On the enumeration of plane bipolar posets and transversal structures

Éric Fusy, Erkan Narmanli, and Gilles Schaeffer

Decorated maps

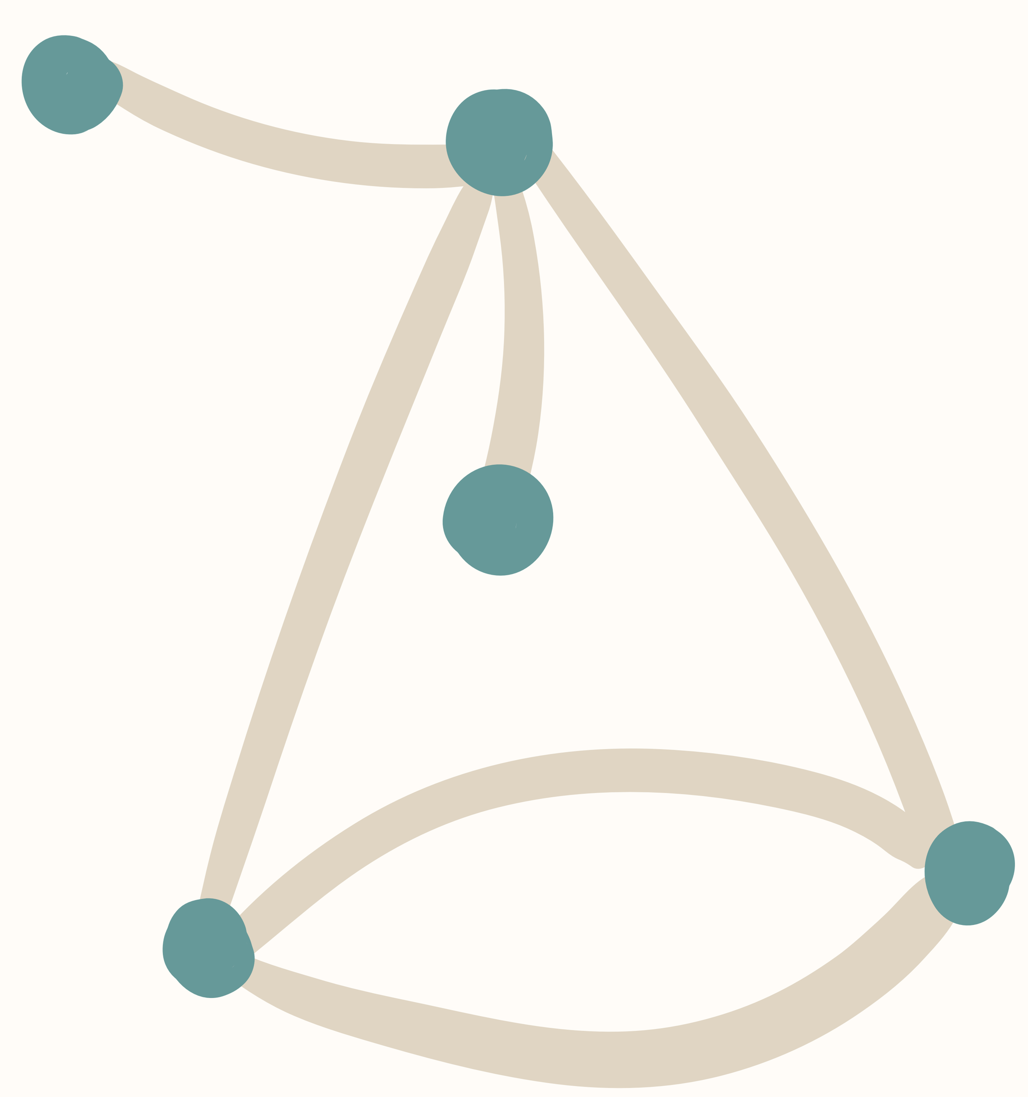
→ *exact enumeration formulas*

→ *universal critical exponent*

 # maps with n edges : $\varkappa \cdot \gamma^n n^{-5/2}$

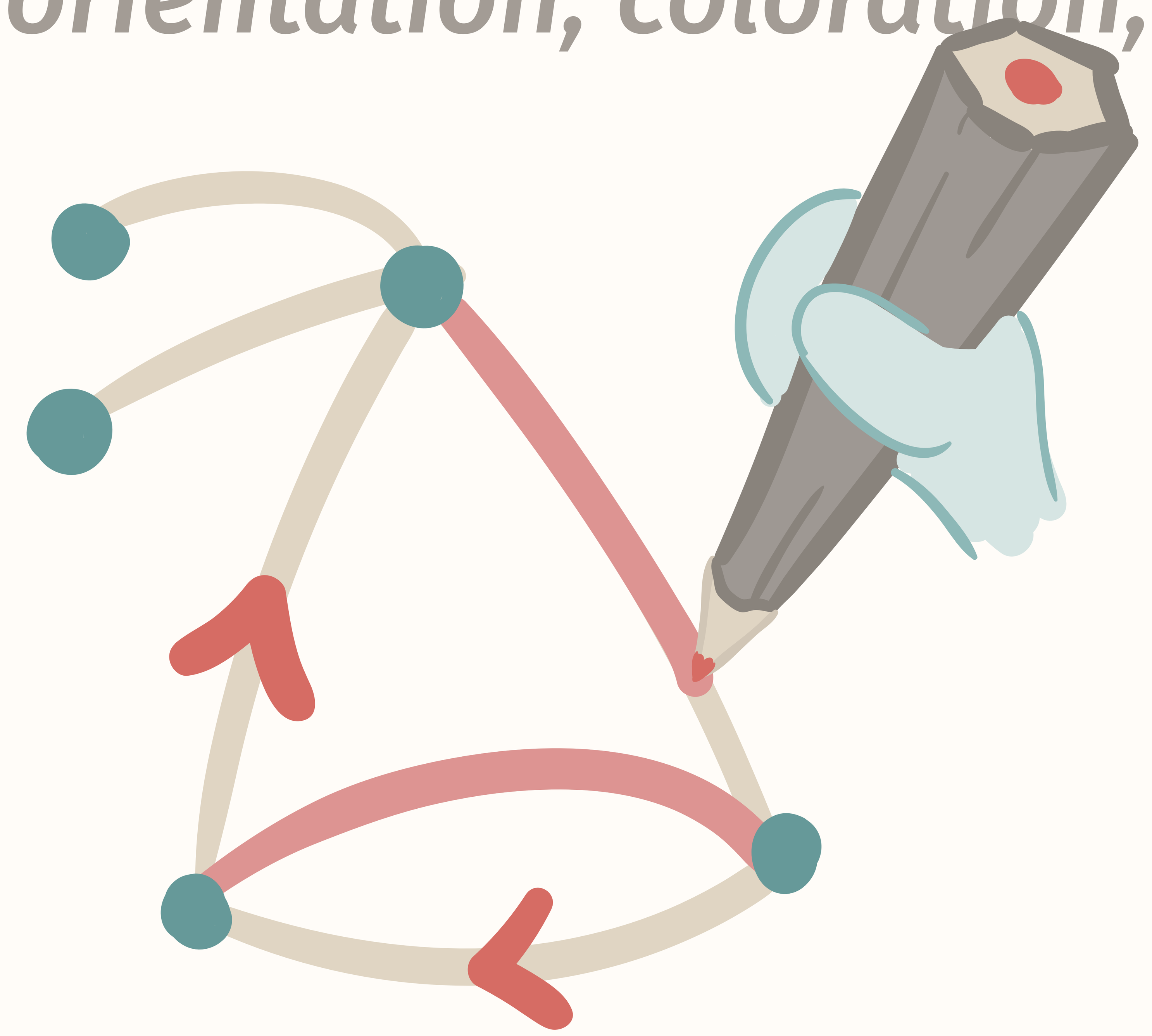
Maps

embedding on the sphere



Decorated maps

orientation, coloration, etc.



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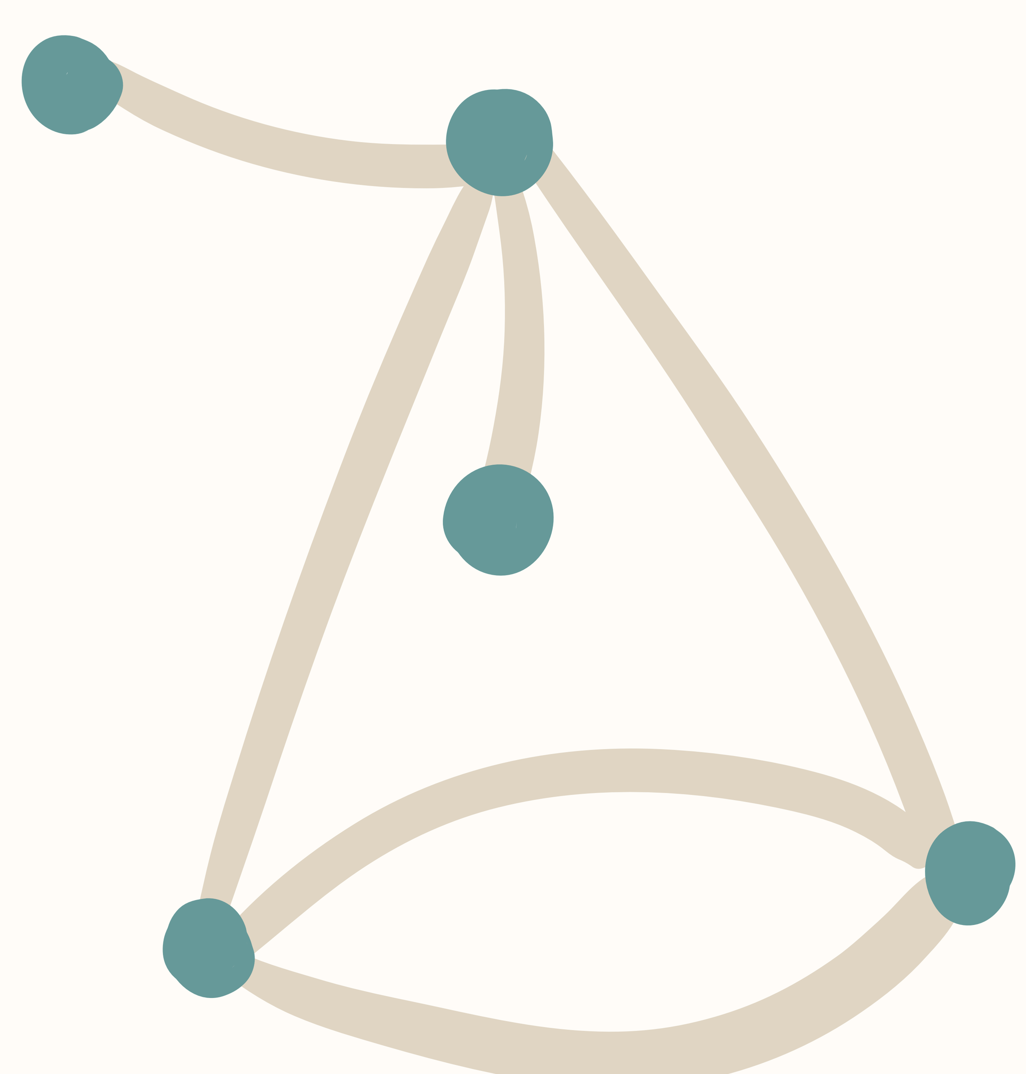
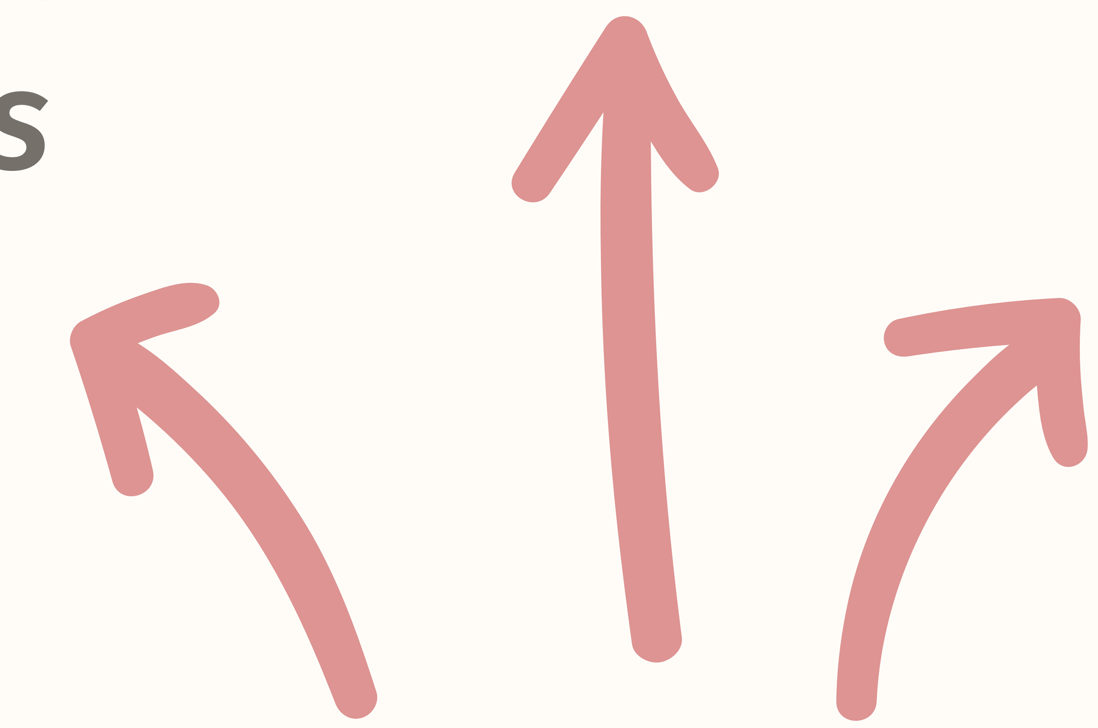


structures transverses

Schnyder woods

posets plans



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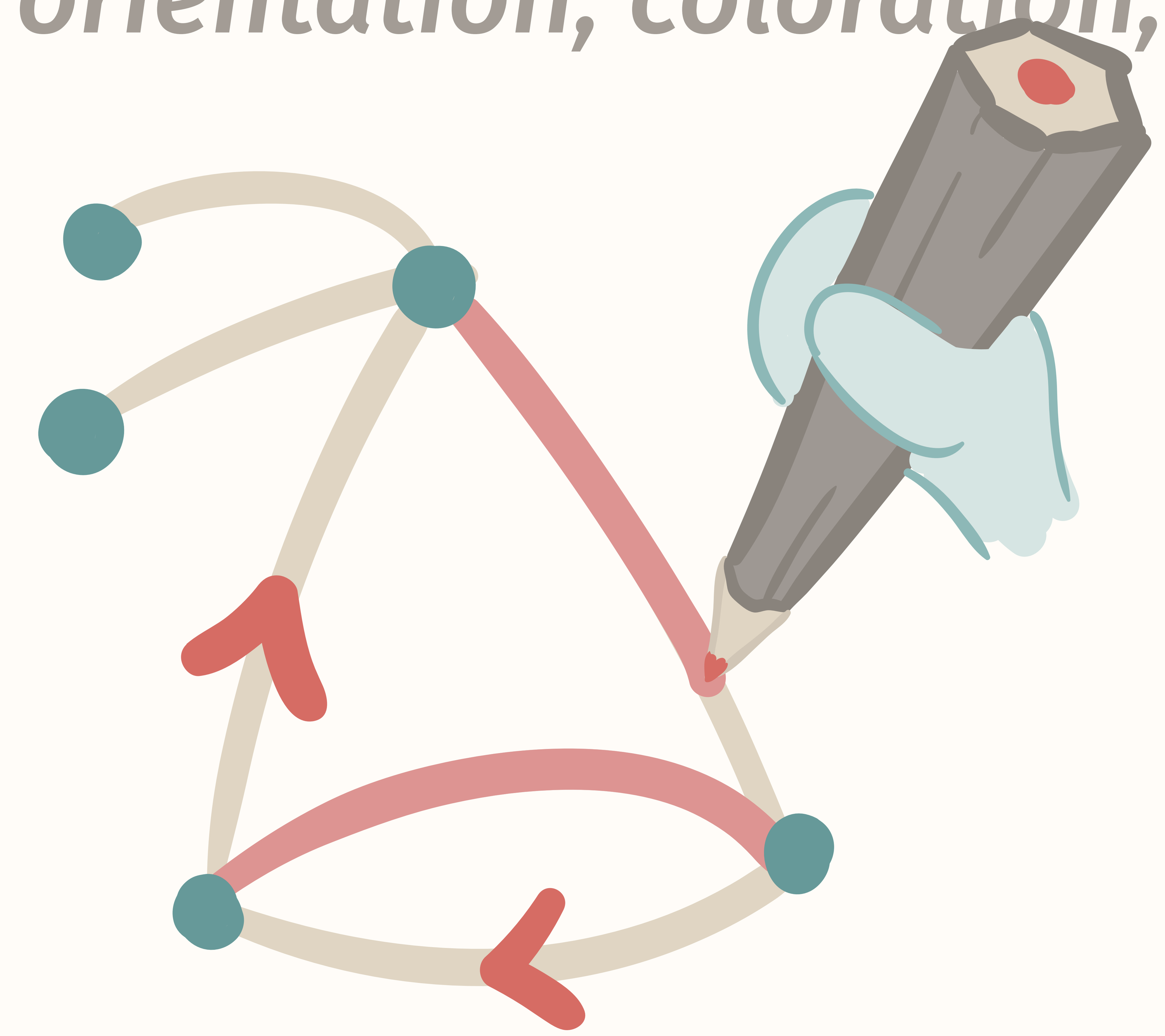
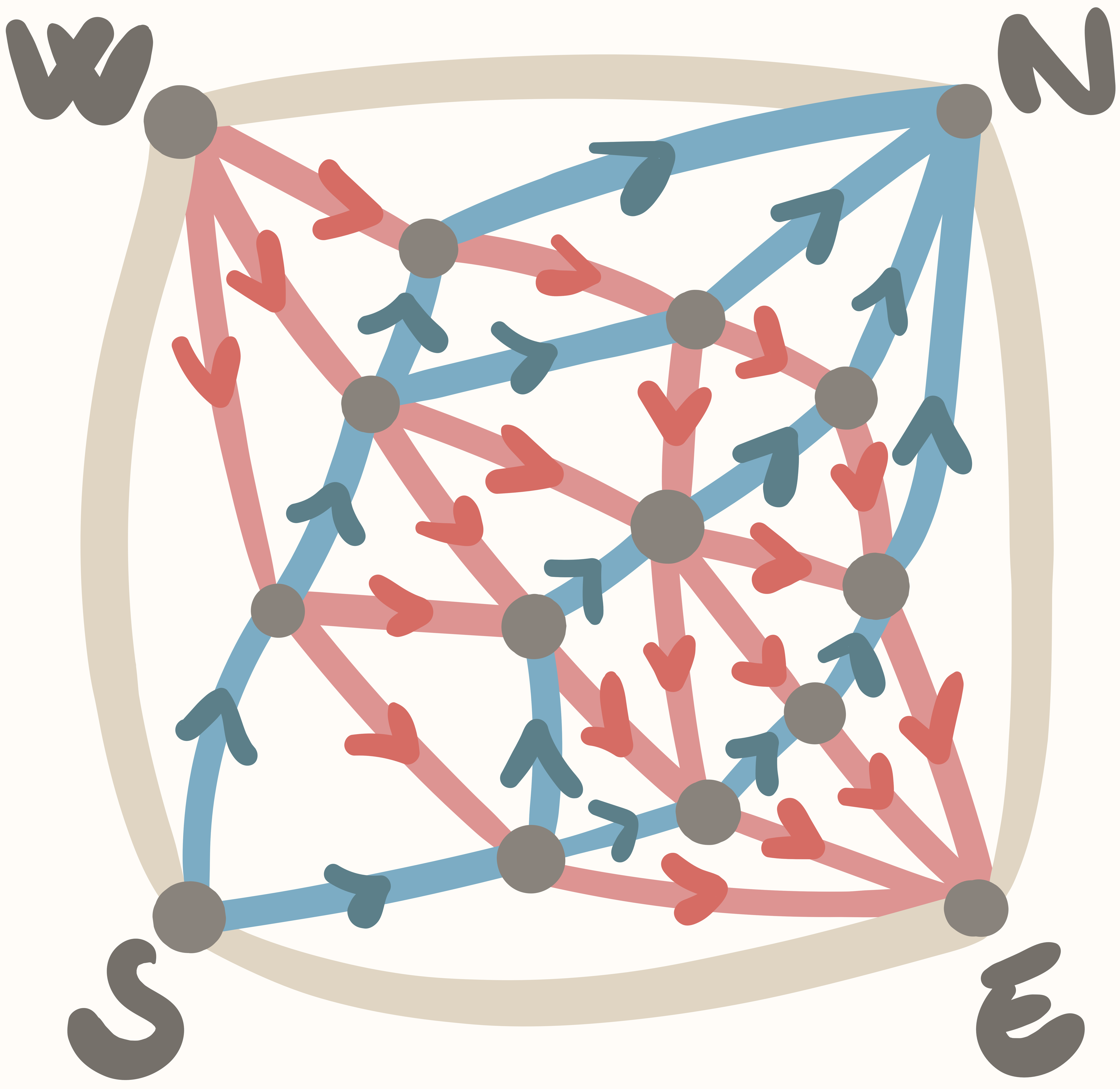


Transversal structures



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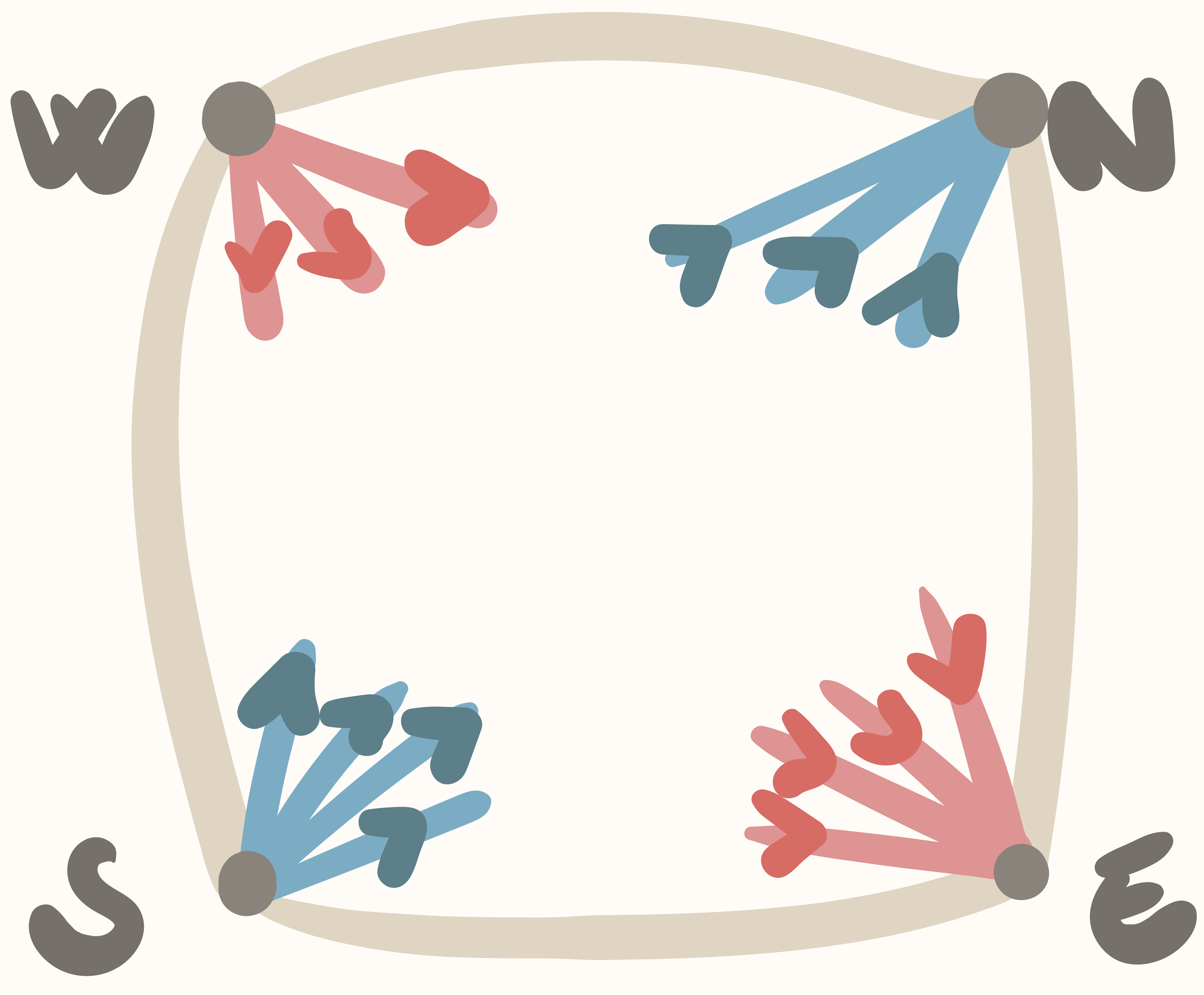
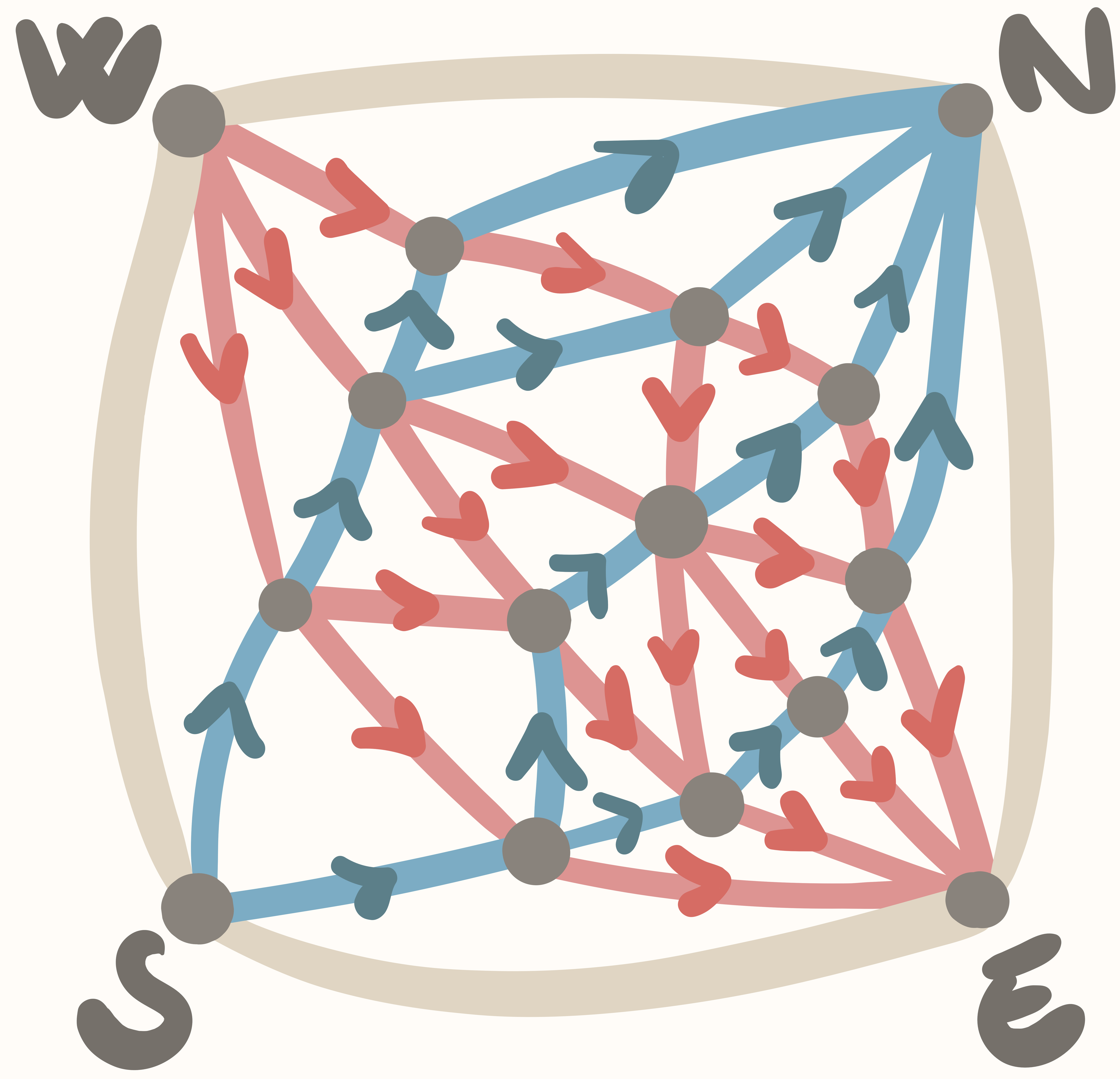
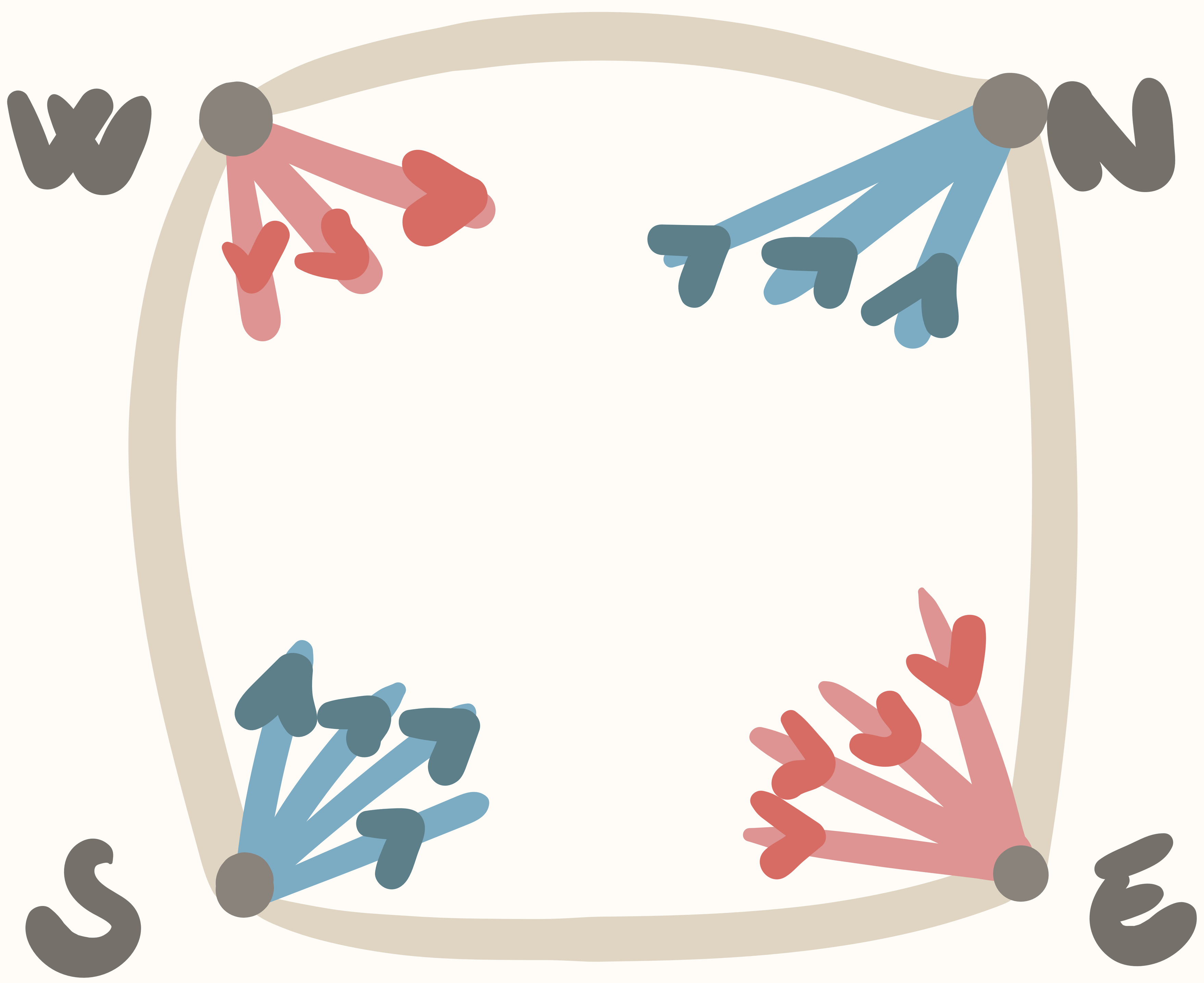
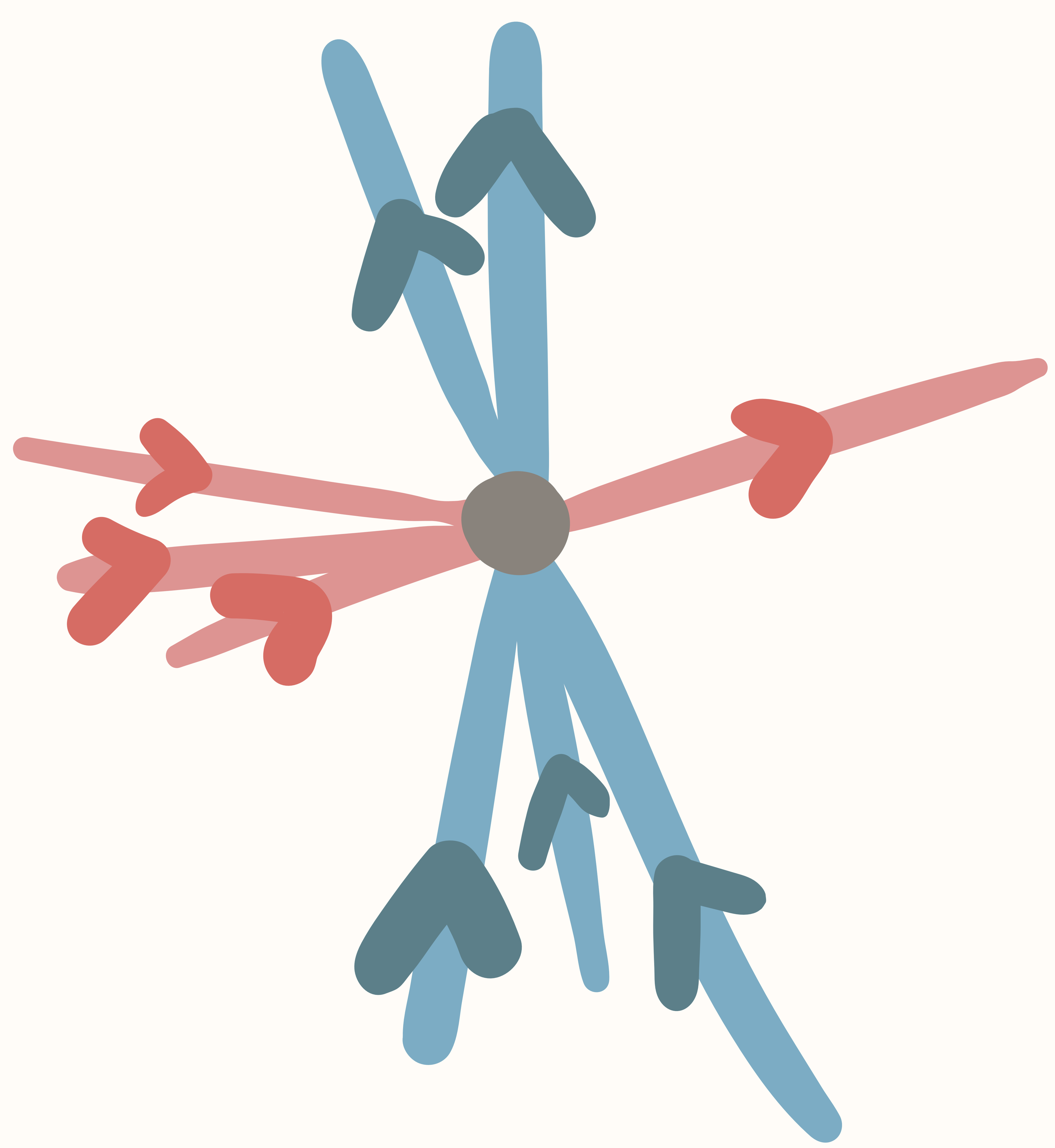
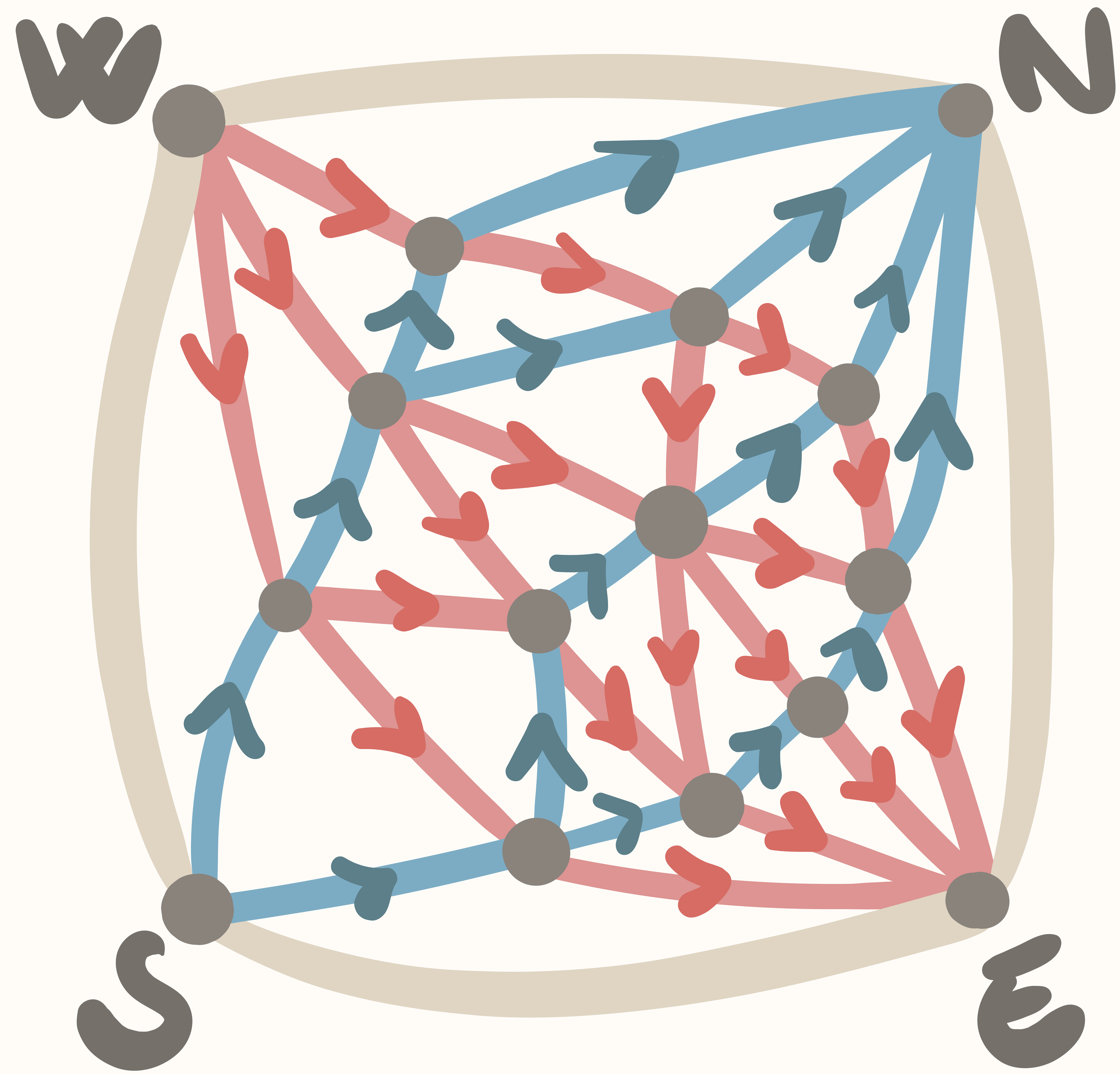




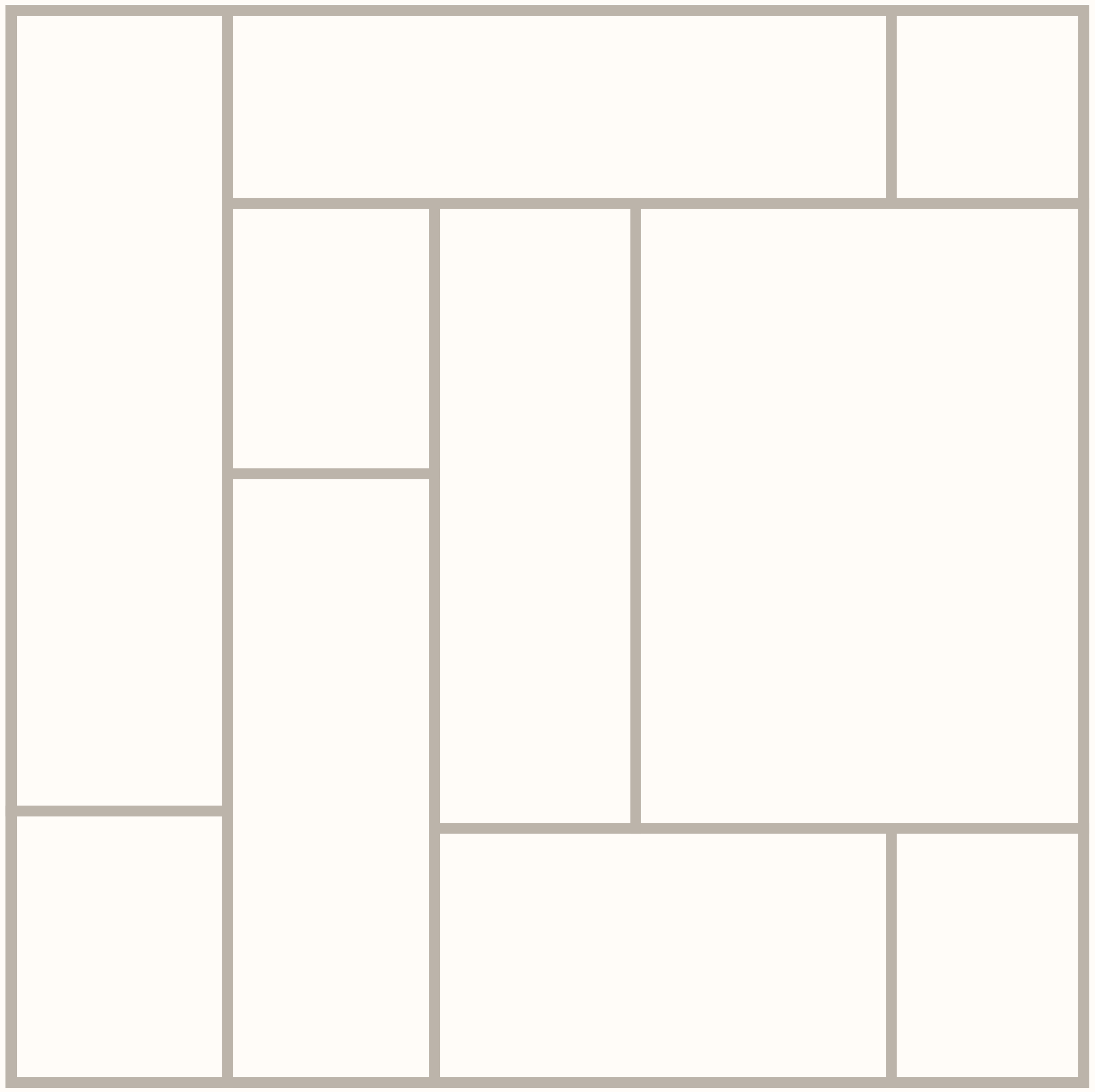
Transversal structures







Link with rectangular tilings



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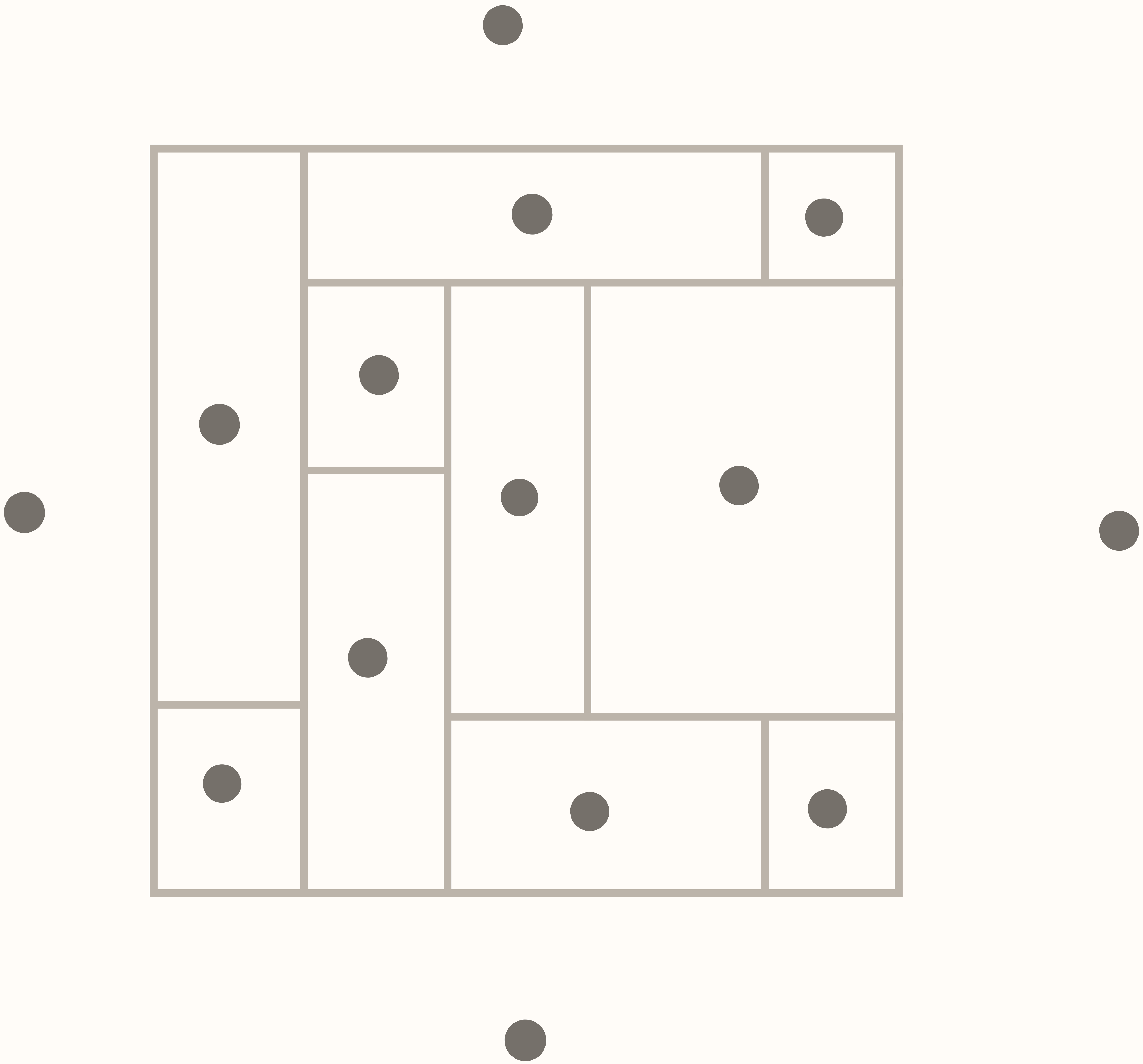

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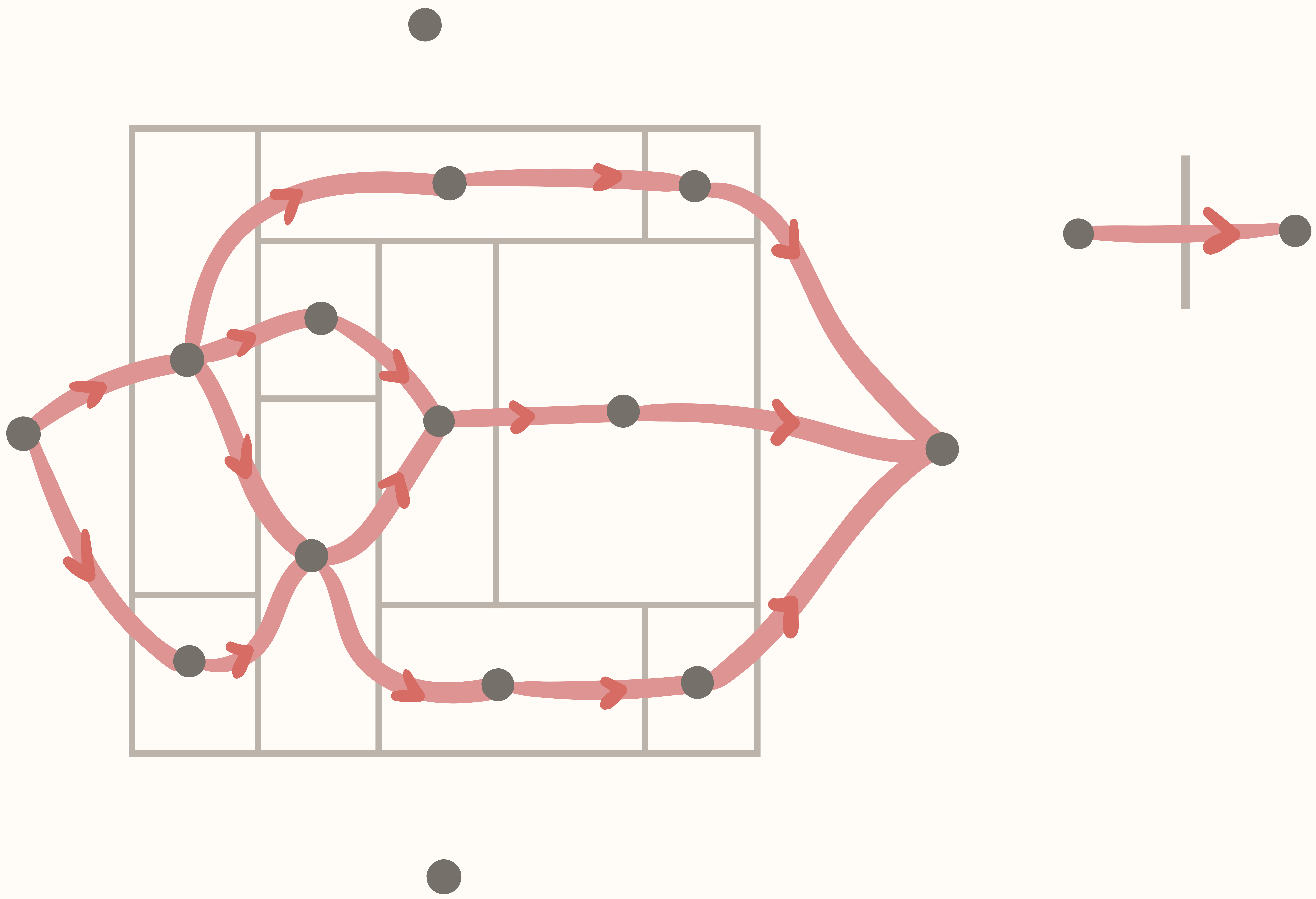

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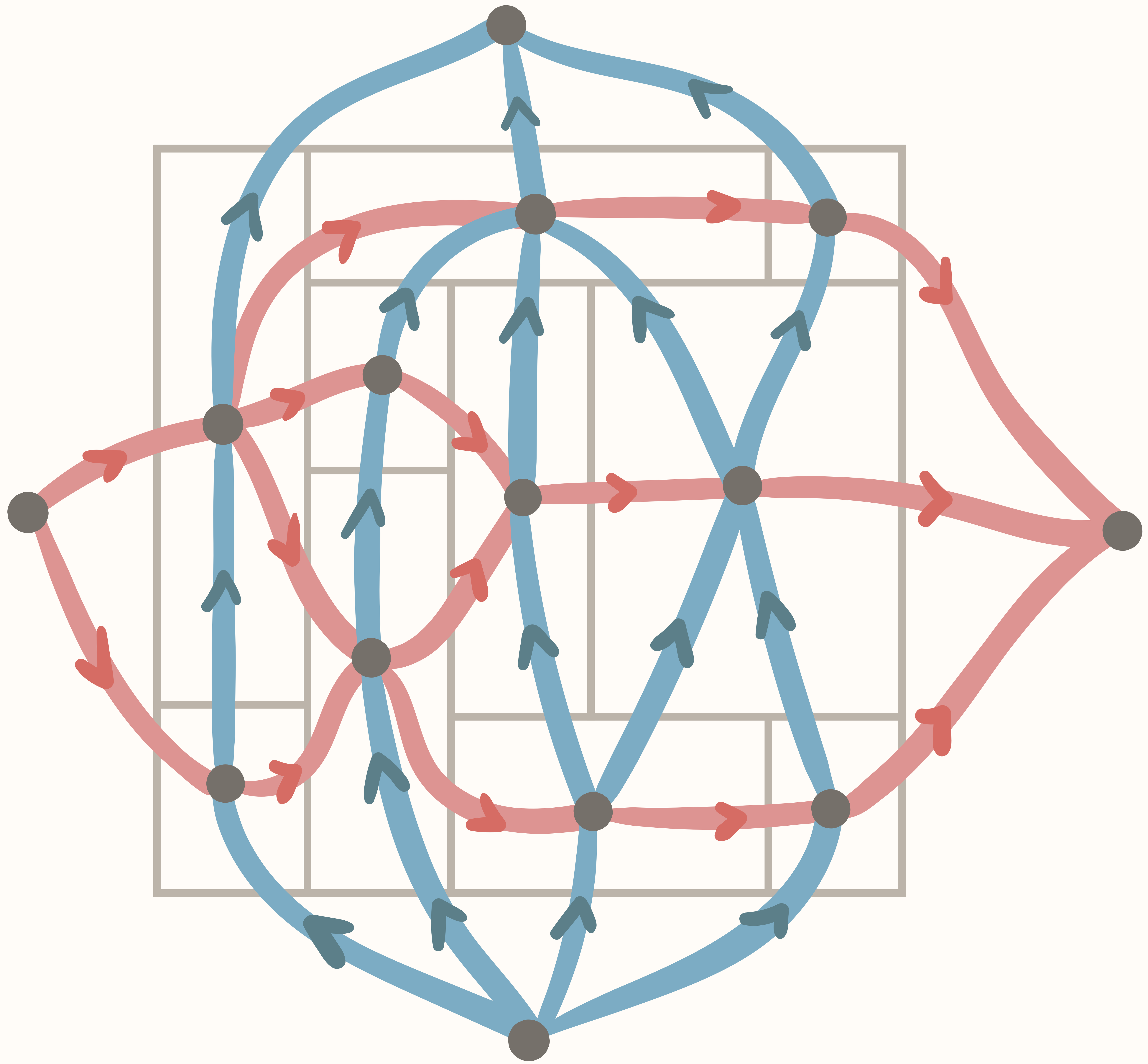

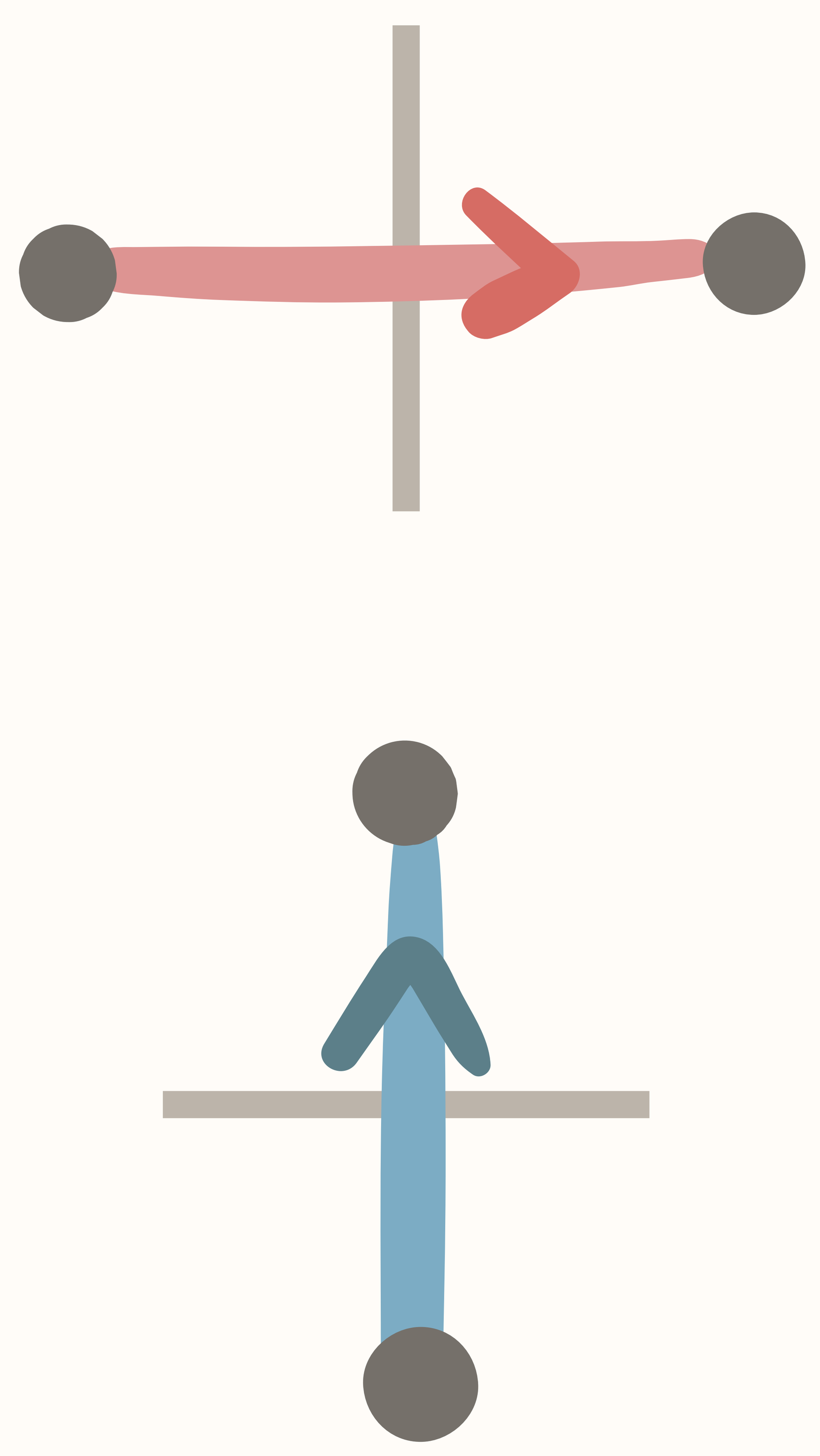

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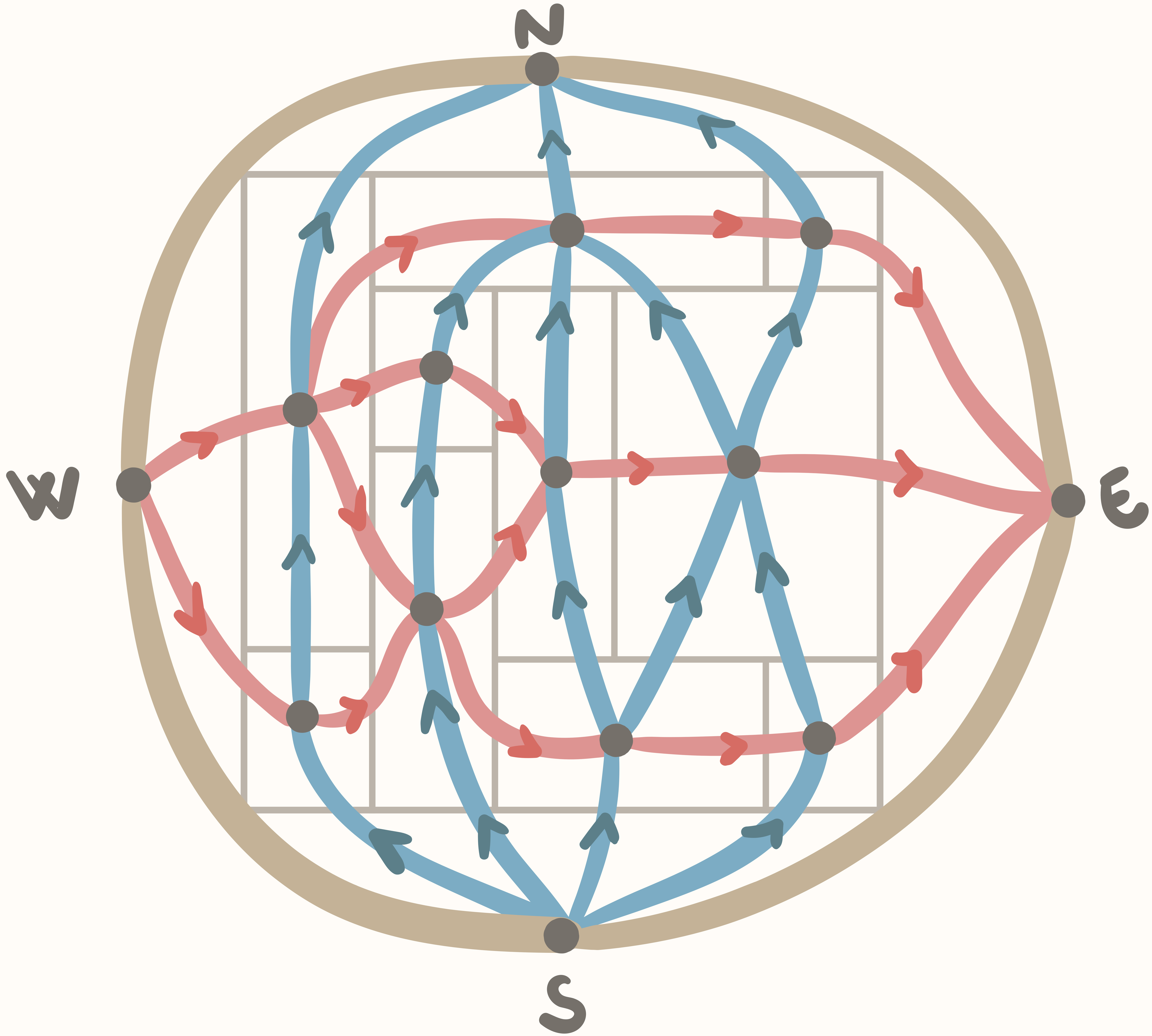

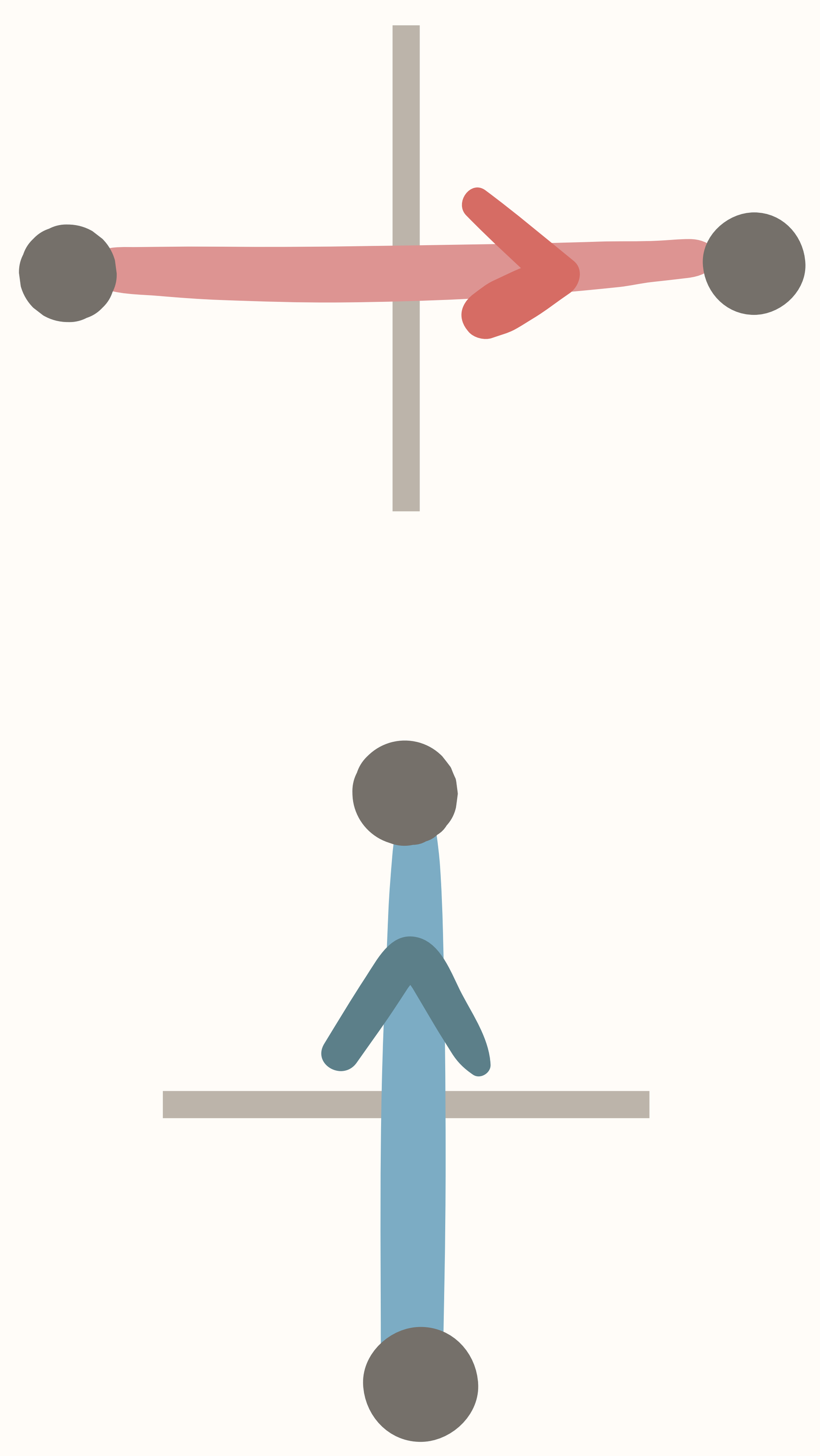

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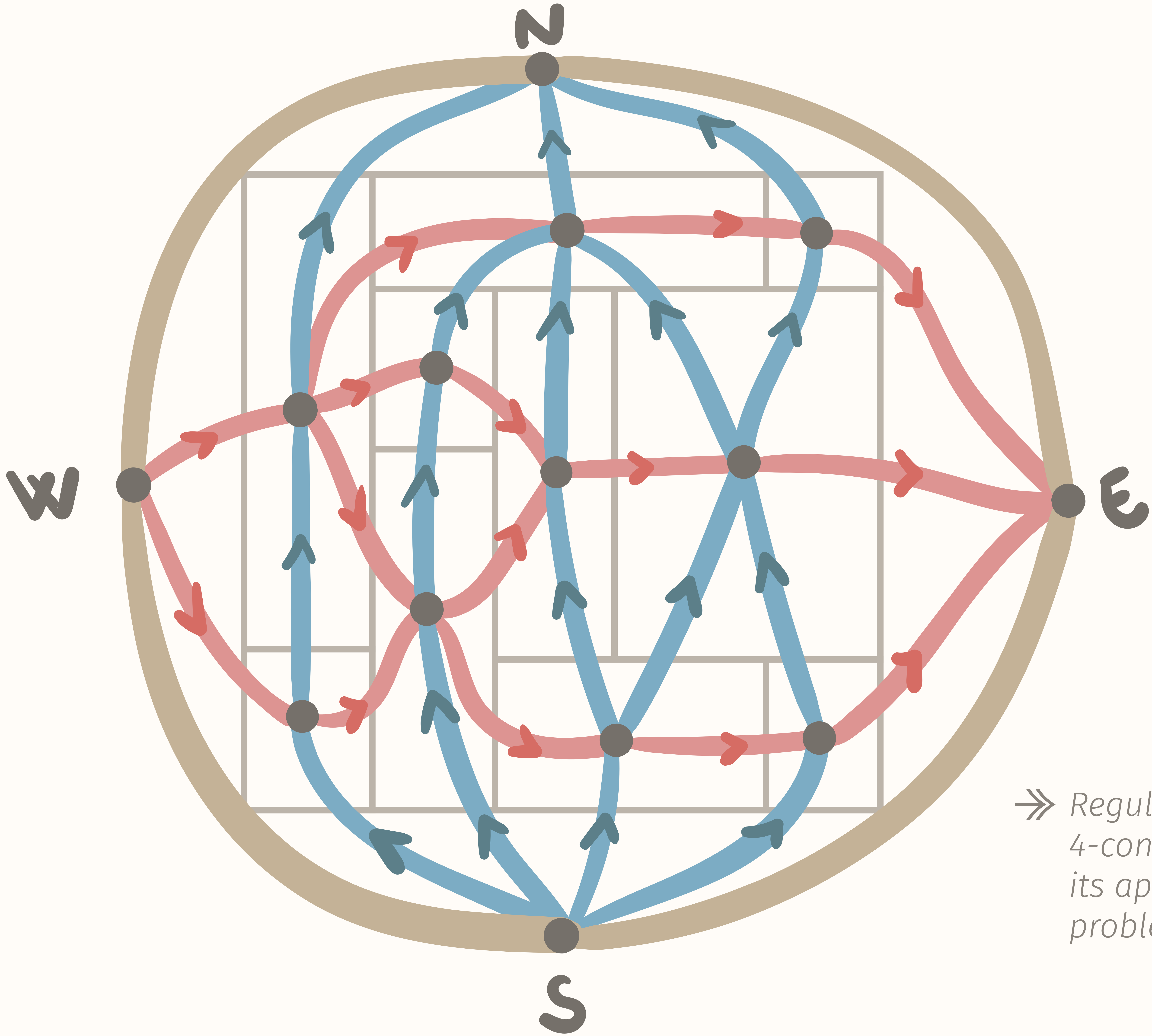
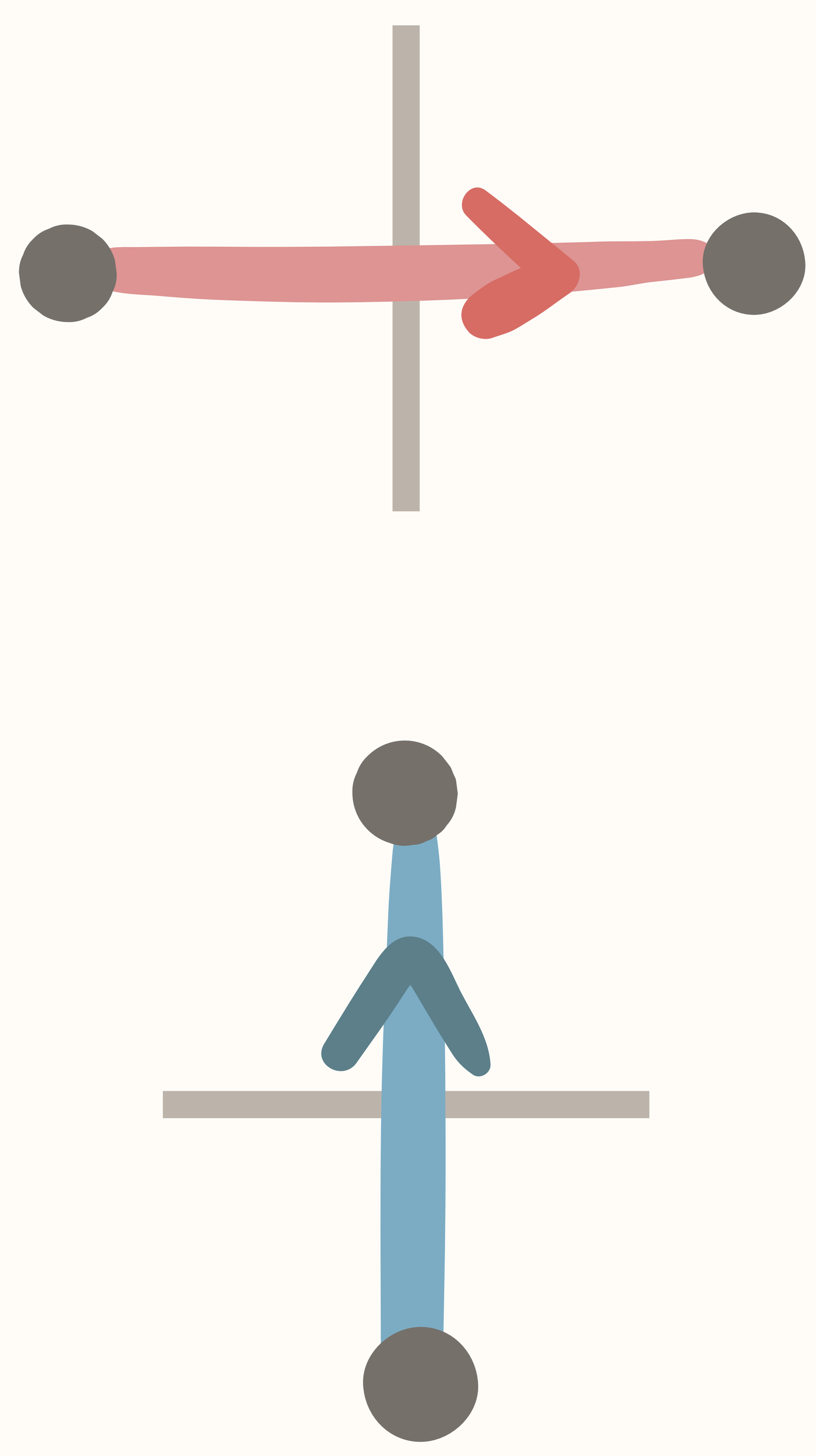




⇛ *Regular edge labellings of 4-connected plane graphes and its applications in graph drawing problems*, G. Kant & X. He (1997)

Summary

The KMSW bijection

1. Application to three map models

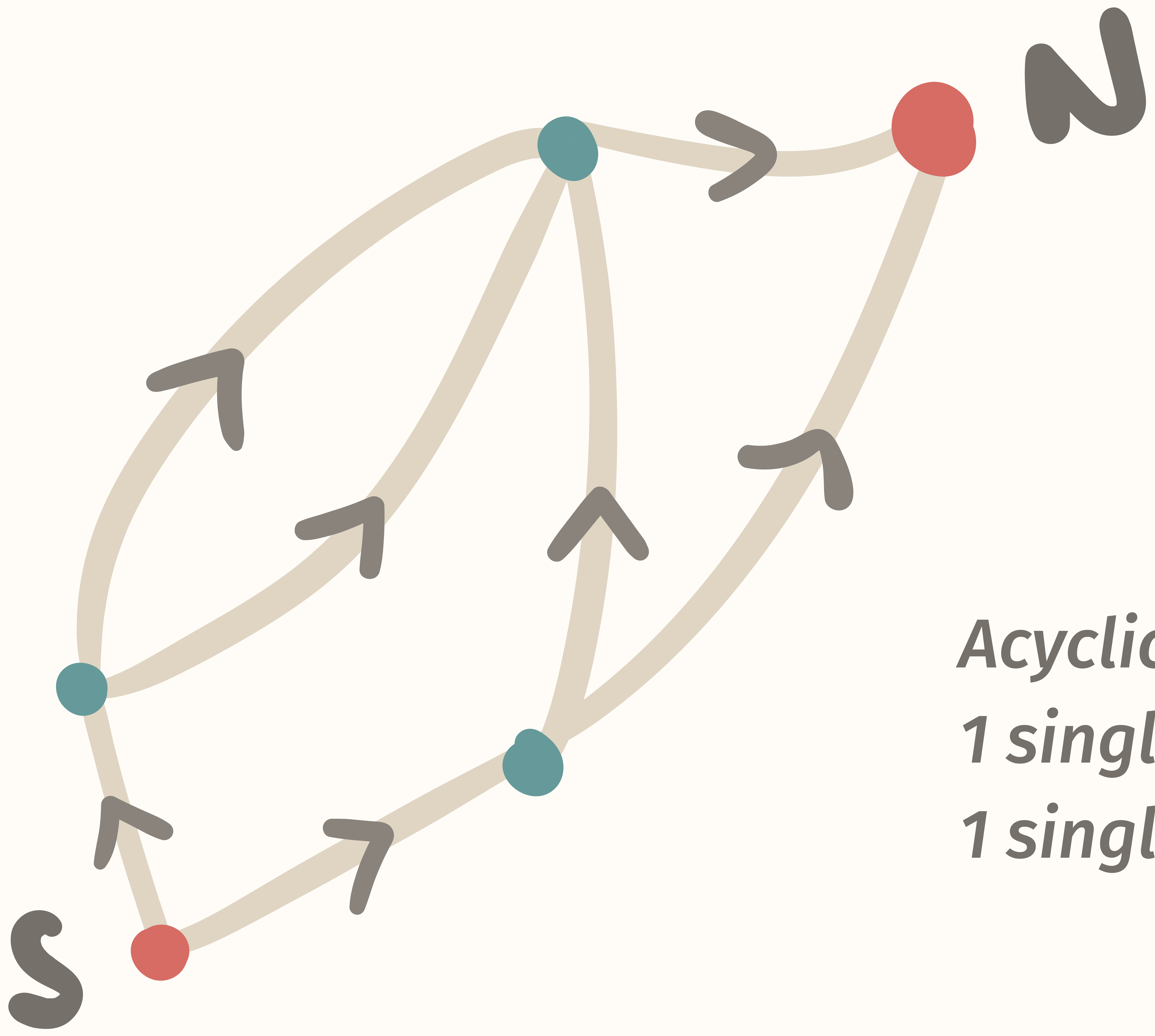
a. Plane bipolar posets
b. Plane bipolar posets by vertices
c. Transversal structures
d. Asymtotics

2. Interlude : plane permutations

3. Application to corner polyhera

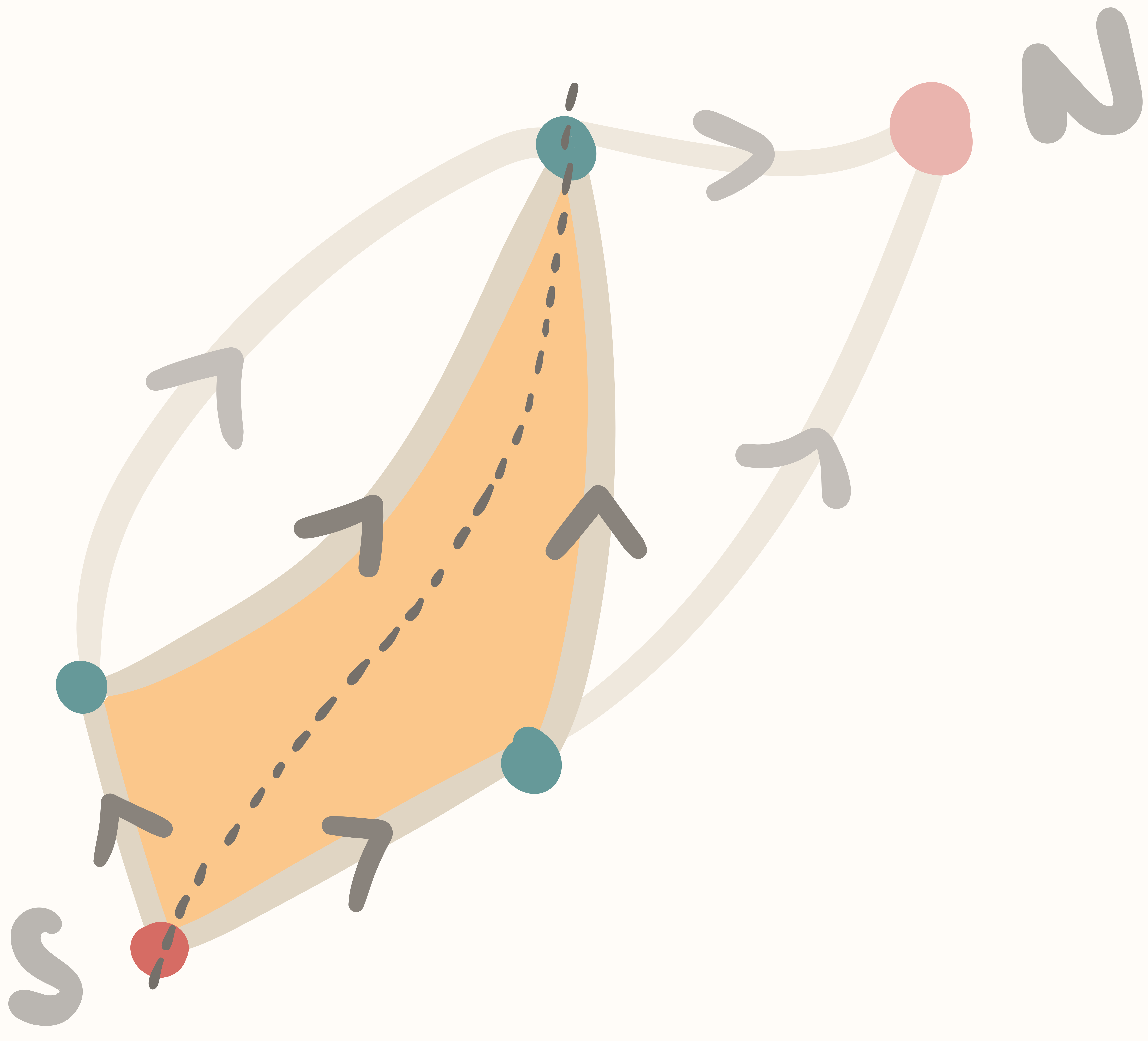
a. Via polyheral orientations
b. Via Schnyder colorings
c. Asymtotics

Plane bipolar orientation



Acyclic
1 single source S
1 single sink N

Plane bipolar orientation



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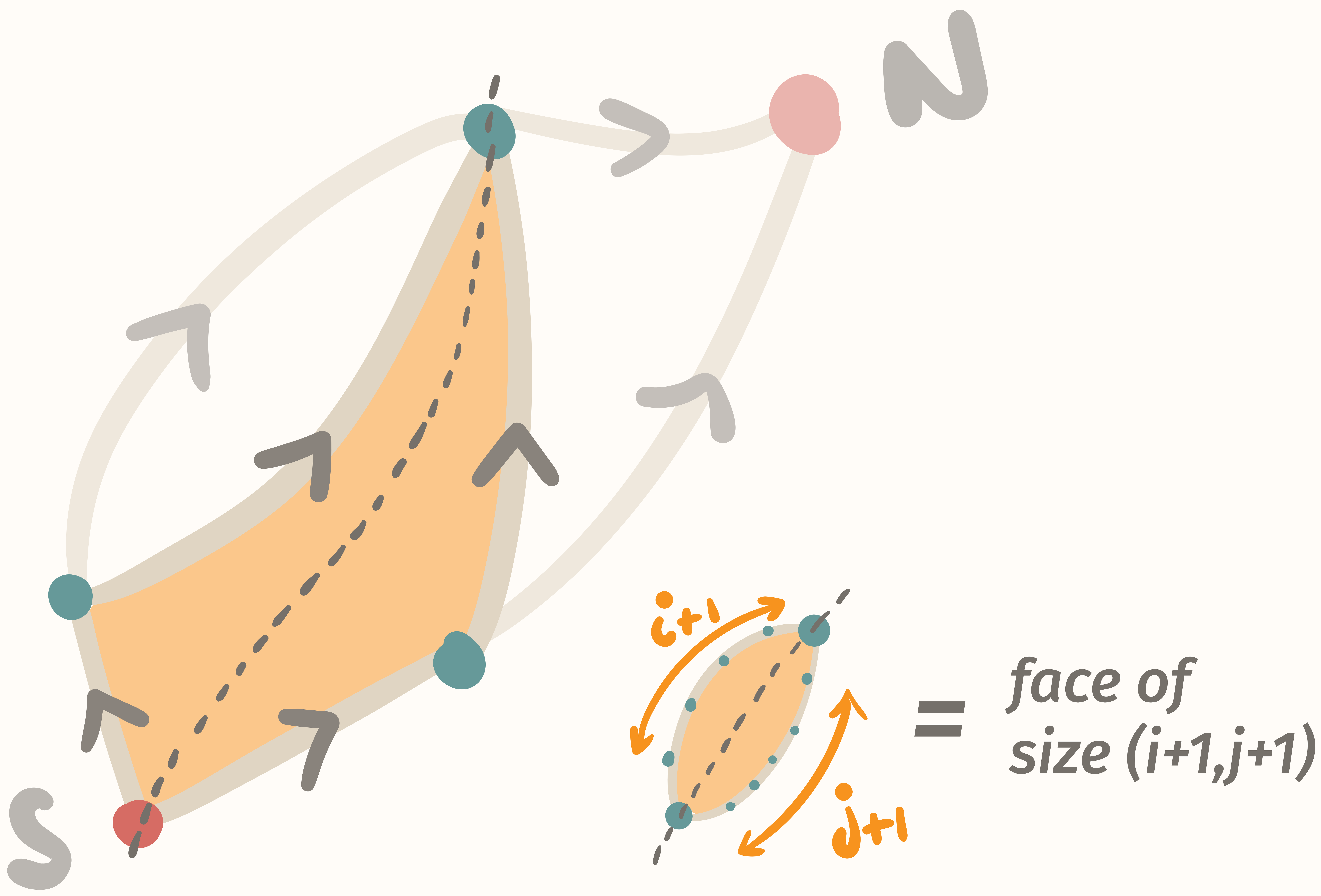

The KMSW bijection

Plane bipolar orientations ⟷ *tandems walks in the quarter plane*

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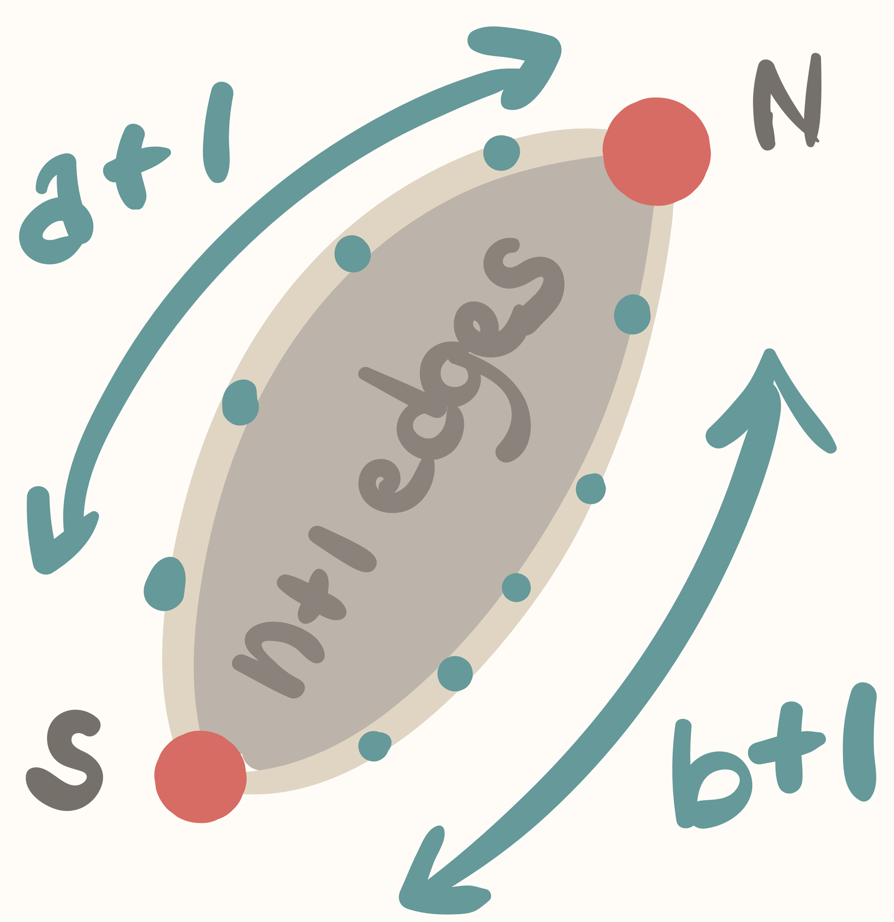

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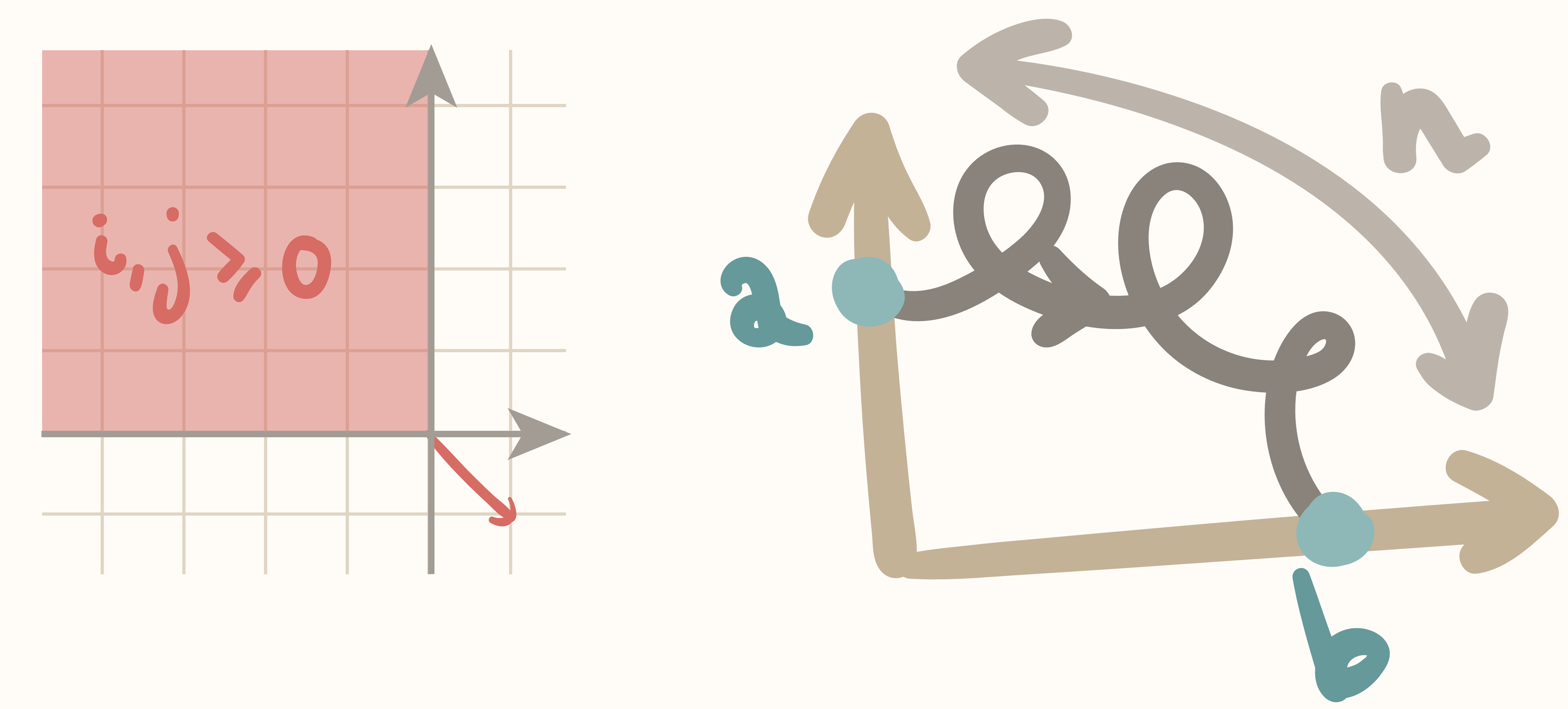

