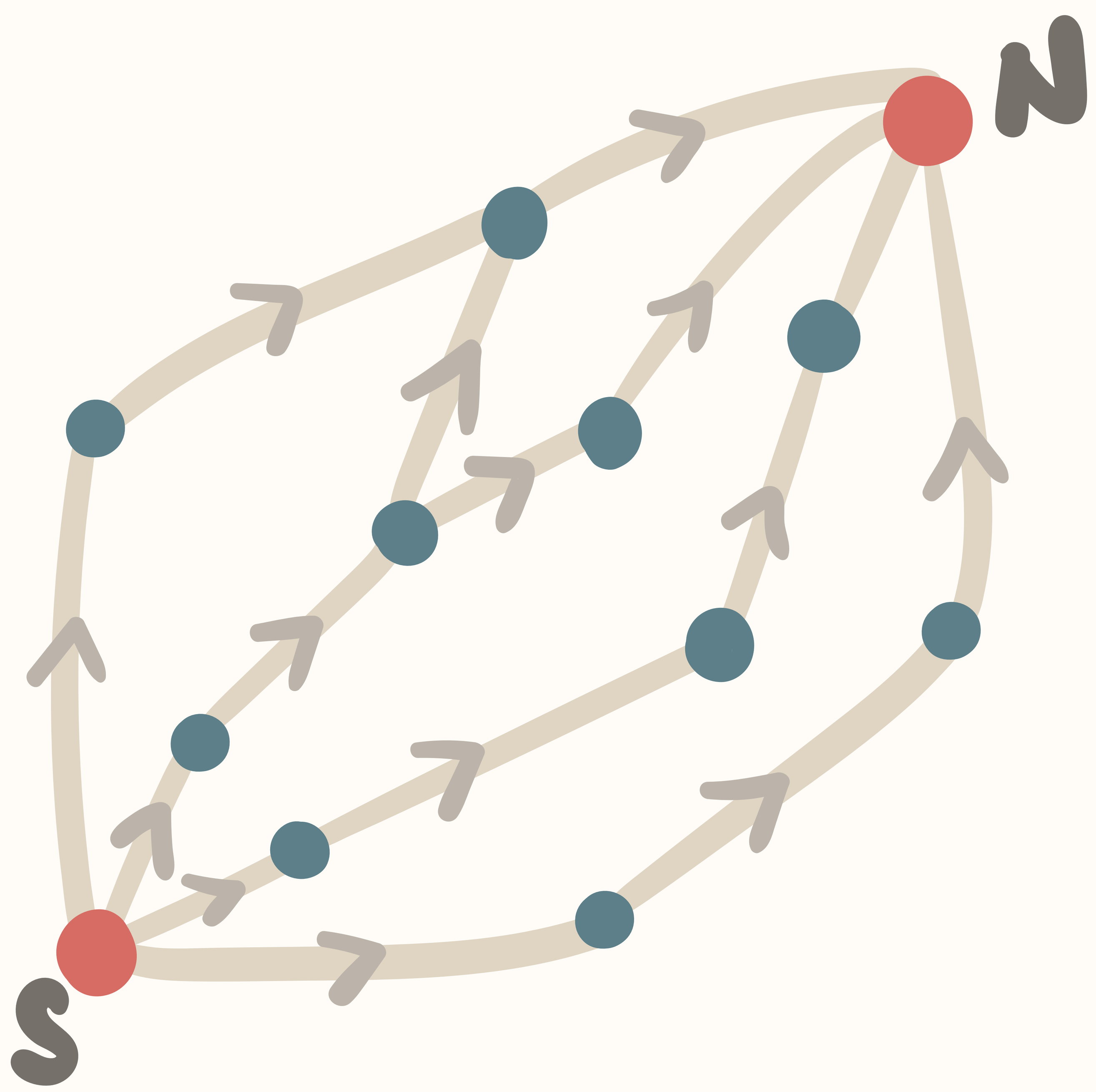
Poset ⟶ *Plane permutation*

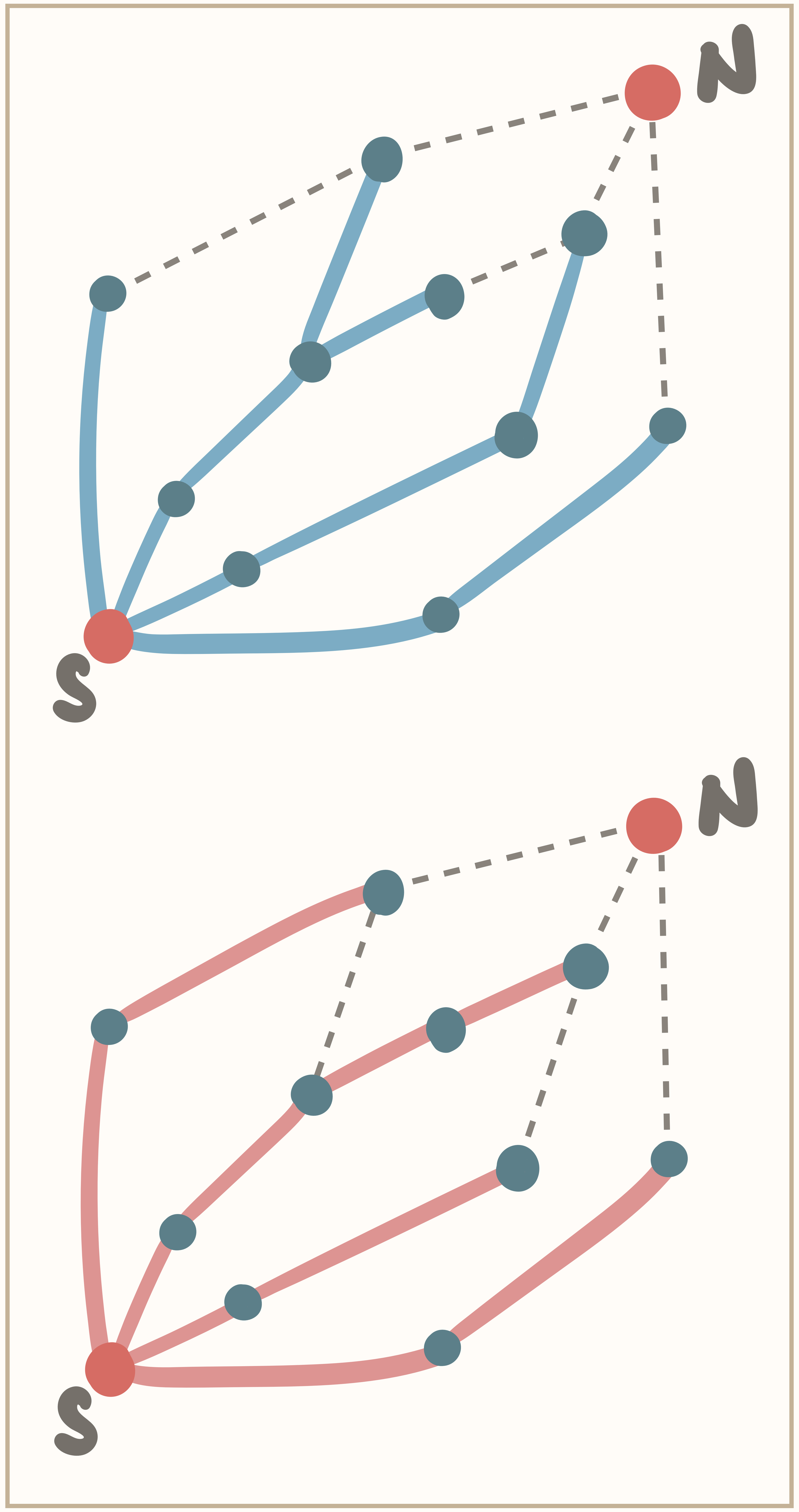




Link with plane permutations

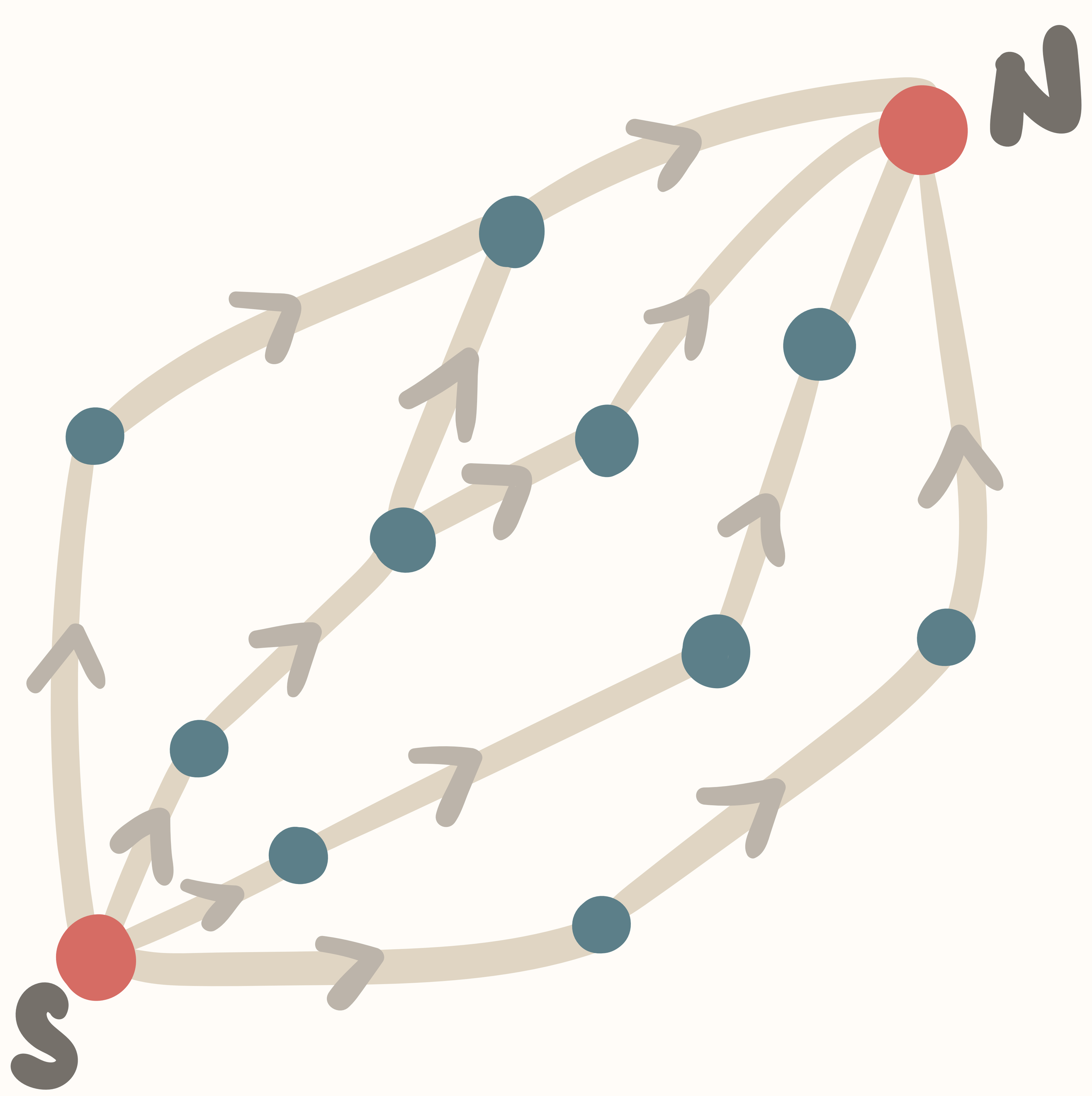
Poset ⟶ *Plane permutation*

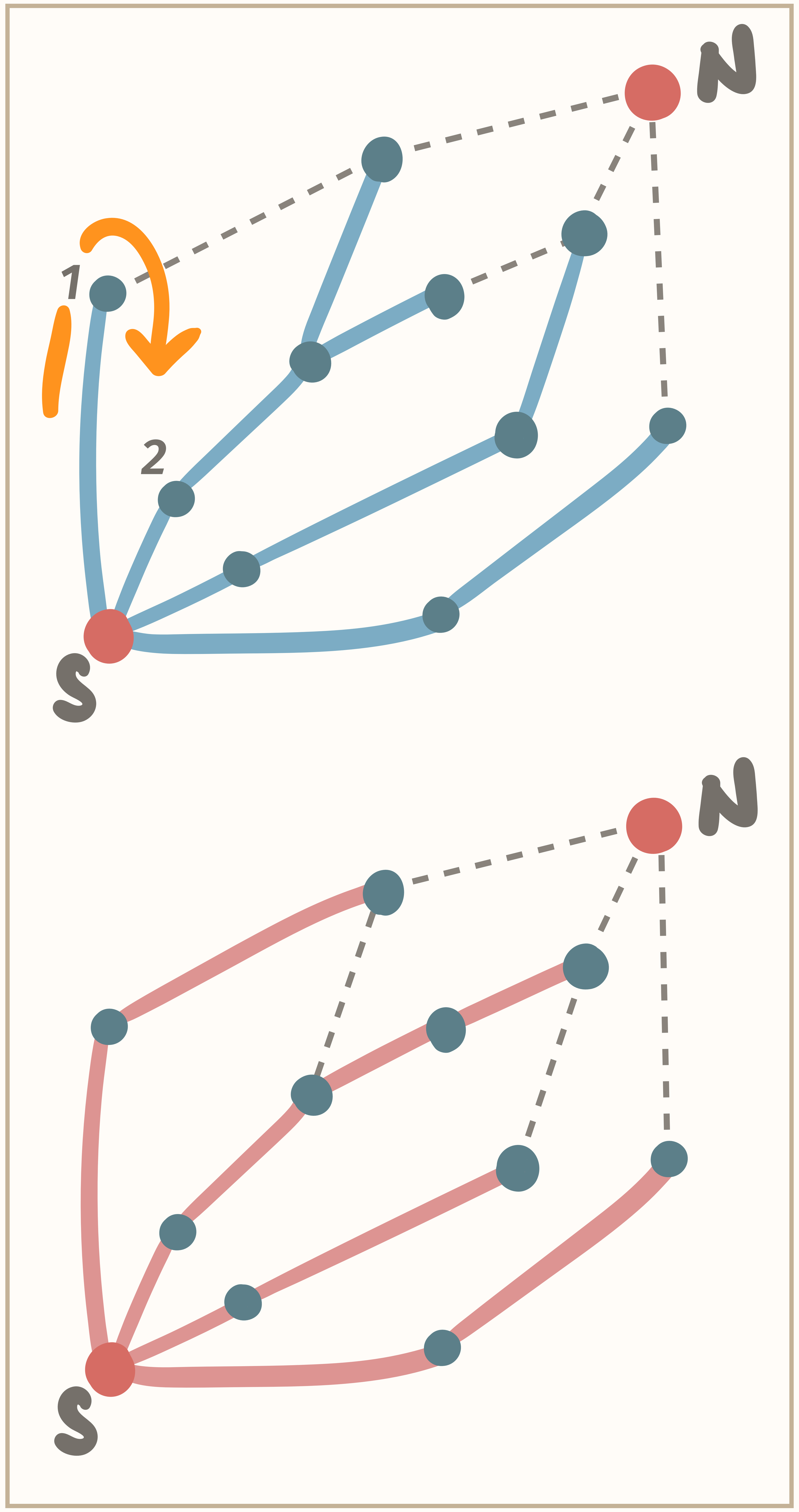




Link with plane permutations

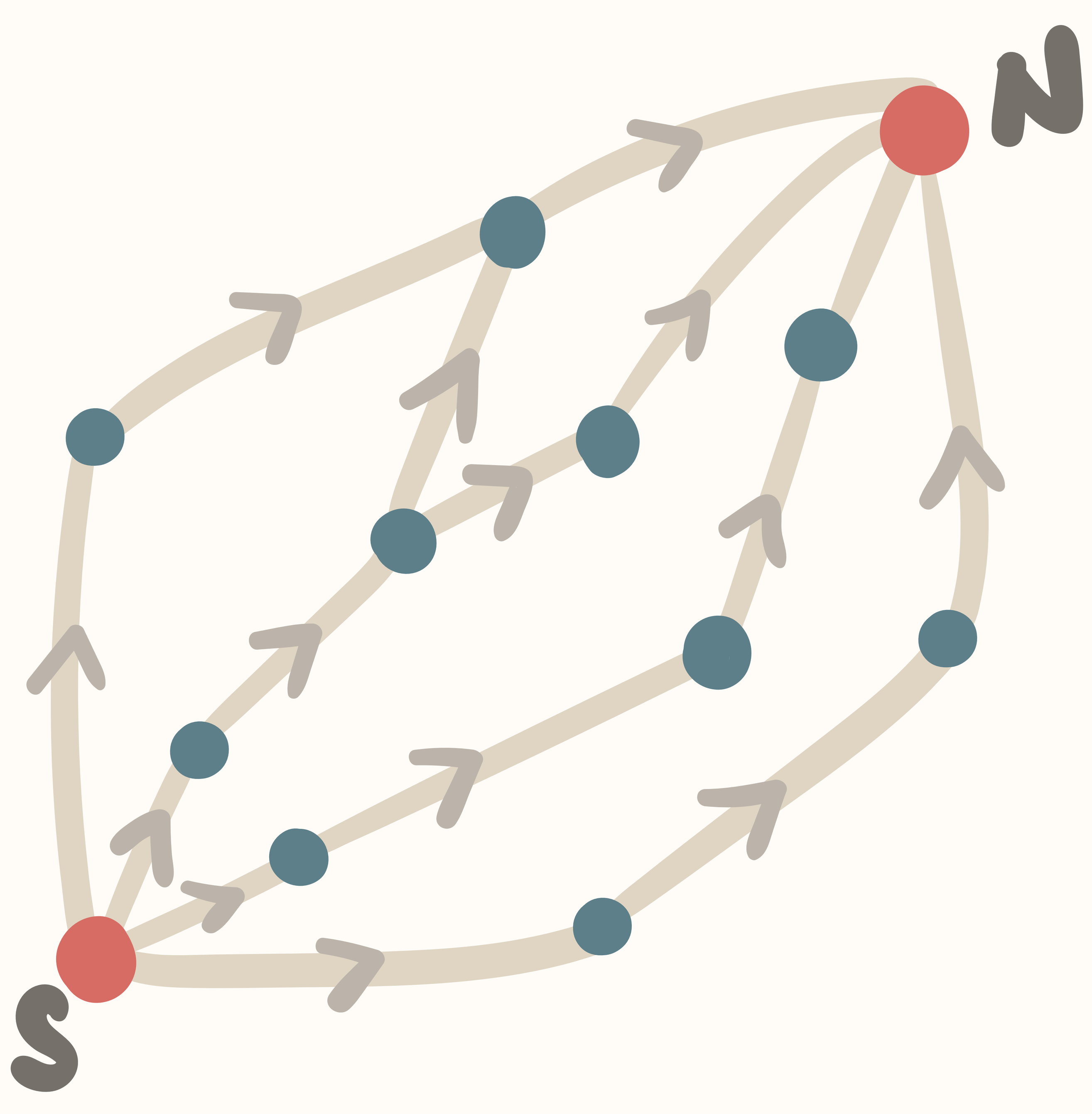
Poset ⟶ *Plane permutation*

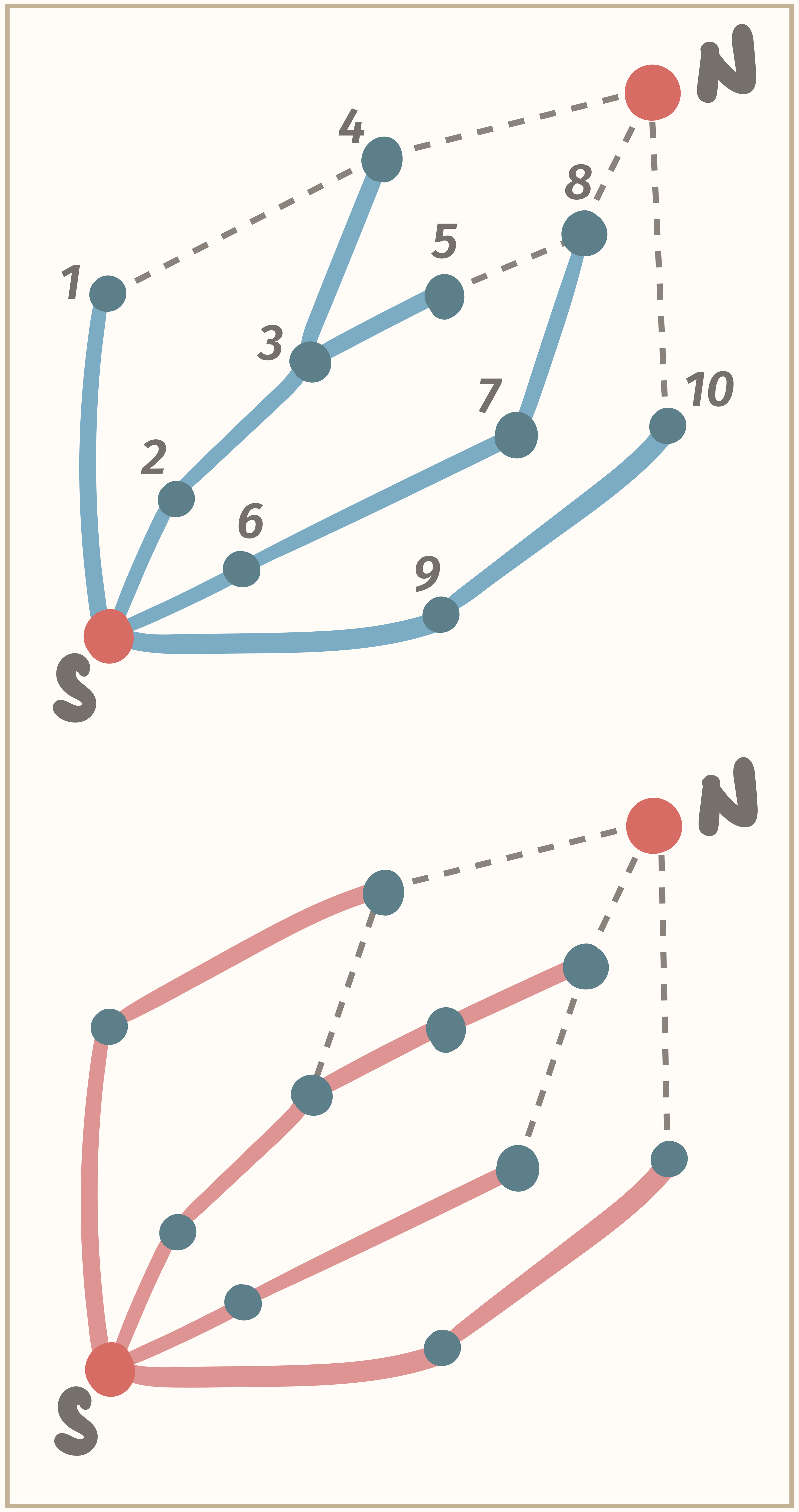