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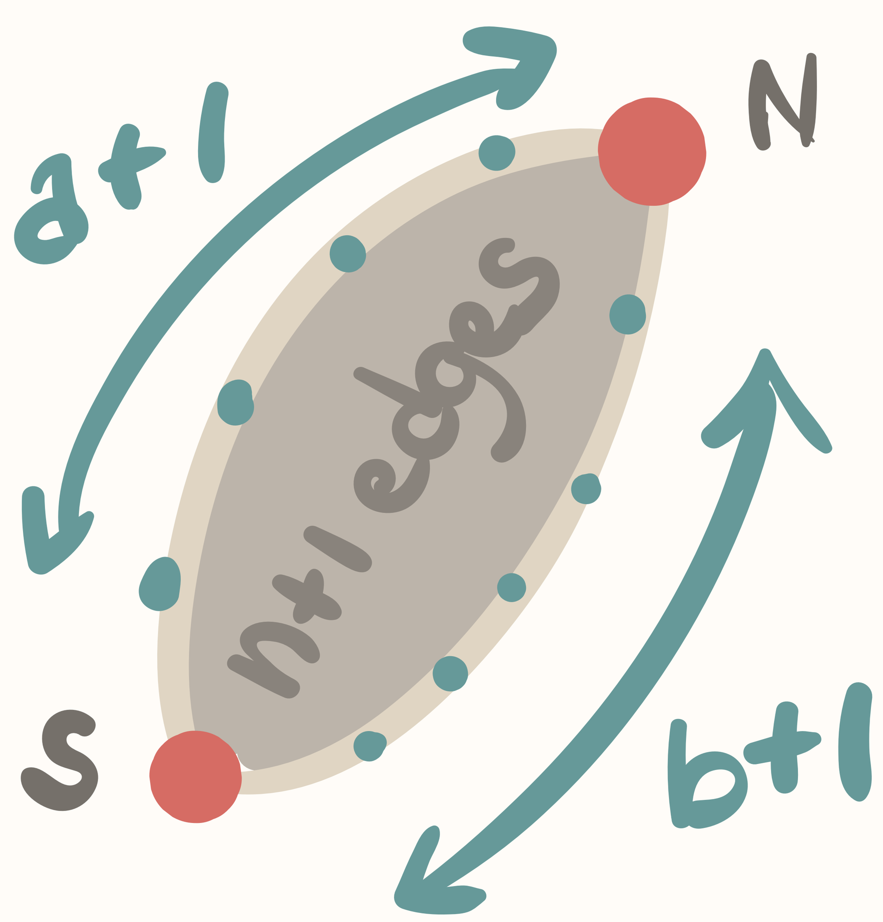

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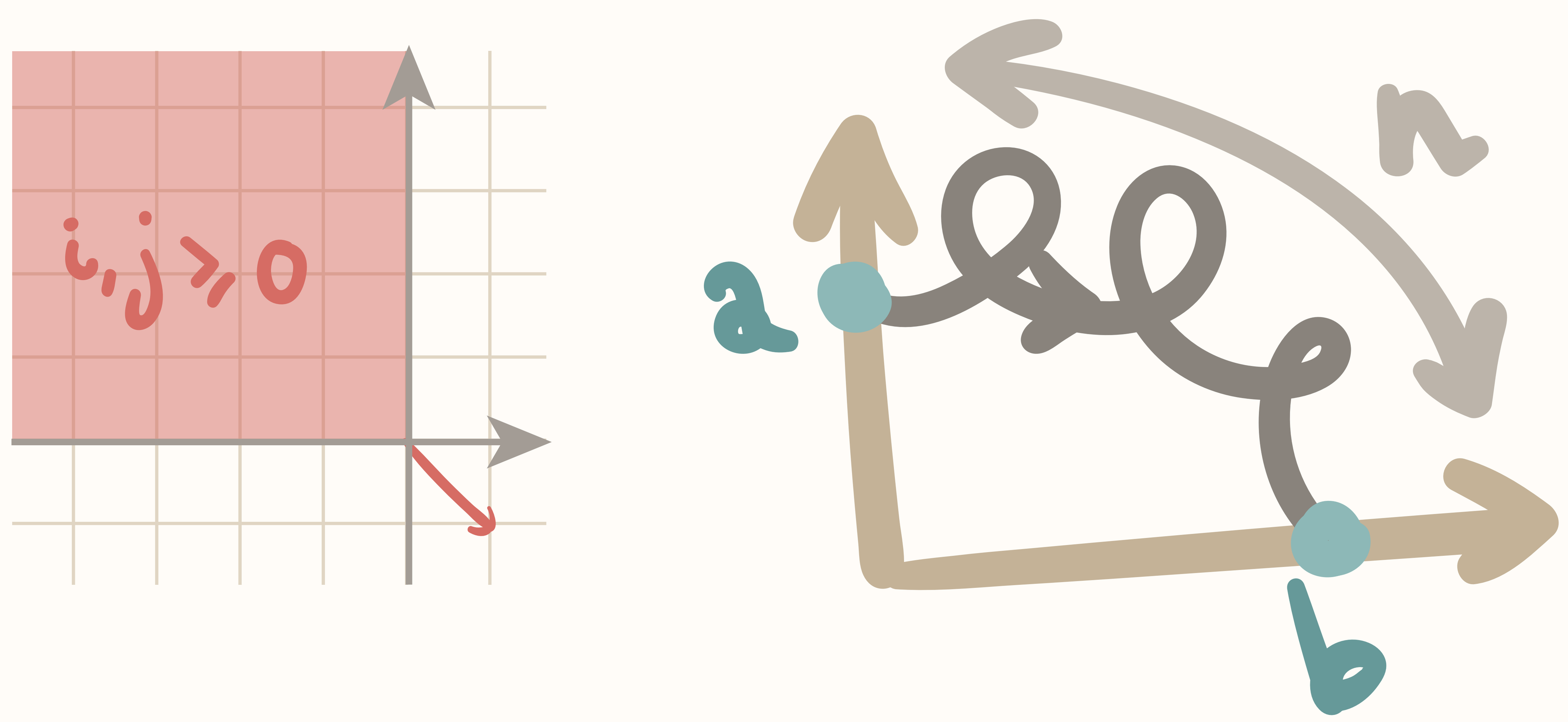

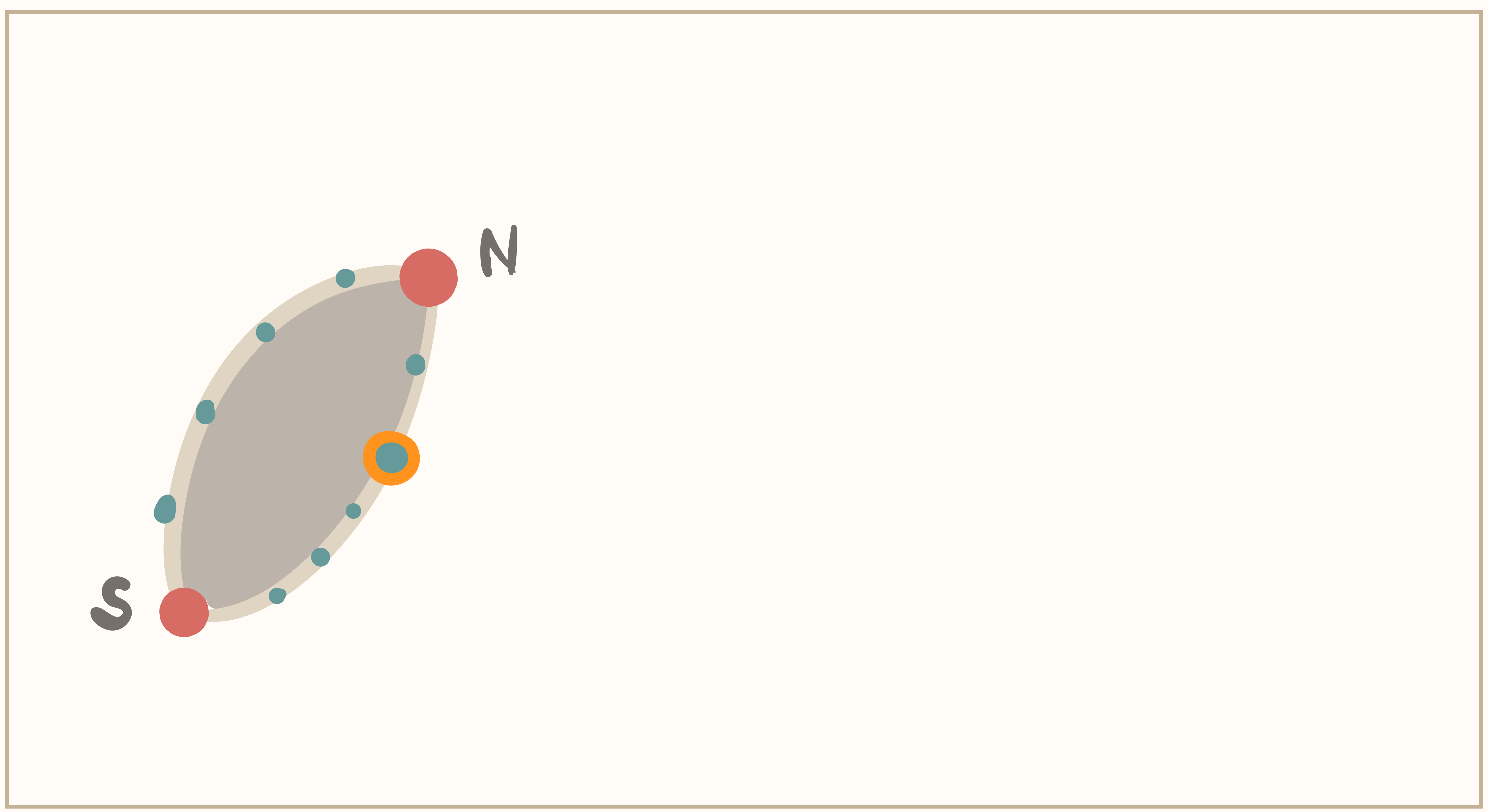

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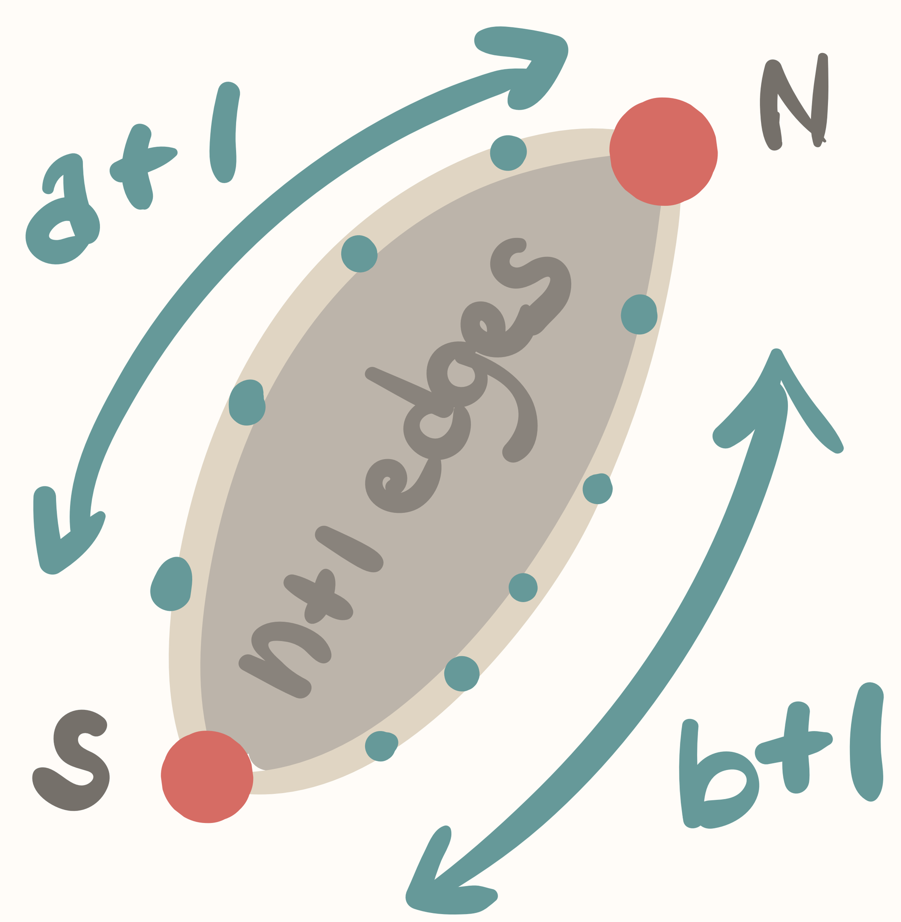

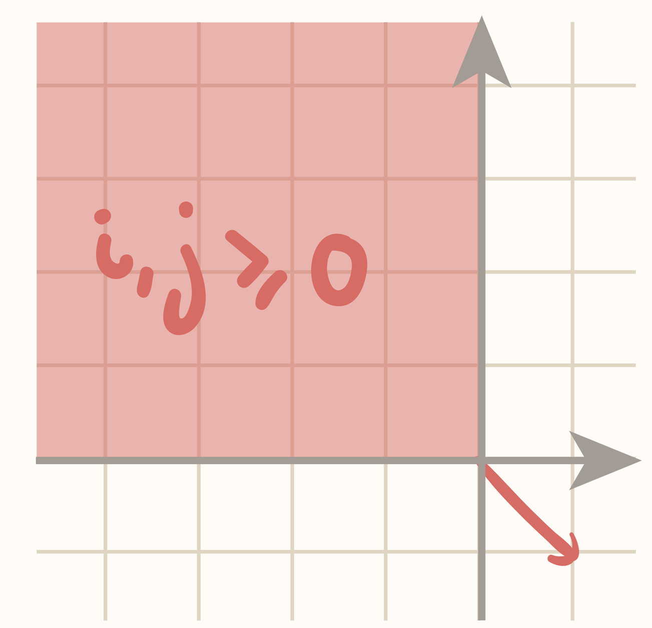

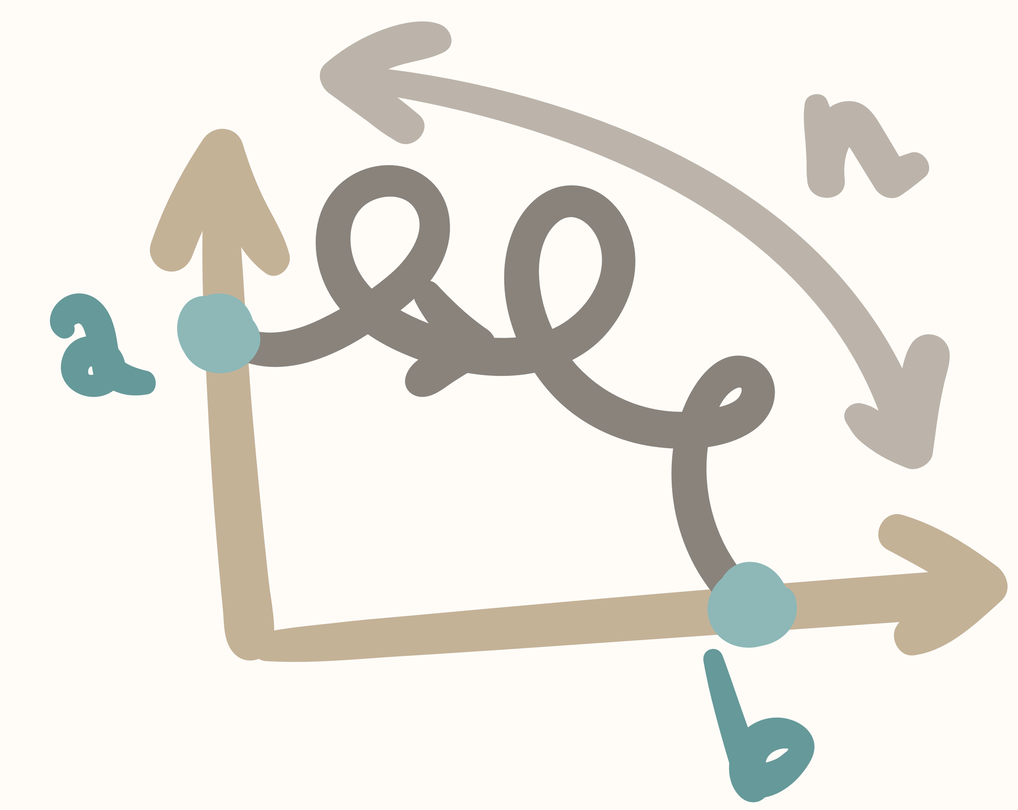

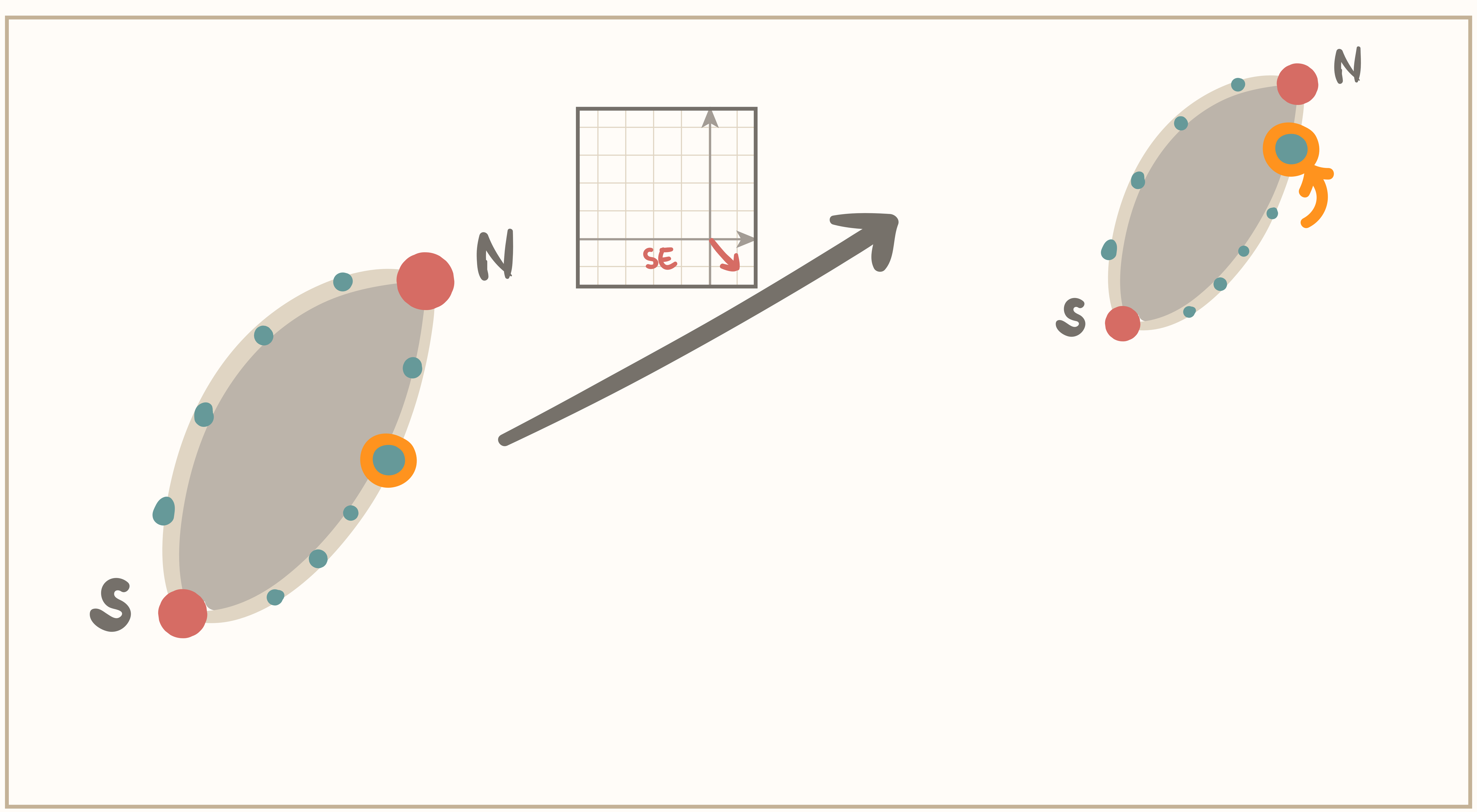

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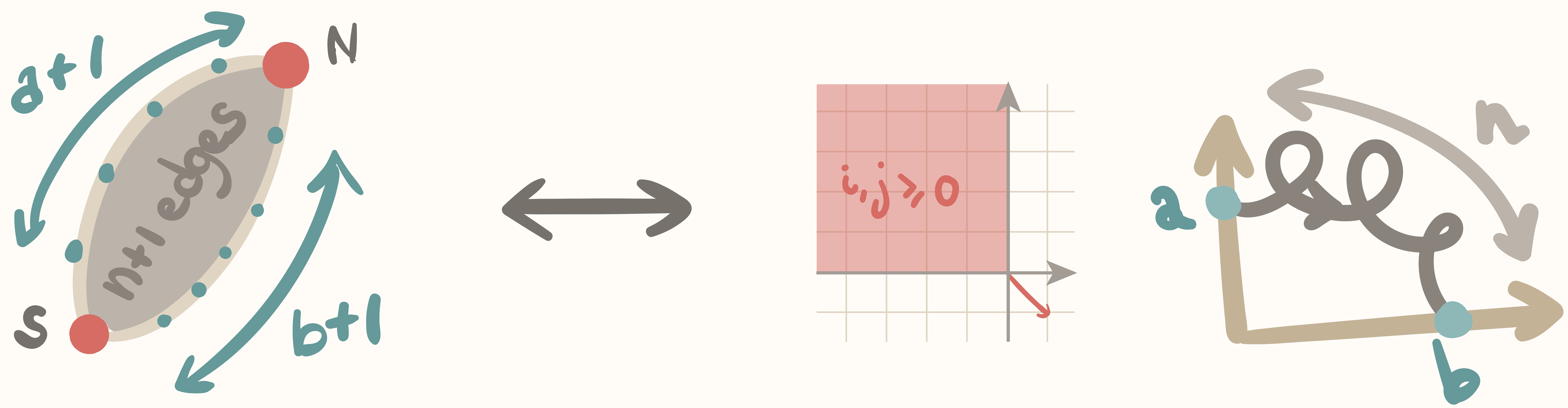

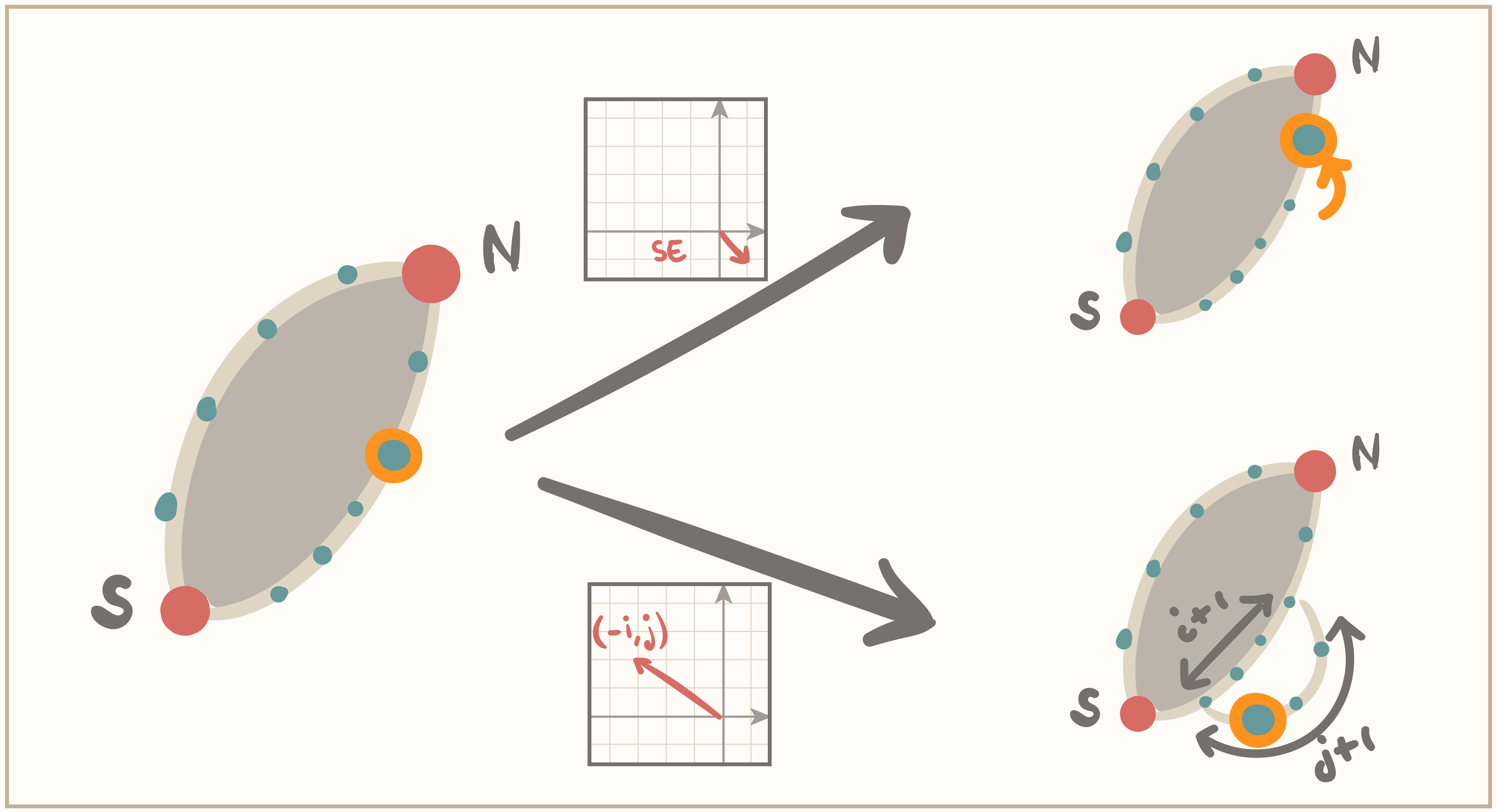

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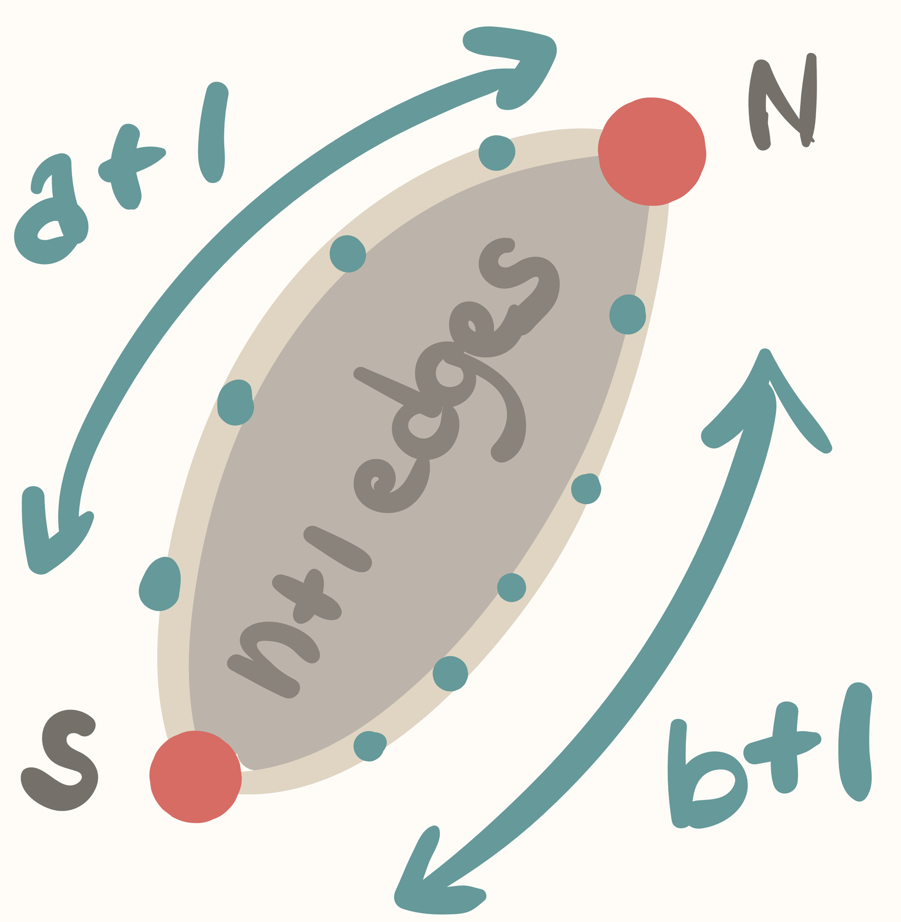

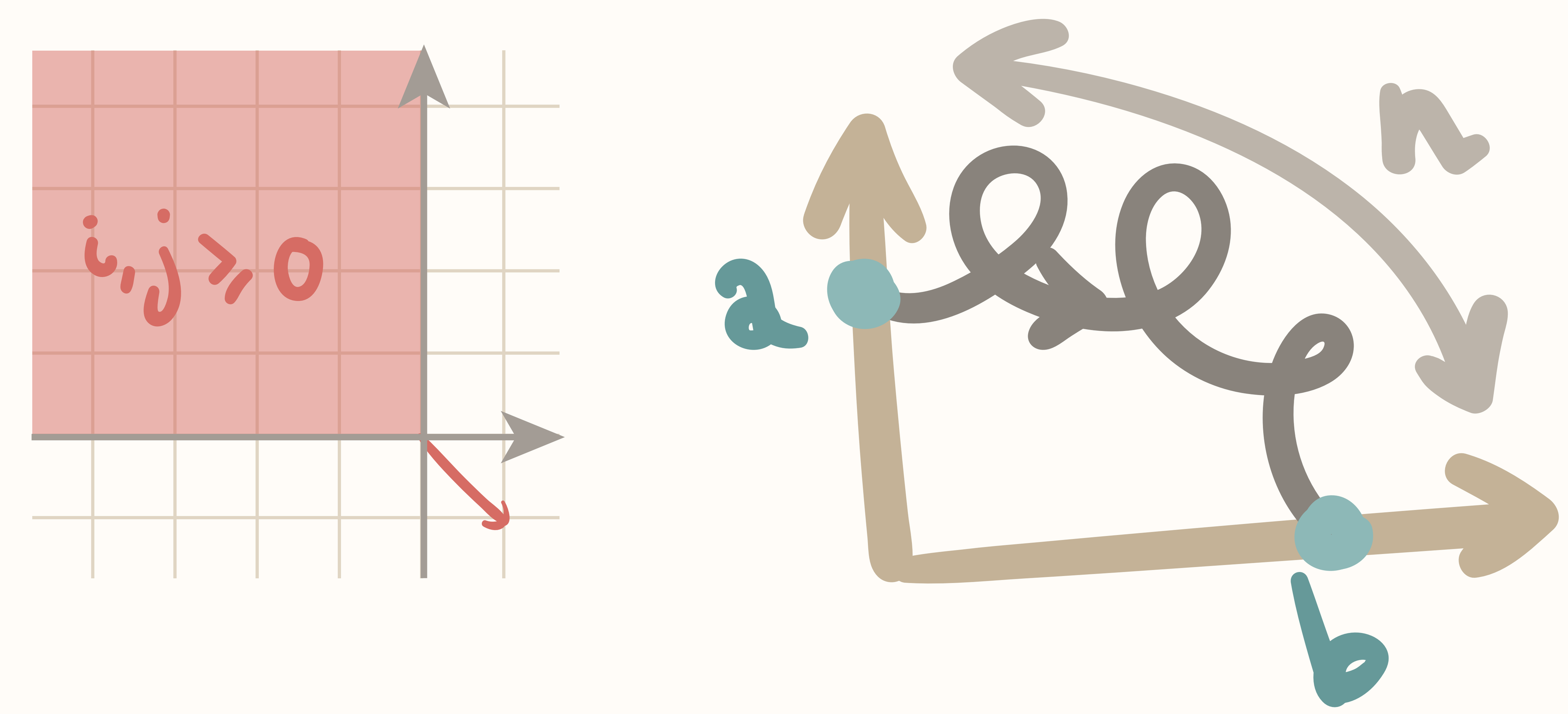

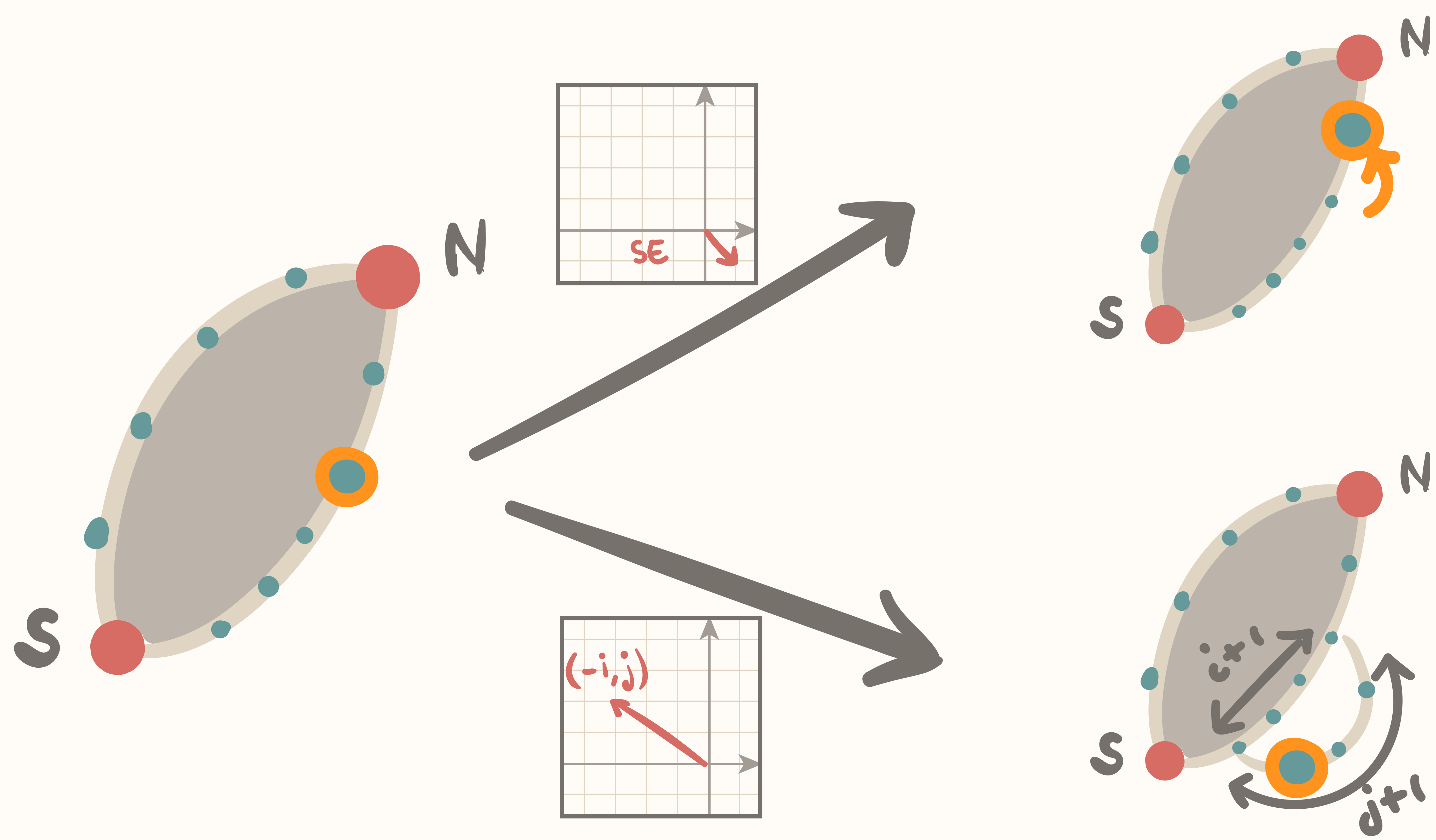

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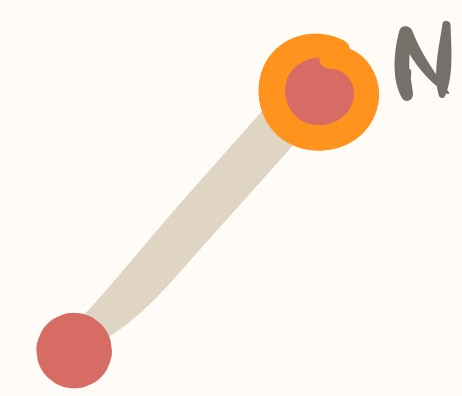

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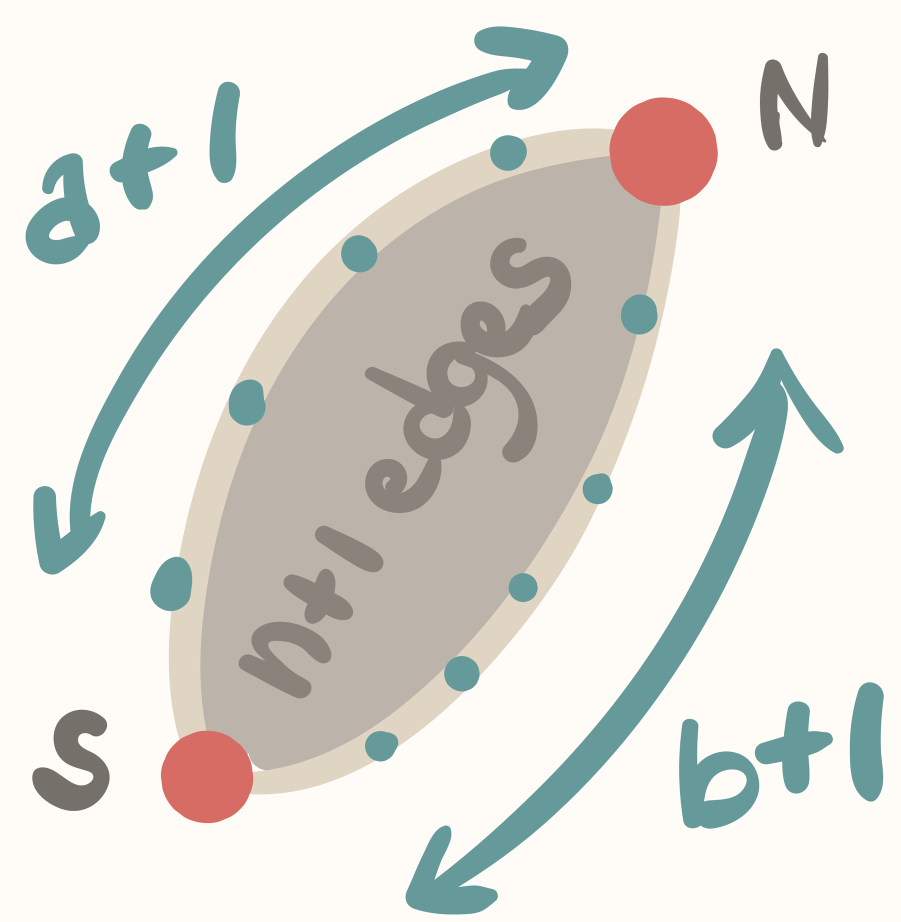

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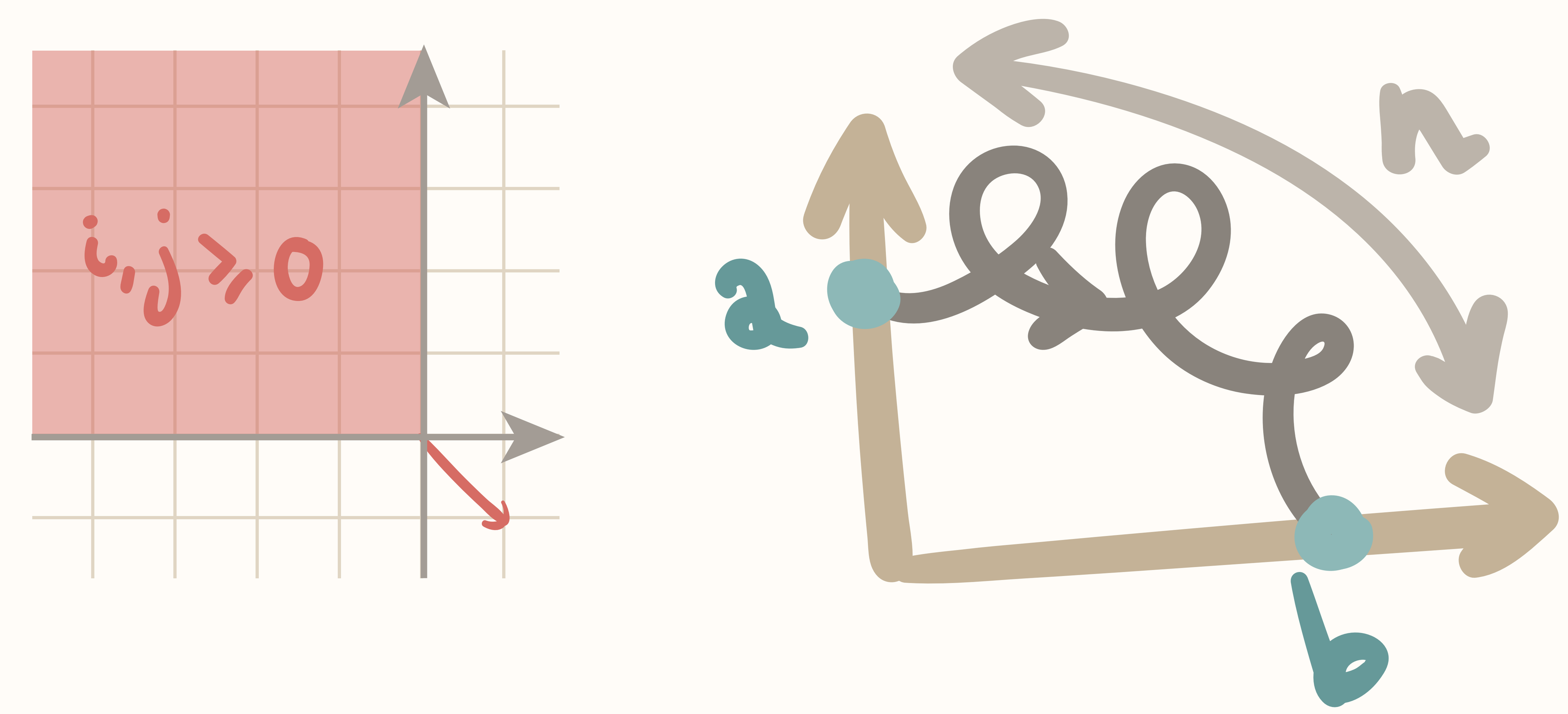

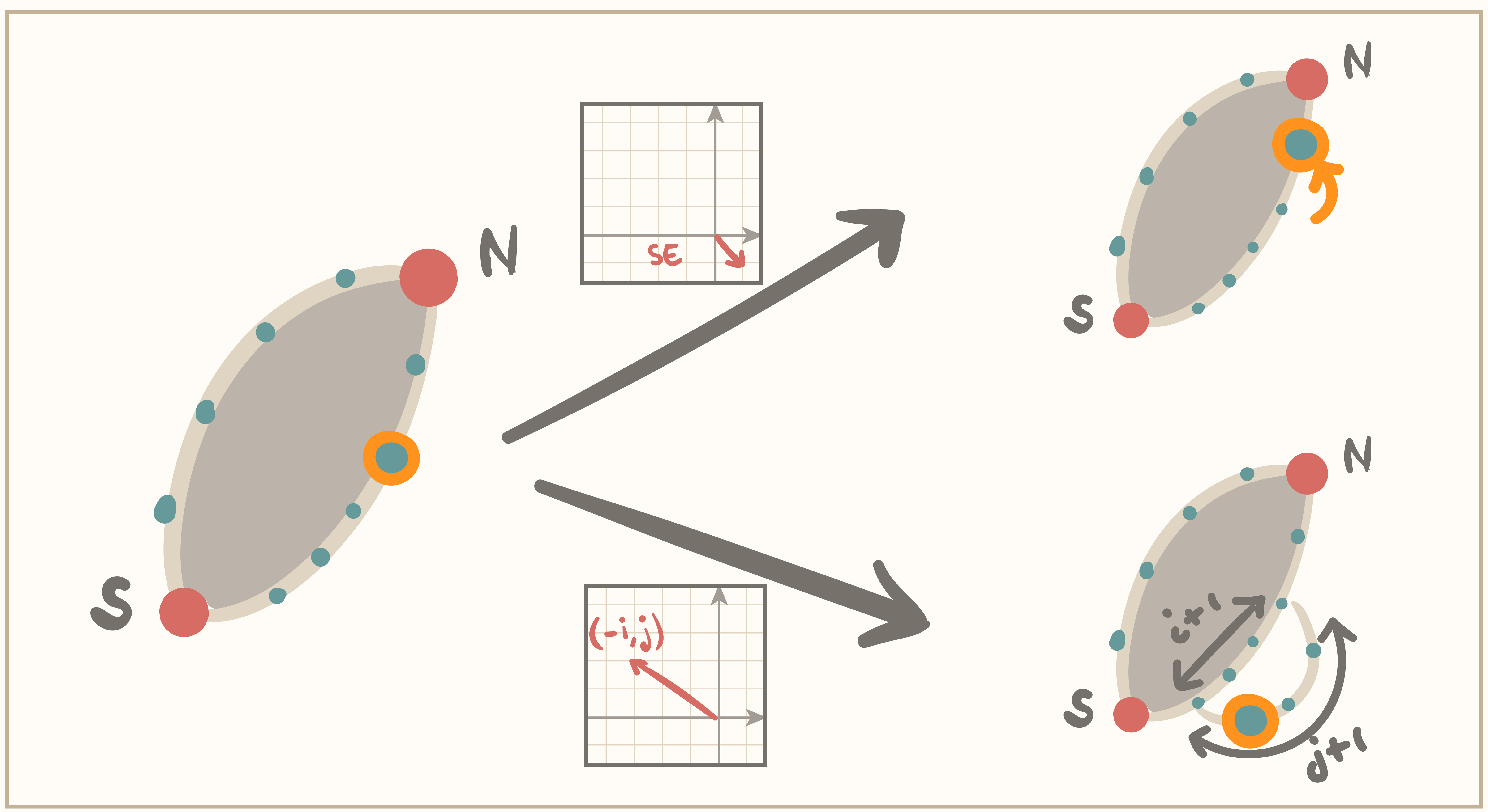

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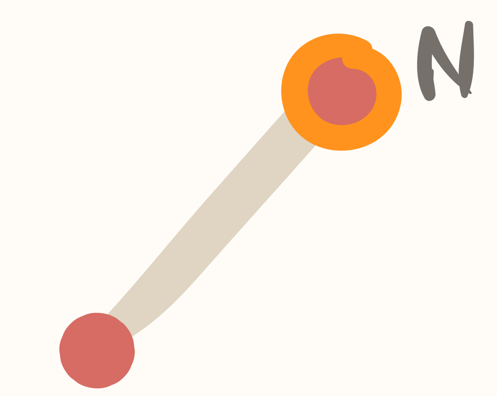

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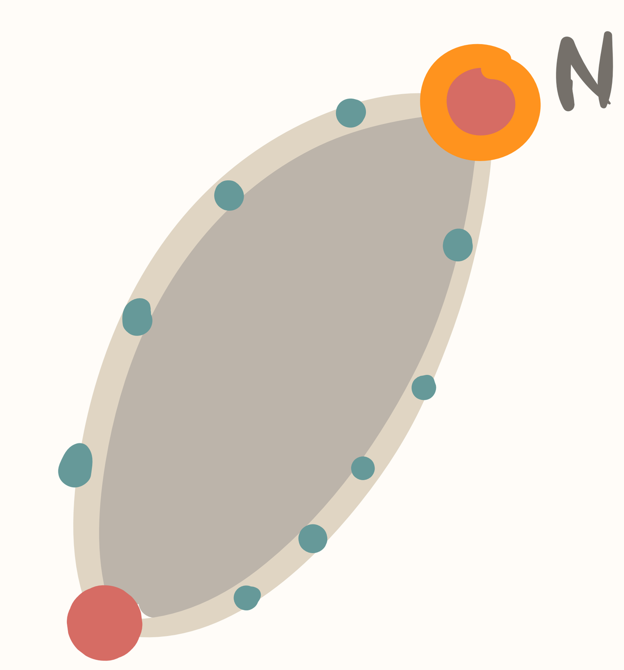

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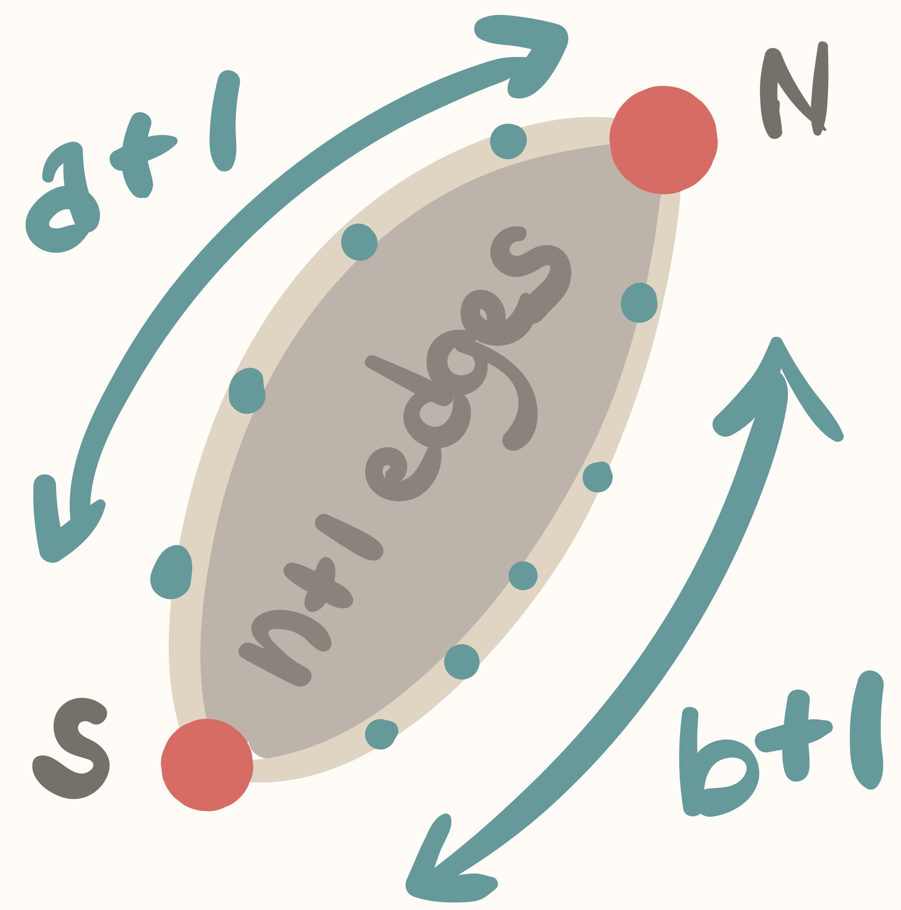

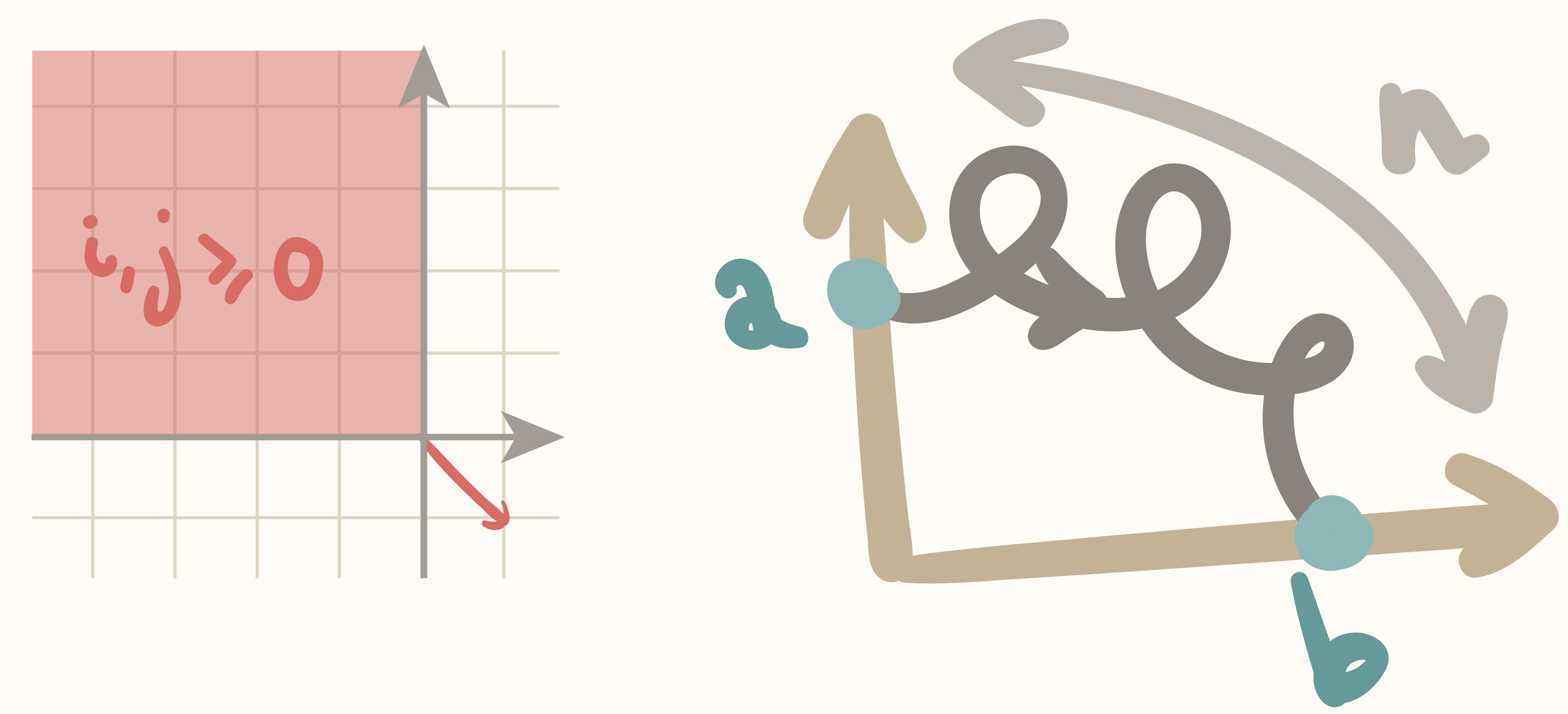

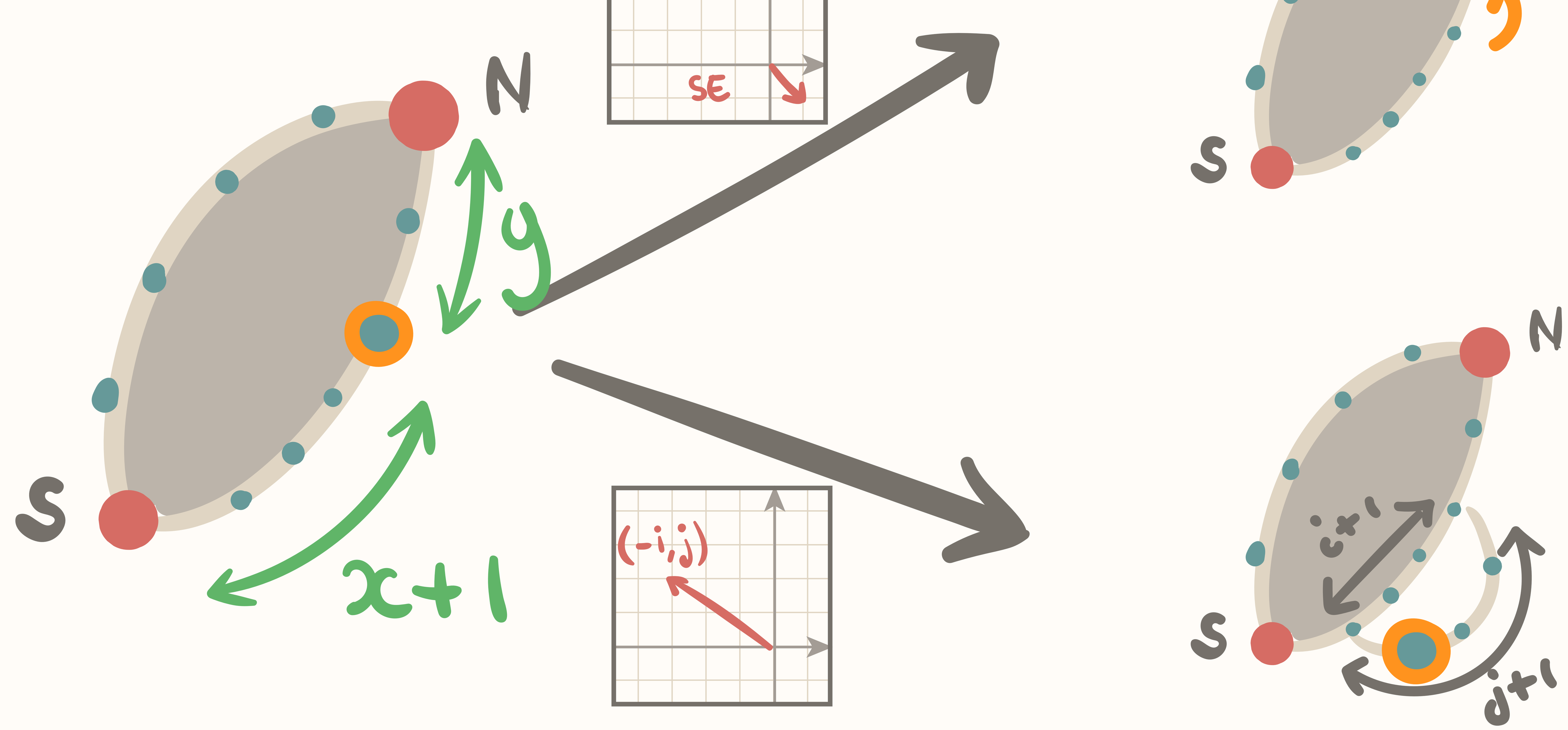

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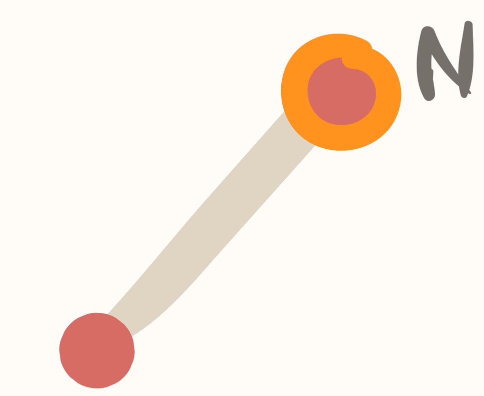

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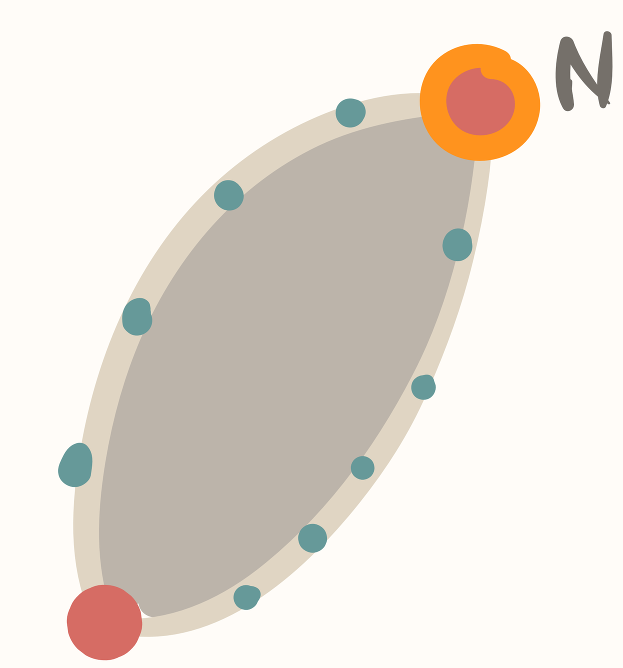
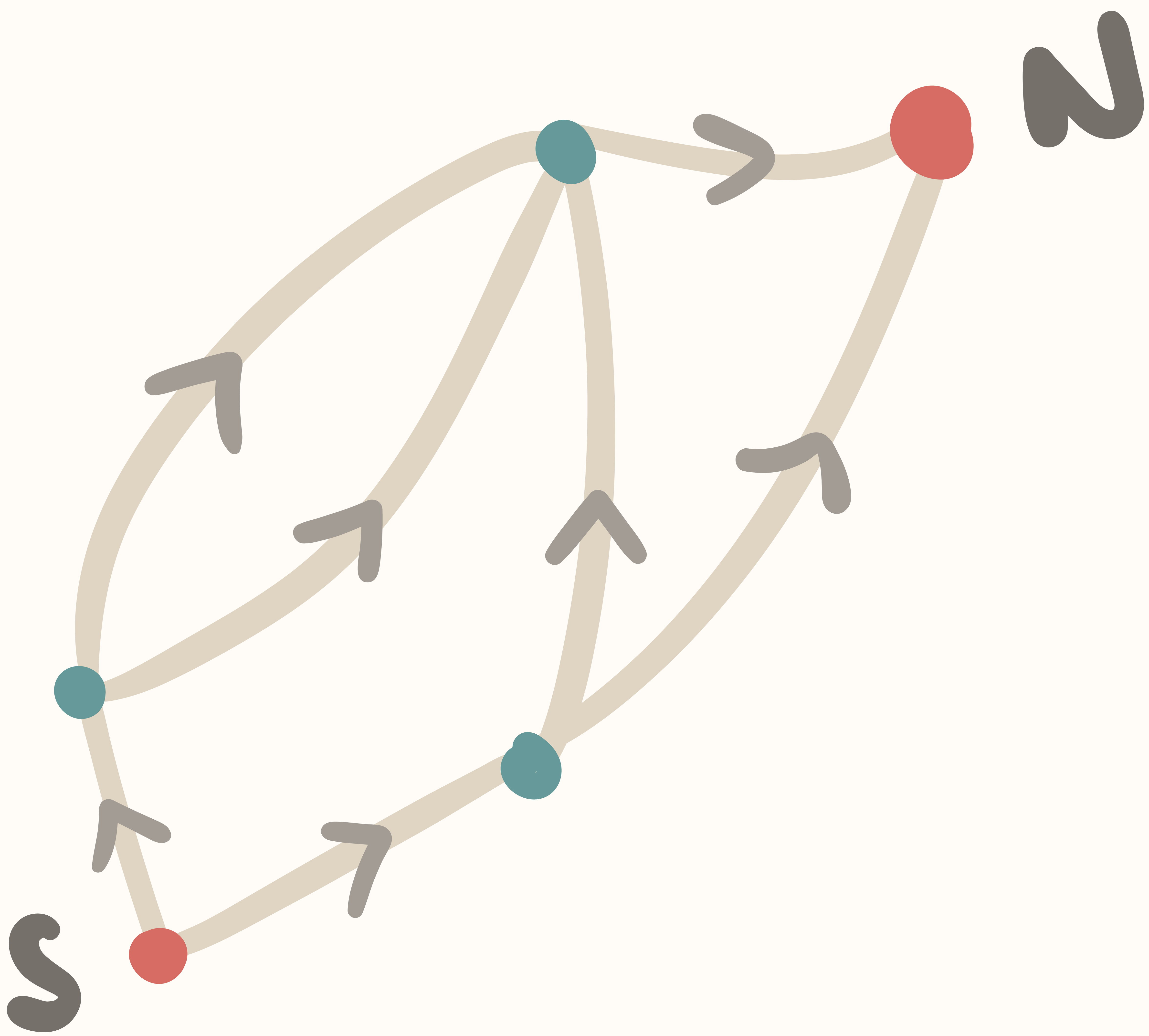
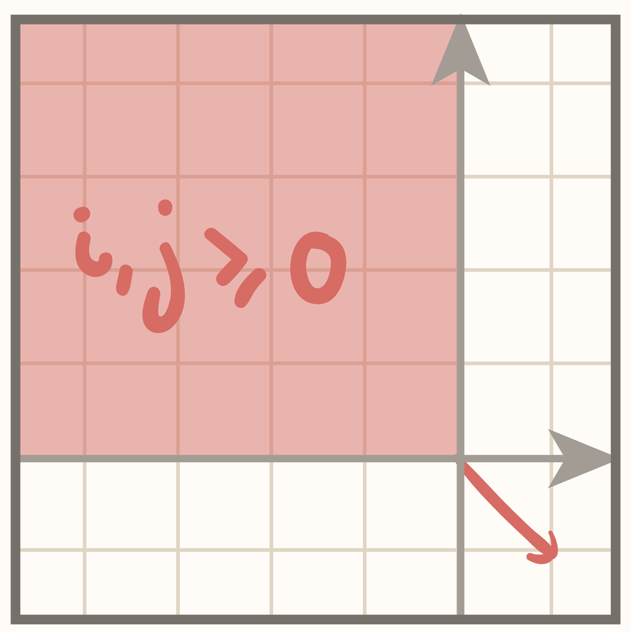
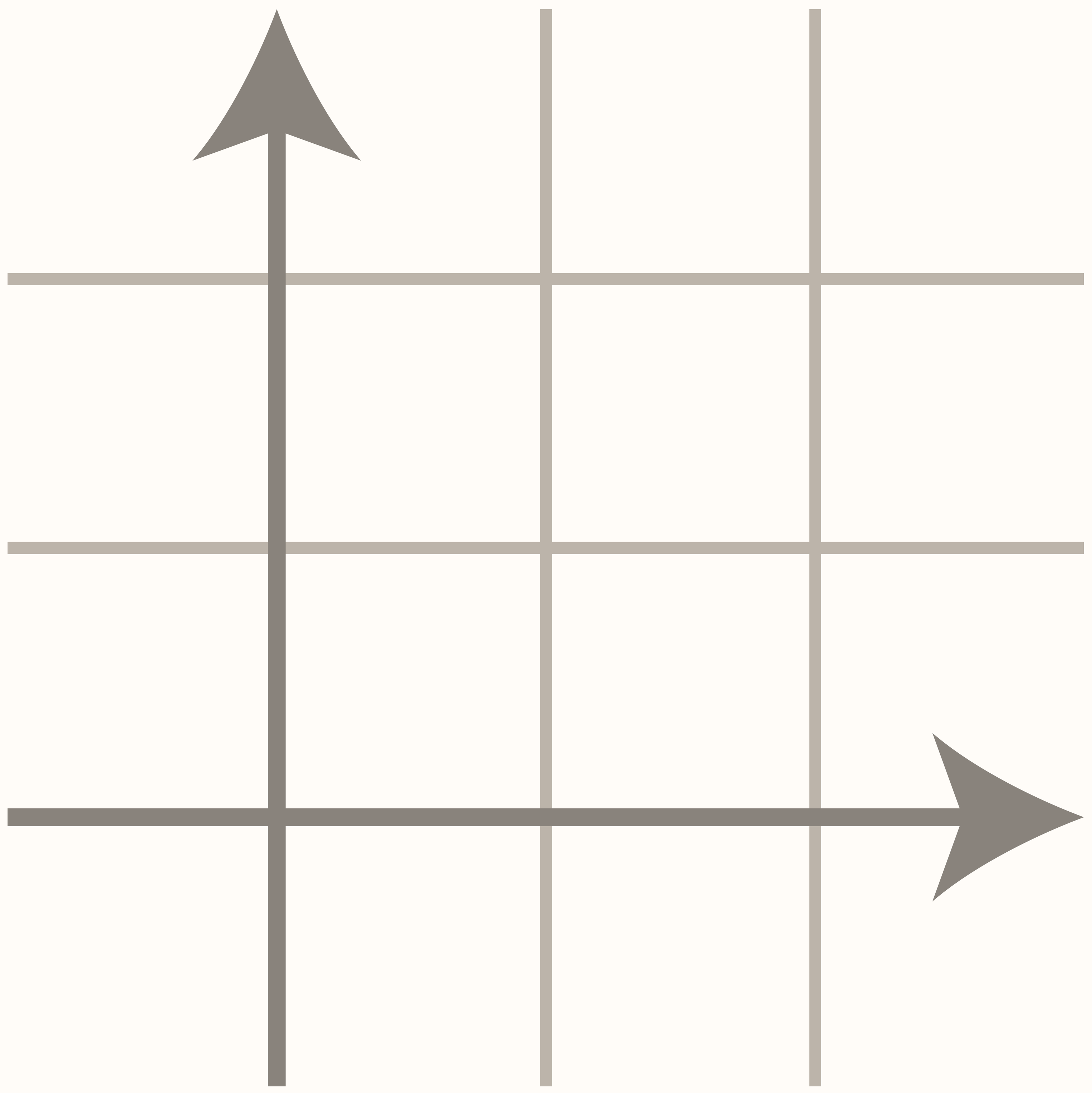


KMSW bijection example







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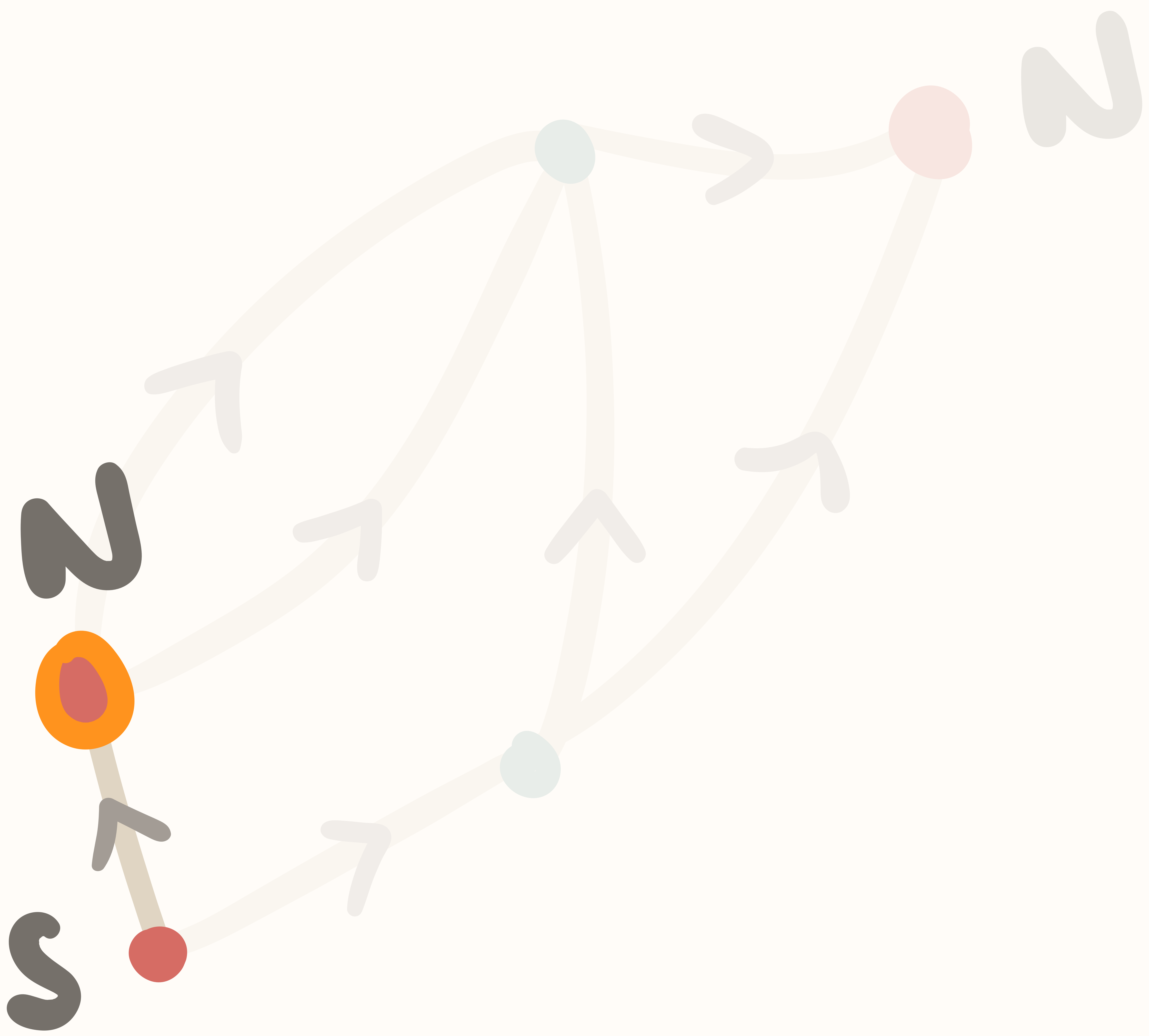

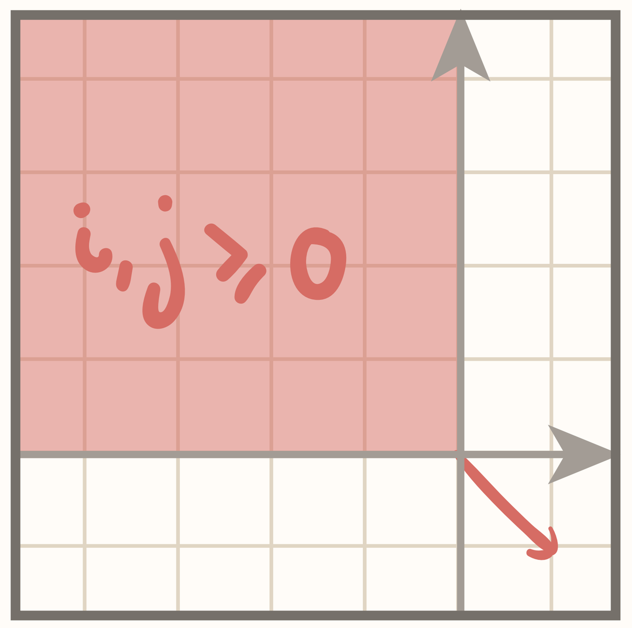

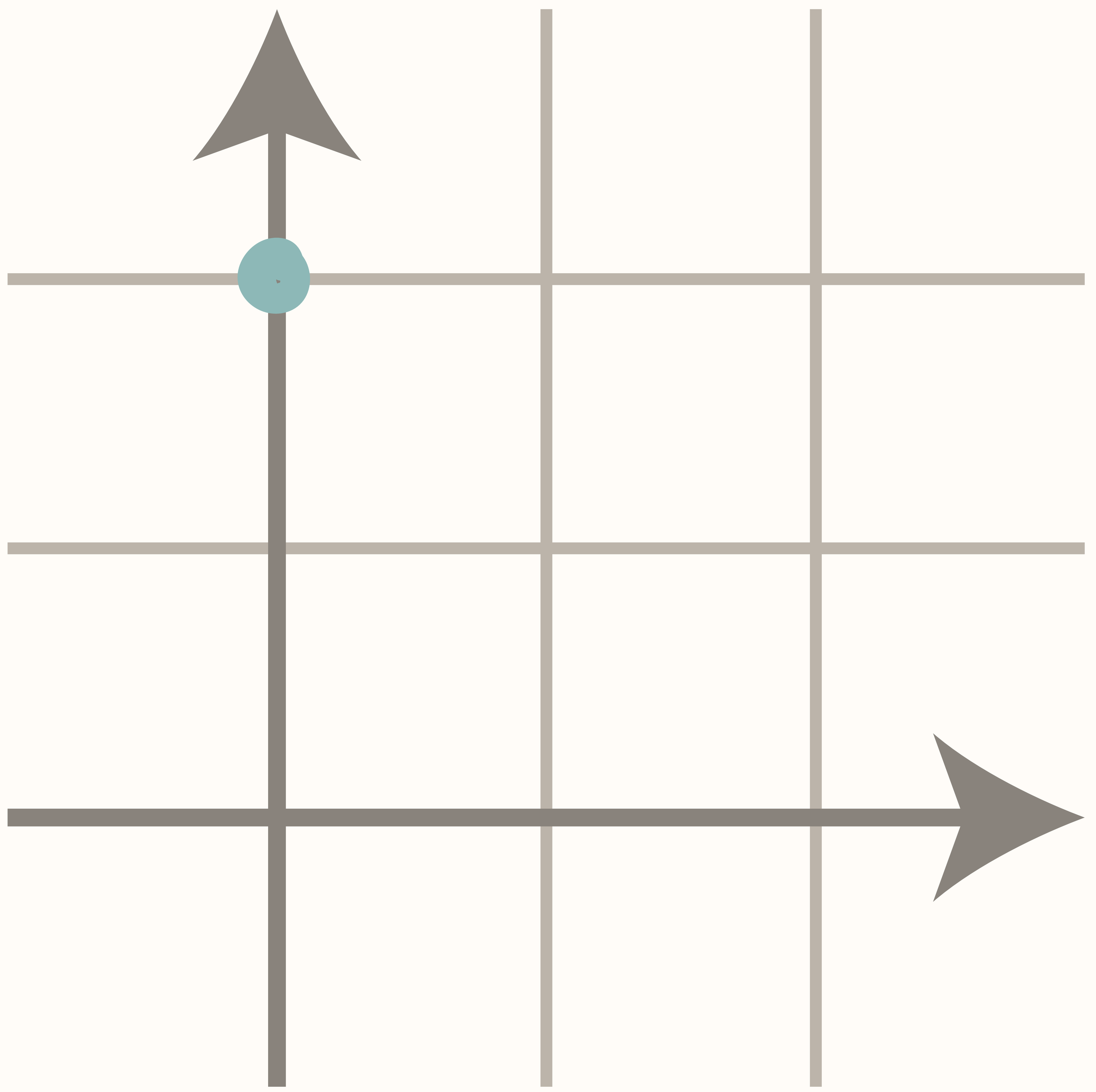

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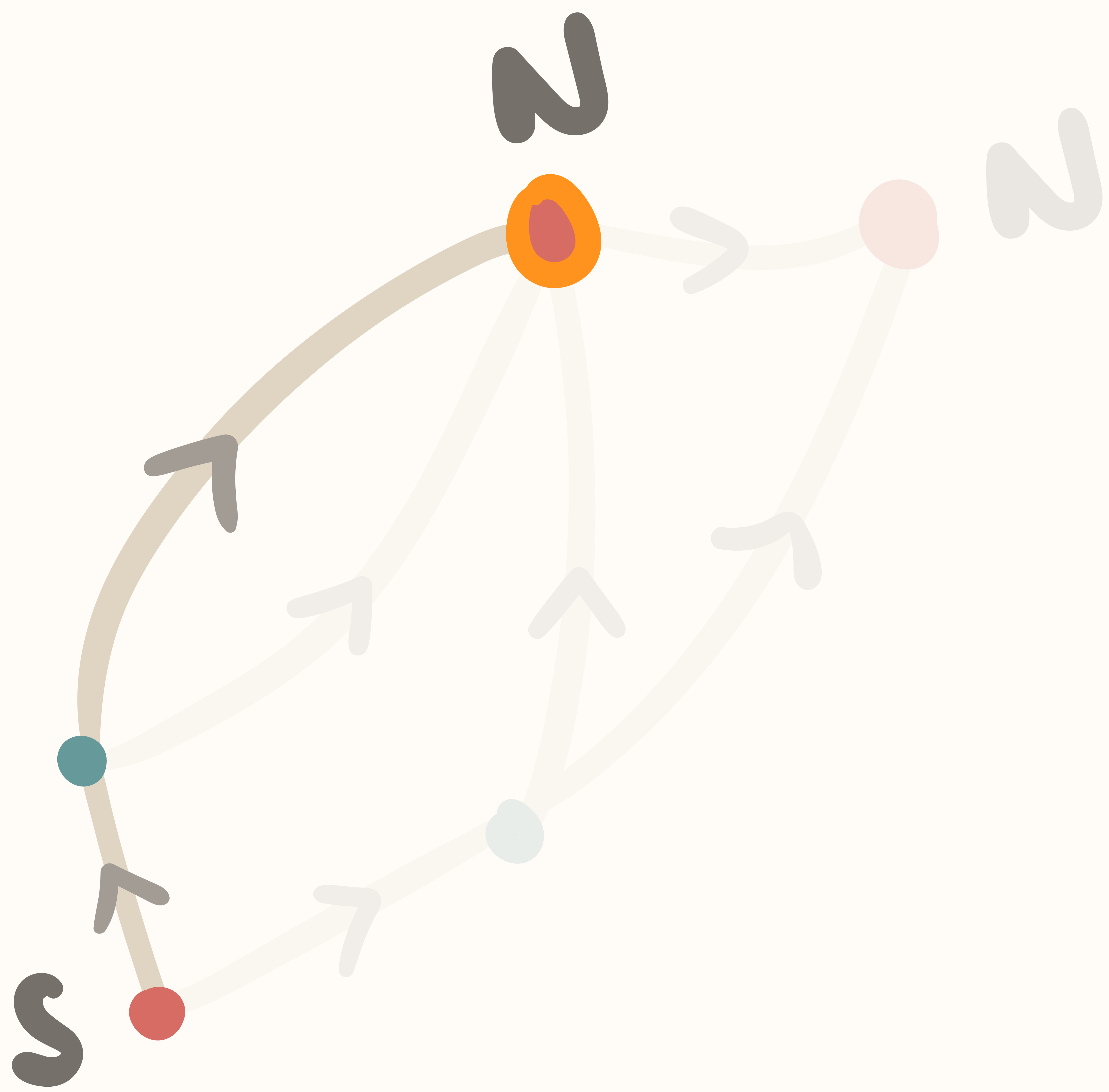

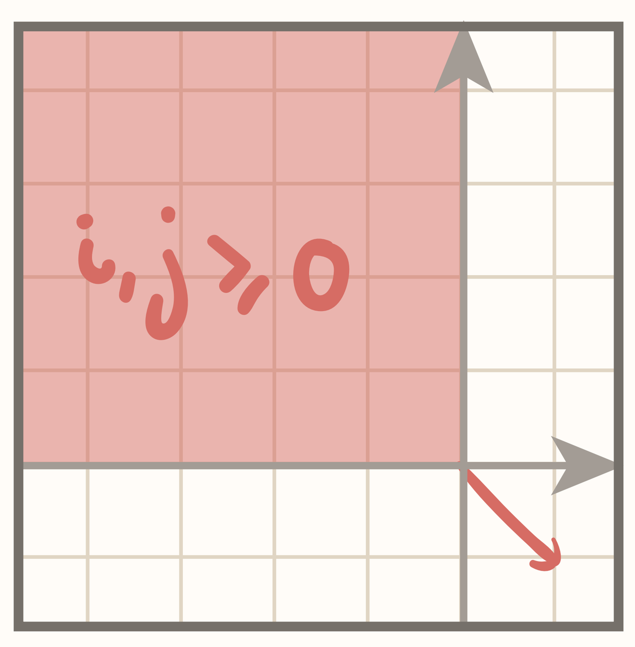

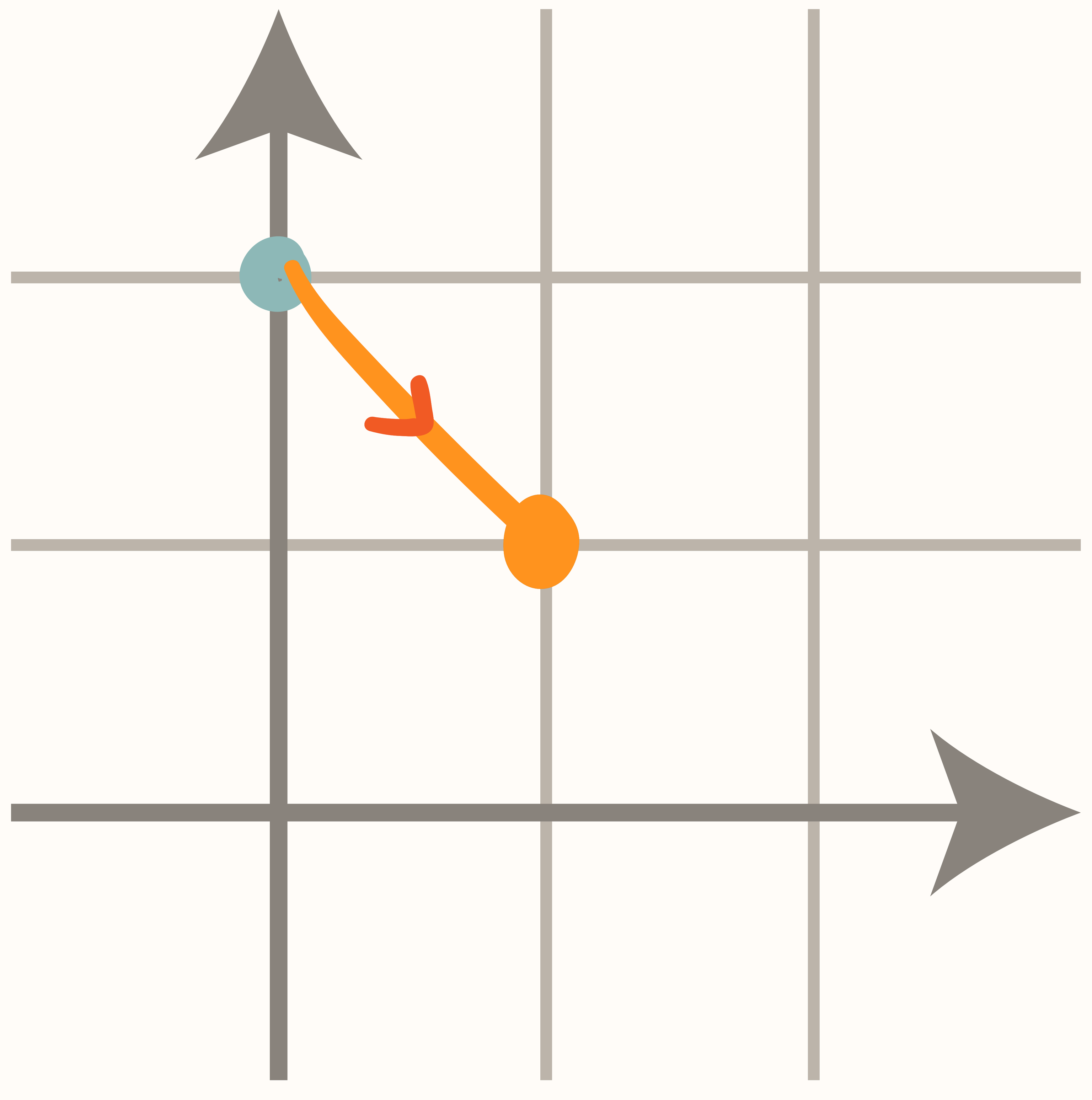

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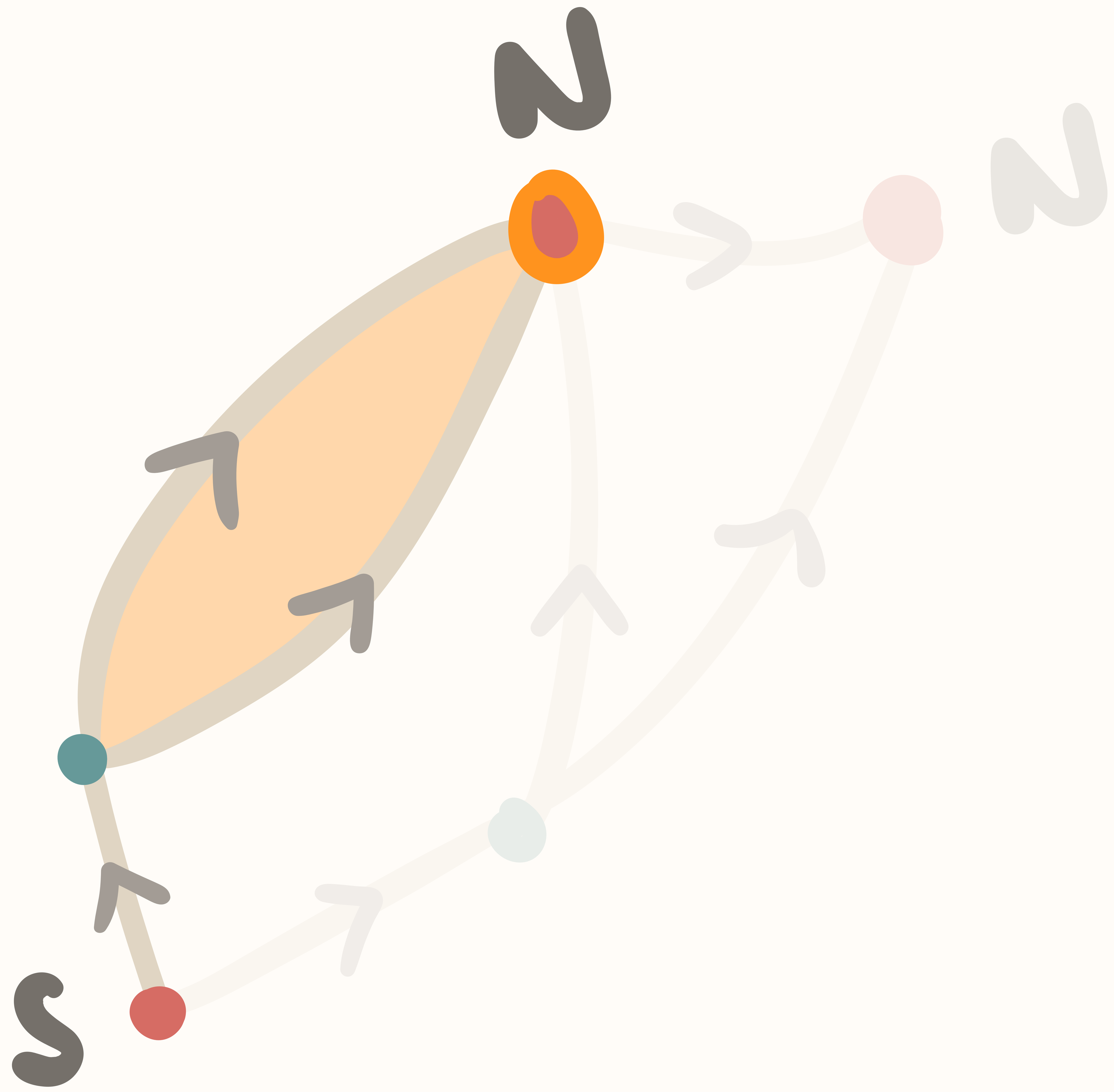