Link with plane permutations

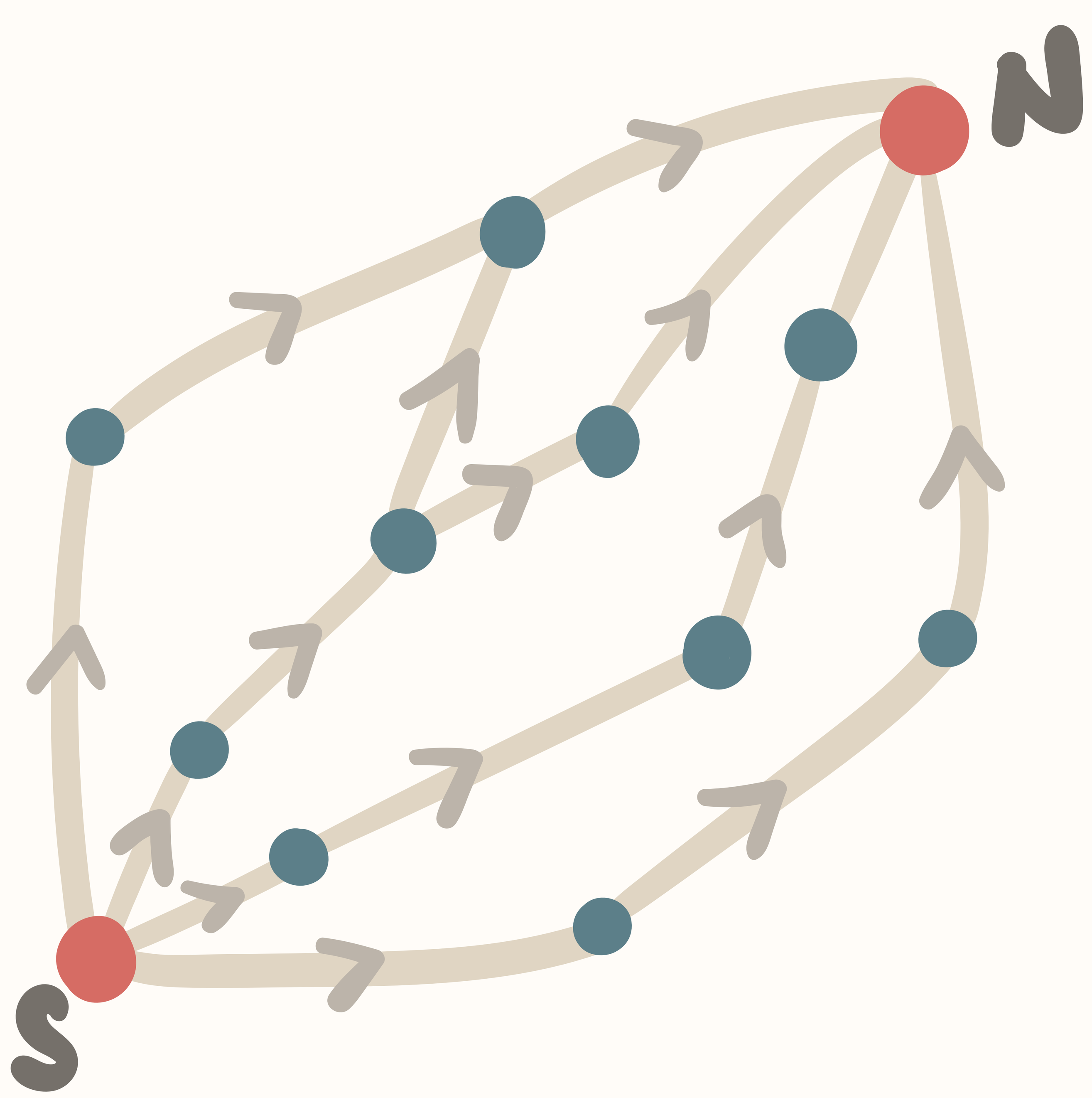
Poset ⟶ *Plane permutation*

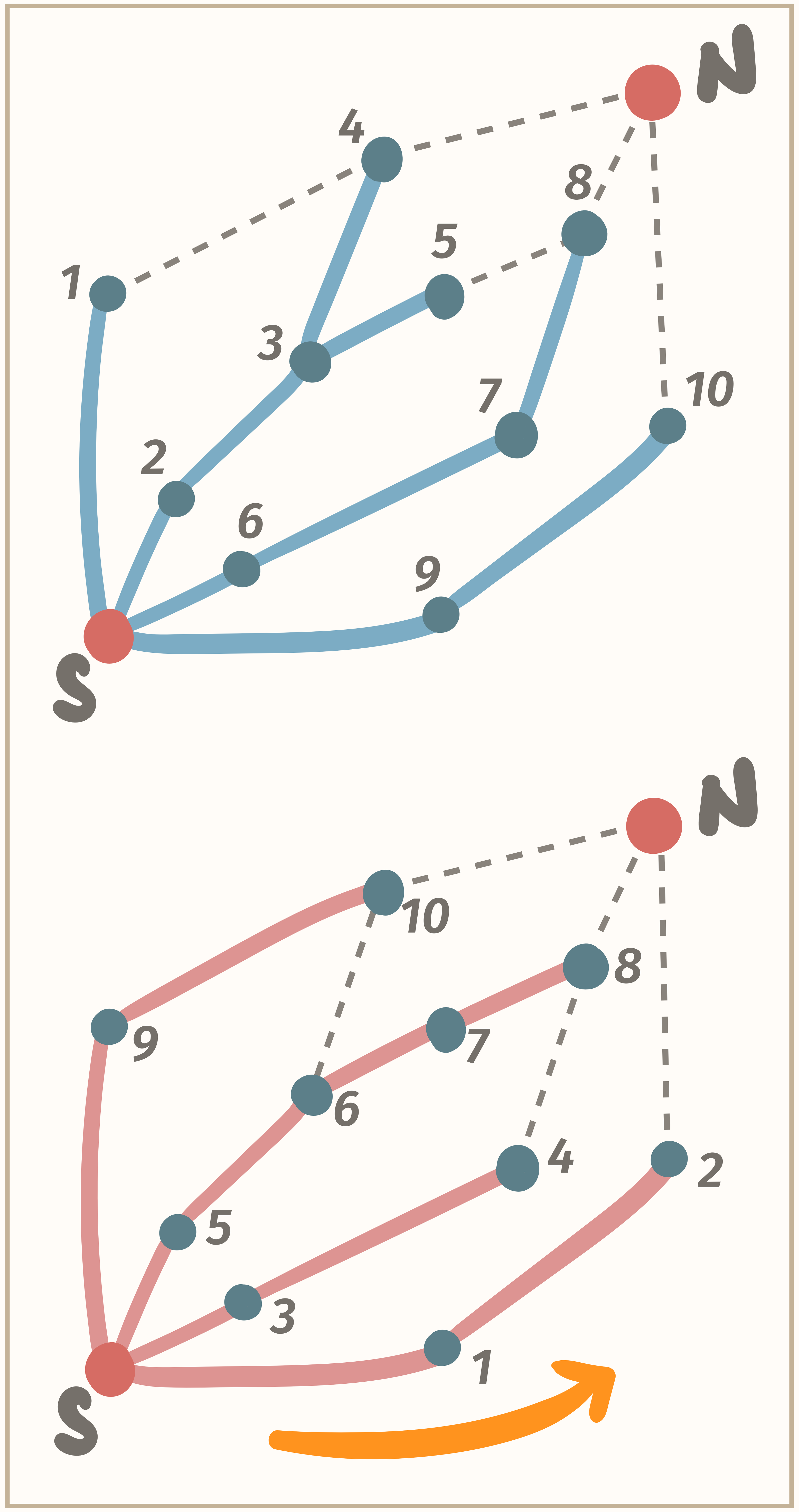




Link with plane permutations

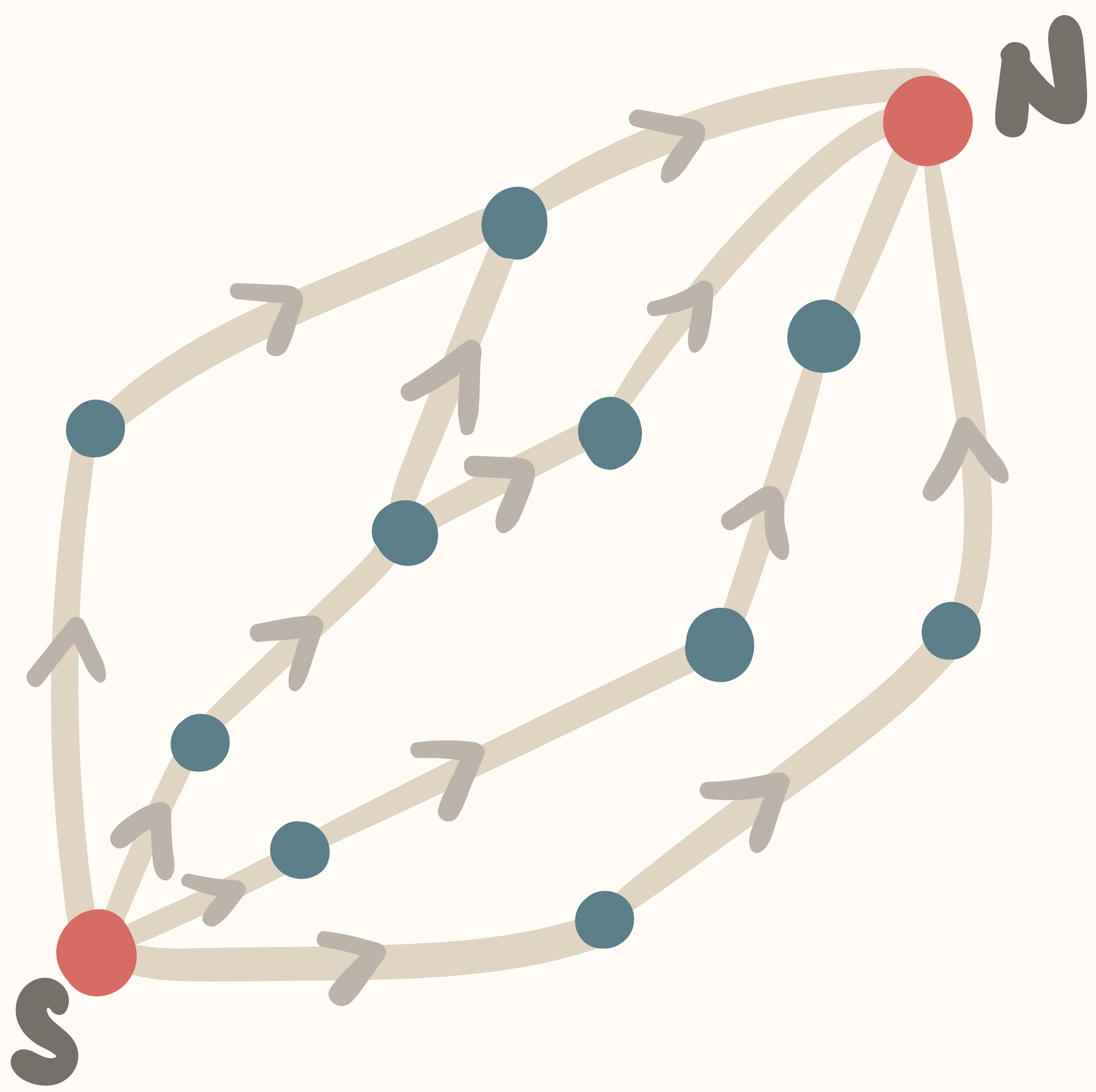
Poset ⟶ *Plane permutation*

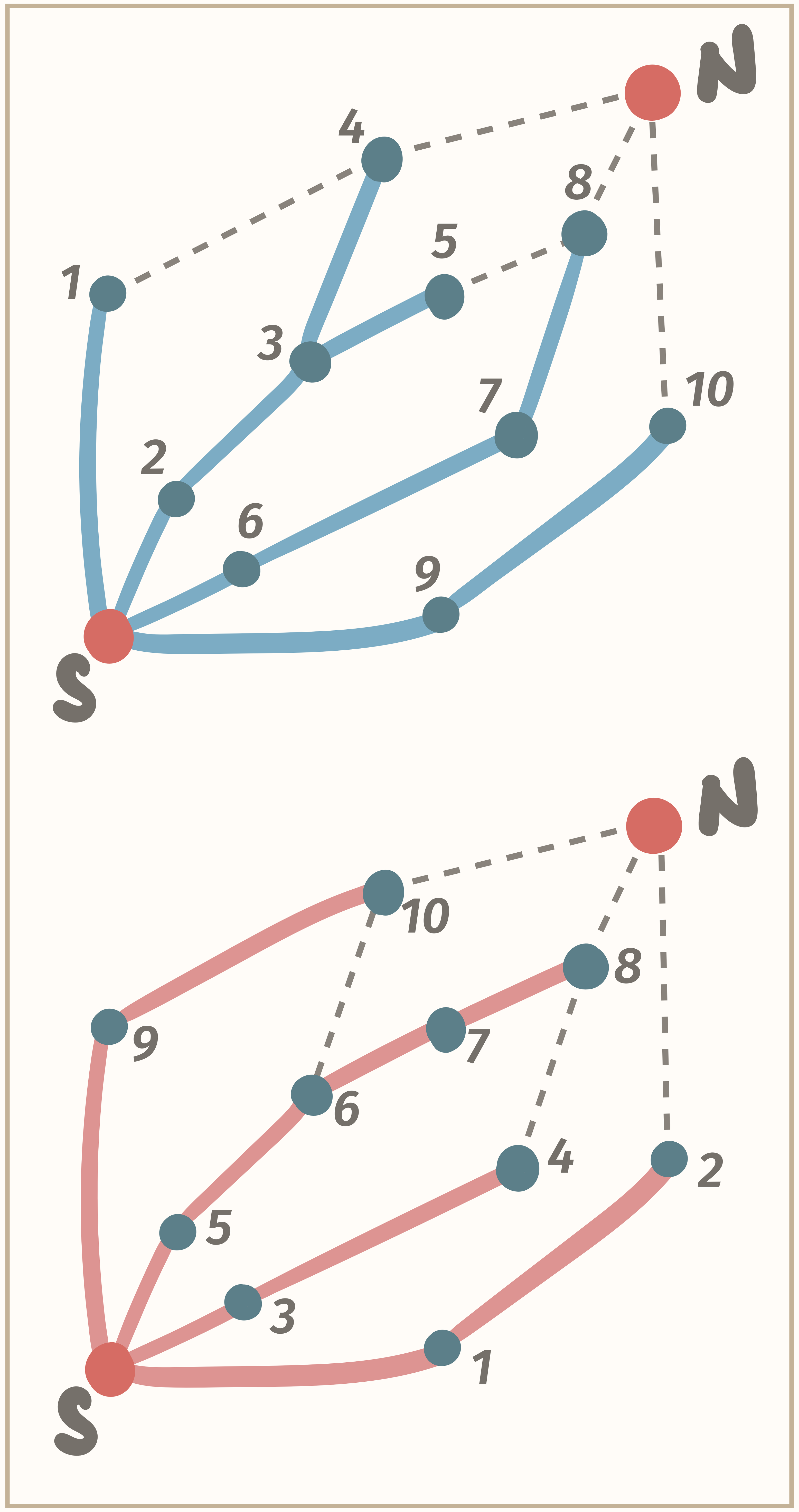


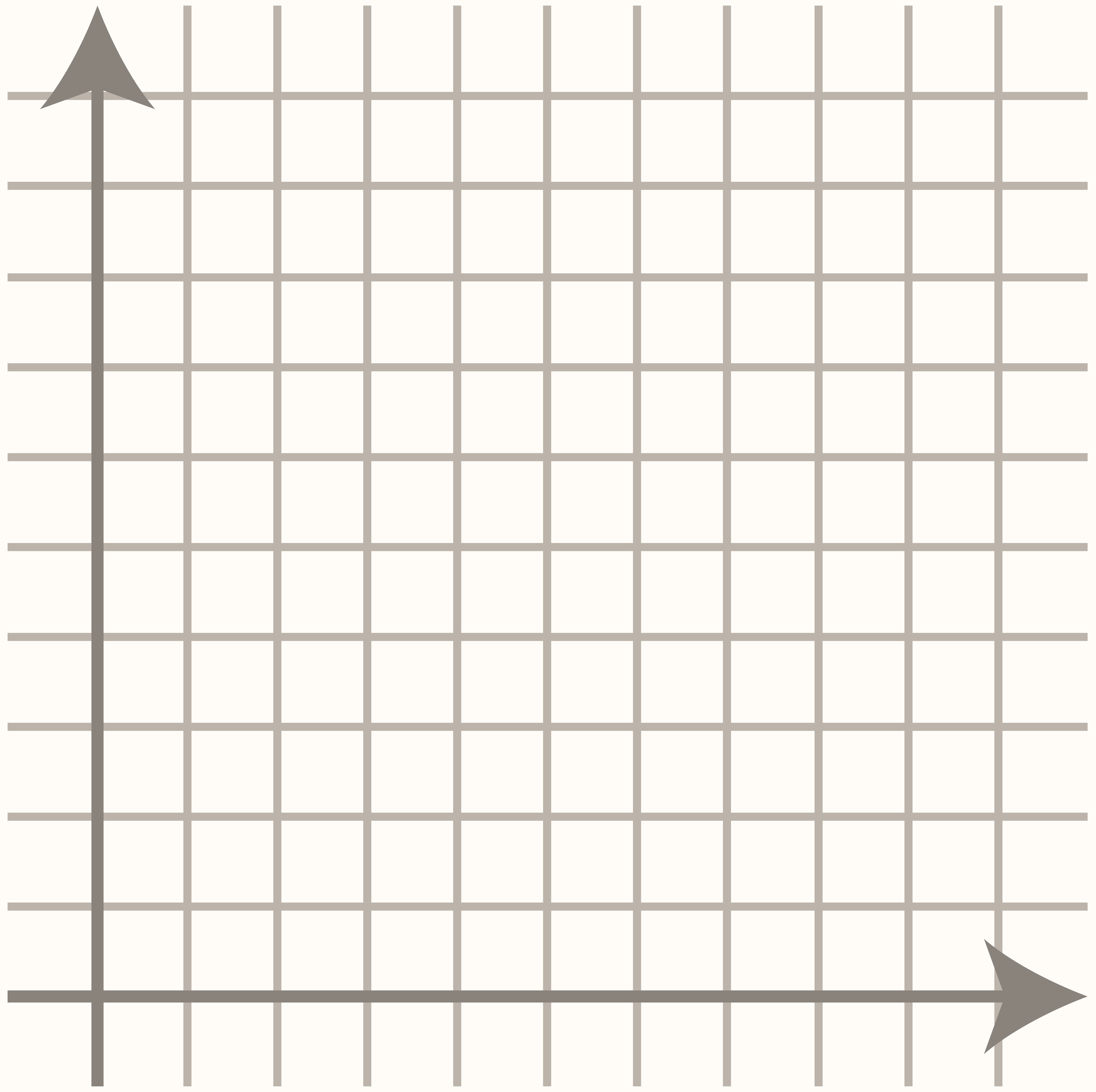


Link with plane permutations

Poset → *Plane permutation*