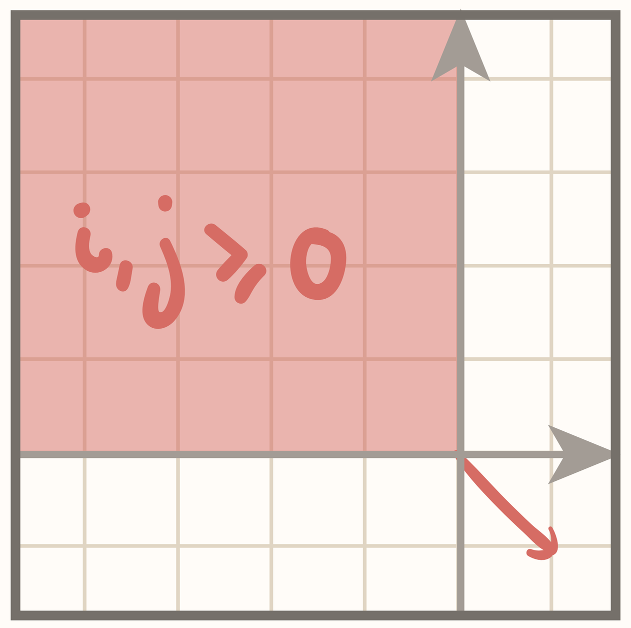

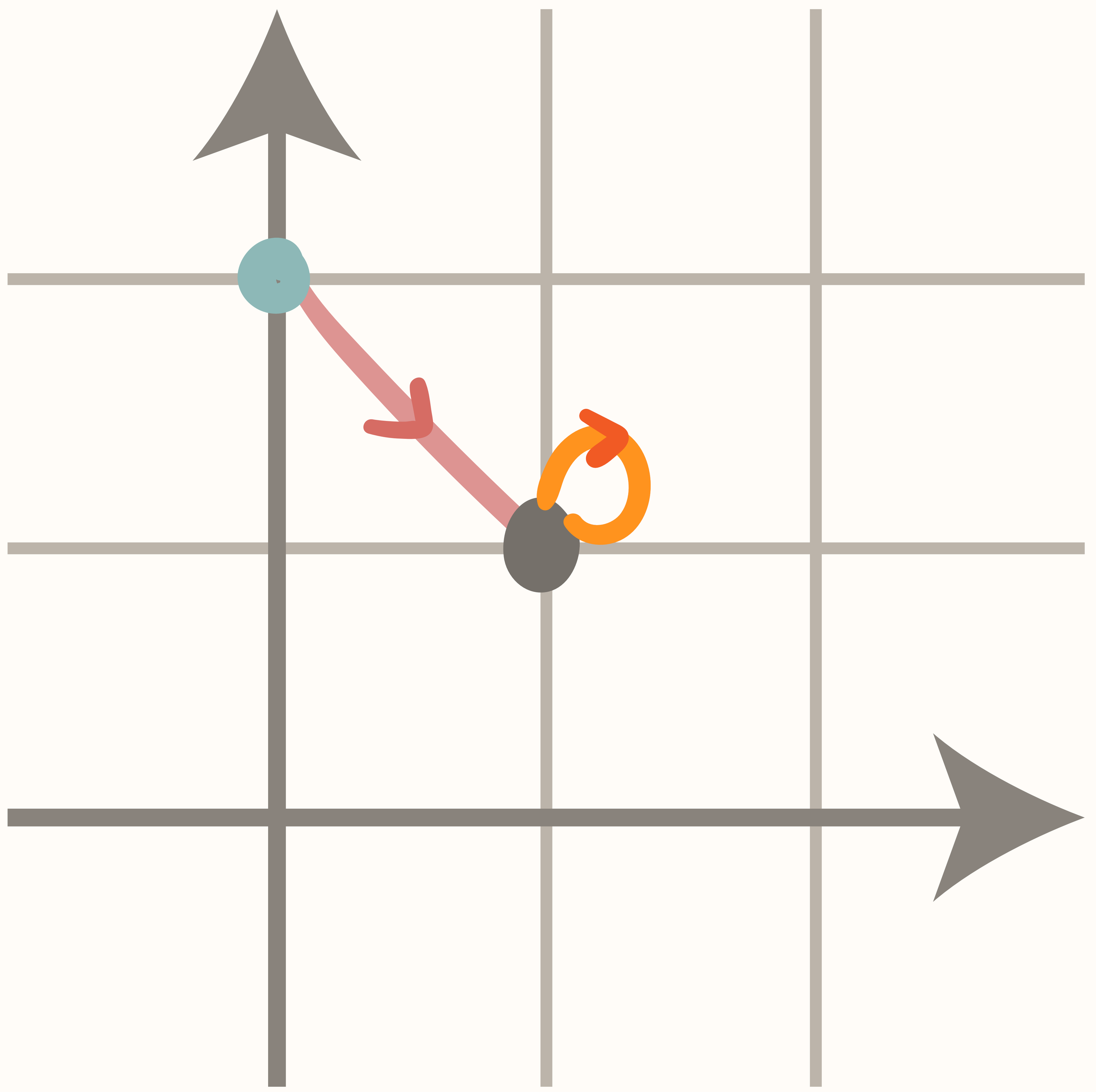

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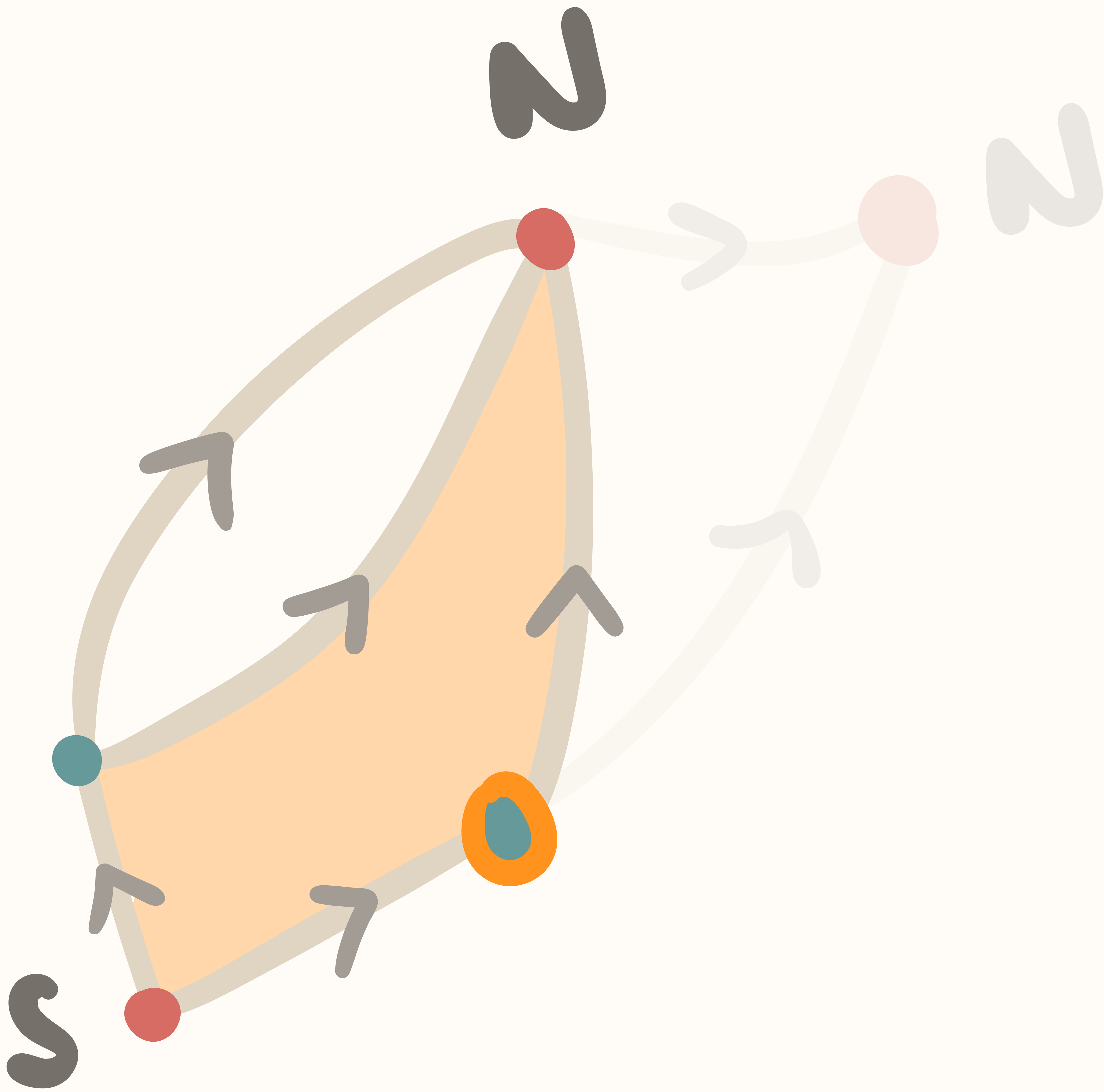

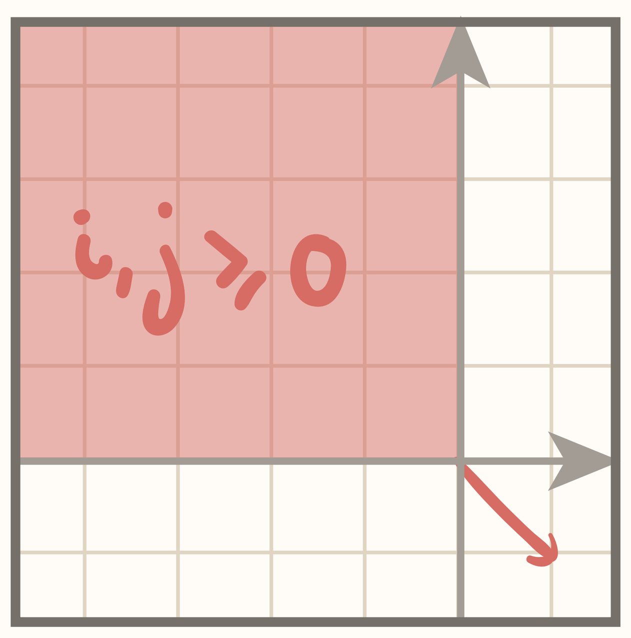

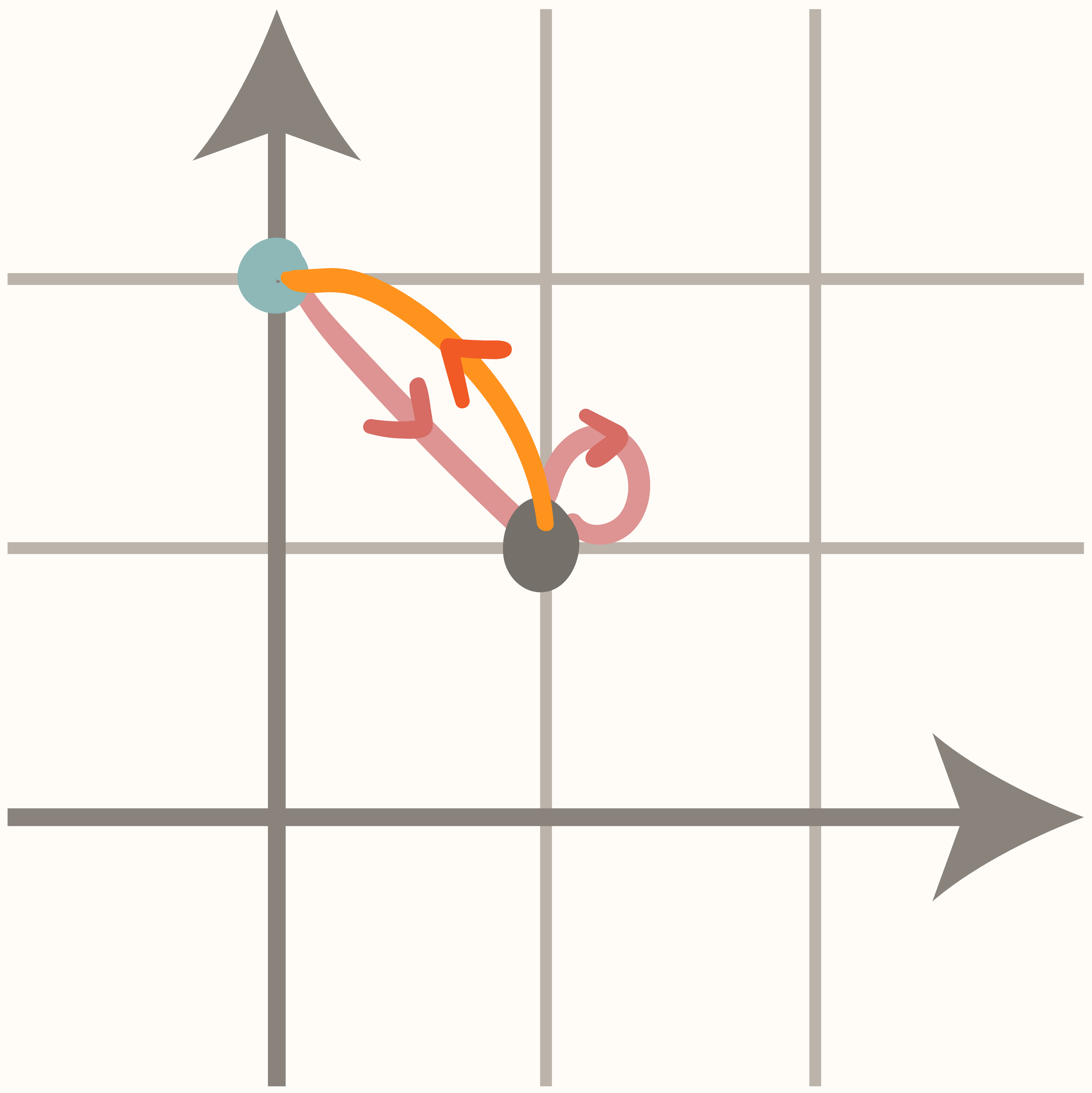

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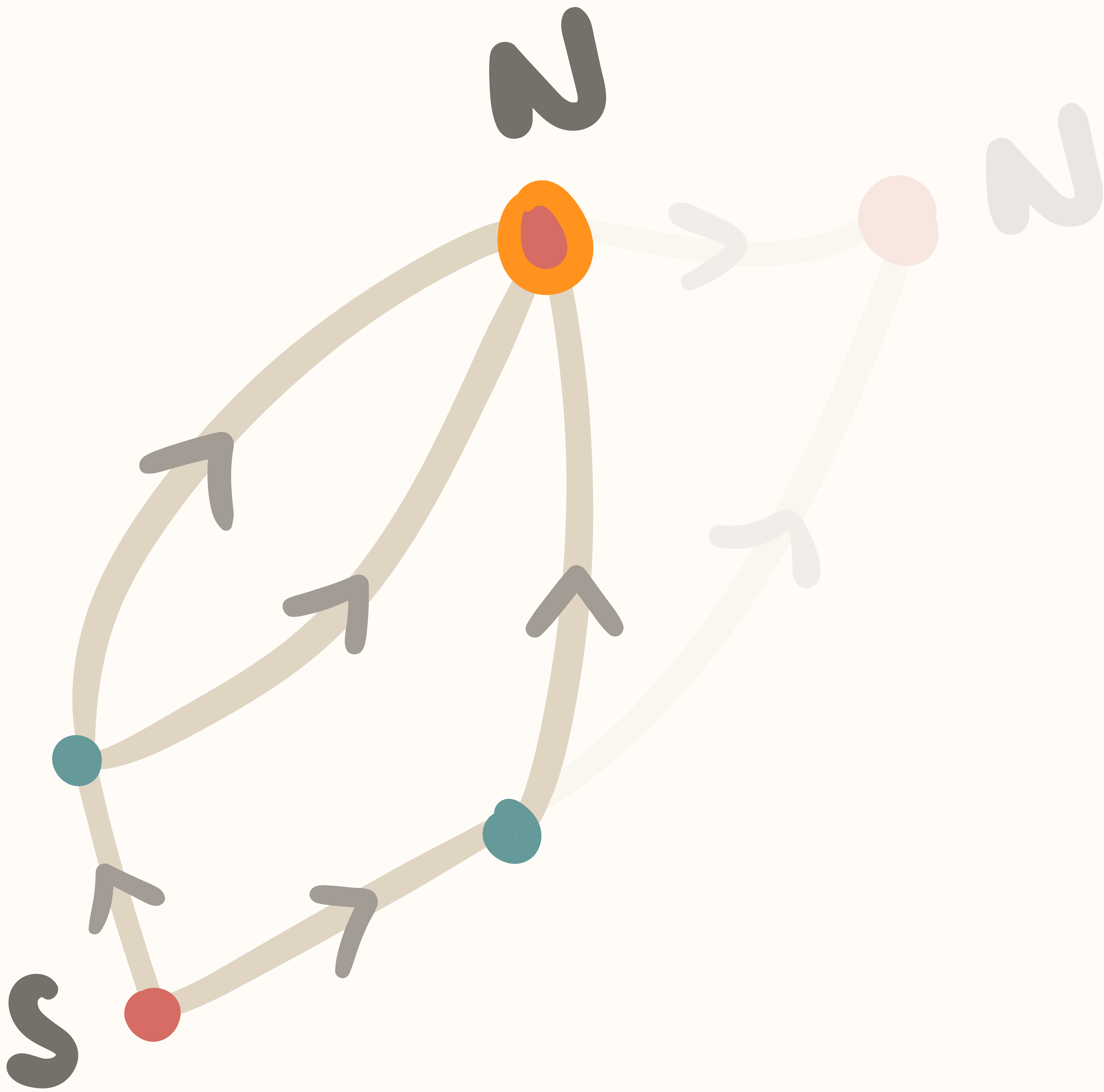

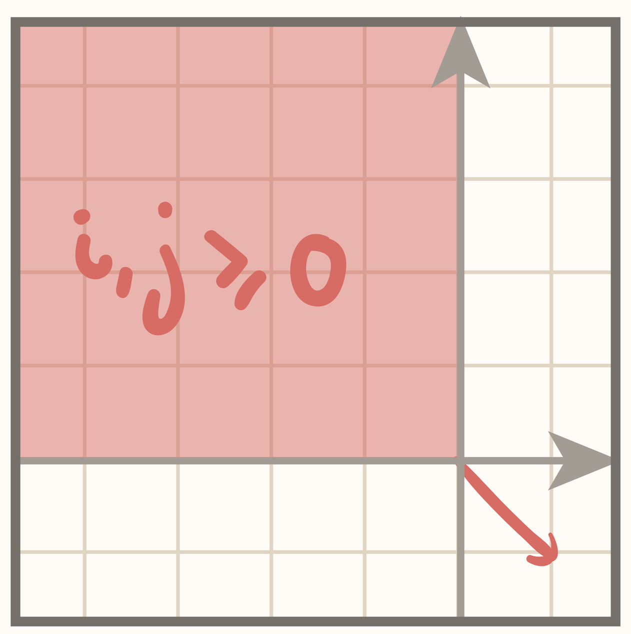

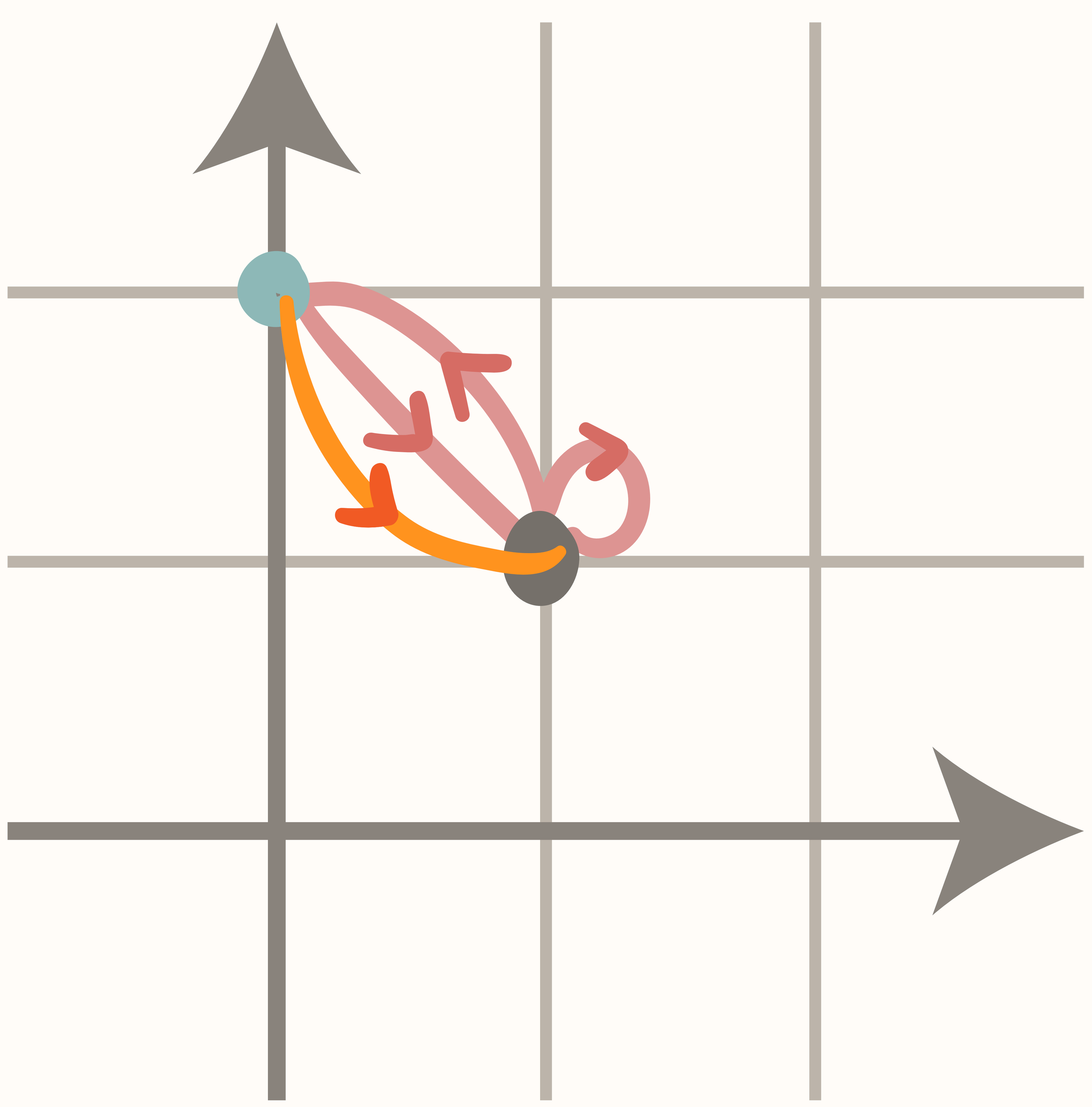

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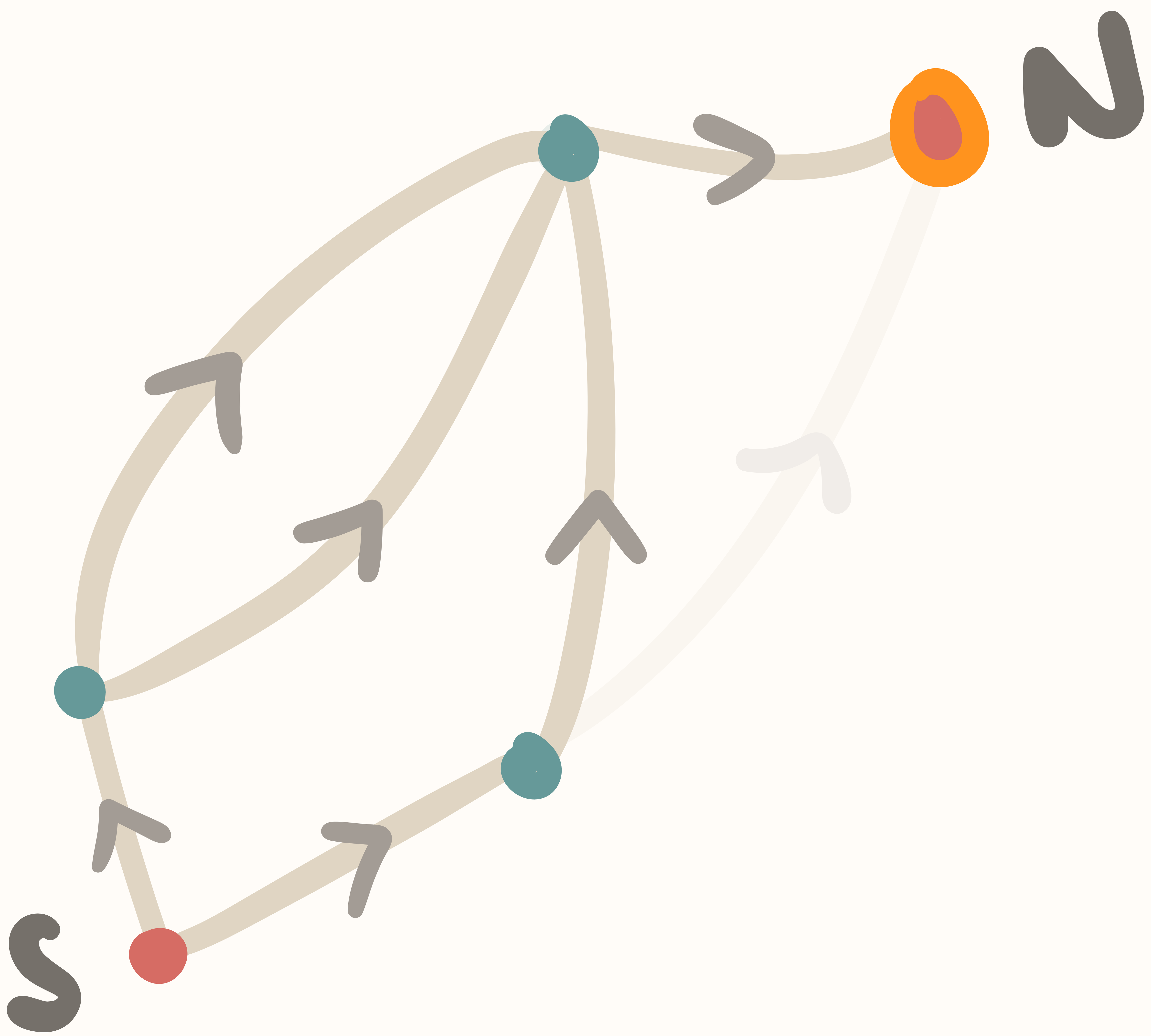

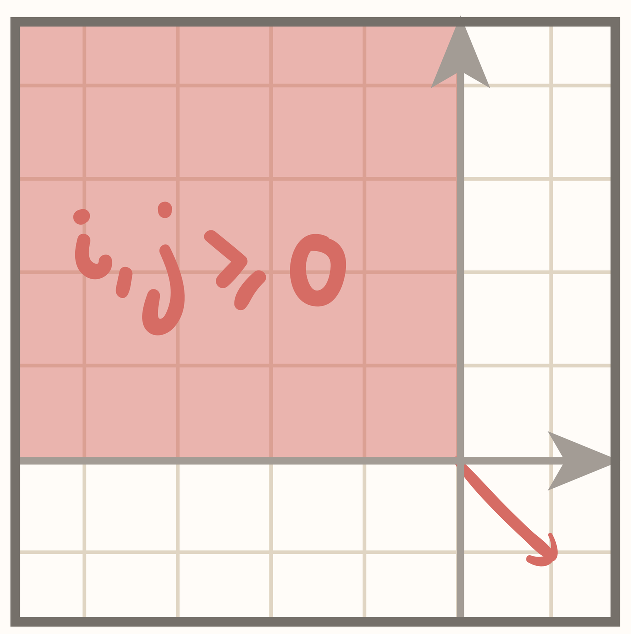

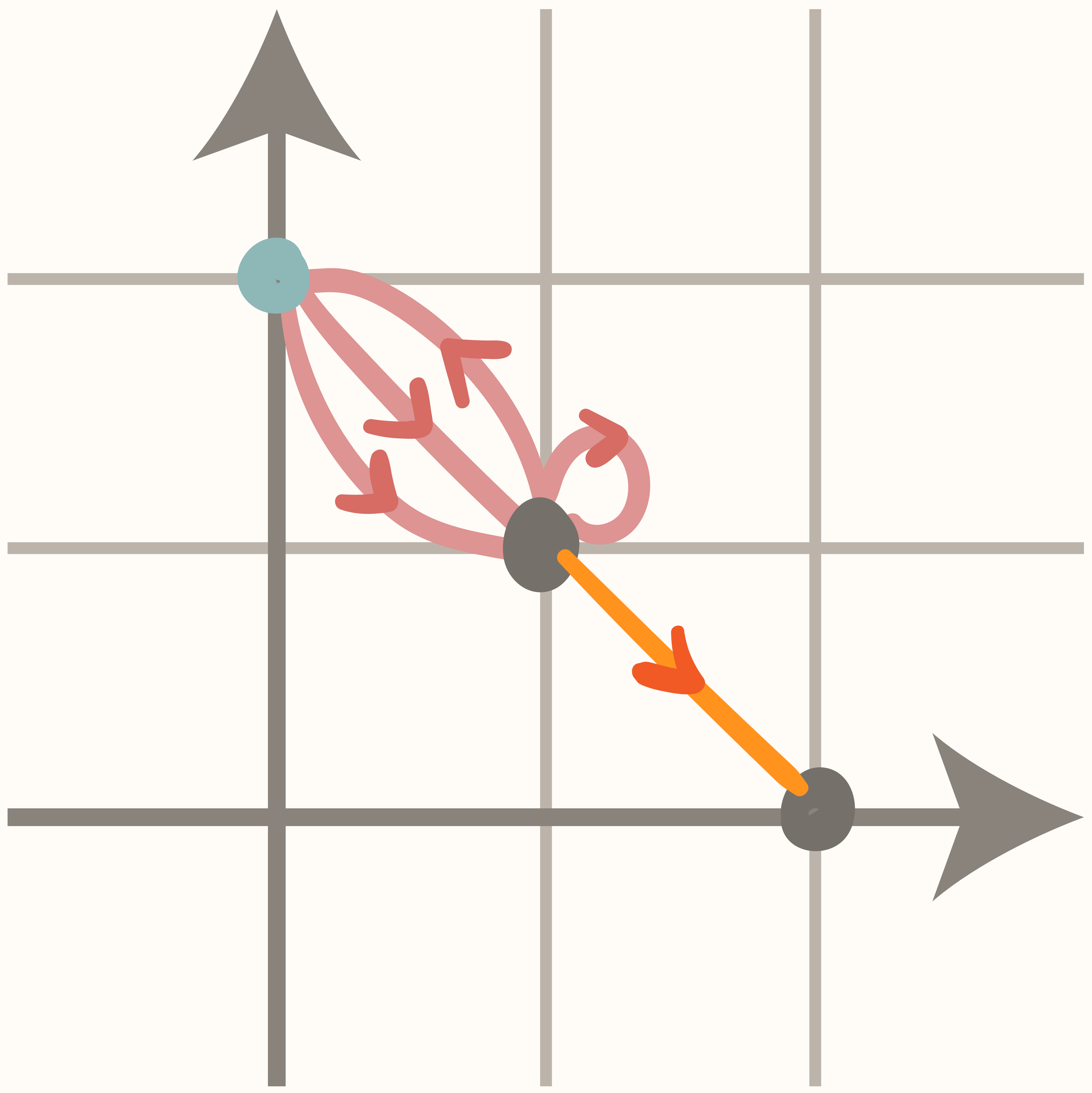

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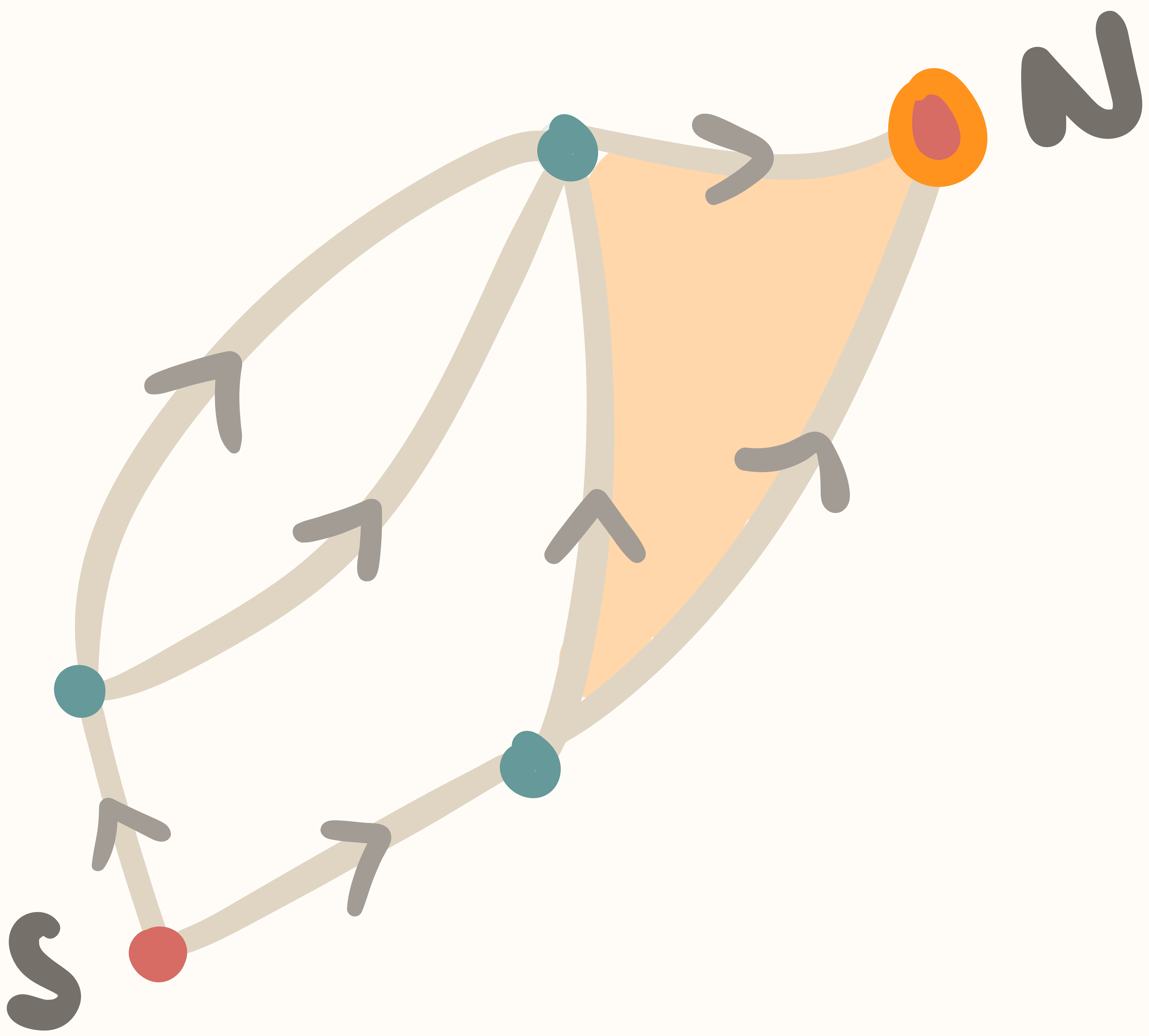
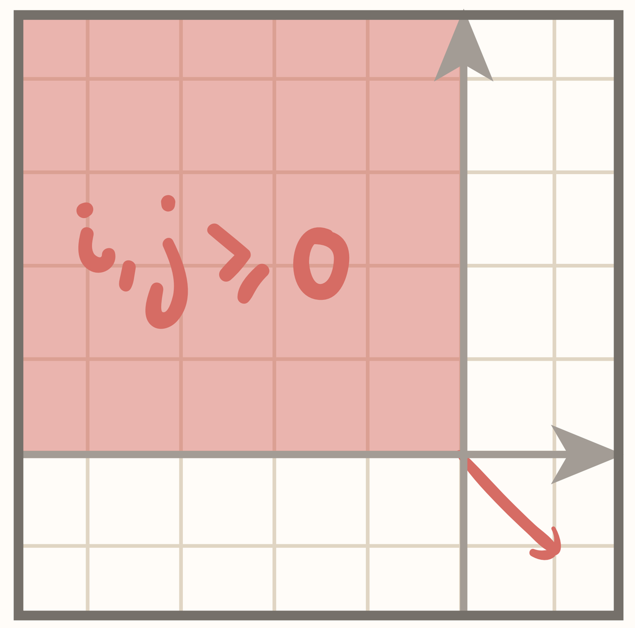
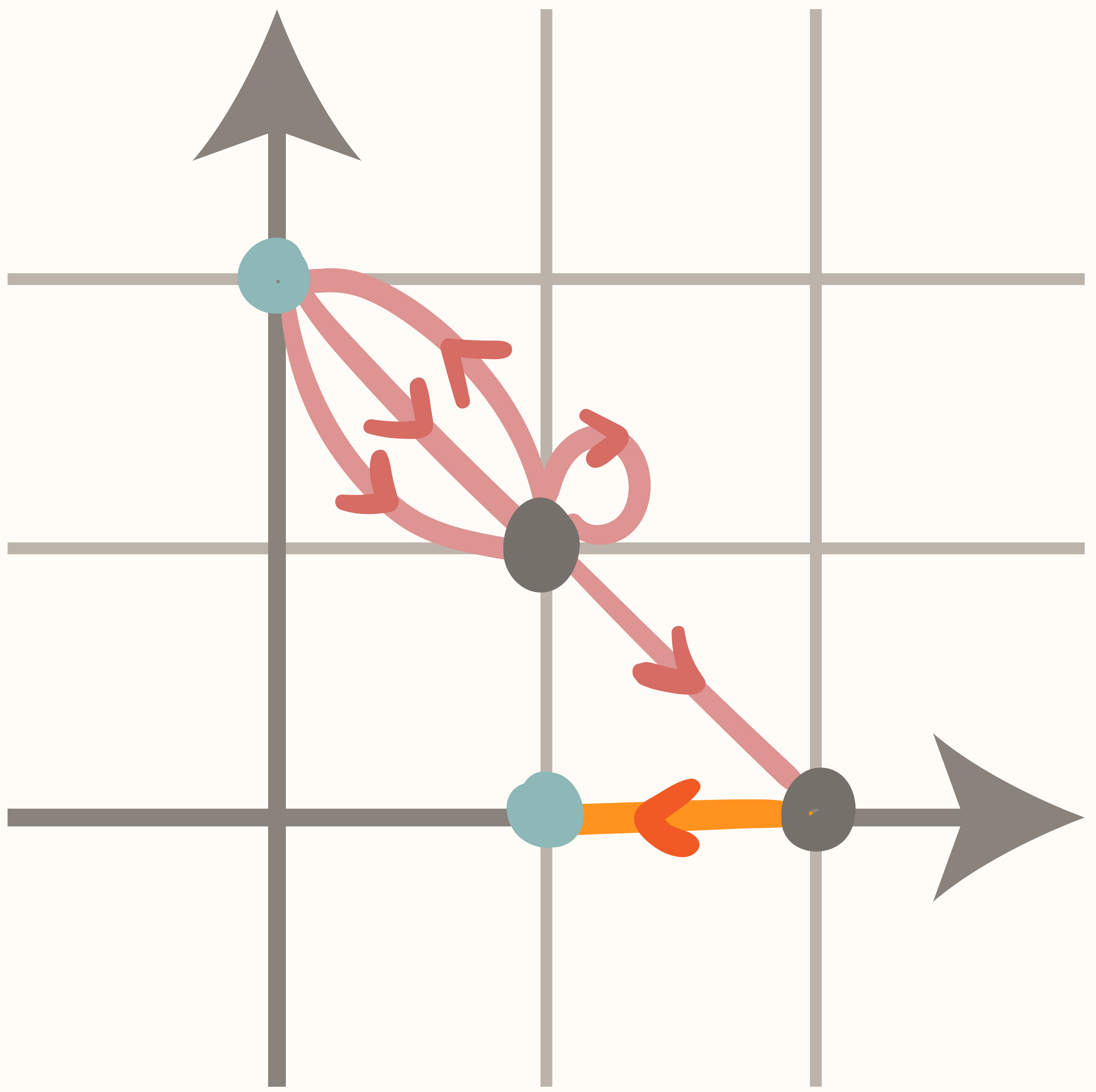
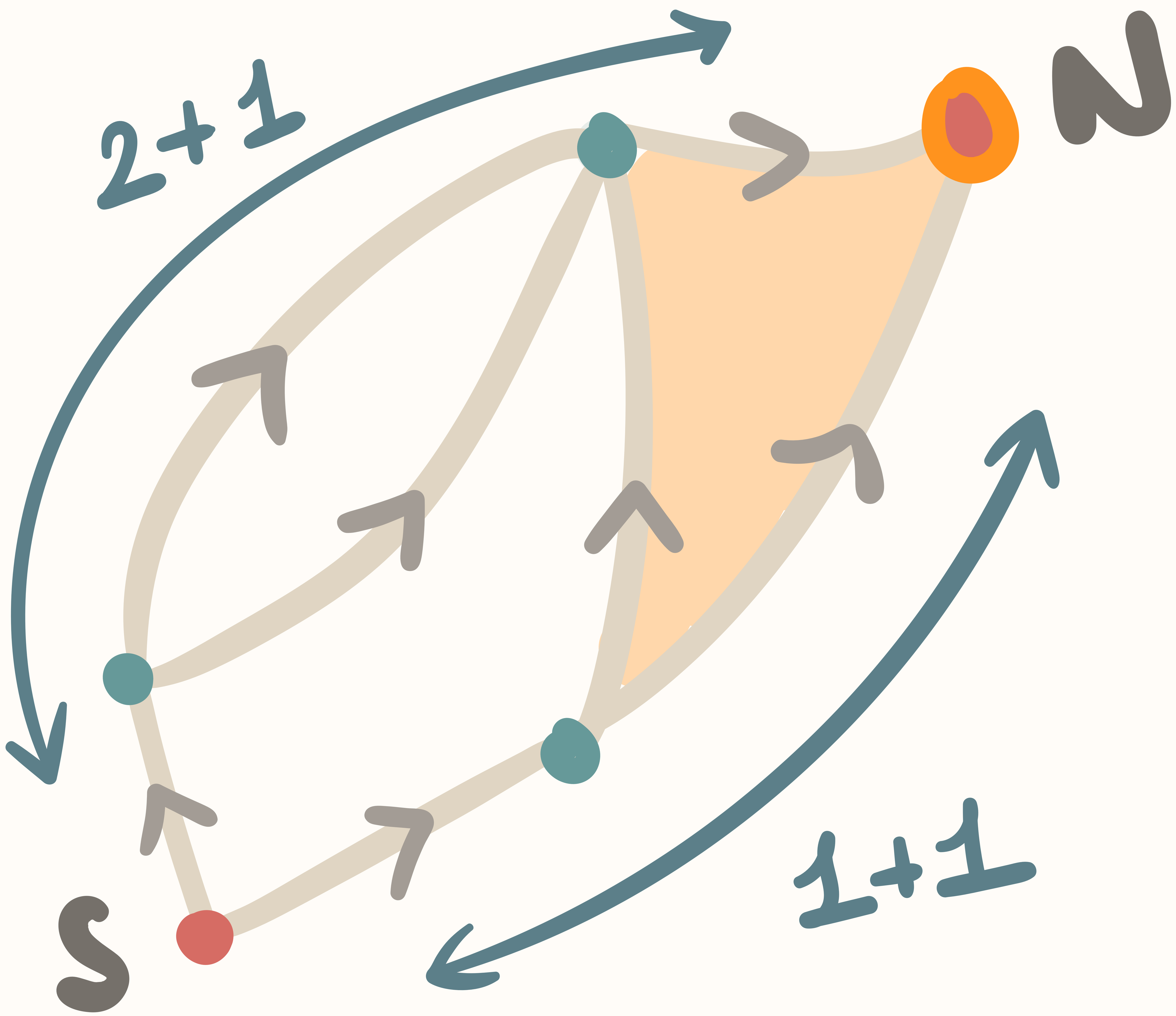
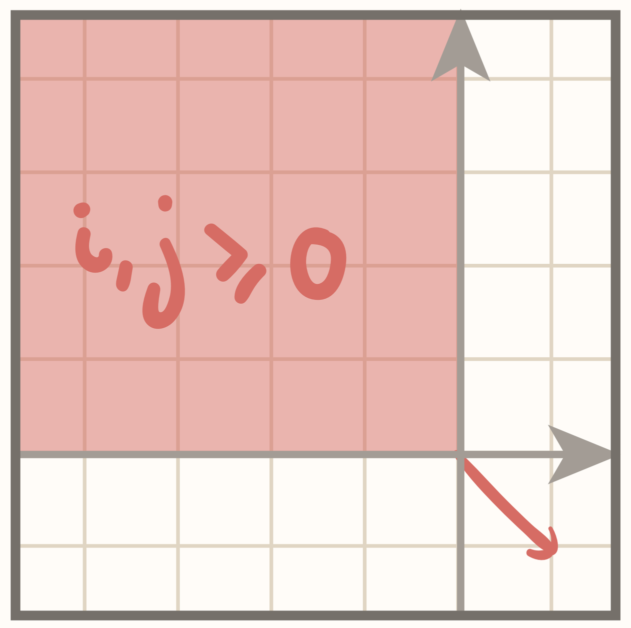


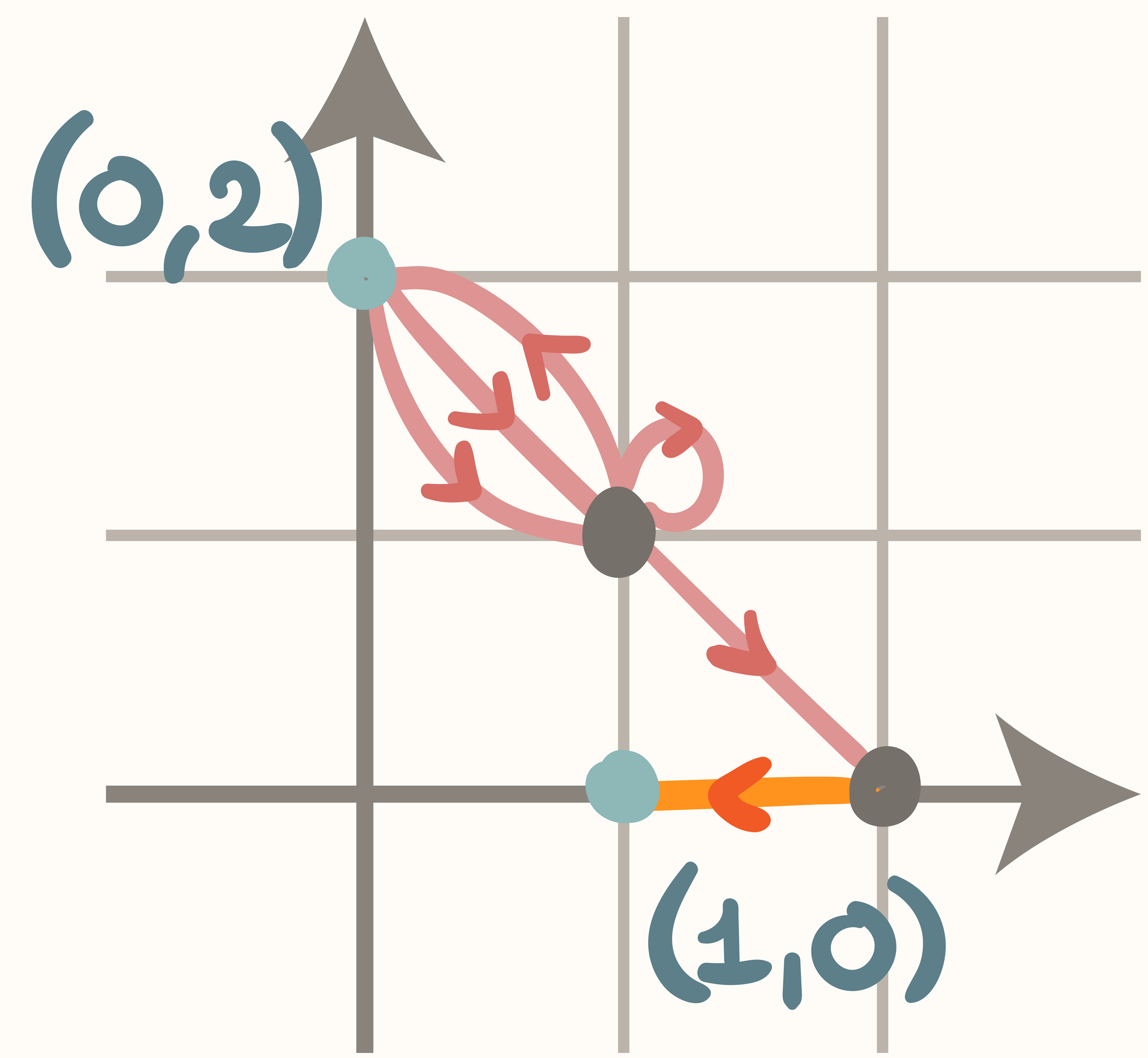




KMSW bijection example

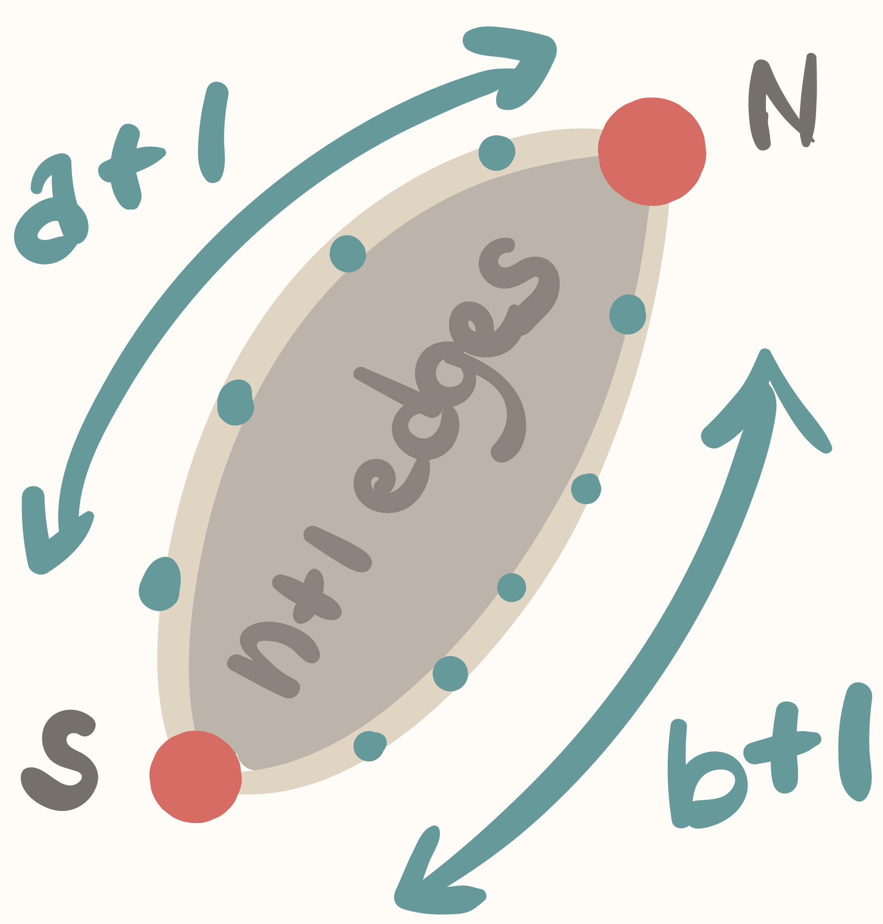




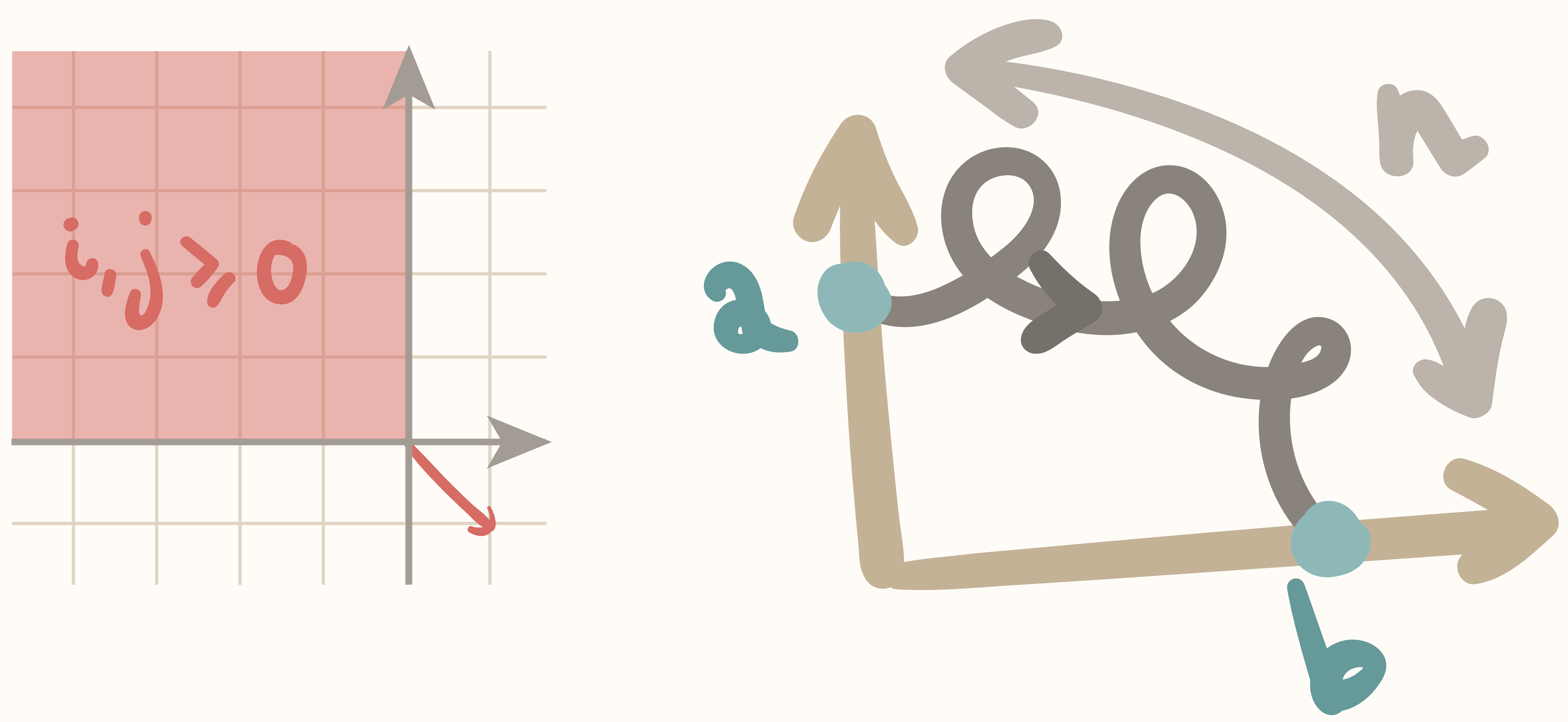


La bijection KMSW

bipolar orientations ⟷ *tandem walks in the quarter plane*

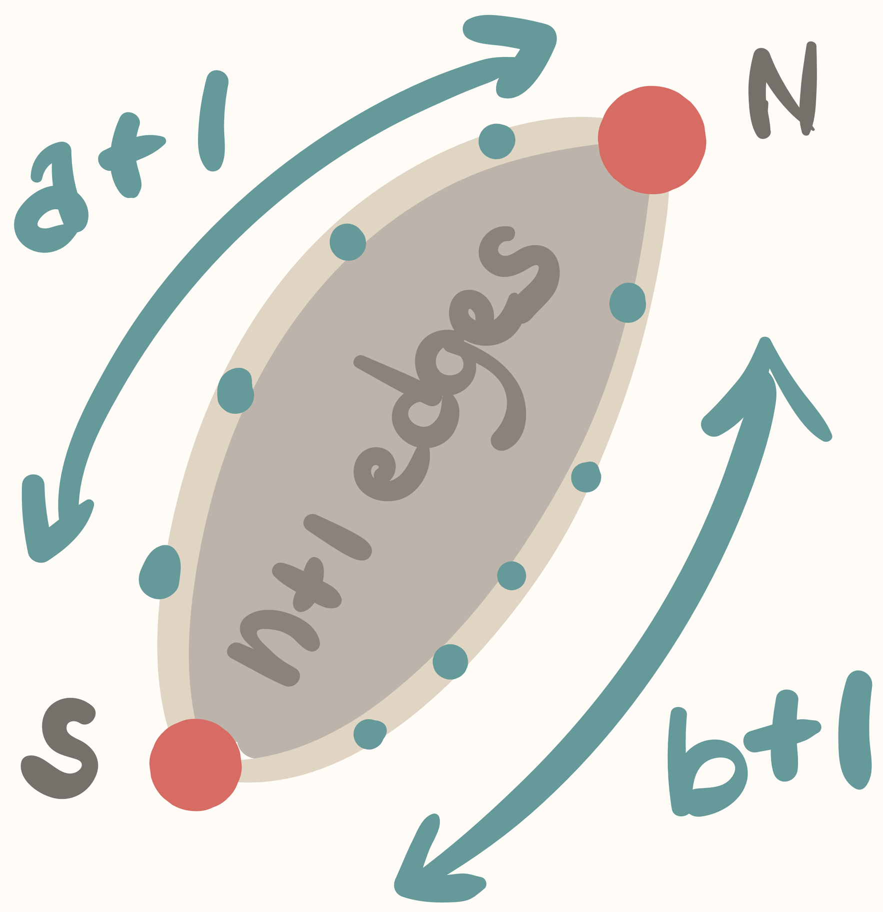


⟷



La bijection KMSW

bipolar orientations ⟷ **tandem walks in the quarter plane**



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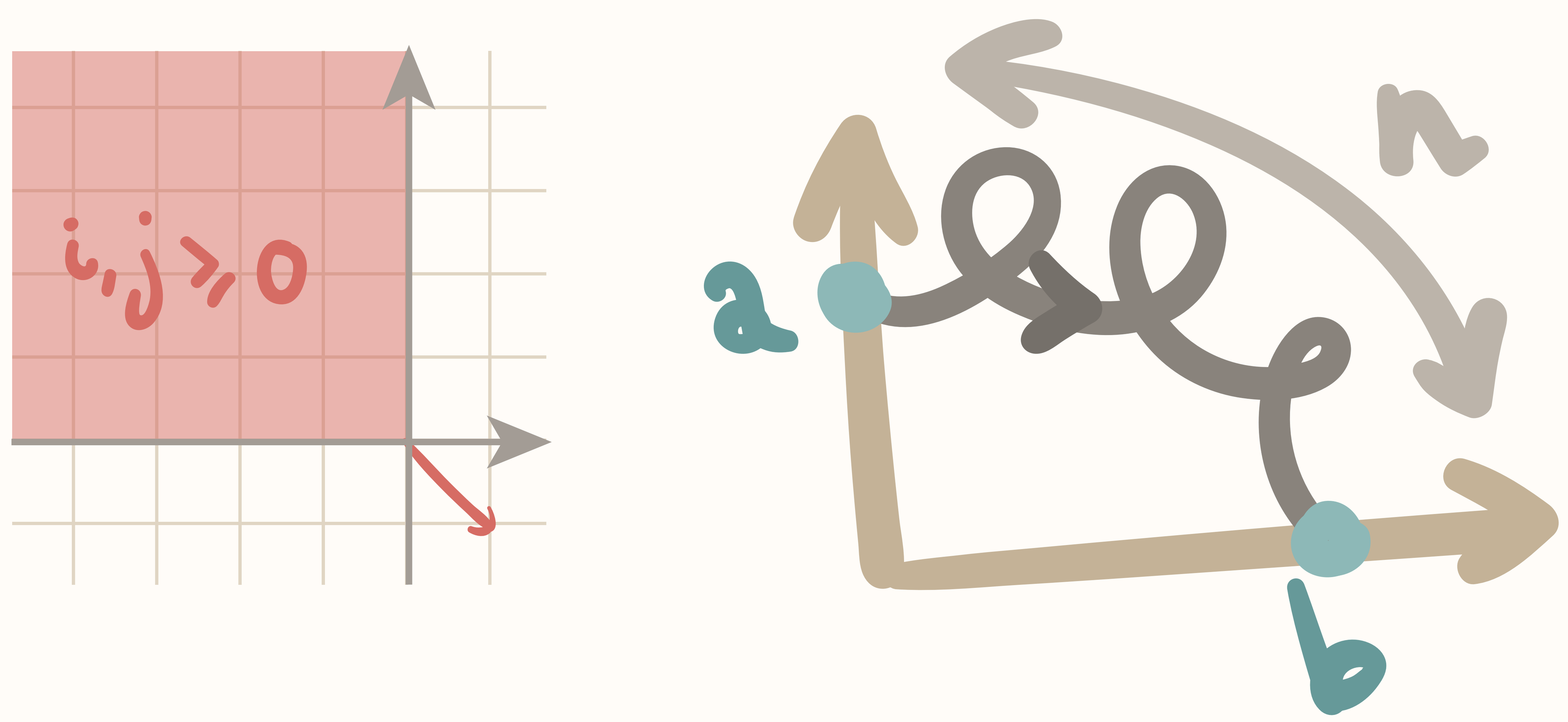
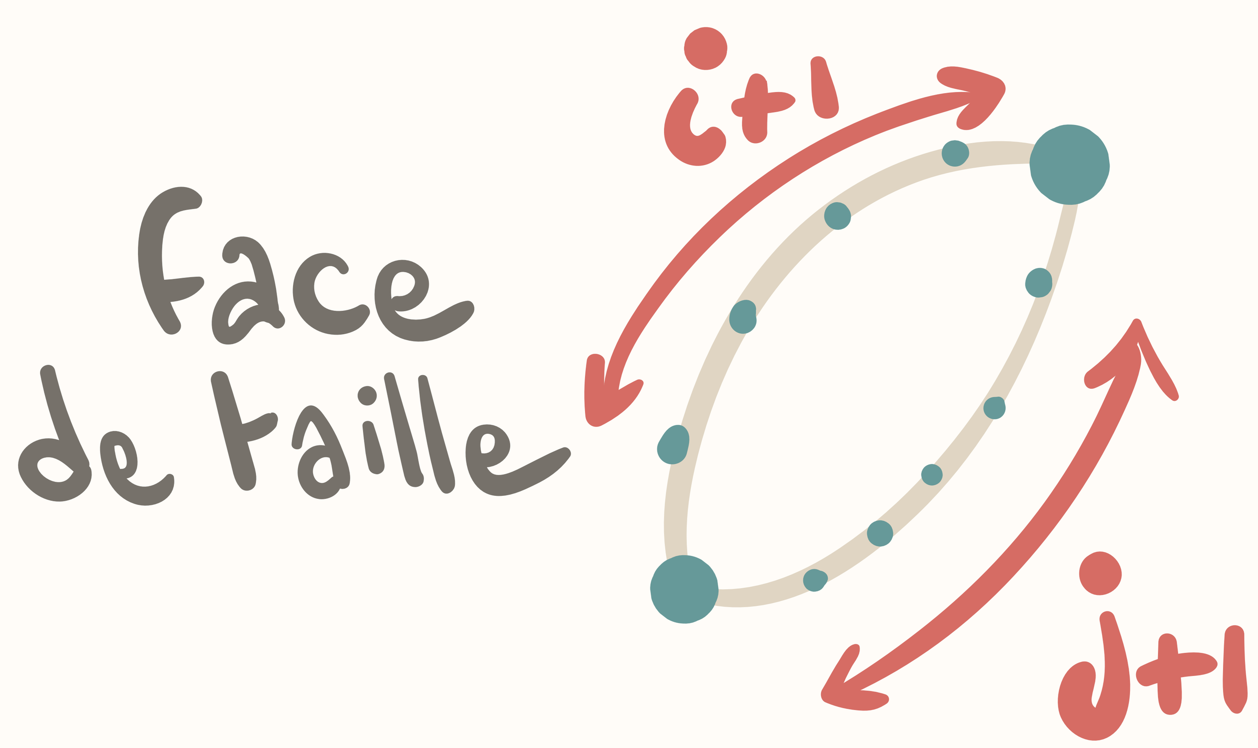
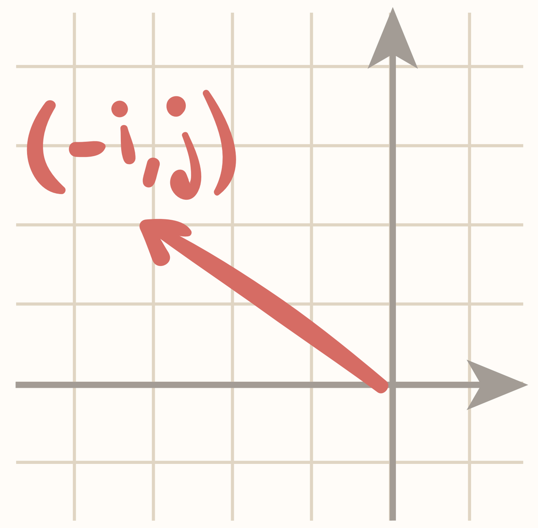


↠ *Bipolar orientations on planar maps and SLE_{12}*, R. Kenyon, J. Miller, S. Sheffield and D. Wilson (2015)



⟷



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b. Plane bipolar posets by vertices

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d. Asymtotics

2. Interlude : plane permutations

3. Application to corner polyhera

a. Via polyheral orientations

b. Via Schnyder colorings

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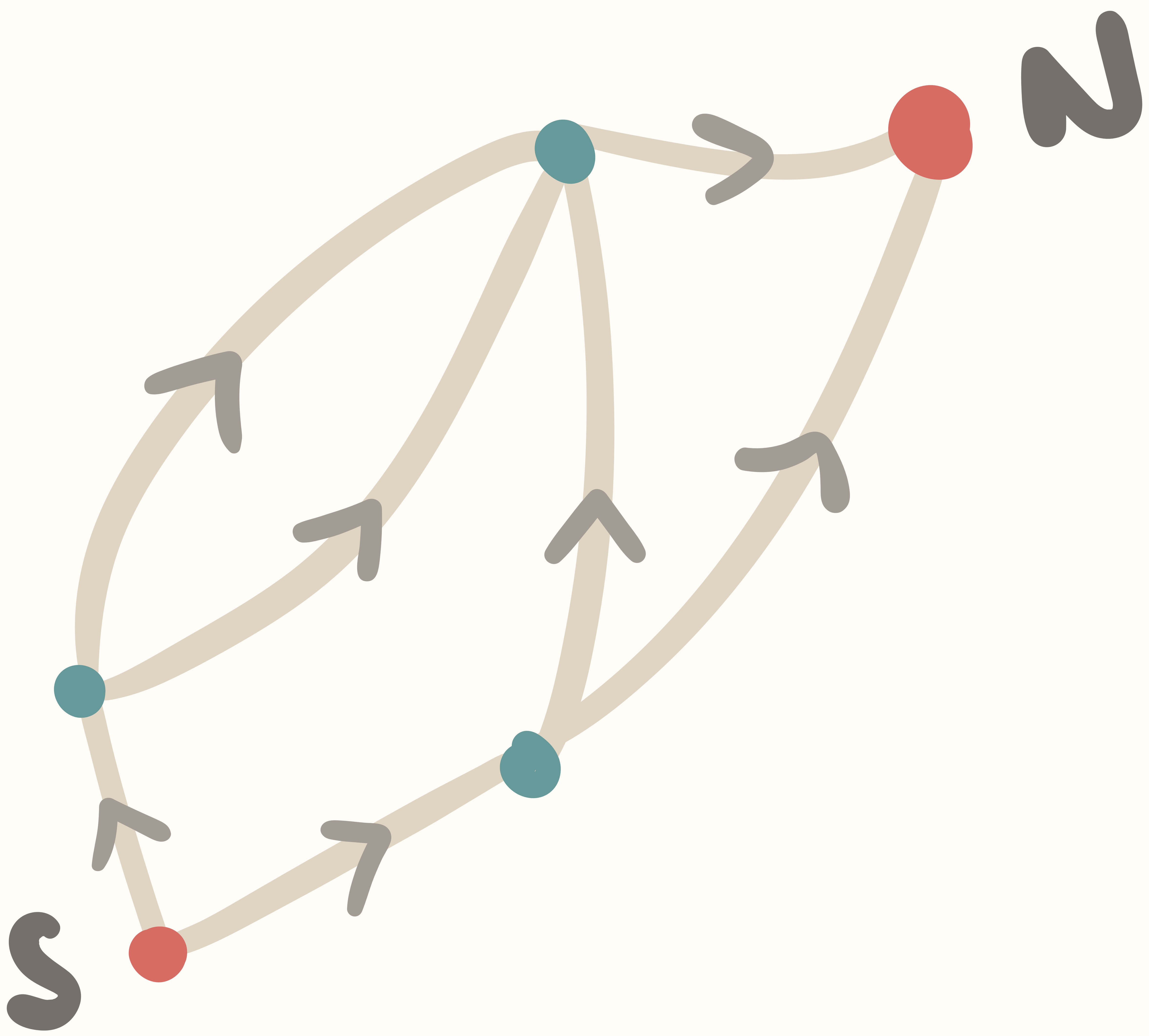
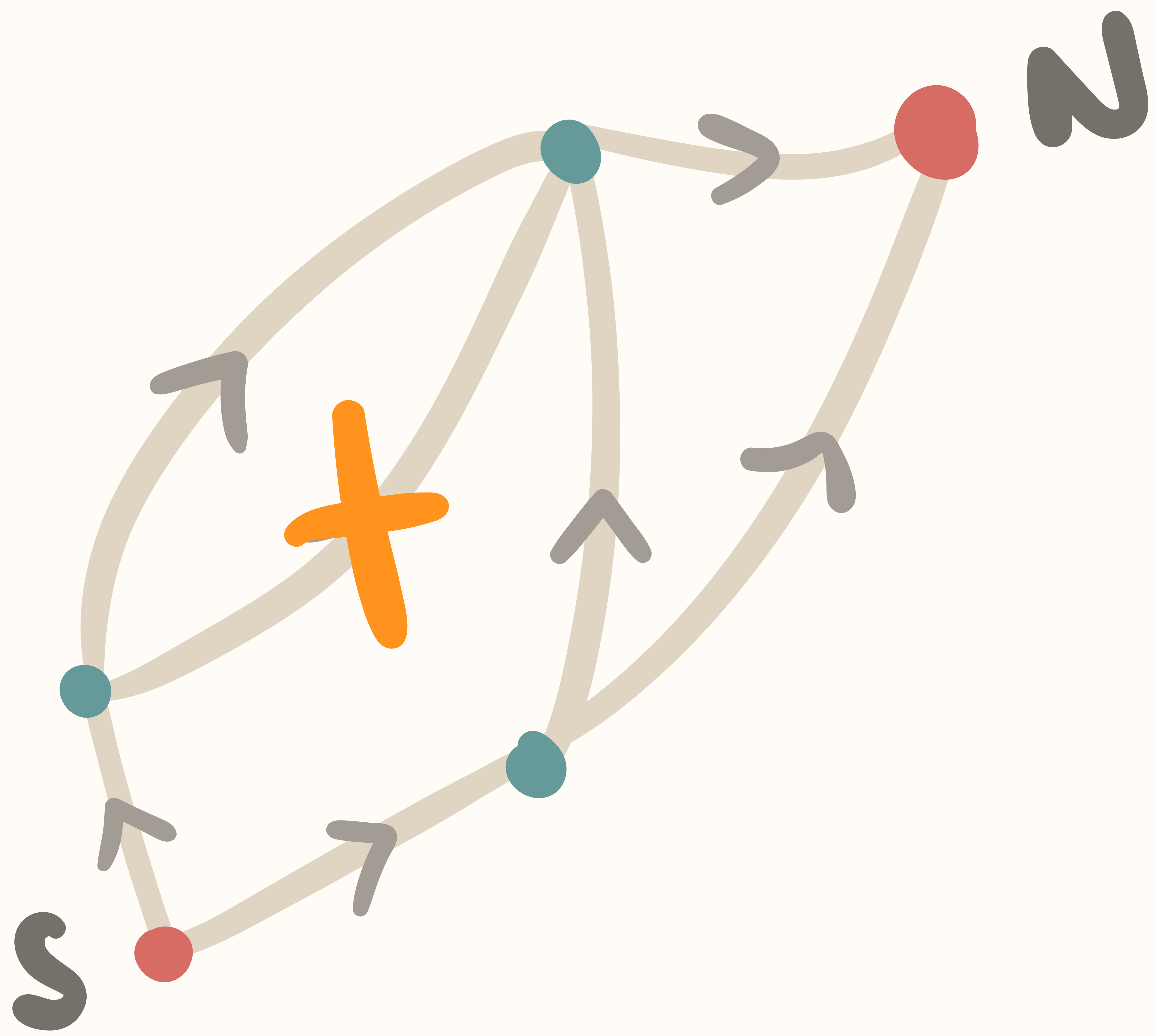
Plane bipolar poset



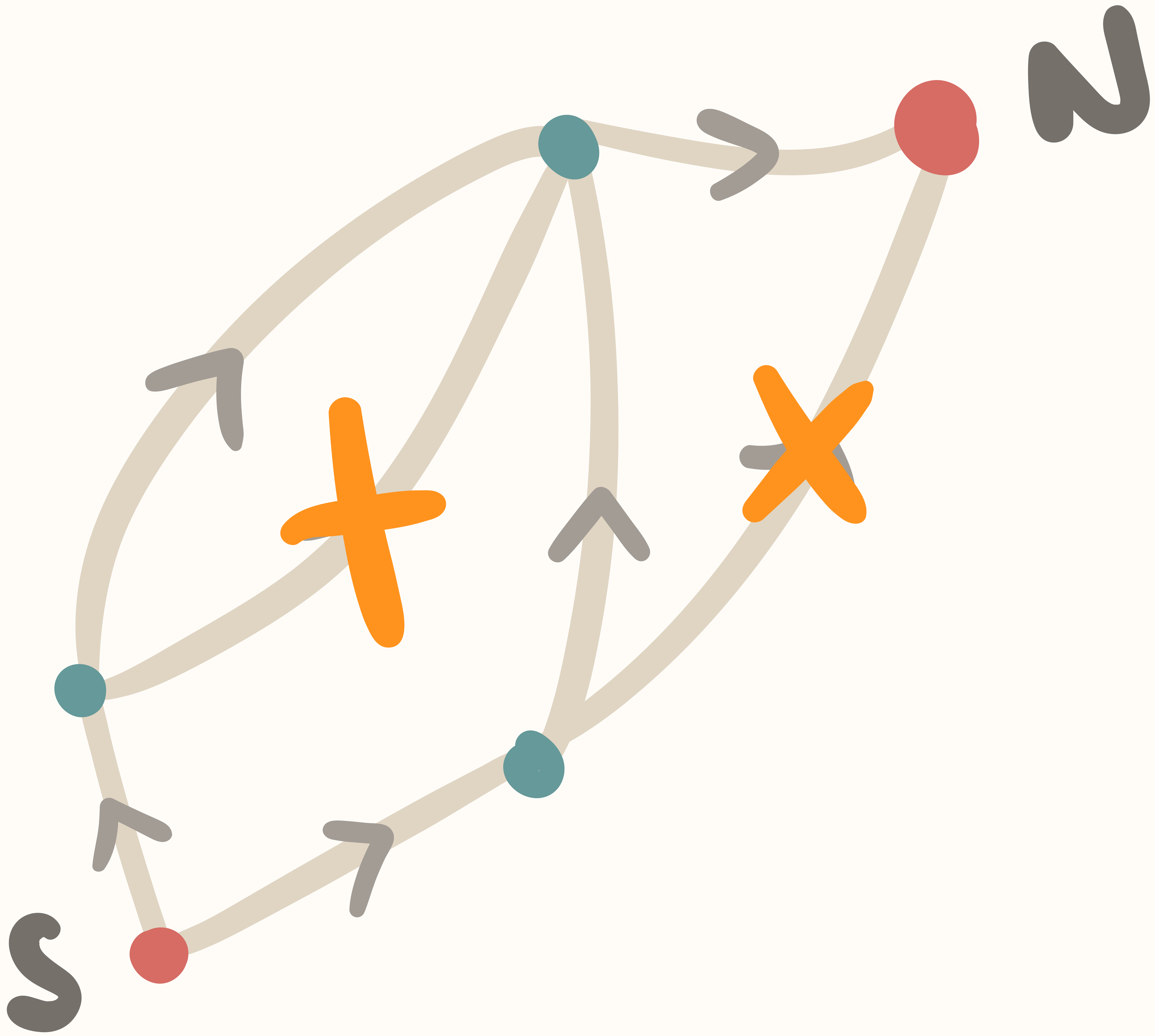
Plane bipolar poset



Poset
(plane bipolar poset)

= ***Bipolar orientation***
No multiple edge

Plane bipolar poset



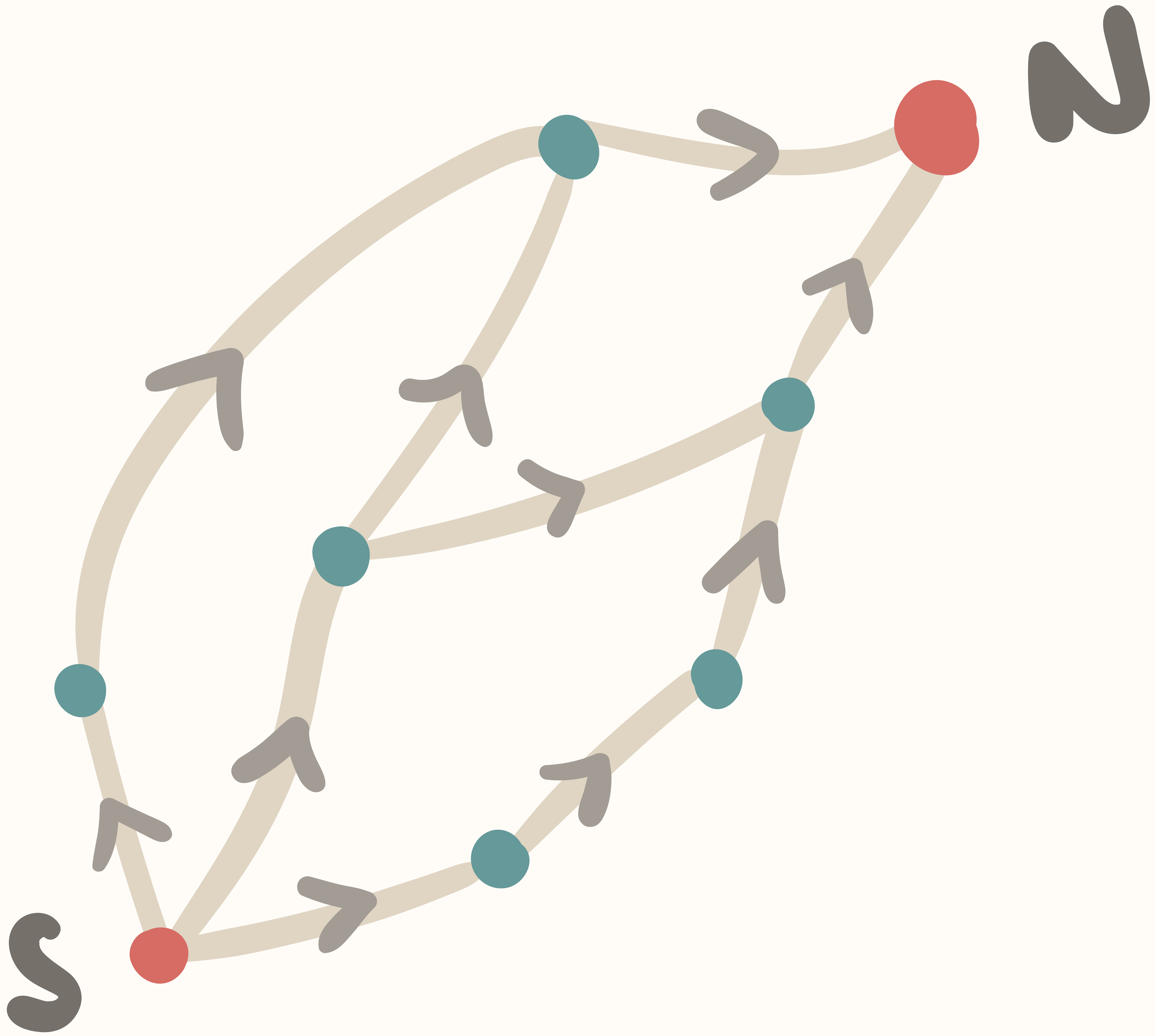
Poset
(plane bipolar poset)

= **Bipolar orientation**

No multiple edge

No transitive edge

Plane bipolar poset



Poset
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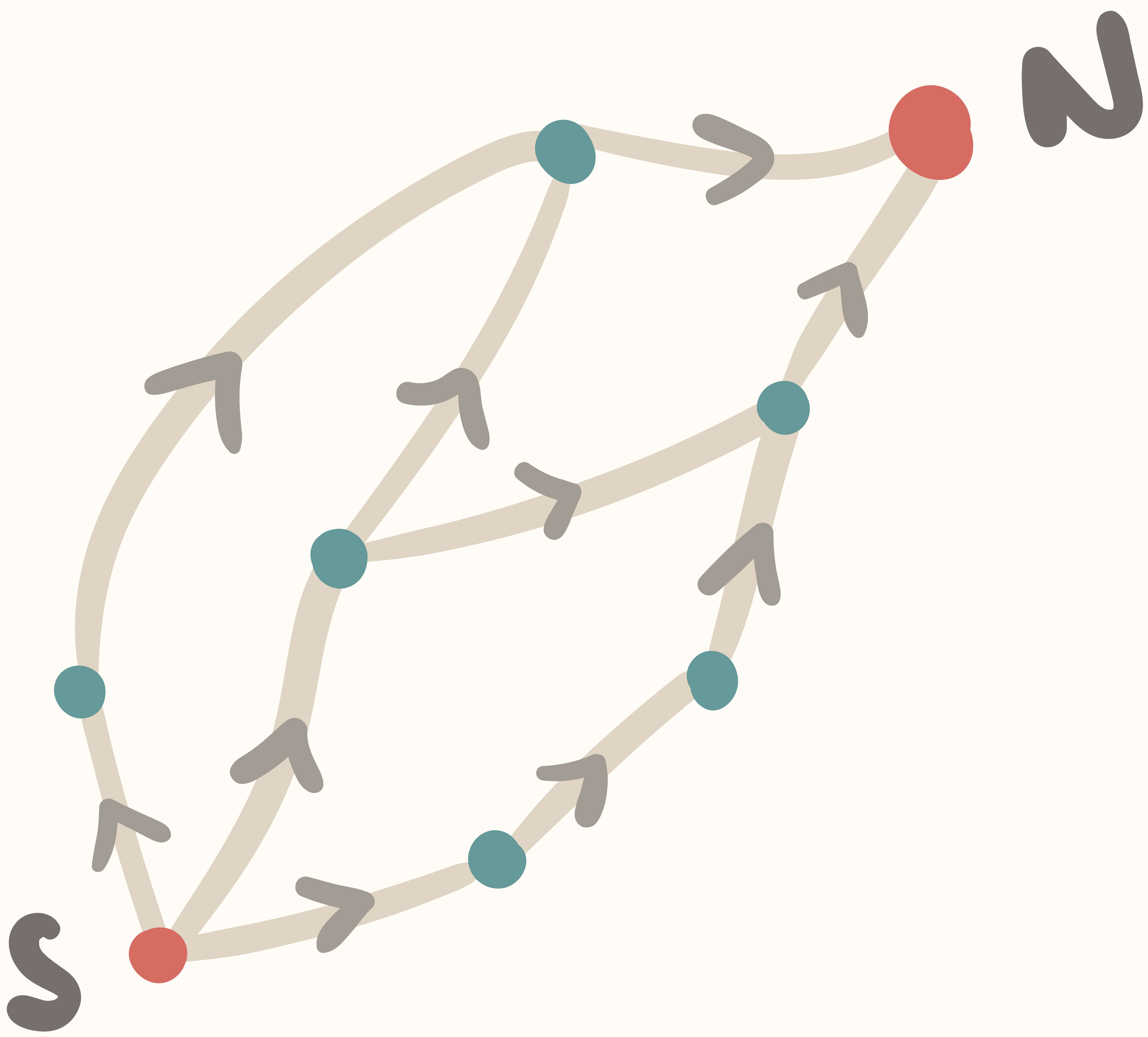
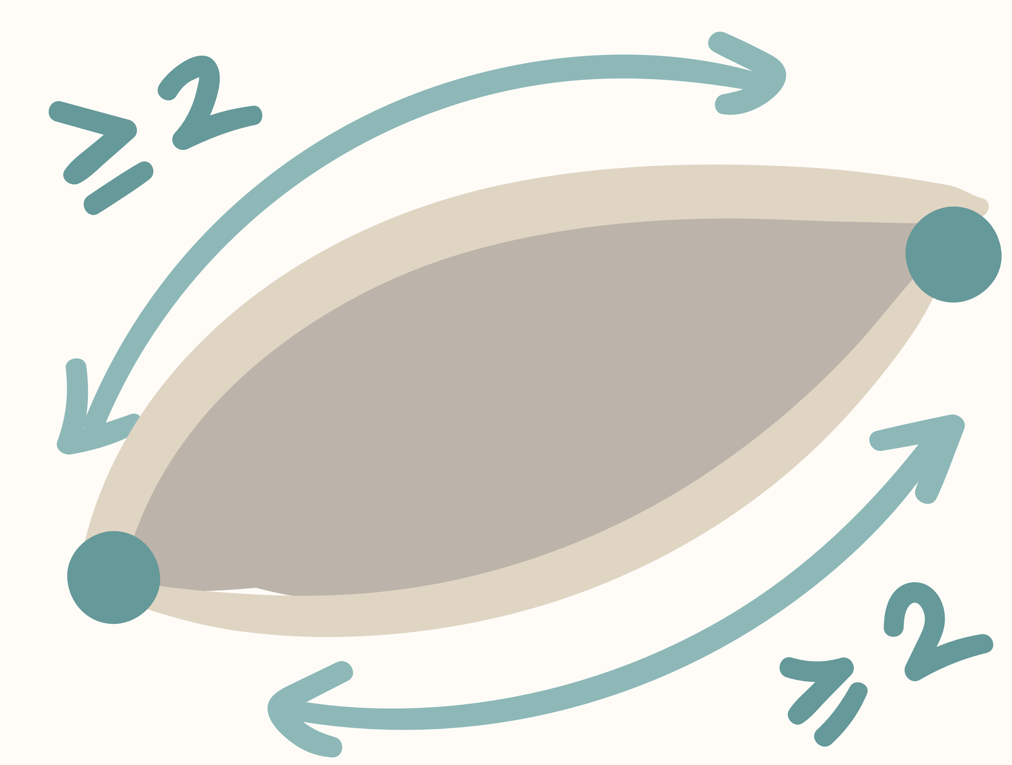
Plane bipolar poset



Poset
(plane bipolar poset)