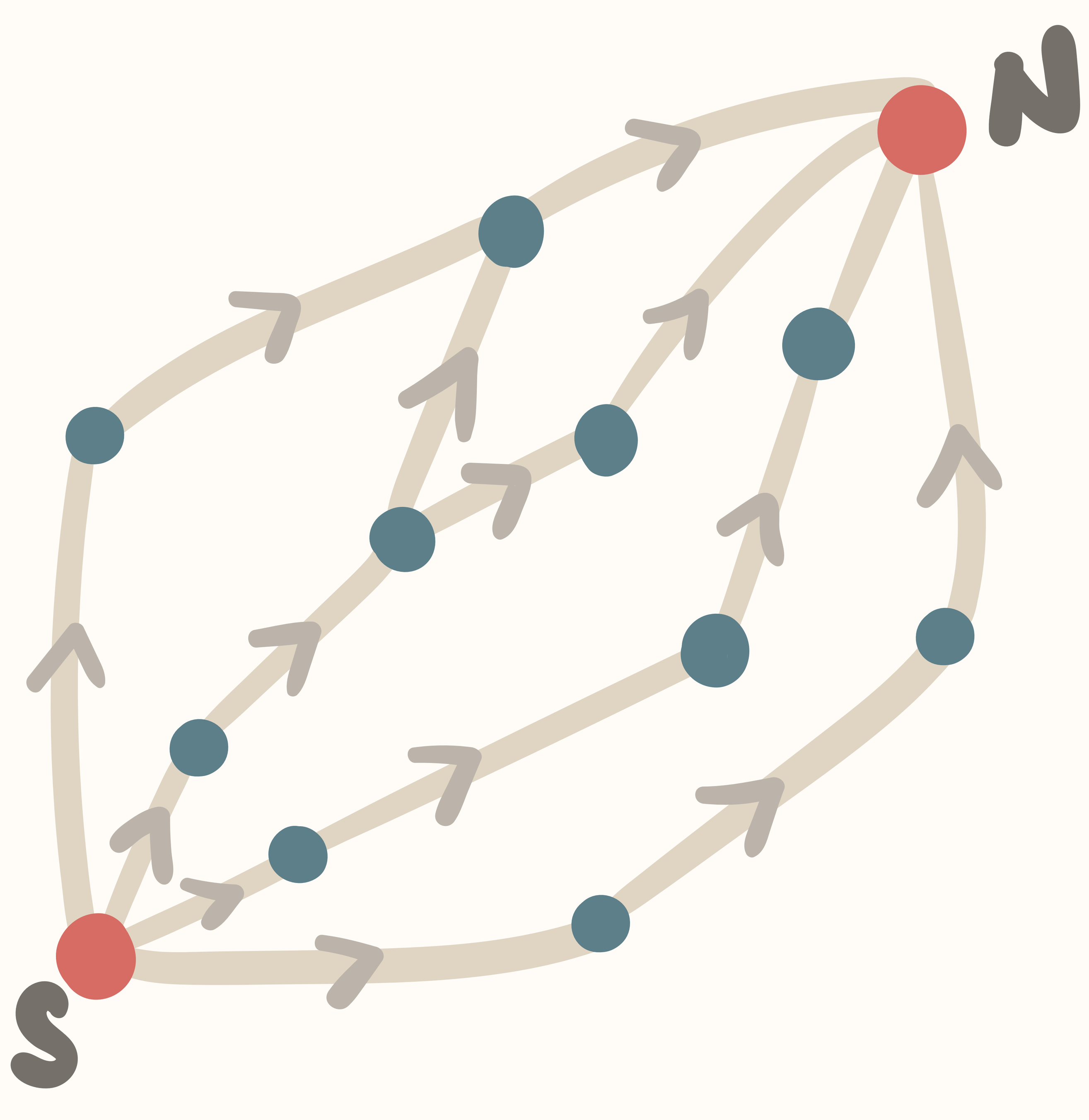


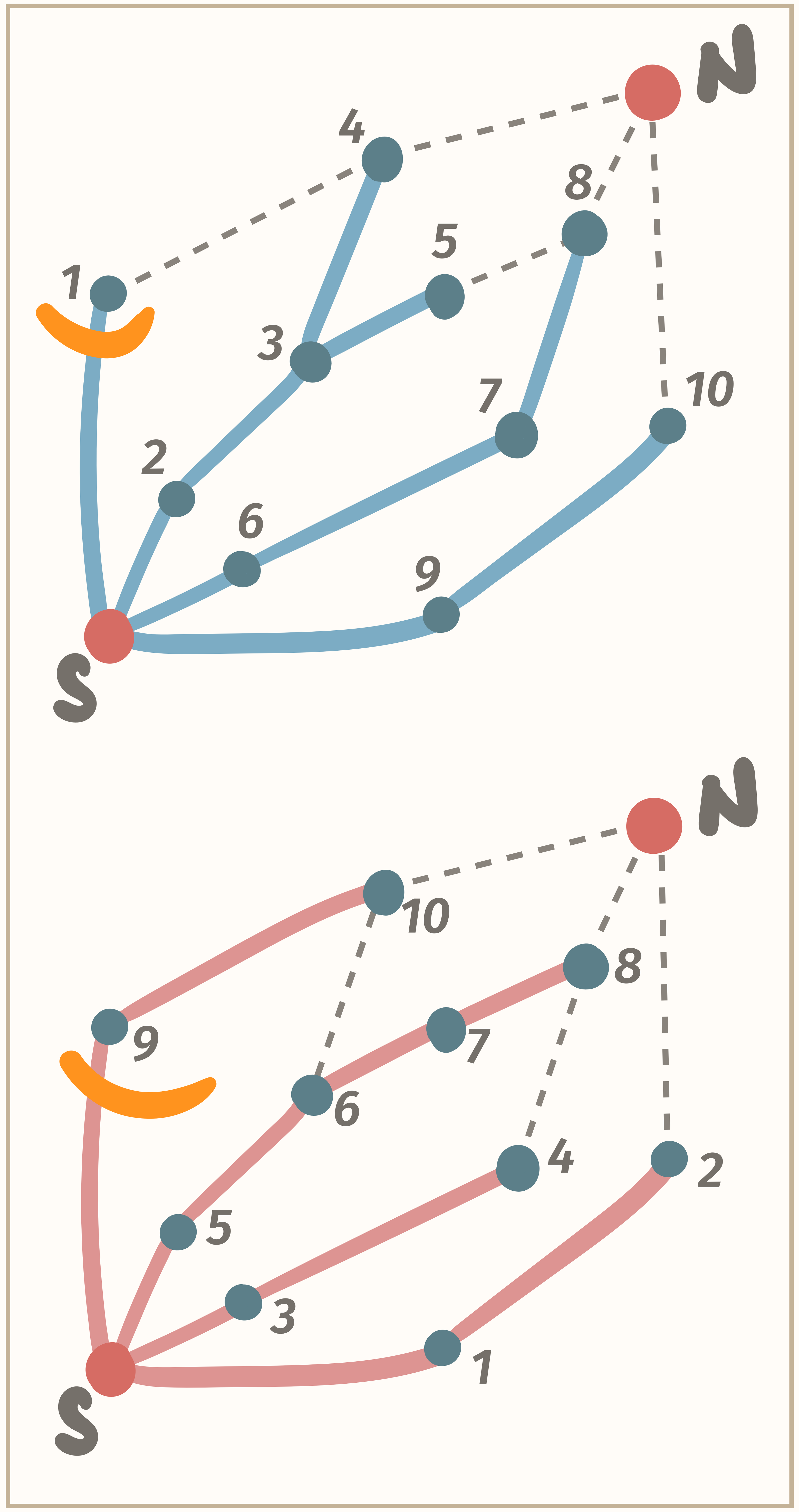


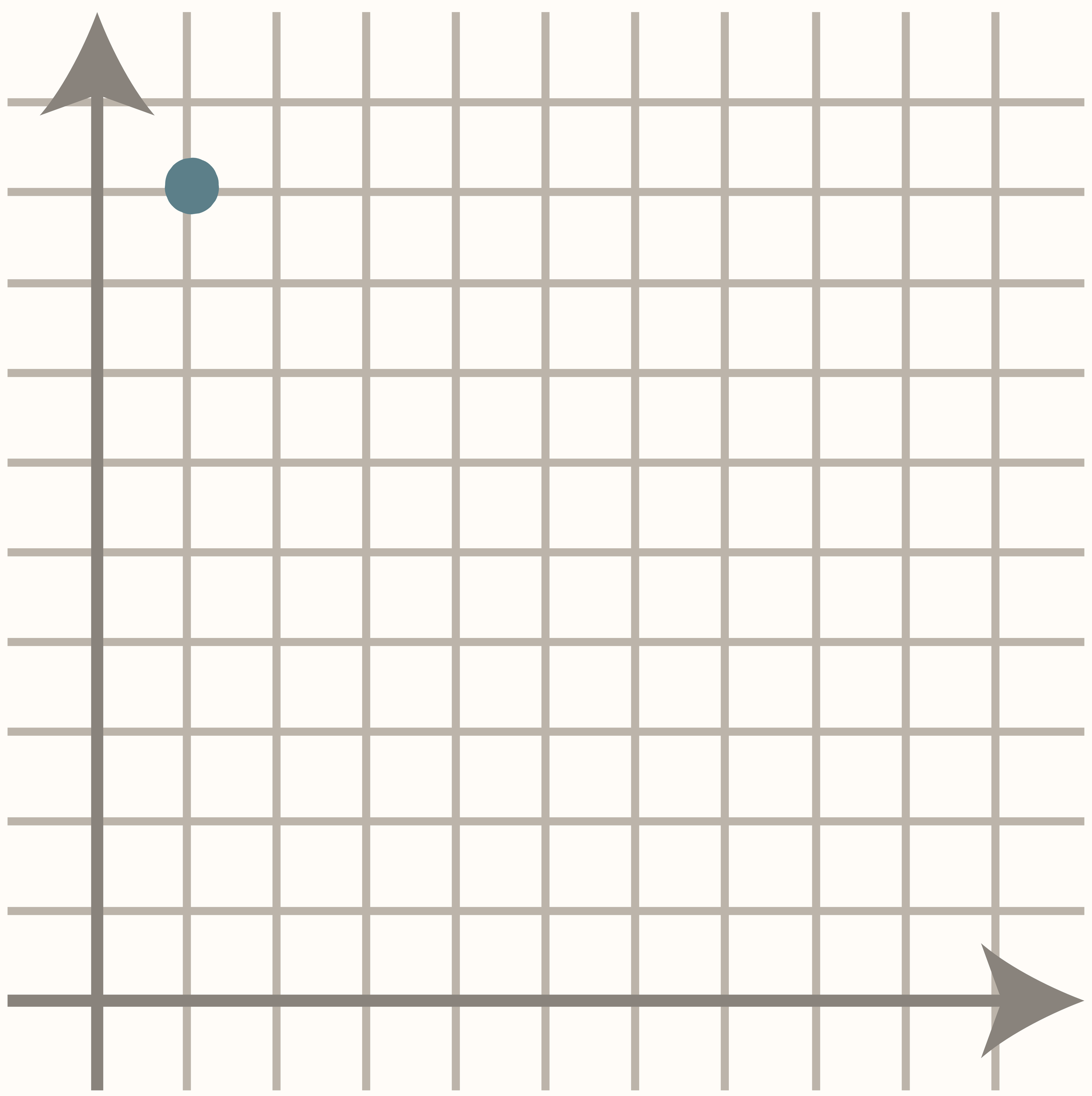


Link with plane permutations

Poset ⟶ *Plane permutation*



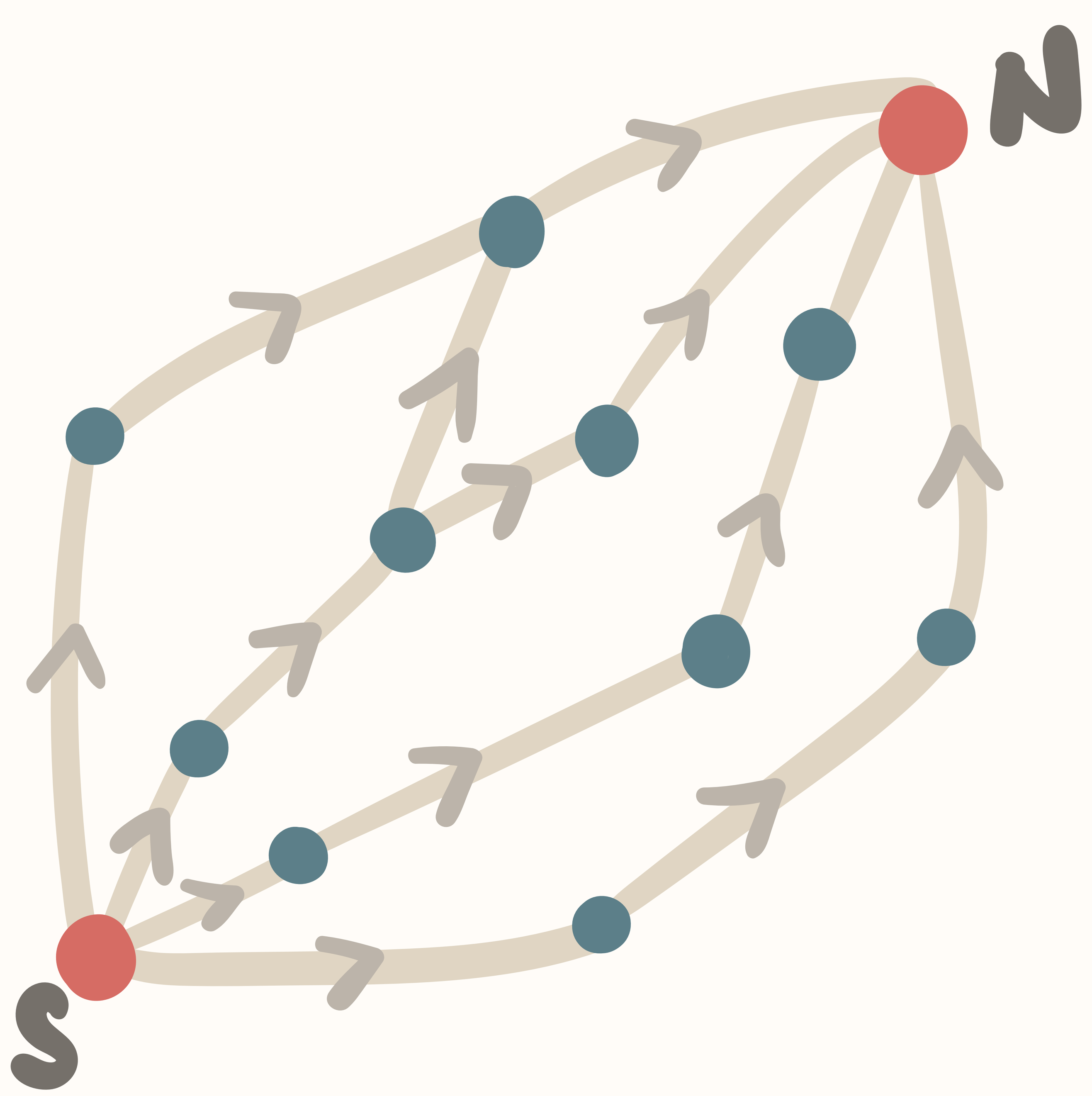


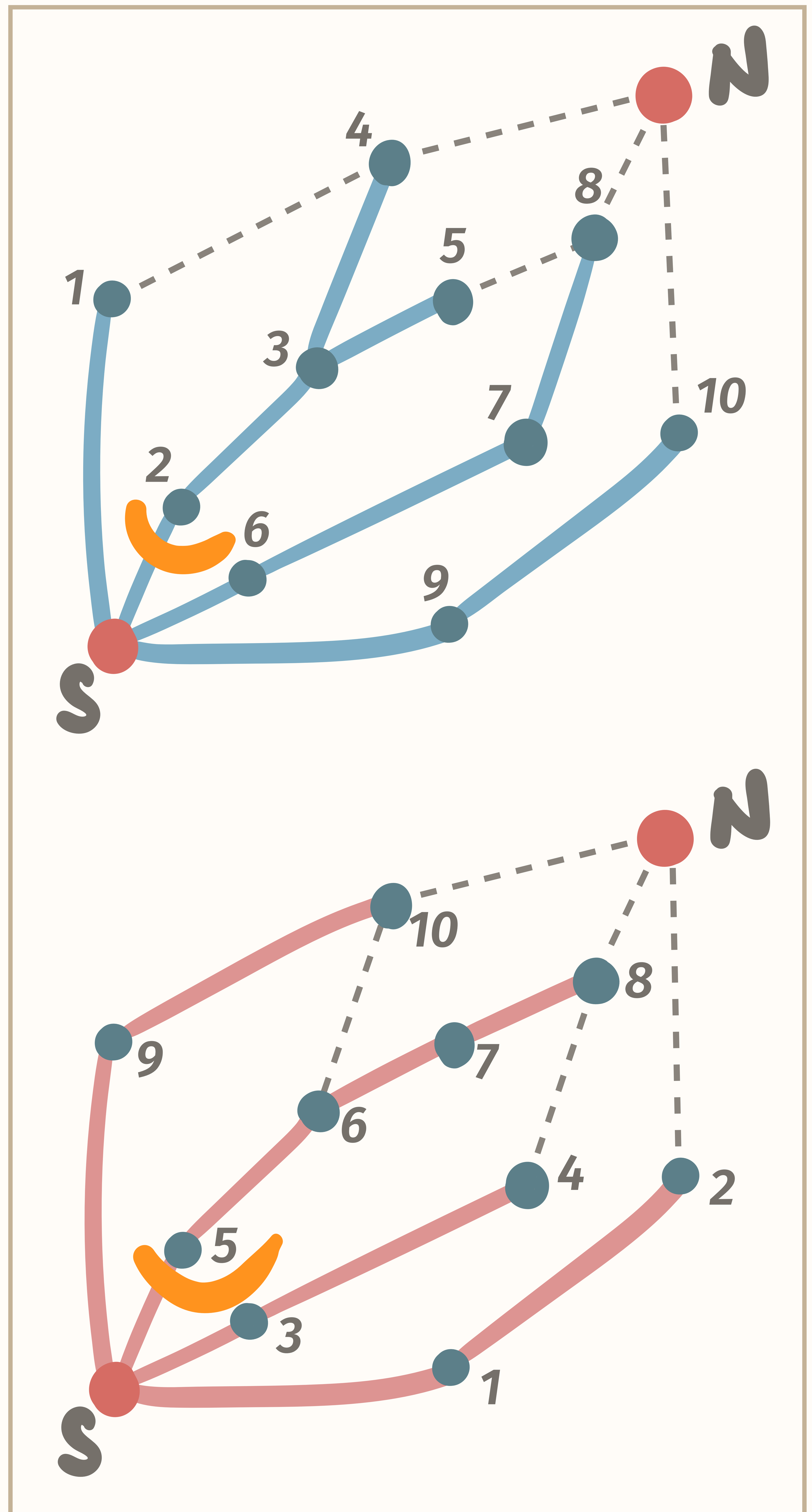


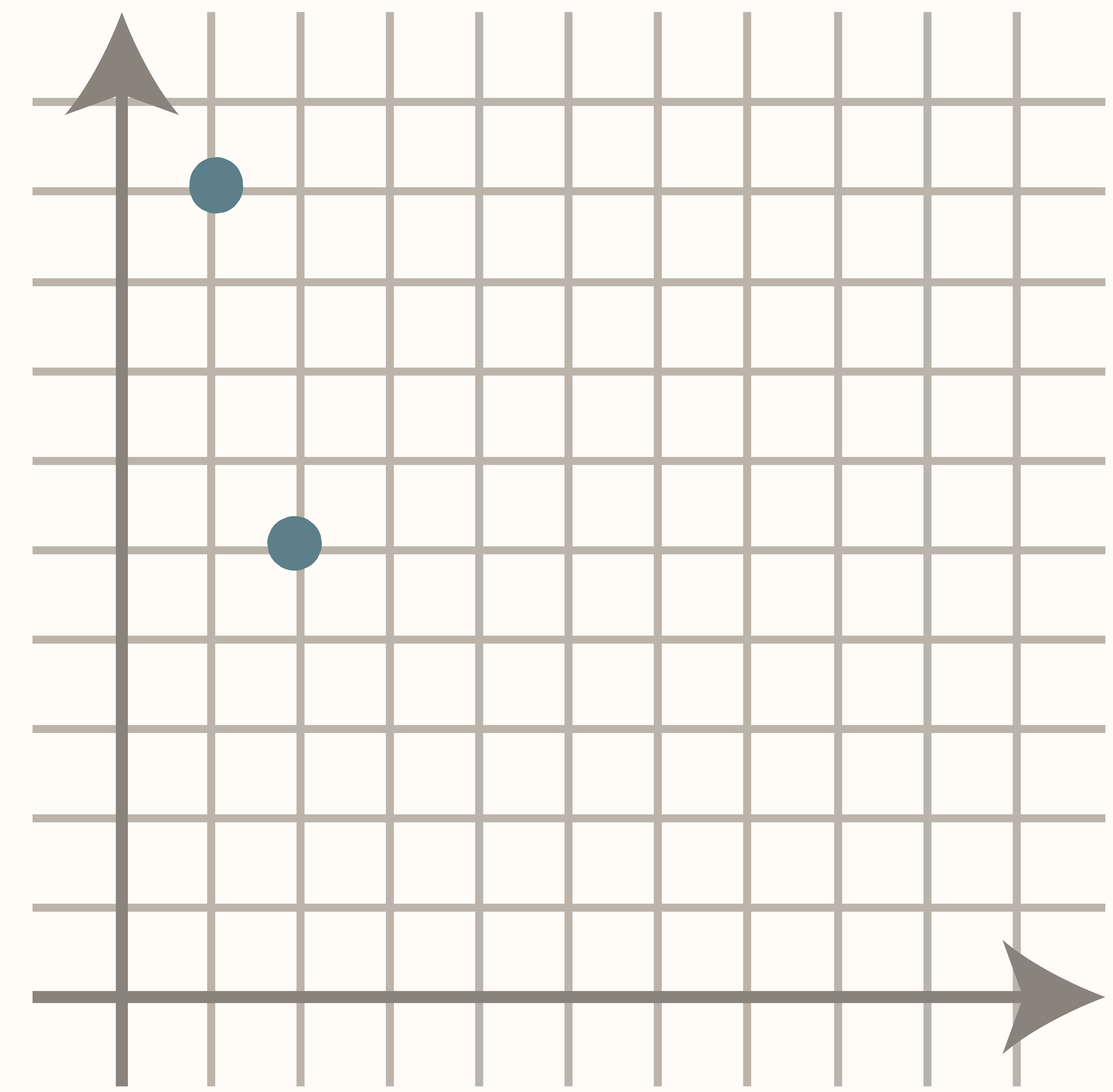
π : 1→9

Link with plane permutations

Poset ⟶ *Plane permutation*





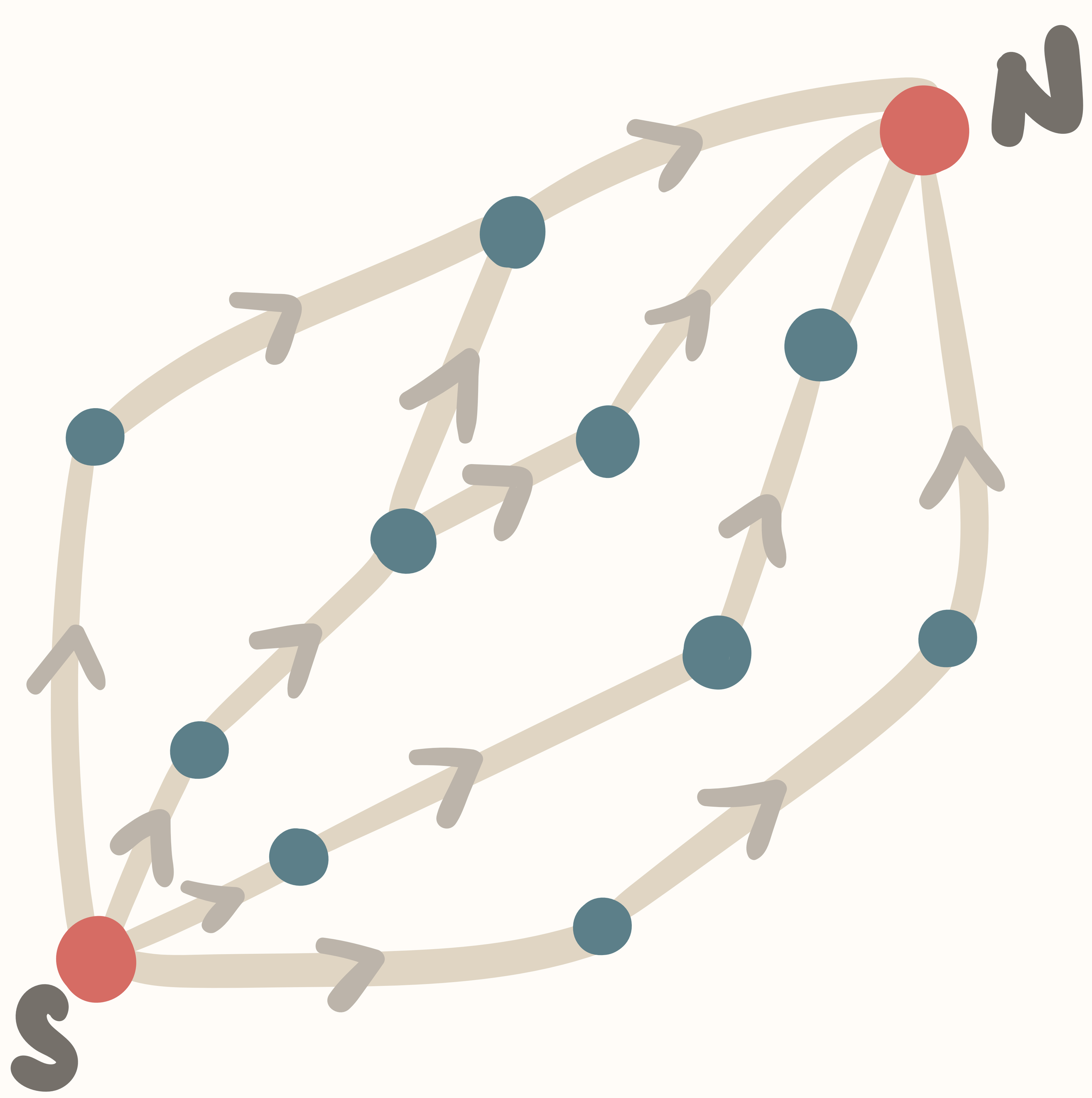


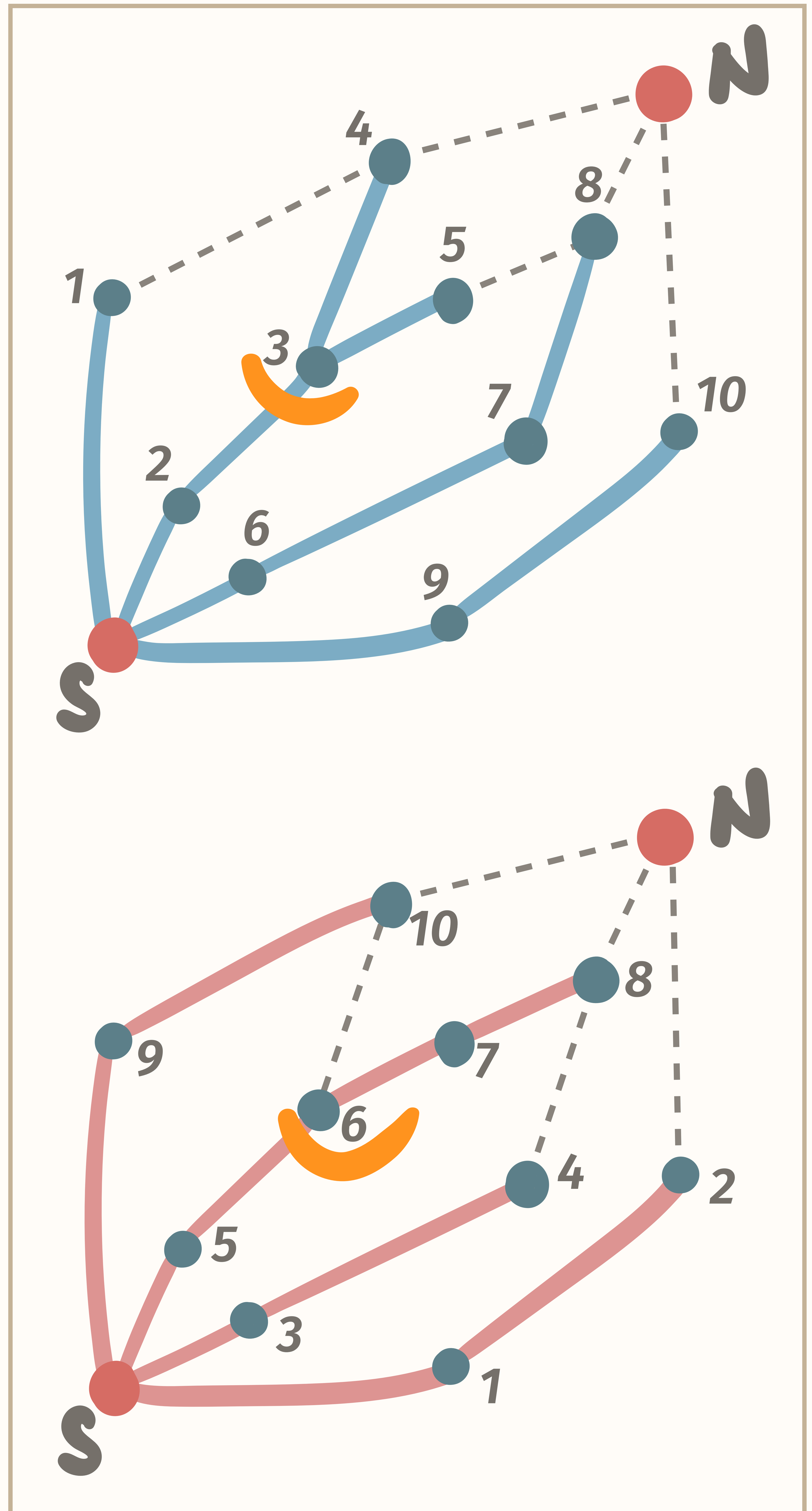
π : $1 \to 9$

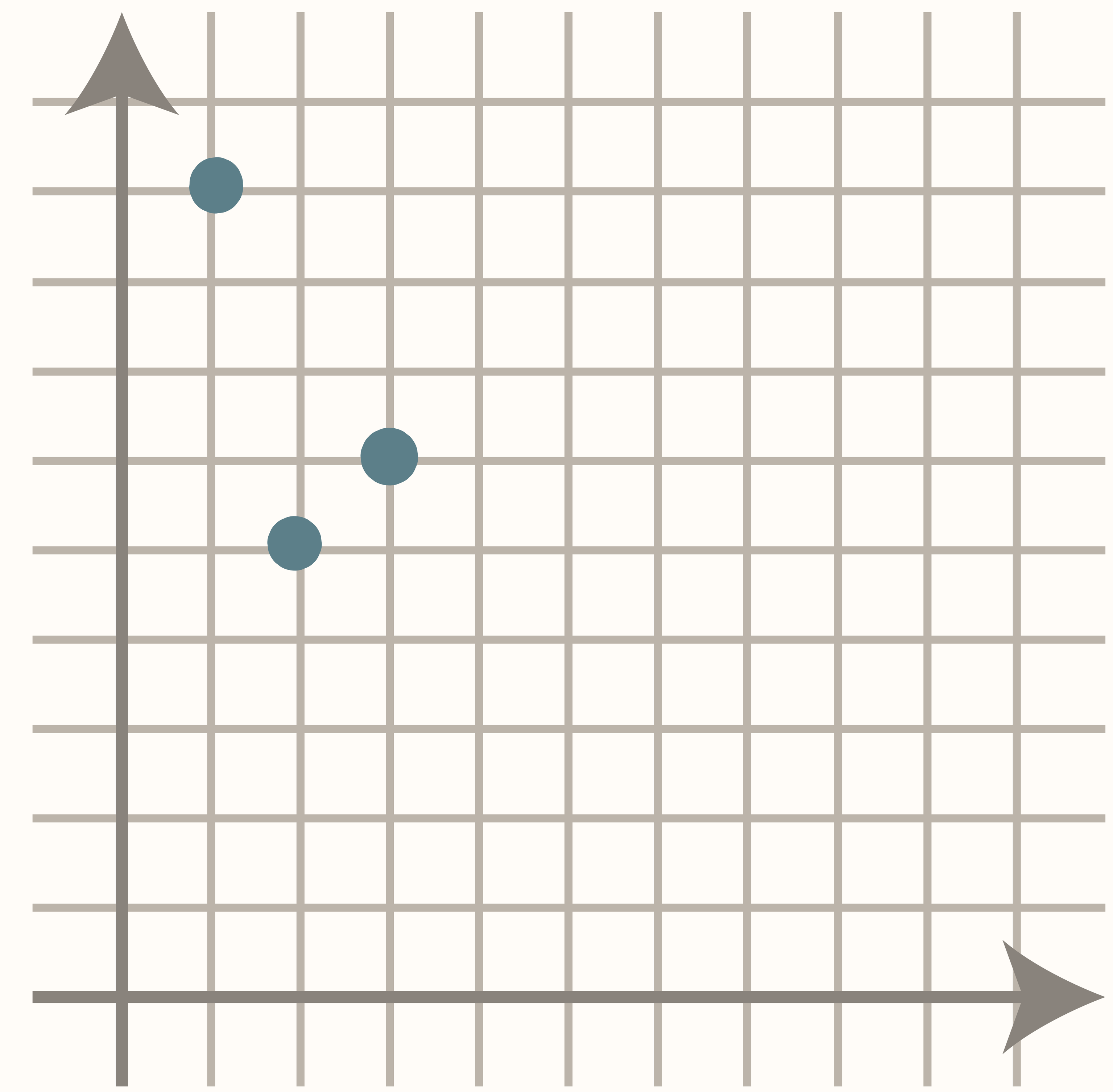
$2 \to 5$

Link with plane permutations

Poset ⟶ *Plane permutation*



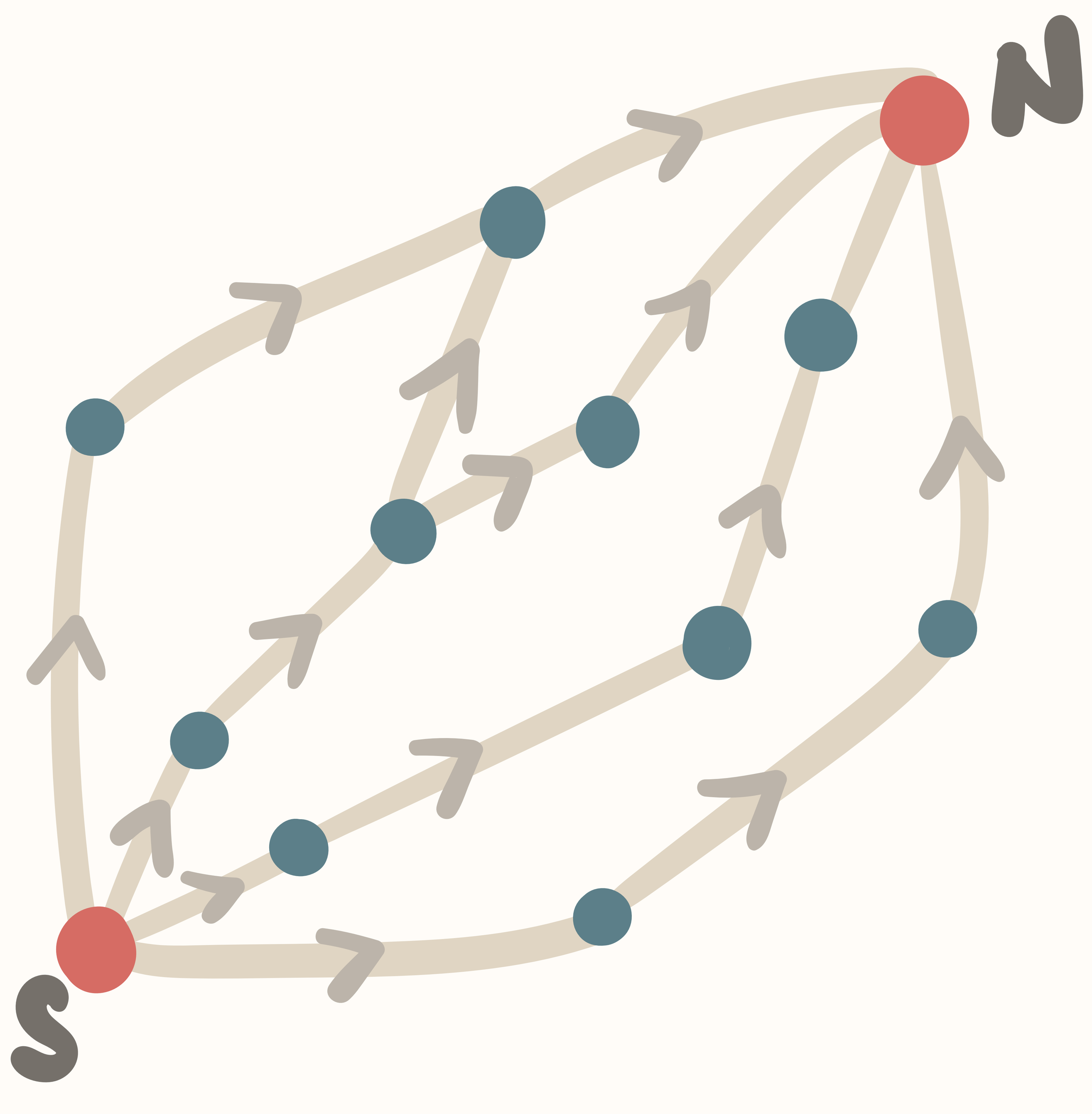


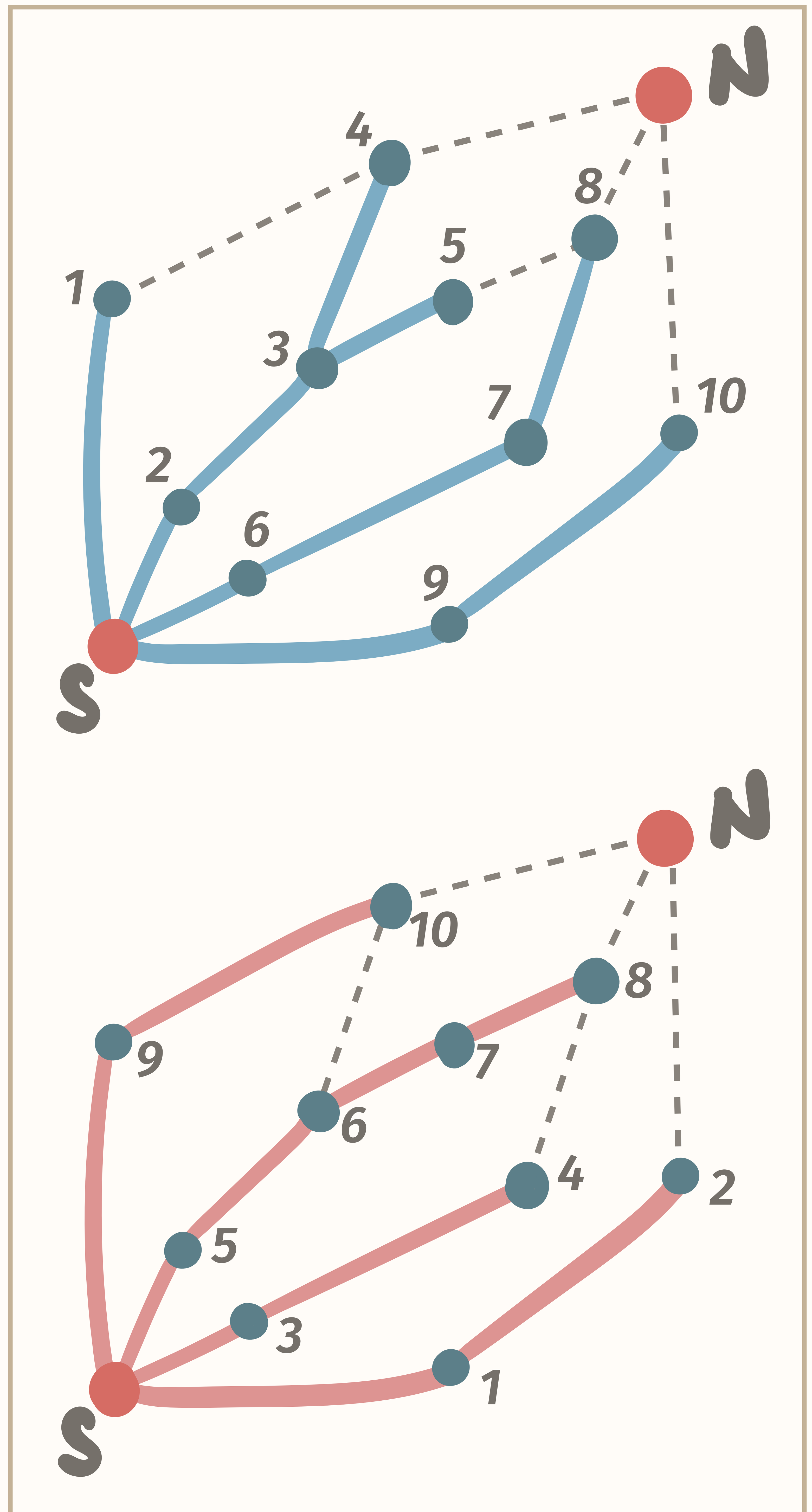


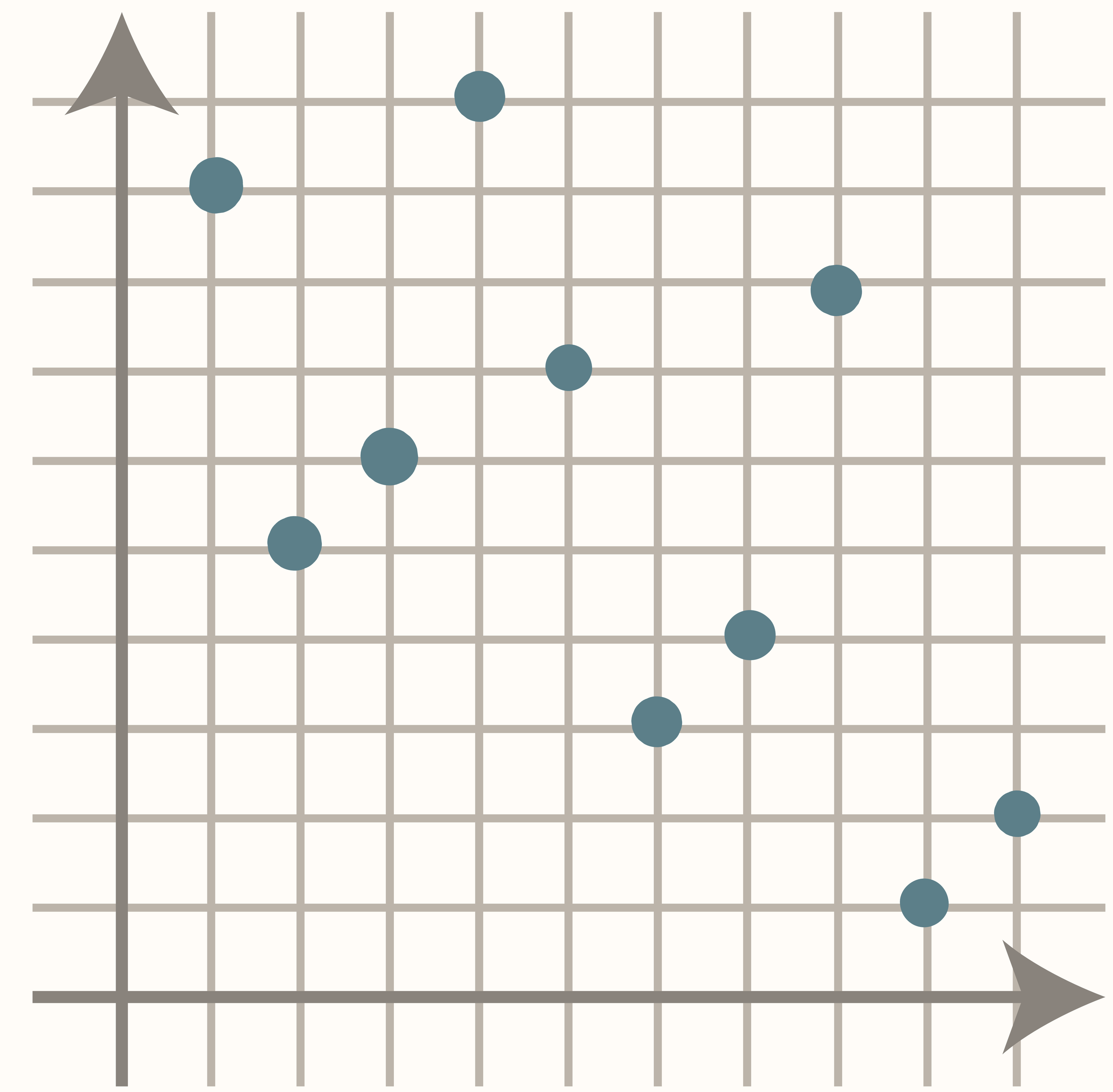
π: 1→9
2→5
3→6

Link with plane permutations