= ***Bipolar orientation***
No multiple edge
No transitive edge

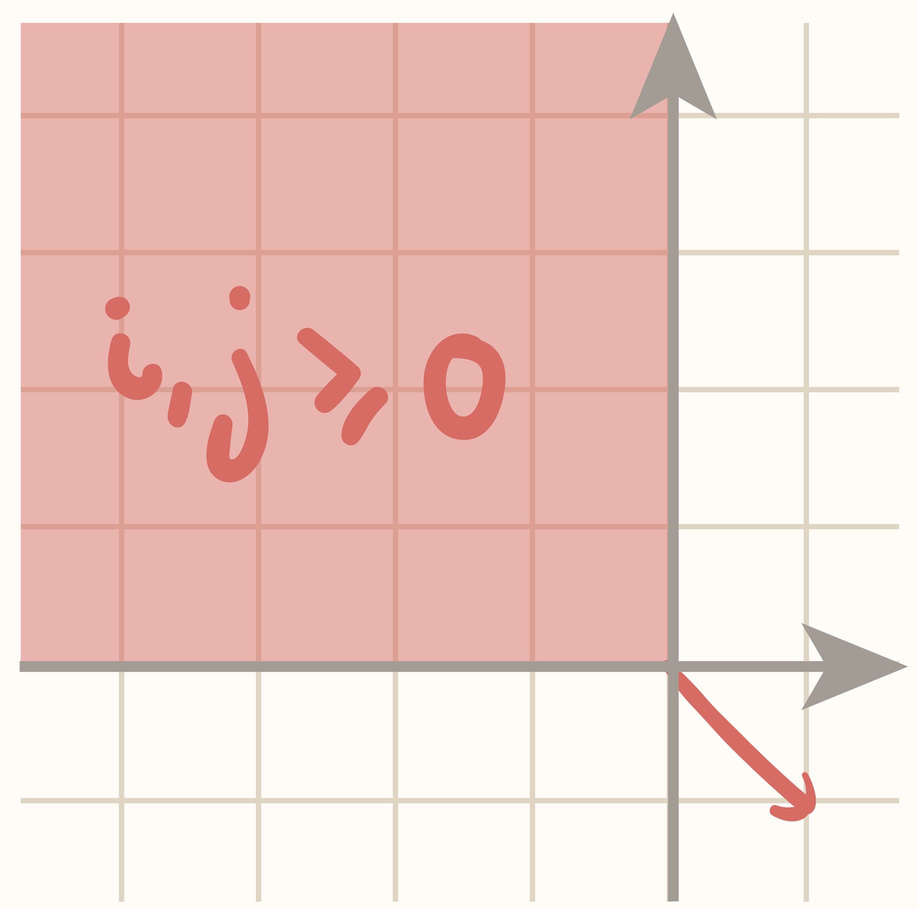
= ***Bipolar orientation***



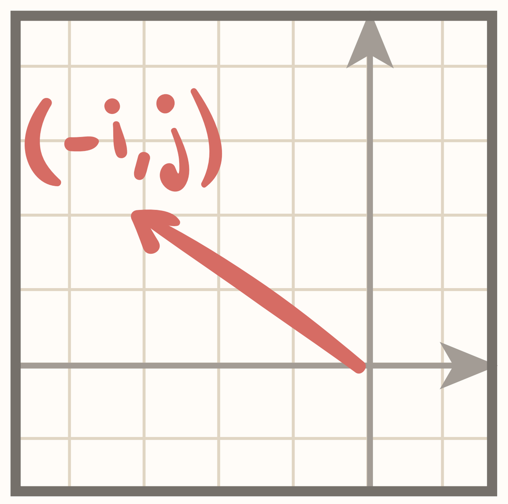
Specialization to Posets

Specialization to Posets

Bipolar orientation =



where



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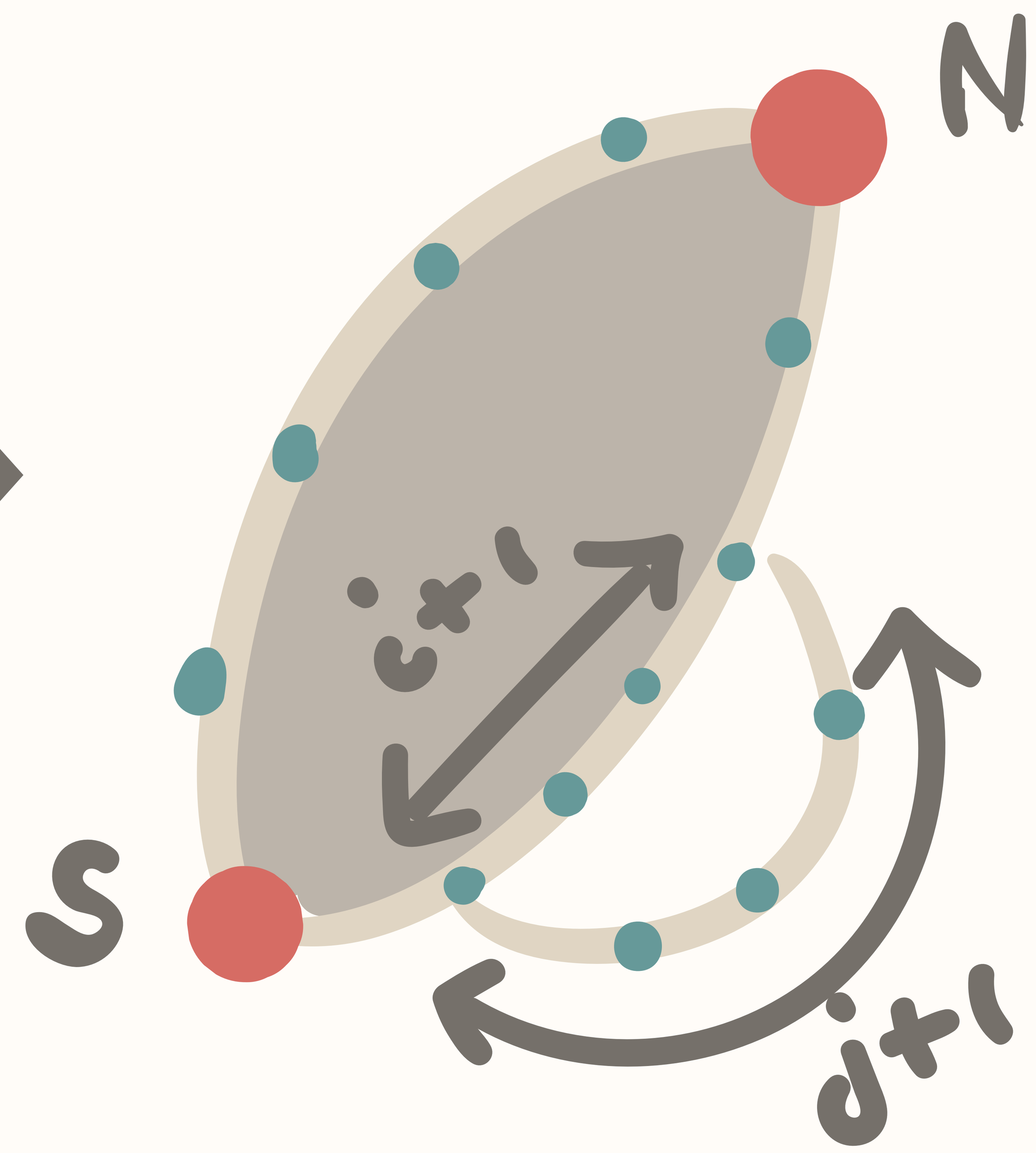
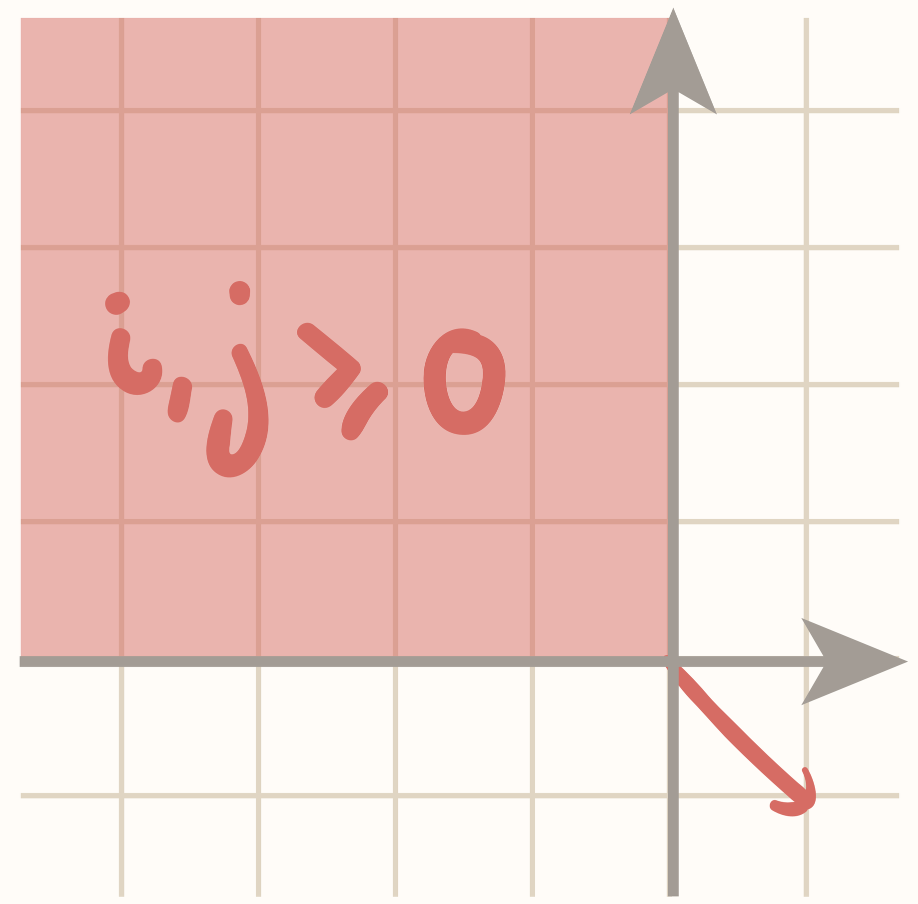
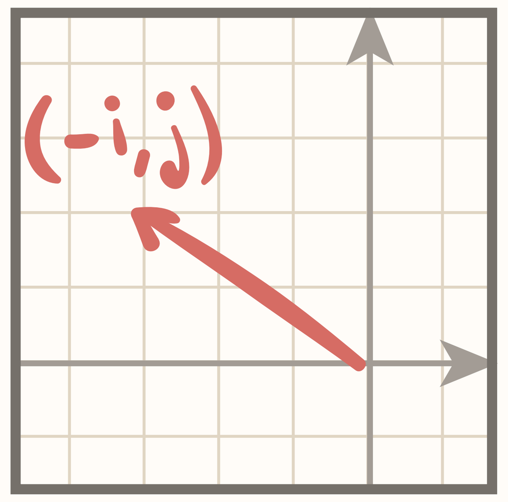


Specialization to Posets

Bipolar orientation =



where



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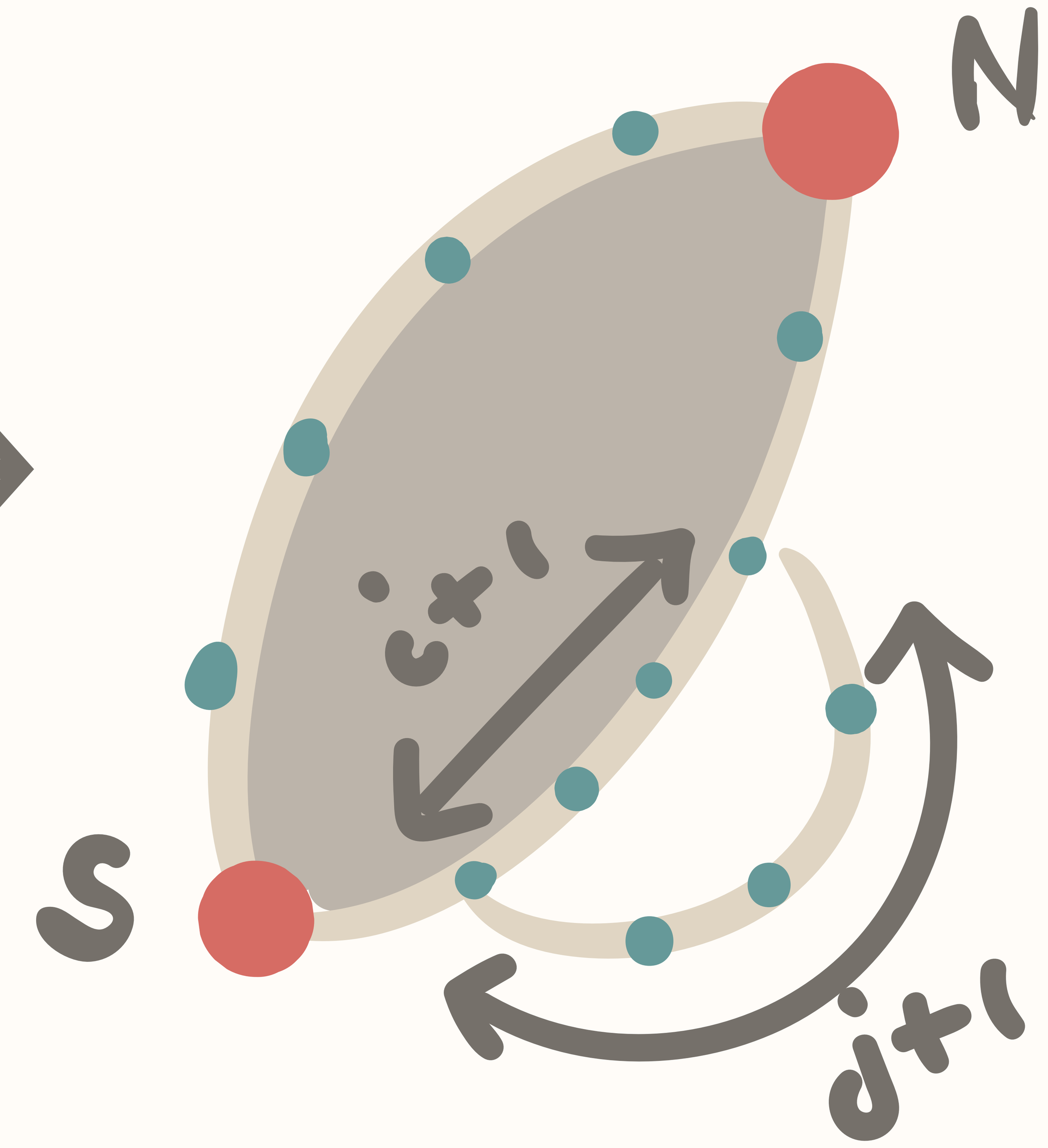
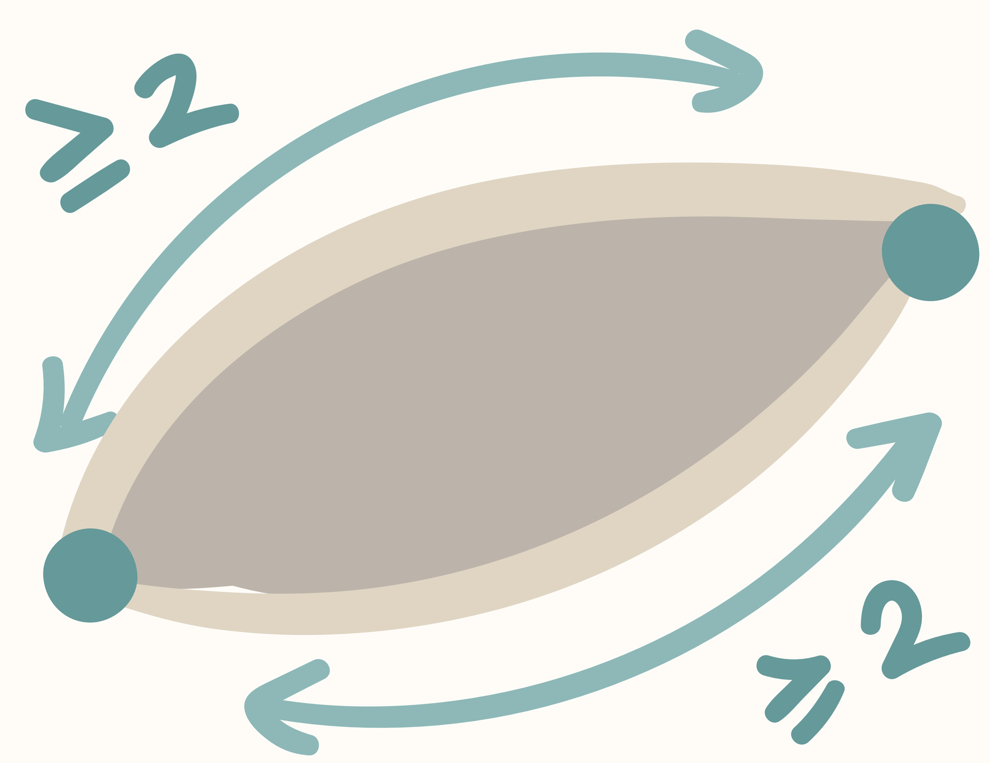


Poset =



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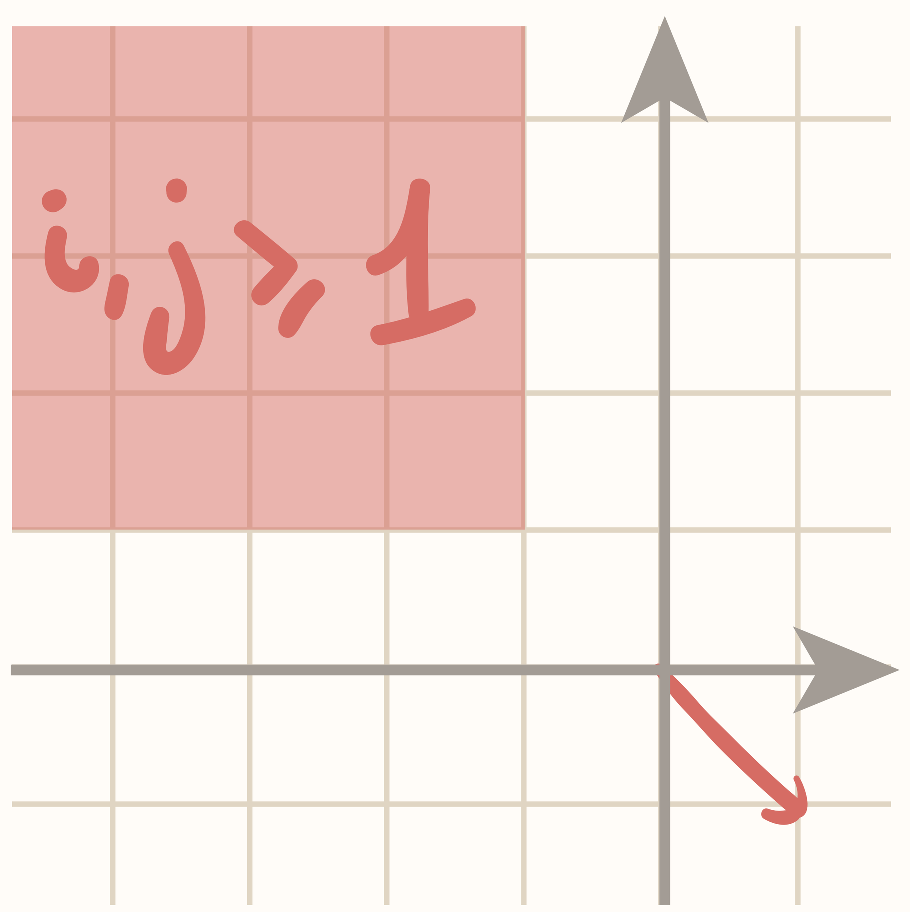

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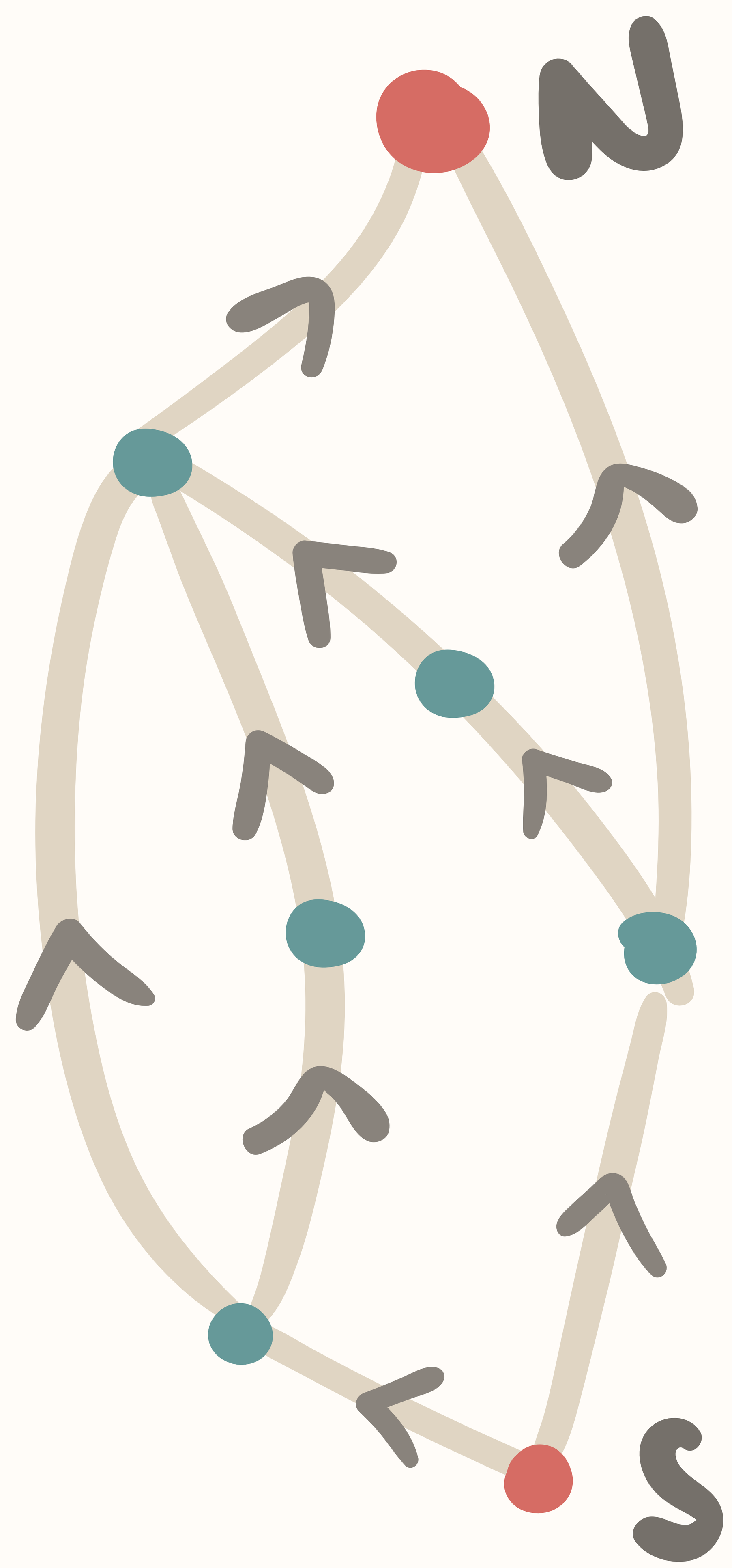
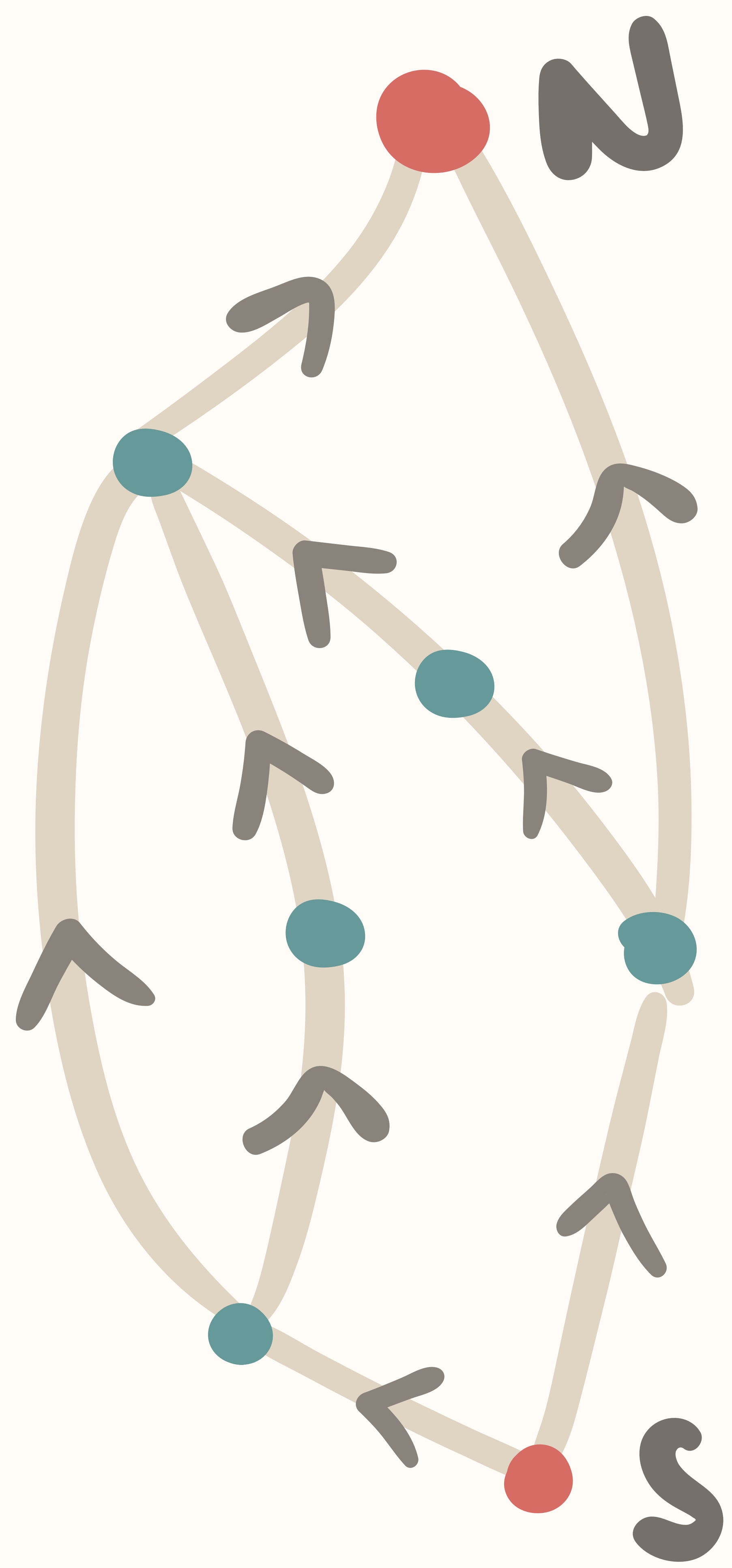
Specialization to Posets by vertices

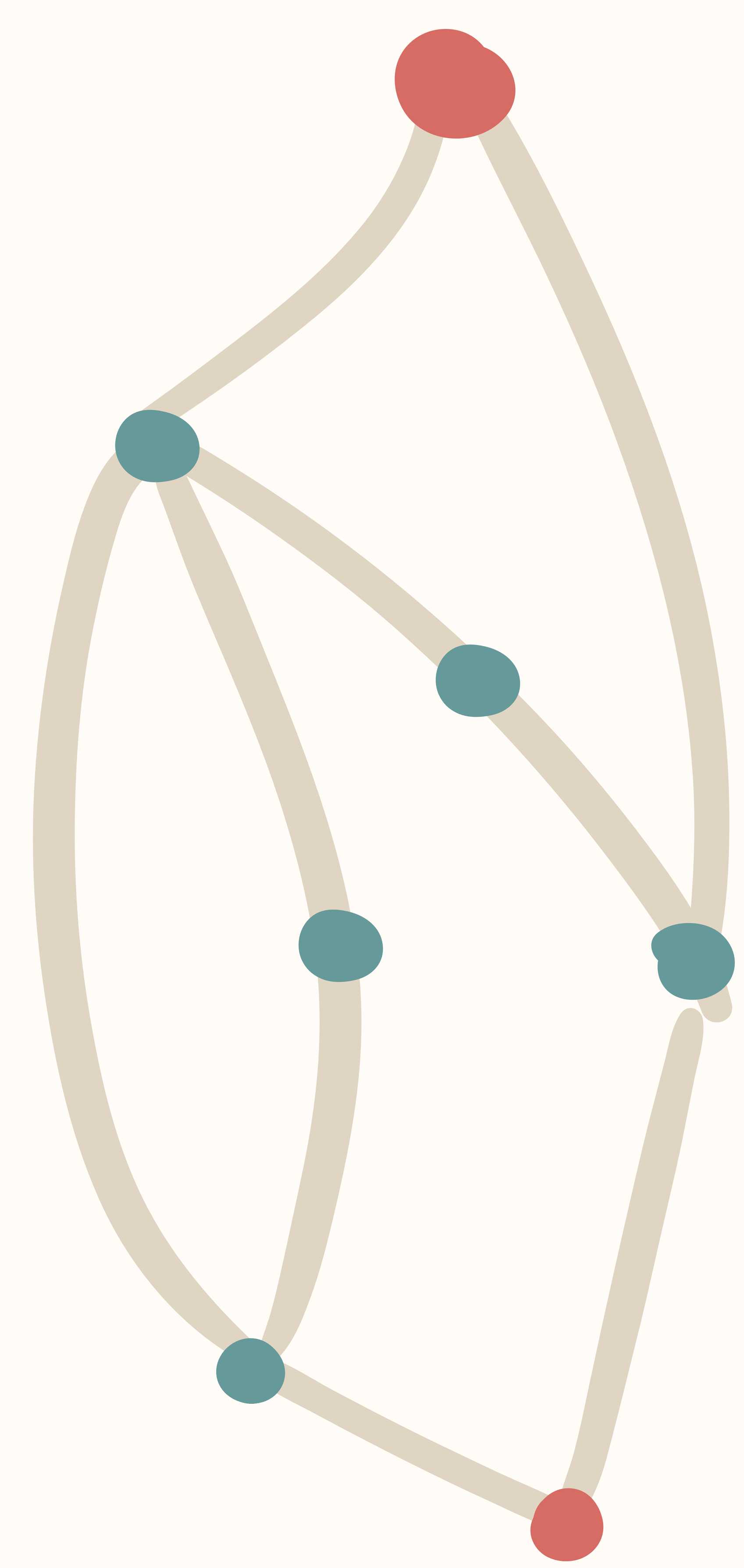
Bipolar orientation



Specialization to Posets by vertices

Bipolar orientation

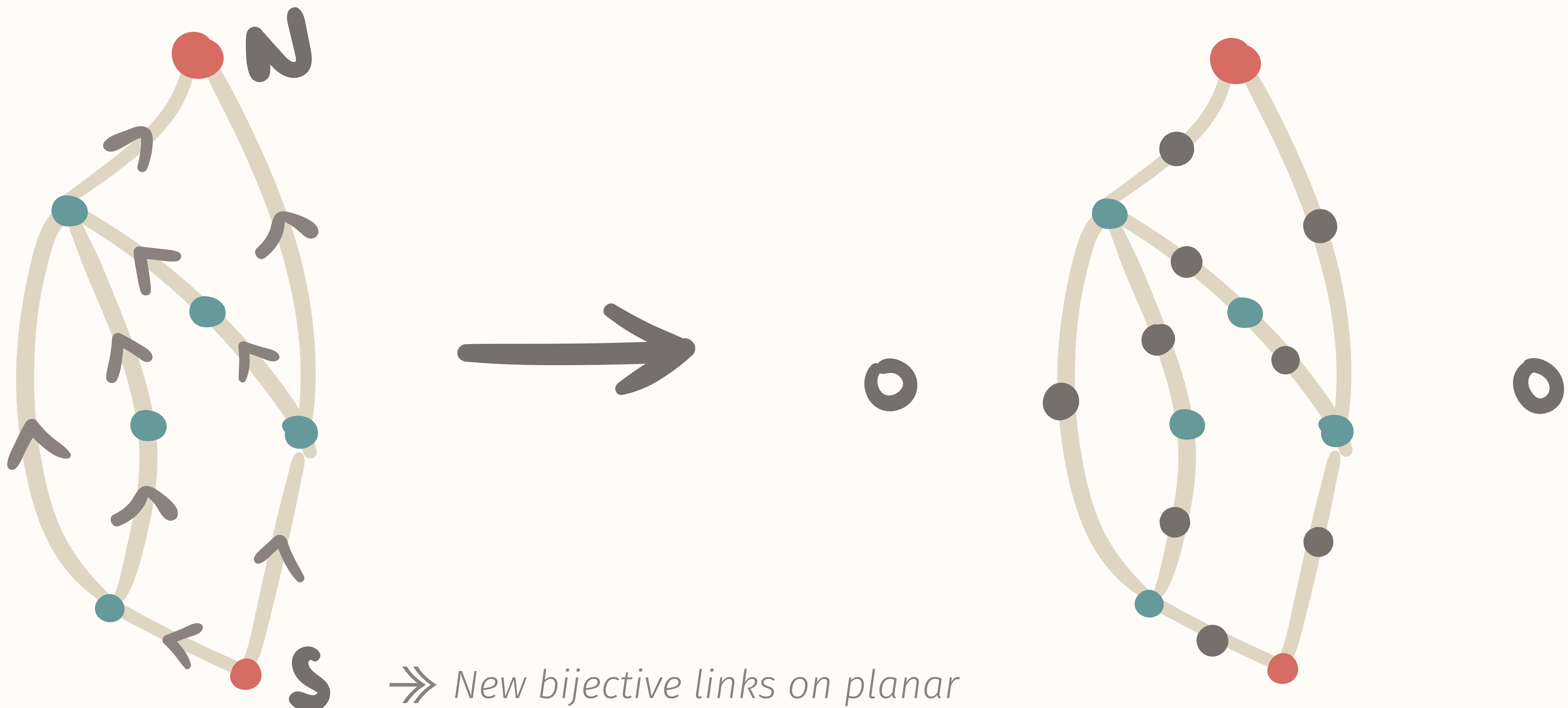




⇛ *New bijective links on planar maps via orientation,* E. Fusy (2010)

Specialization to Posets by vertices

Bipolar orientation



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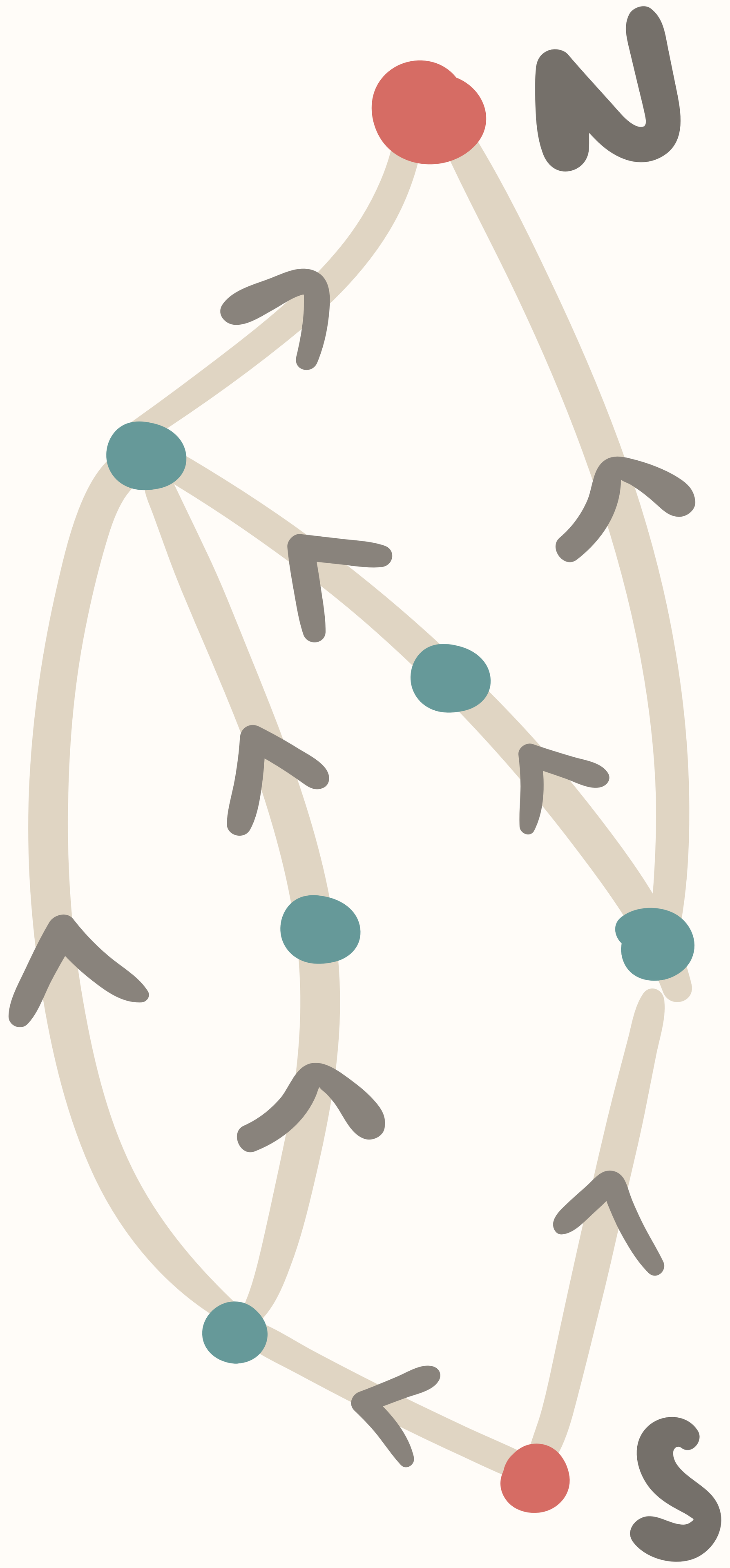
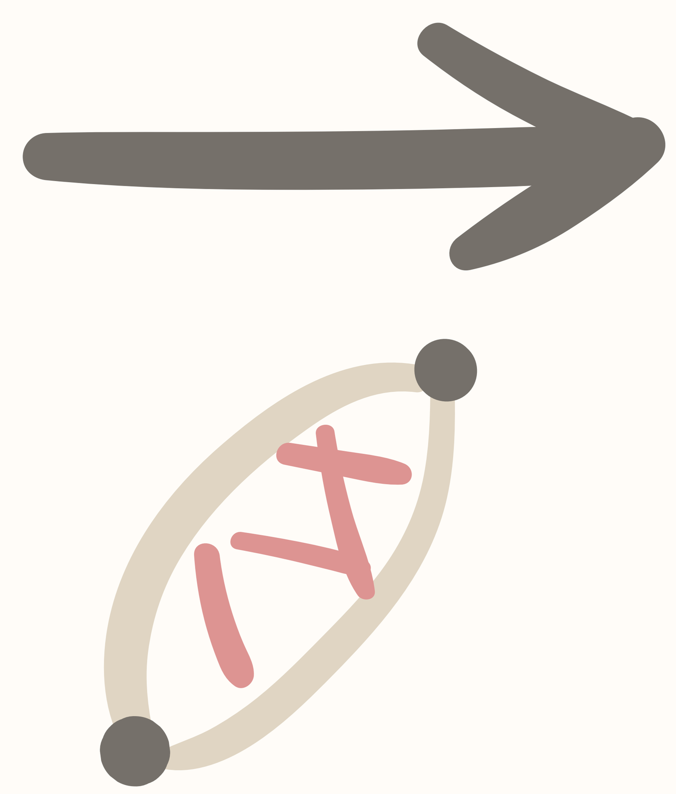
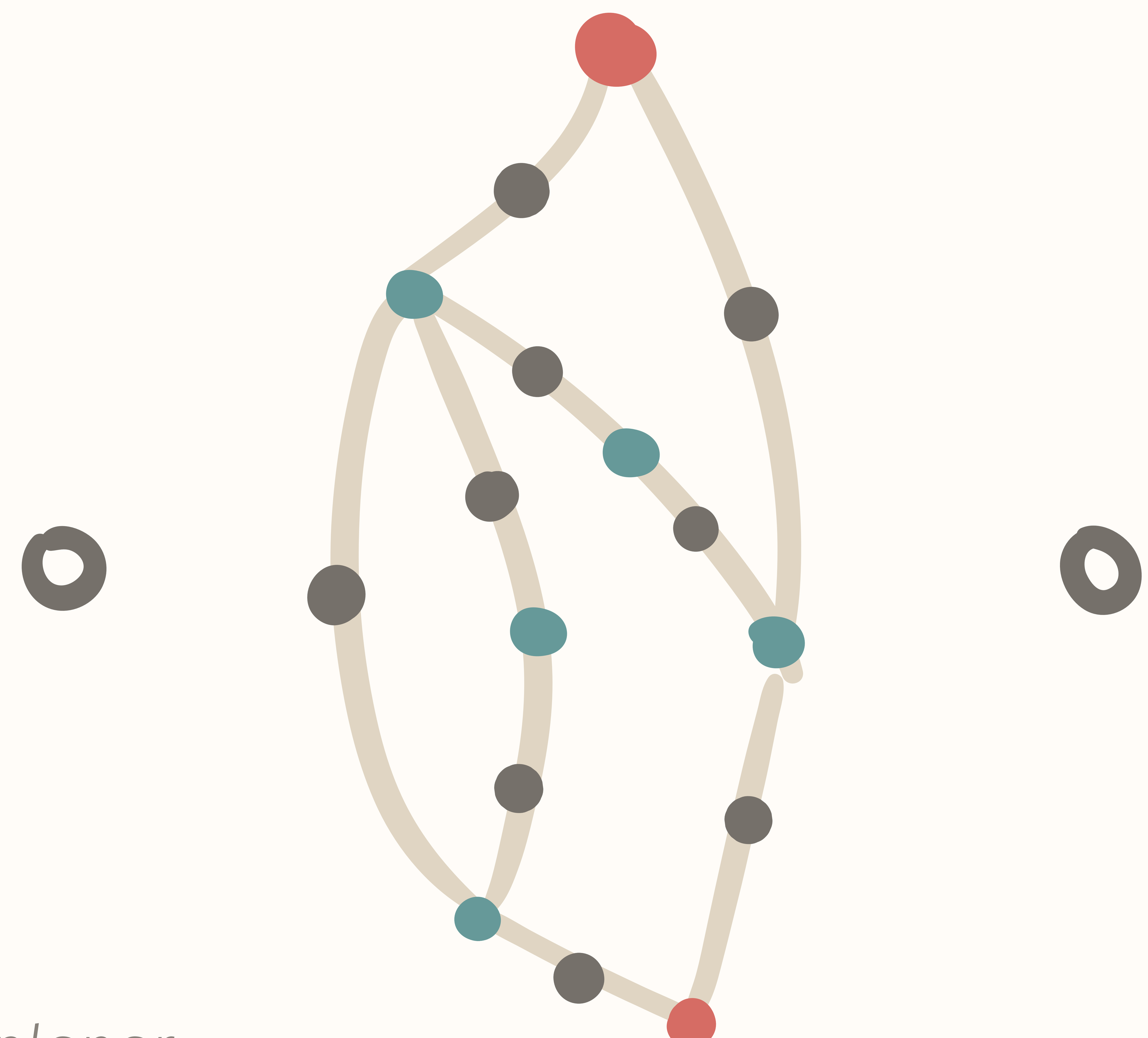
Bipolar orientation







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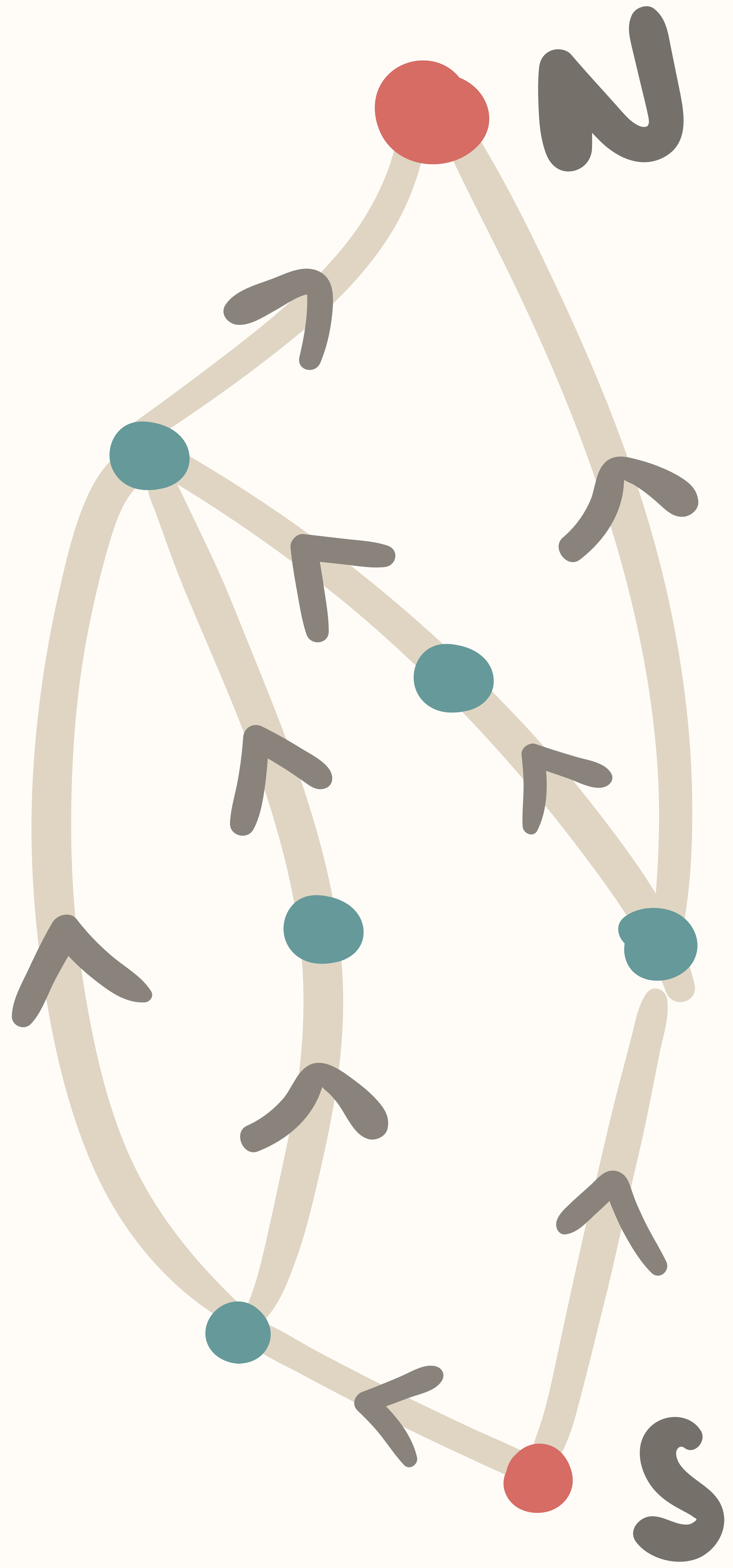
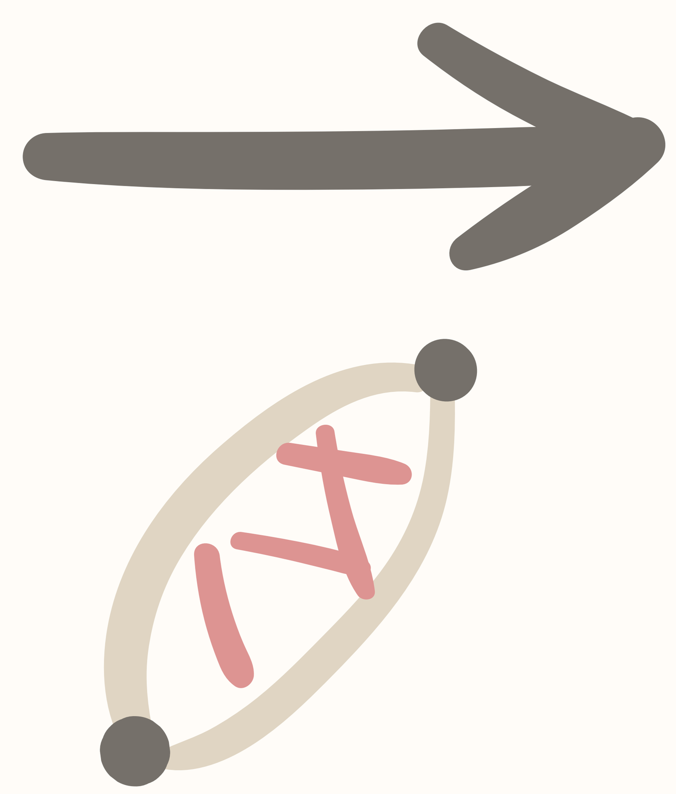
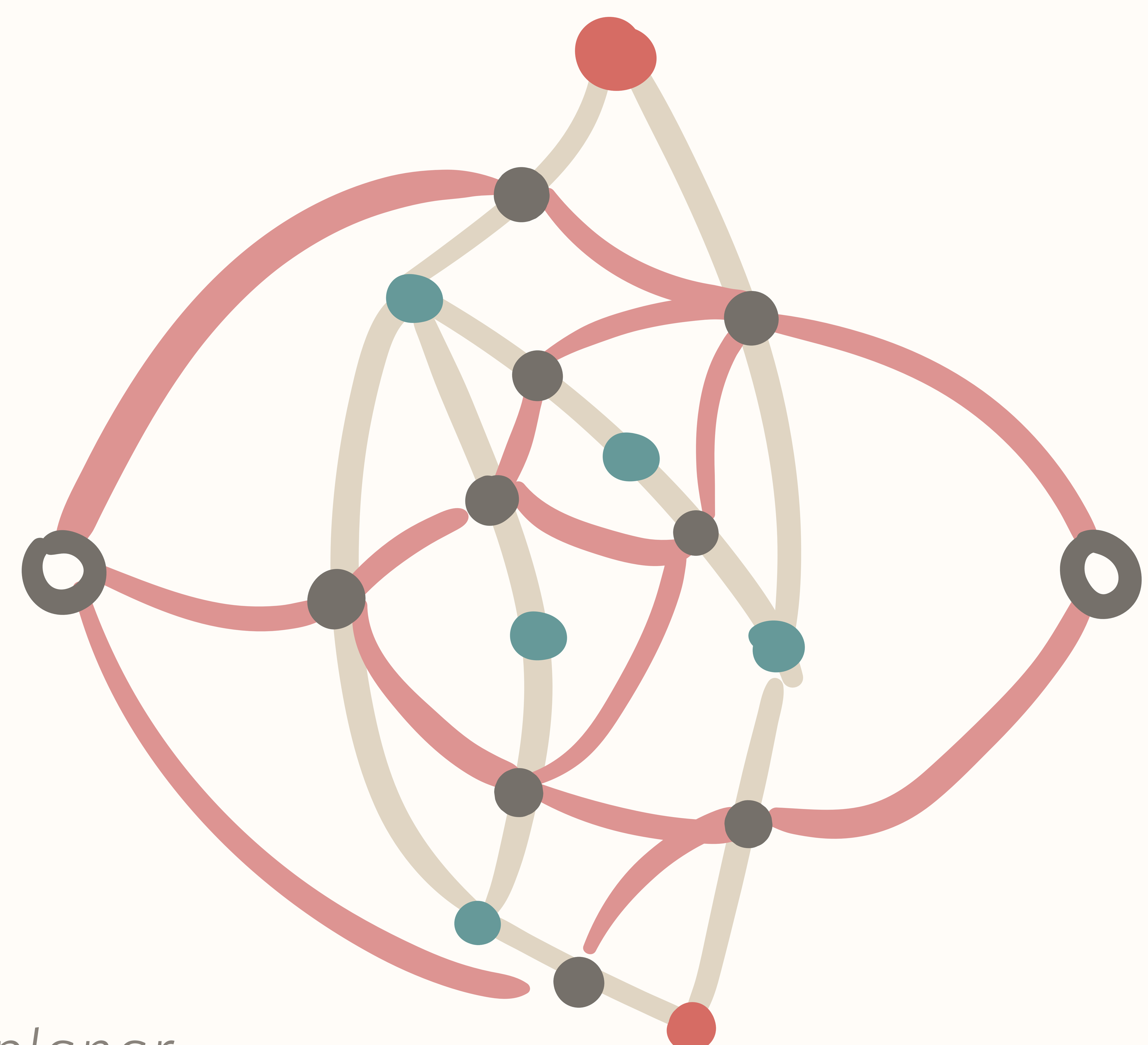
Bipolar orientation







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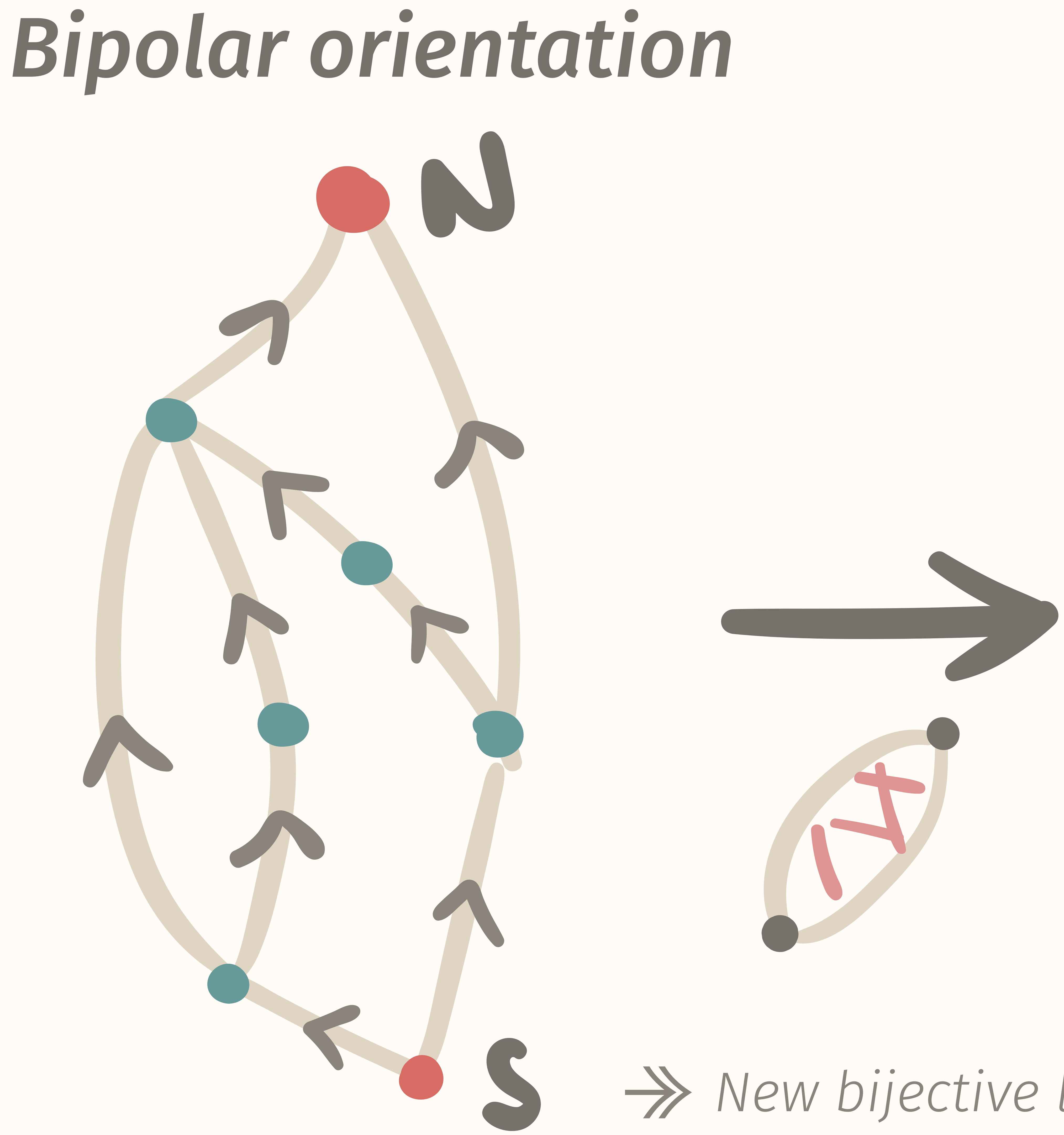

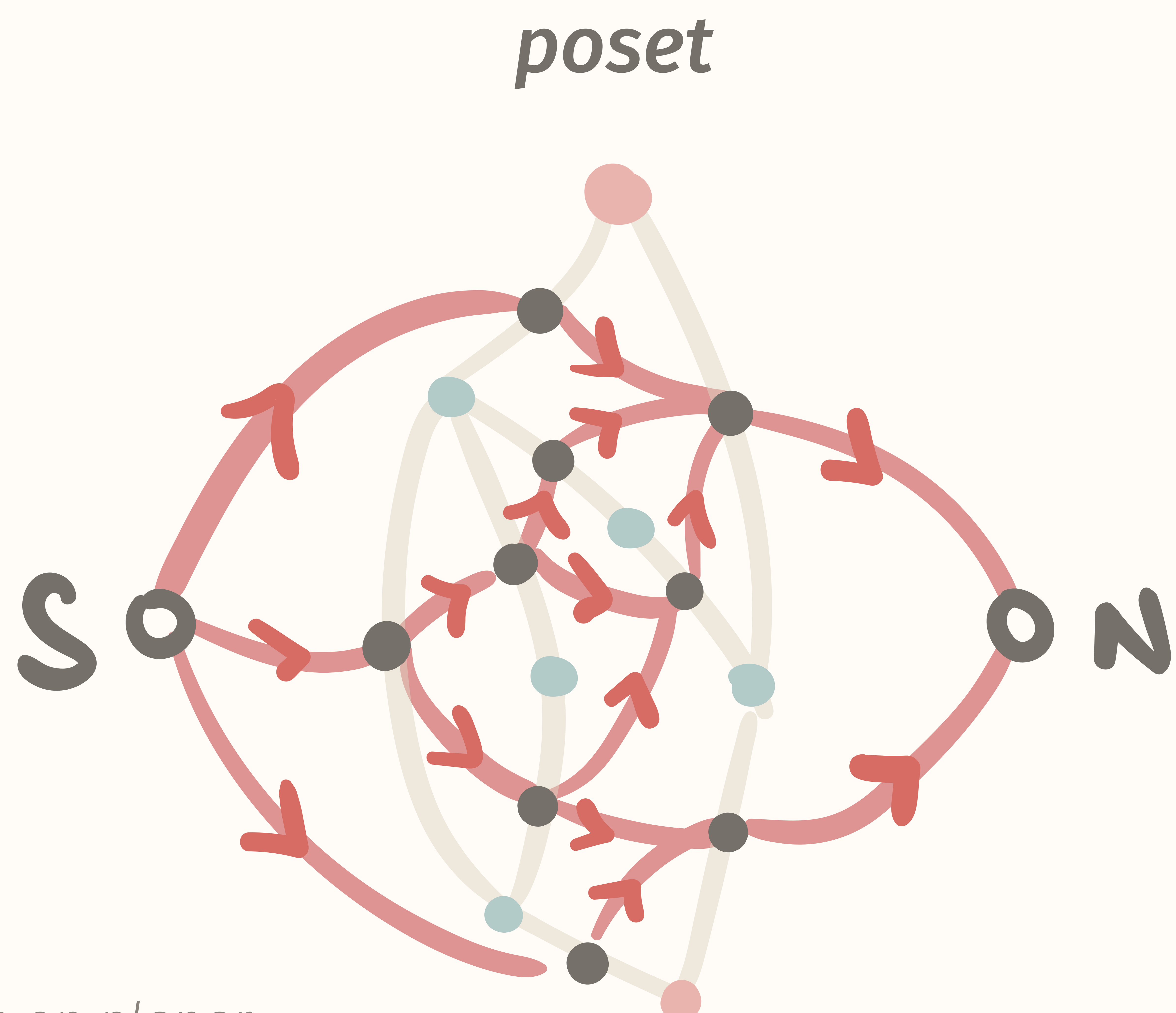

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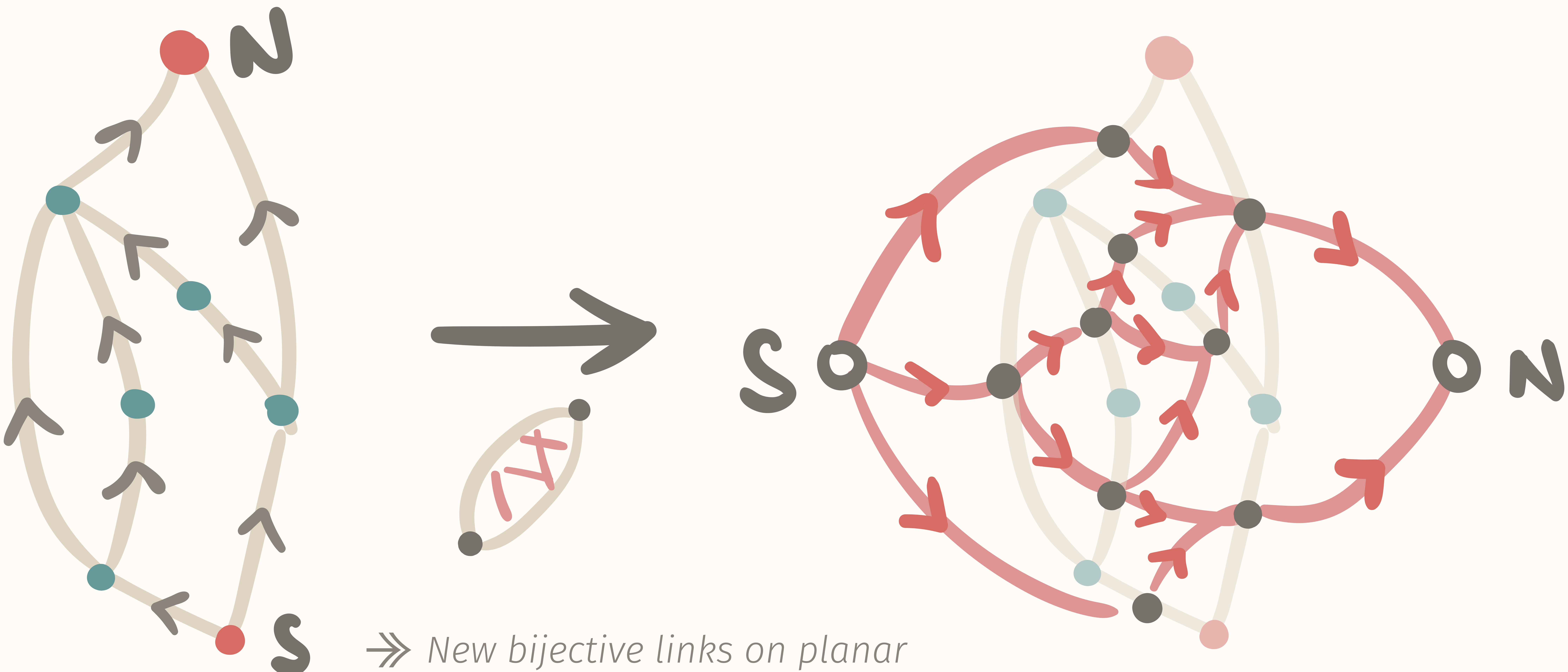
Specialization to Posets by vertices

Bipolar orientation *poset*



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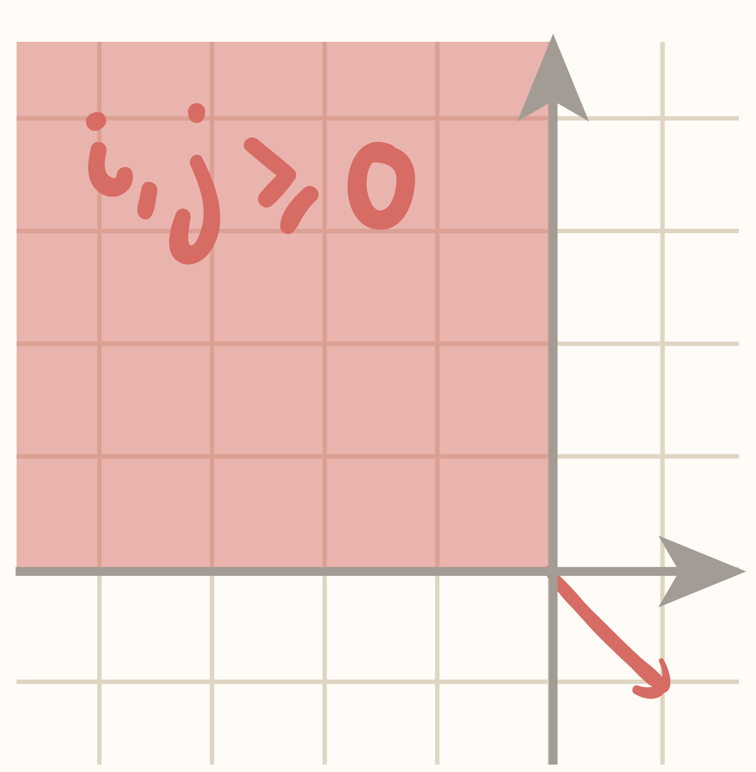
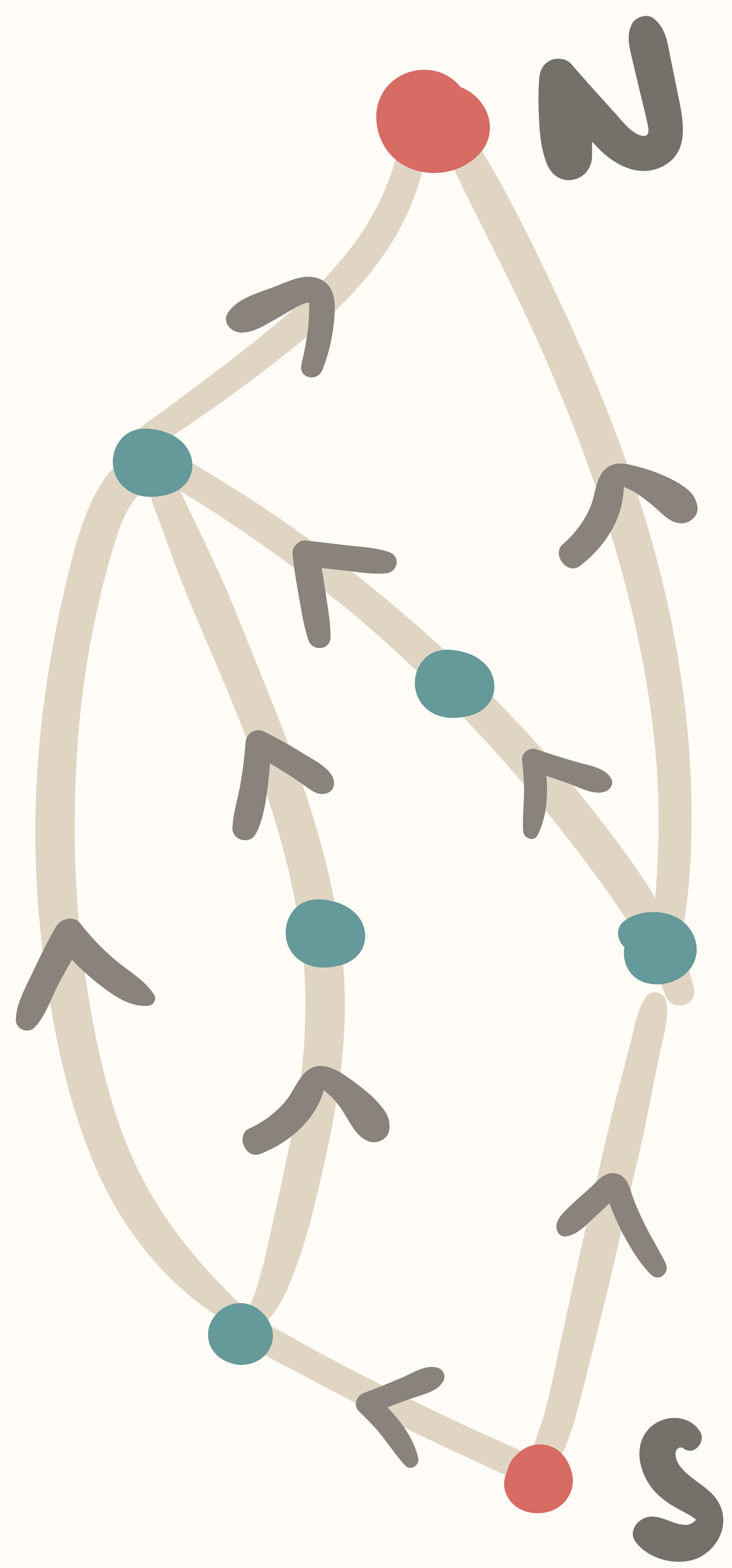
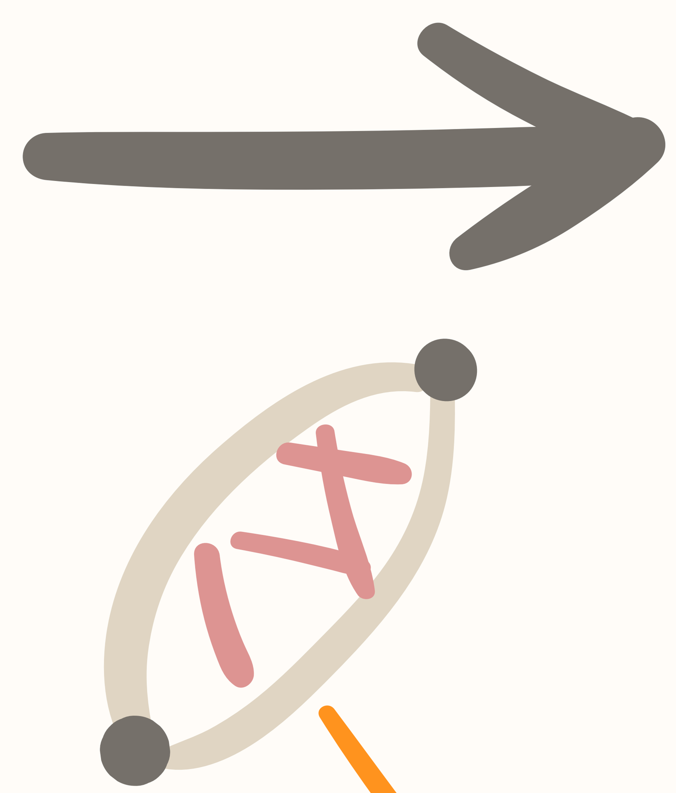


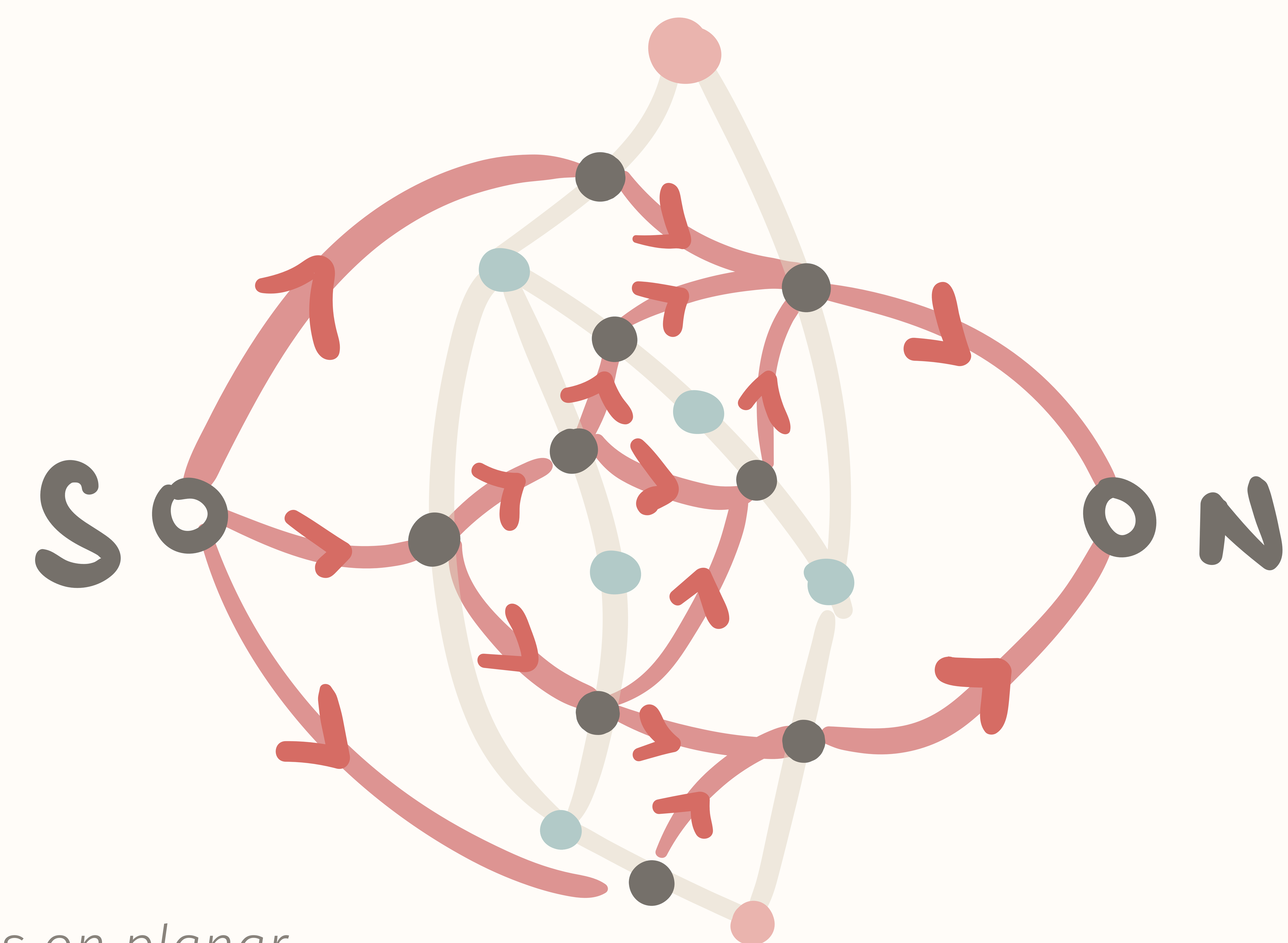
Specialization to Posets by vertices

Bipolar orientation





poset



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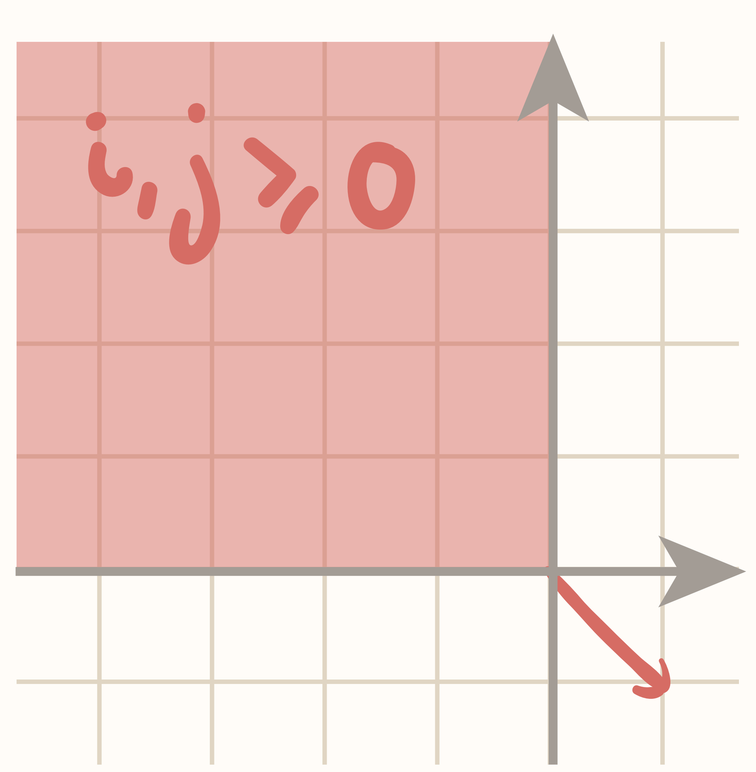
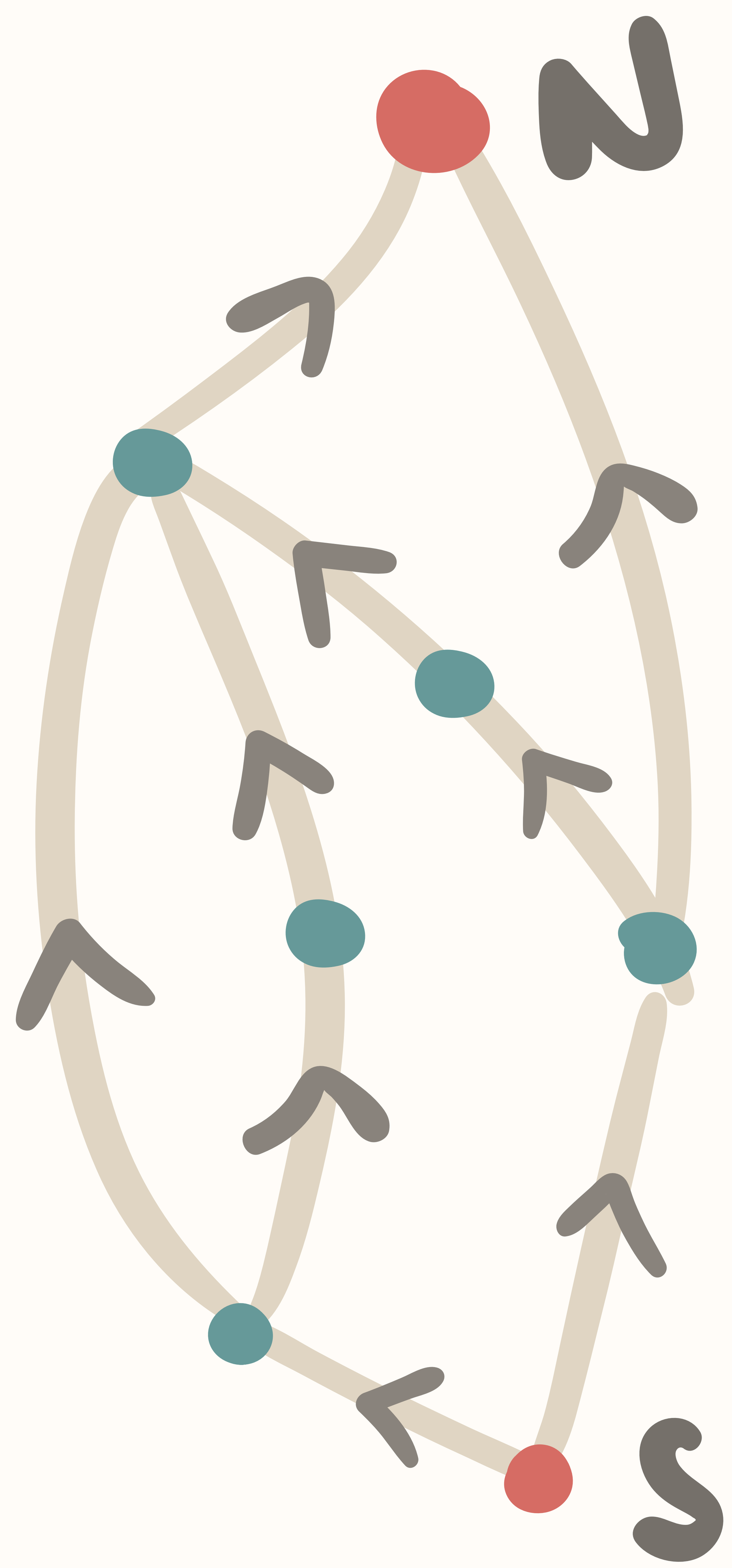
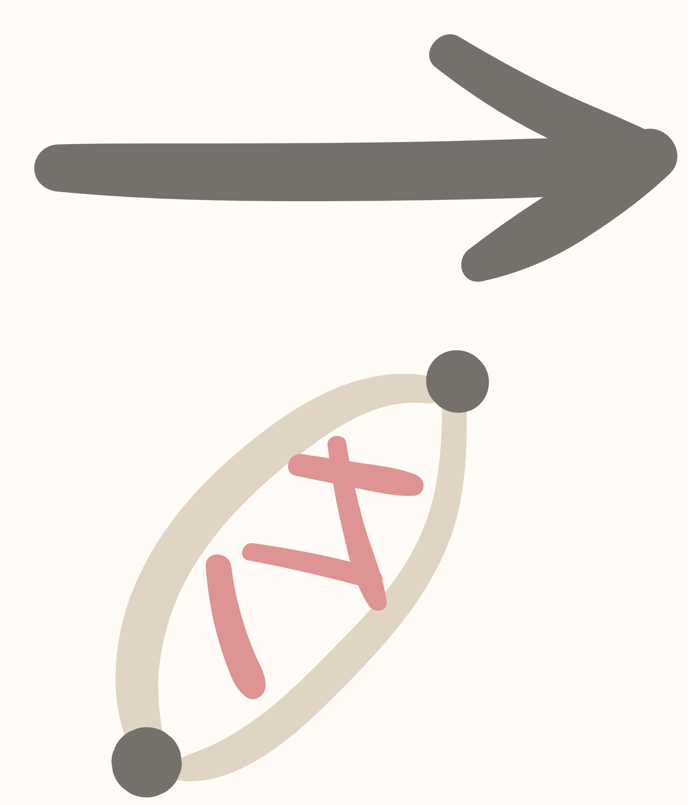
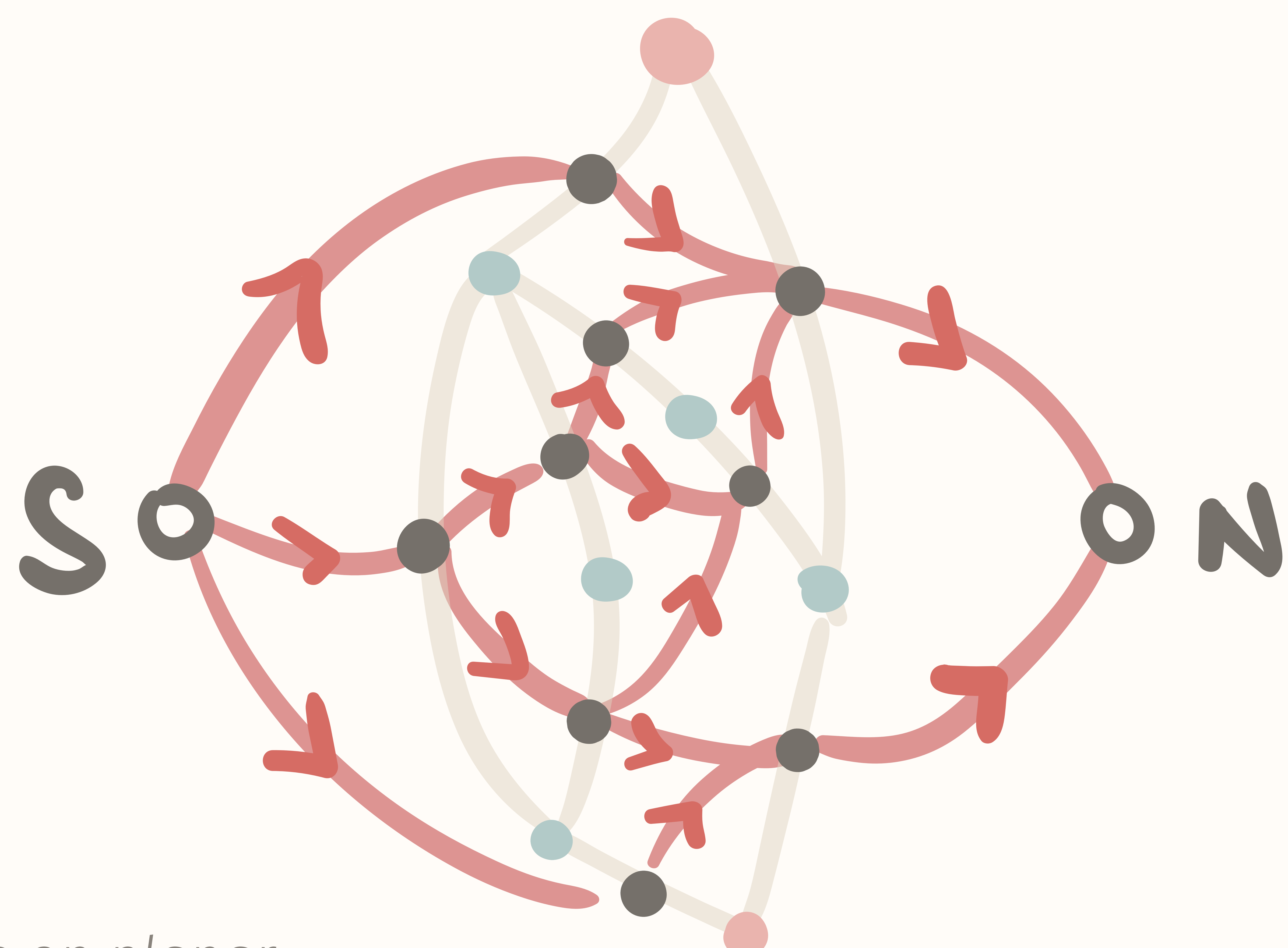


Specialization to Posets by vertices

Bipolar orientation *poset*







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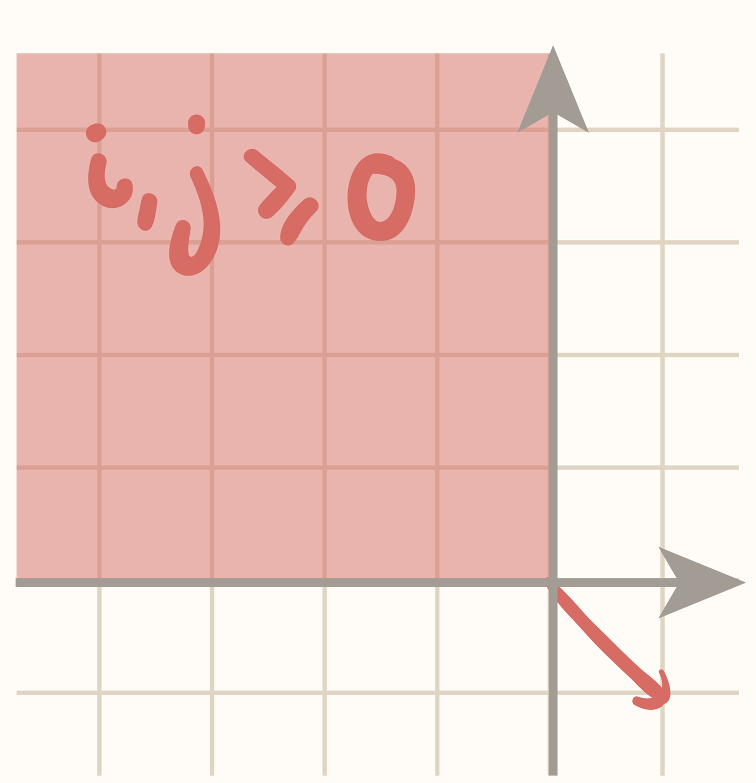

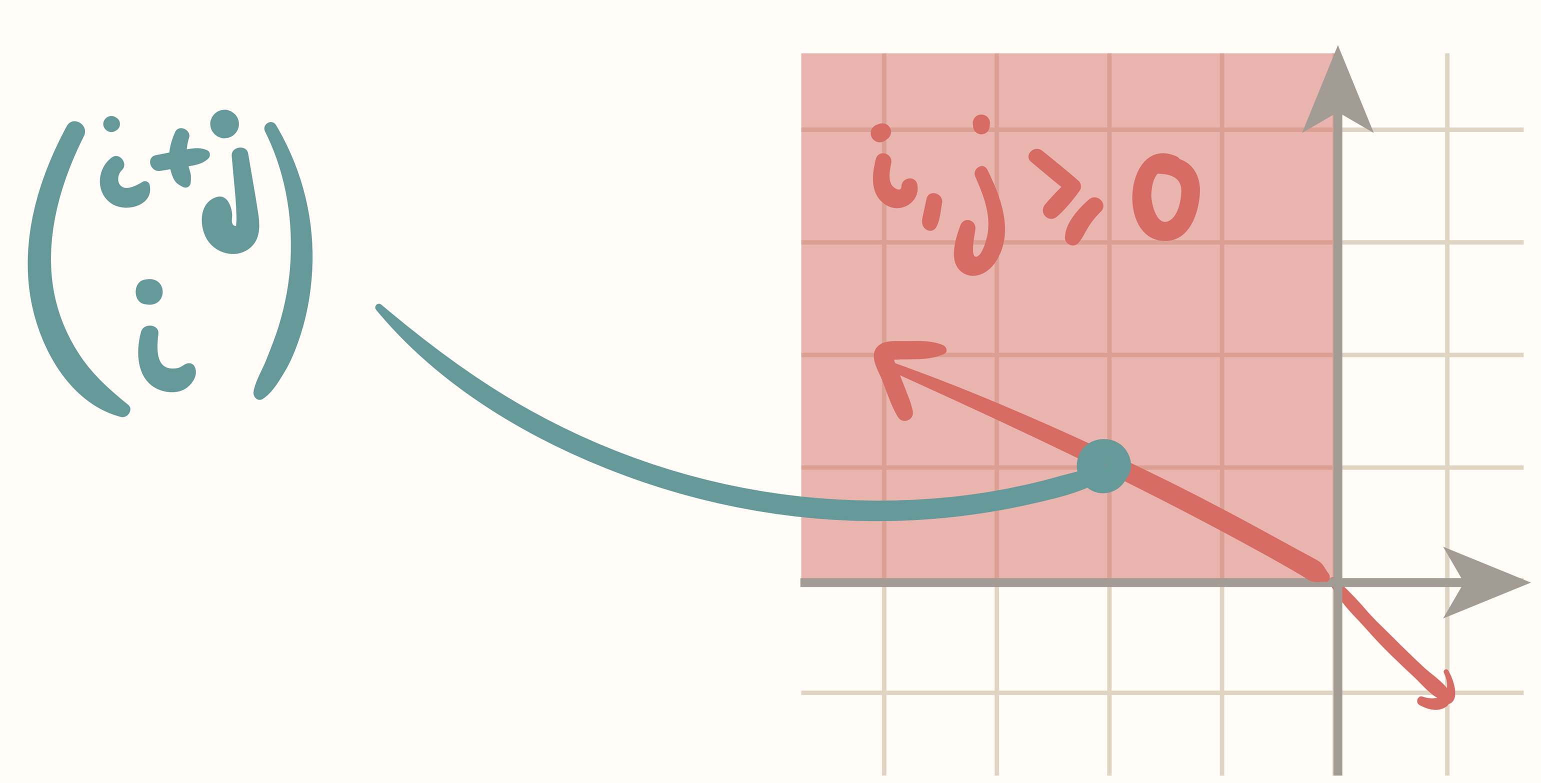

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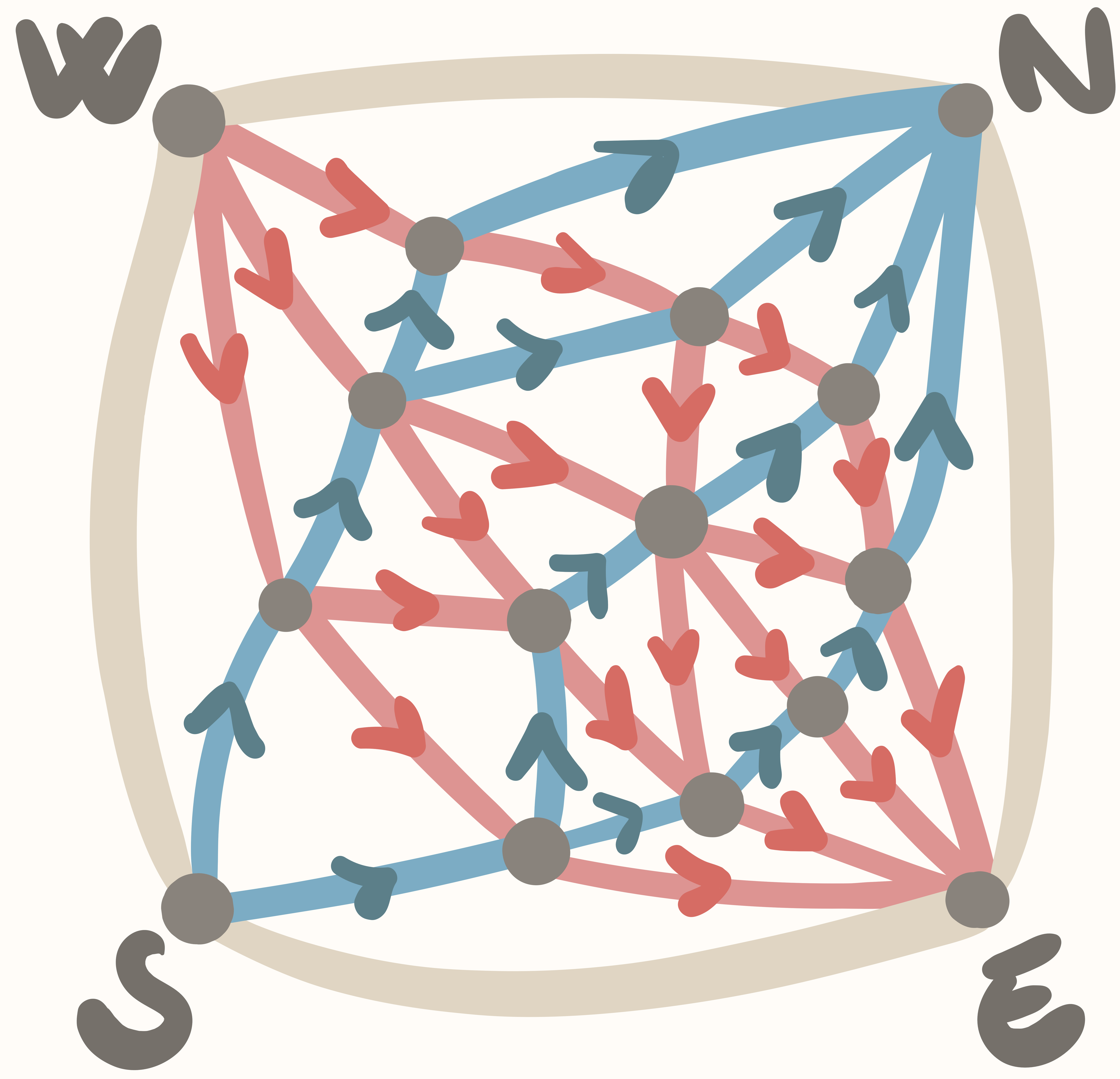
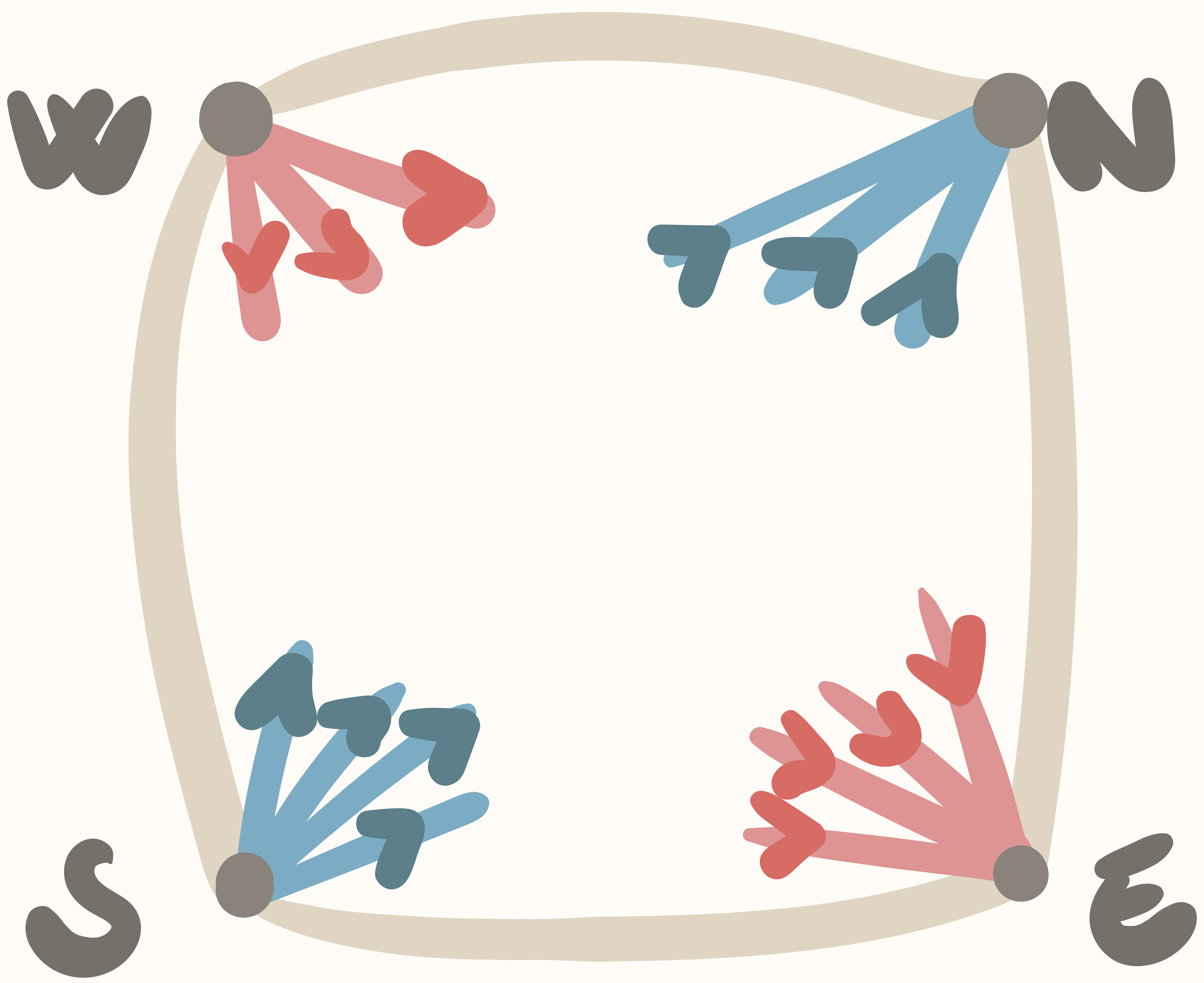
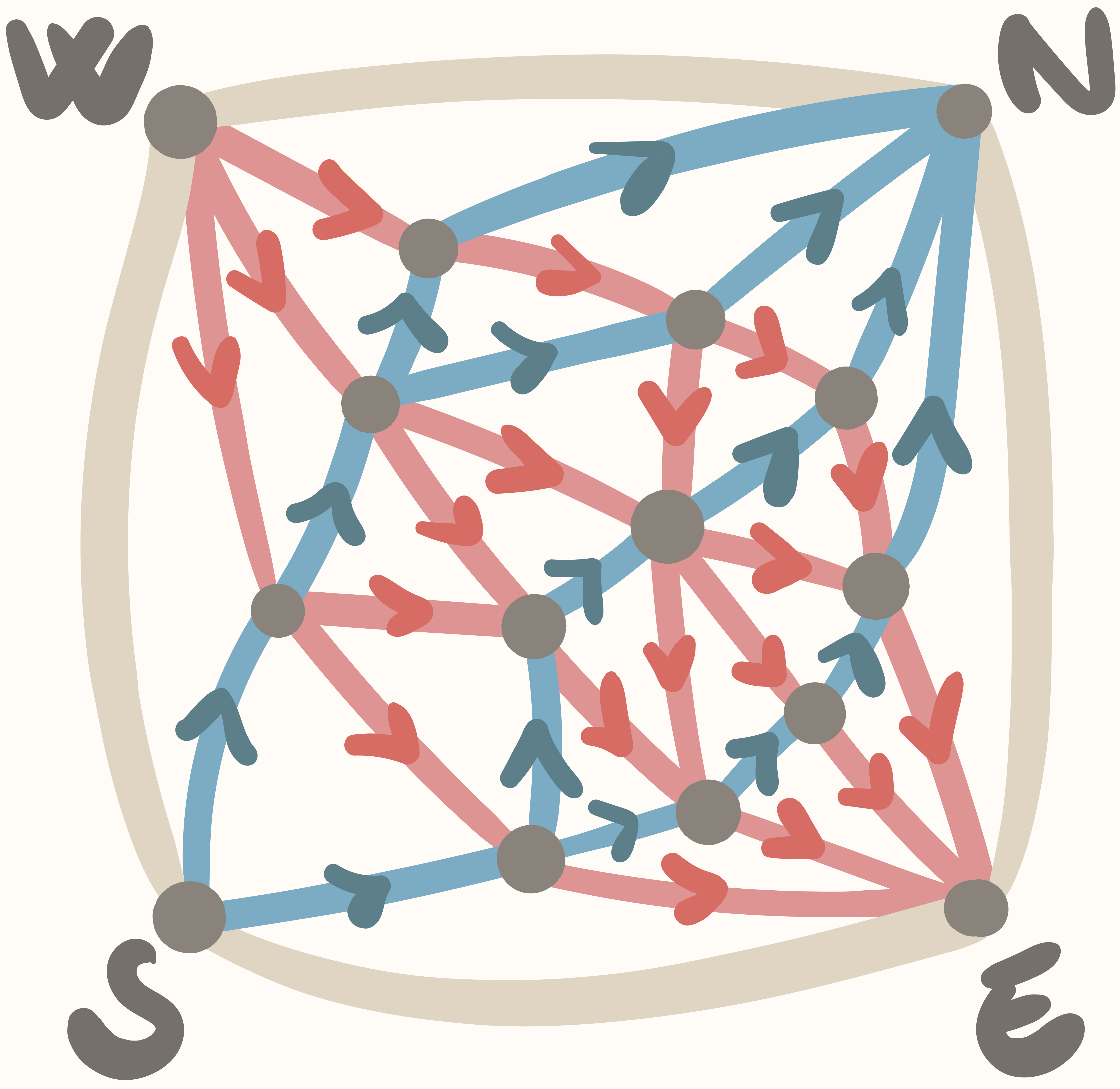
Transversal structures

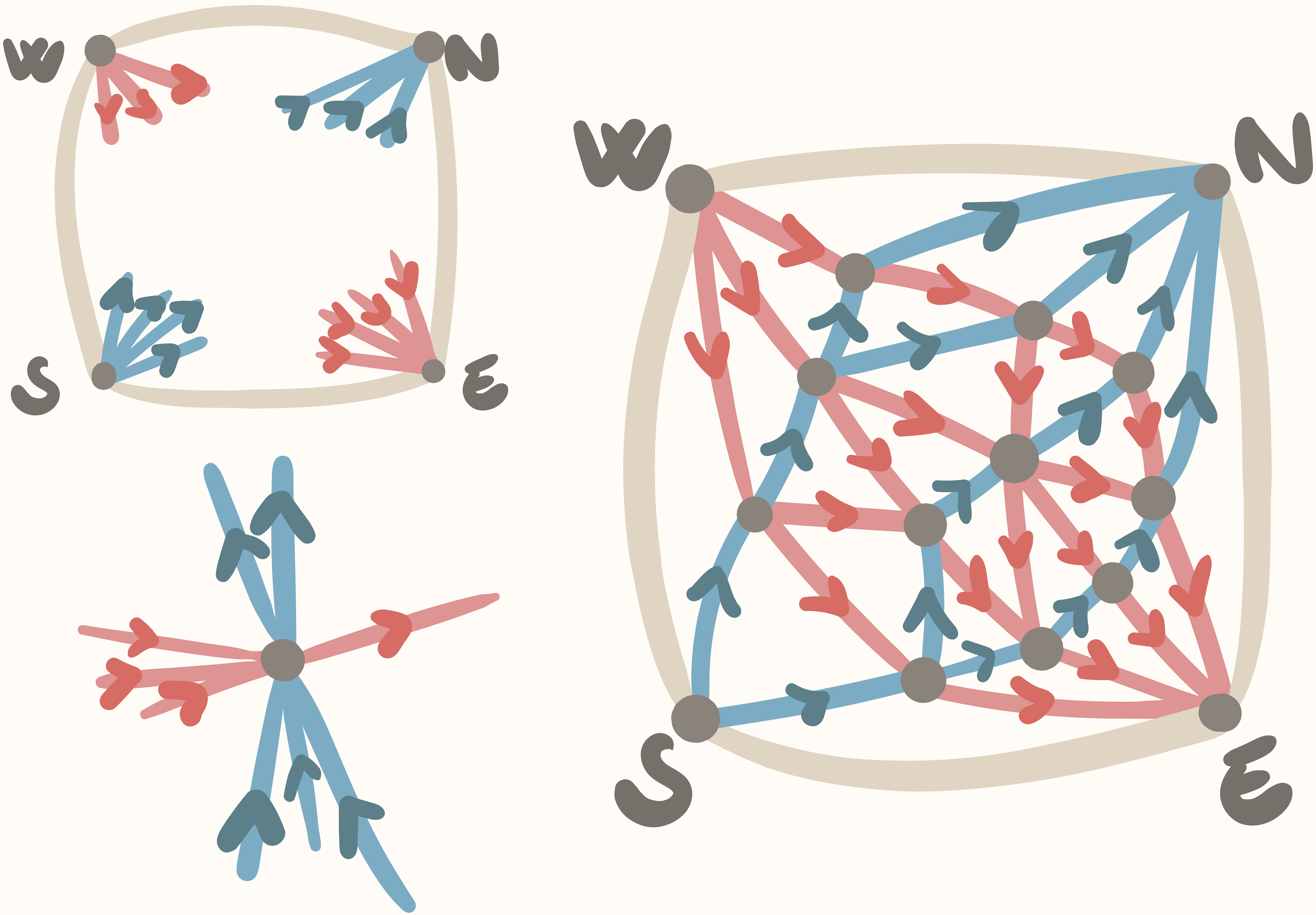


Transversal structures

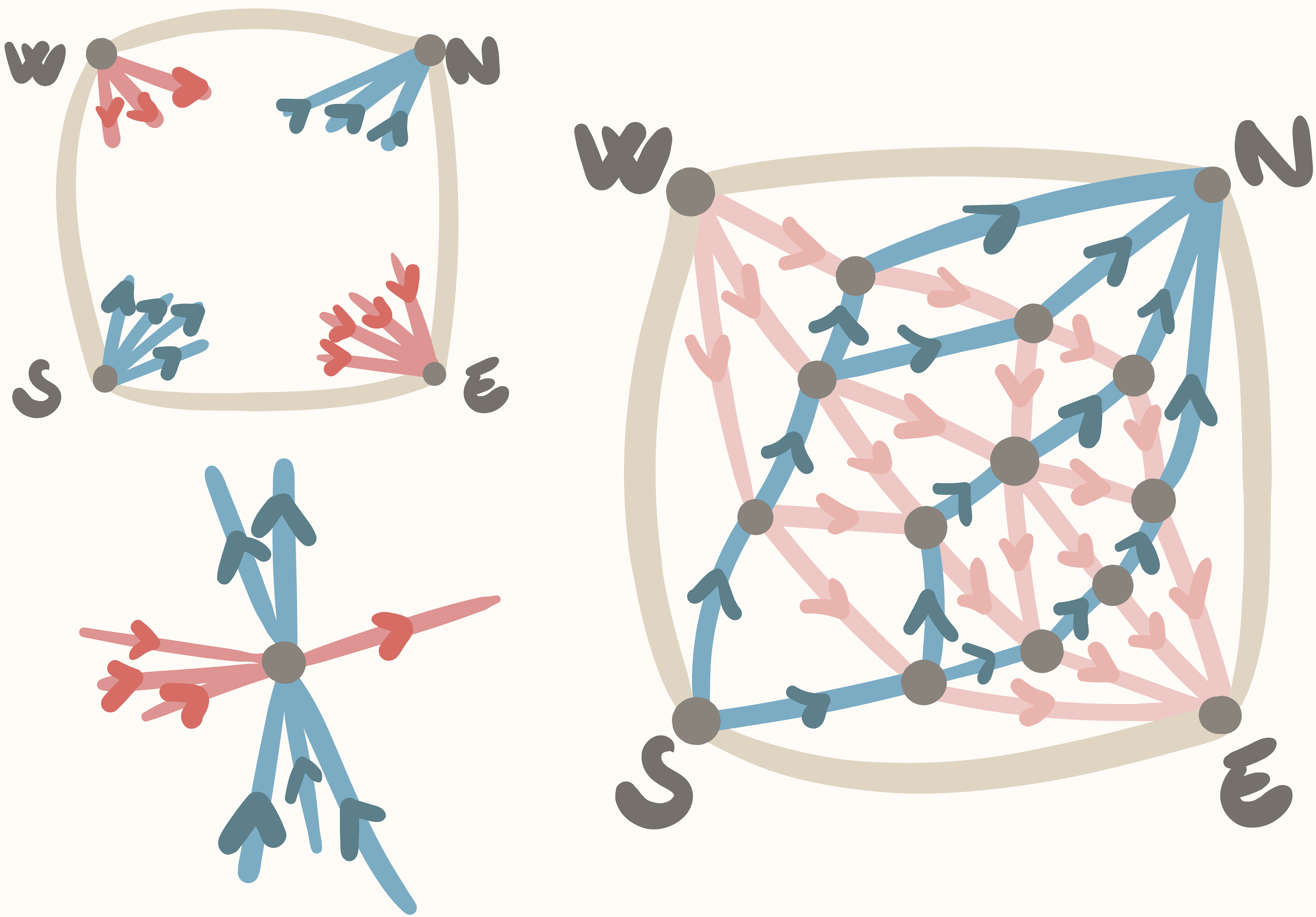




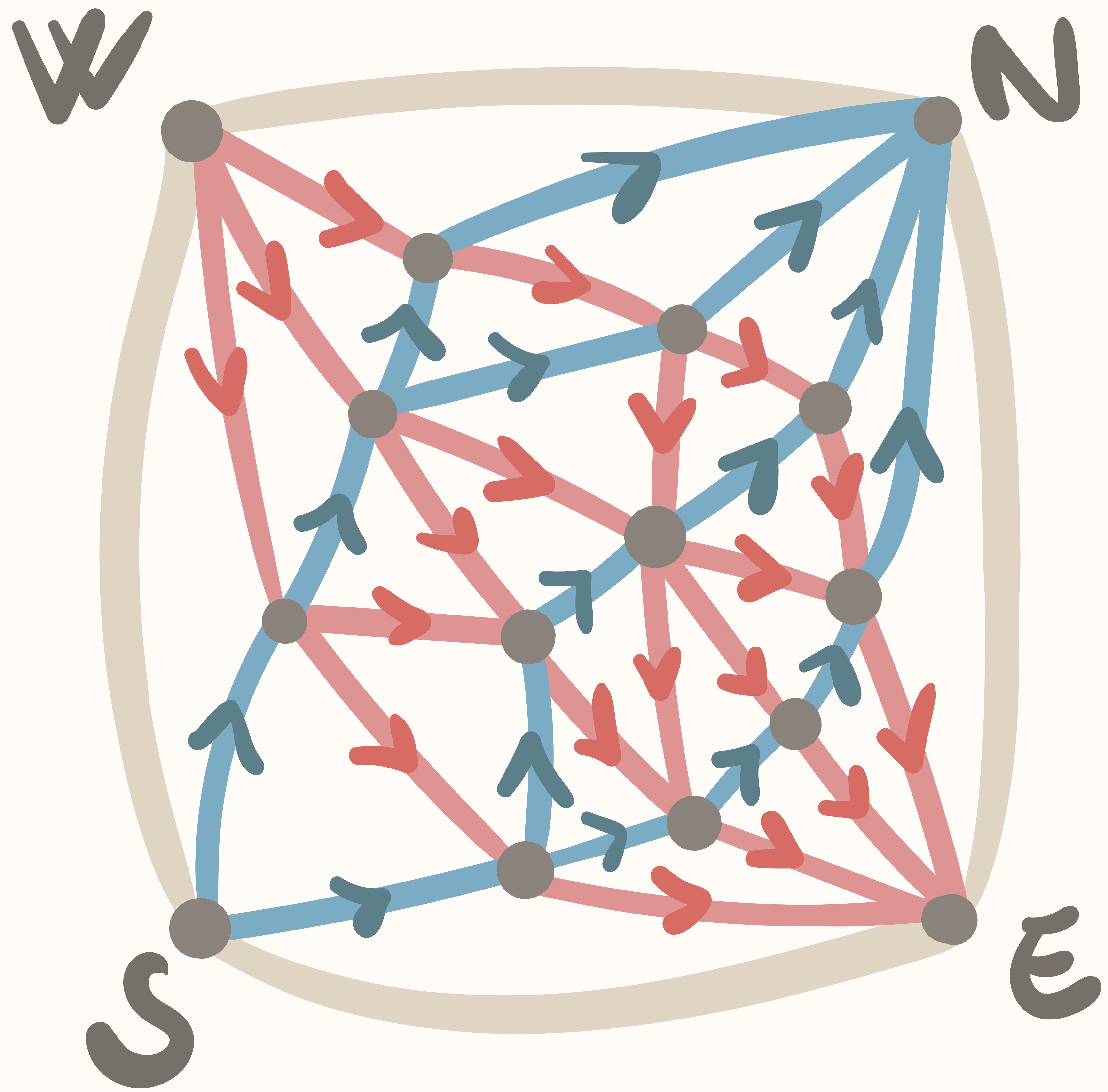
Transversal structures



Transversal structures



Specialization to transversal structures



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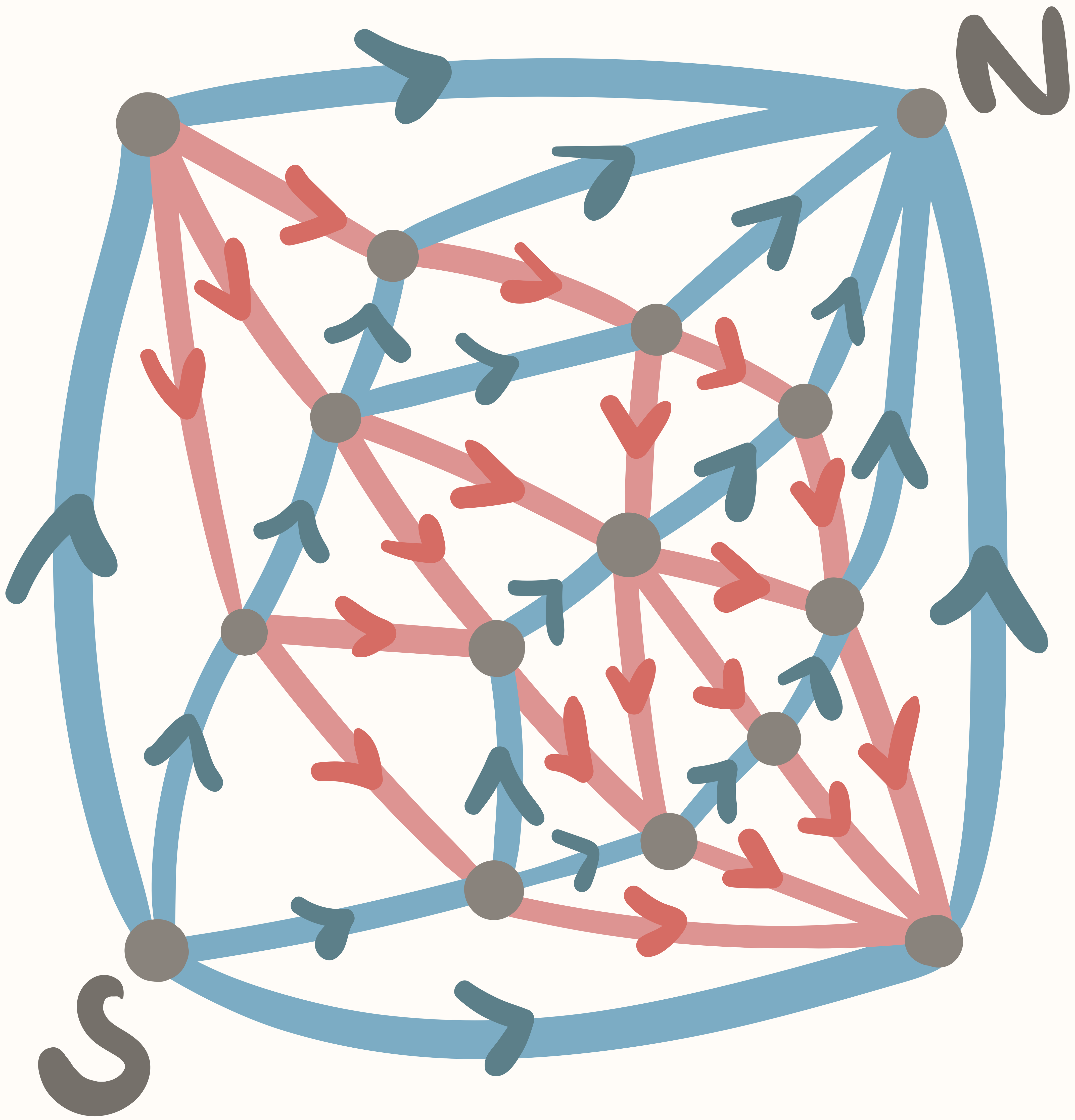

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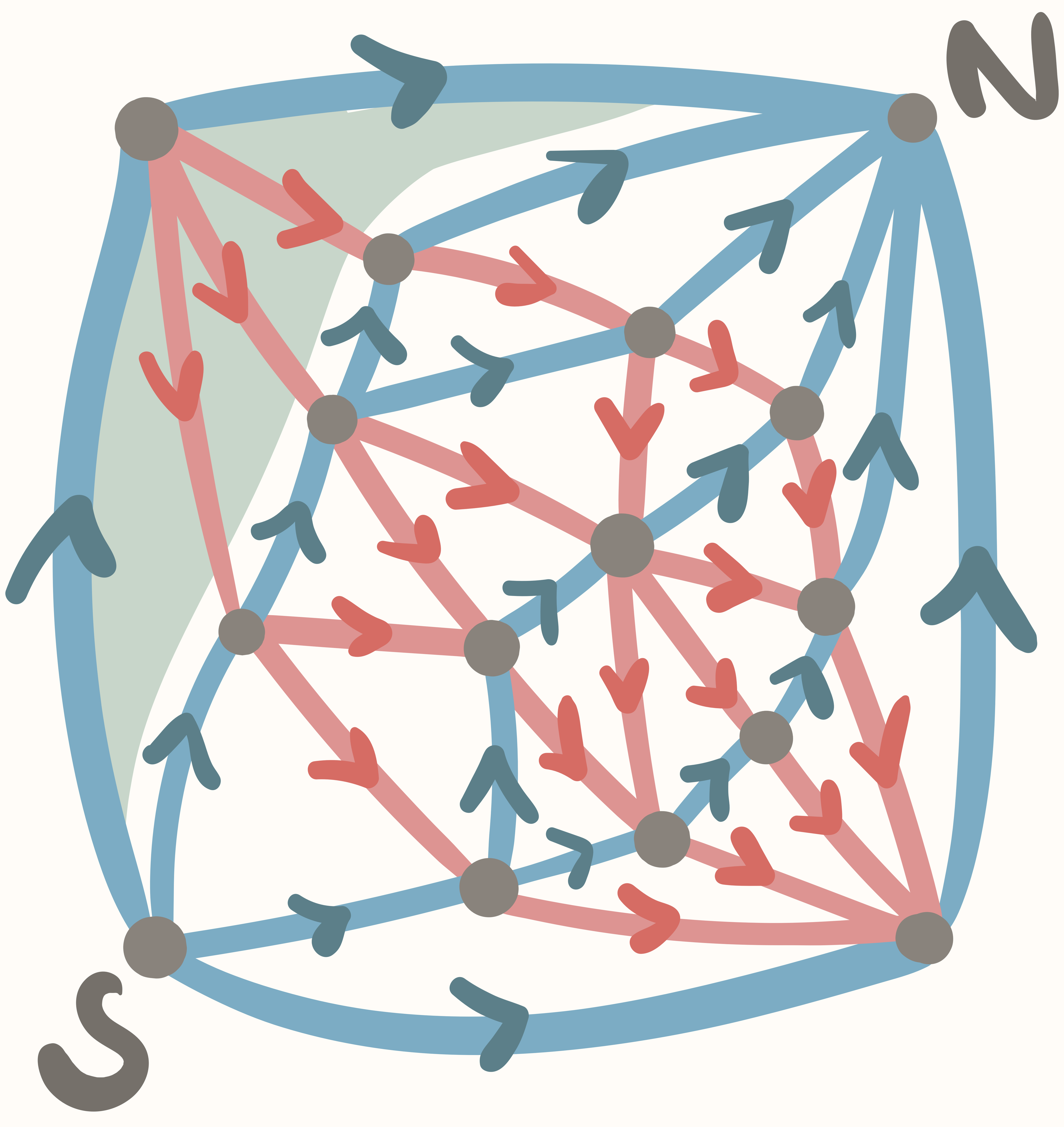
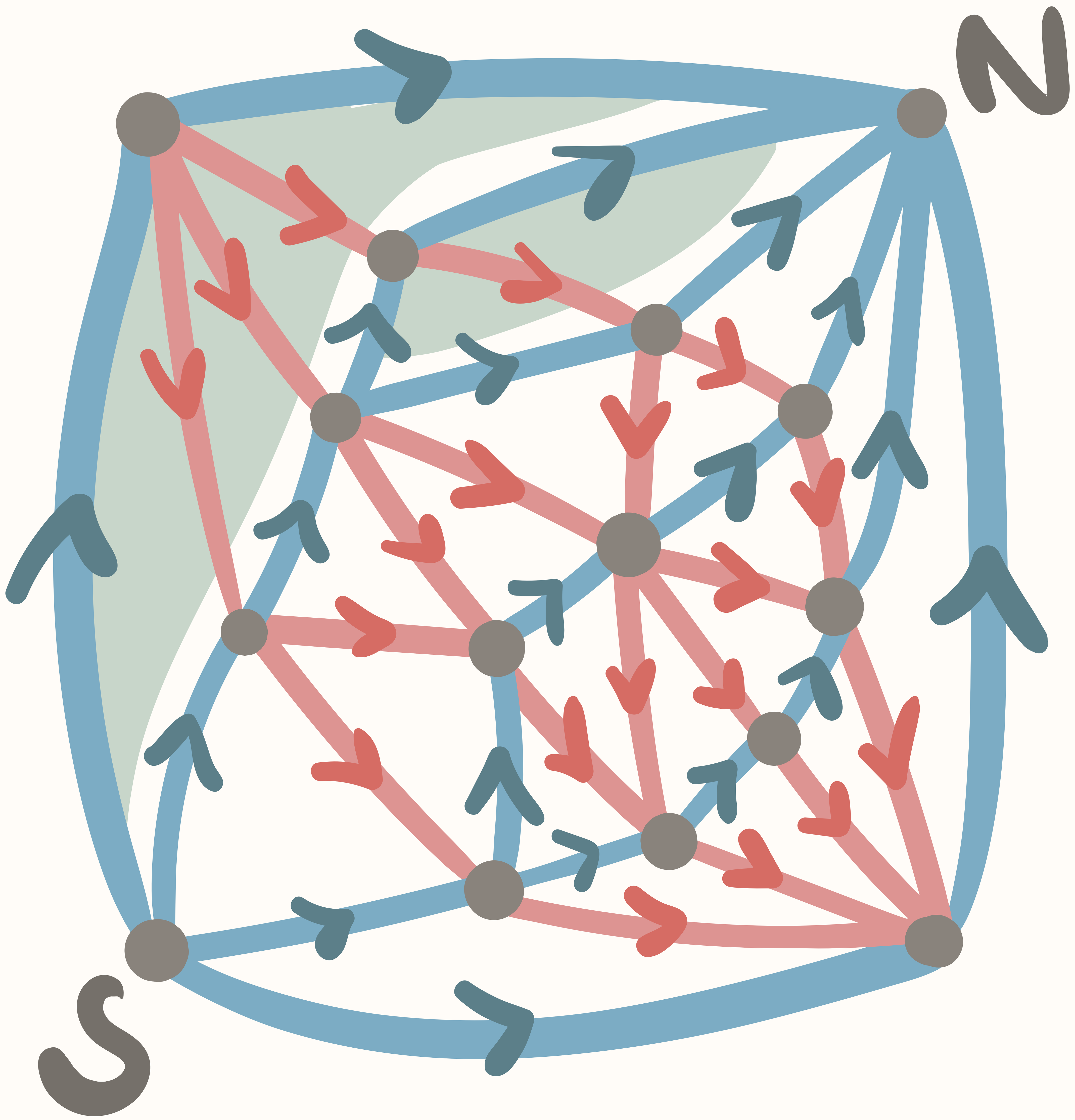
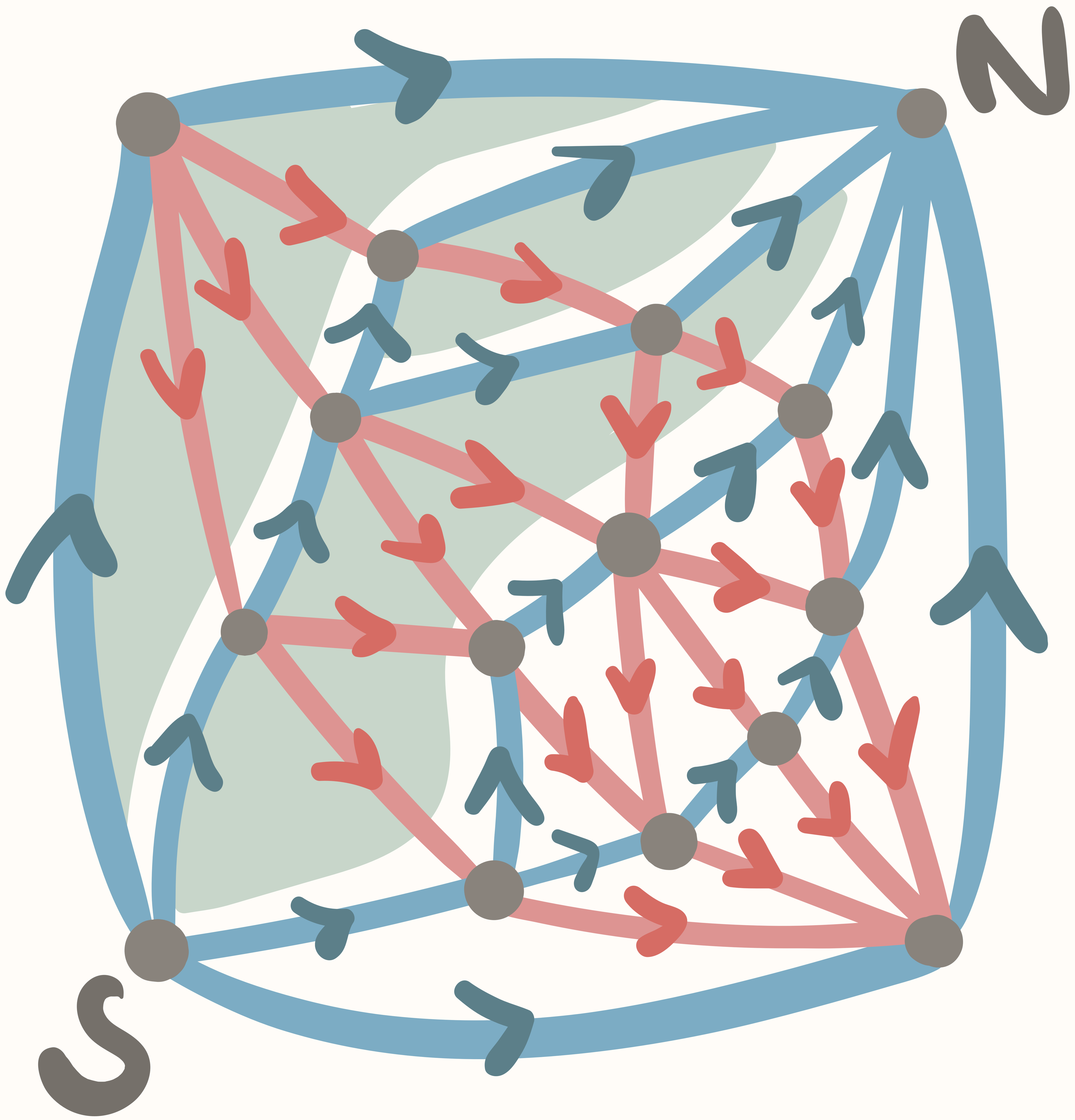


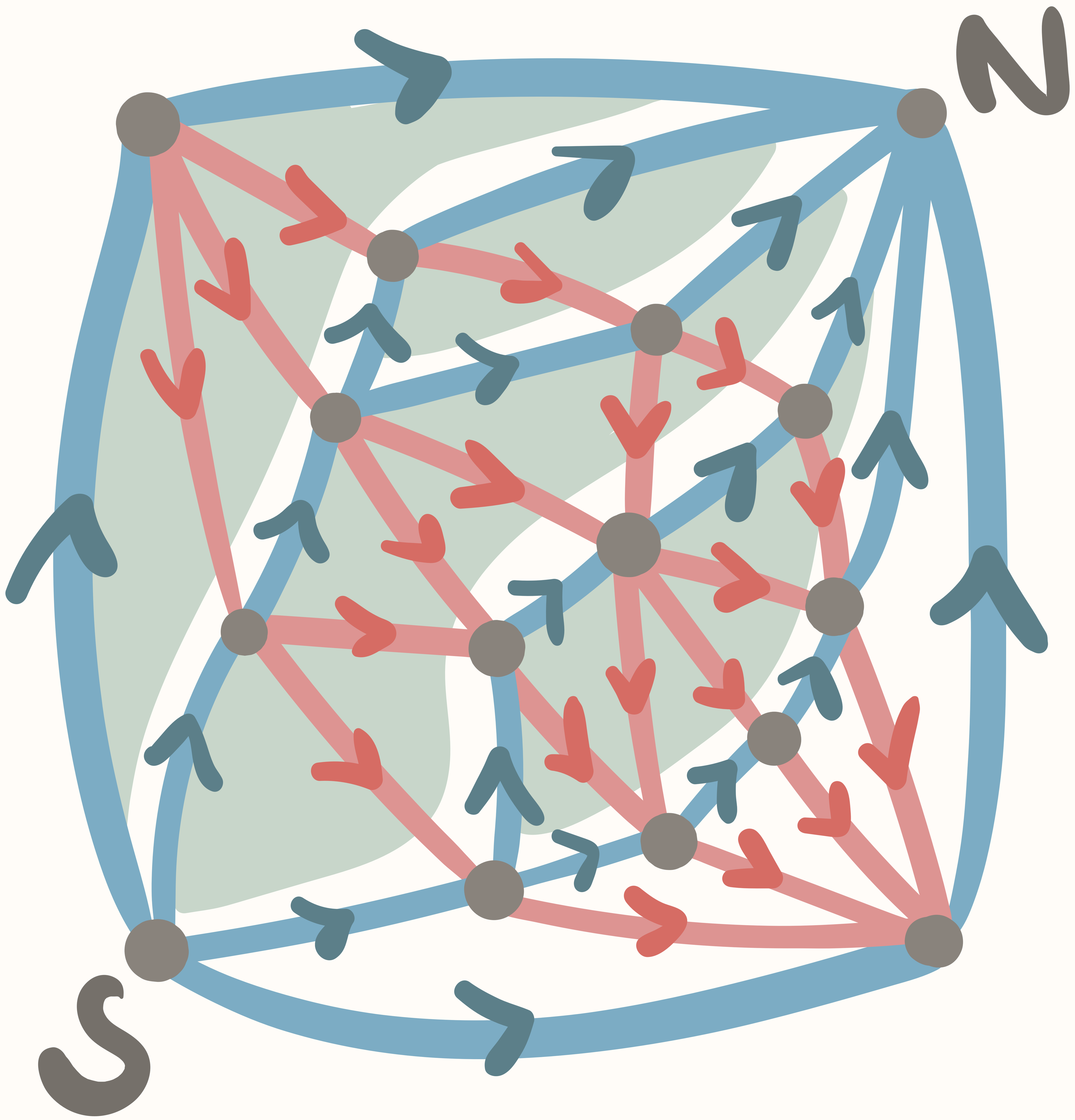
Specialization to transversal structures



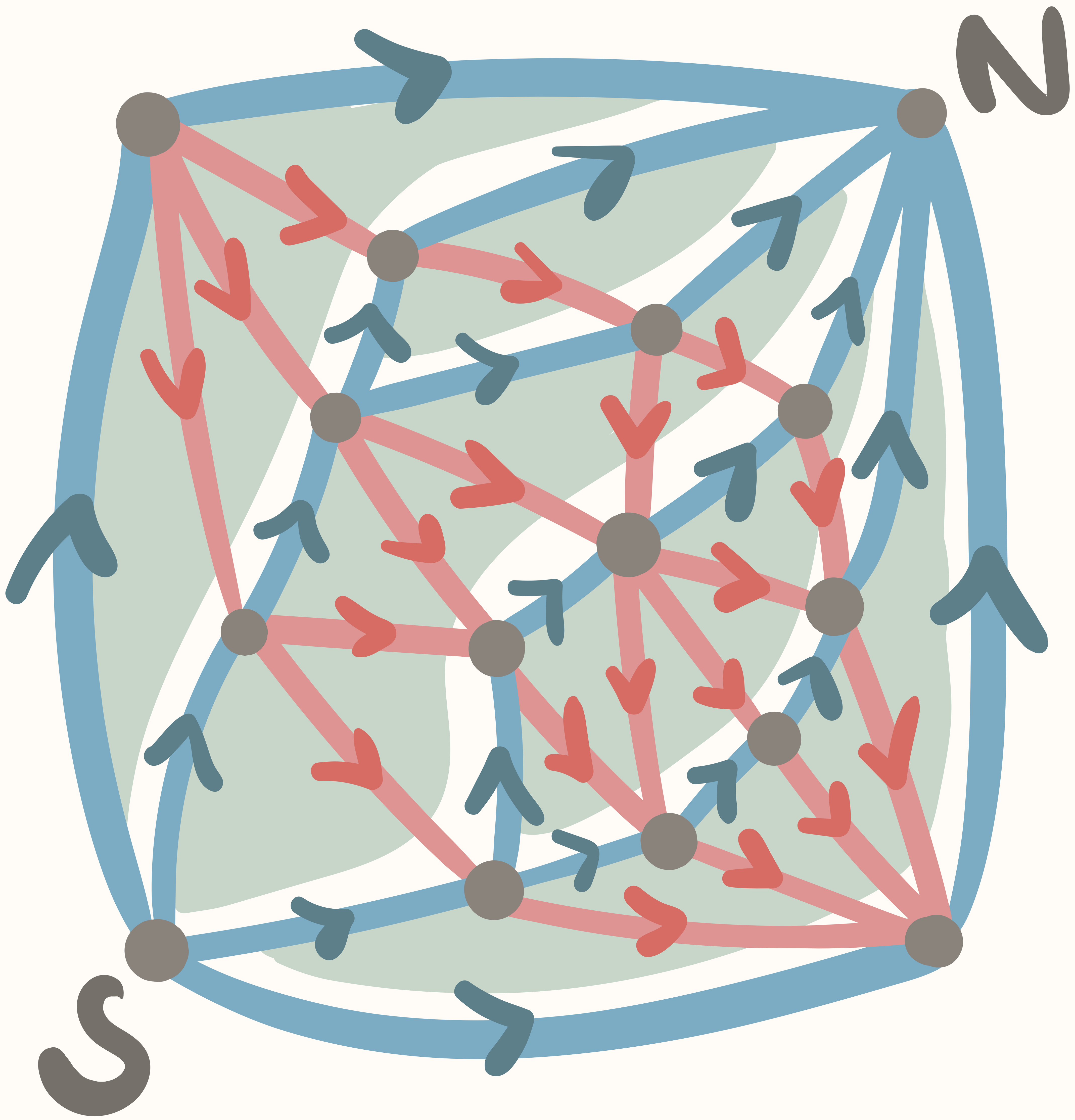
Specialization to transversal structures



Specialization to transversal structures



Specialization to transversal structures



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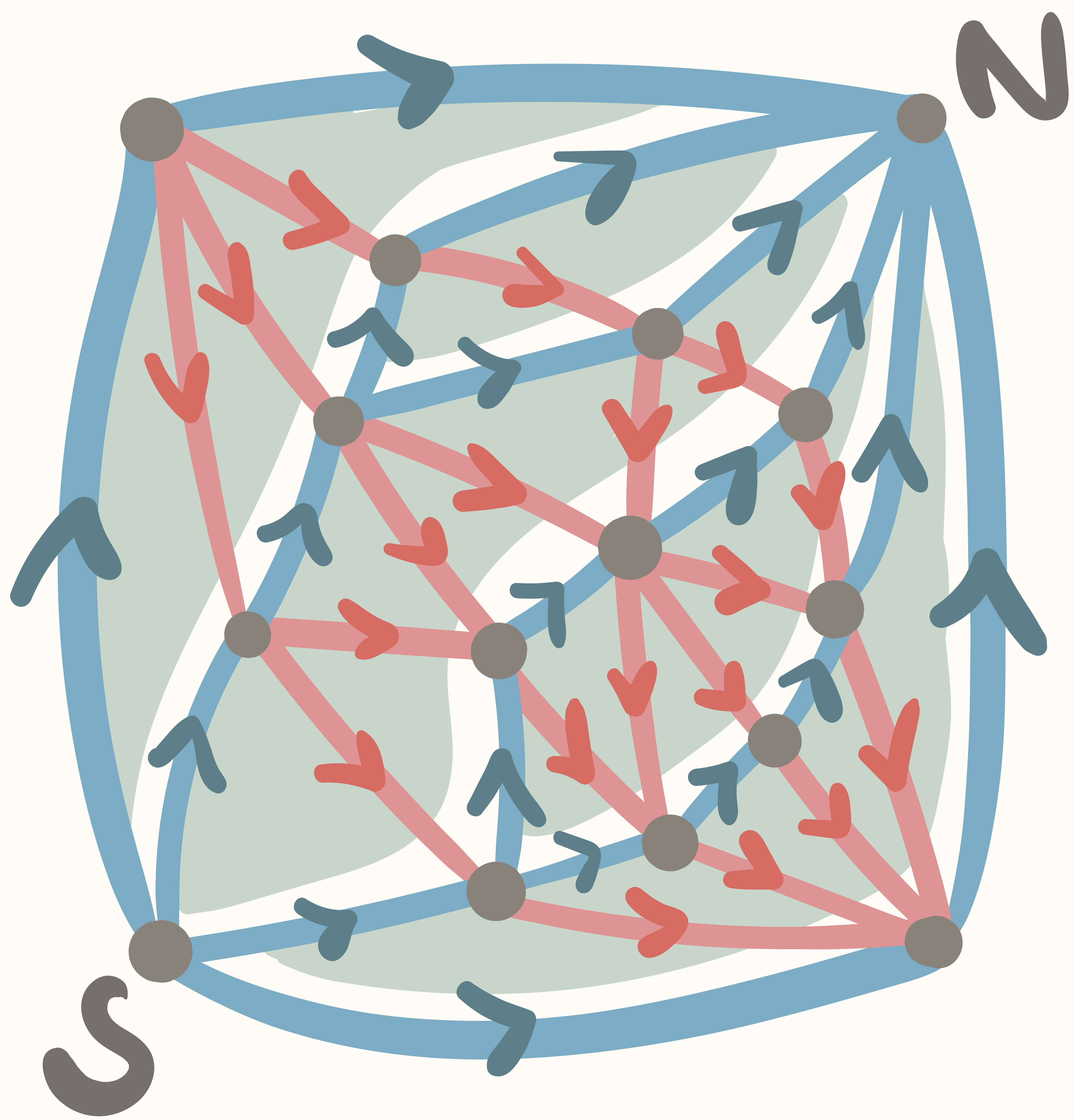
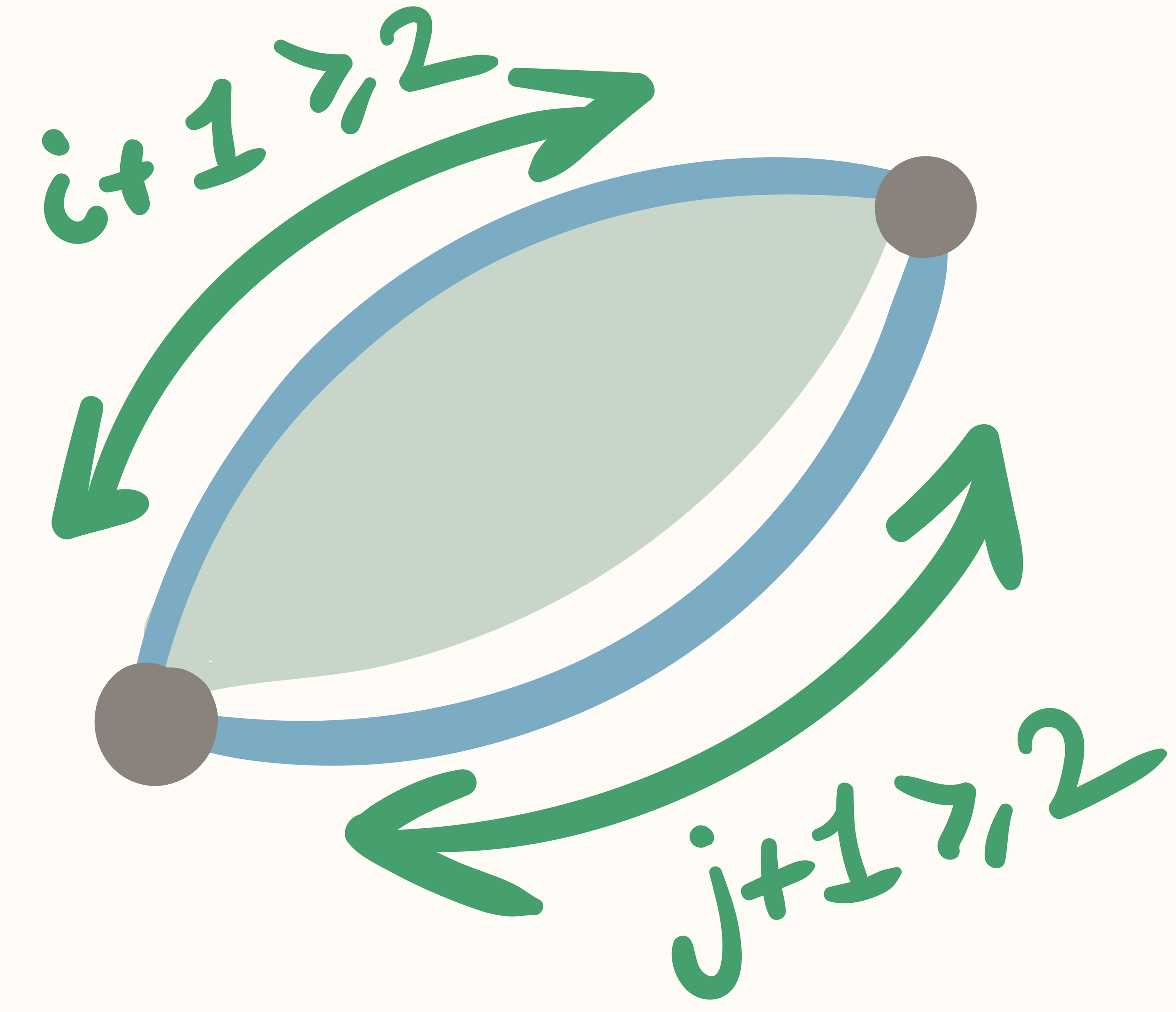
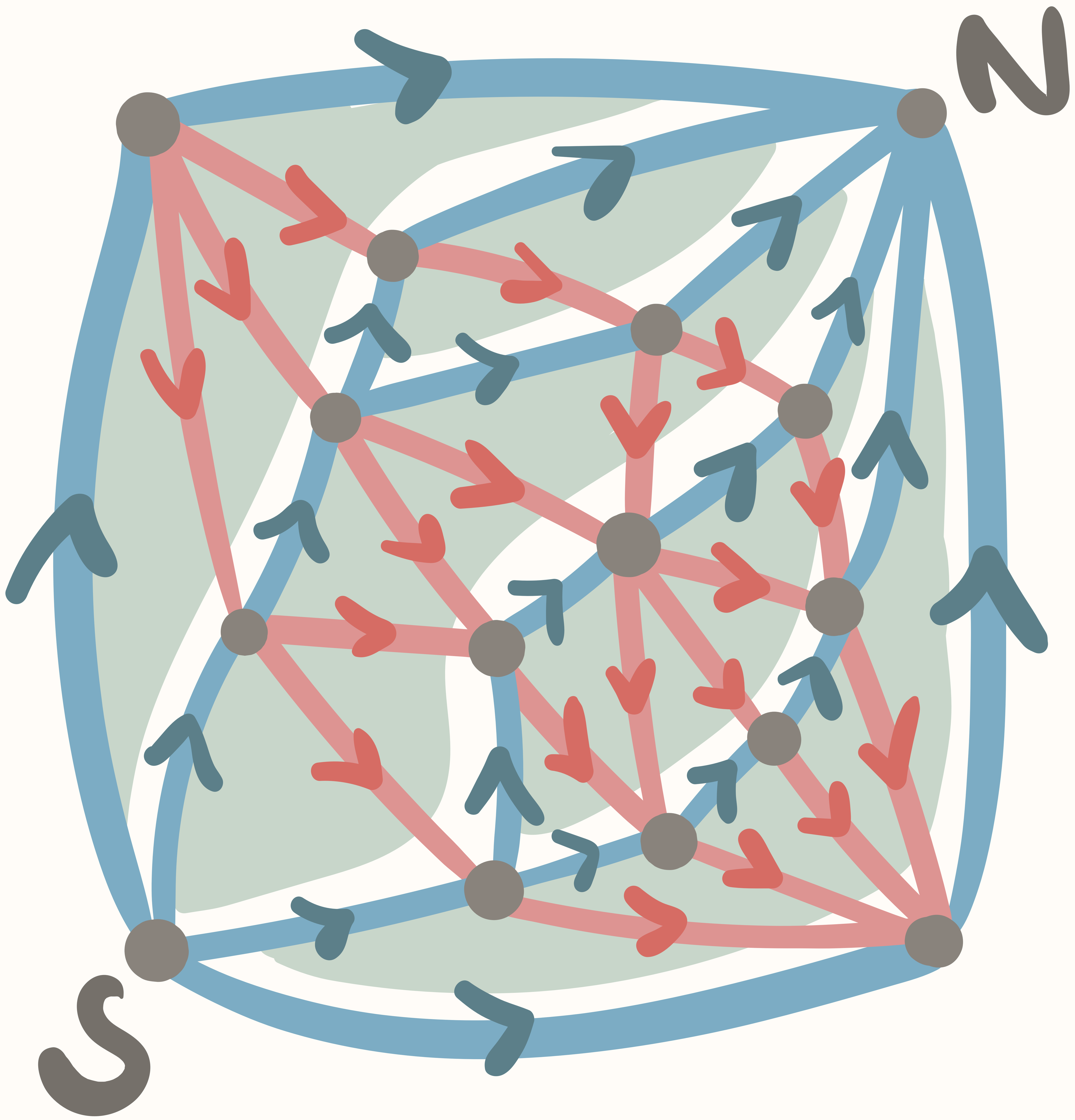
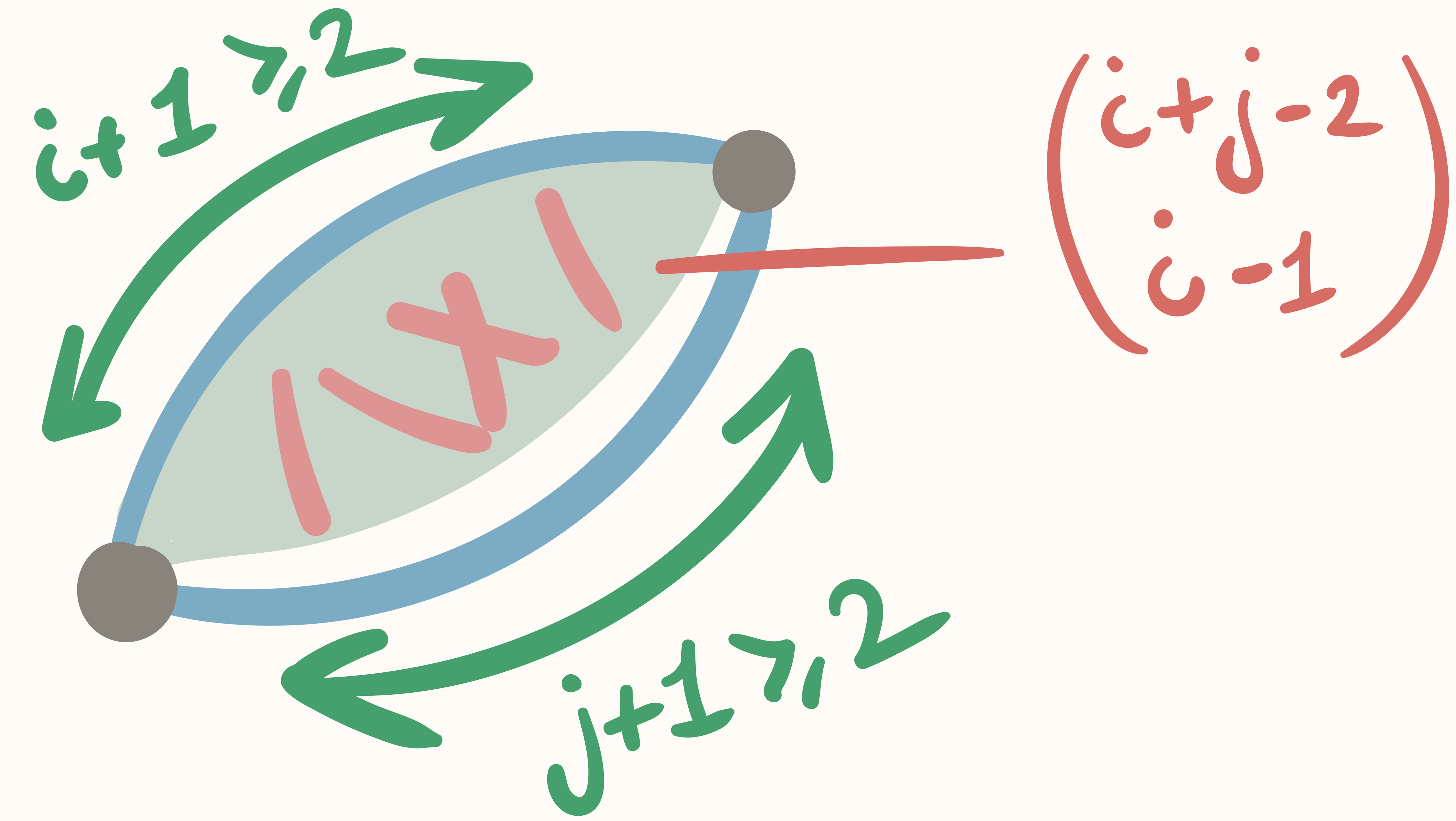




Specialization to transversal structures





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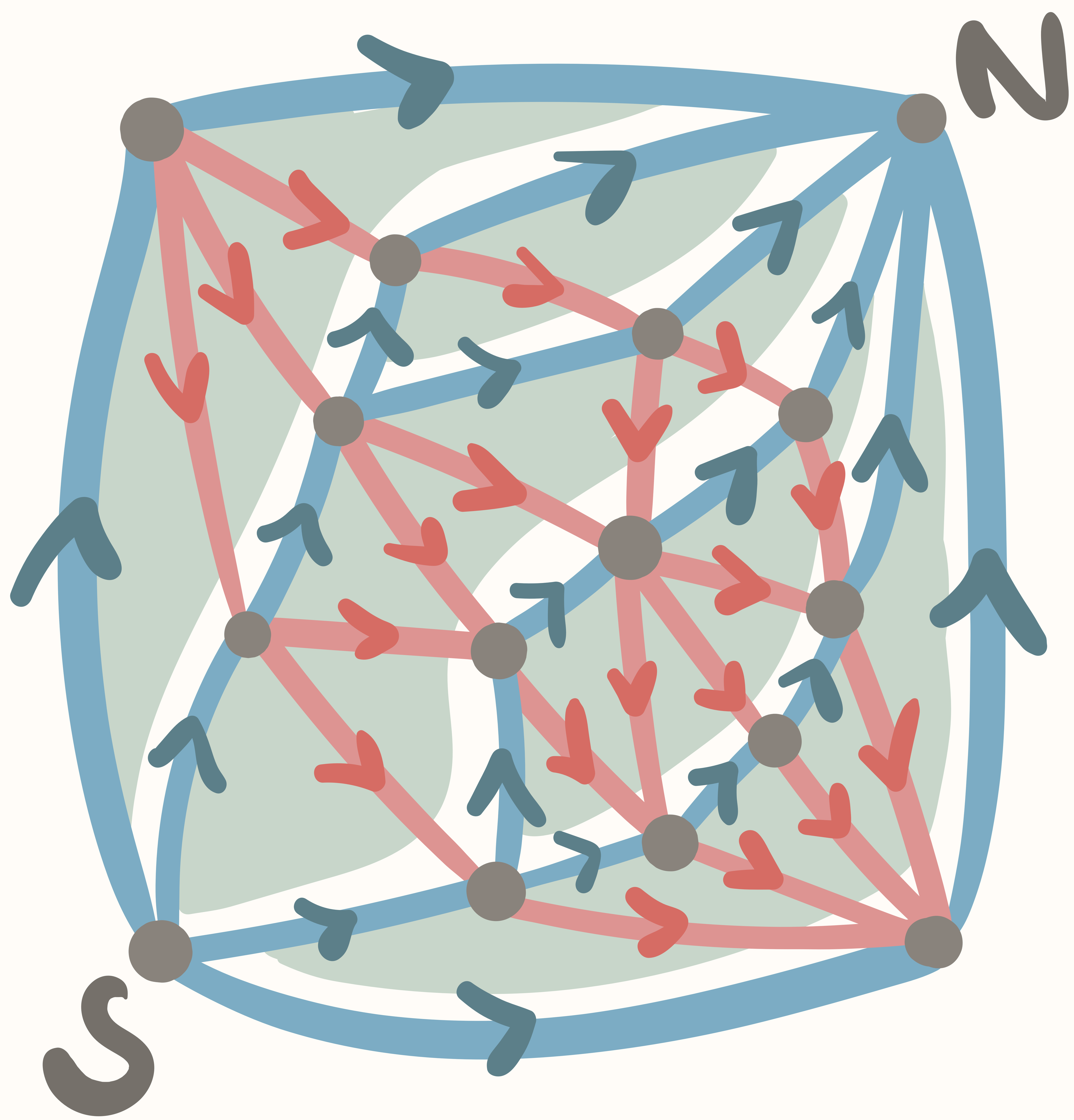
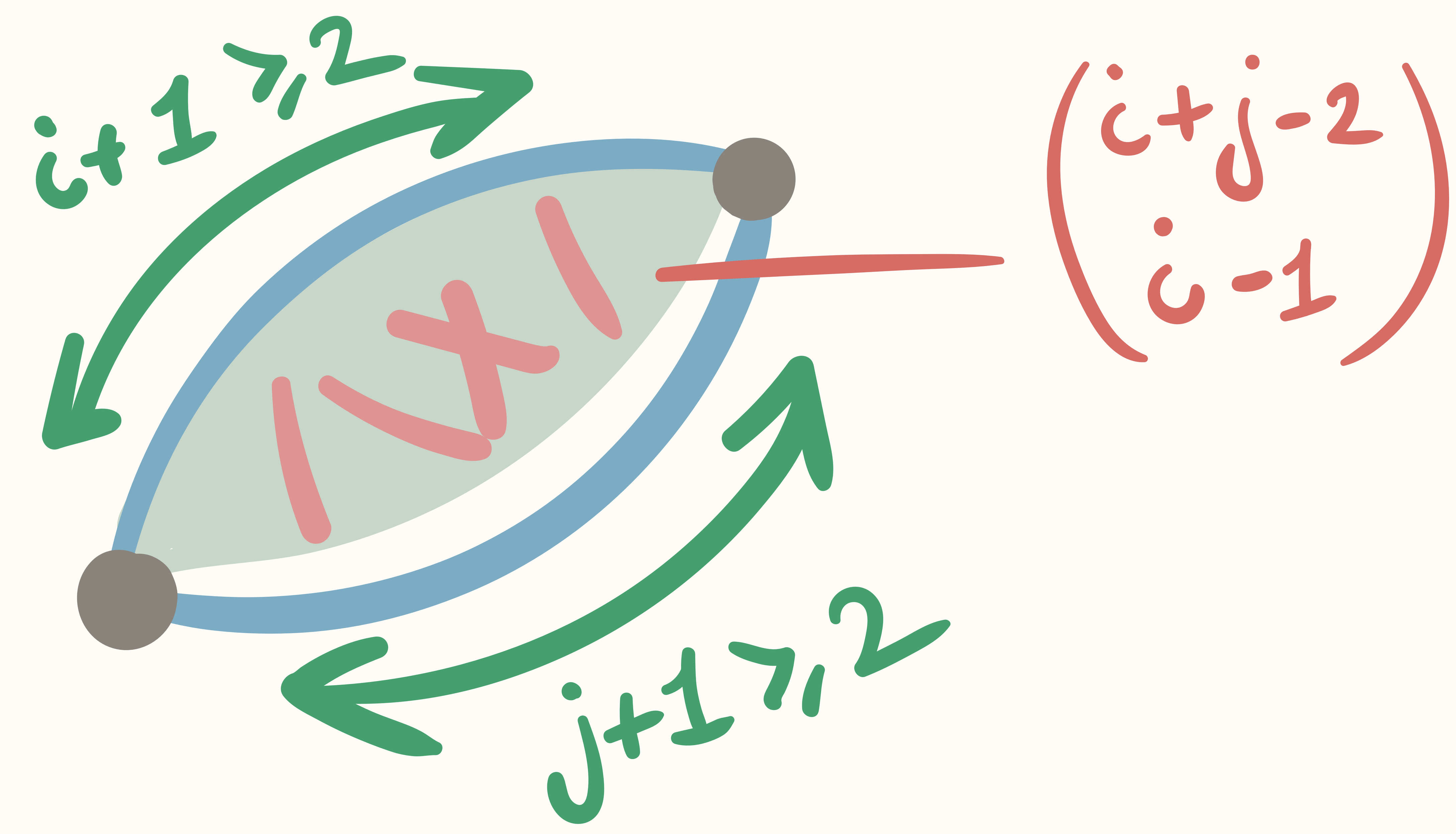
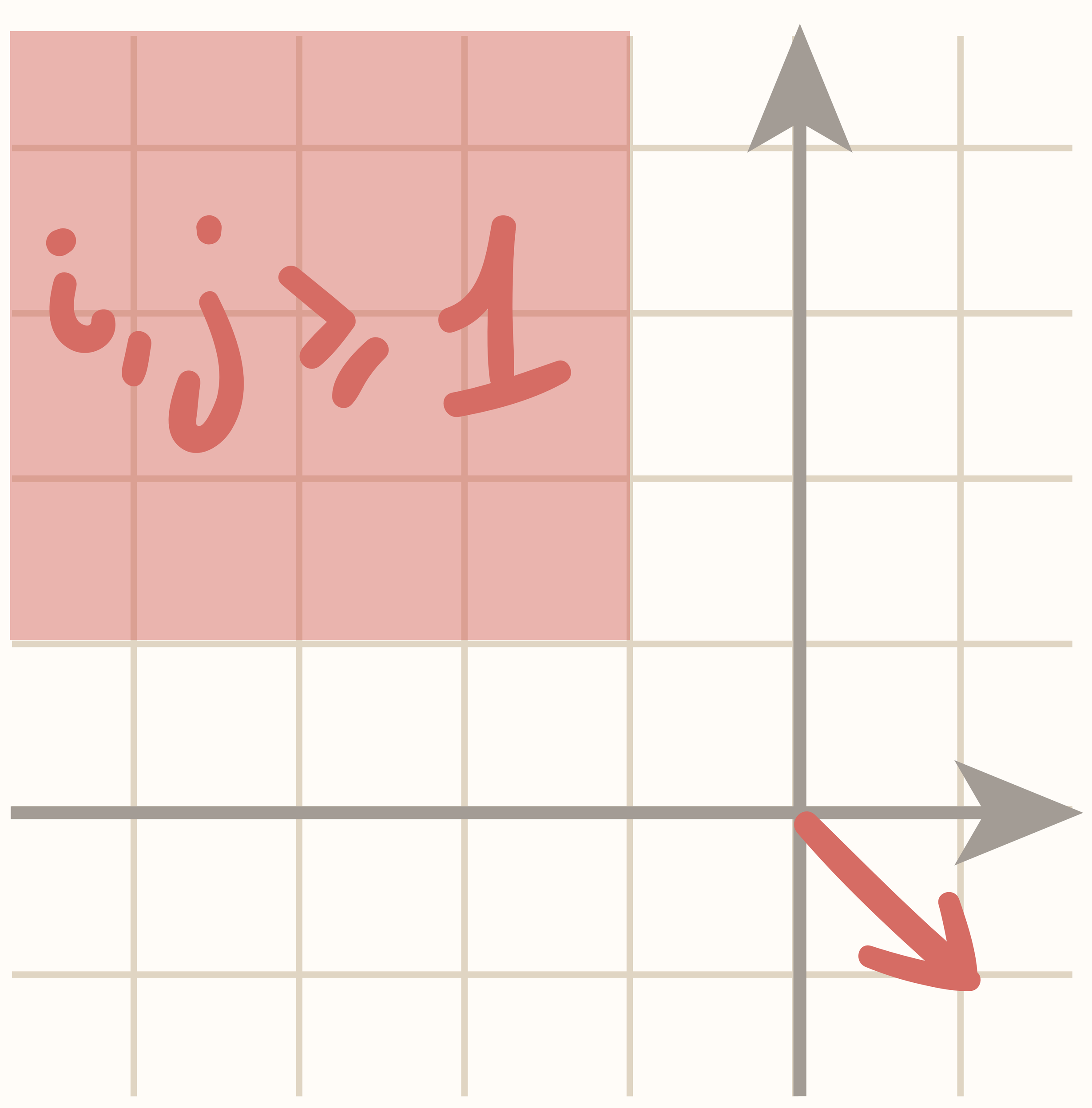
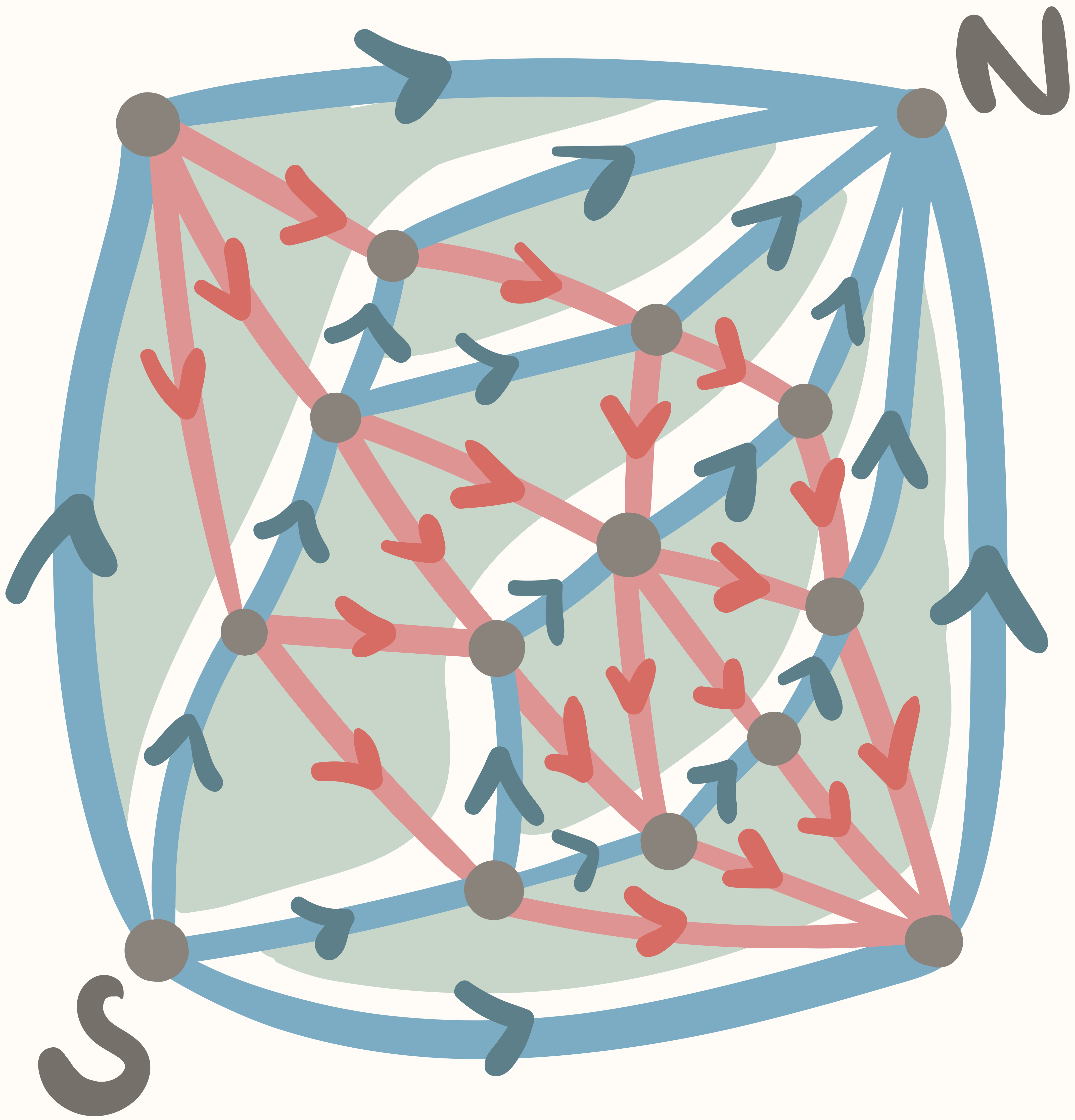
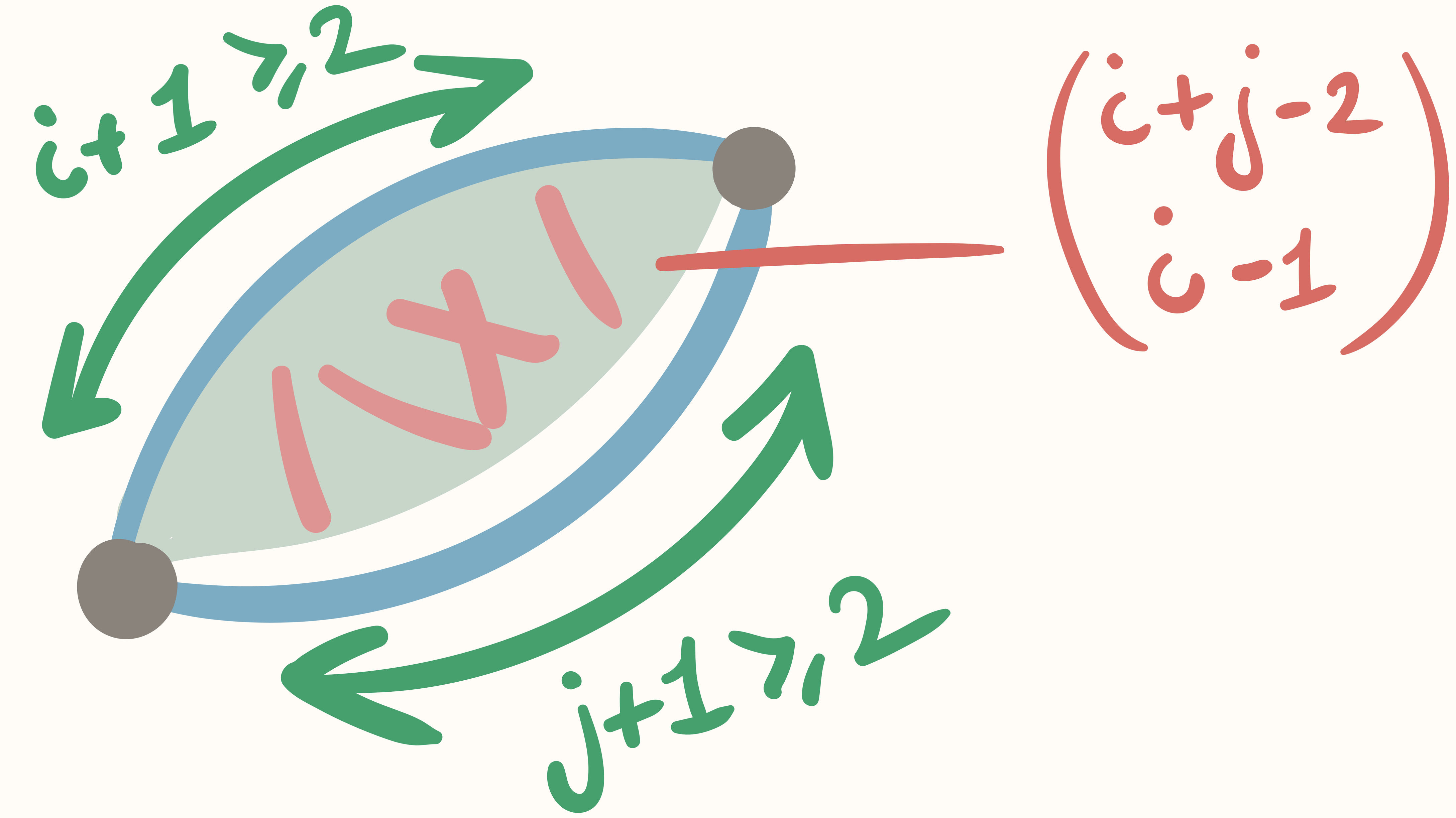


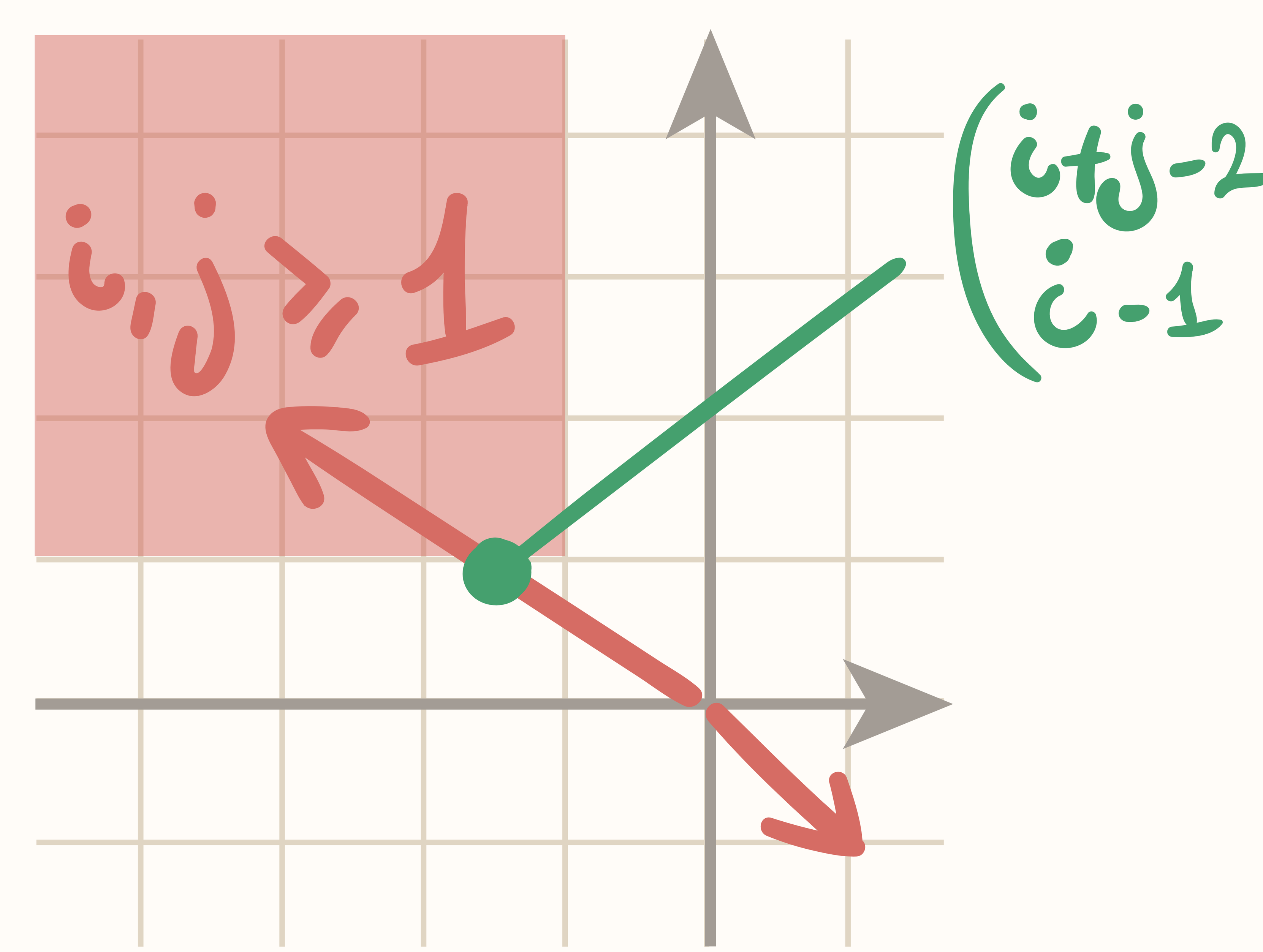




Specialization to transversal structures



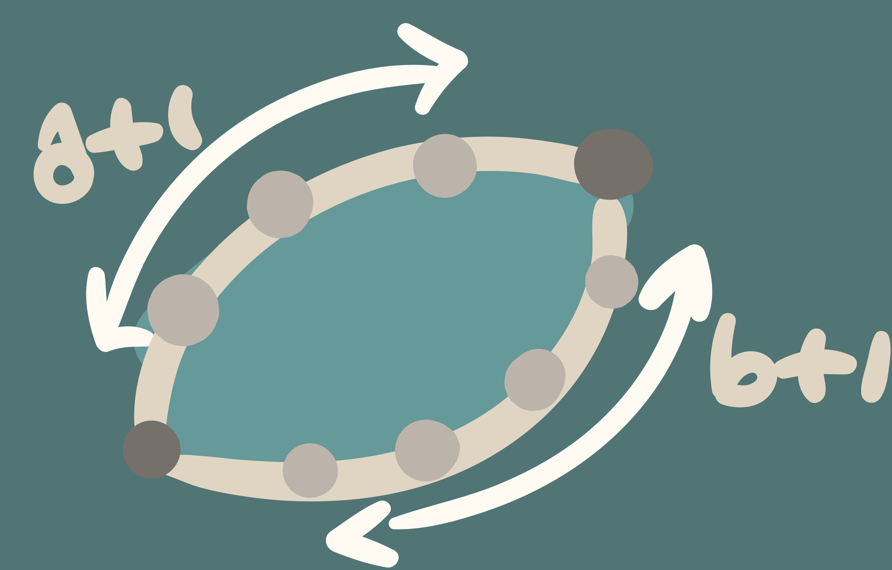
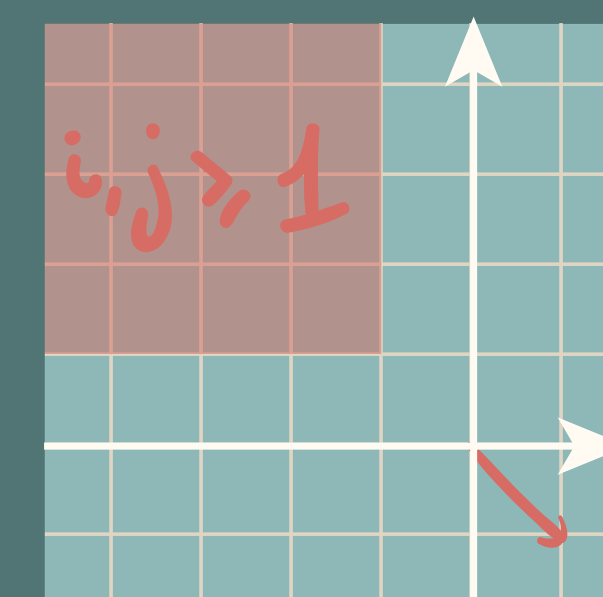
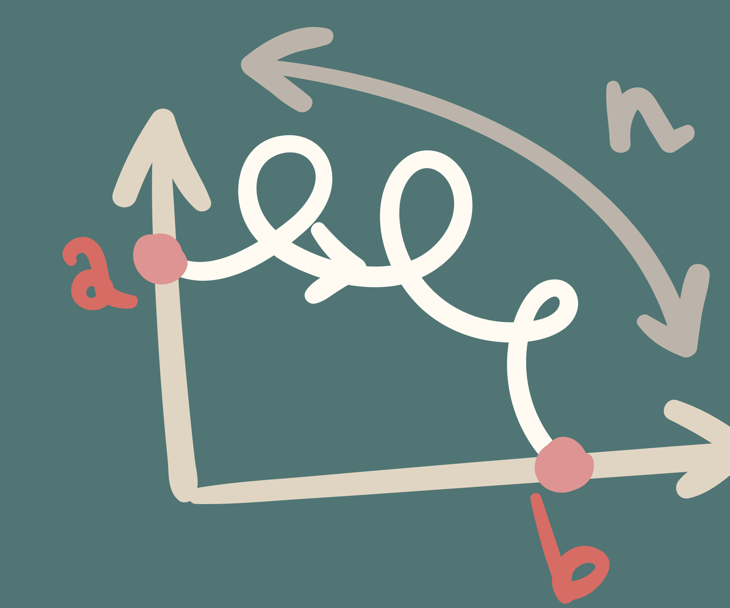