Poset ⟶ *Plane permutation*



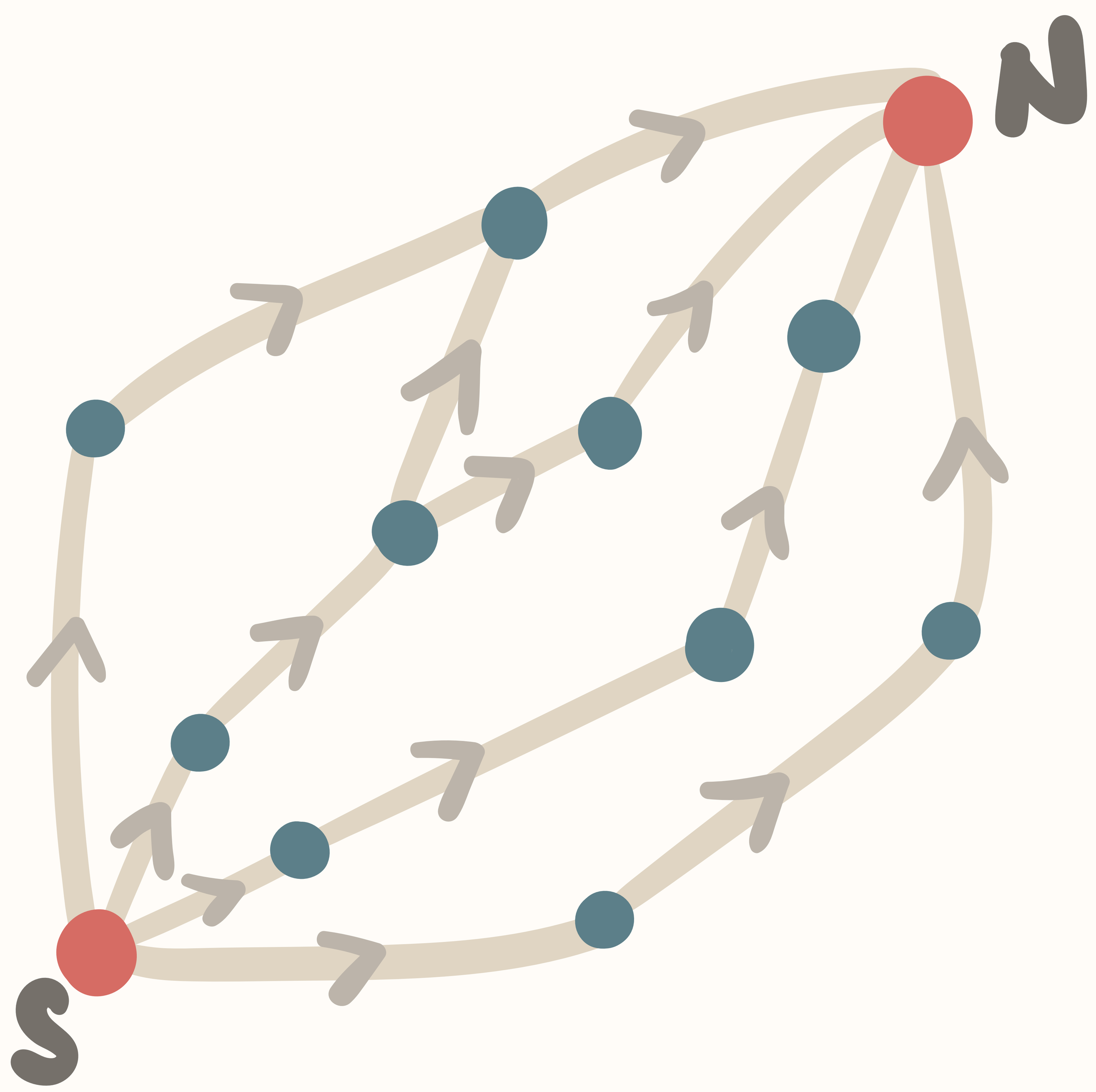


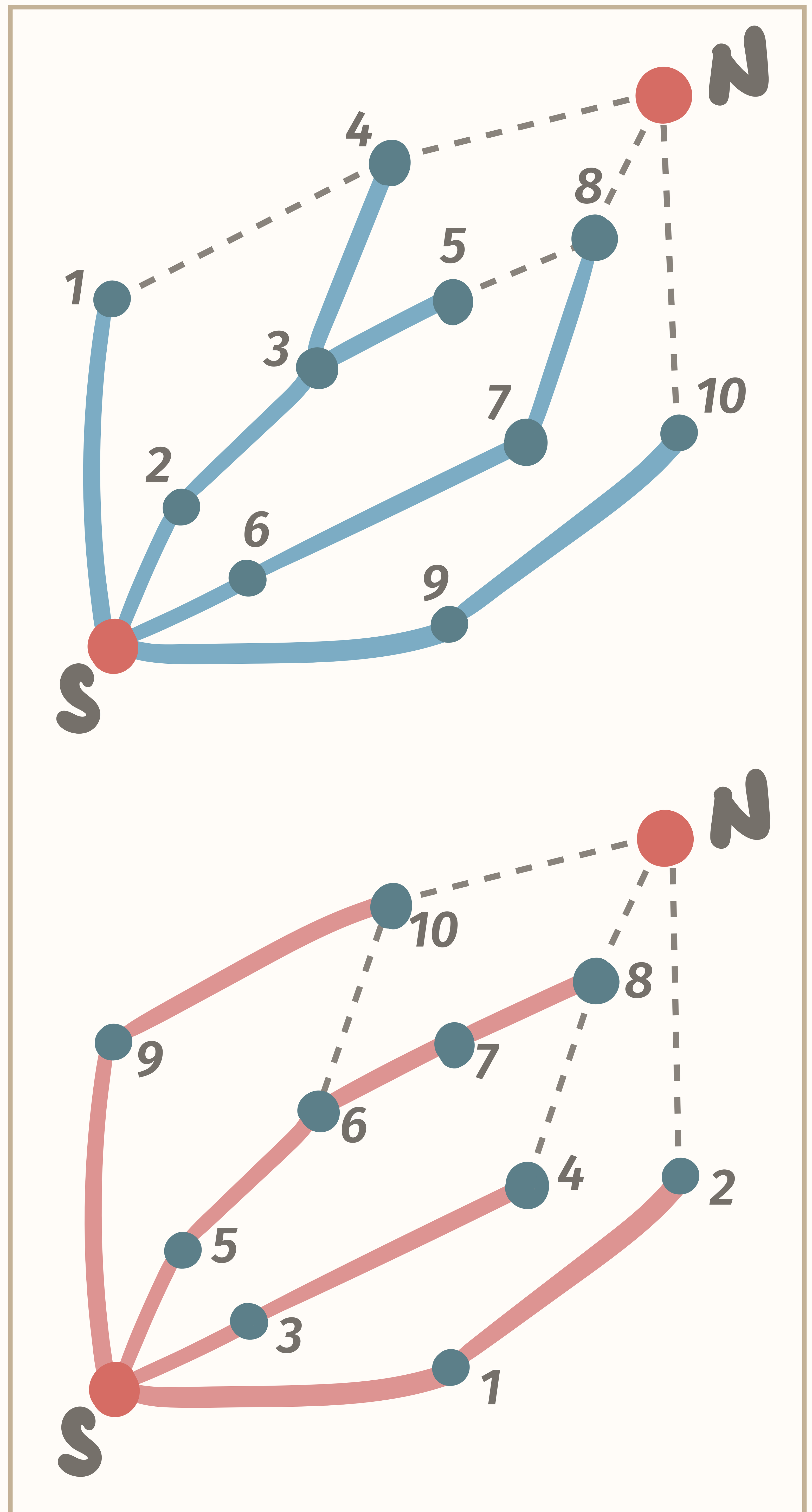


π :
1→9   6→3
2→5   7→4
3→6   8→8
4→10   9→1
5→7   10→2

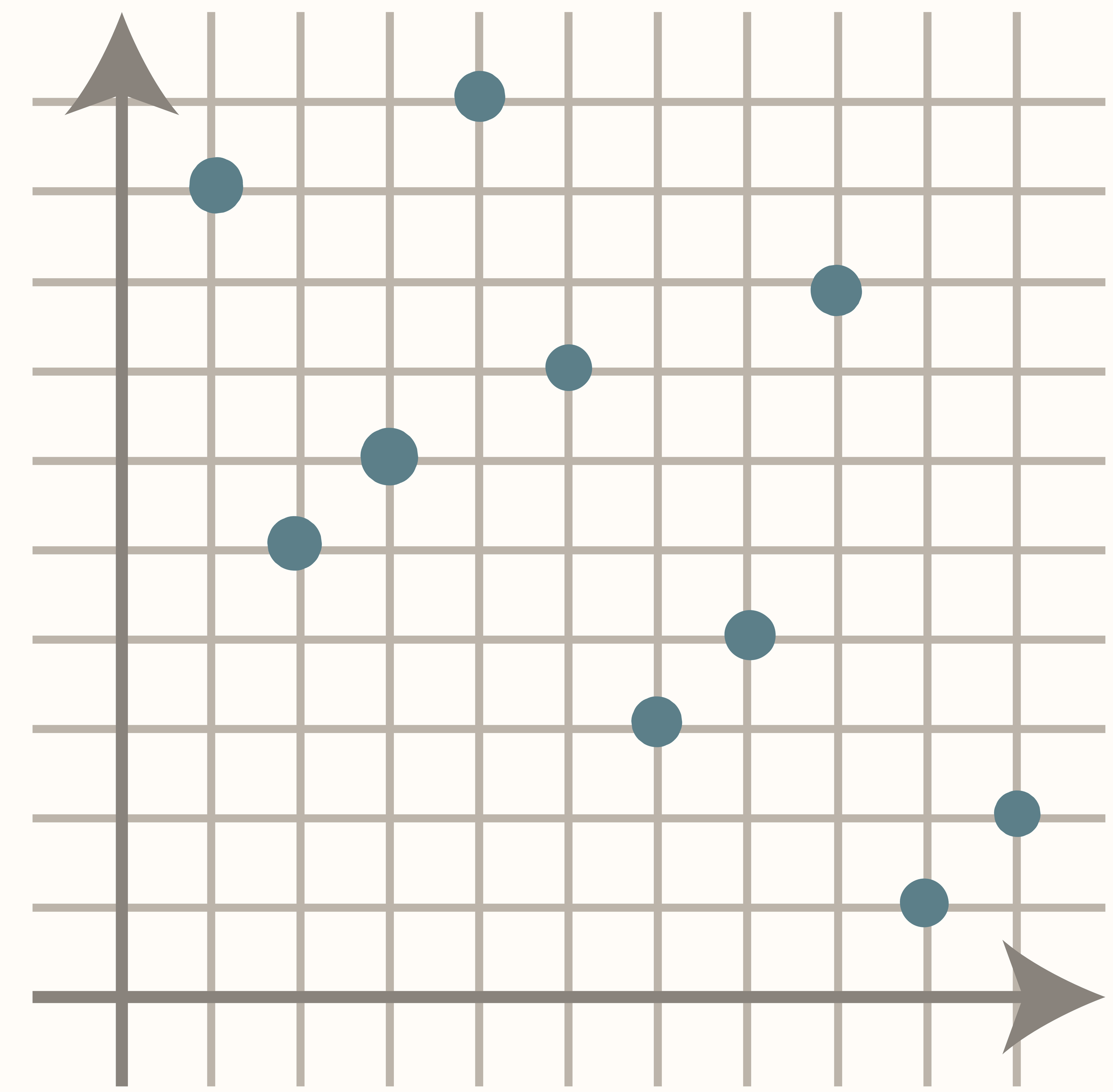
Link with plane permutations

Poset ⟶ Plane permutation





≫ *Area requirement and symmetry display of planar upward drawings,* G. Di Battista, R. Tamassia, and I. G. Tollis (1992)



π :