Specializations summary

Model	*Tandem walk*

Specializations summary

Model	*Tandem walk*
Posets *n+2 edges* 	←bijection→  

Specializations summary

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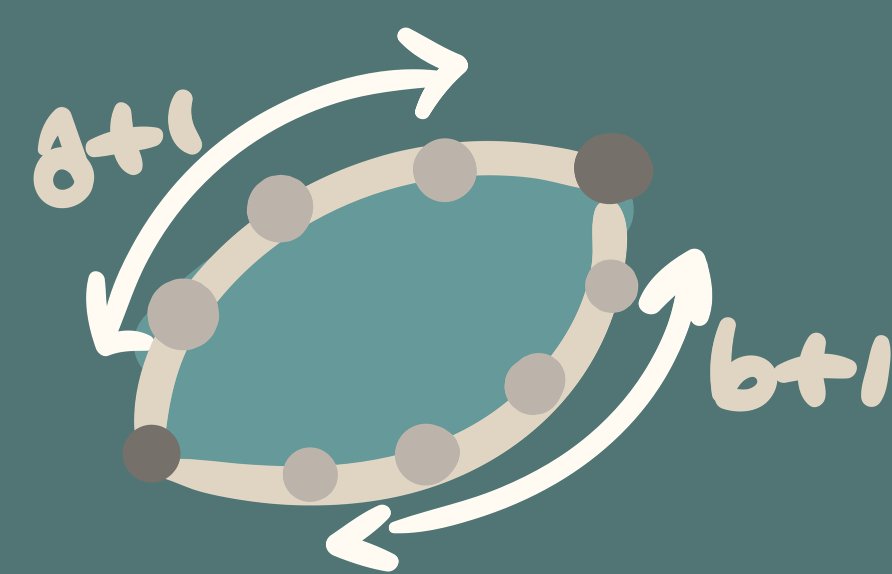
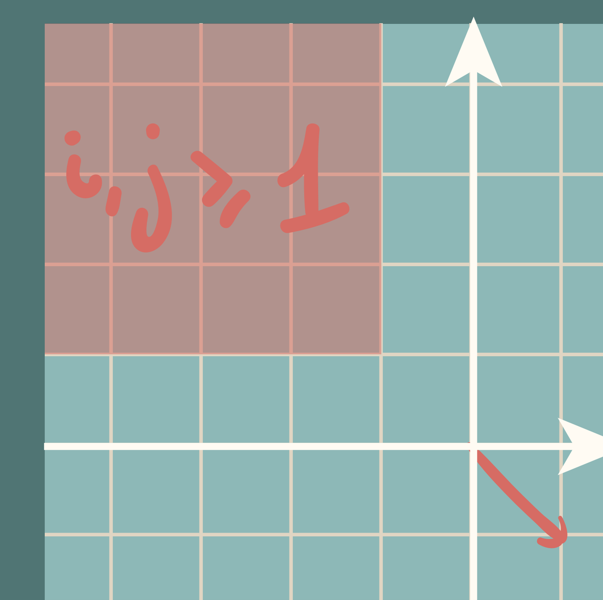
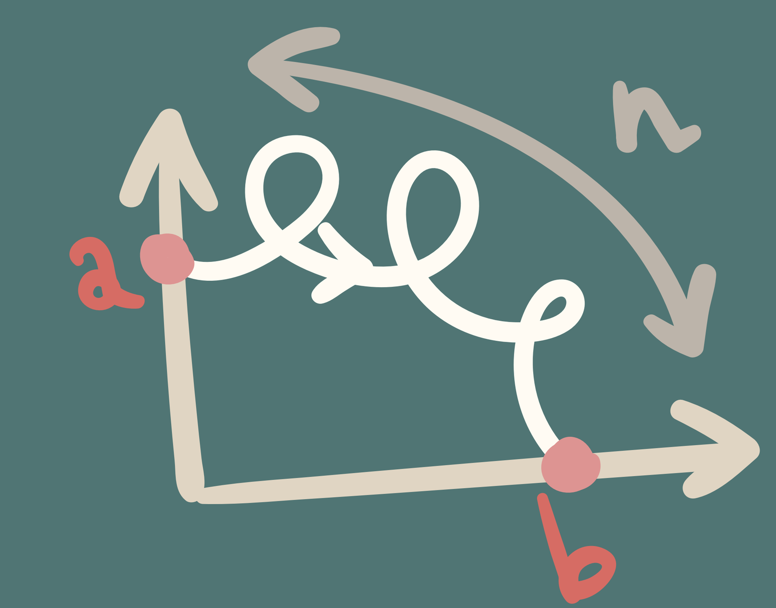
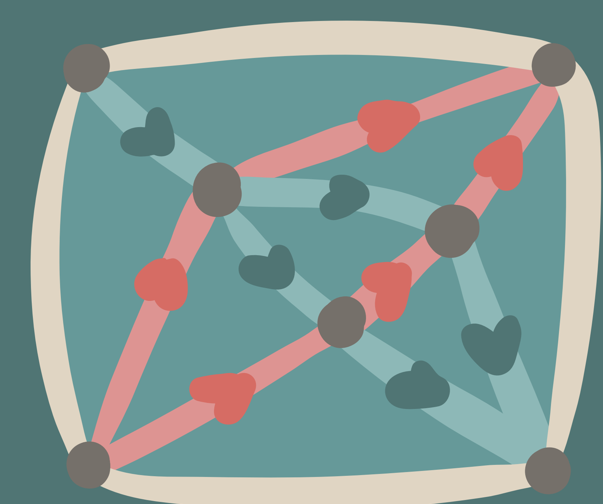
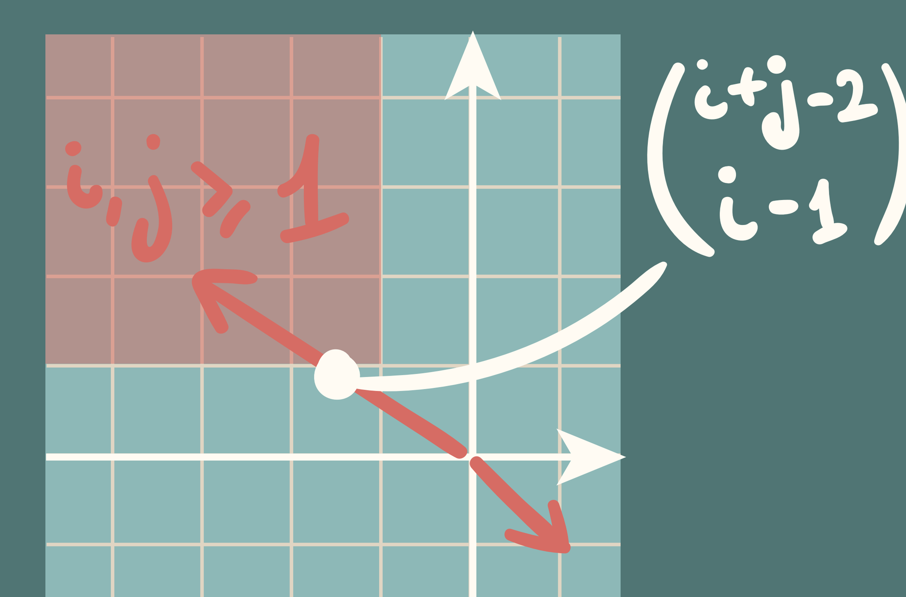
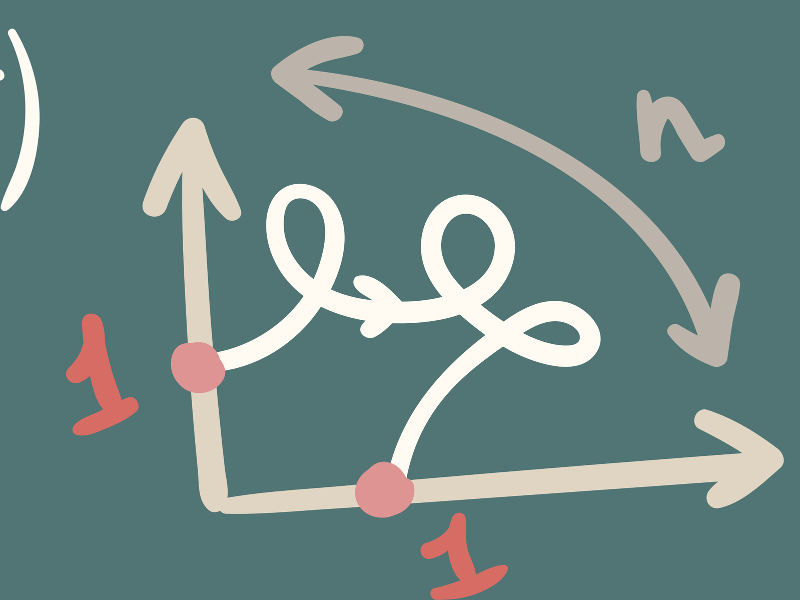
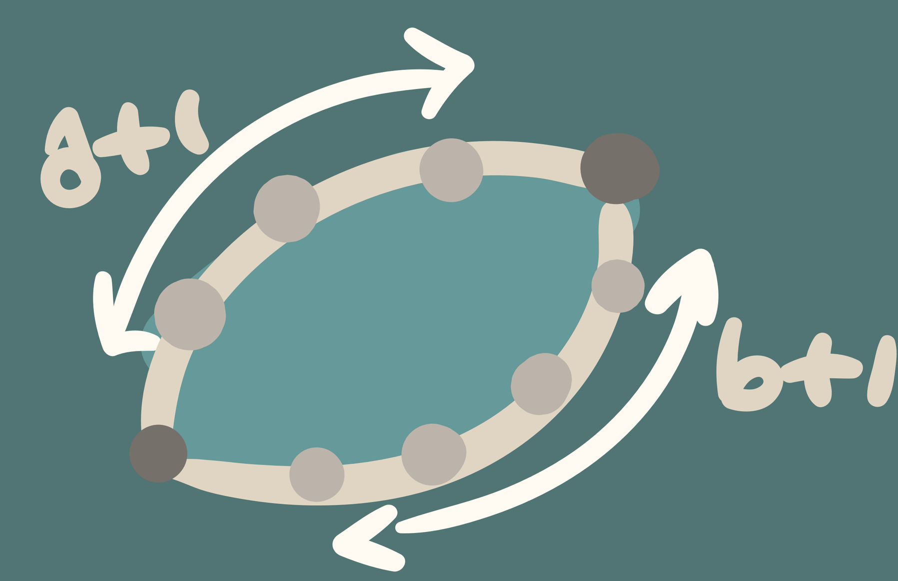
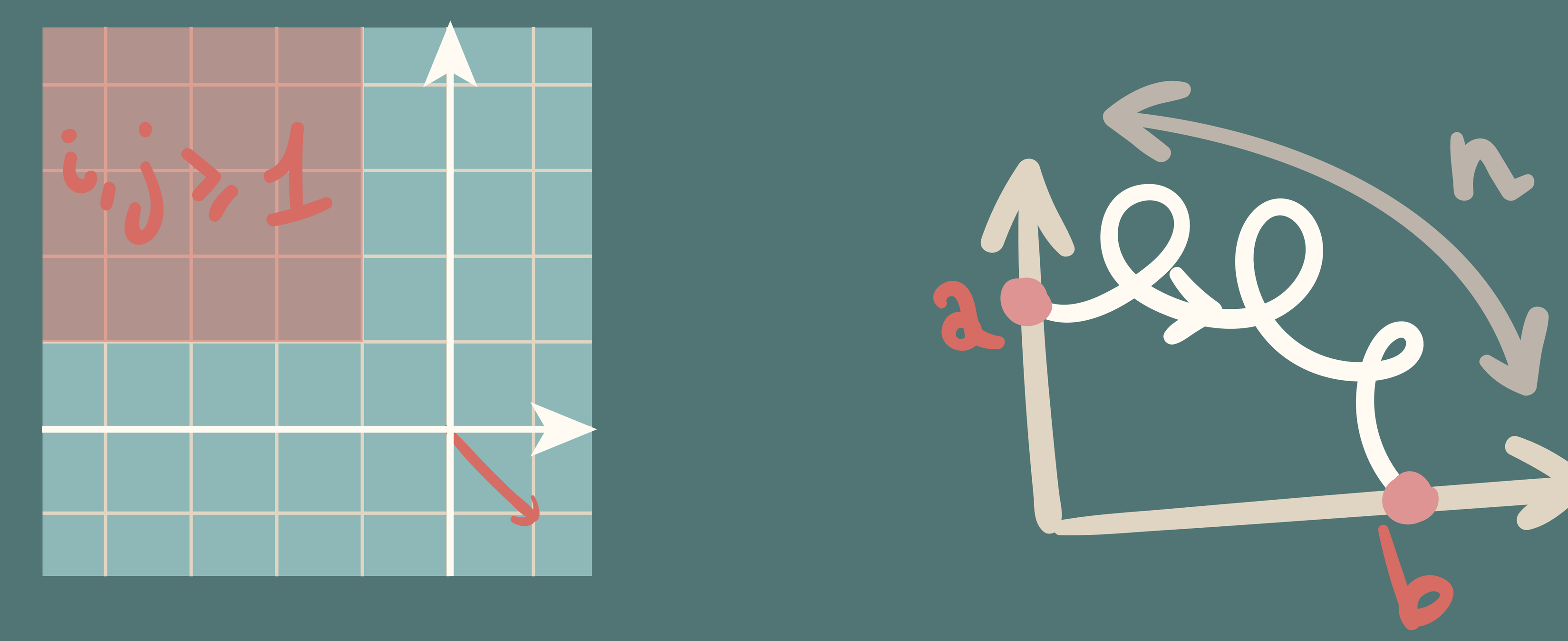
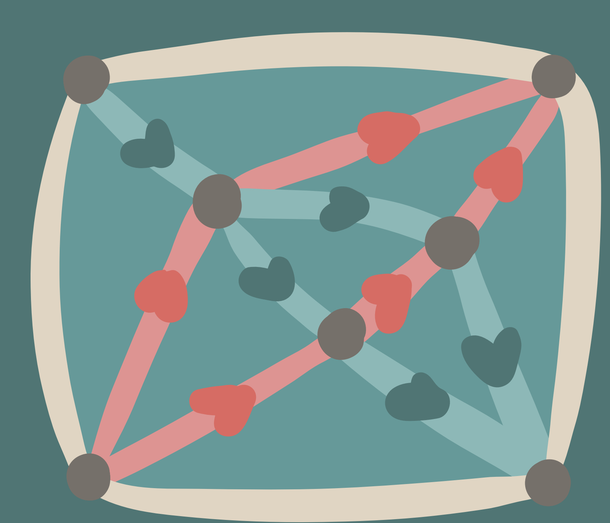
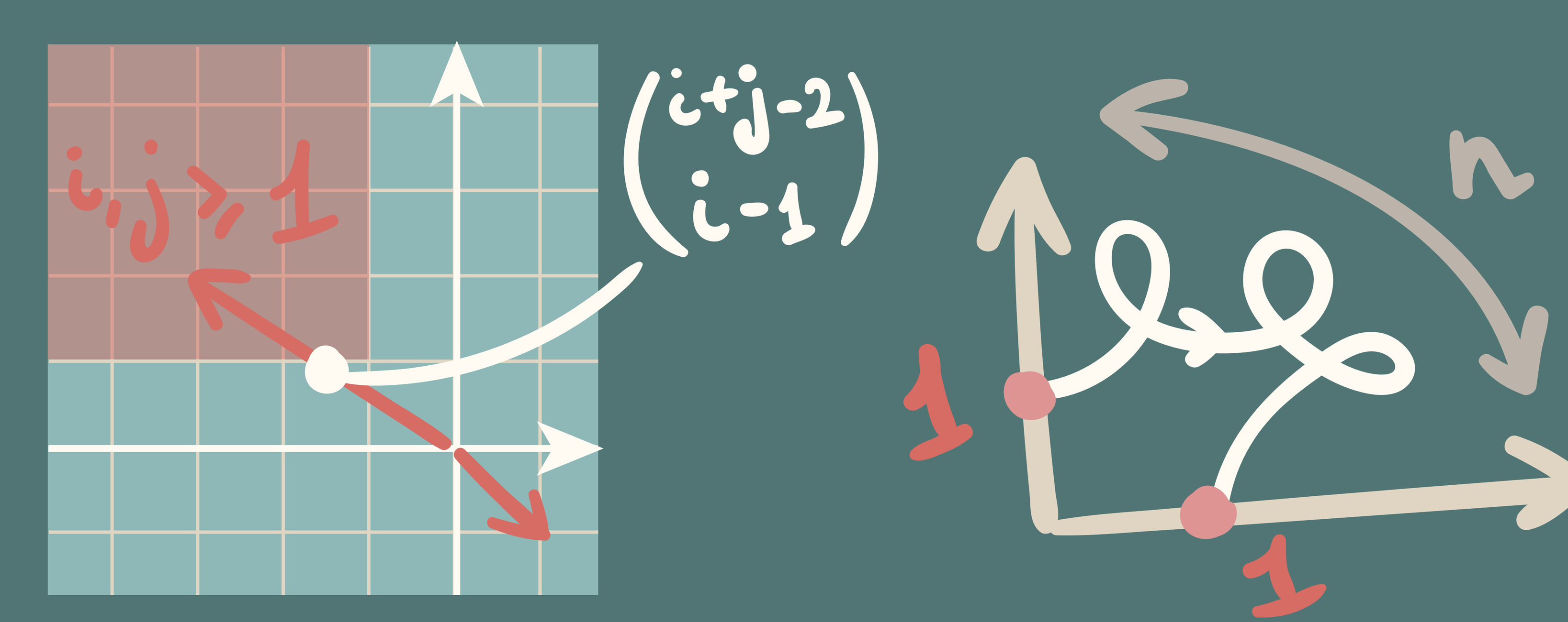
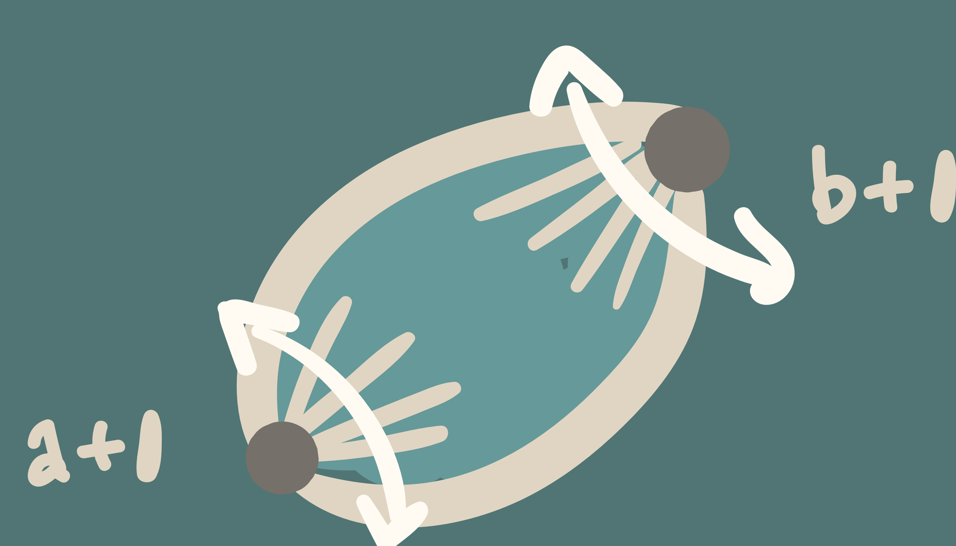
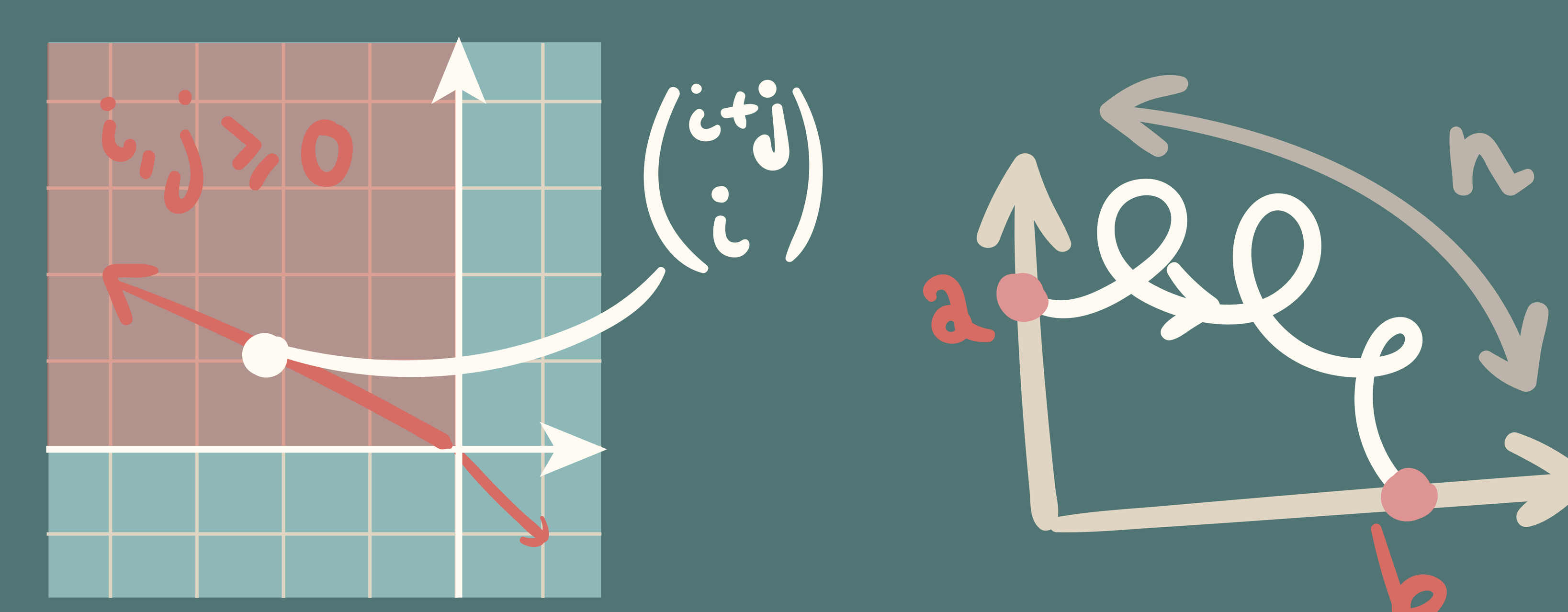
Posets *n+2 edges* 	←*bijection*→	 
Transversal structures *n blue edges* 	←*bijection*→	 

Specializations summary

Model			Tandem walk
Posets *n+2 edges*		←bijection→	
Transversal structures *n blue edges*		←bijection→	
Posets *n vertices*		←bijection→	

Summary

The KMSW bijection

1. Application to three map models

a. Plane bipolar posets
b. Plane bipolar posets by vertices
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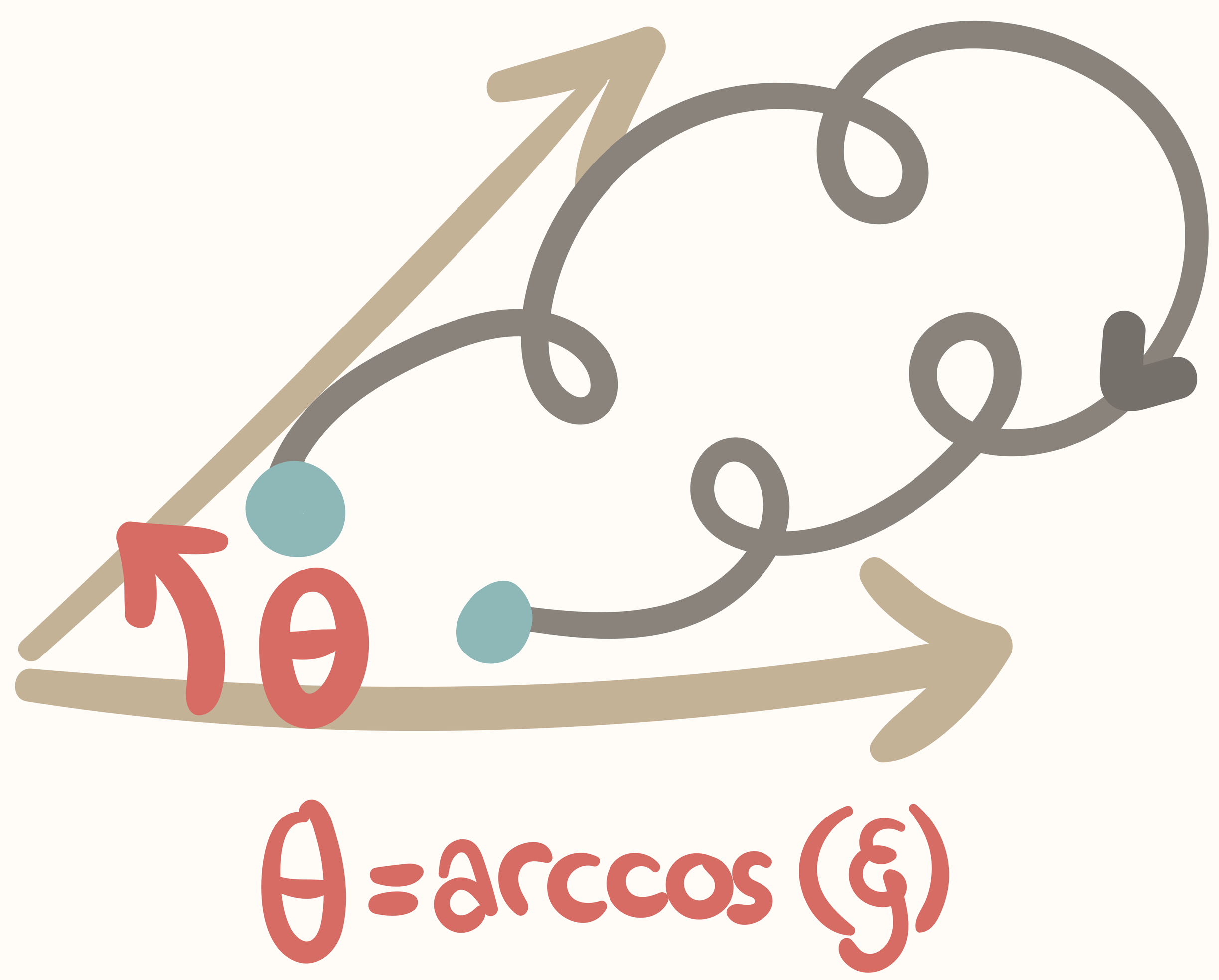
Asymptotic counting results



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$$a_n \sim \varkappa \cdot \gamma^n n^{-1-\frac{\pi}{\arccos(\xi)}}$$

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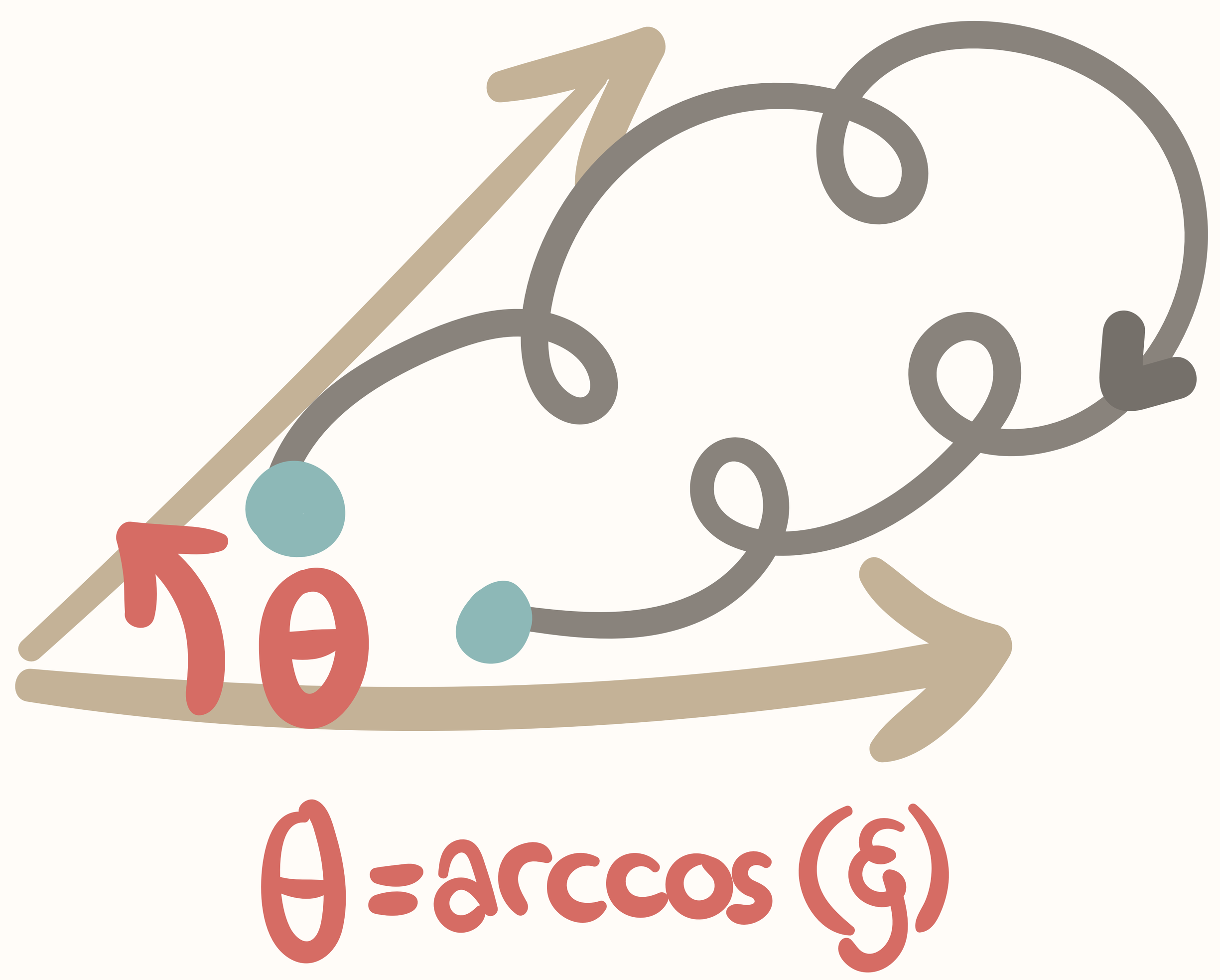

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If the drift is zero, *i.e.* :

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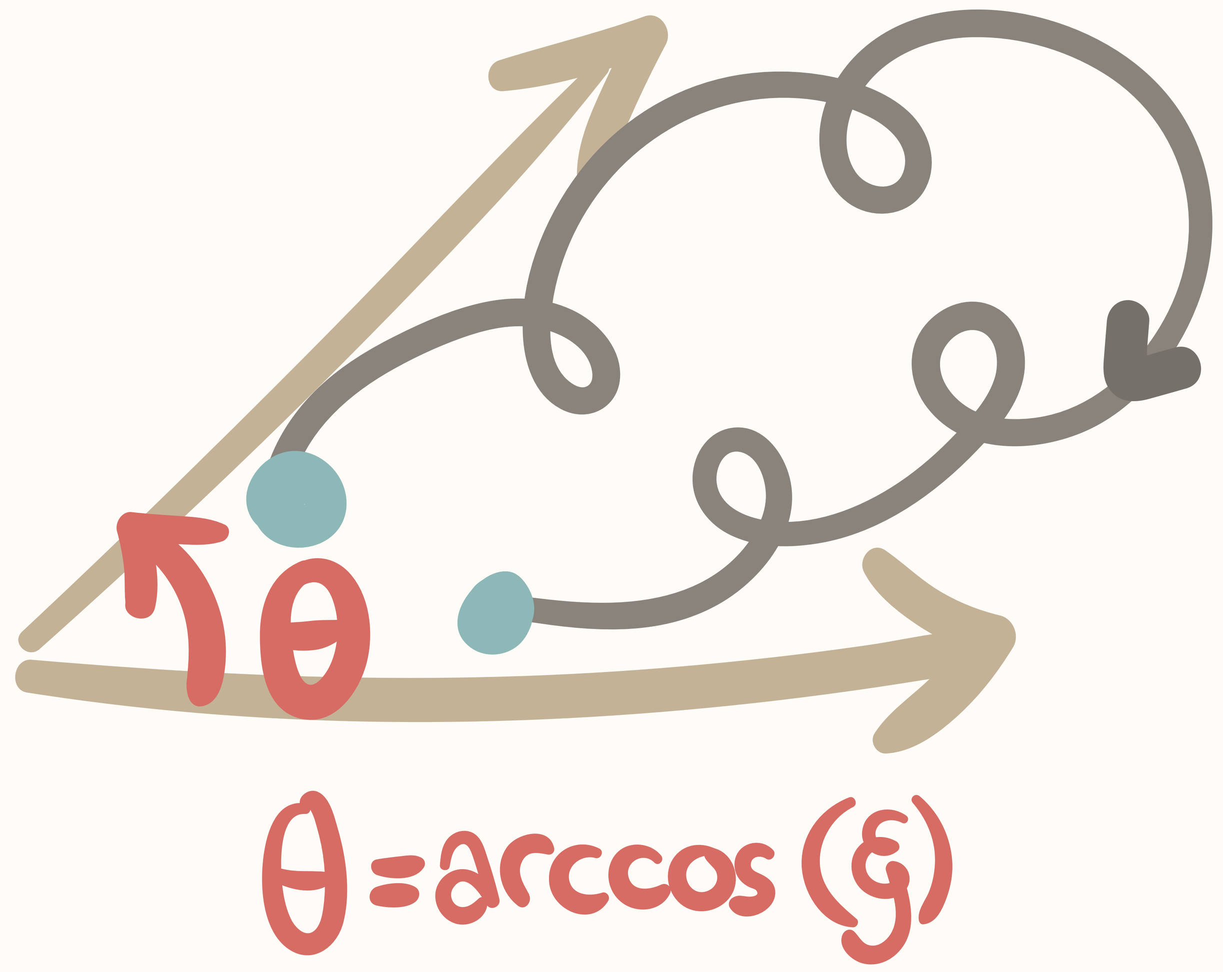

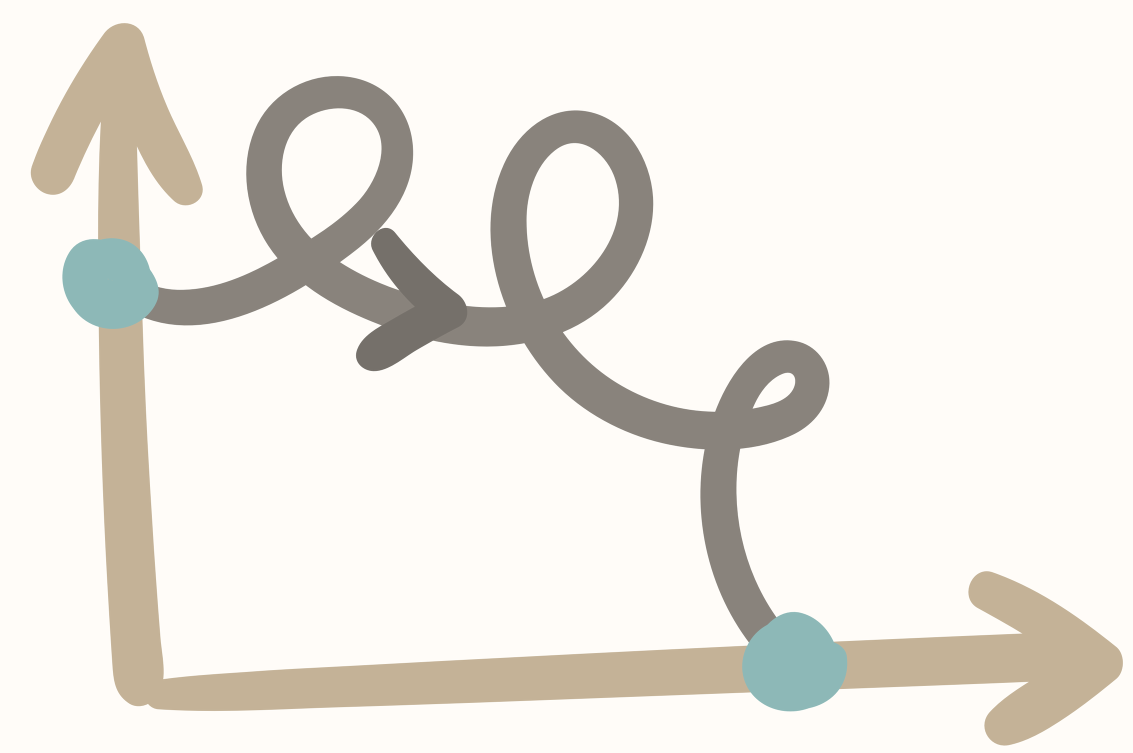

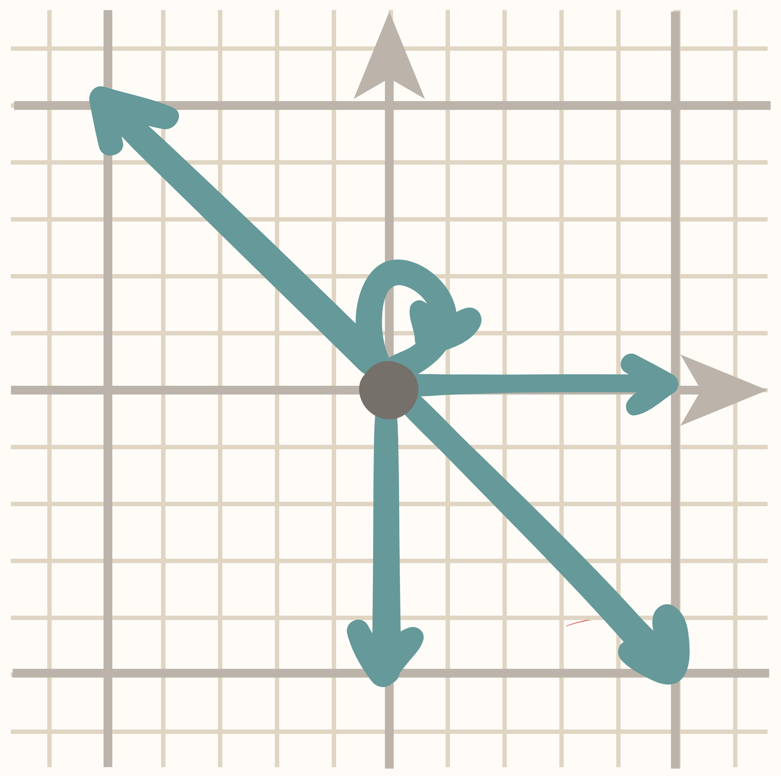

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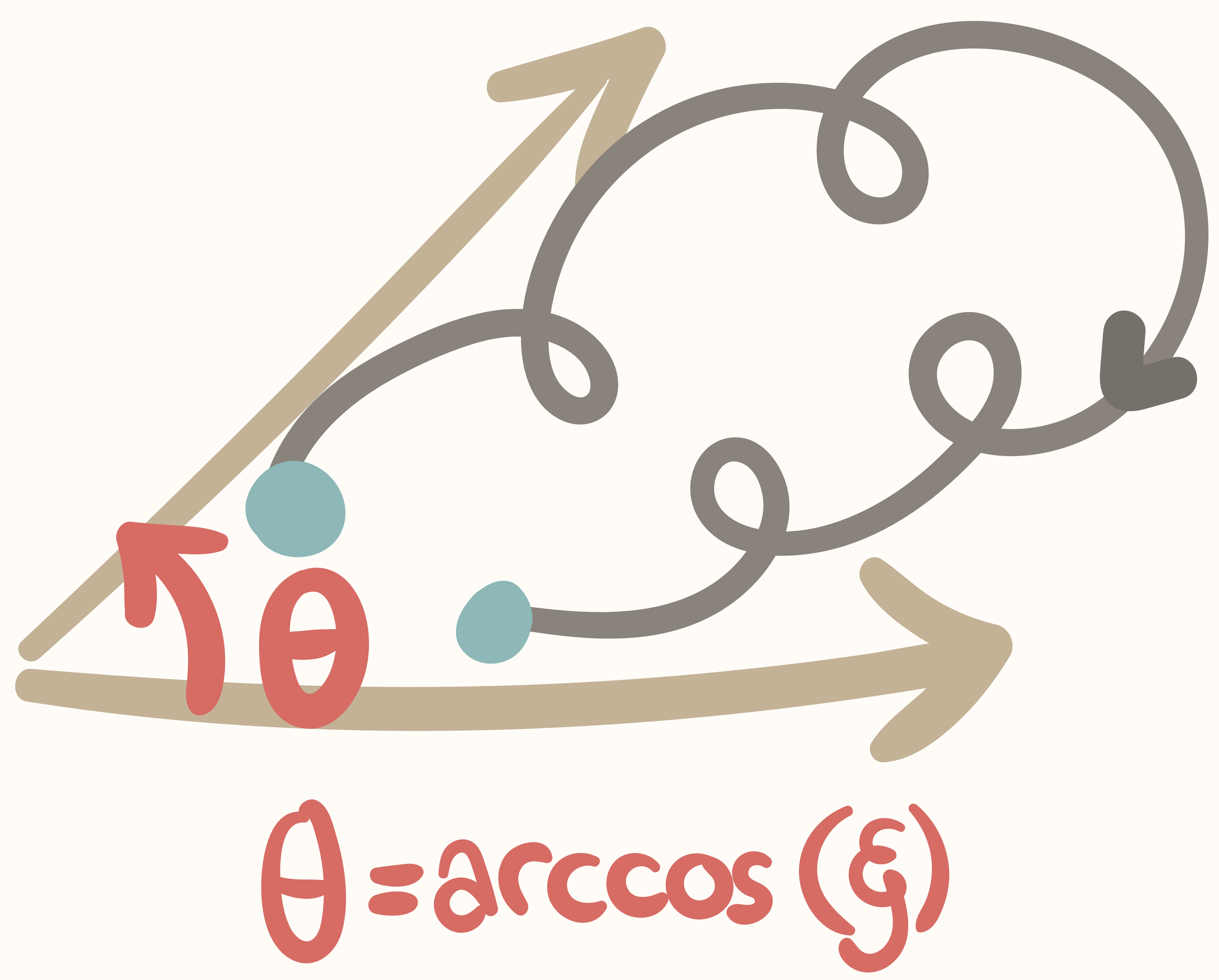

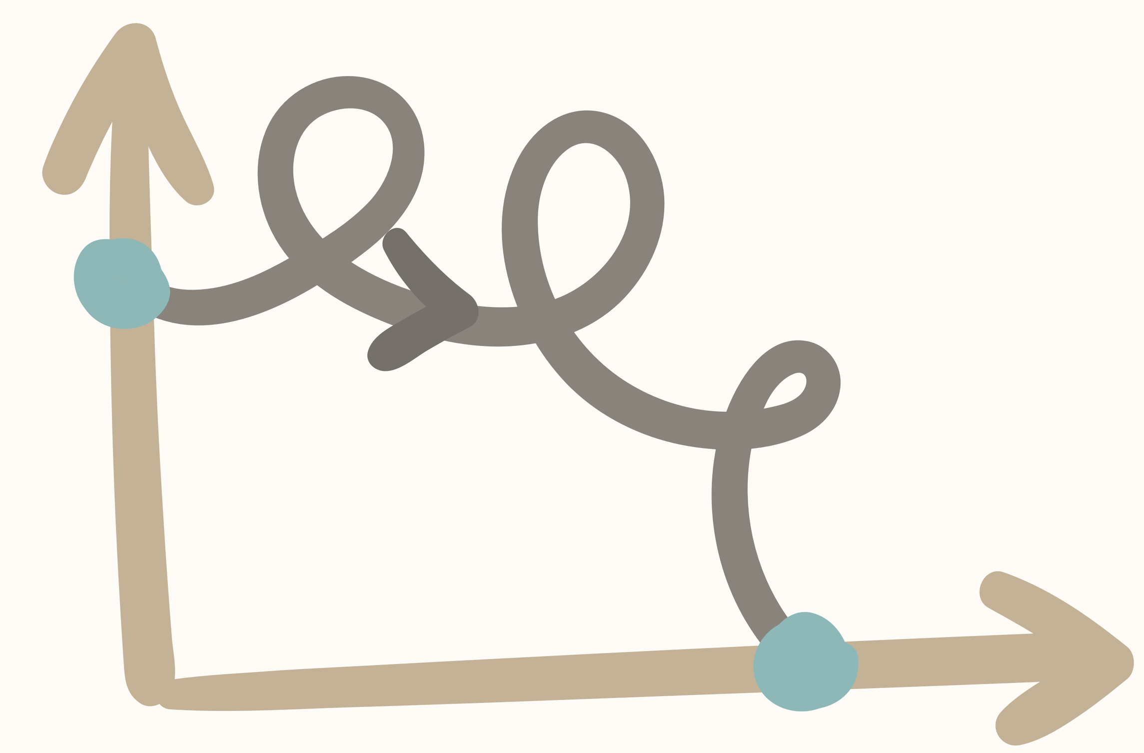

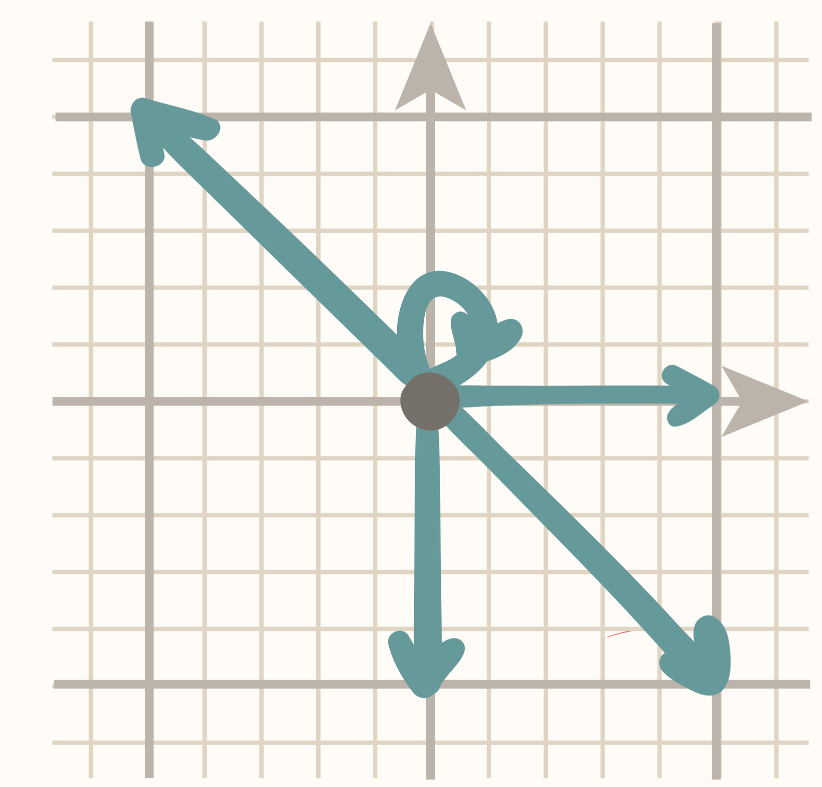

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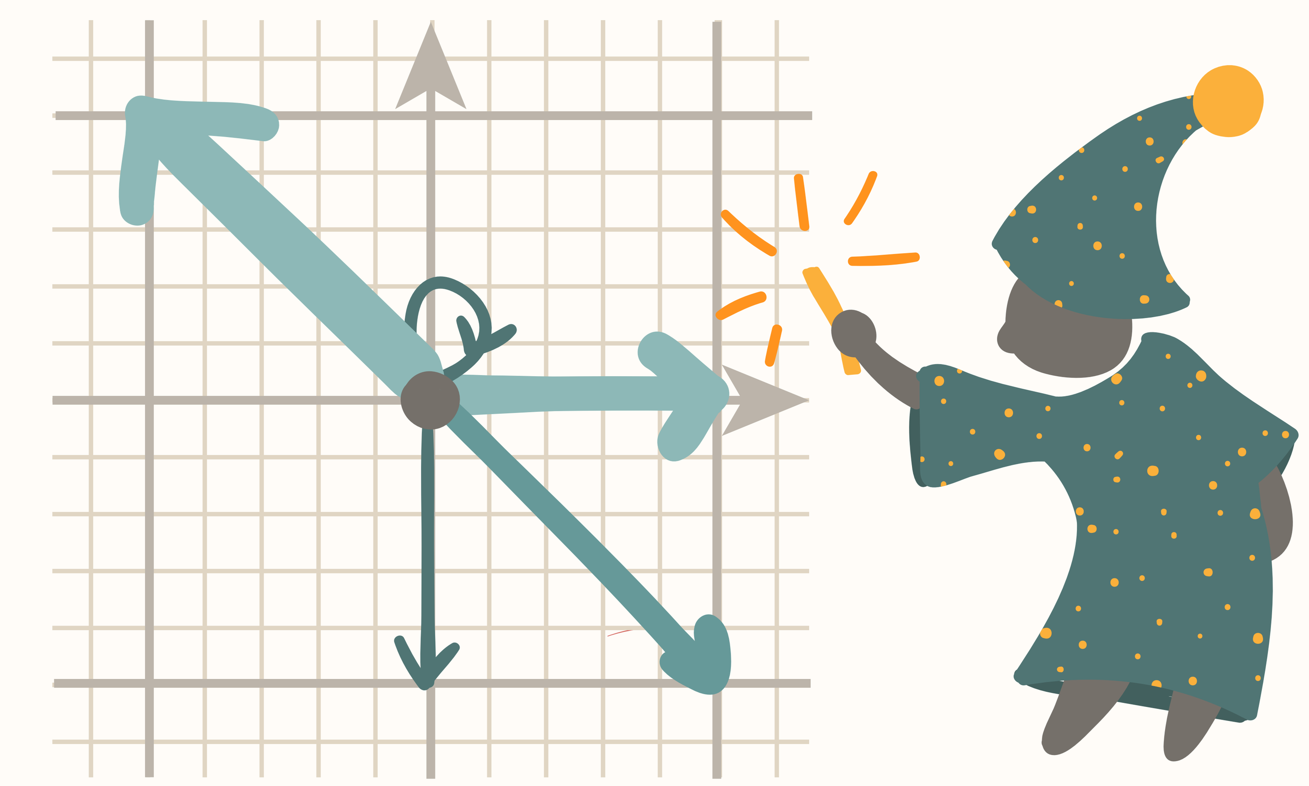

Weighted steps

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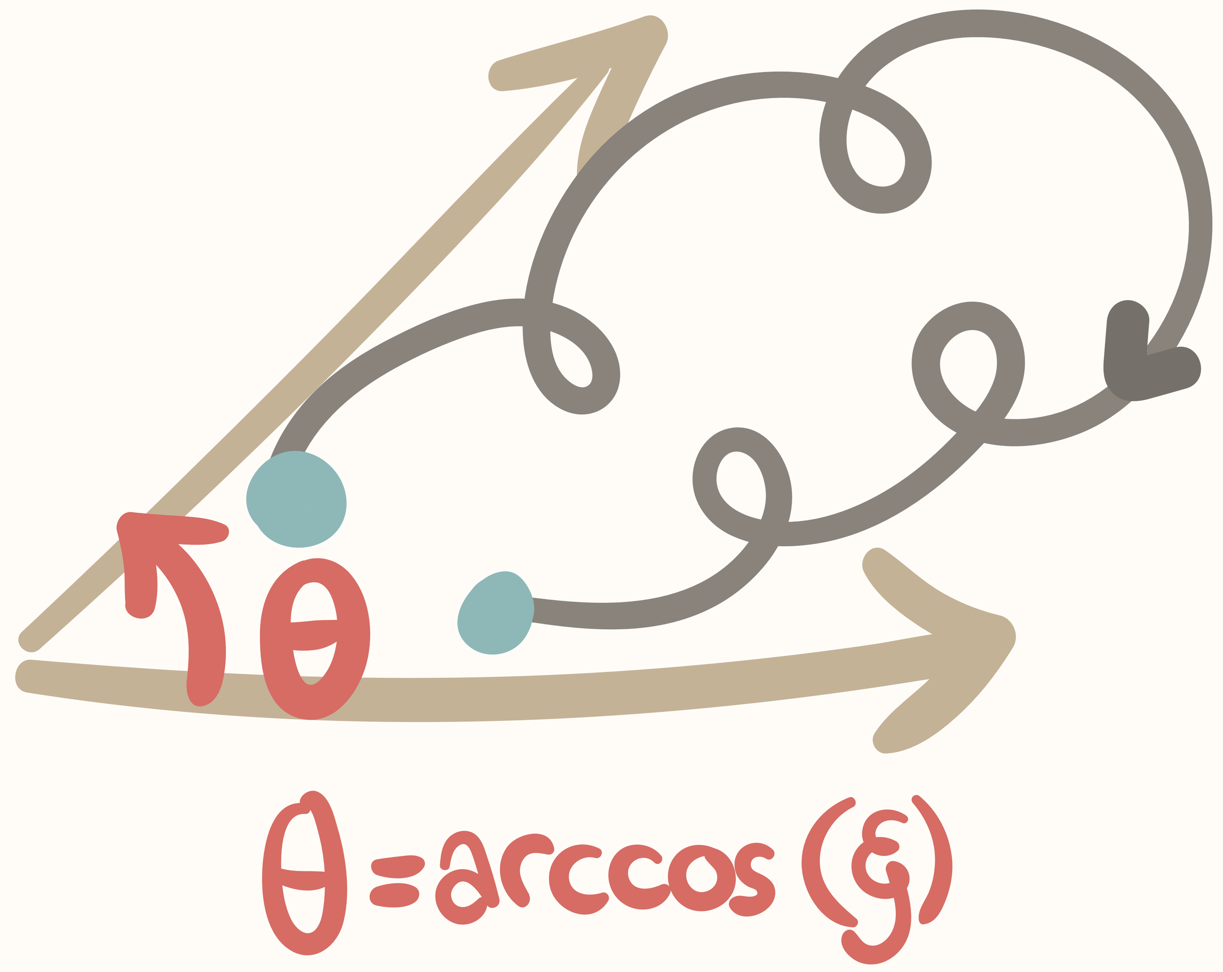
Asymptotic counting results



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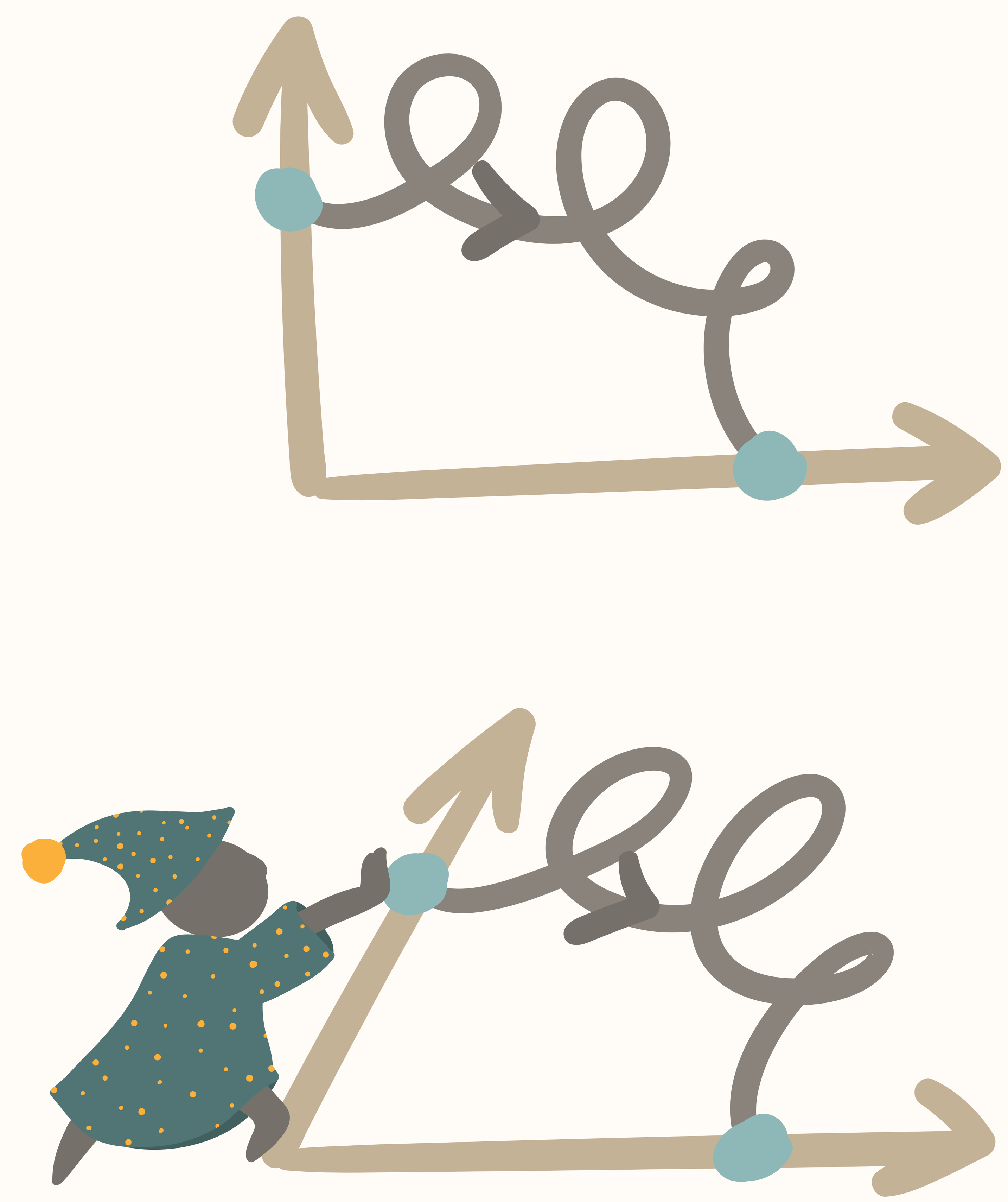
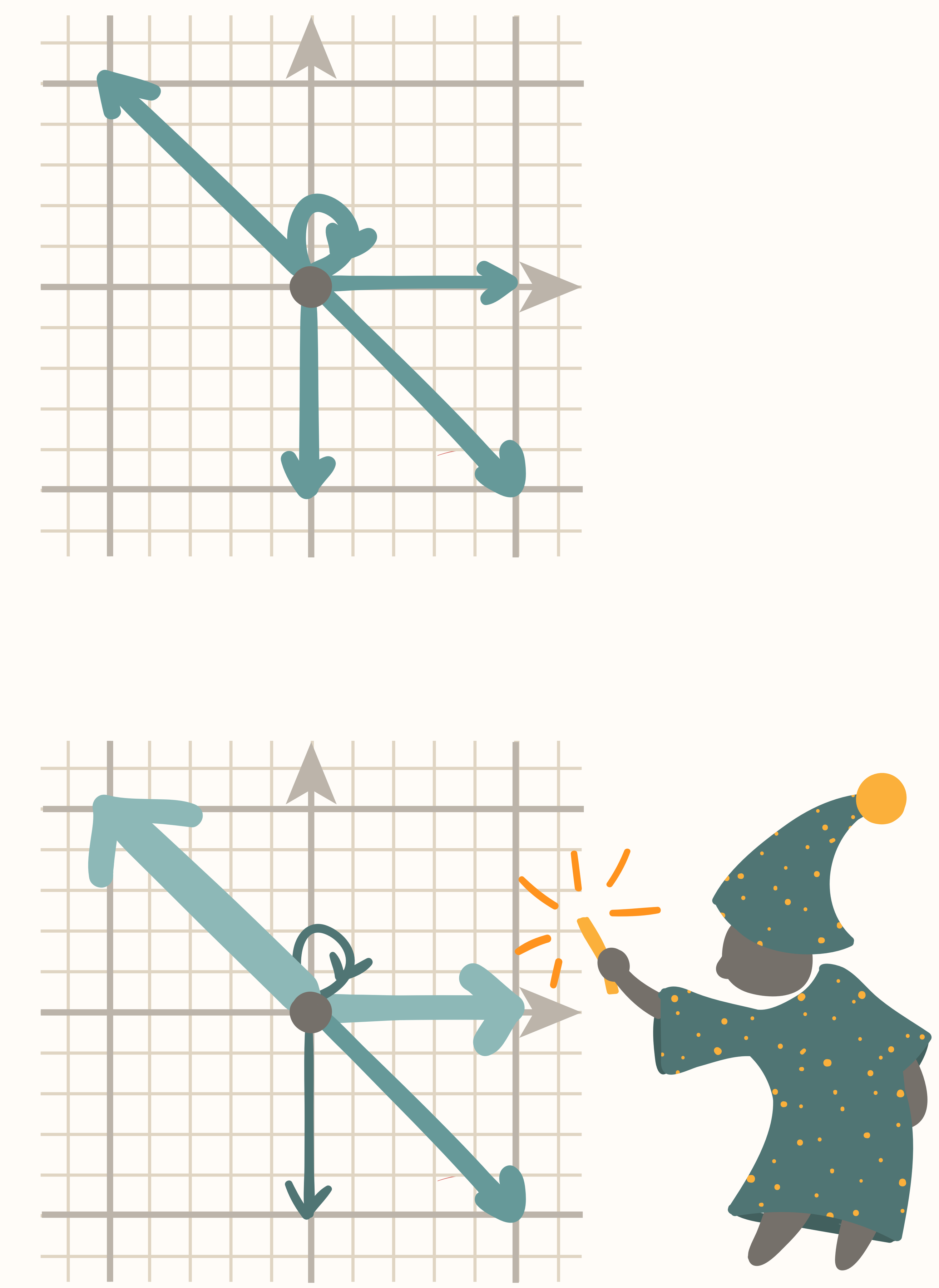


Shear transformation



Weighted steps

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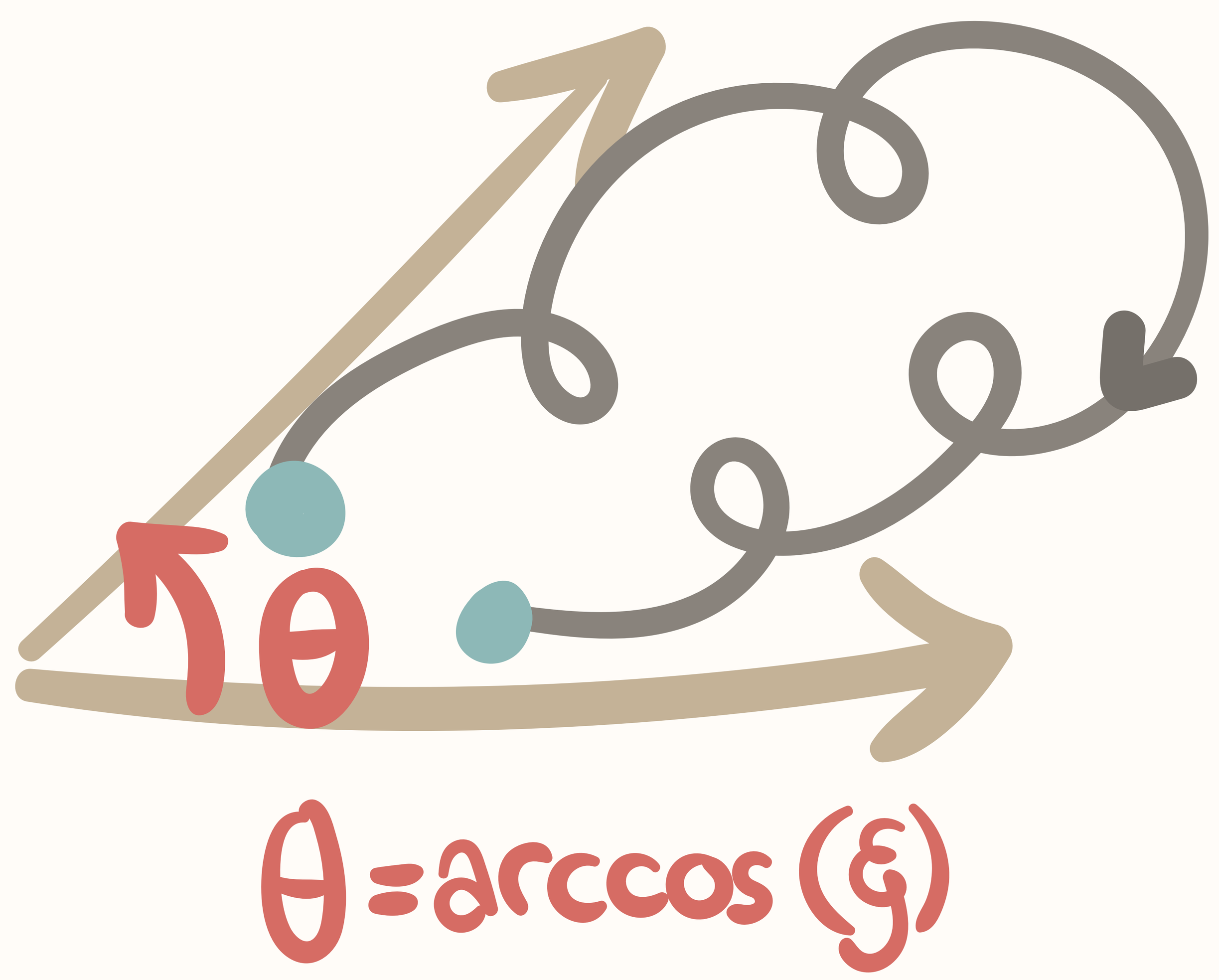

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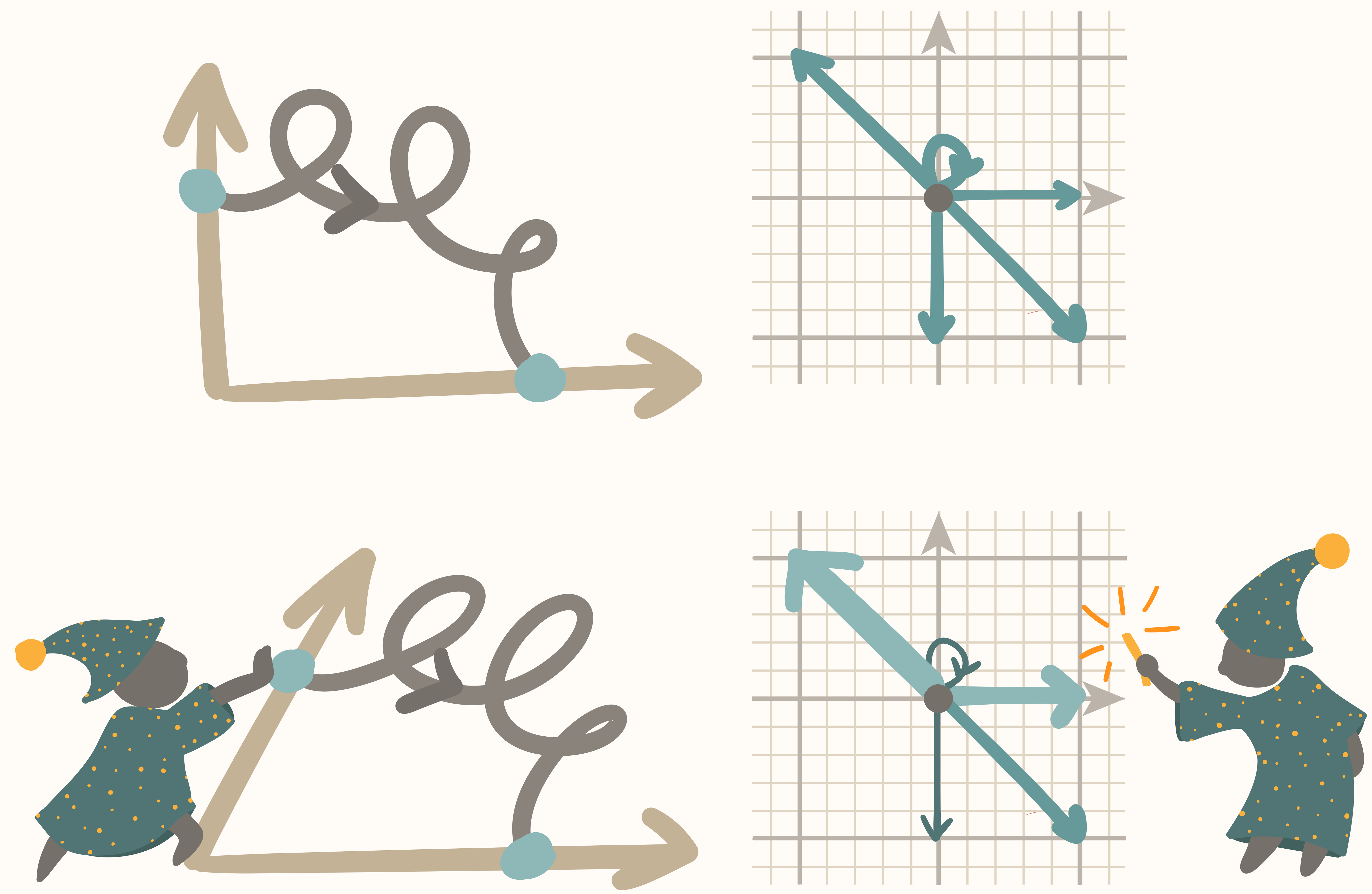
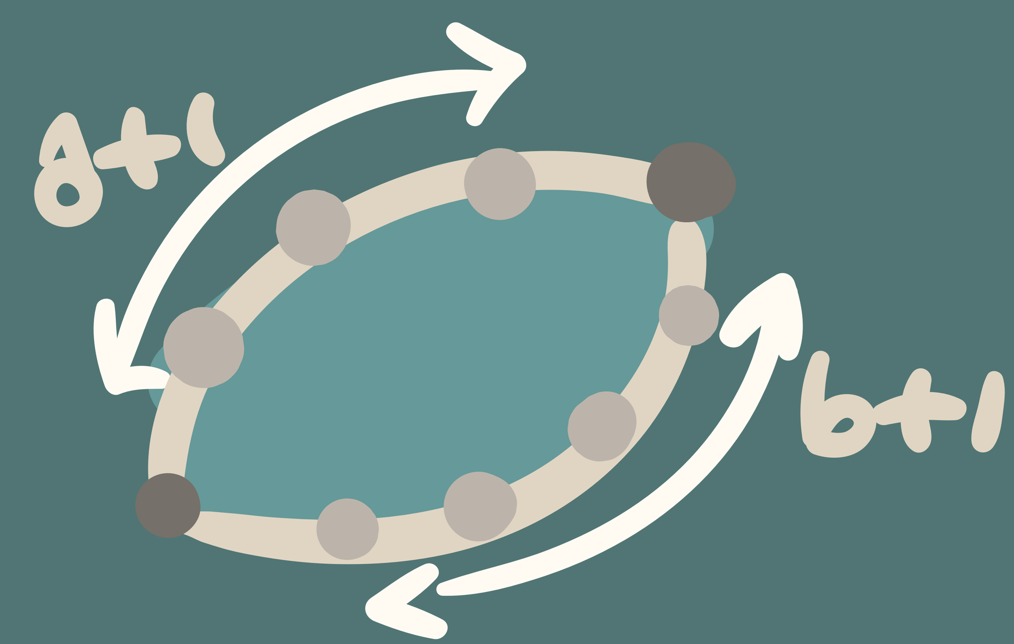


Shear transformation

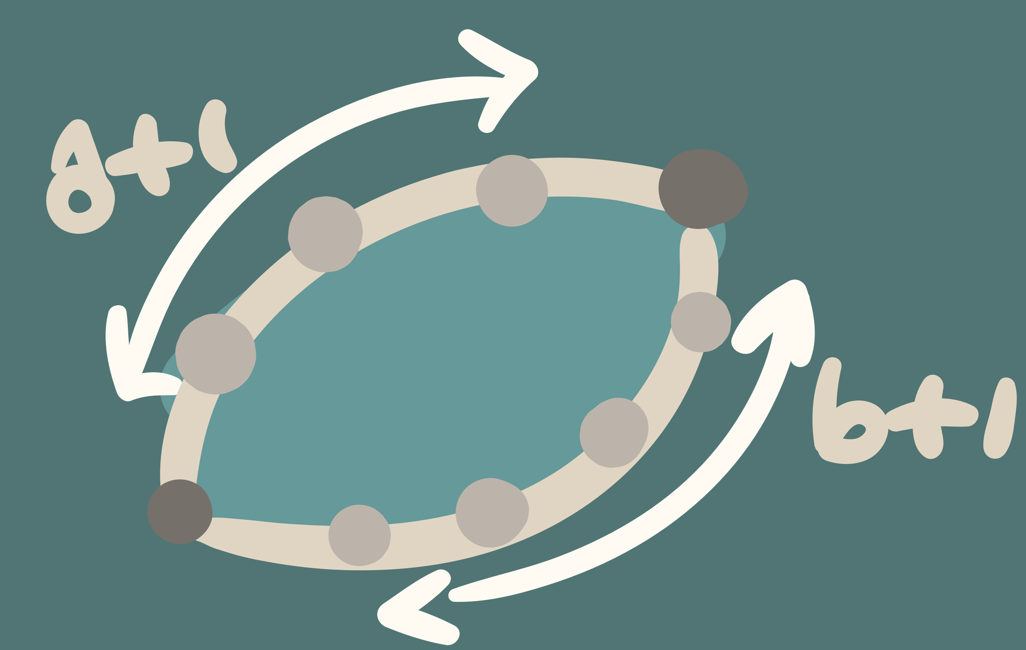
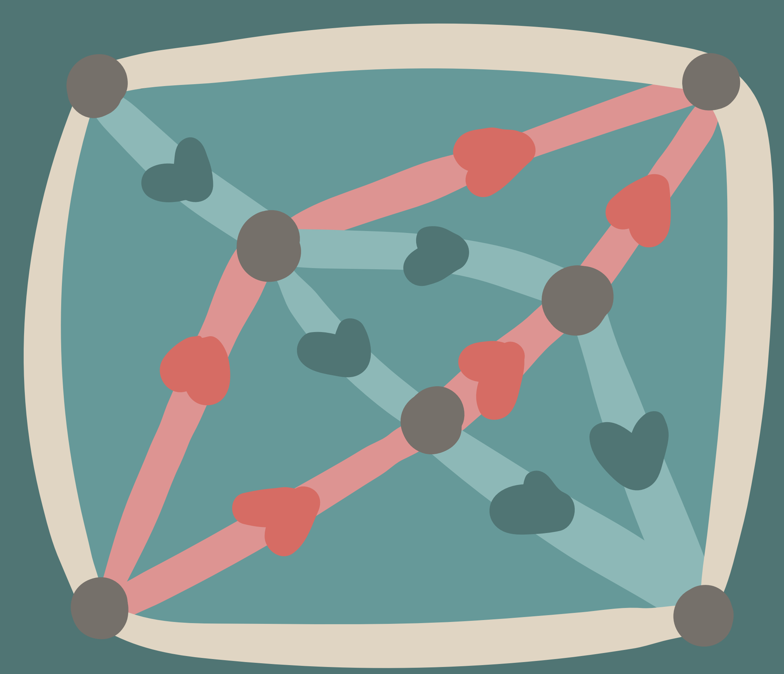
Weighted steps

Asymptotics

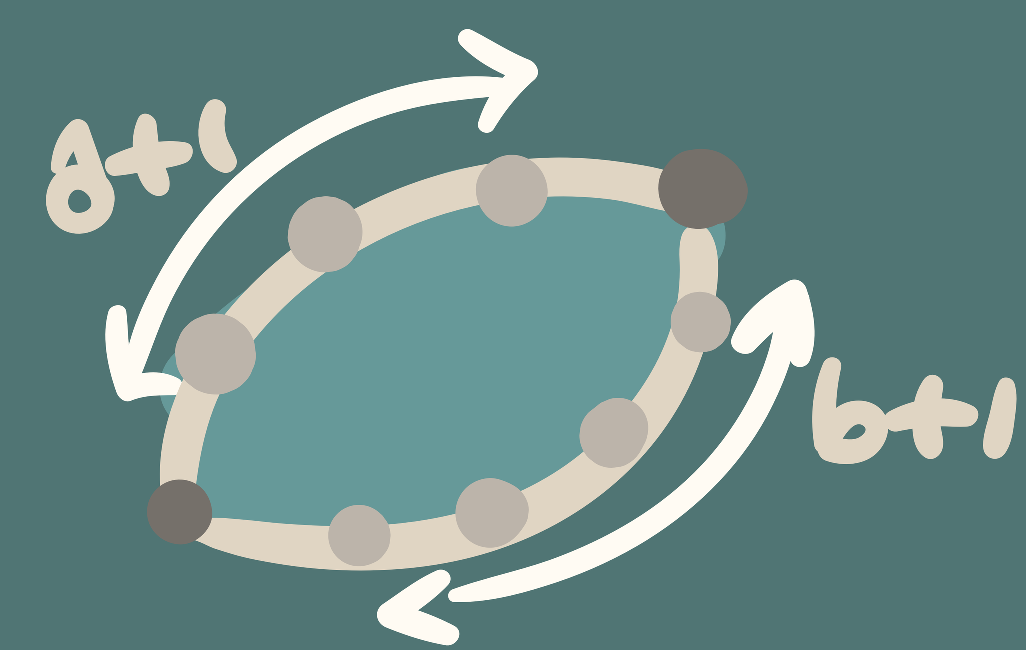
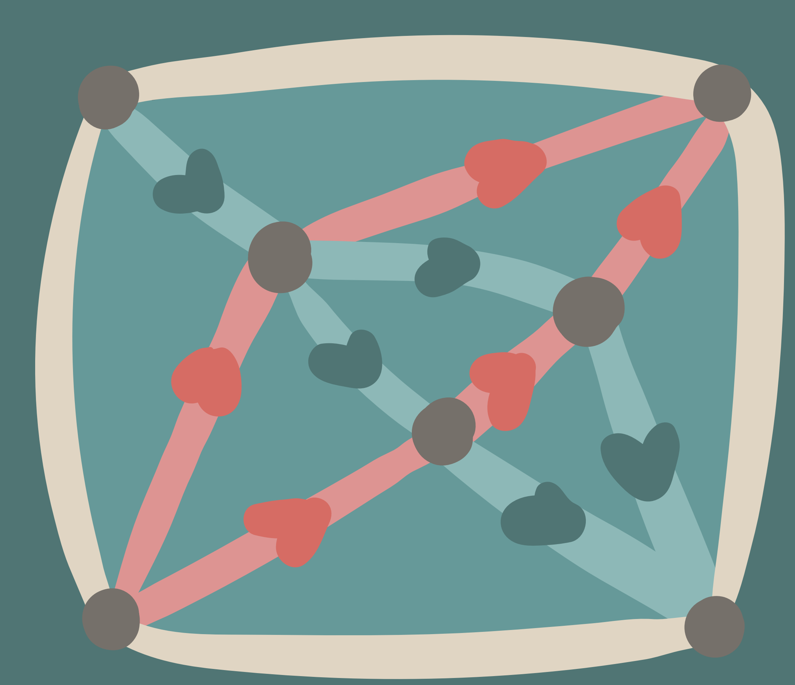
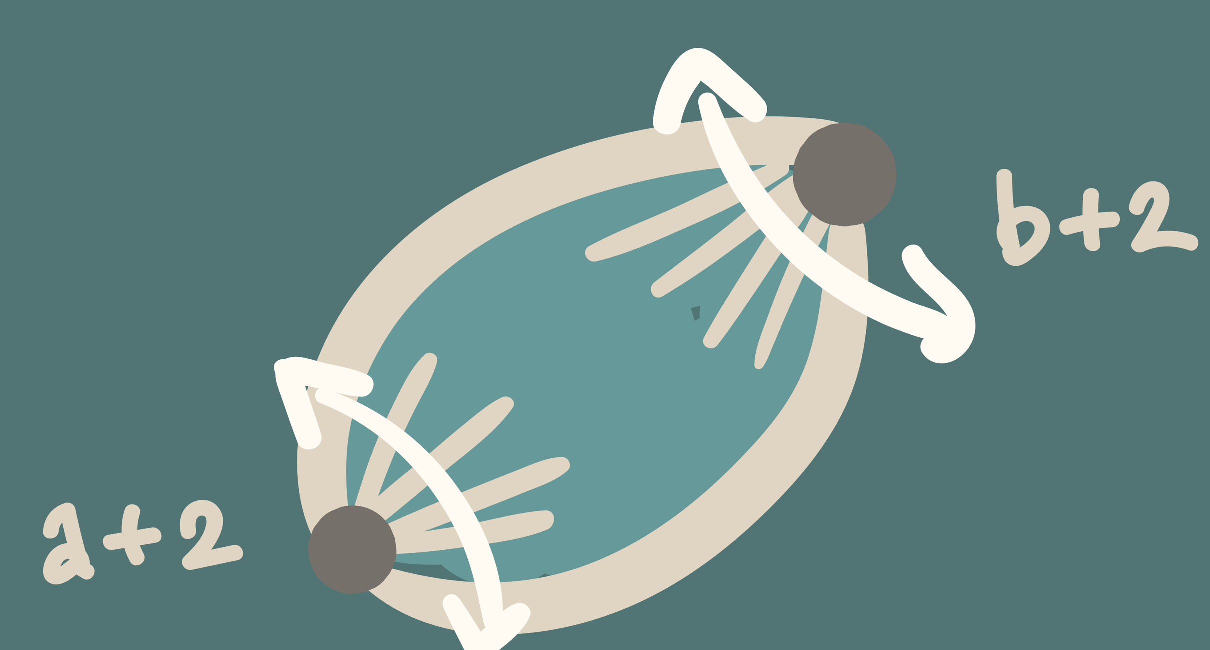
Asymptotic counting results

Model	*Asymptotics*
Posets *n+2 edges*  	$e_n \sim \varkappa\, \gamma^n n^{-\alpha}$ γ *and* α *are algebraic* $\gamma \approx 4.80\ldots$ $\alpha \approx 5.14\ldots$

Asymptotic counting results

Model		*Asymptotics*
Posets *n+2 edges*		$e_n \sim \varkappa \, \gamma^n n^{-\alpha}$ γ *and* α *are algebraic* $\gamma \approx 4.80\ldots \quad \alpha \approx 5.14\ldots$
Transversal structures *n vertices*		$t_n \sim \varkappa \left(\frac{27}{2}\right)^n n^{-1-\frac{\pi}{\arccos(7/8)}}$ ↠ *Counting rectangular drawings,* Y. Inoue, T. Takahashi & R. Fujimaki (2009)

Asymptotic counting results

Model	*Asymptotics*
Posets *n+2 edges* 	$e_n \sim \varkappa\, \gamma^n n^{-\alpha}$ γ *and* α *are algebraic* $\gamma \approx 4.80\ldots$ $\alpha \approx 5.14\ldots$
Transversal structures *n vertices* 	$t_n \sim \varkappa \left(\frac{27}{2}\right)^n n^{-1-\frac{\pi}{\arccos(7/8)}}$ → *Counting rectangular drawings,* Y. Inoue, T. Takahashi & R. Fujimaki (2009)
Posets *n vertices* 	$b_n \sim \varkappa \left(\frac{11+\sqrt{5}}{2}\right)^n n^{-6}$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bn ~ ϰ((11 + √5)/2)^n · n^-6

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POSETS PER VERTEX

$$b_n \sim \kappa \left(\frac{11+\sqrt{5}}{2}\right)^n \cdot n^{-6}$$

PLANE PERMUTATIONS

$$p_n \sim \kappa \left(\frac{11+\sqrt{5}}{2}\right)^n \cdot n^{-6}$$

⇛ *Semi-Baxter and strong-Baxter : two relatives of Baxter Sequences,*
M. Bouvel, V. Guerrini, A. Rechnitzer & S. Rinaldi (2018)

Asymptotic coun

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POSETS PER VERTEX

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1, 1, 2, 6, 23, 104, 530, 2958, 17734, 112657, ...

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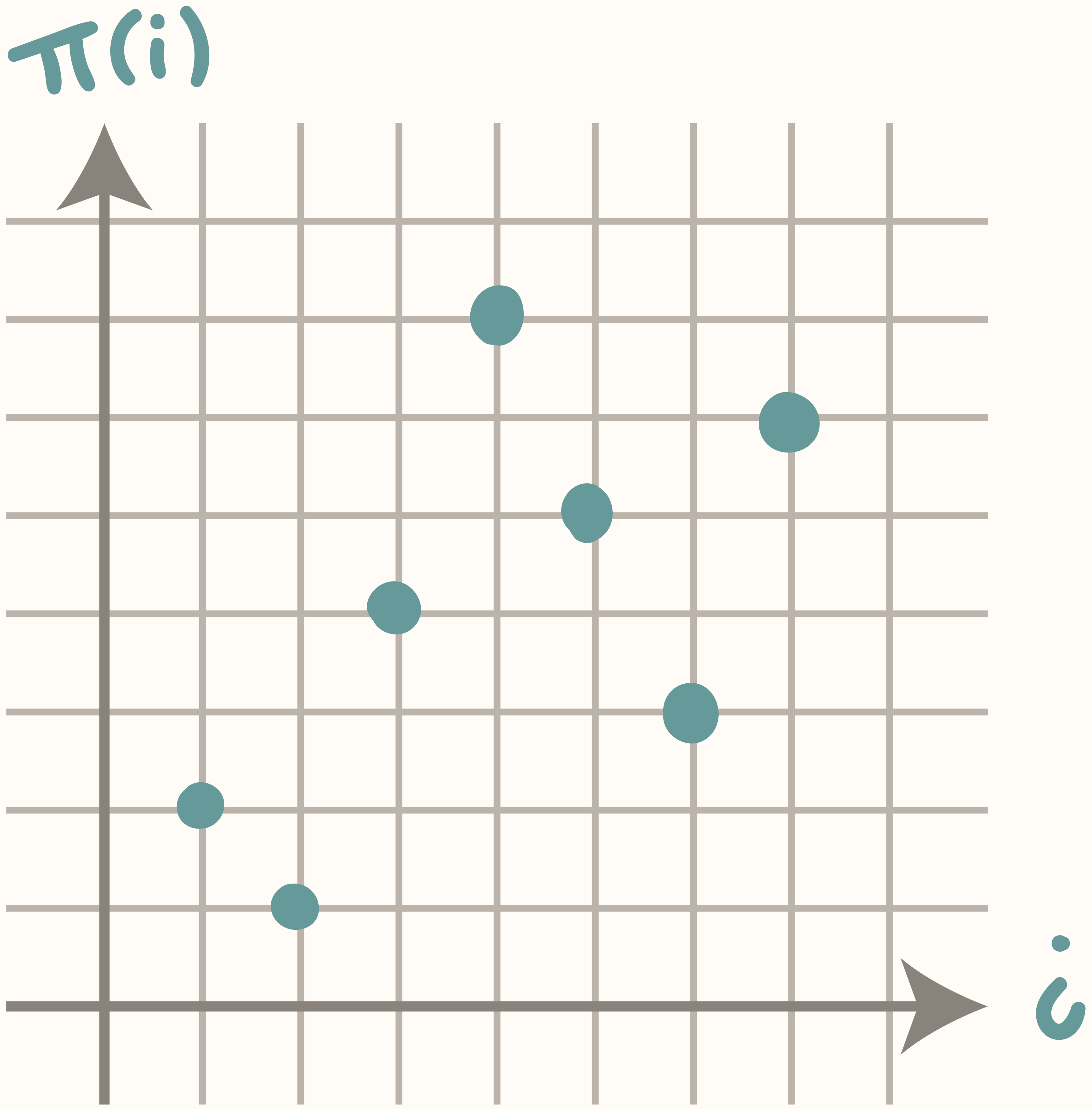
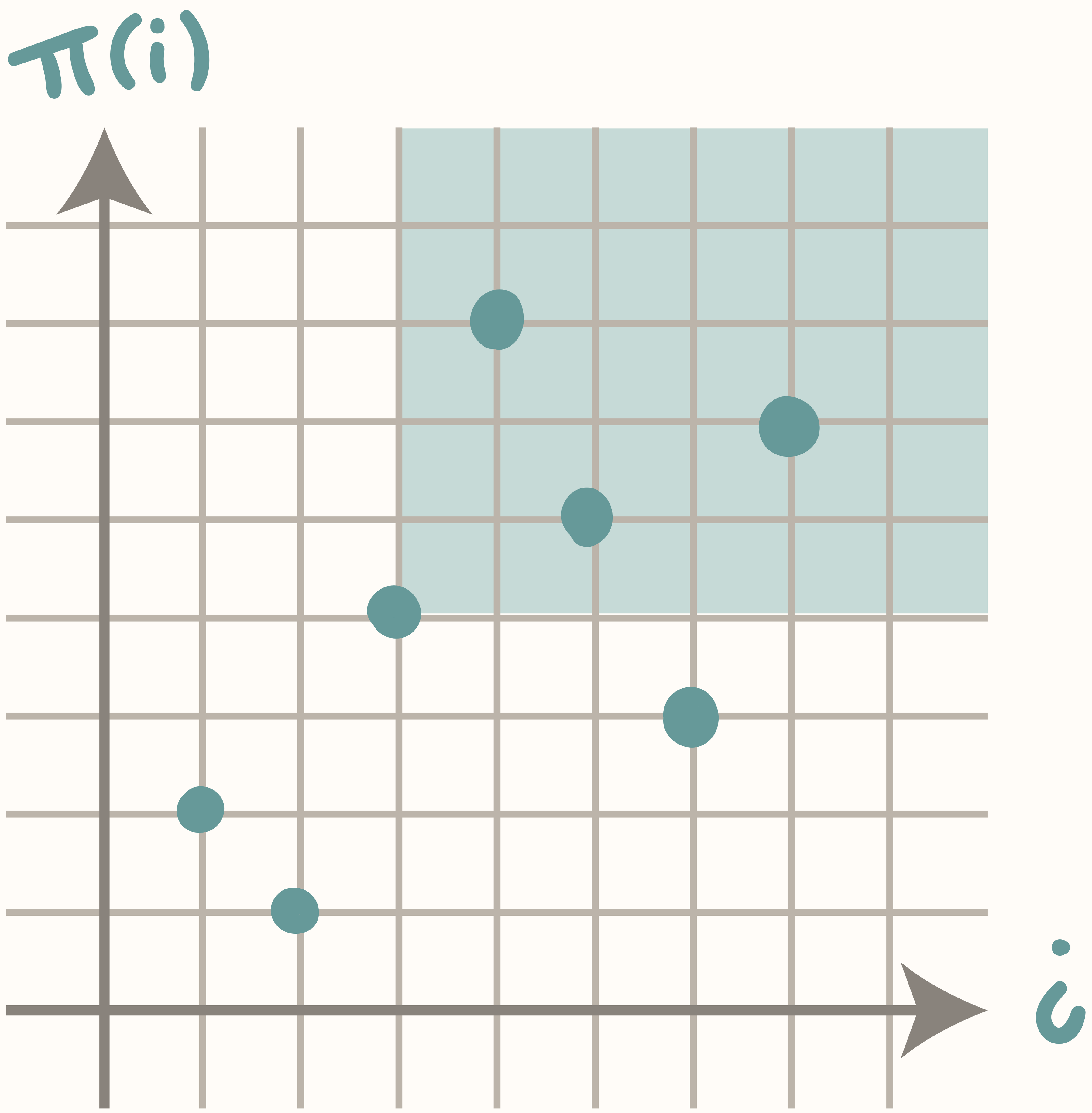
Plane permutations

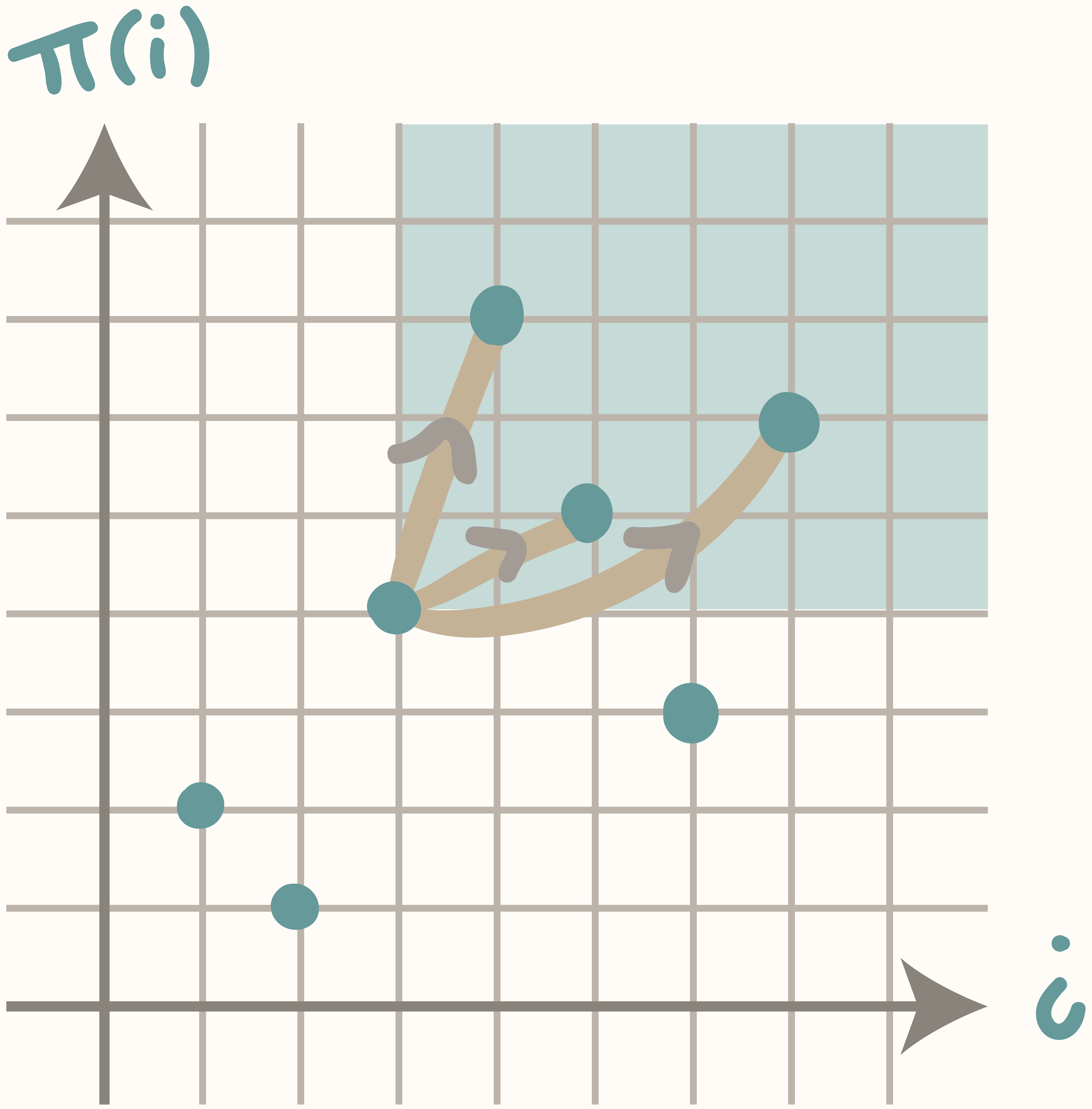


Plane permutations



Dominance relation

Plane permutations



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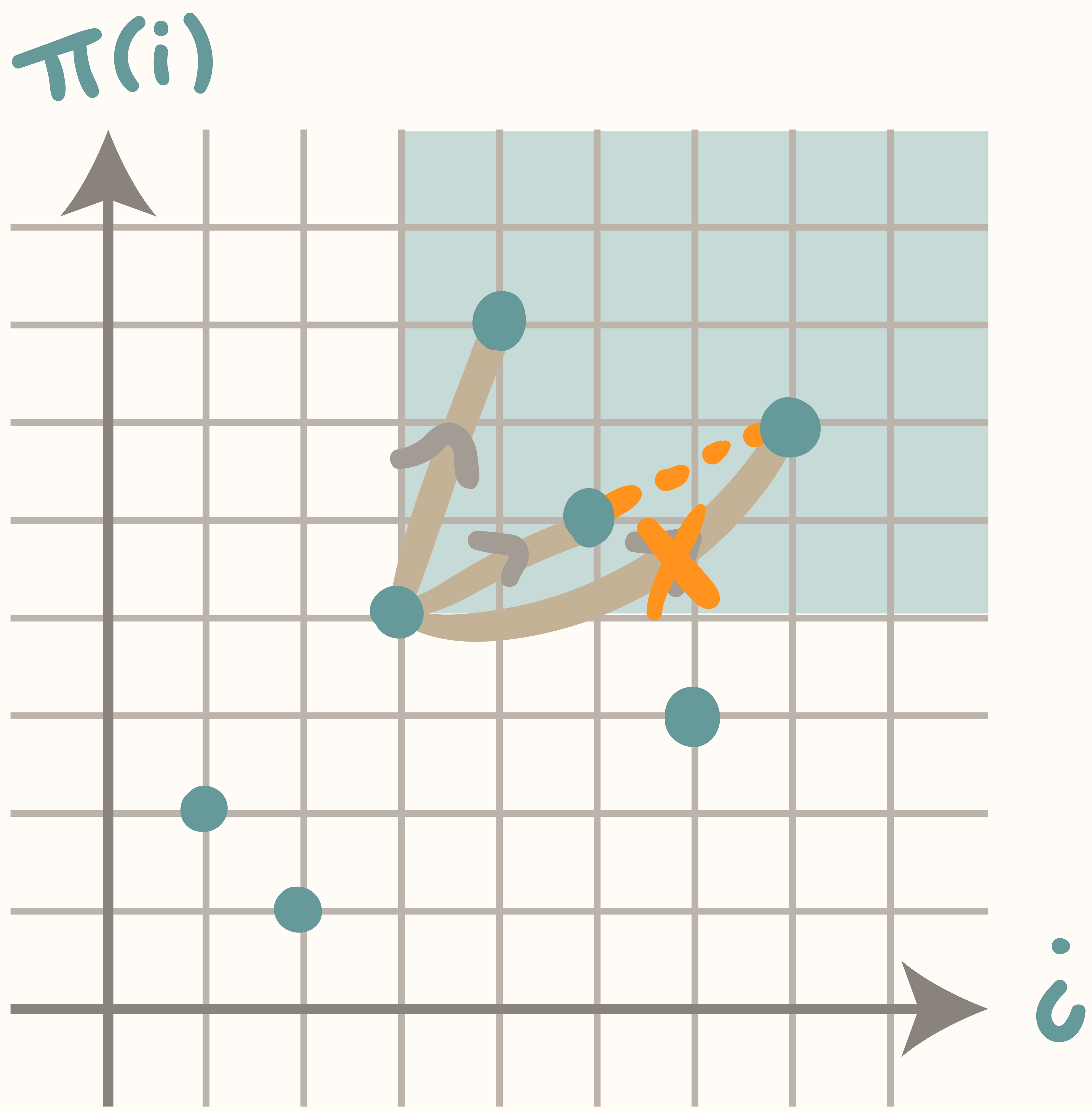
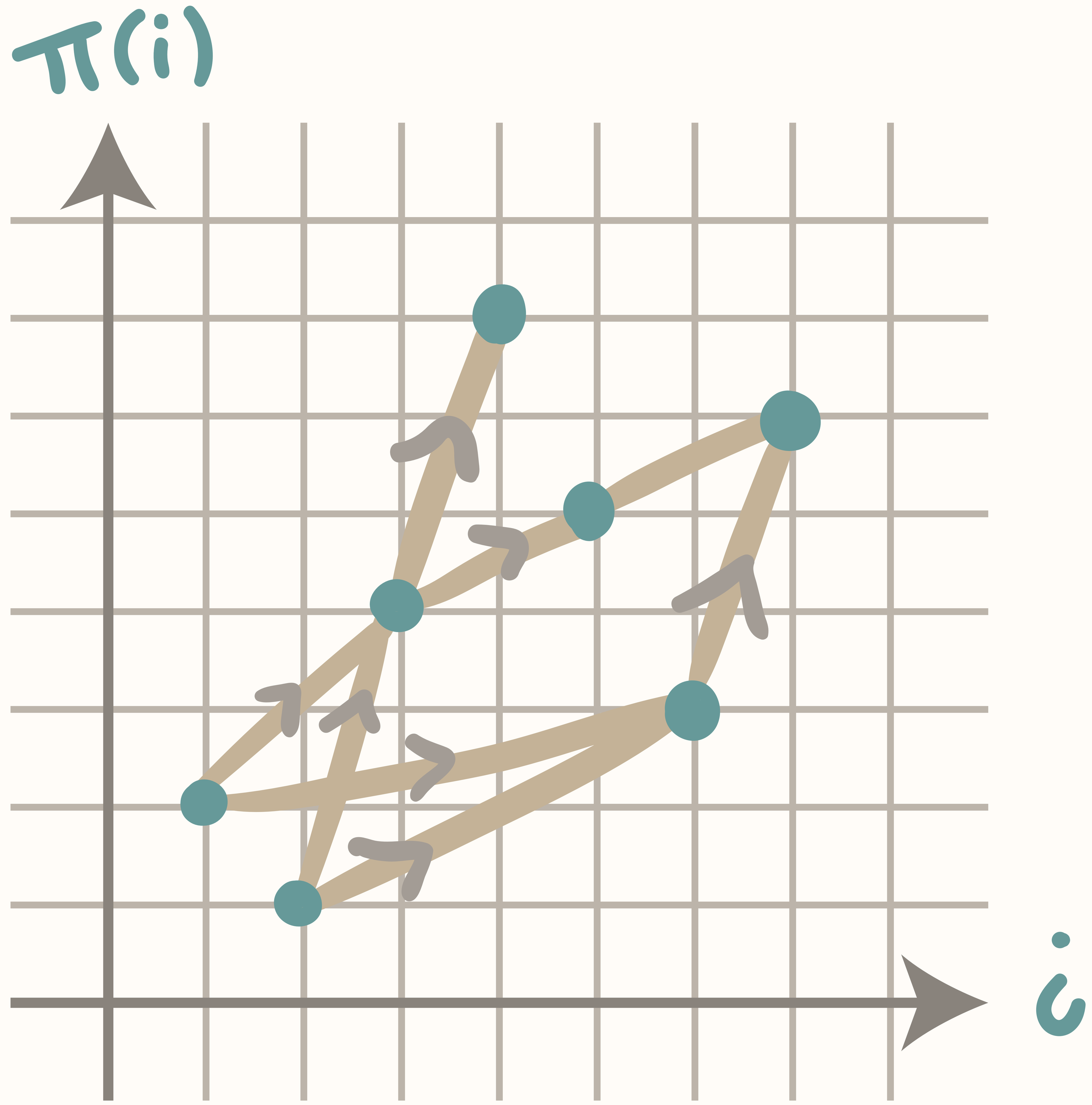


Dominance diagram

= *Dominance relation with no transitive edges*

Plane permutations



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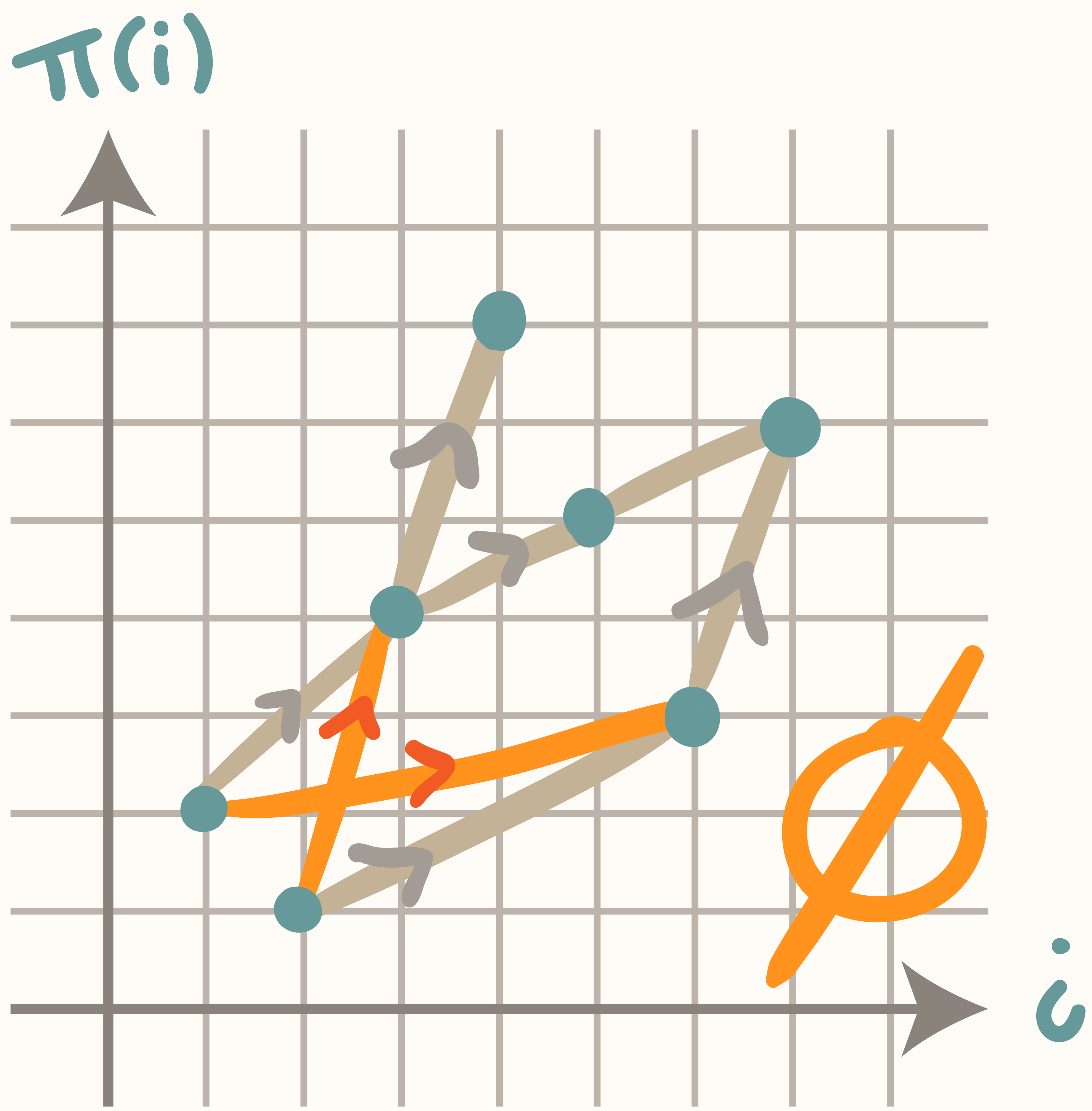
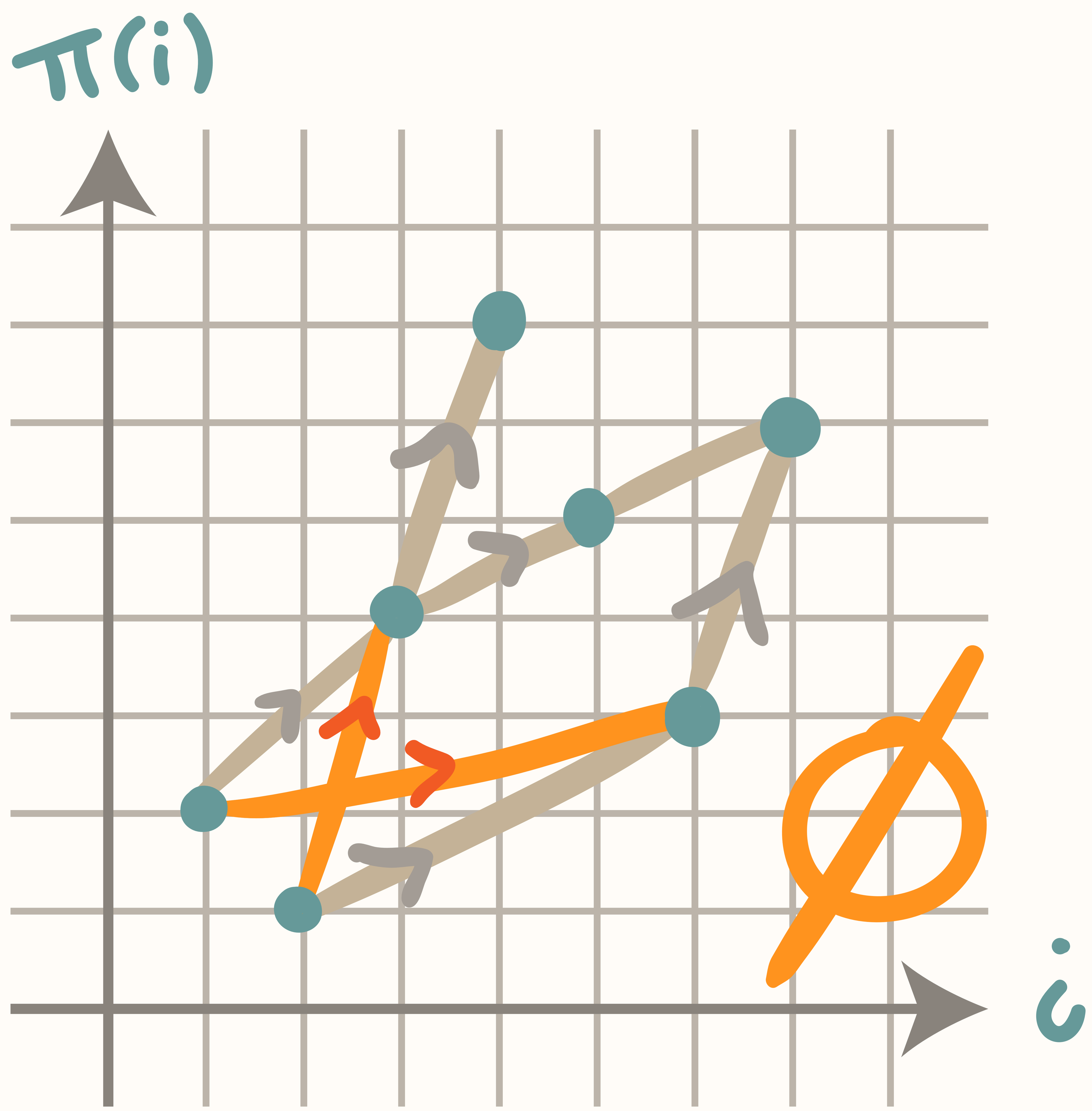


Plane permutation

= *No edge crossing in the dominance diagram*

Plane permutations

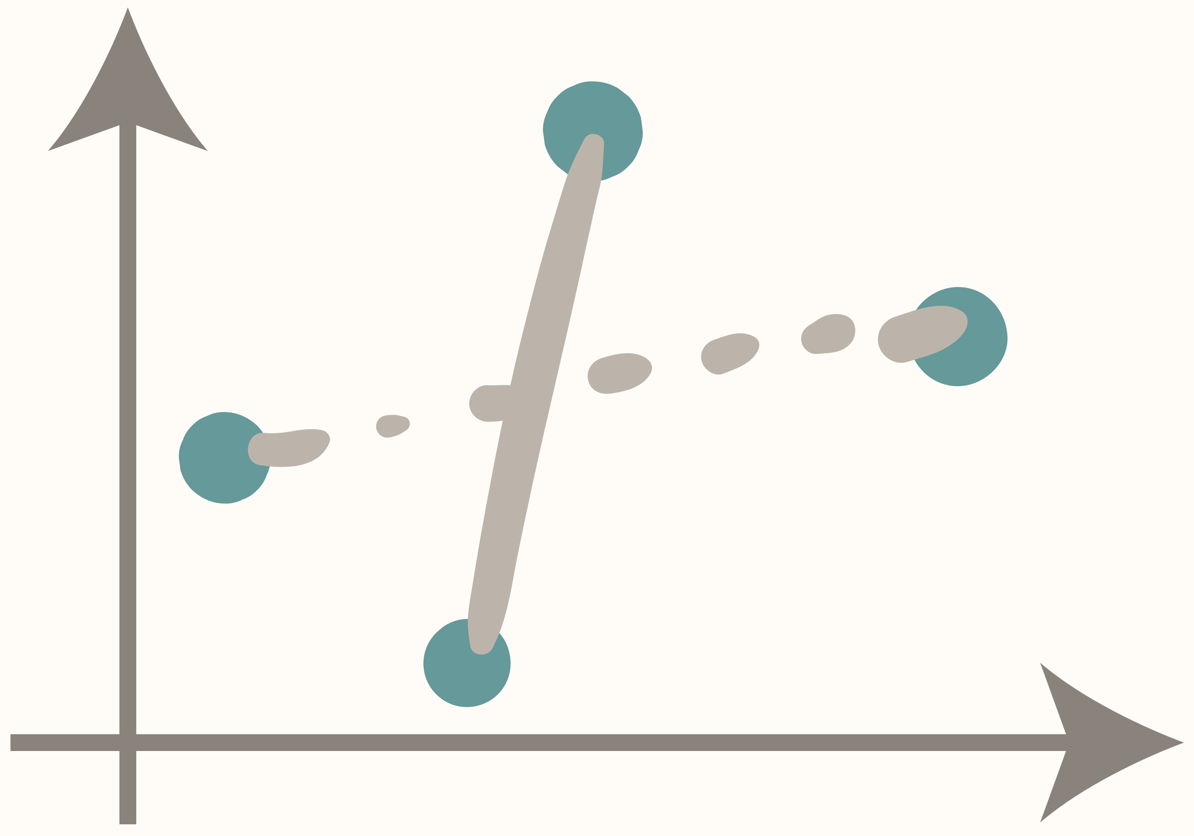


Plane permutation

= *No edge crossing in the dominance diagram*

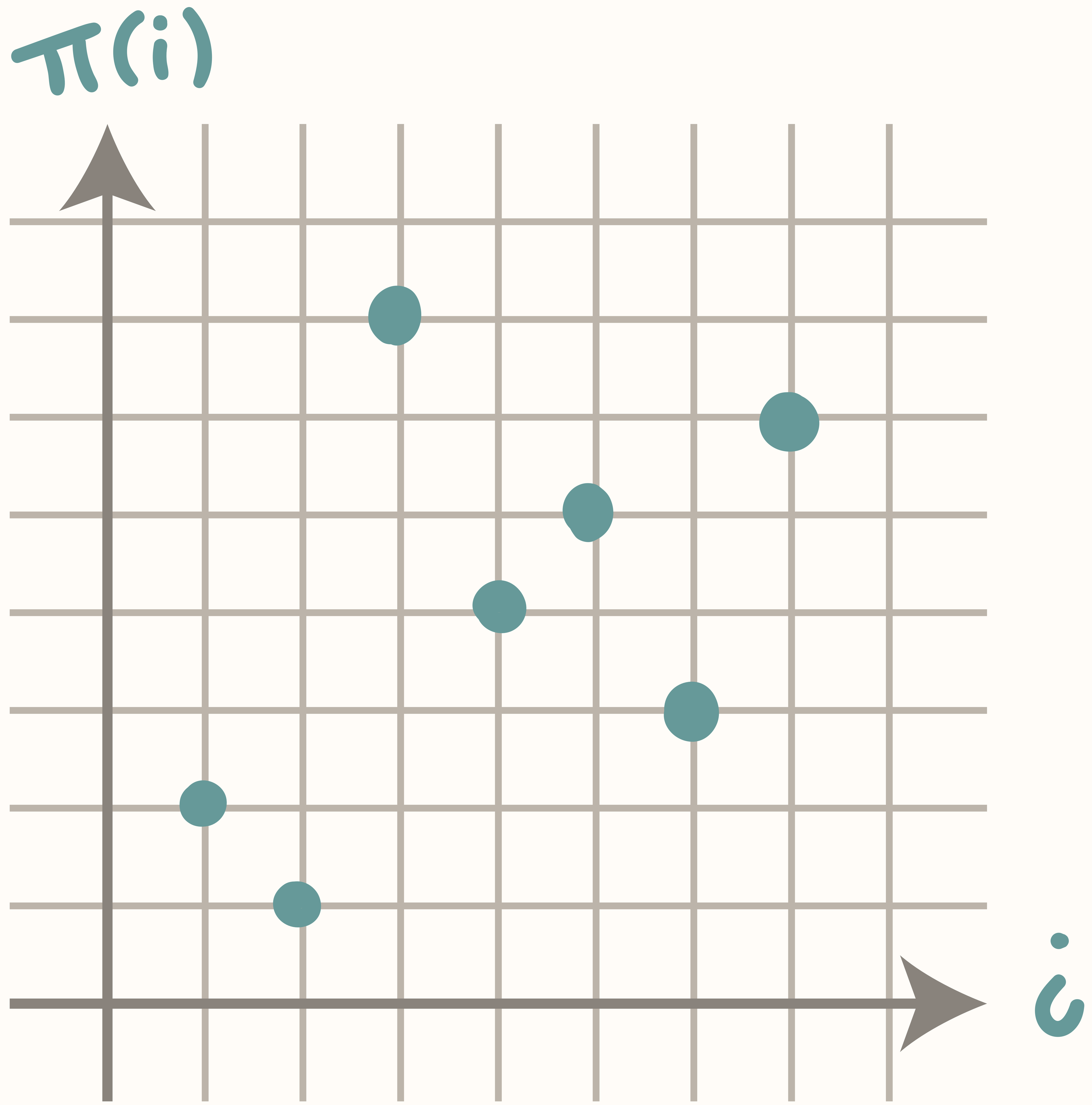
= *Avoid the vincular pattern :*

2 <u>1 4</u> 3



Link with plane permutations

Plane permutation → *Poset*



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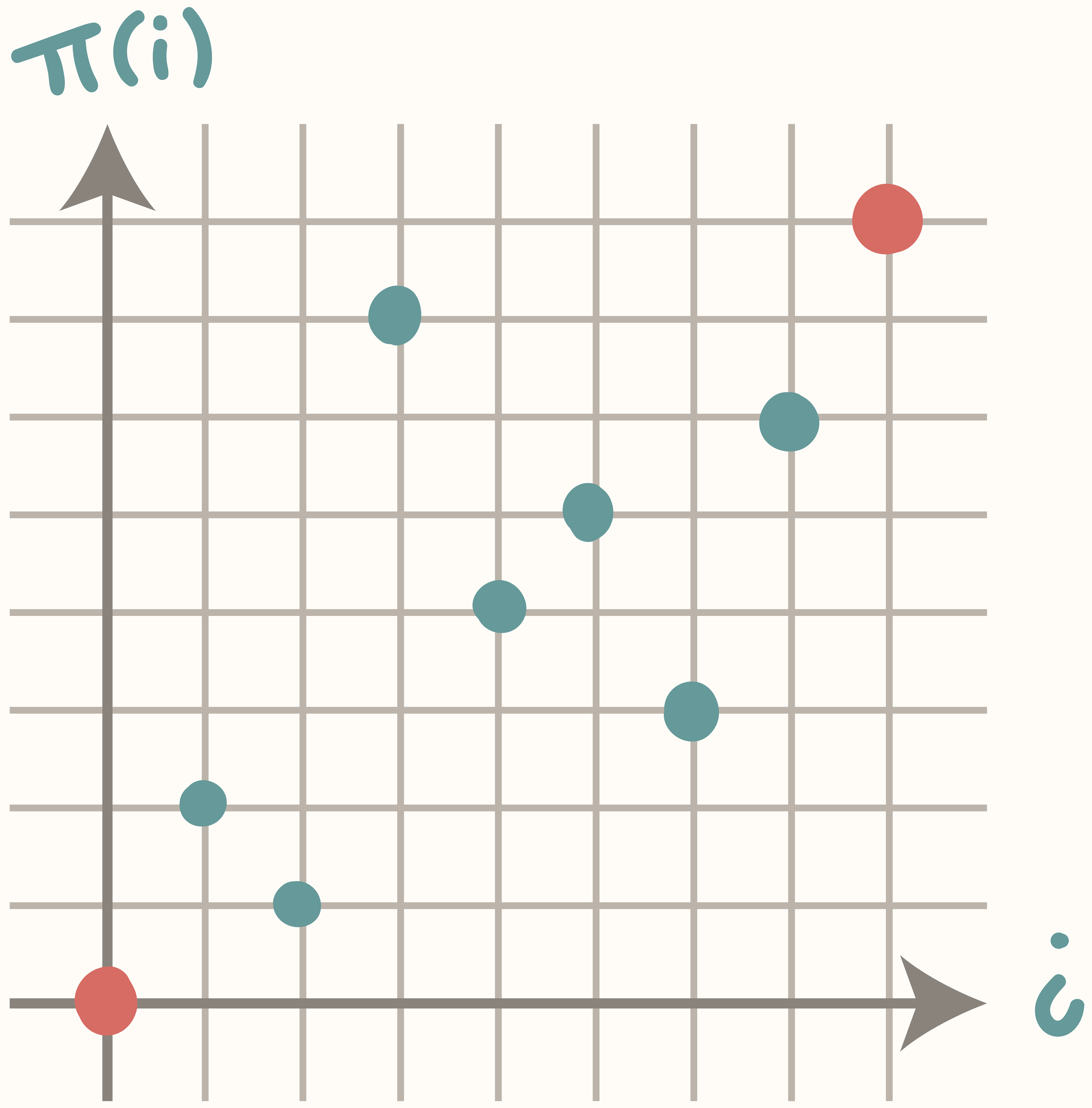

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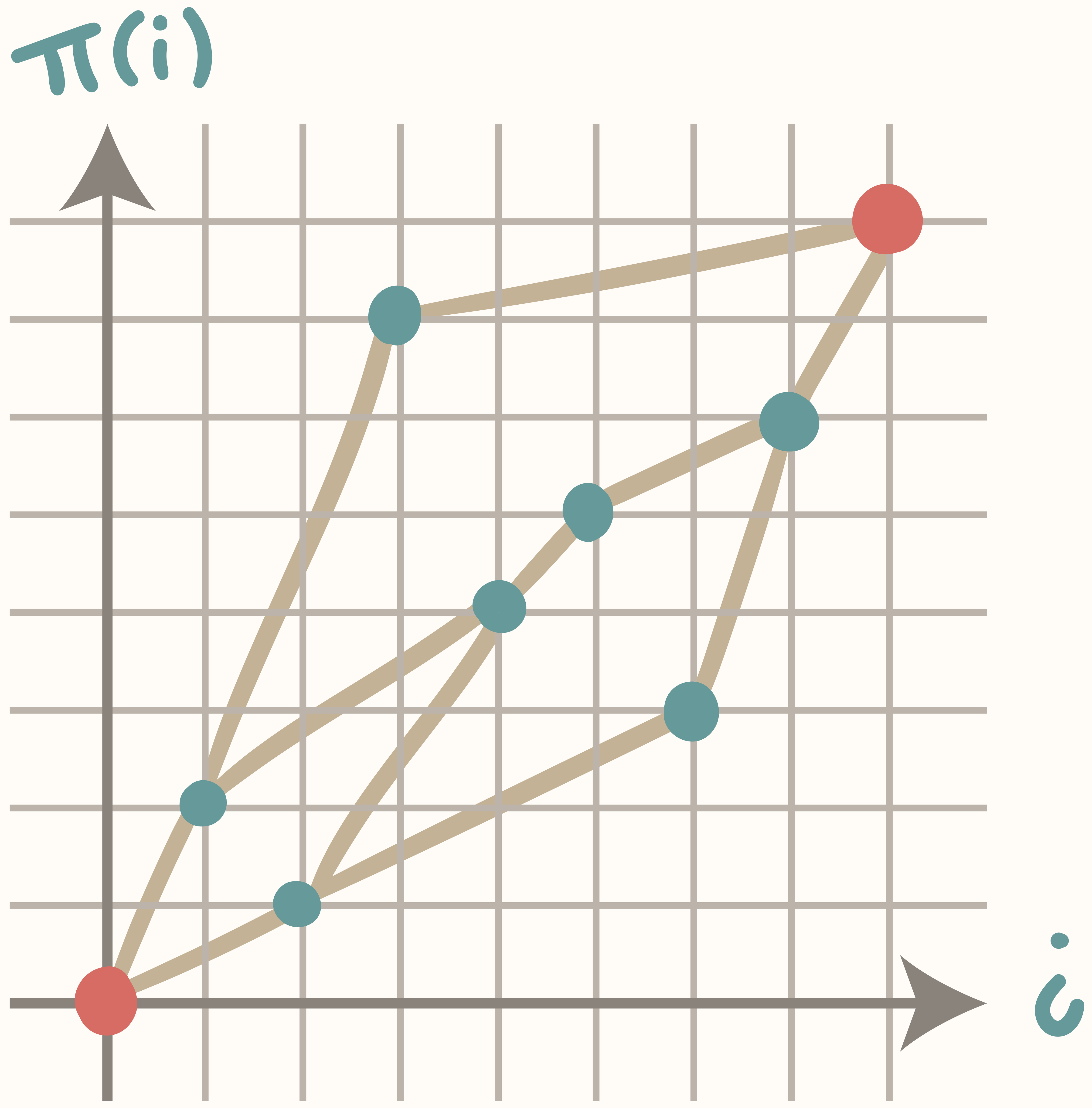

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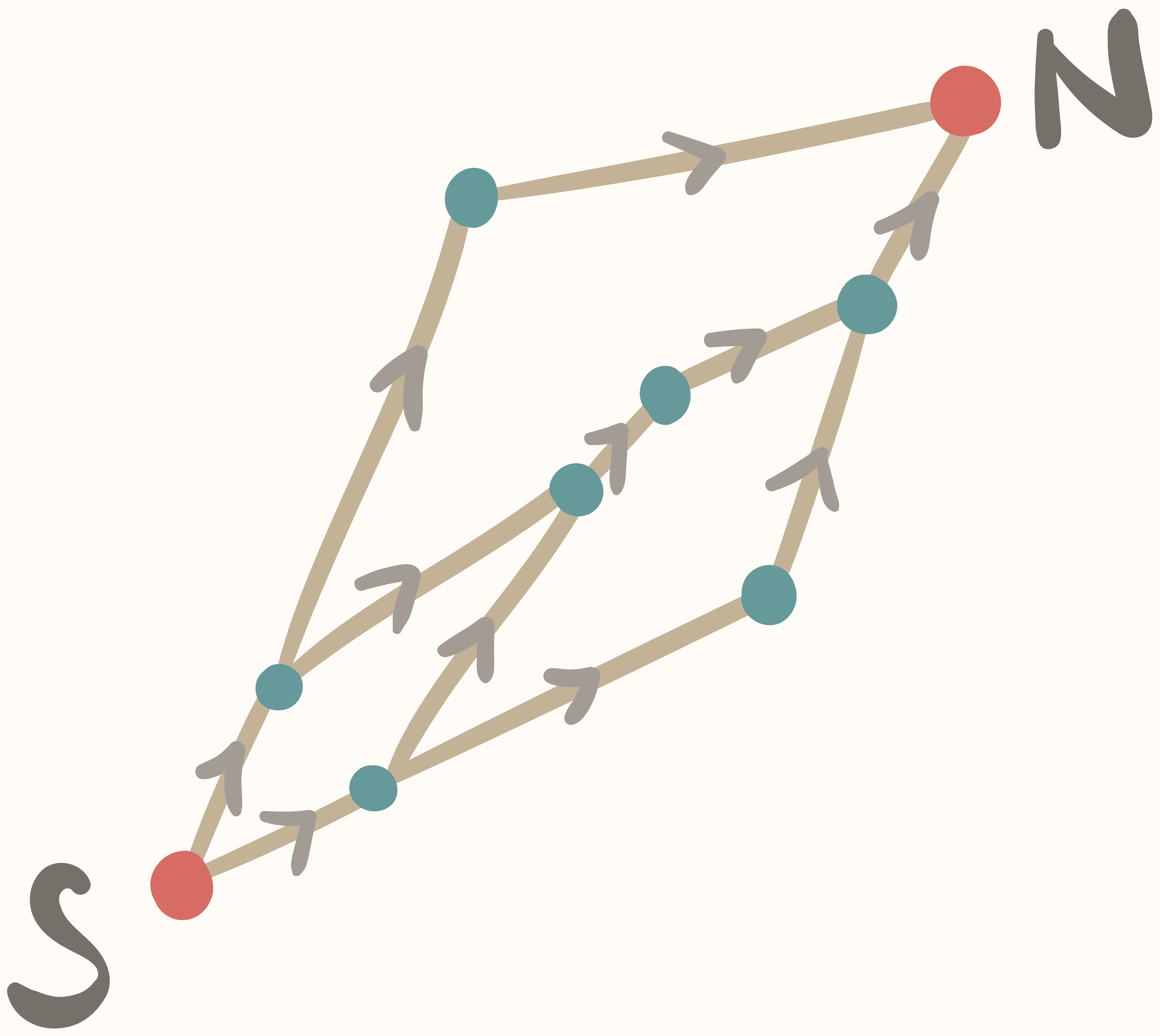

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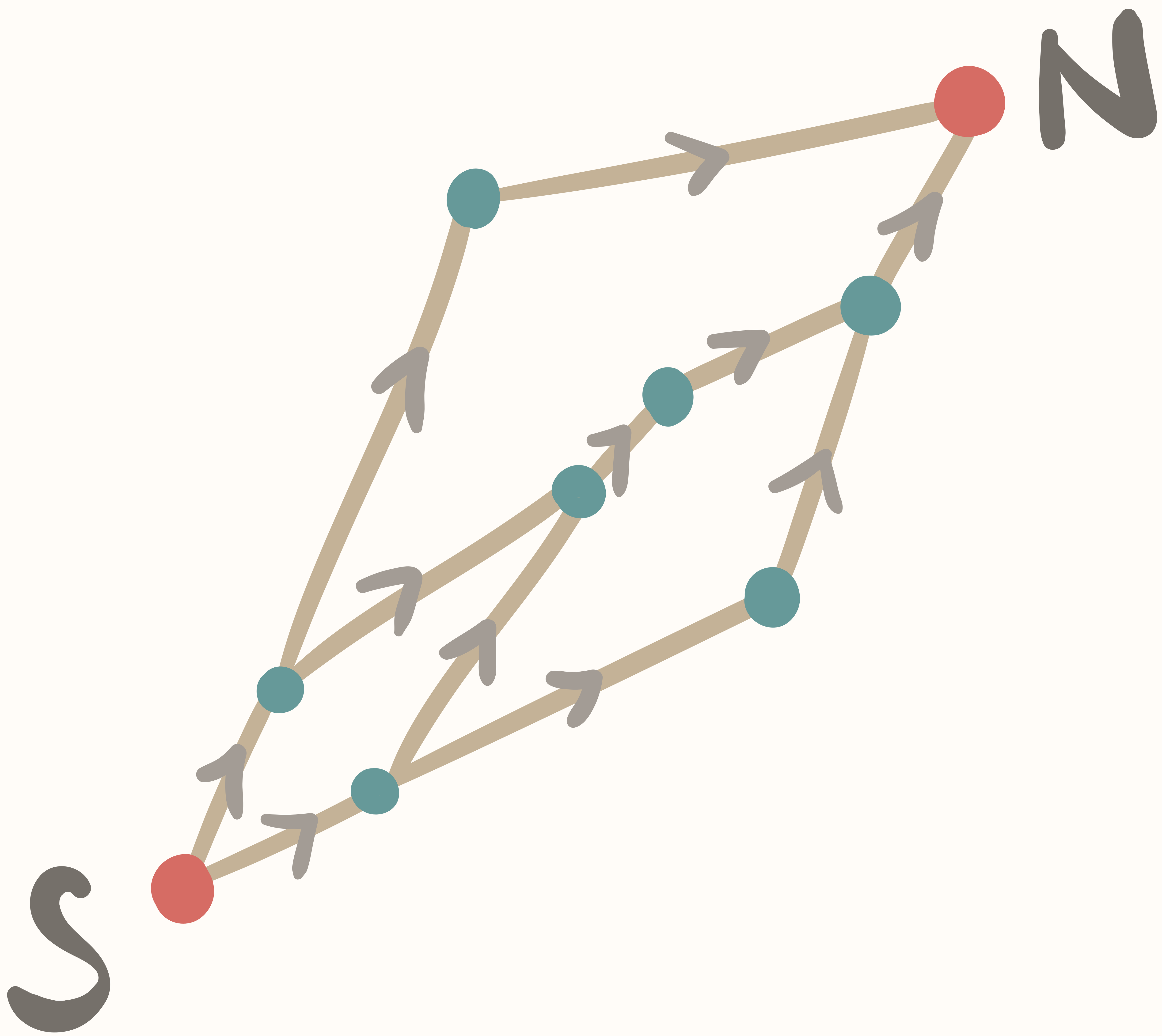
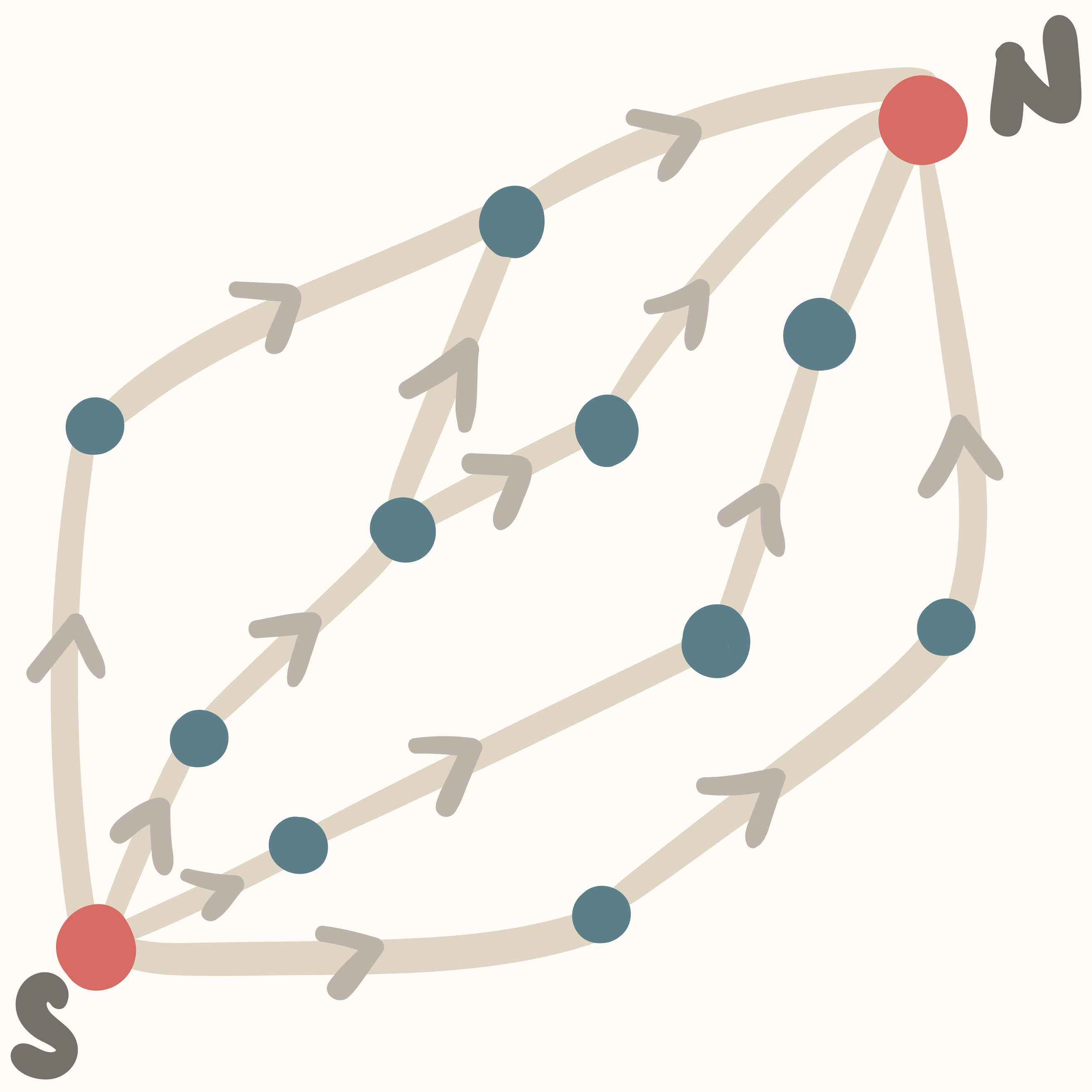


↠ *Baxter permutations and plane bipolar orientations,*
N. Bonichon, M. Bousquet-Mélou, & E. Fusy (2010)

Link with plane permutations

Poset → *Plane permutation*



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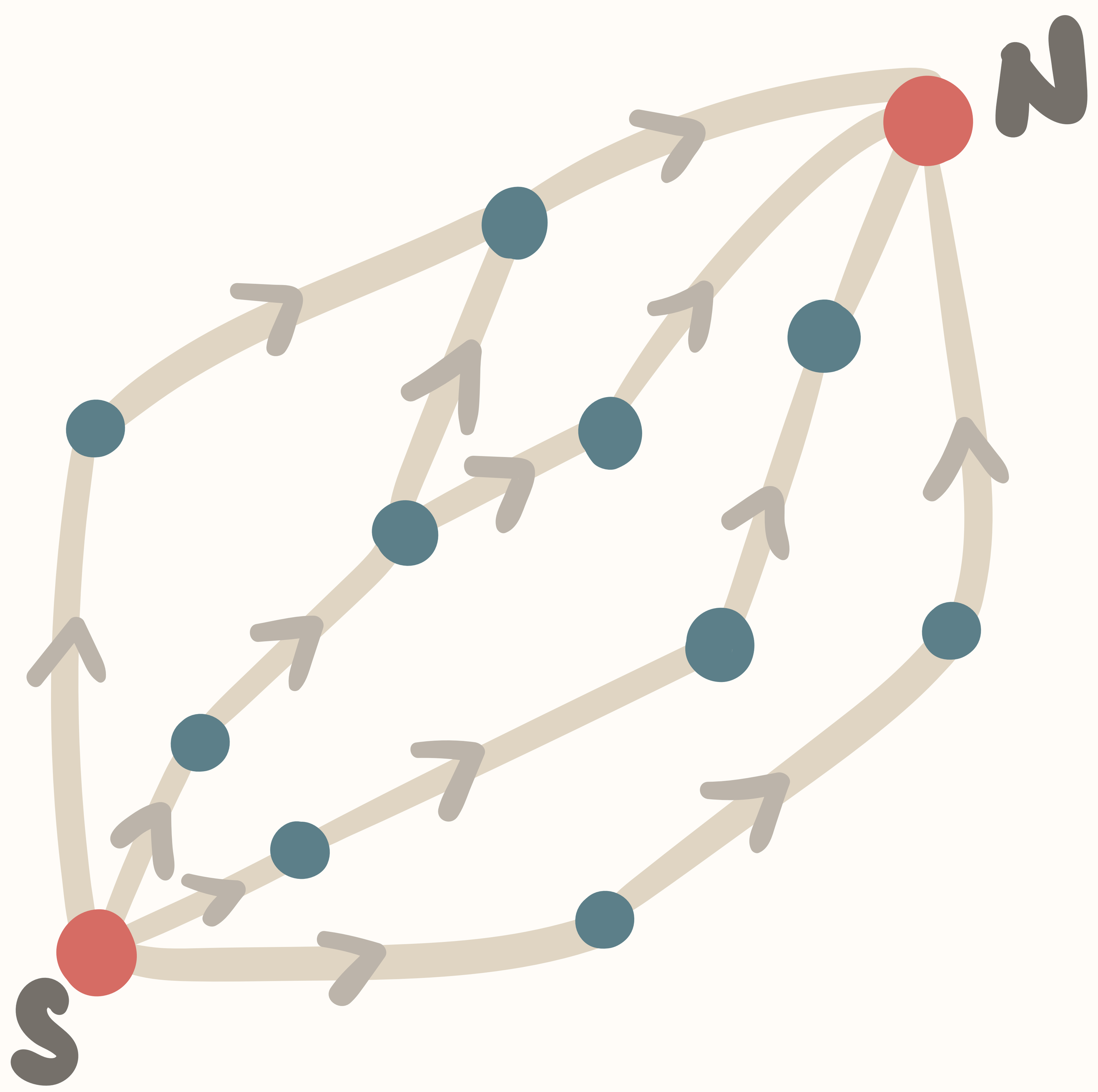

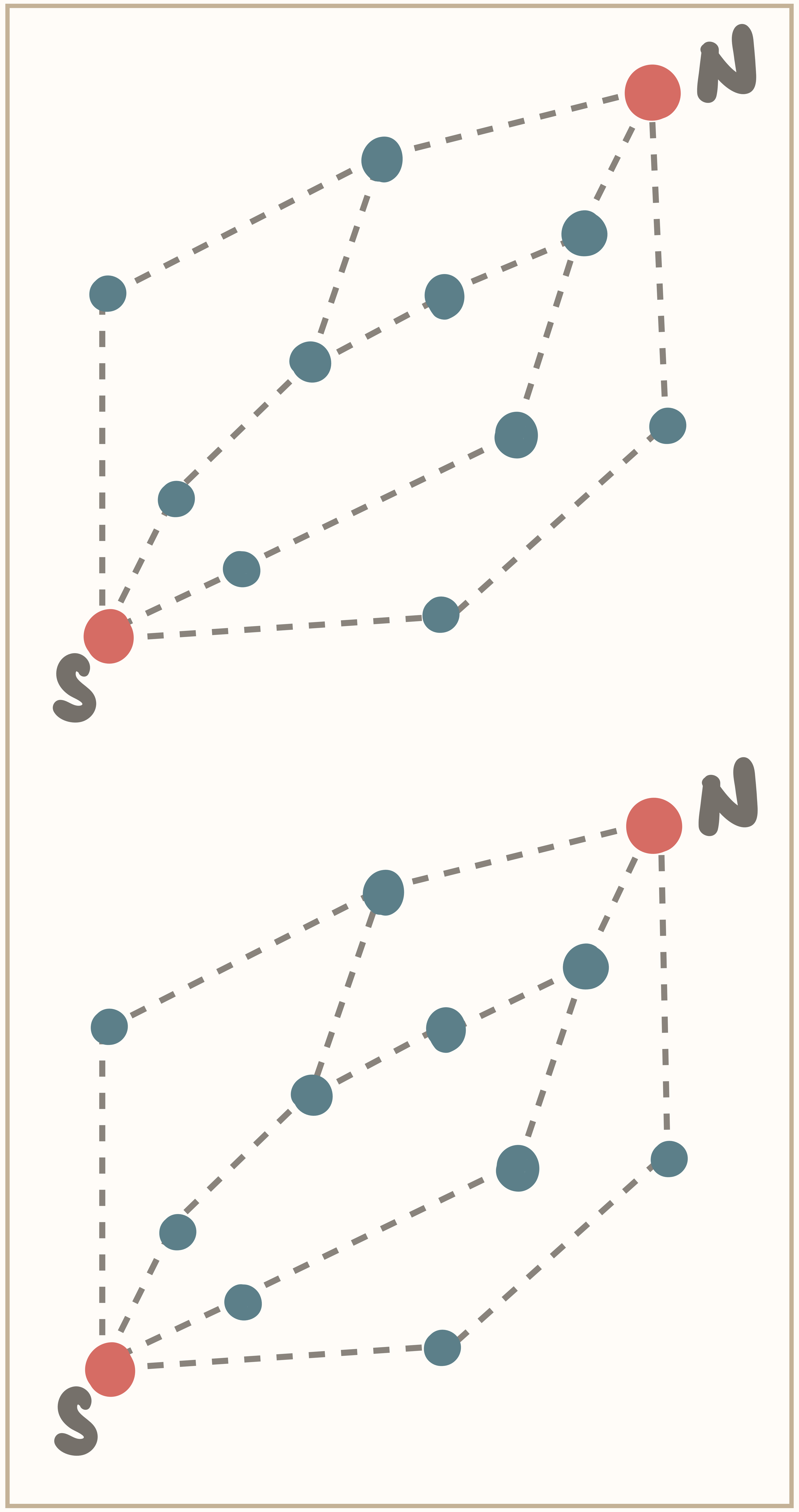

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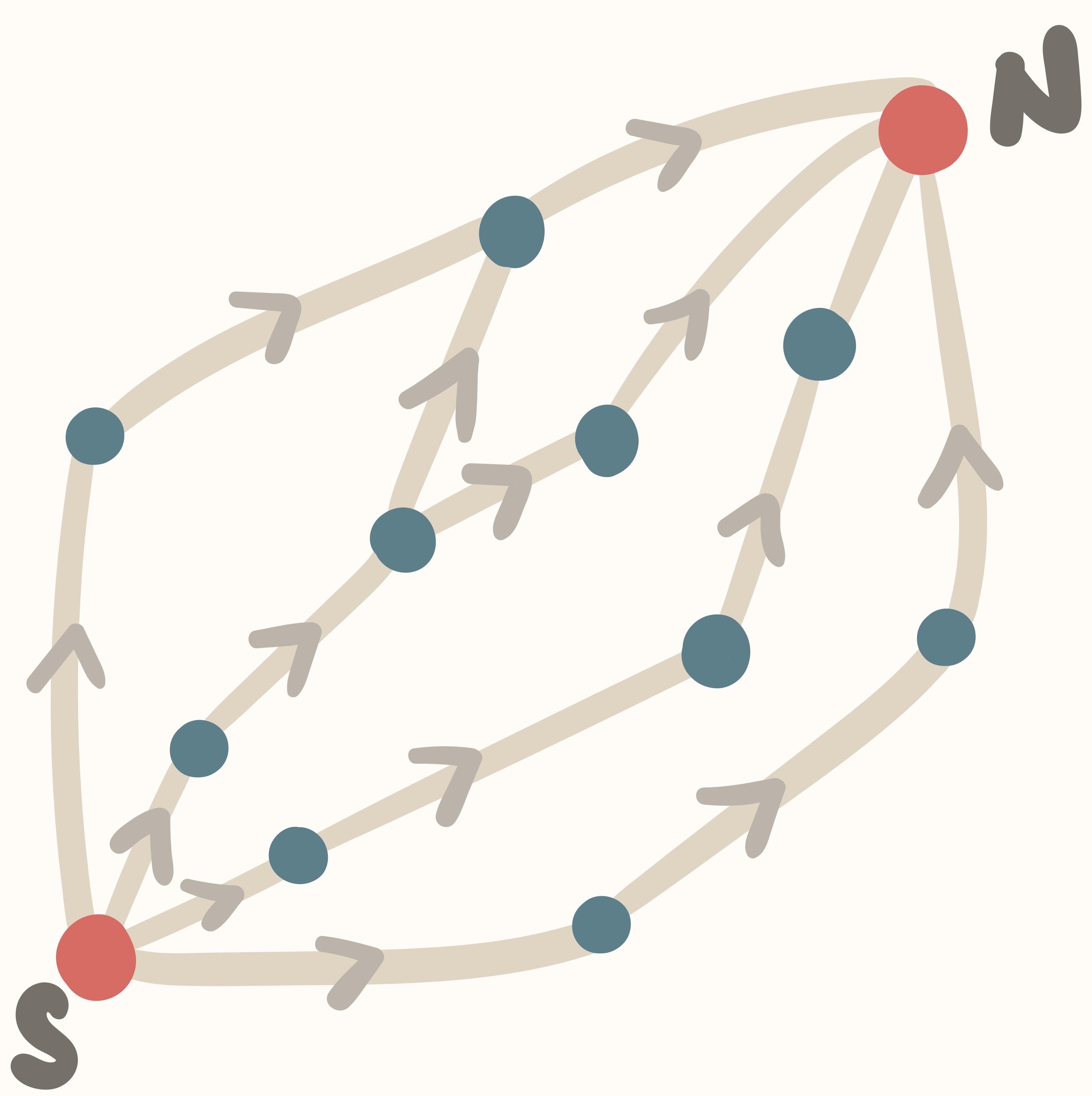
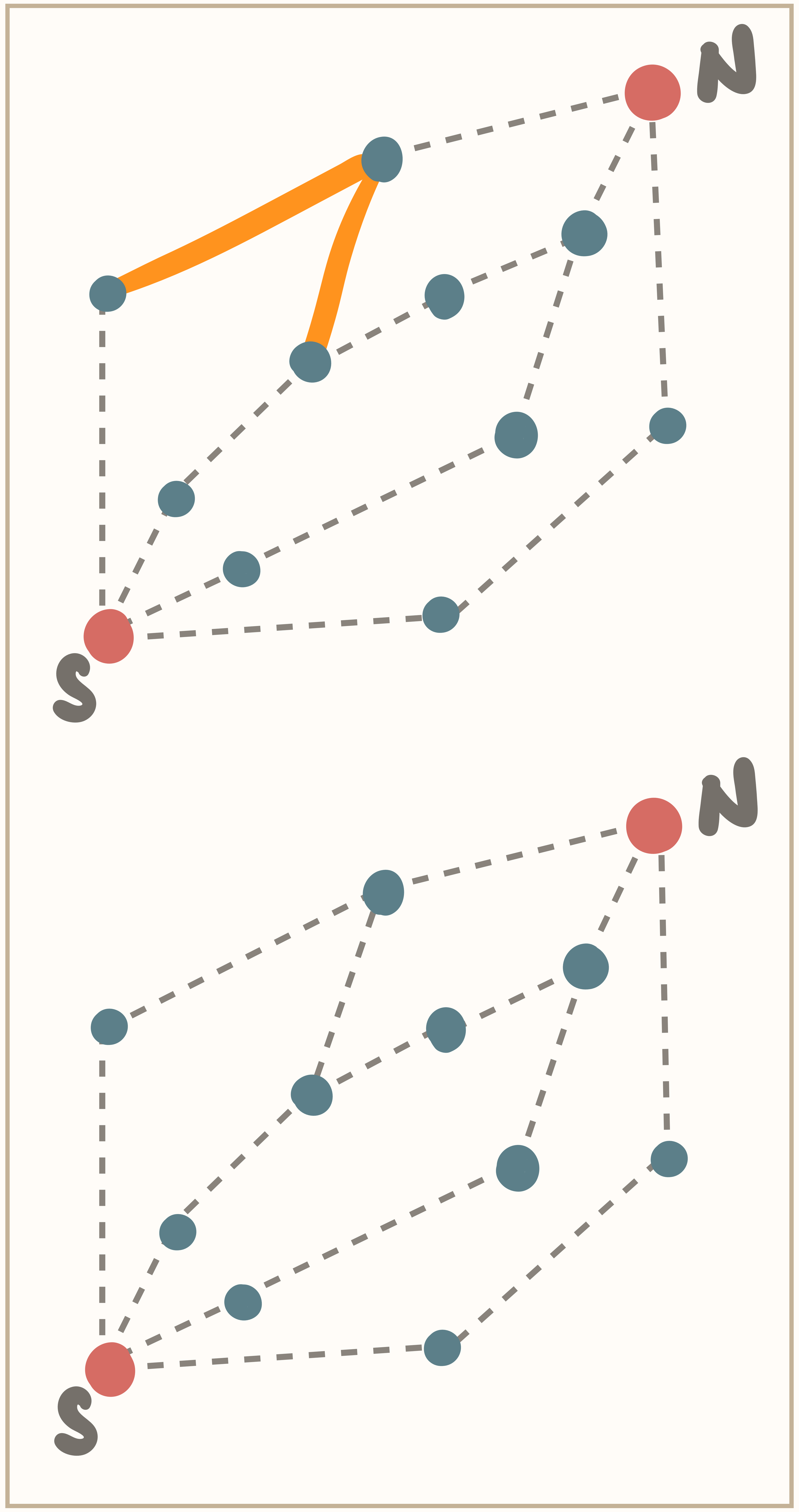
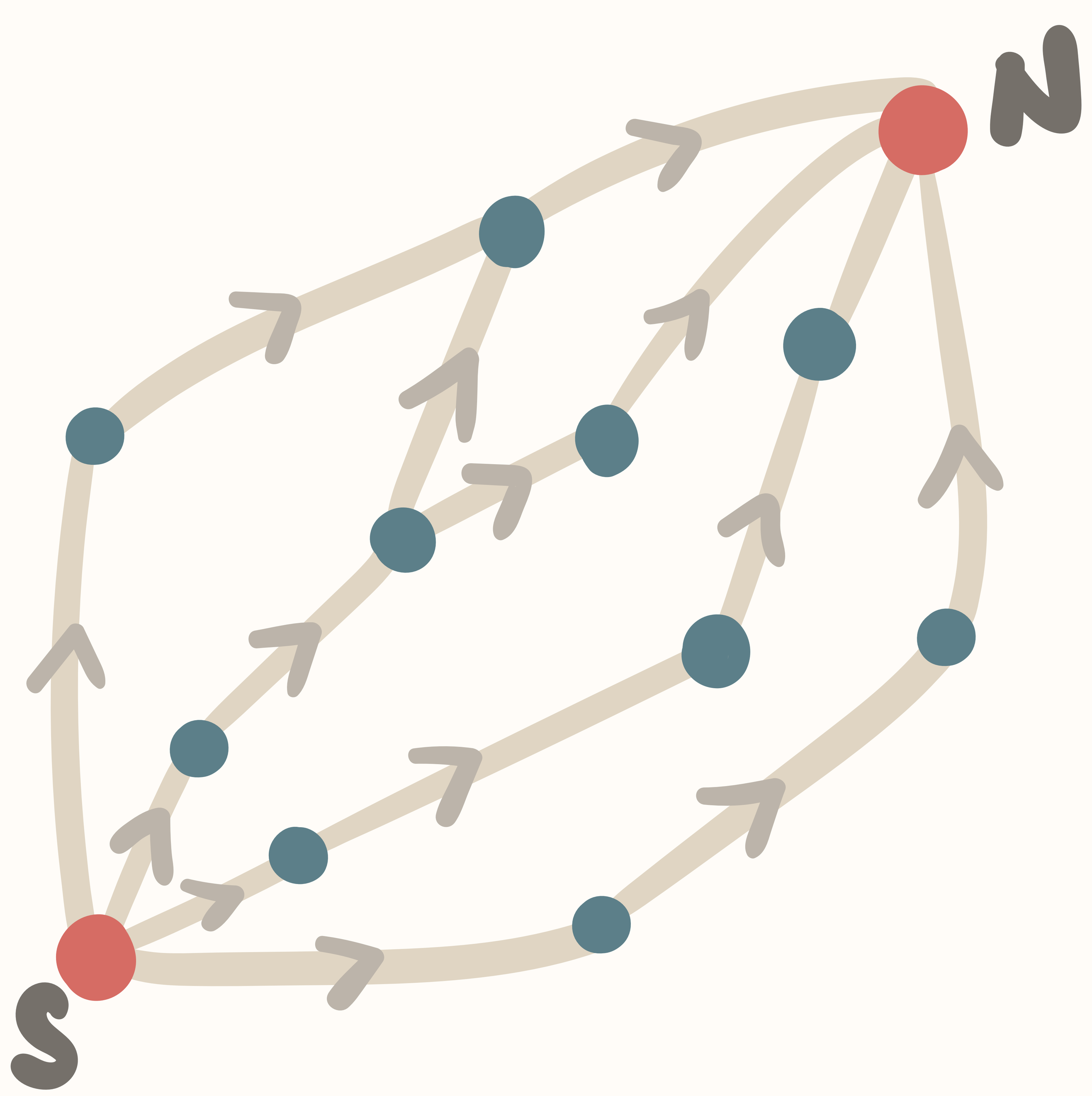
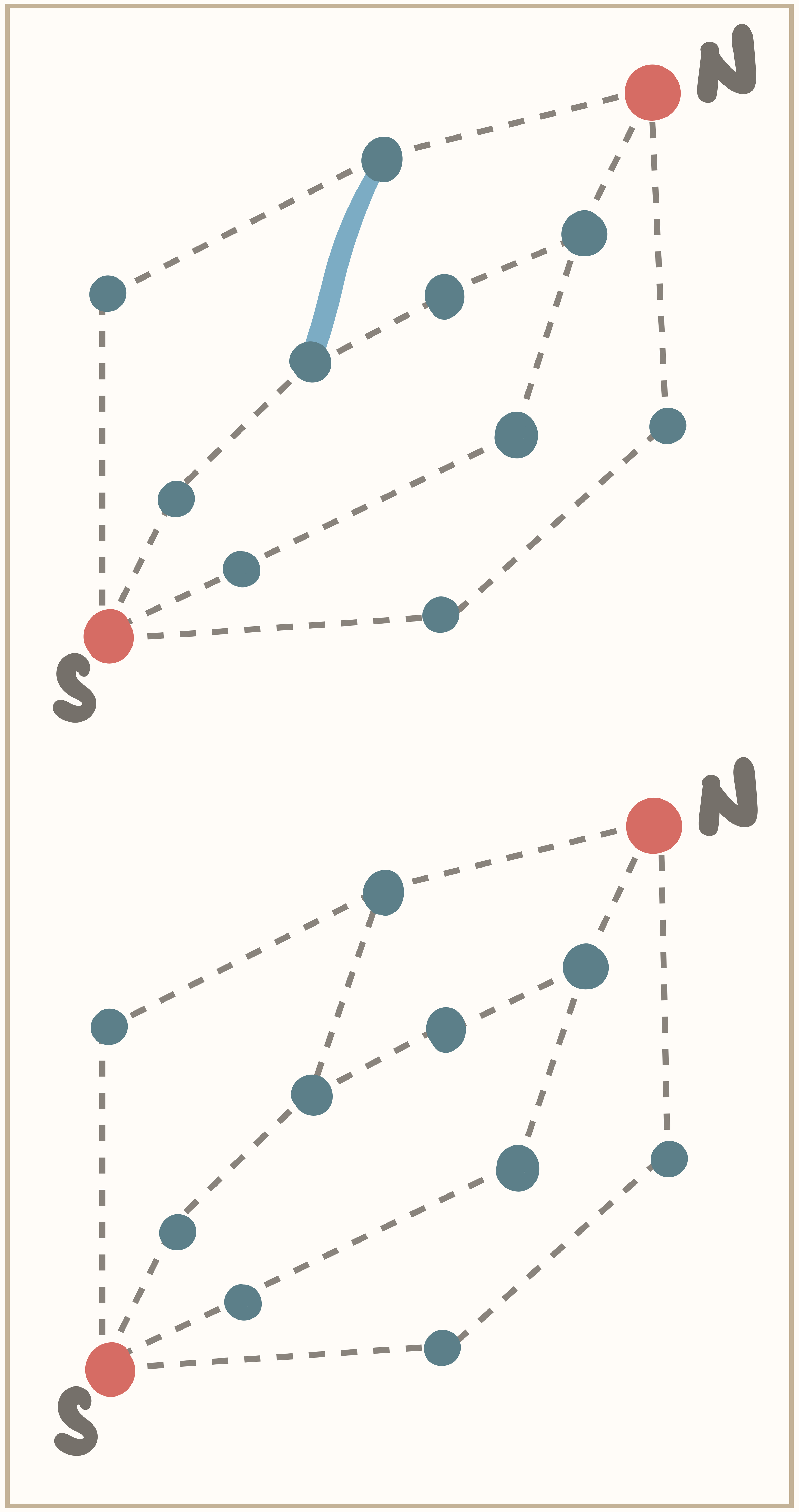




Link with plane permutations

Poset ⟶ *Plane permutation*





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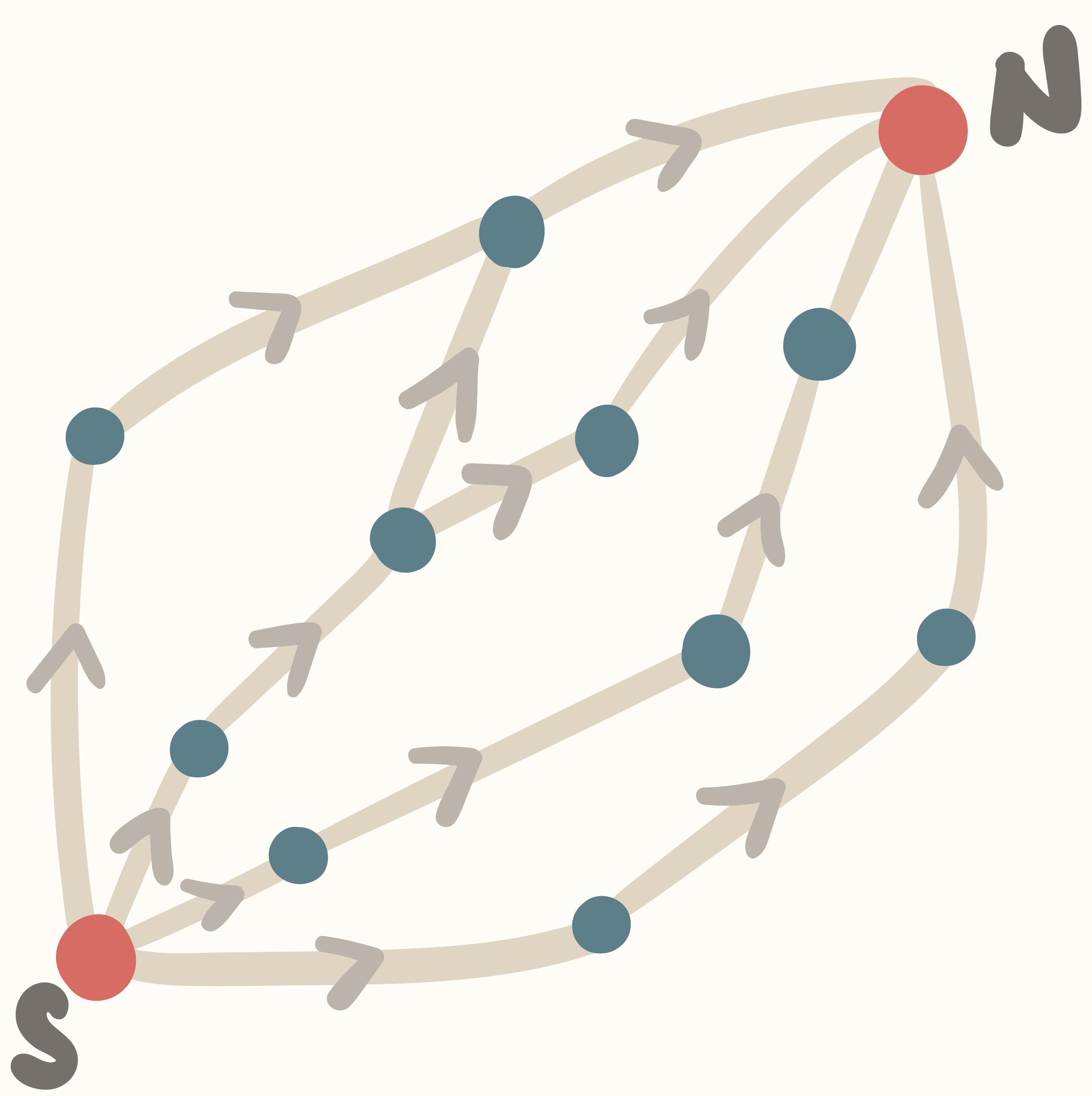

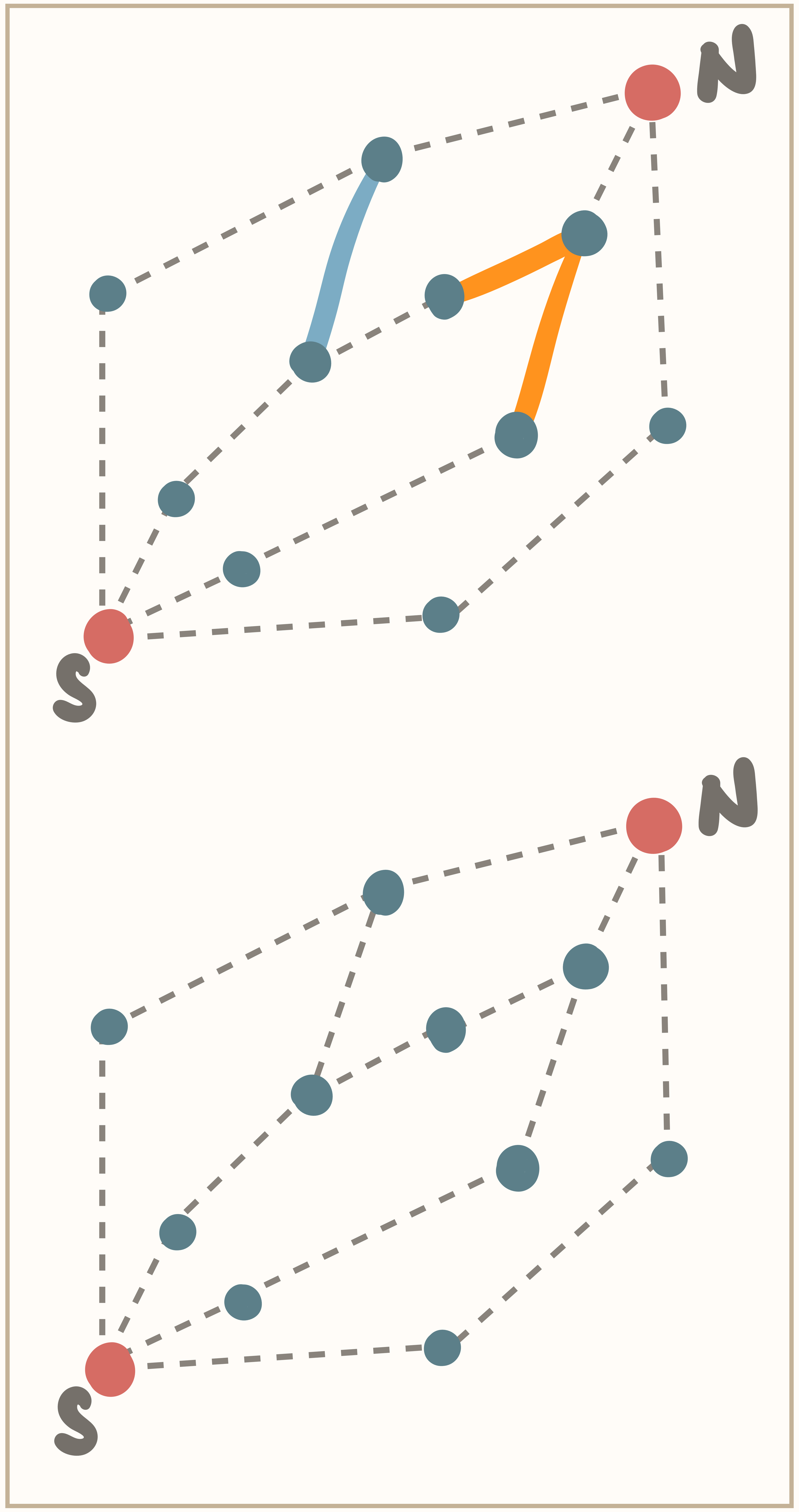

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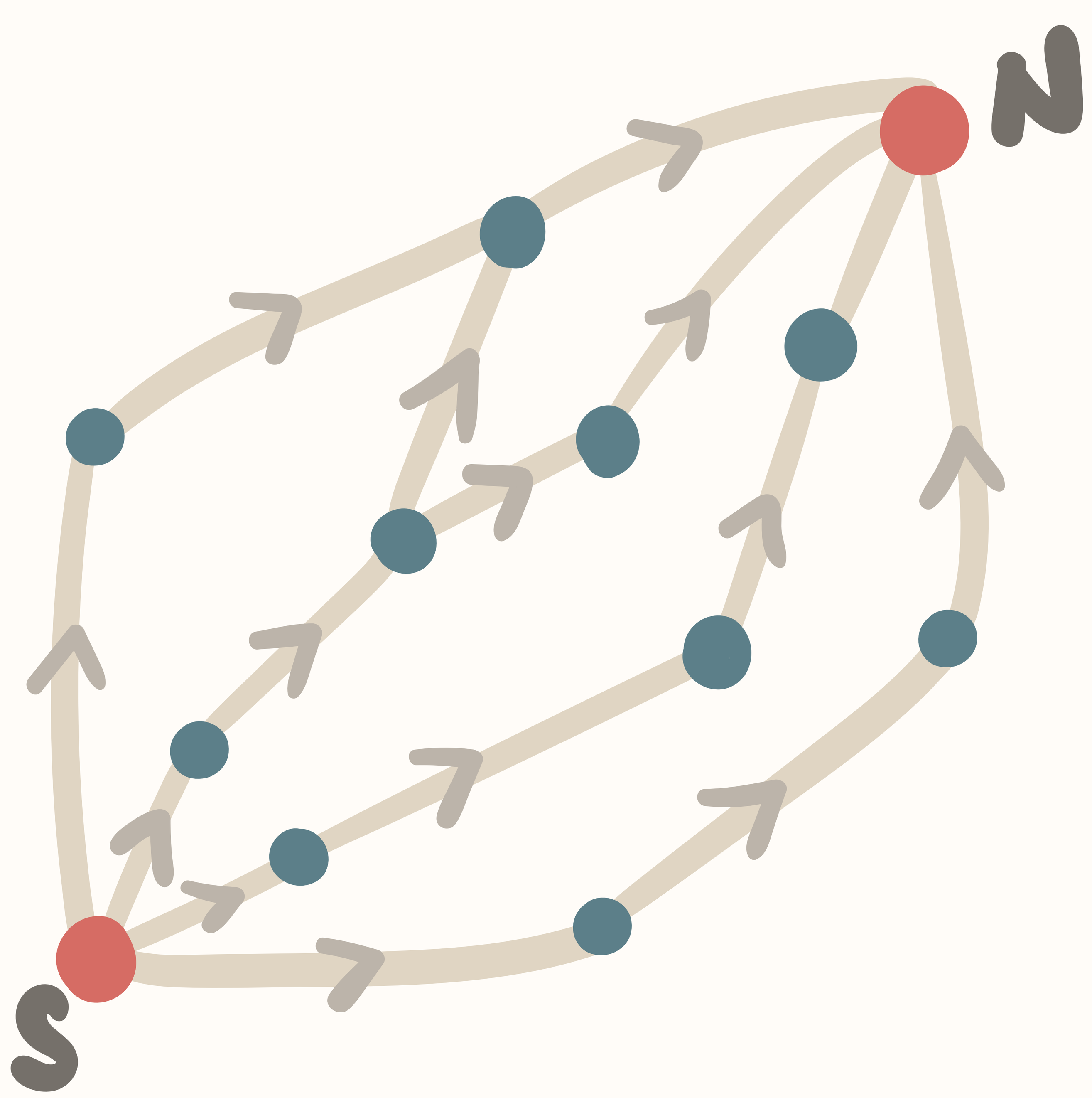

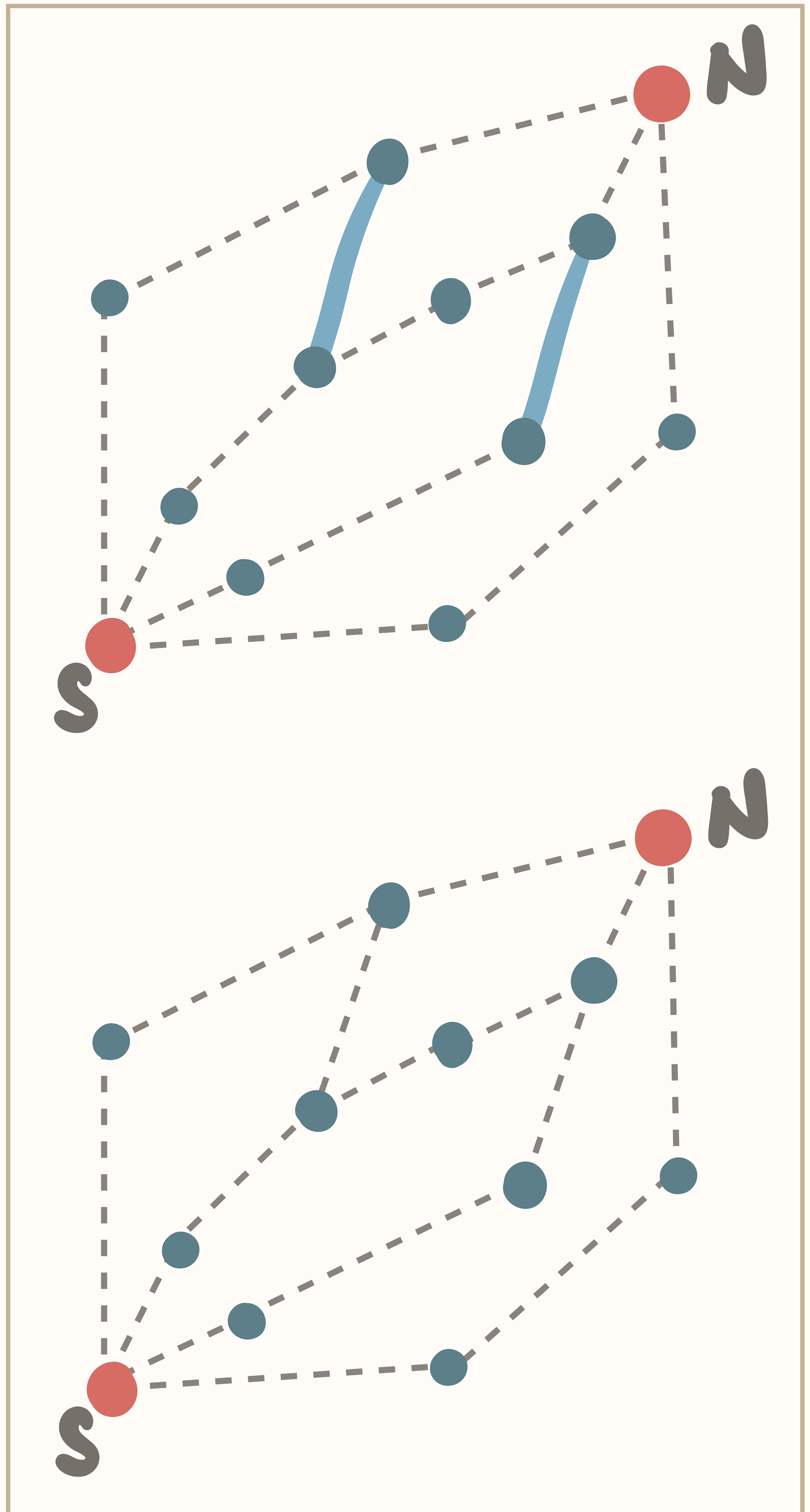

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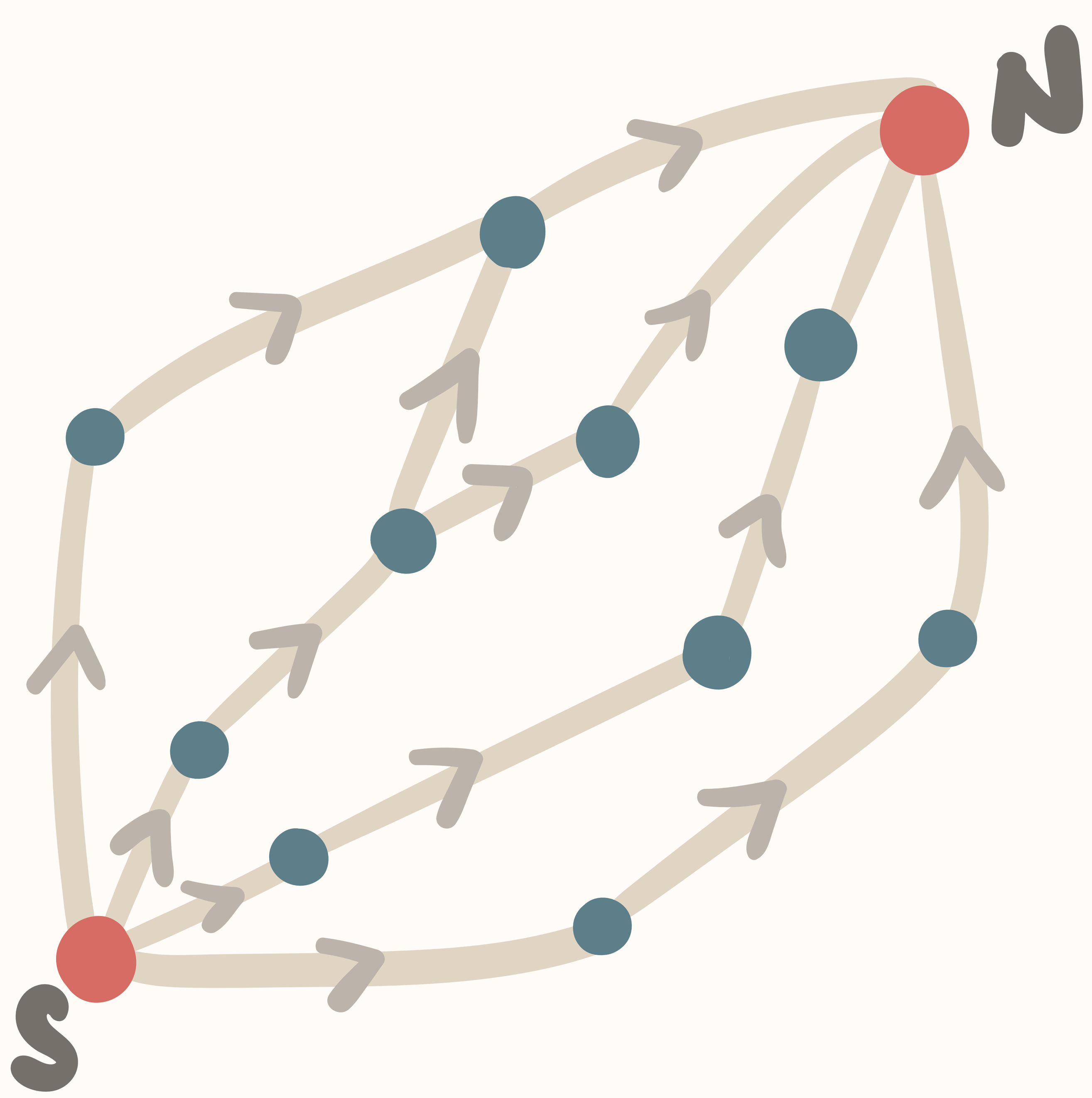

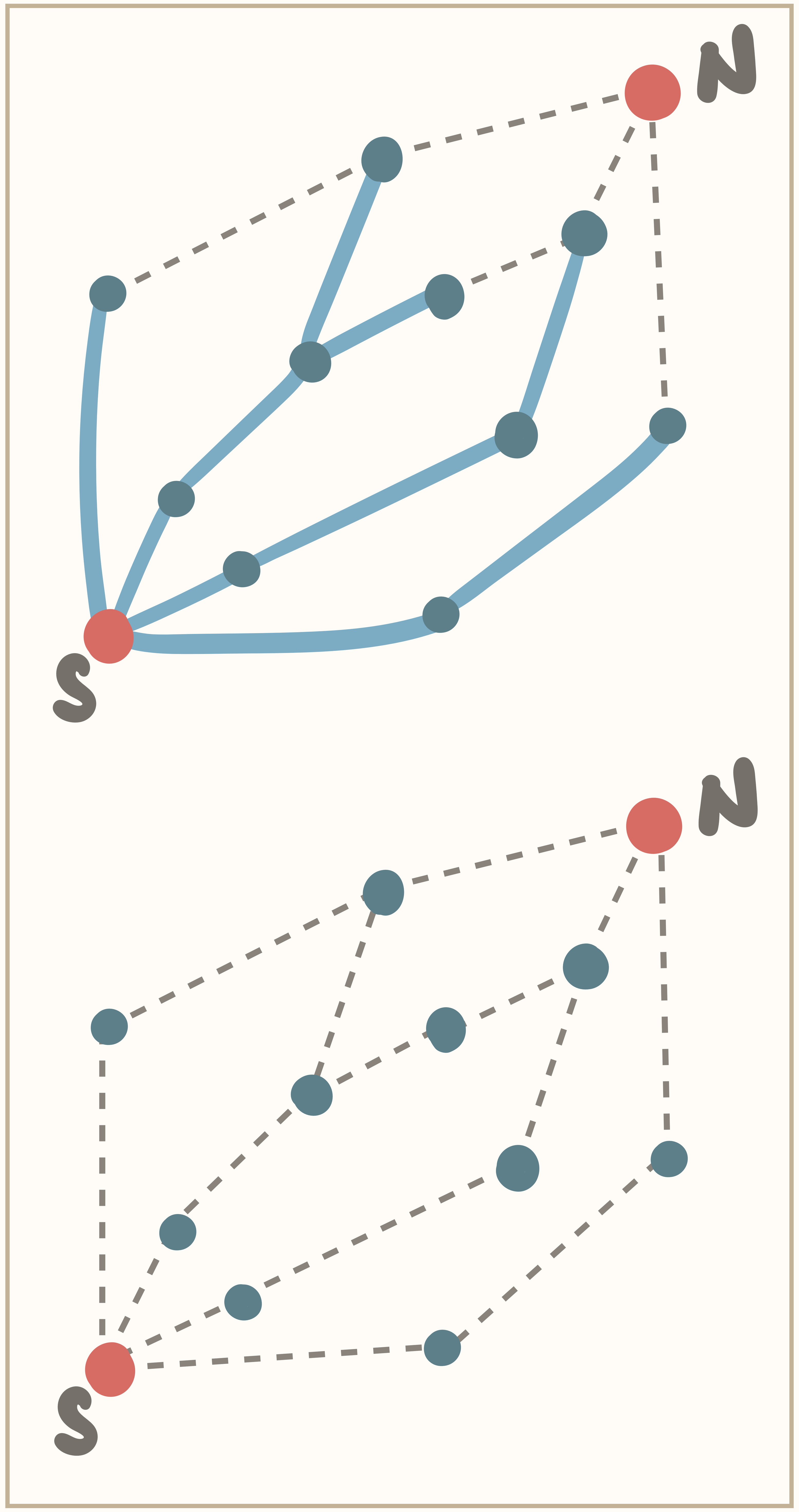

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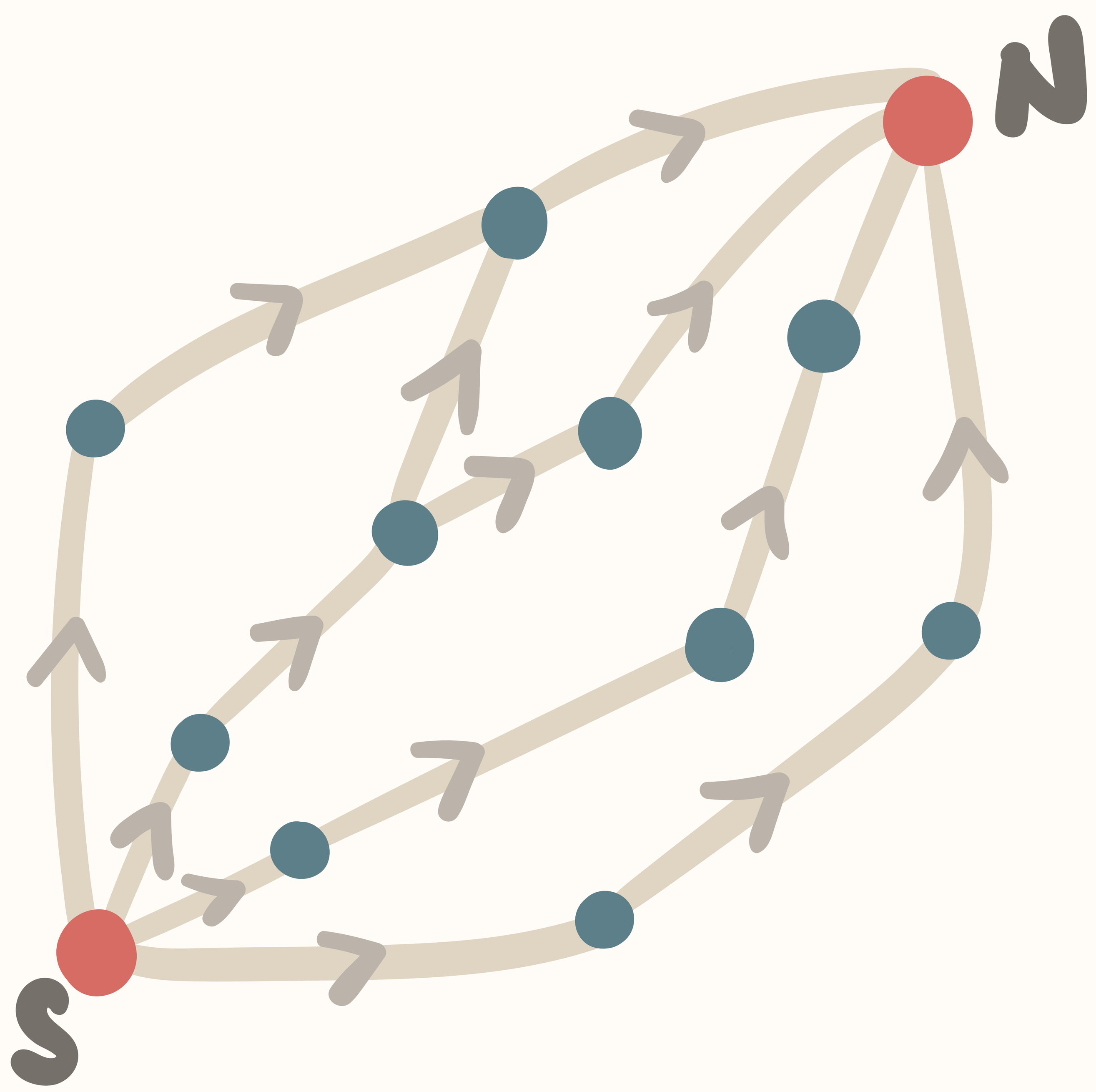