1→9	6→3
2→5	7→4
3→6	8→8
4→10	9→1
5→7	10→2

Summary

Maps and decorated maps

1. Bijection with quadrant tandem walks

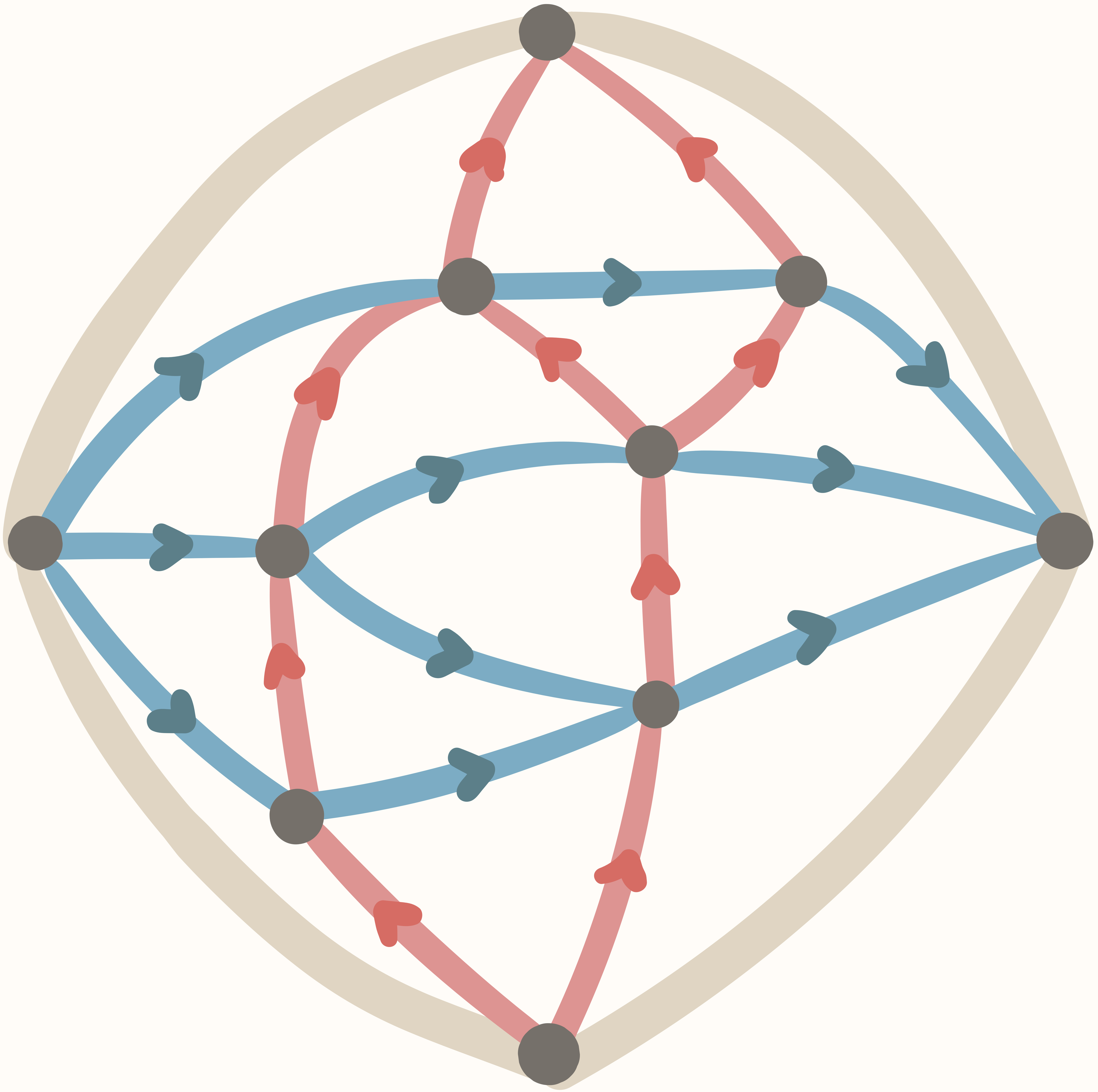
a. The KMSW bijection
b. Plane bipolar posets
c. Plane bipolar posets by vertices
d. Transversal structures

2. Asymptotic enumeration

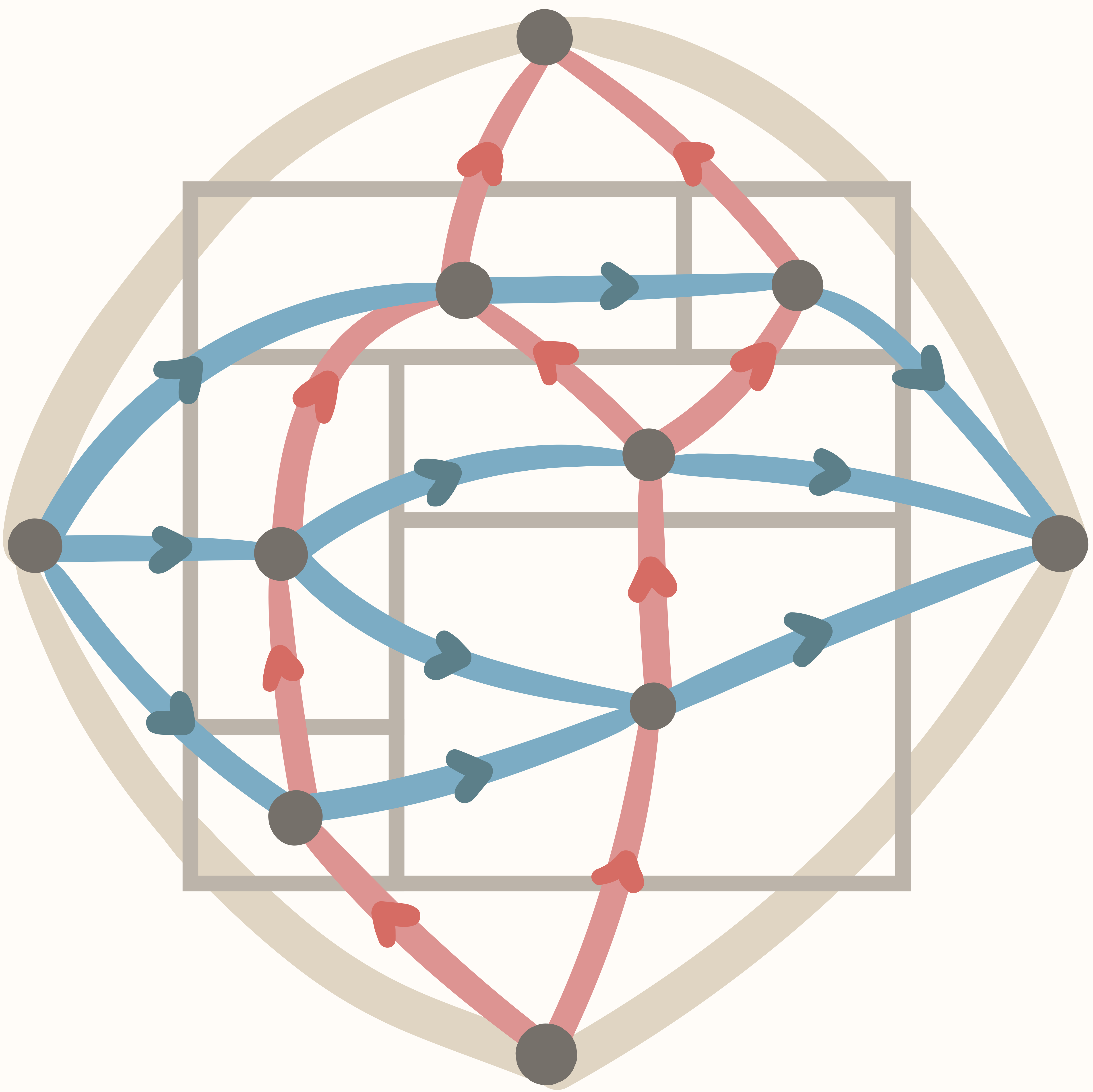
(Digression on plane permutations)

3. Generic transversal structures

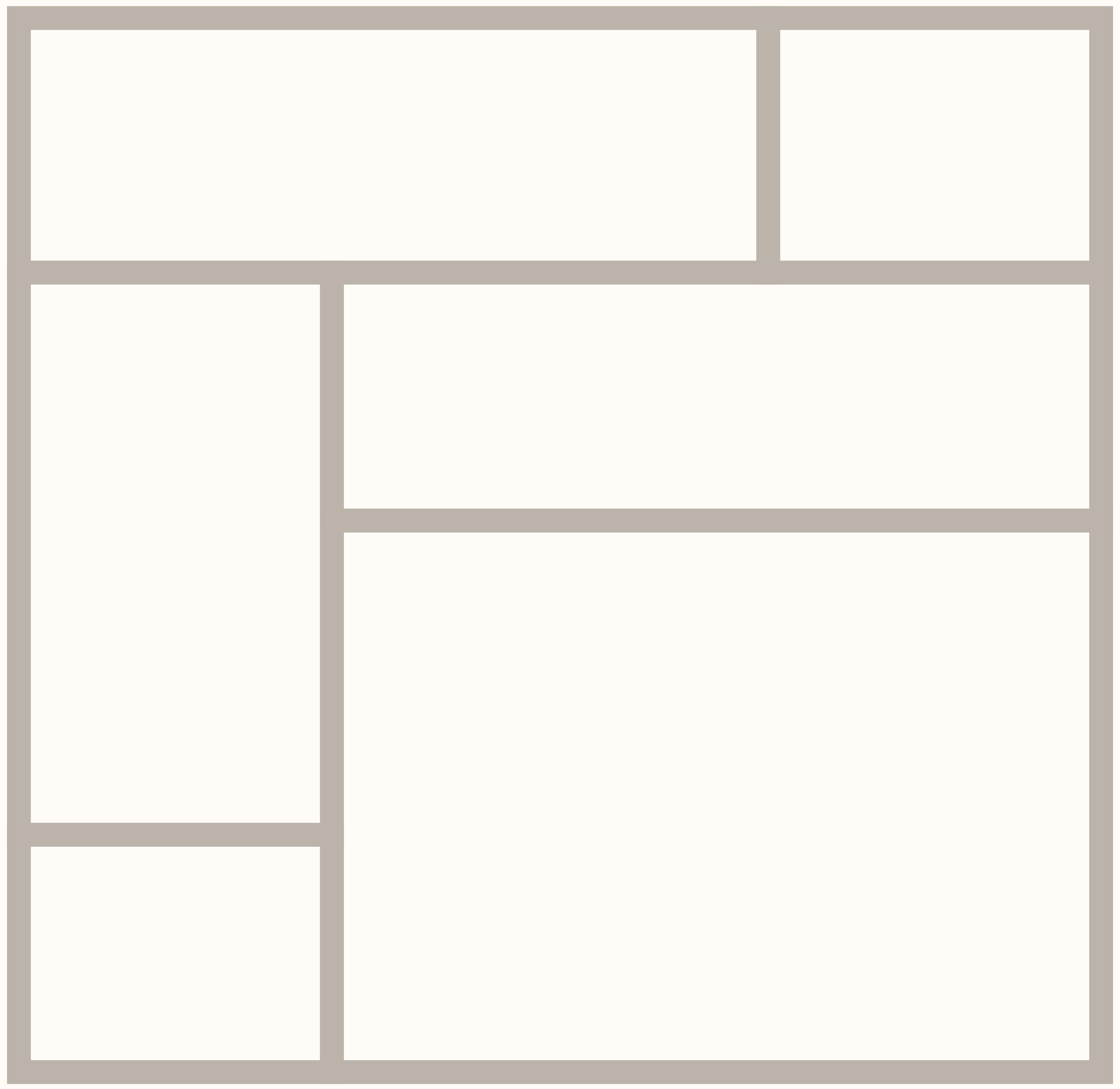
Generic transversal structures



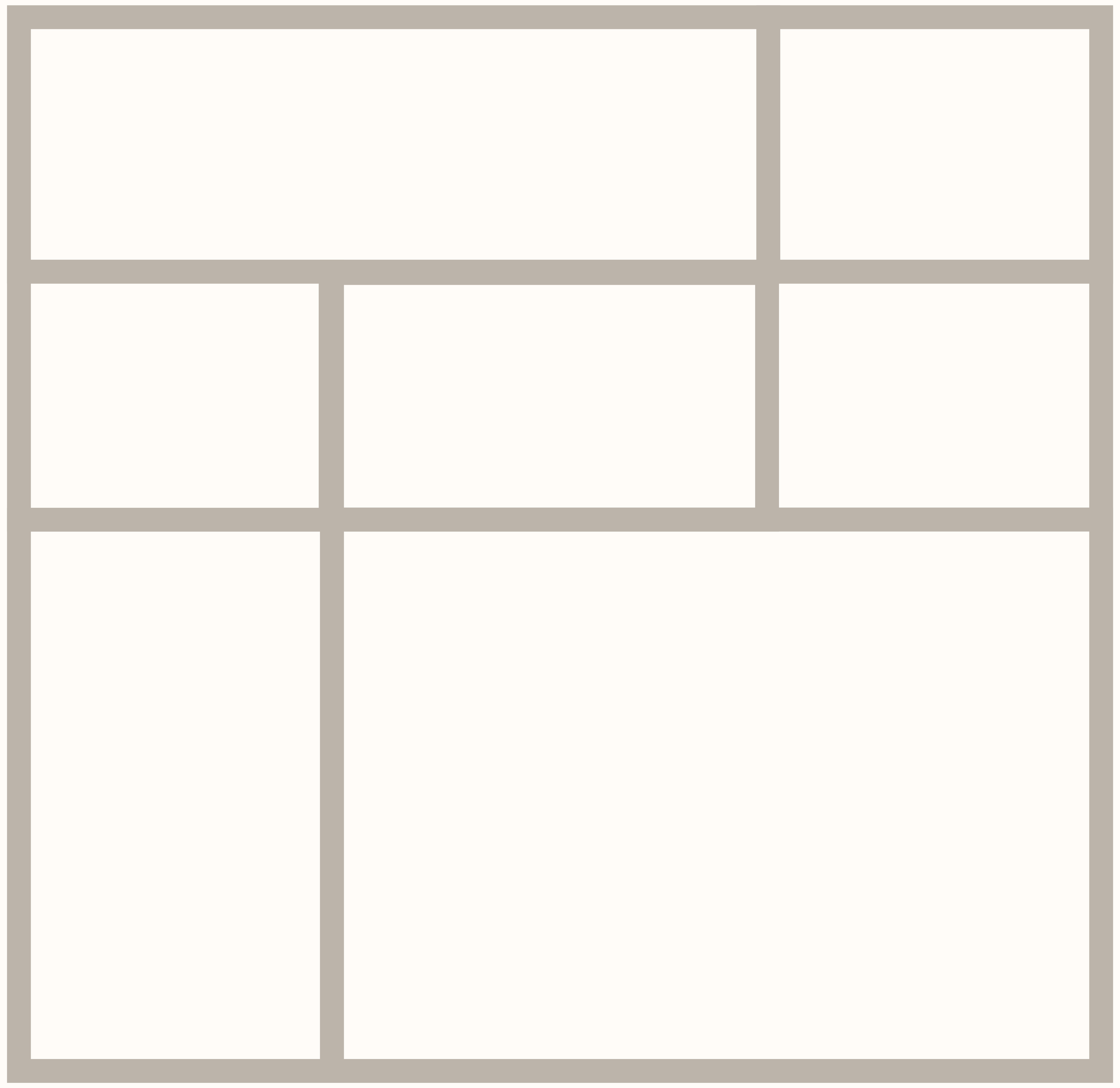
Generic transversal structures



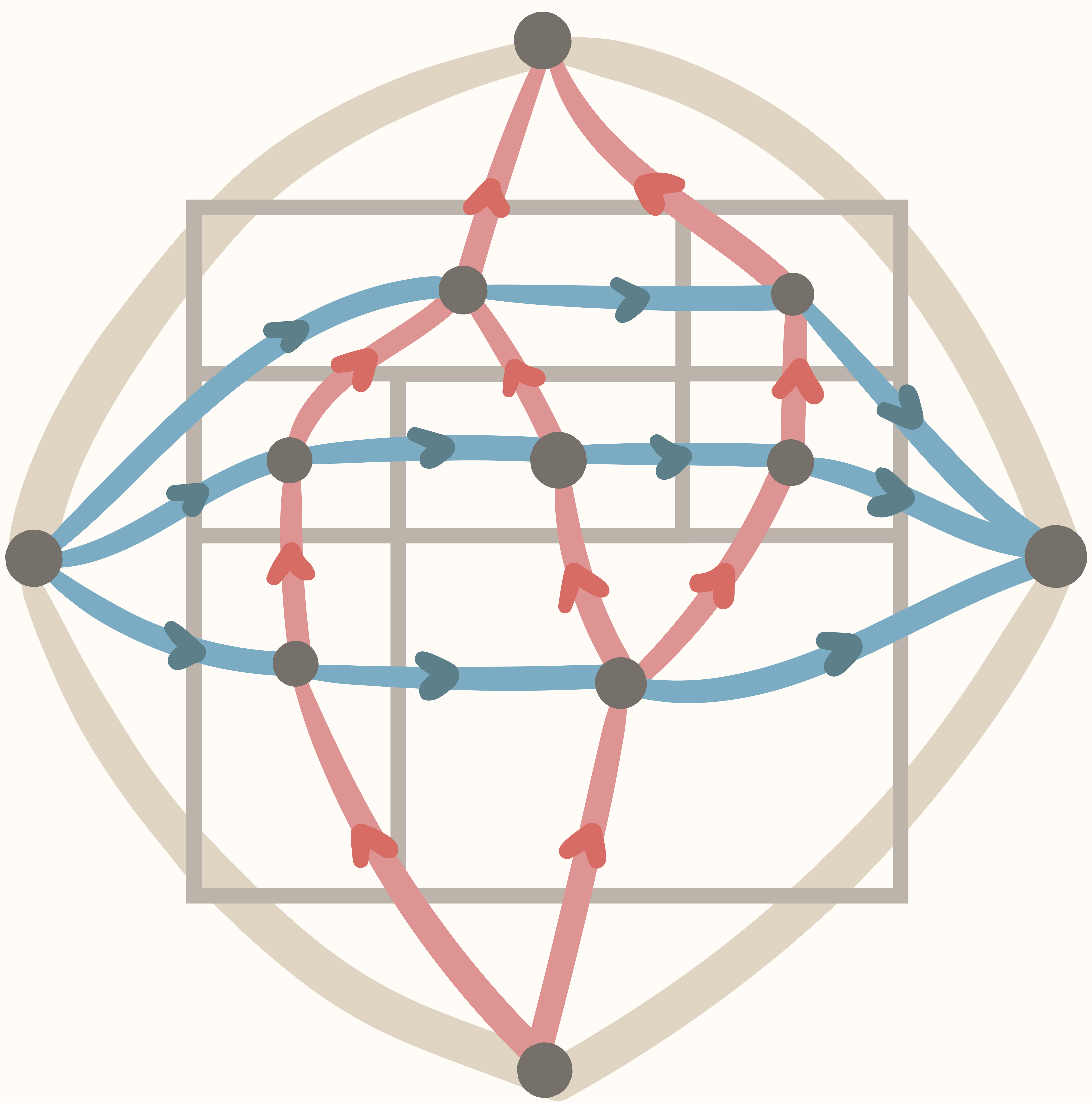
Generic transversal structures



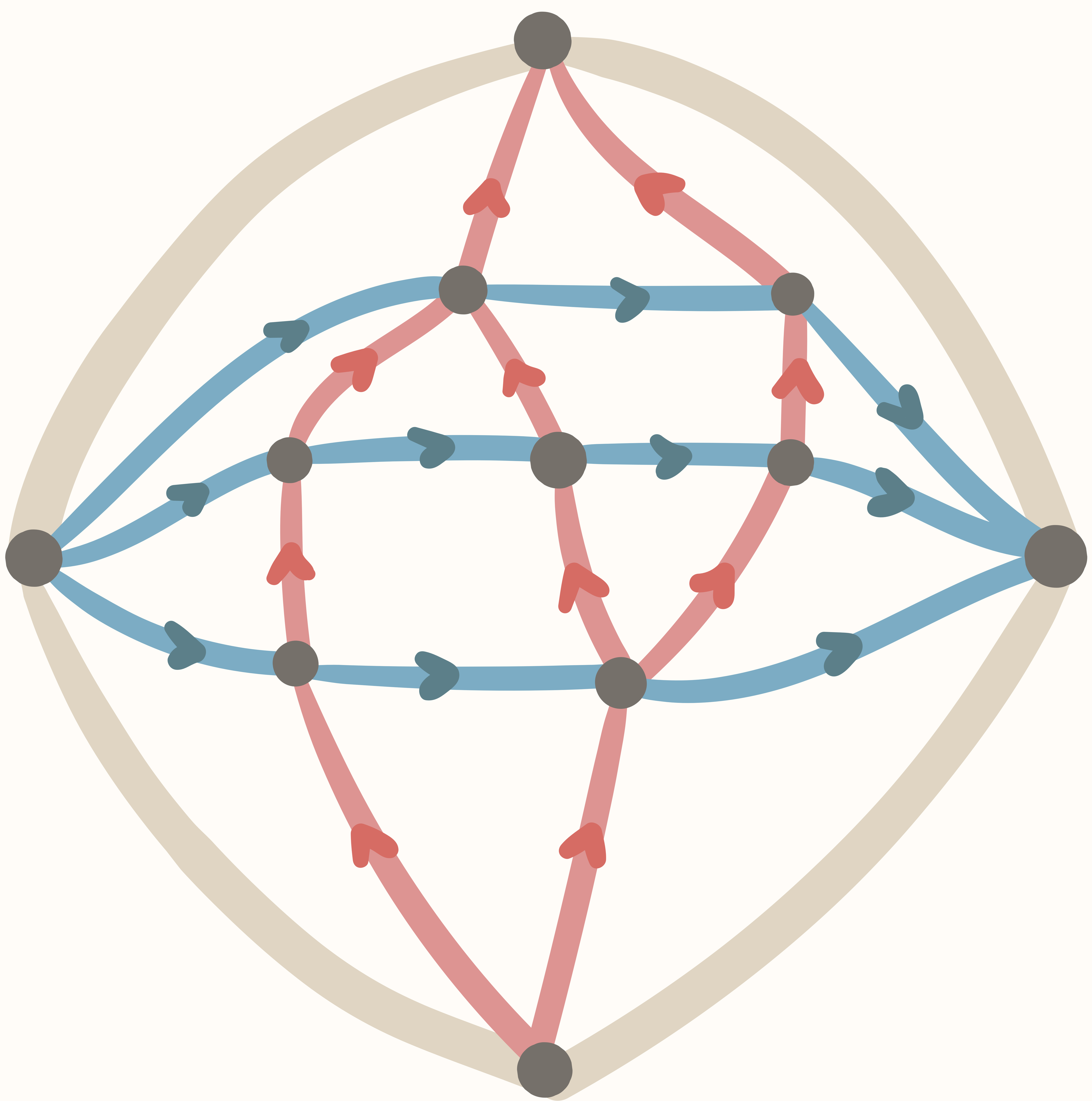
Generic transversal structures

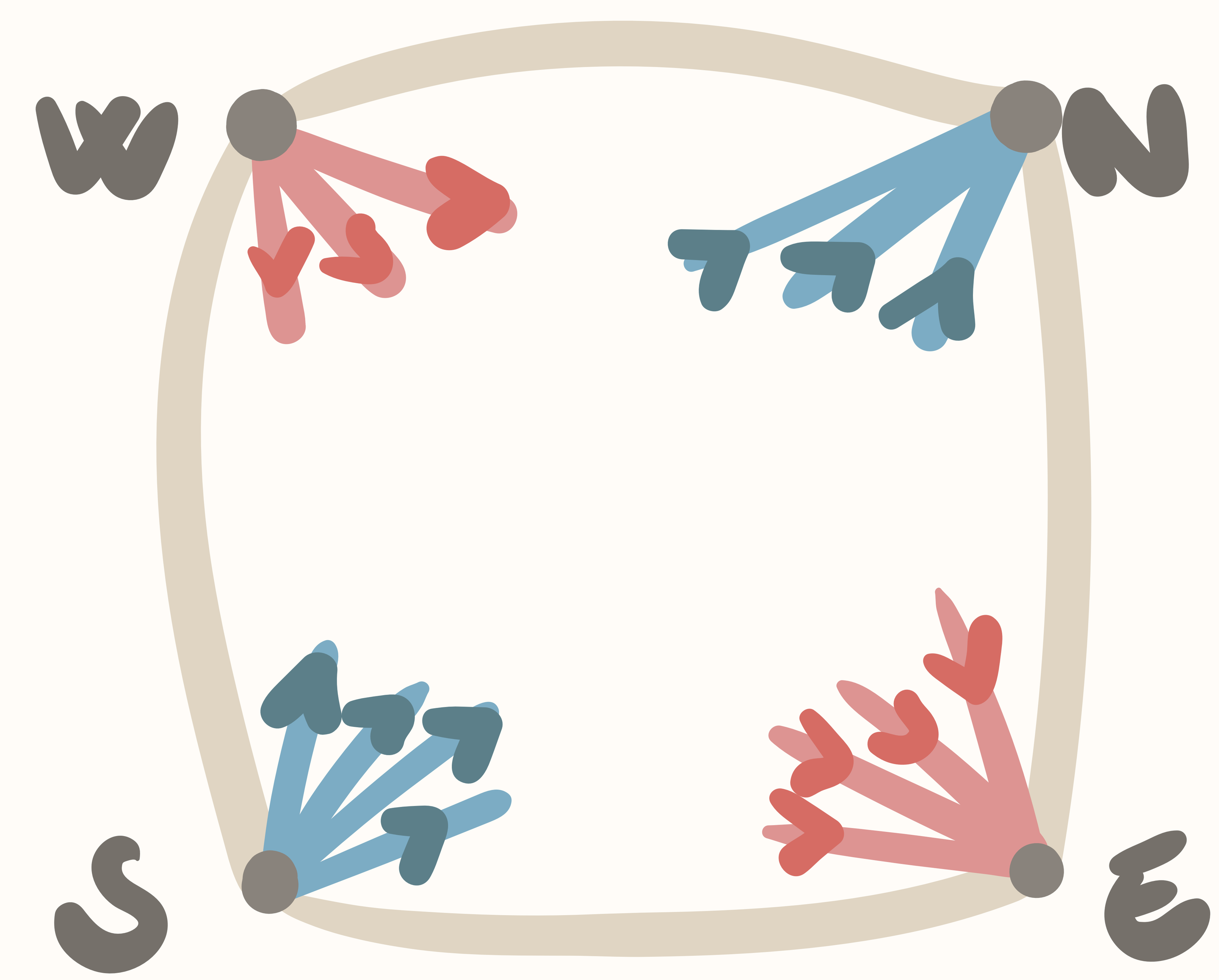


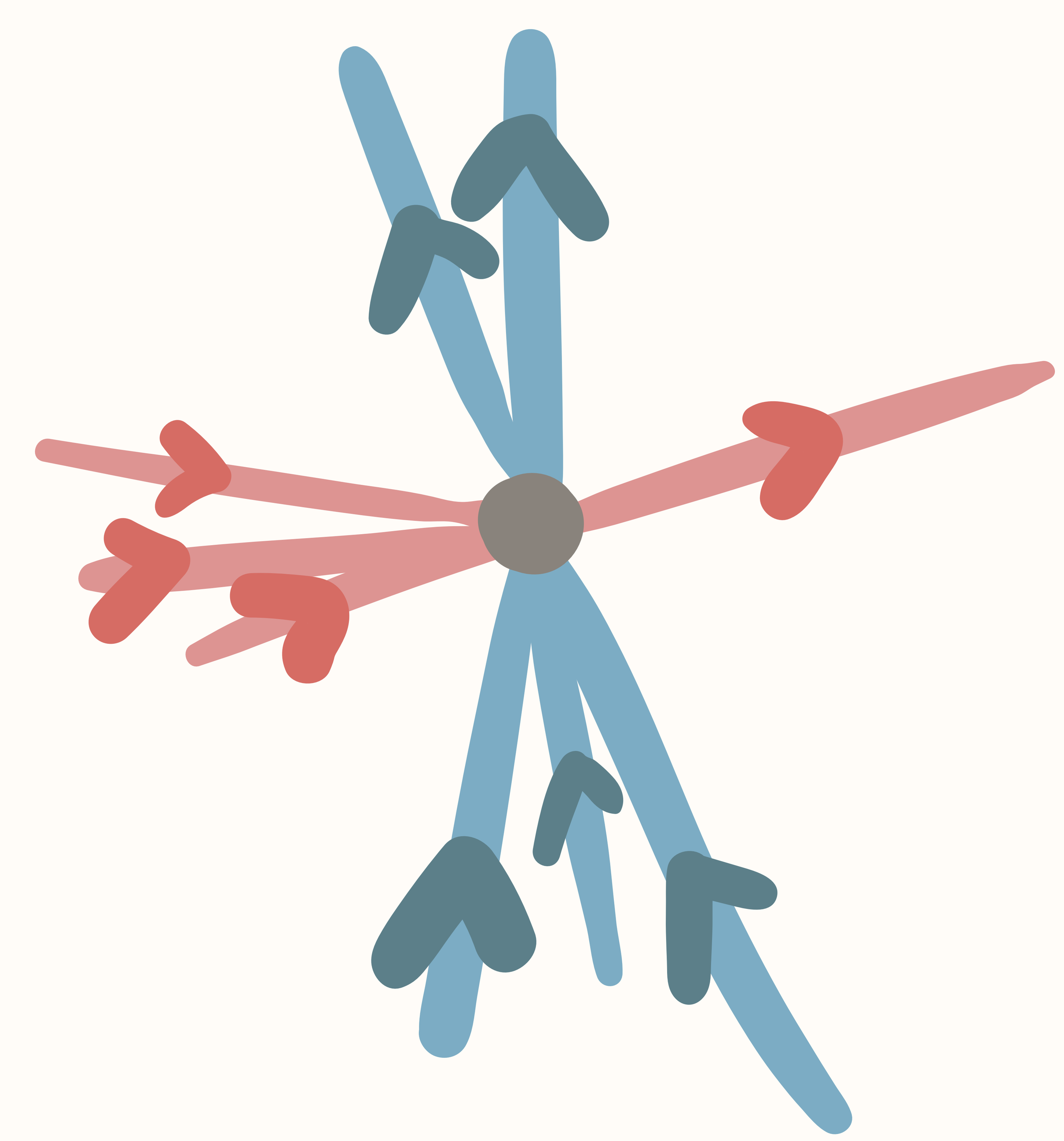
Generic transversal structures



Generic transversal structures

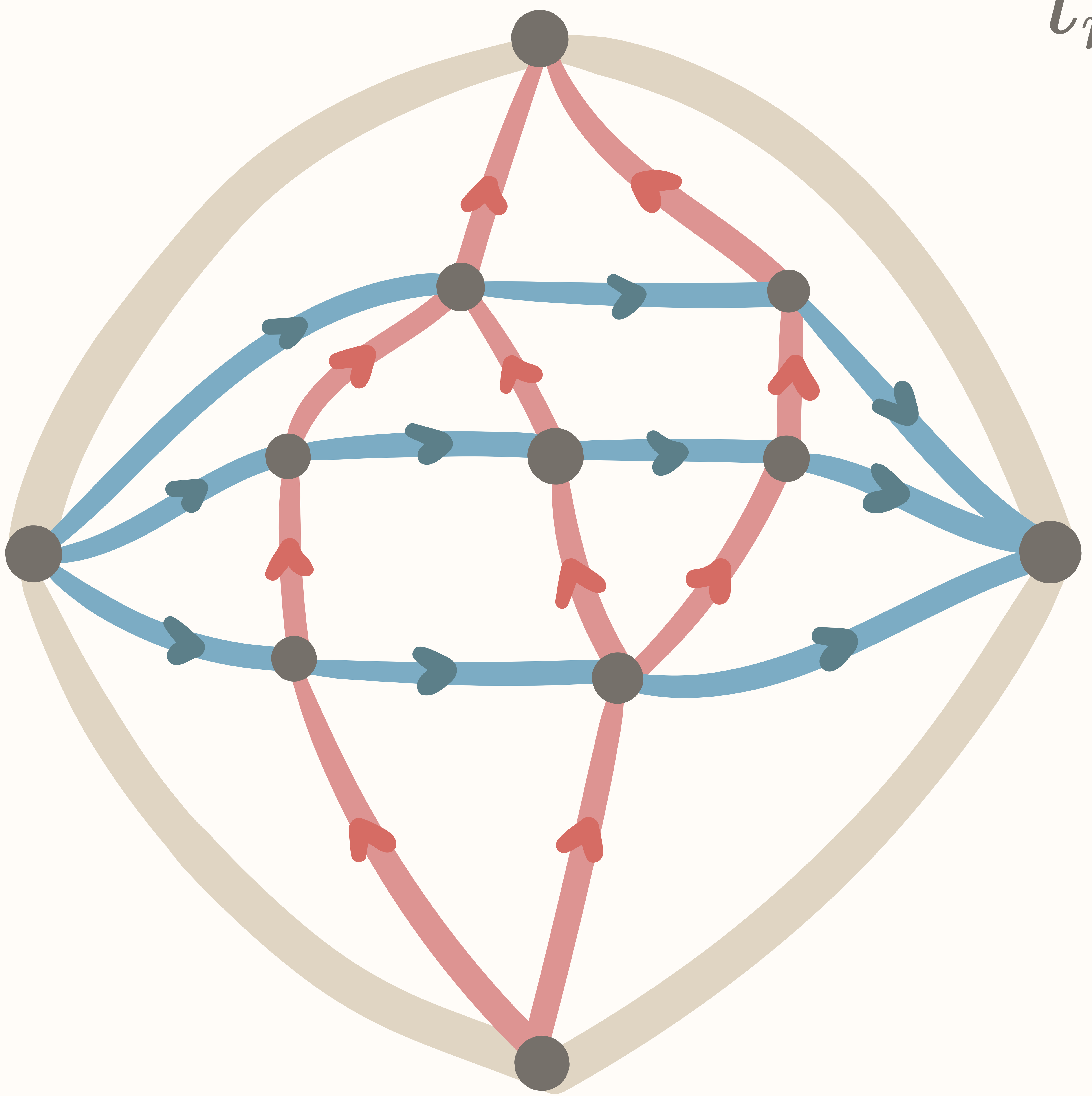




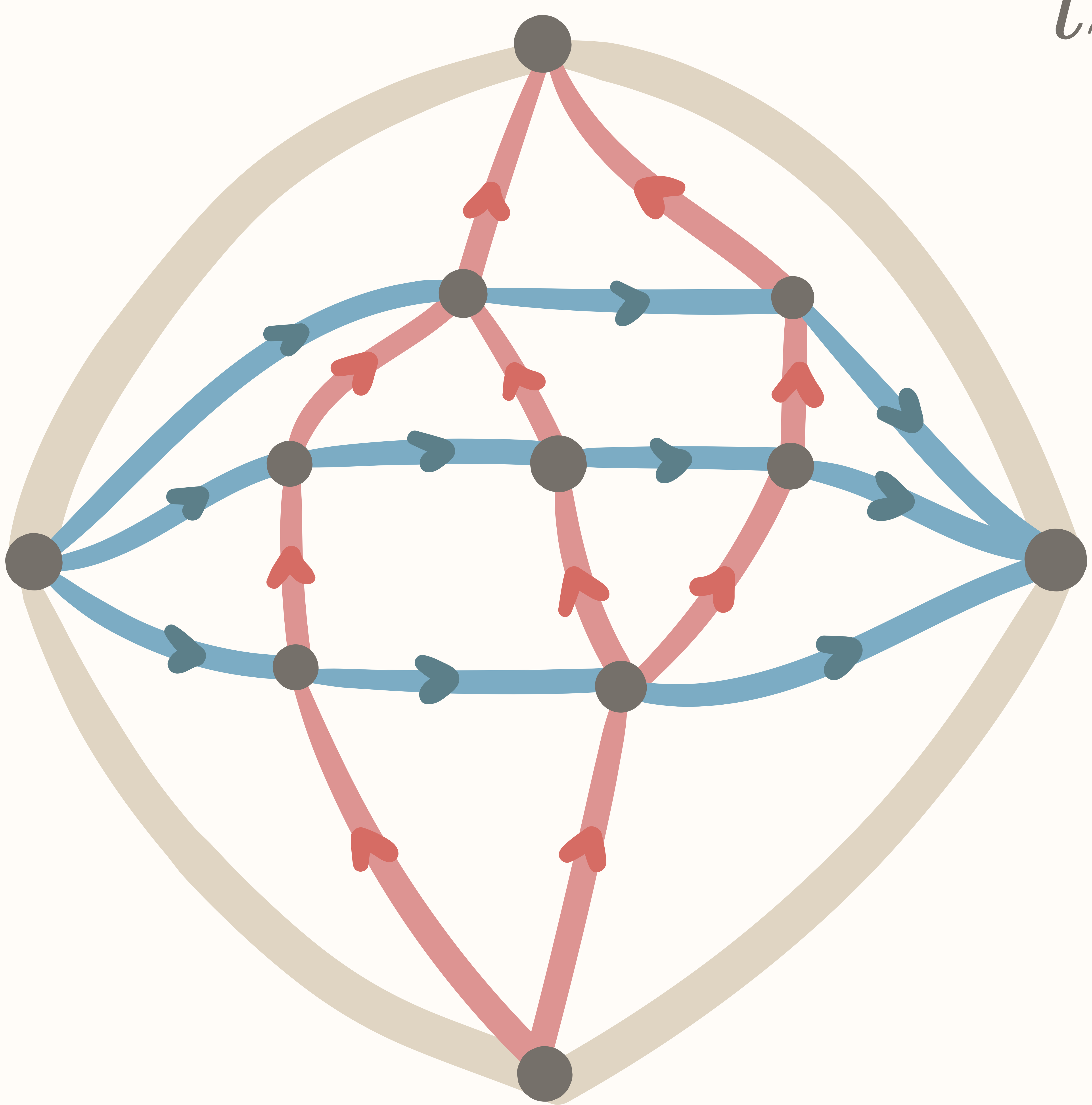


Generic transversal structures

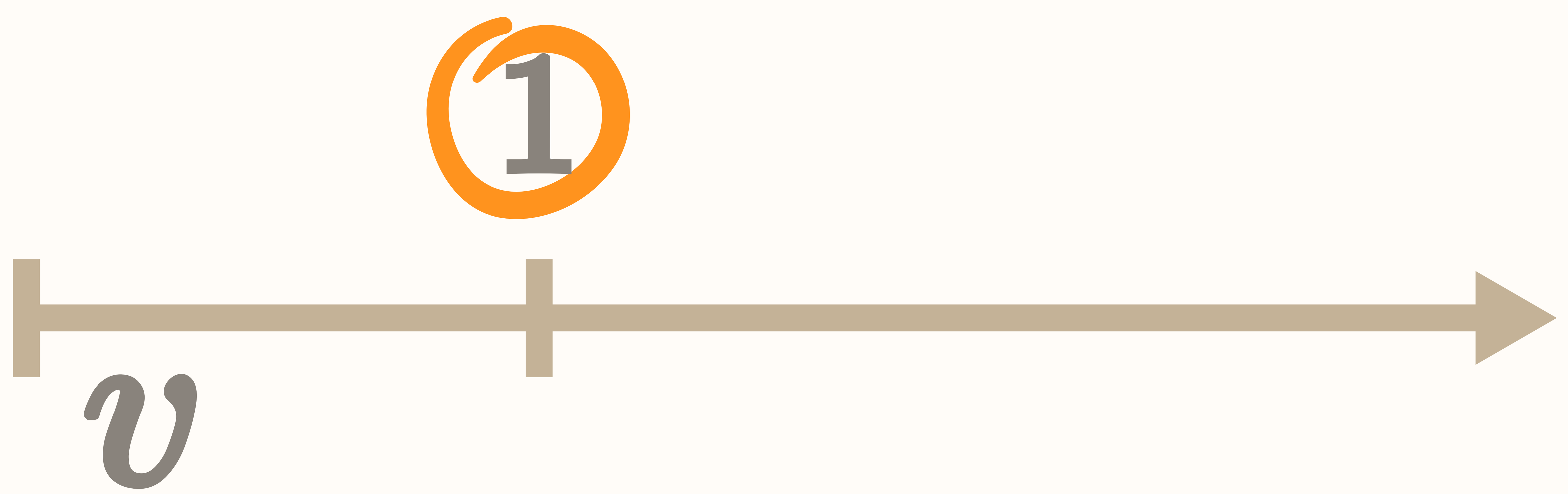
$$t_n \sim \varkappa\ \gamma(v)^n\, n^{-1-\frac{\pi}{\arccos(\xi(v))}}$$



Generic transversal structures

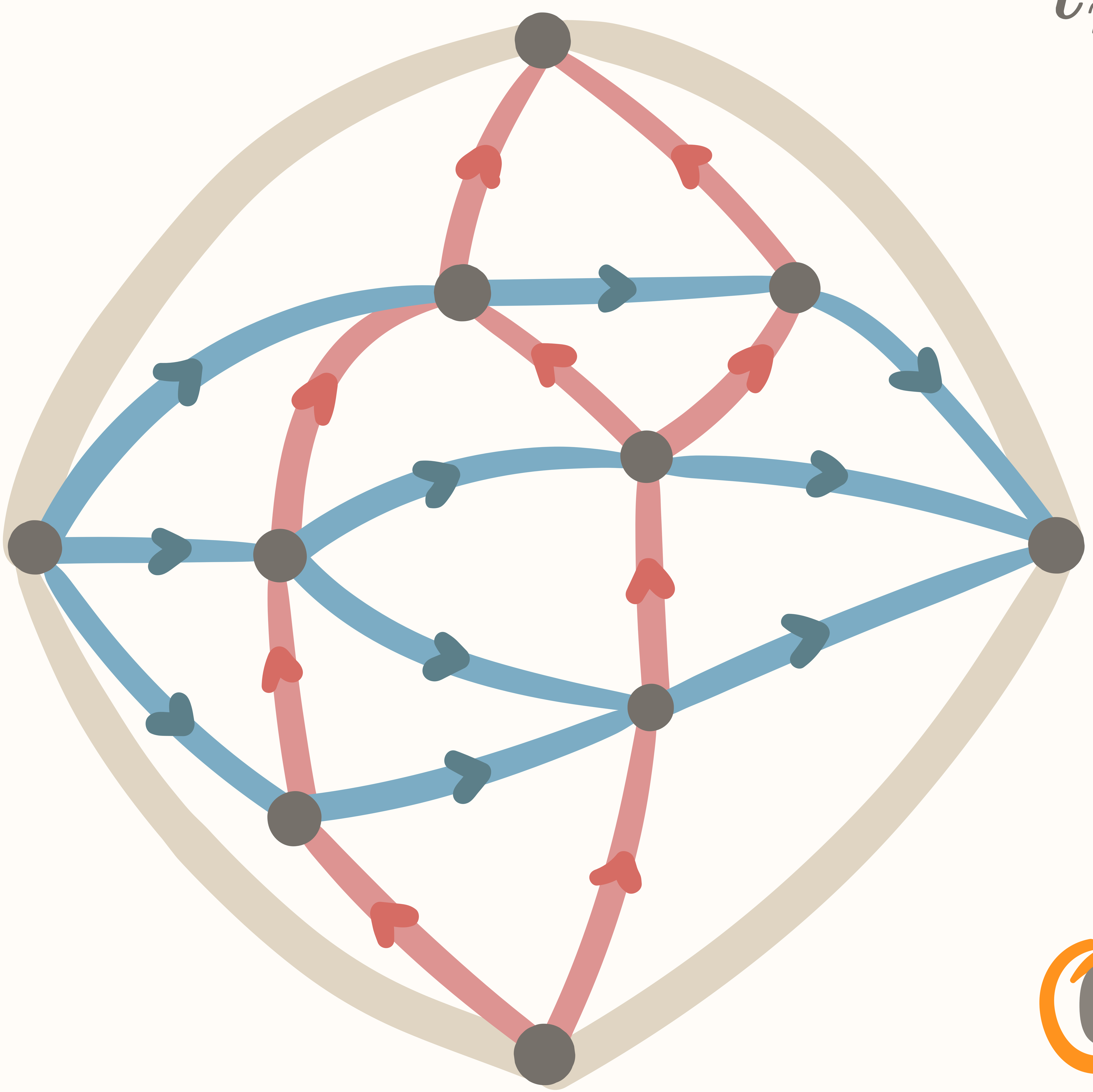


$$t_n \sim \varkappa\ \gamma(v)^n\, n^{-1-\frac{\pi}{\arccos(\xi(v))}}$$



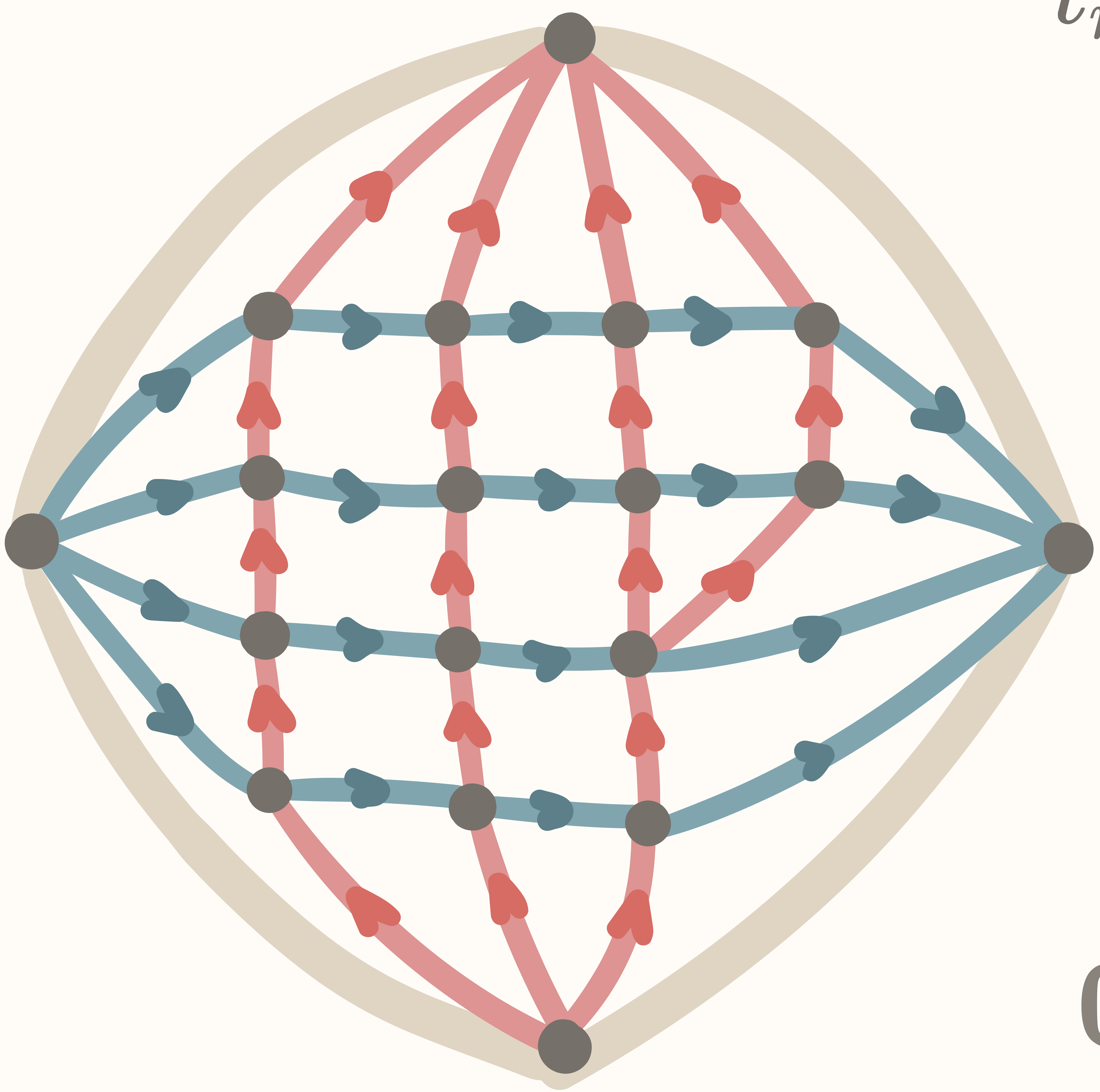
Generic transversal structures

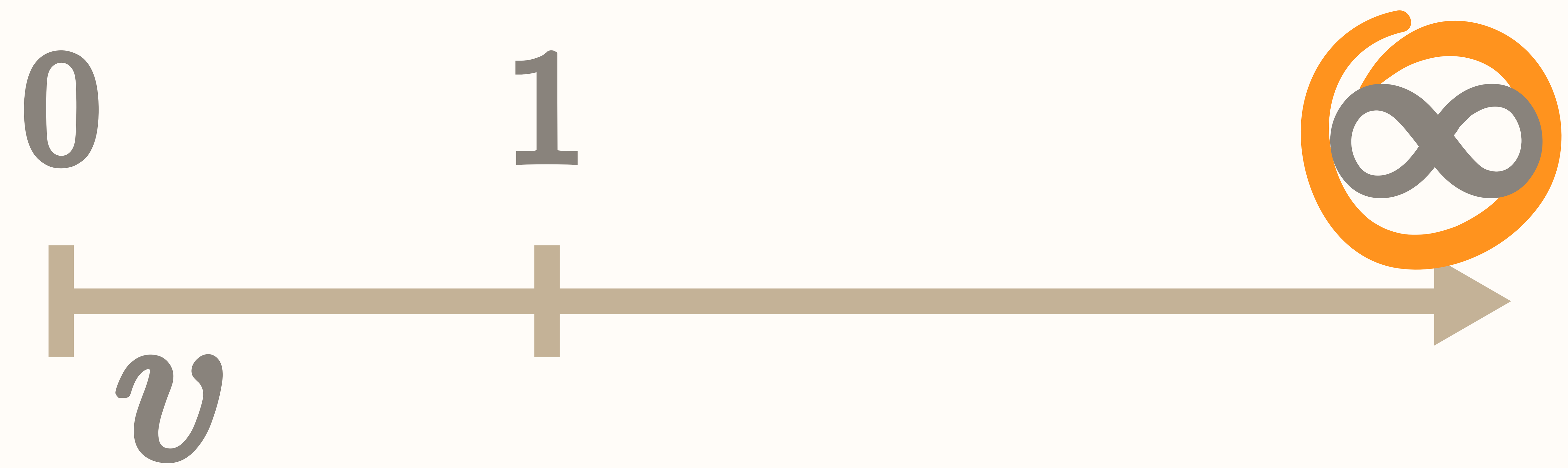
$$t_n \sim \varkappa \left(\frac{27}{2}\right)^n n^{-1-\frac{\pi}{\arccos(7/8)}}$$



Generic transversal structures

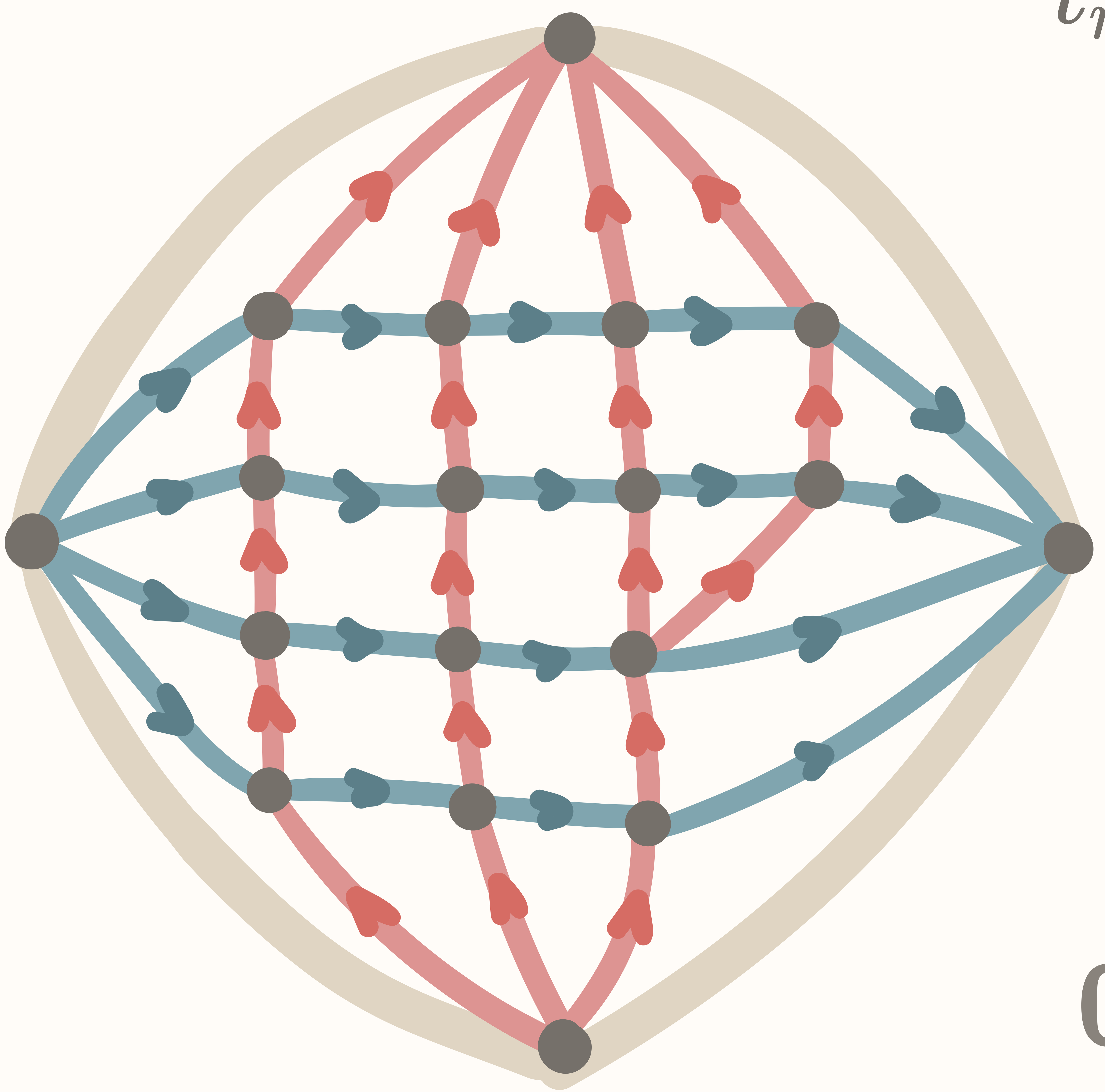
$$t_n \sim \varkappa \, \gamma(v)^n \, n^{-1-\frac{\pi}{\arccos(\xi(v))}}$$

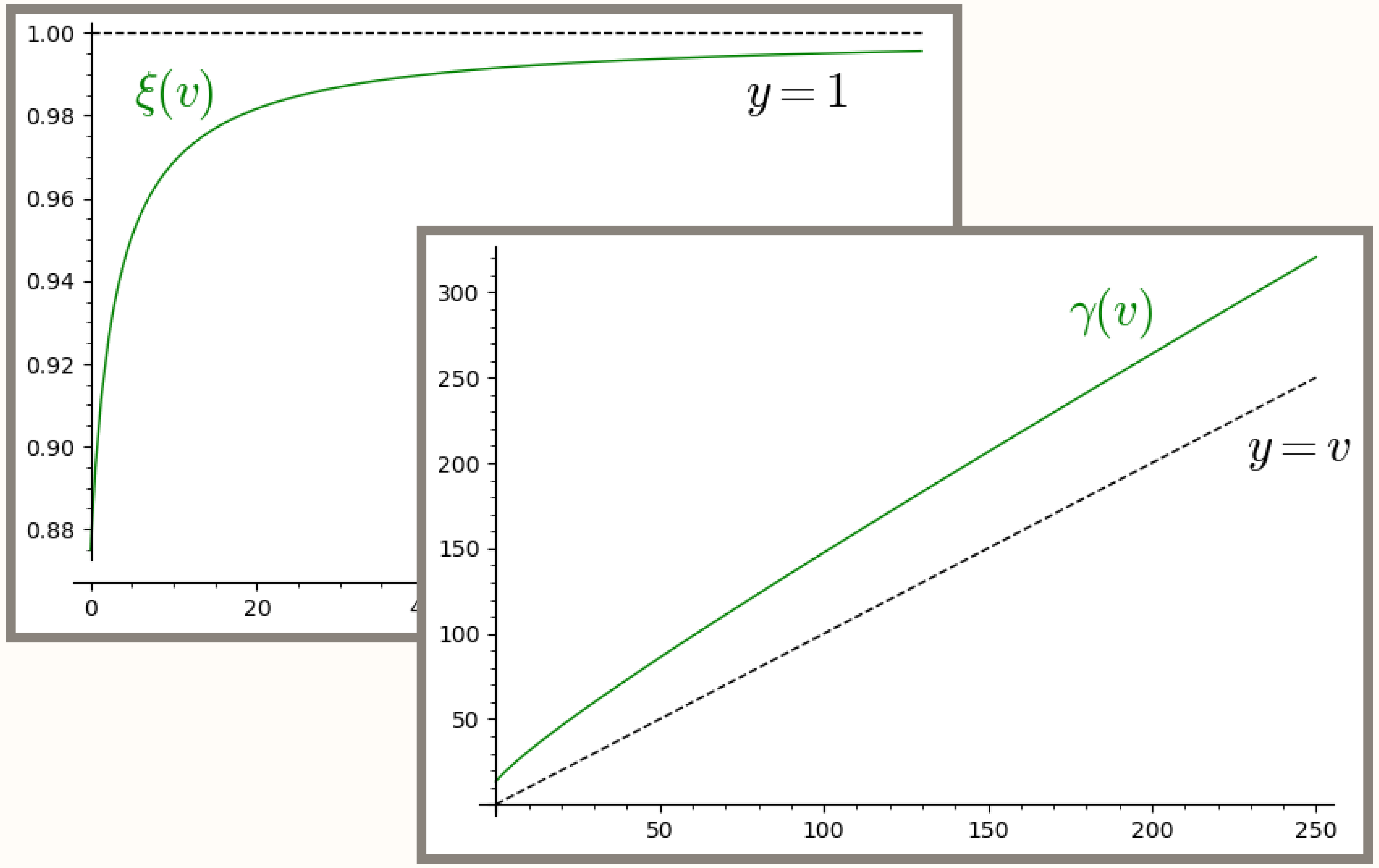


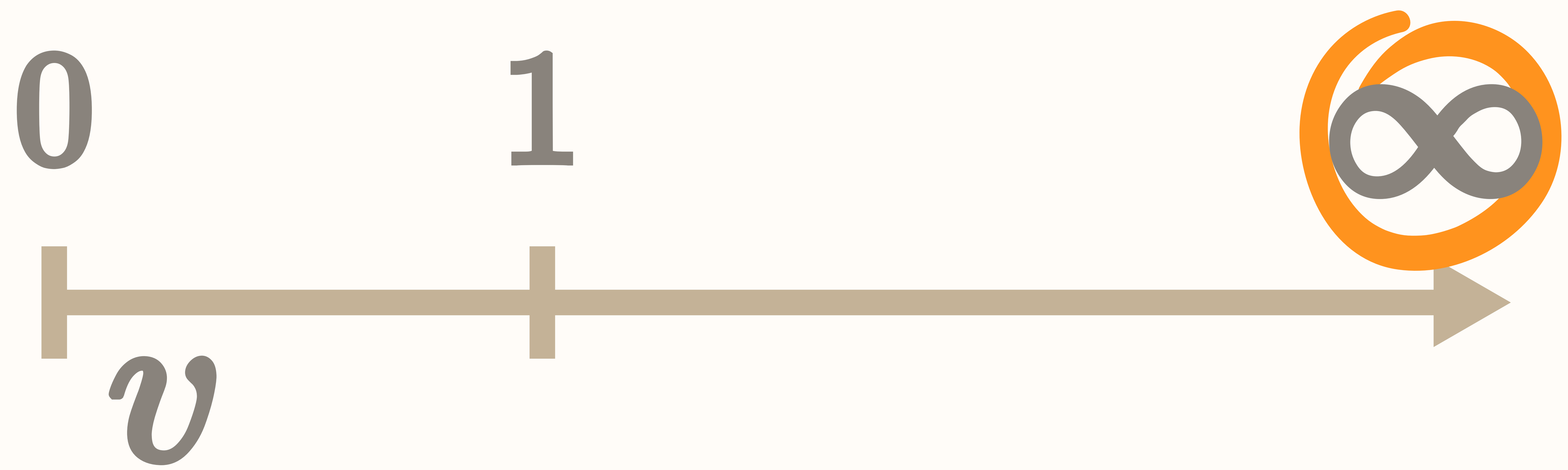


Generic transversal structures

$$t_n \sim \varkappa\, \gamma(v)^n\, n^{-1-\frac{\pi}{\arccos(\xi(v))}}$$







Merci