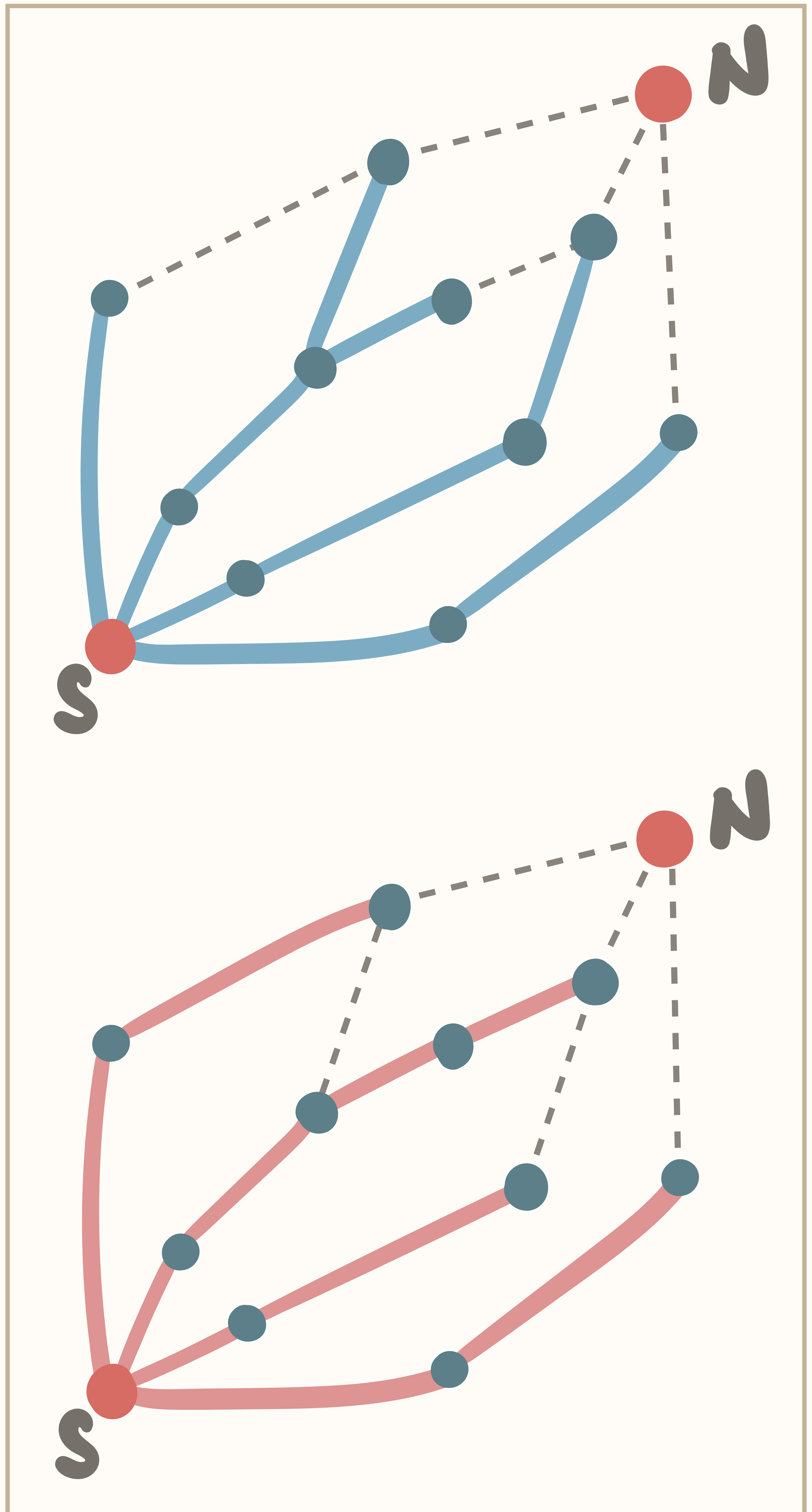

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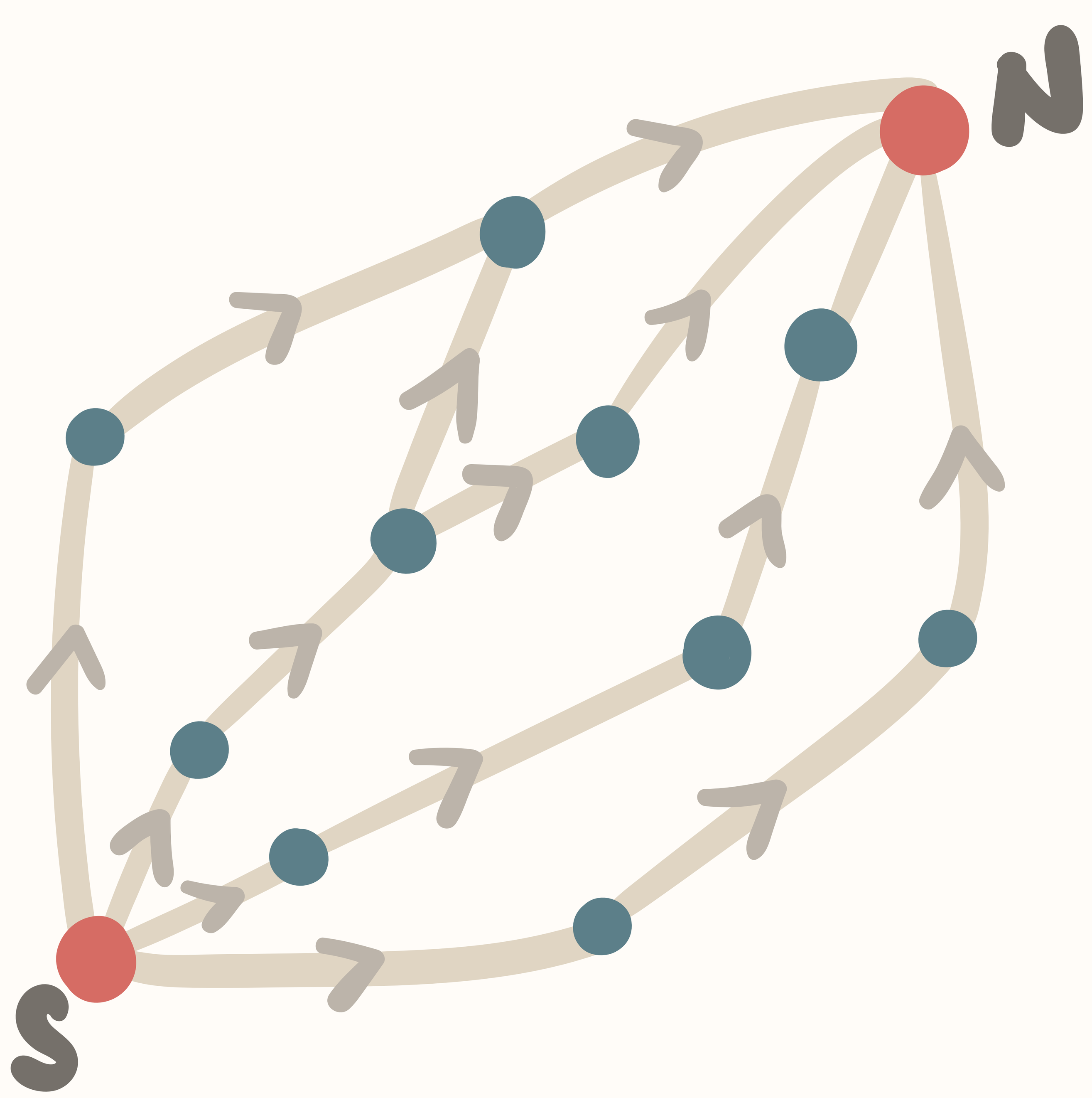

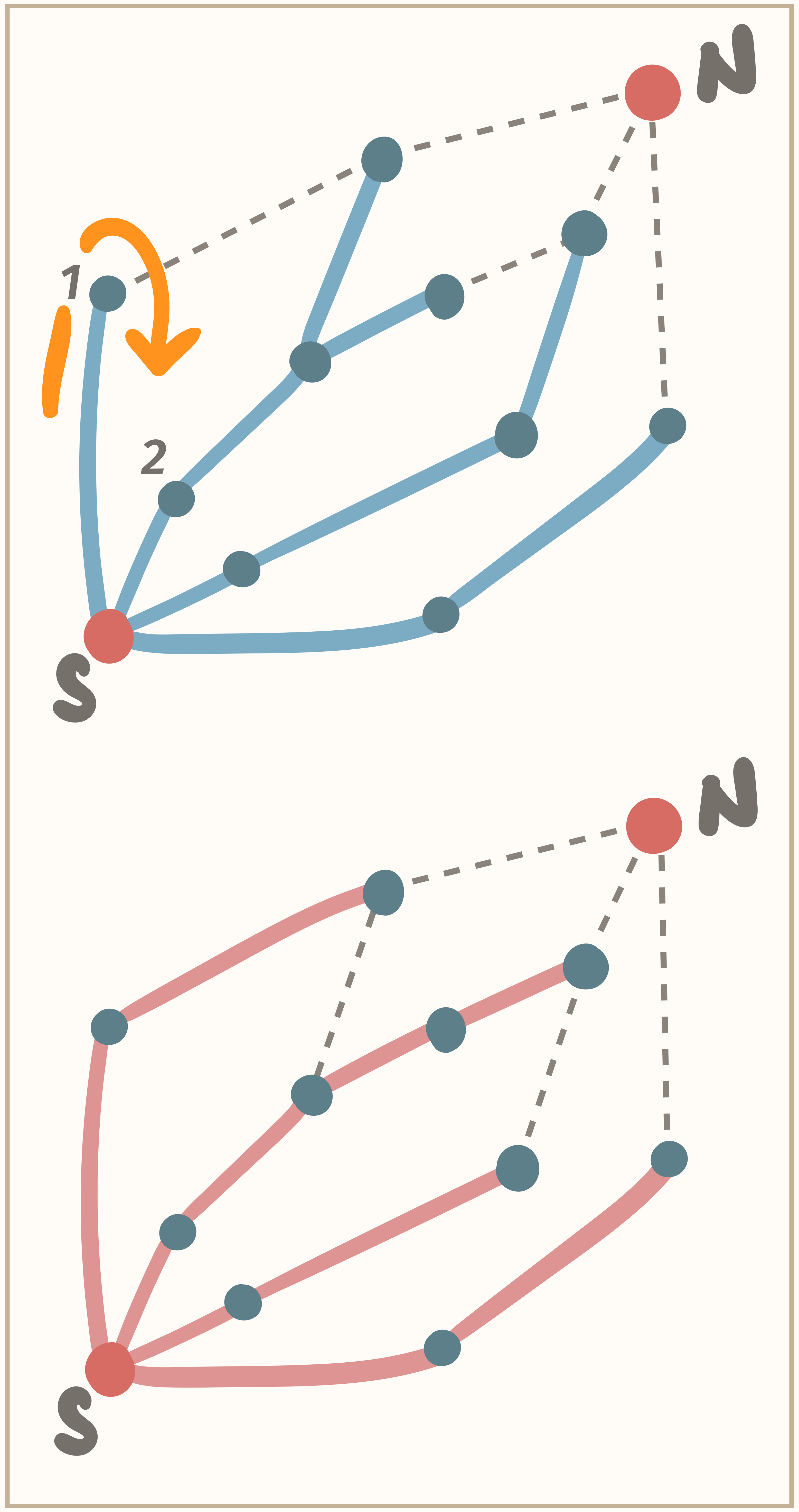

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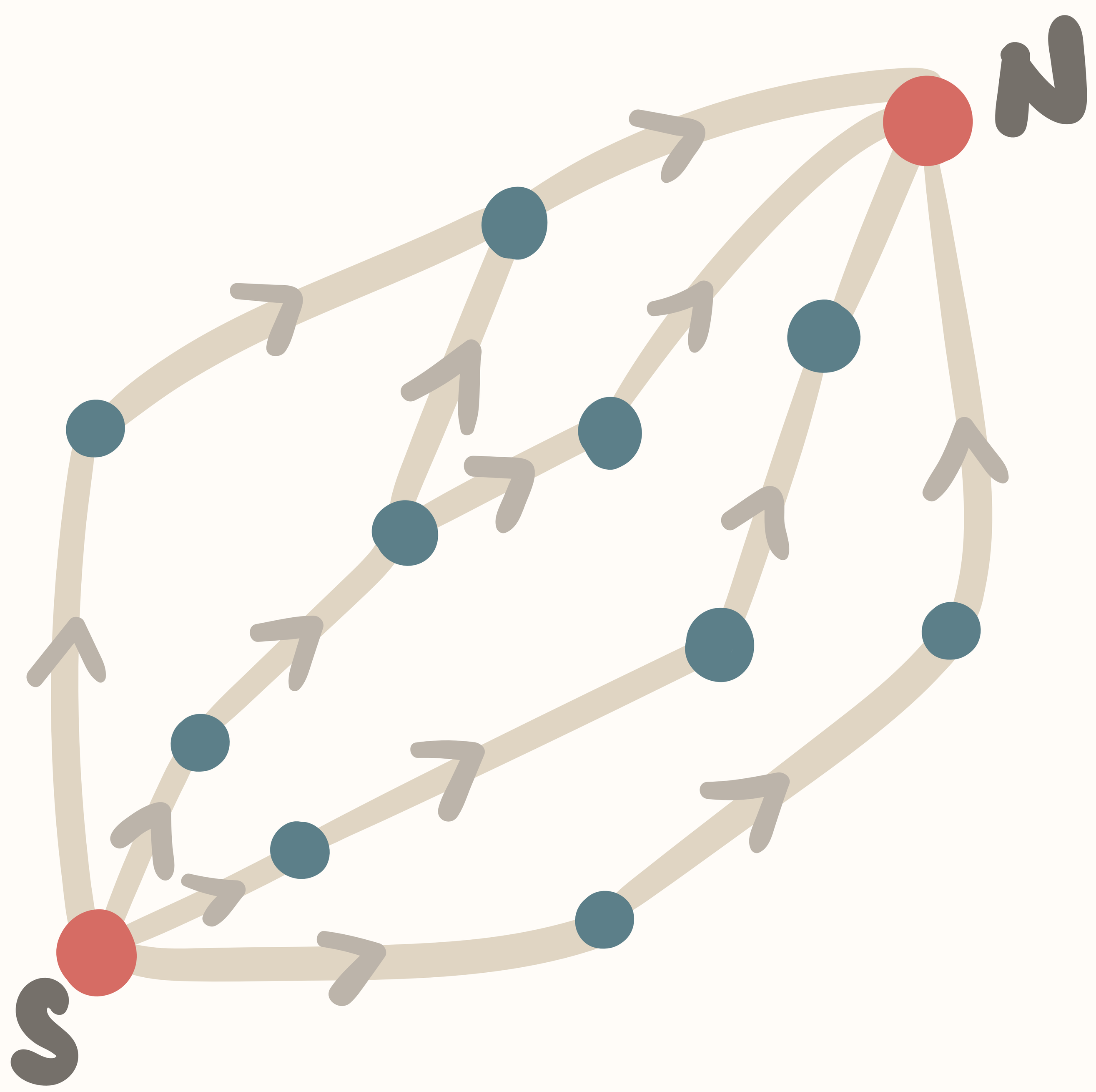

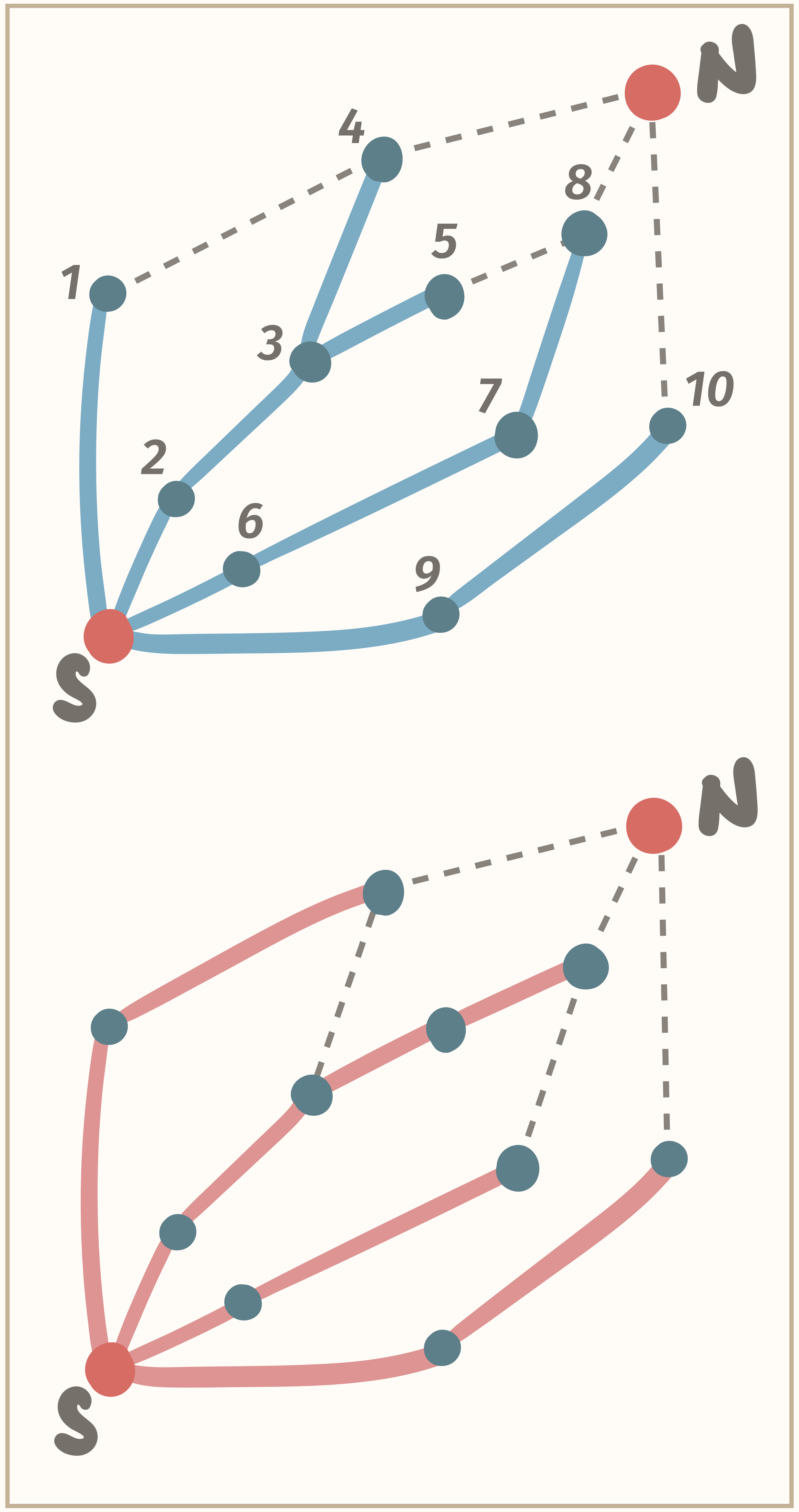

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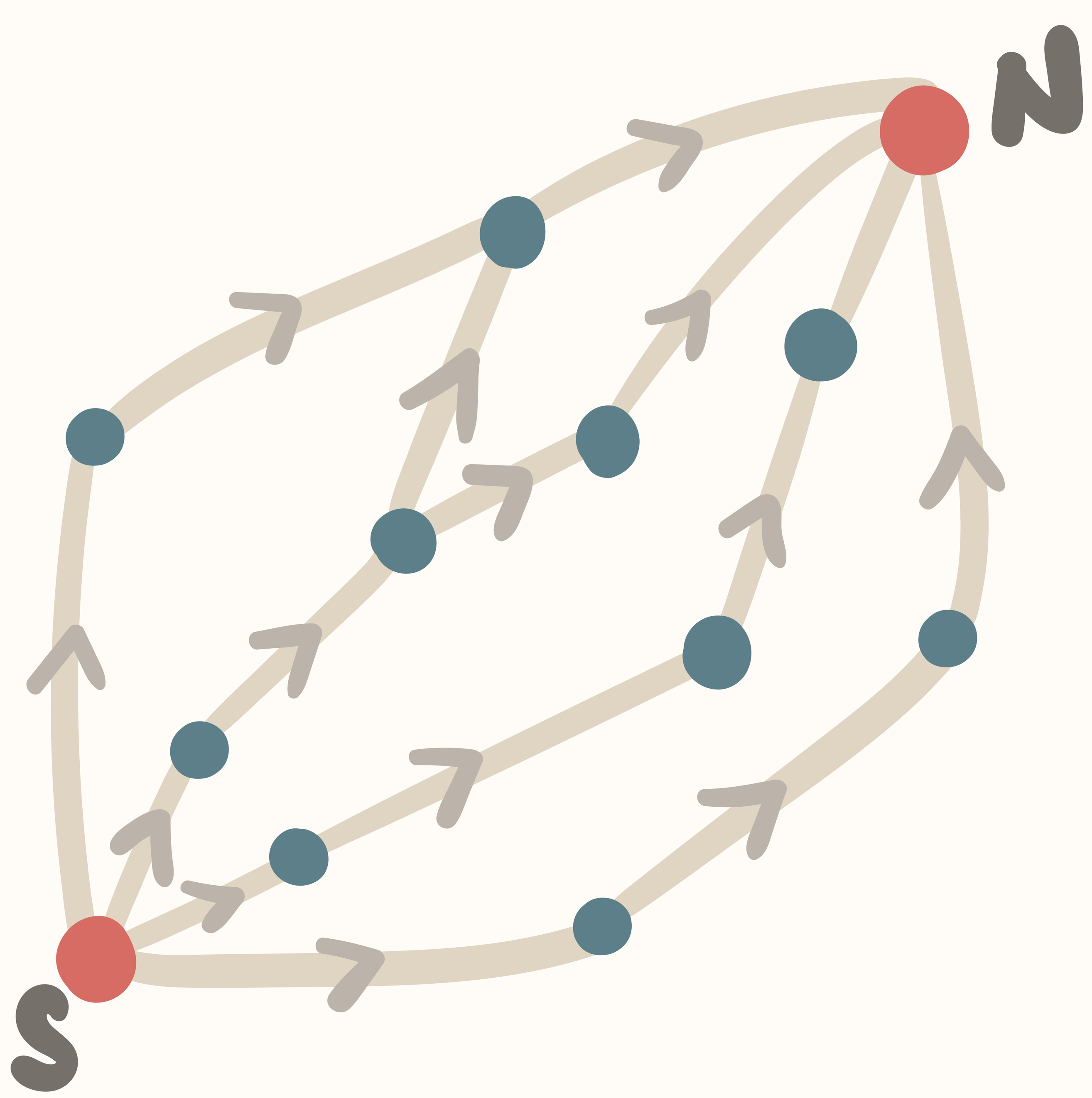
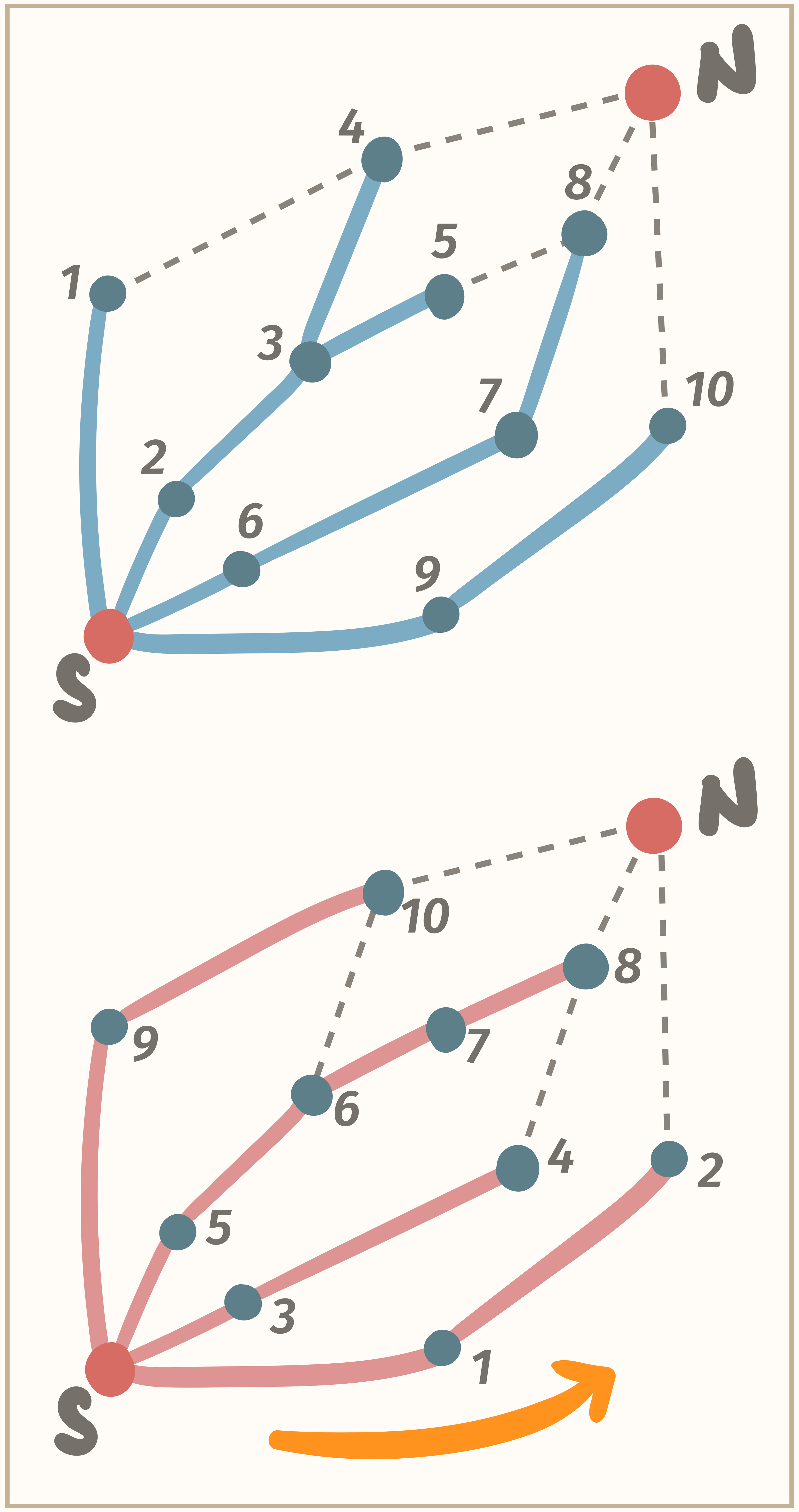
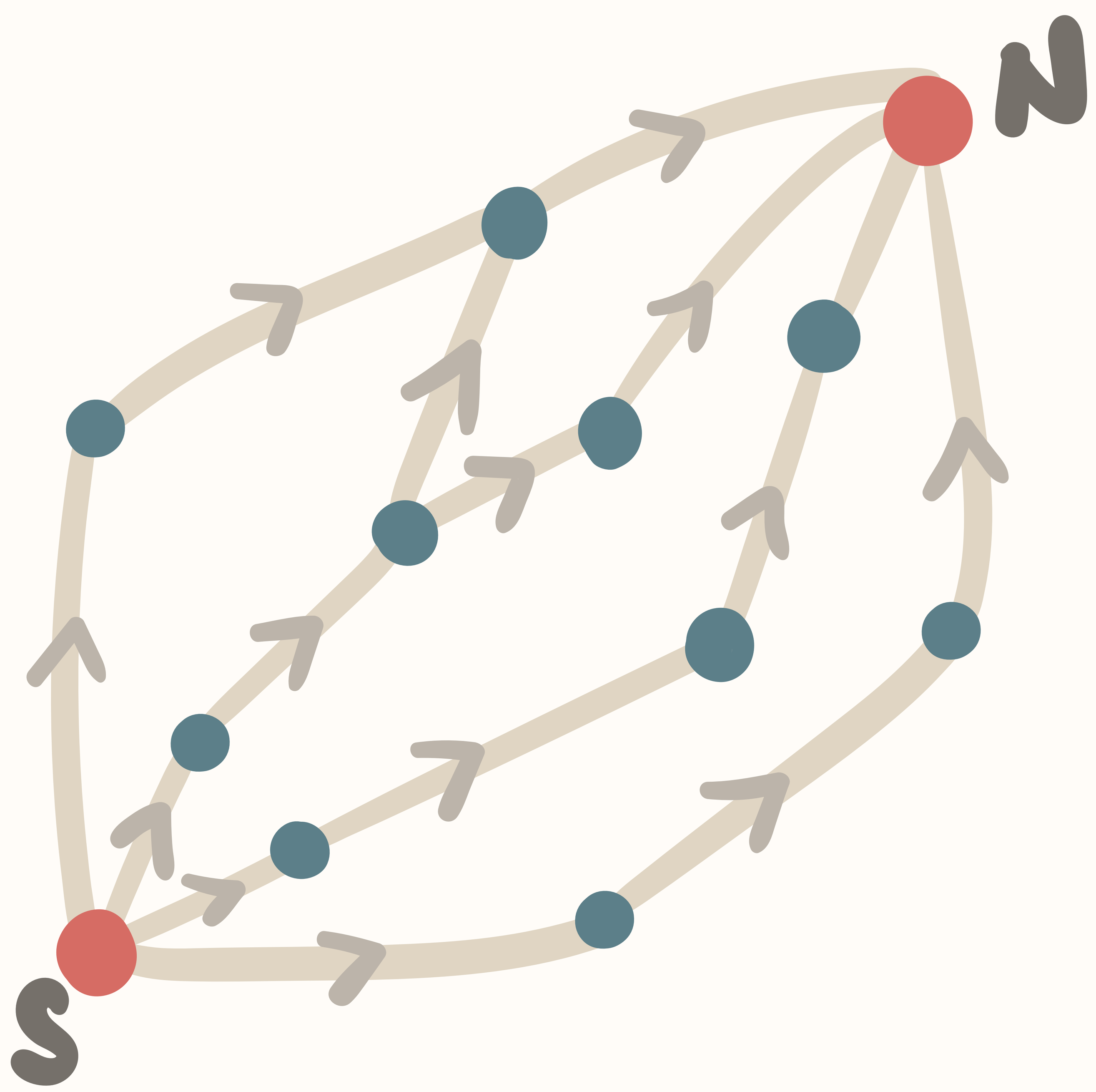
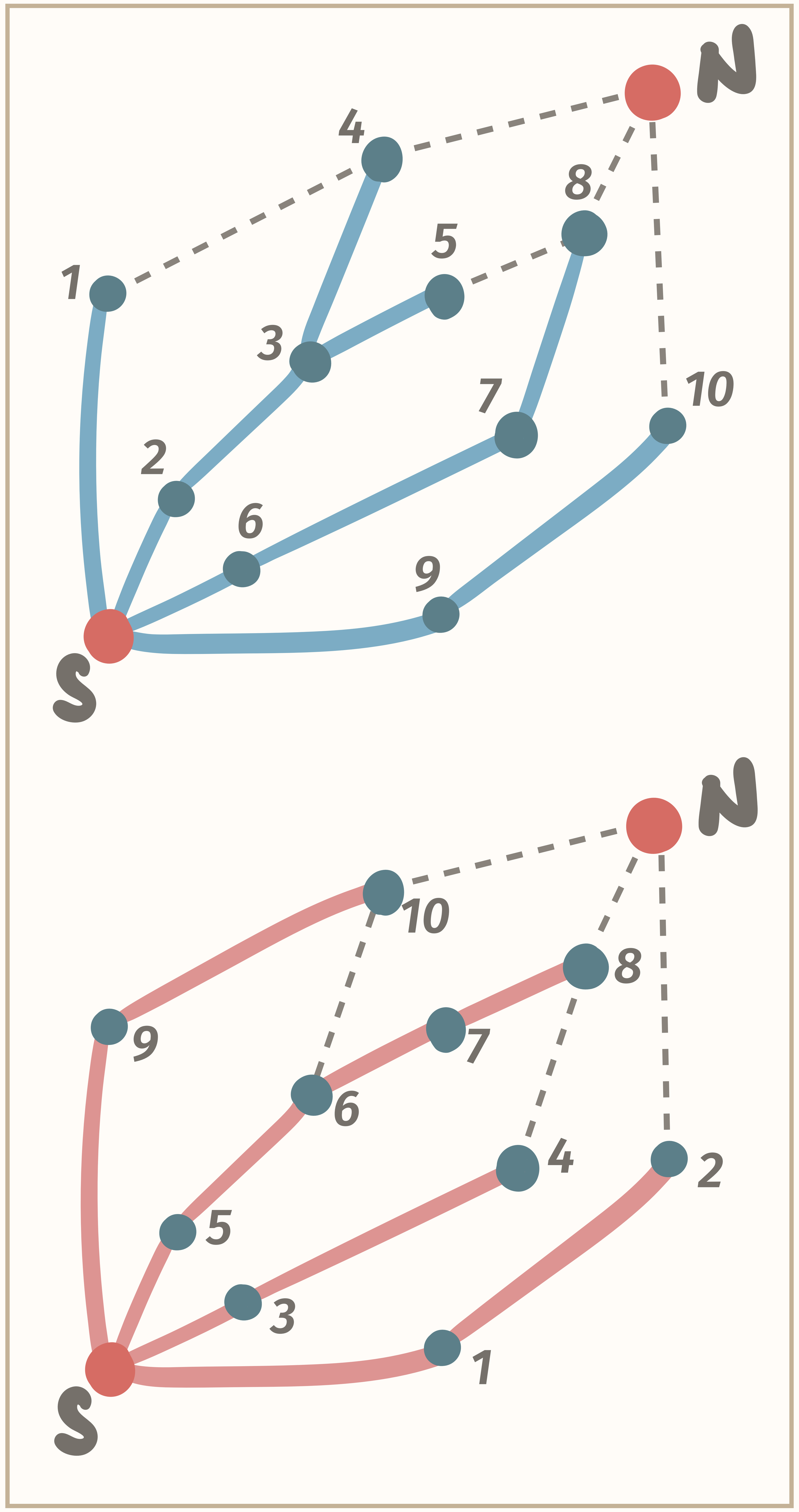


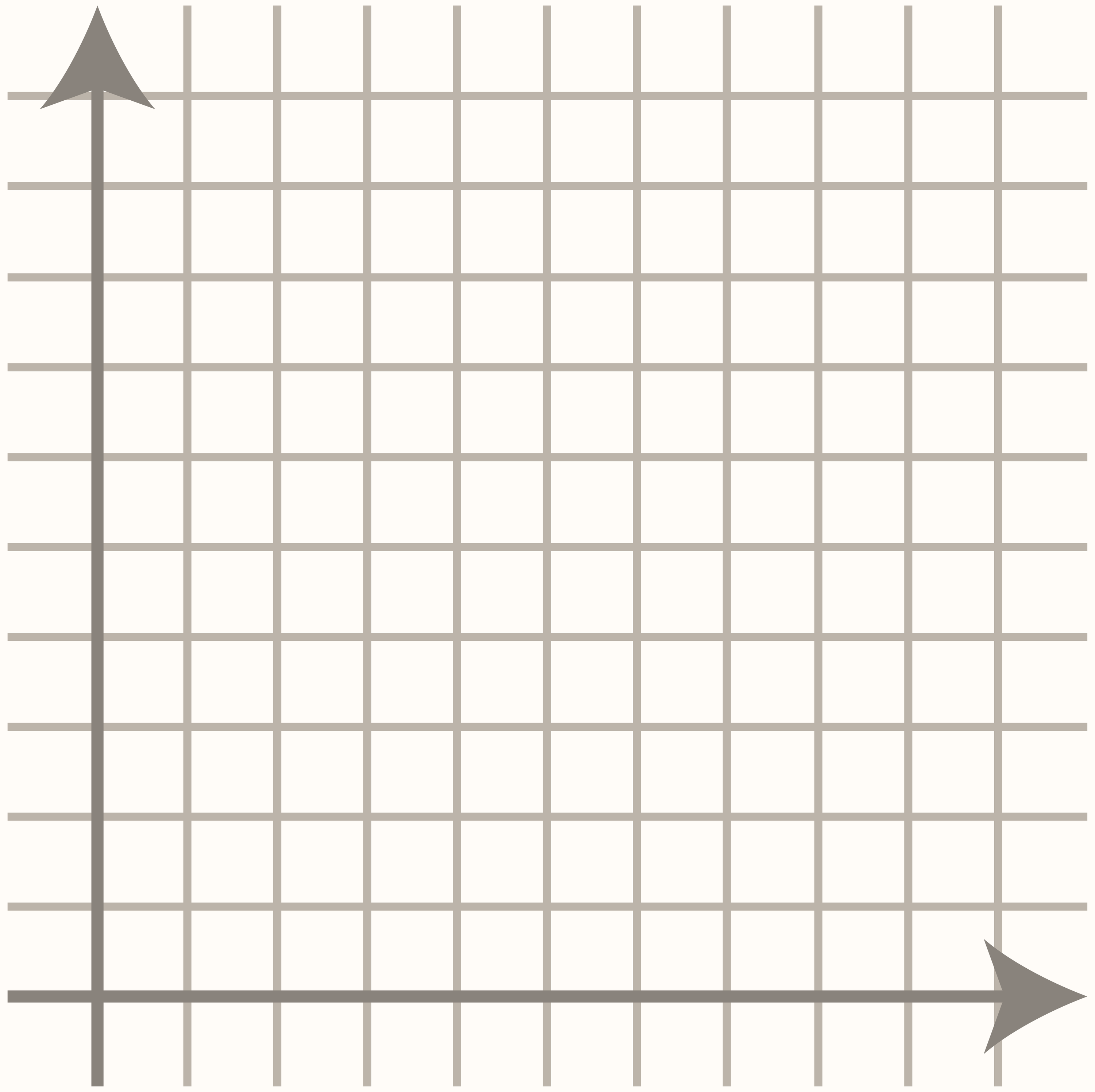


Link with plane permutations

Poset → *Plane permutation*







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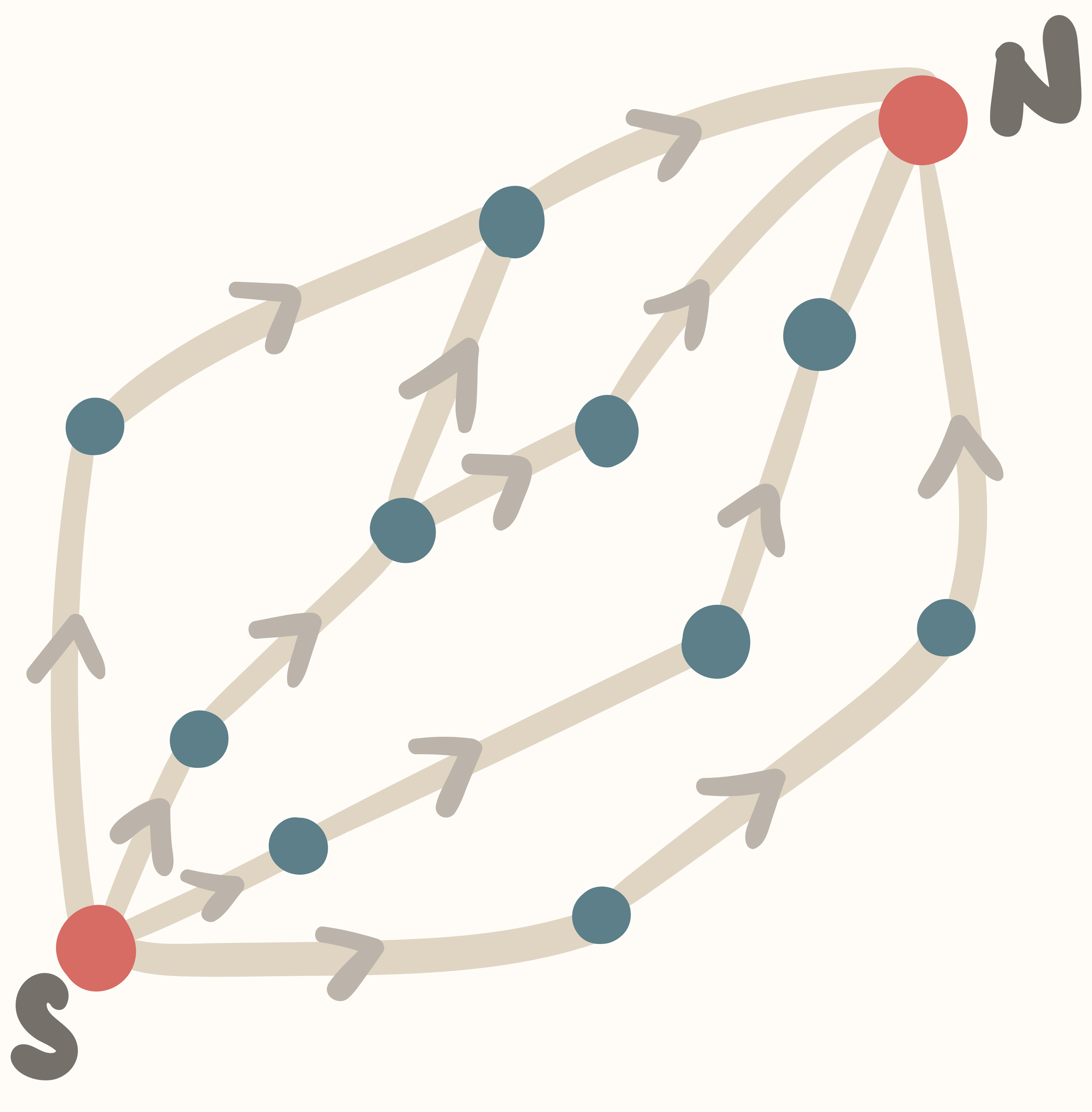

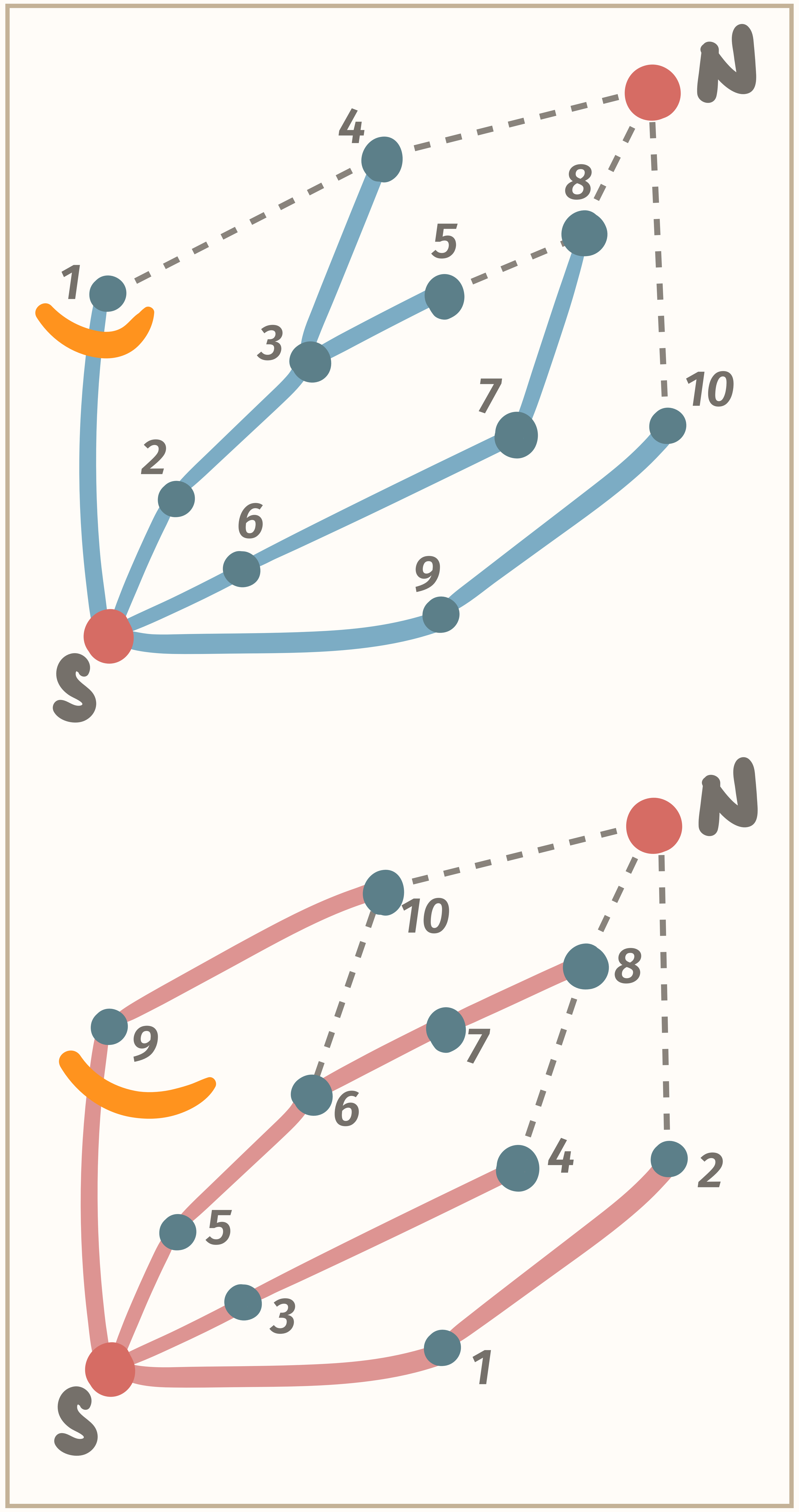

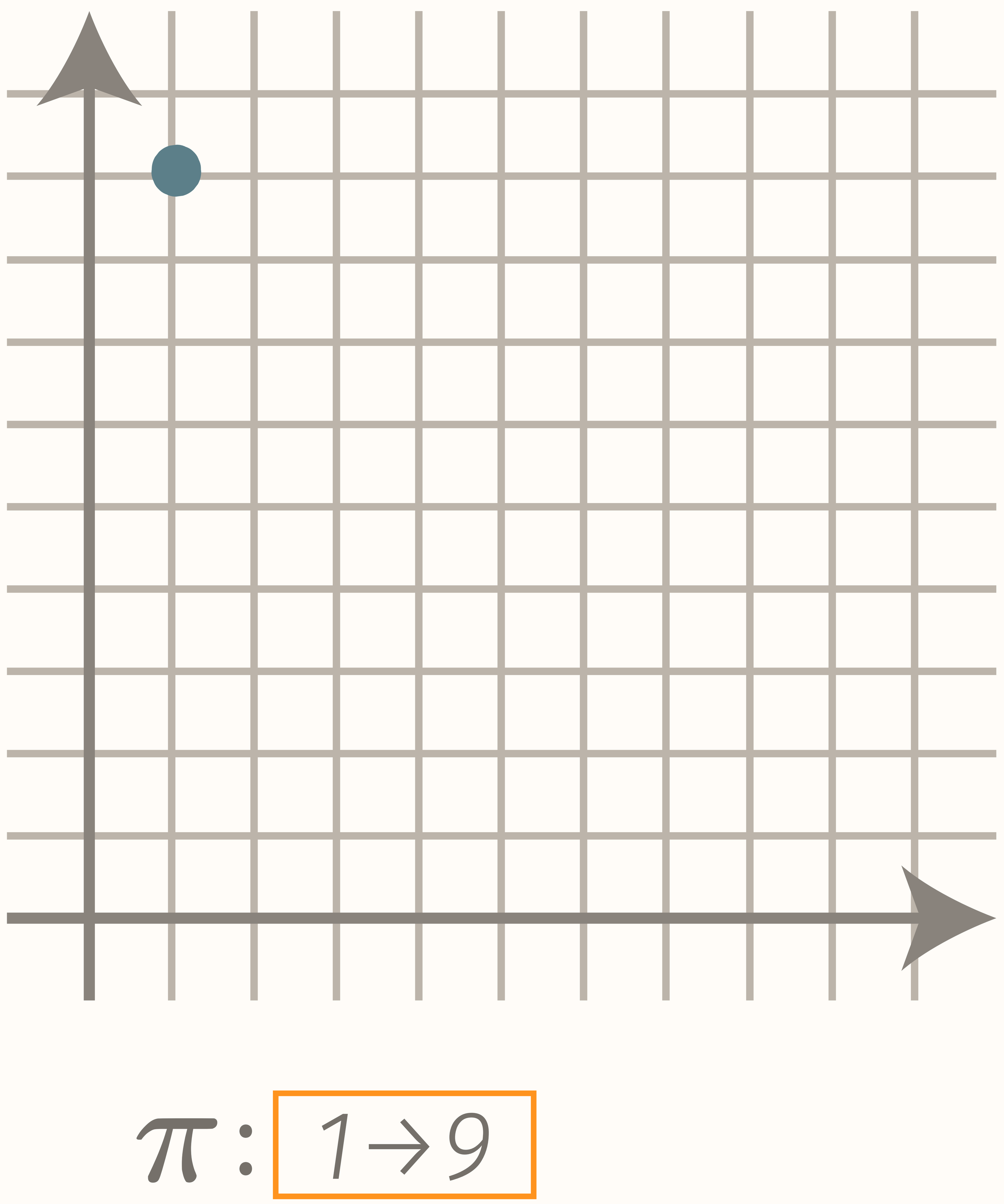

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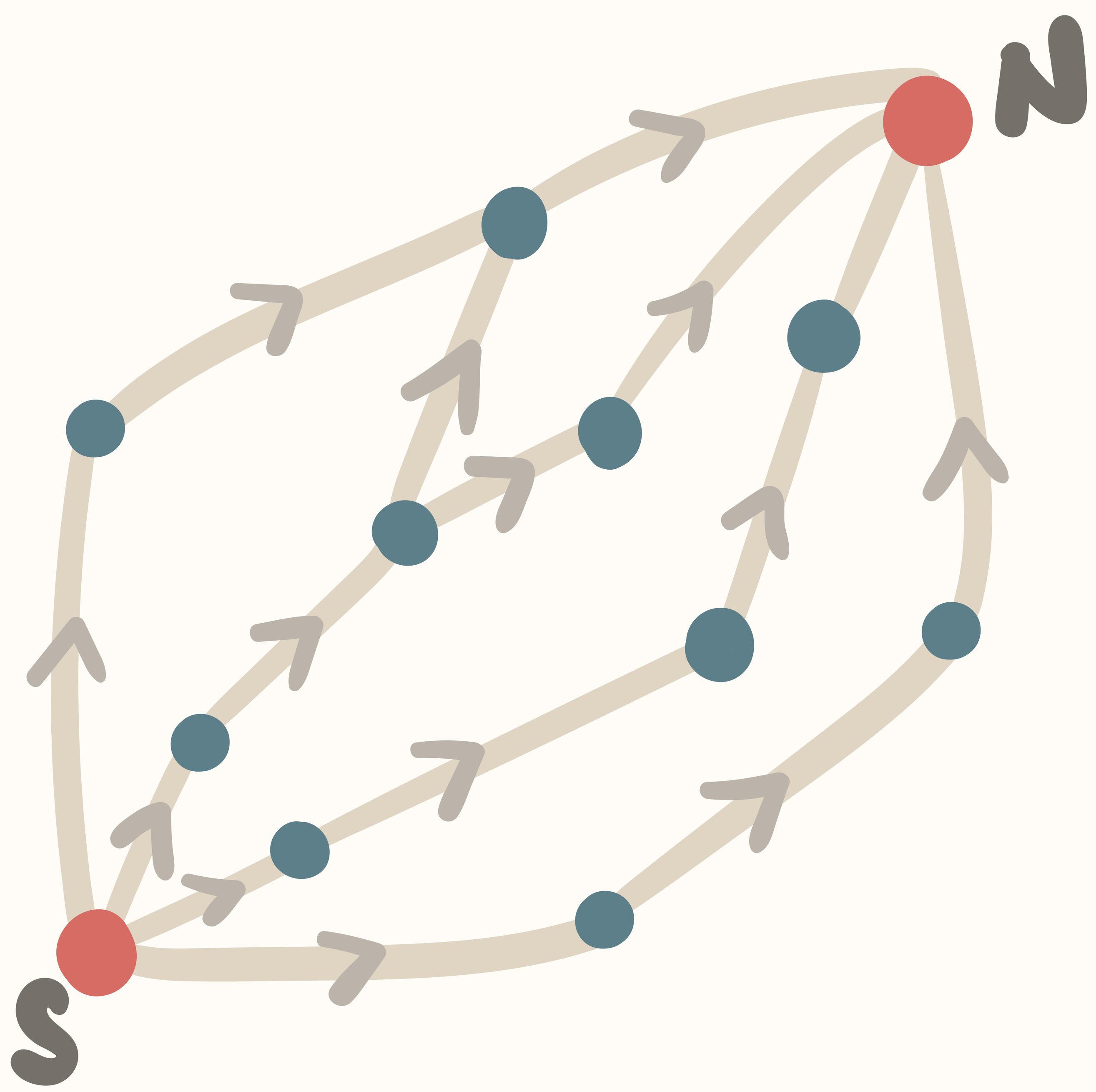
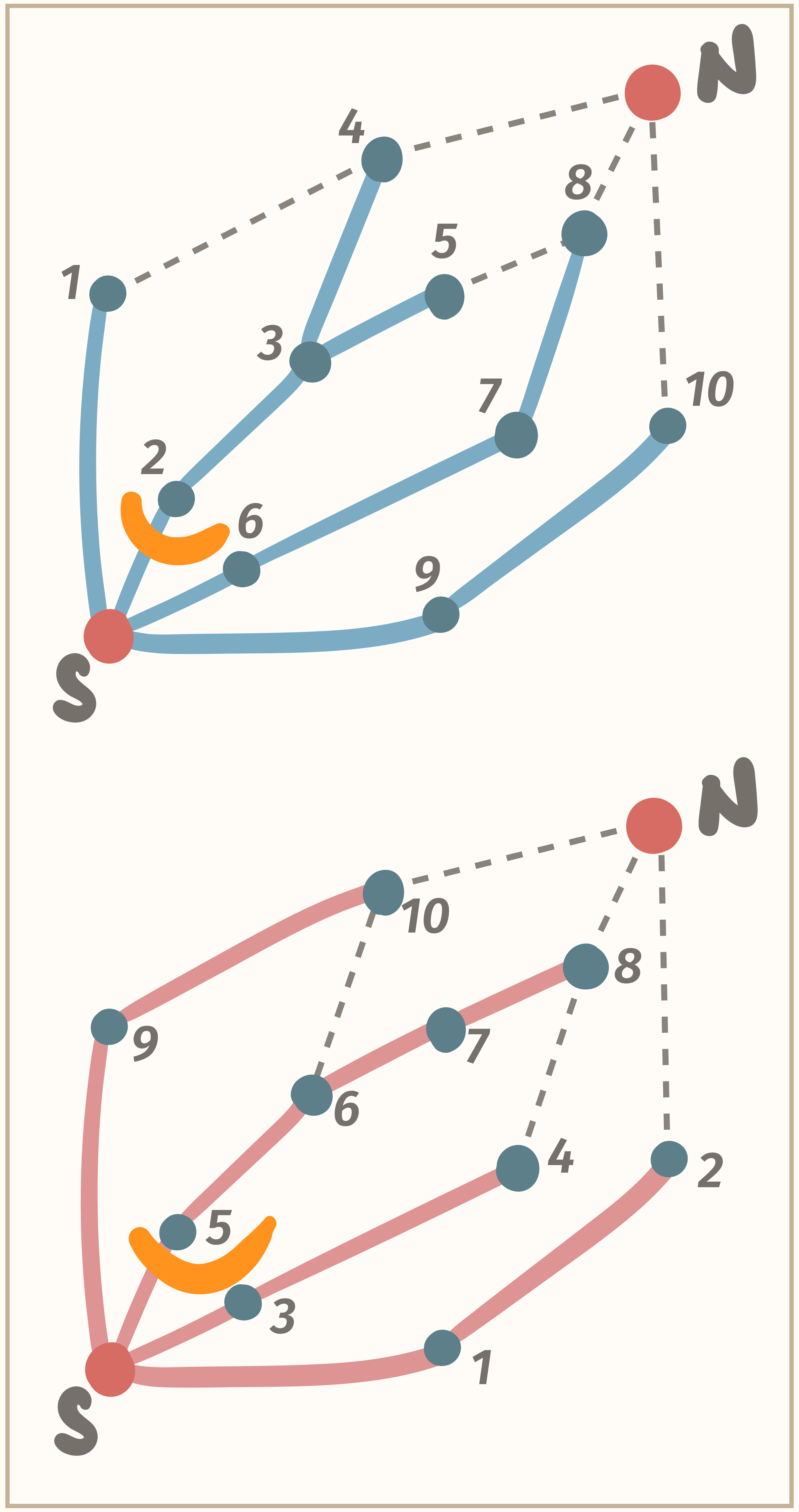
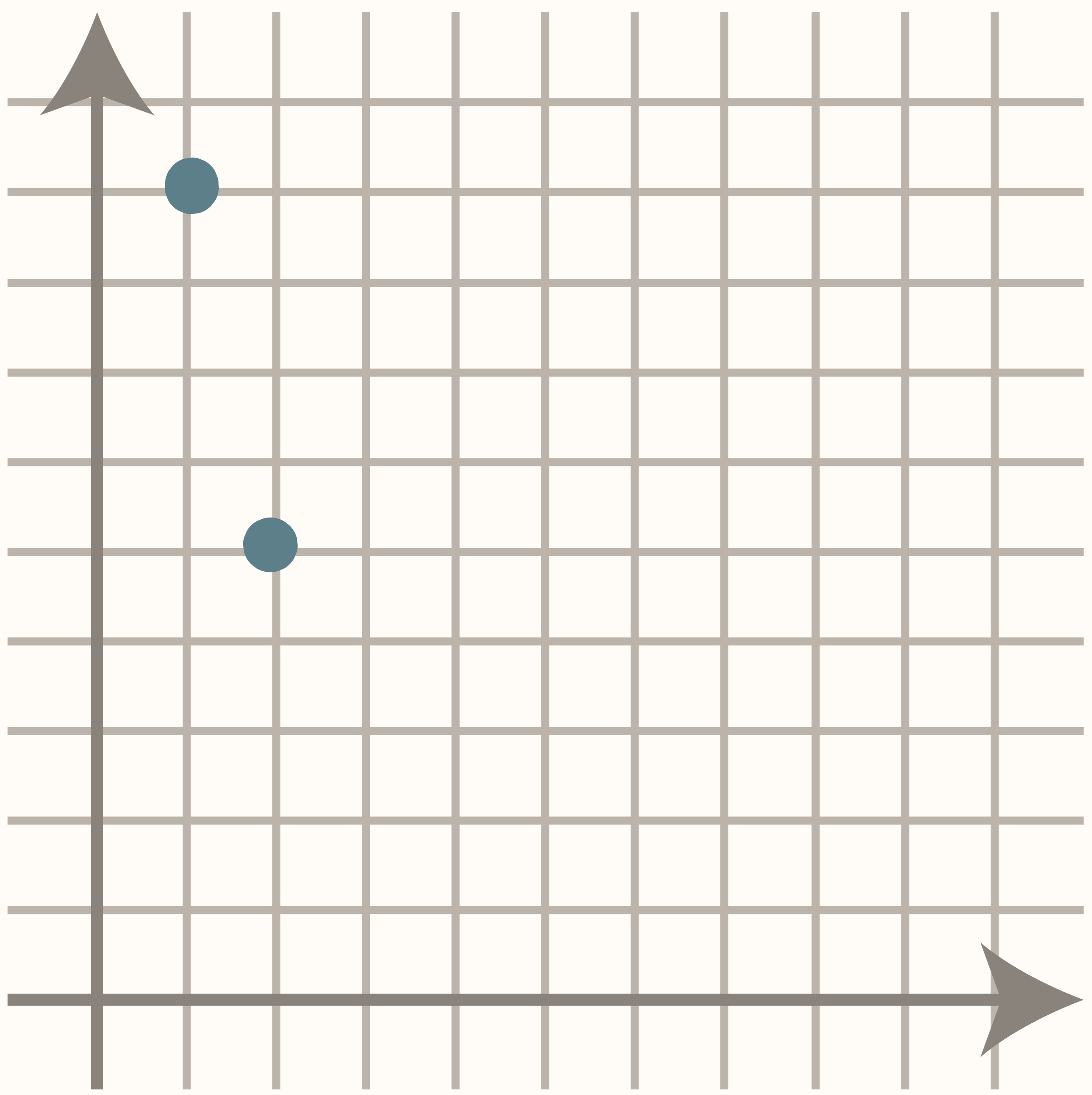




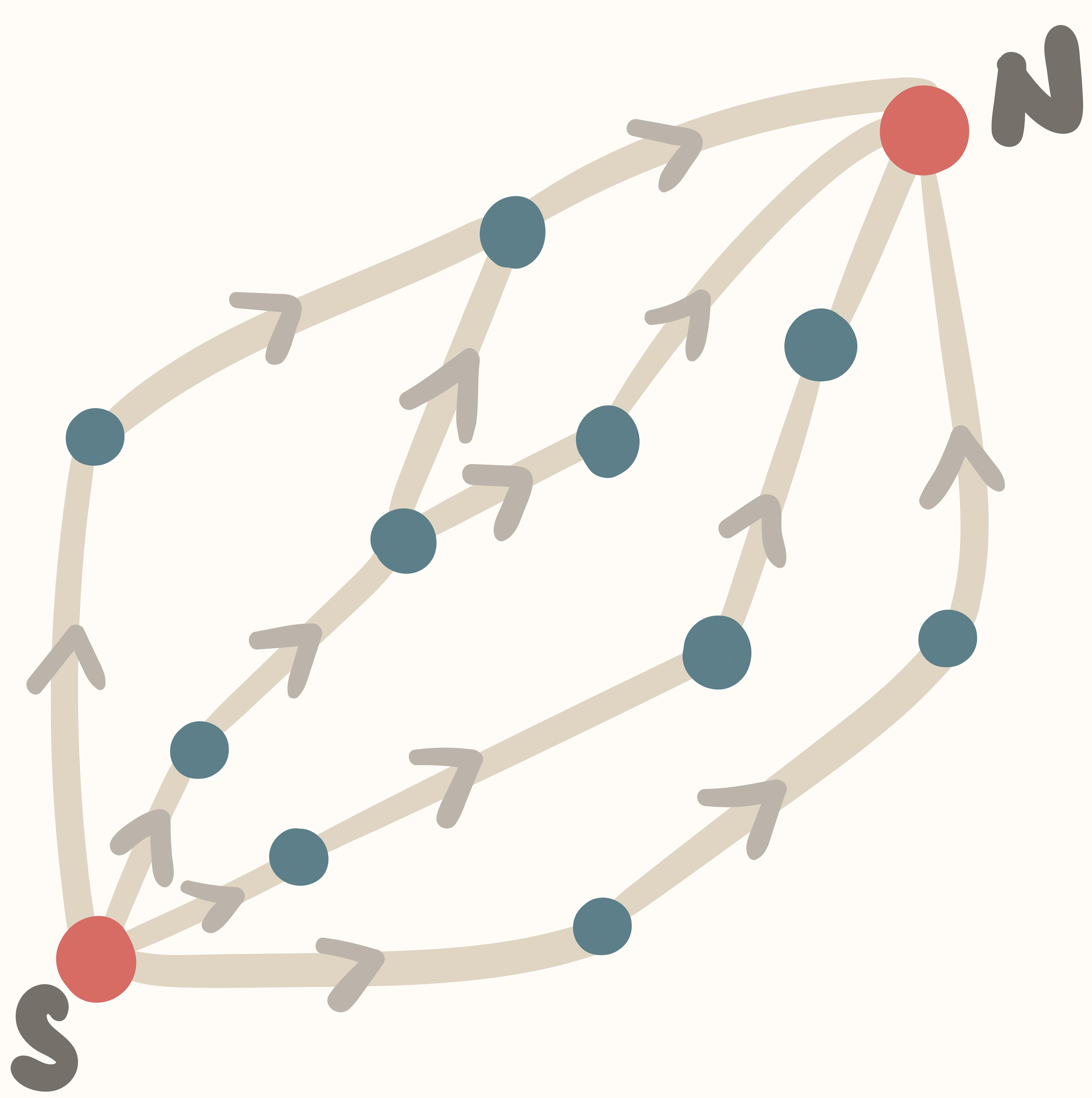


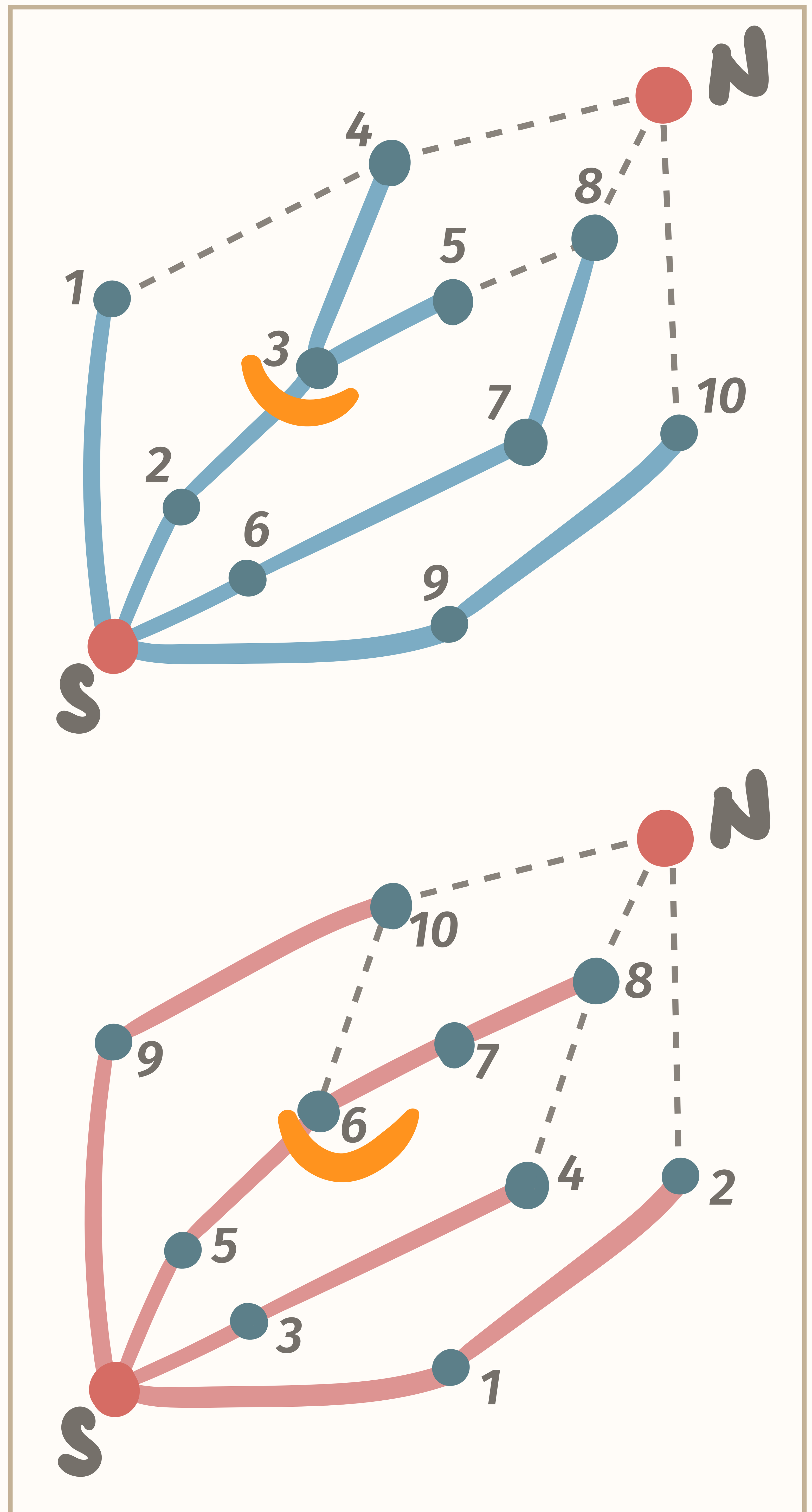
π : $1 \to 9$

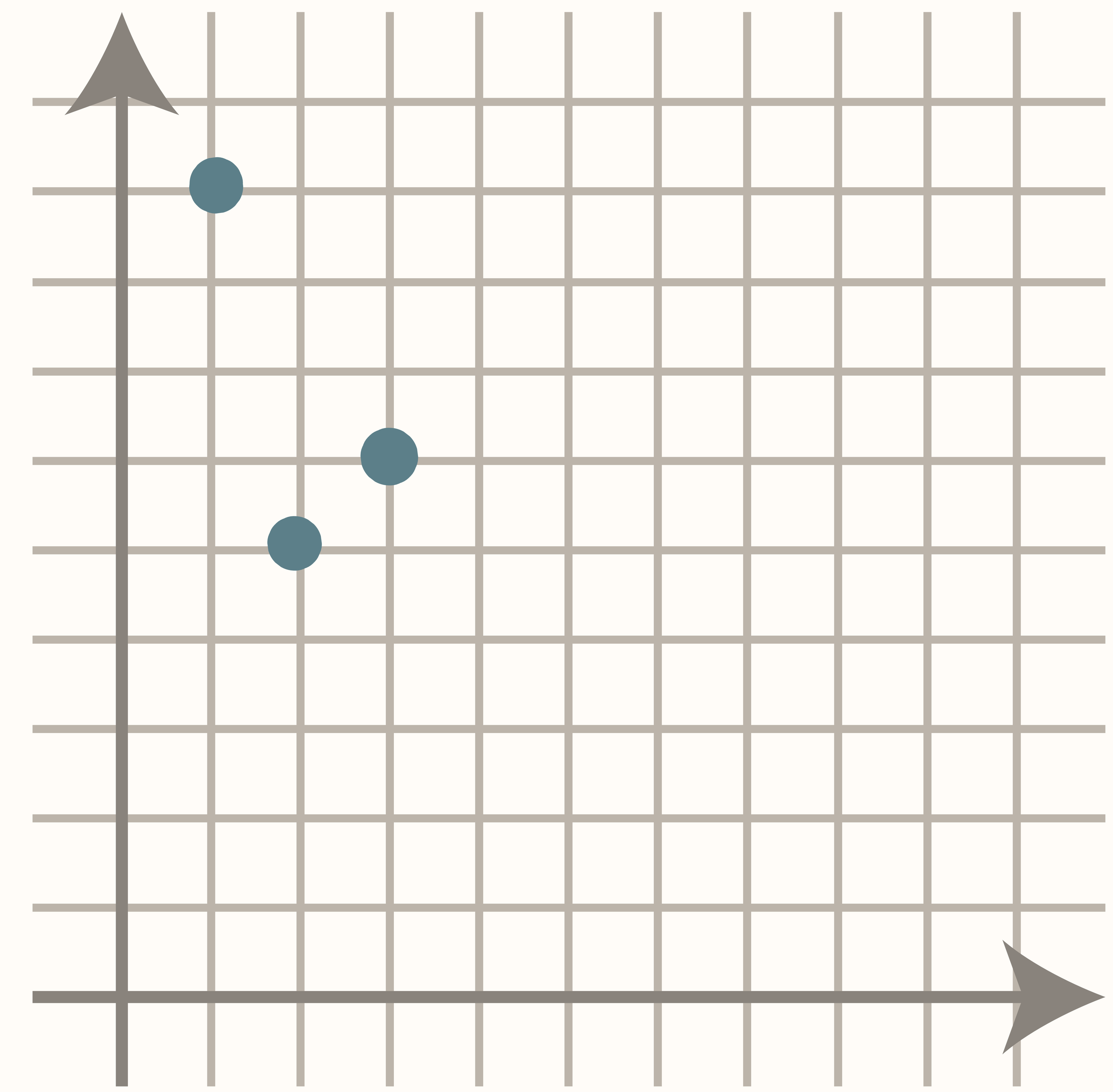
$2 \to 5$

Link with plane permutations

Poset ⟶ *Plane permutation*



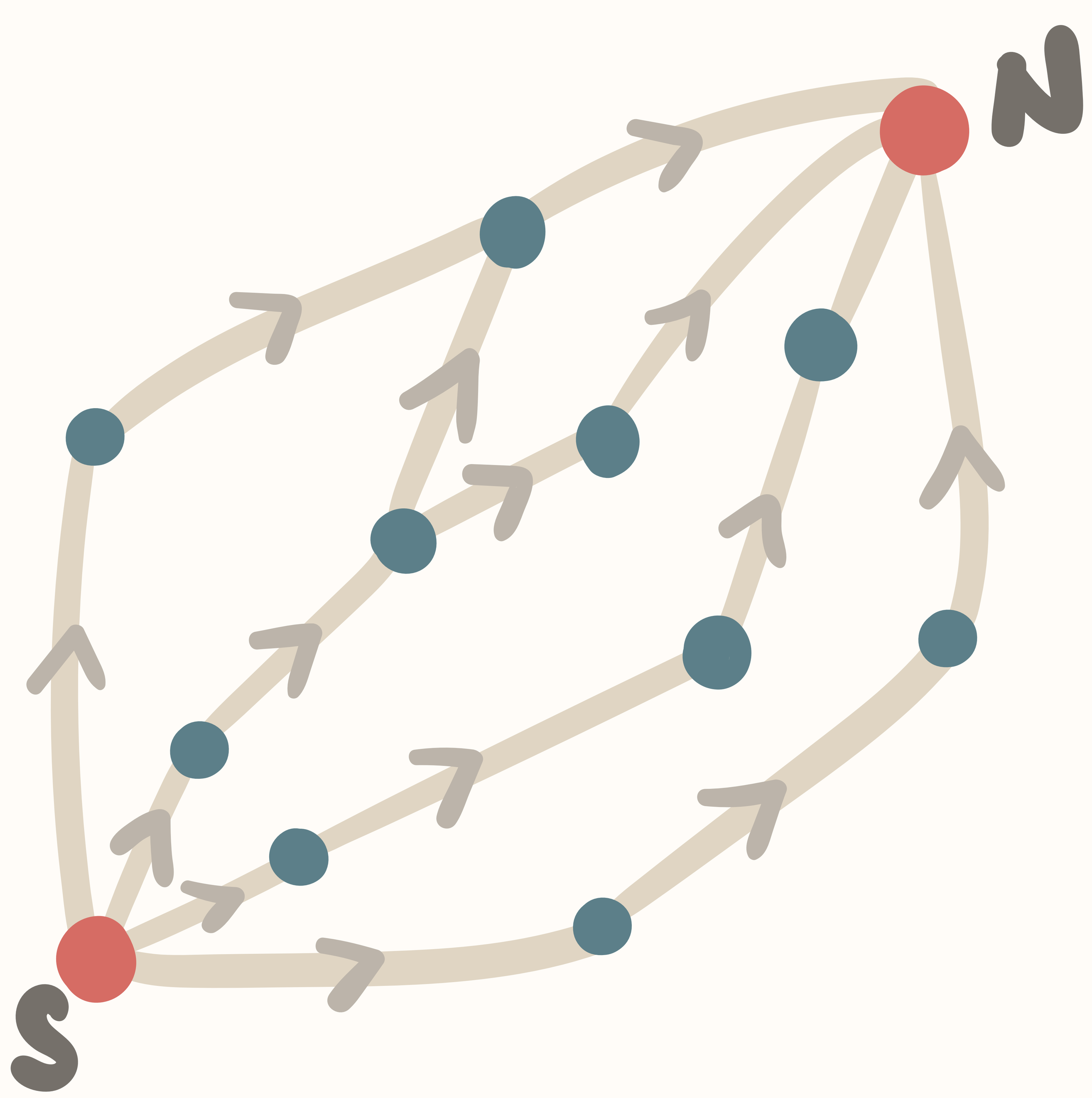


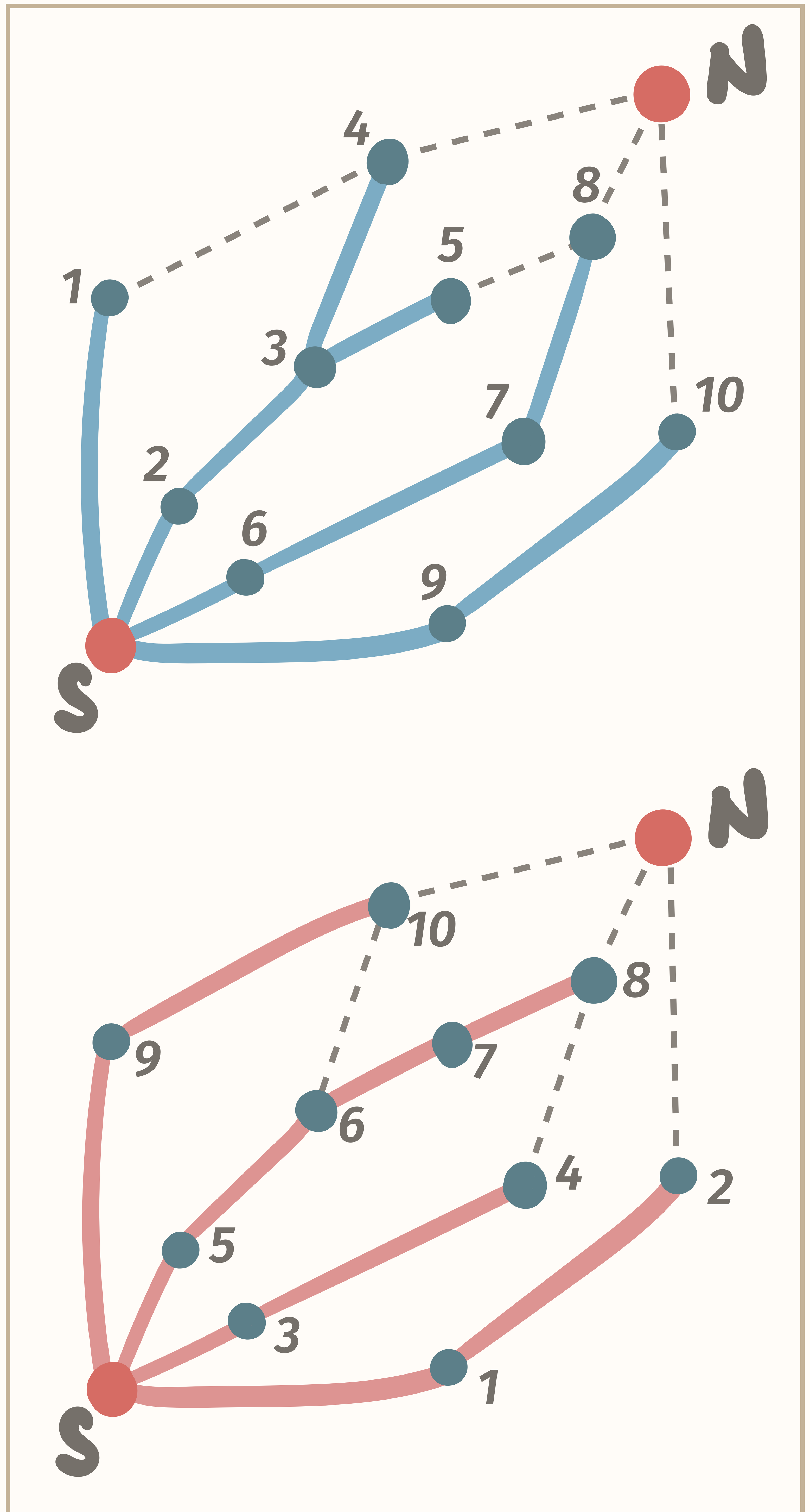


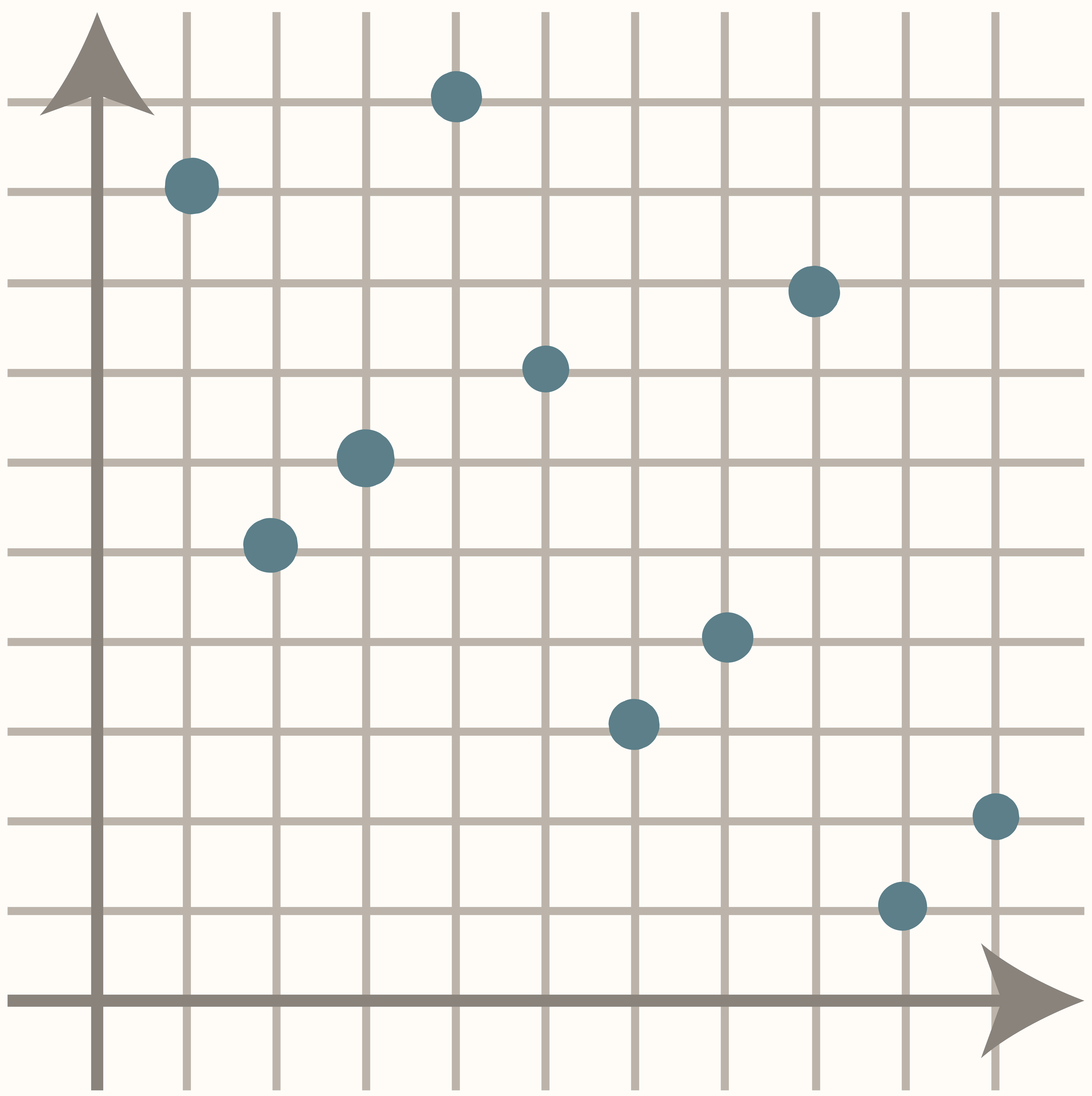
π : 1→9
2→5
3→6

Link with plane permutations

Poset ⟶ *Plane permutation*





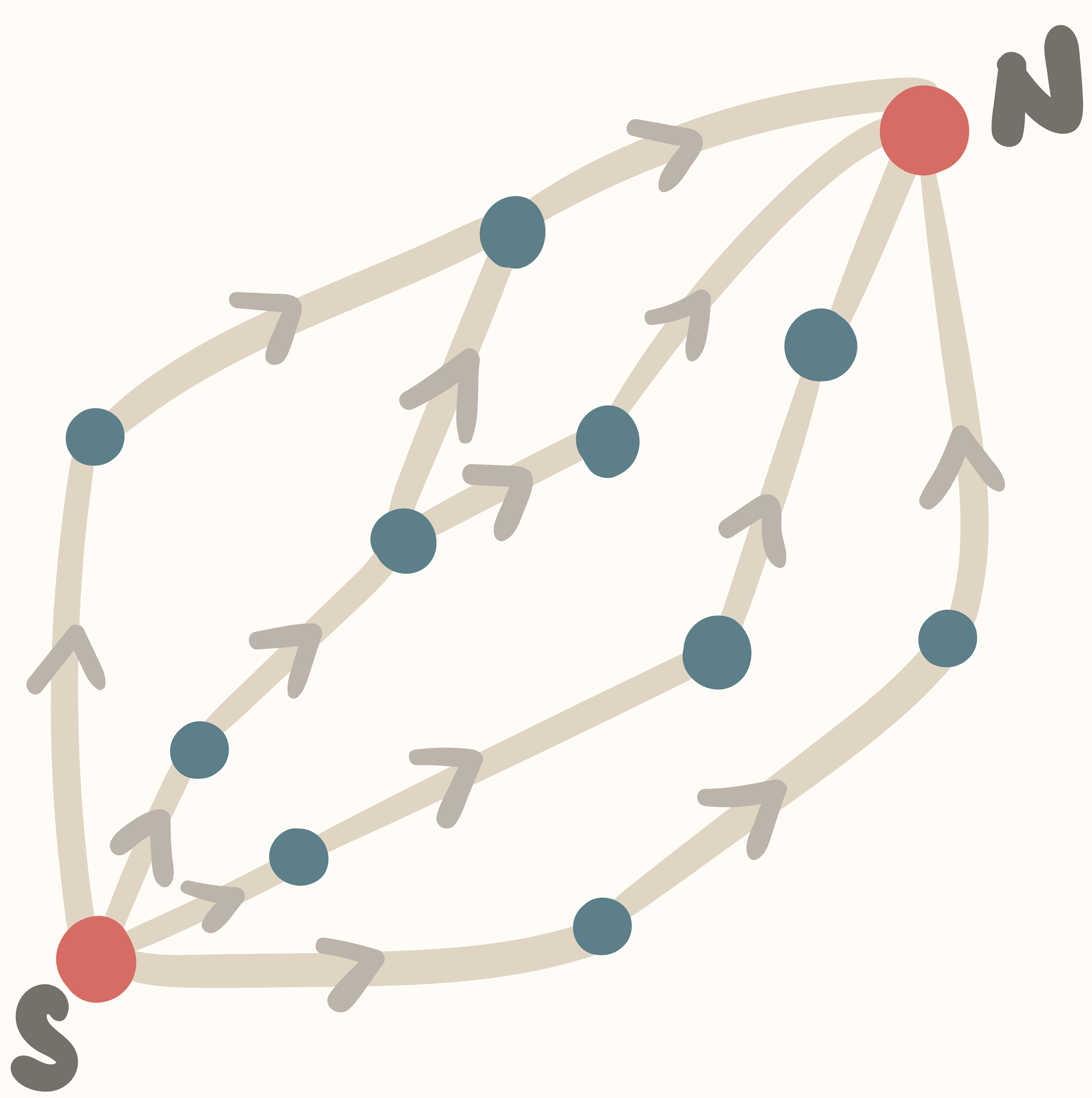


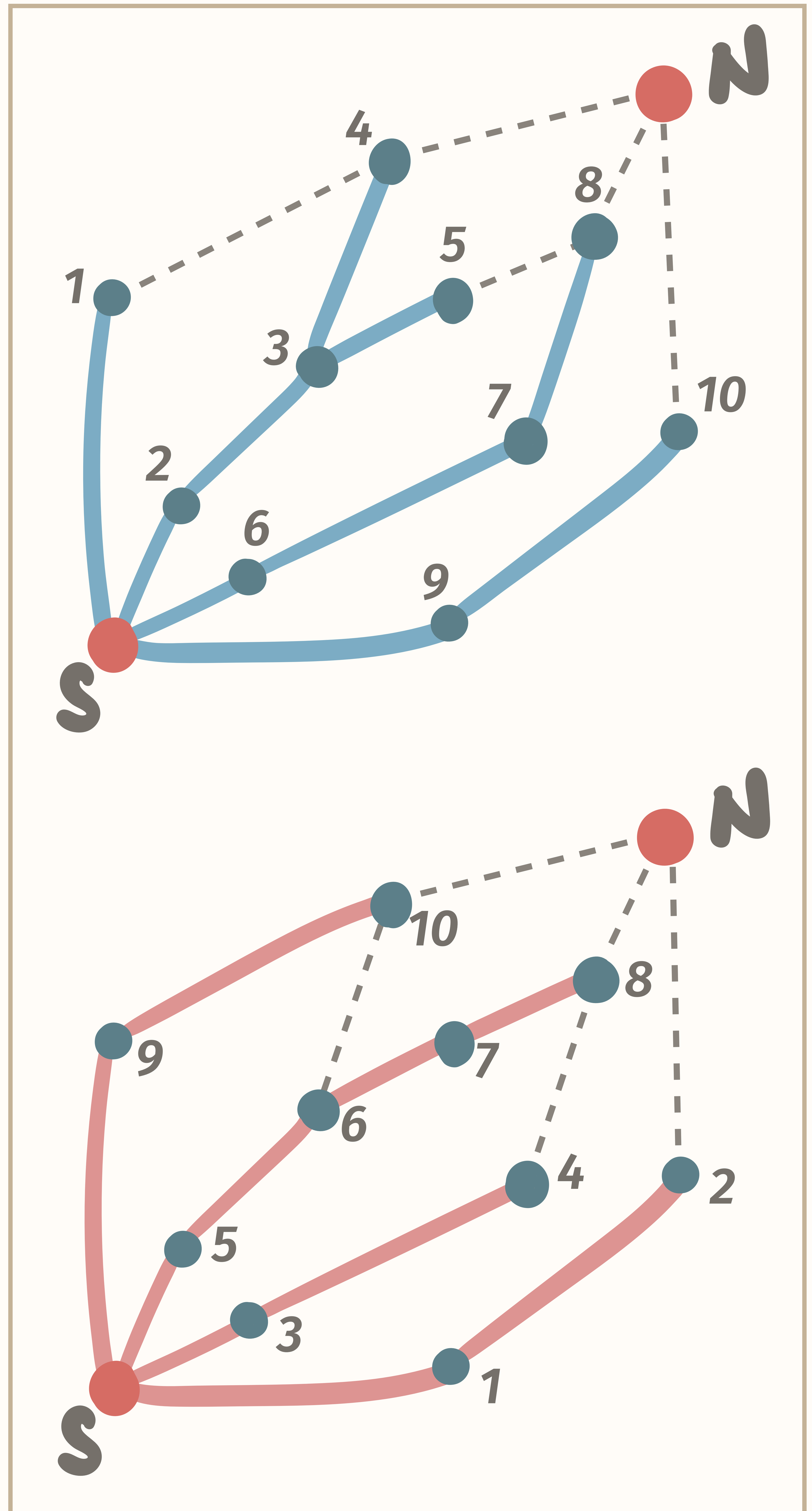
π :

1→9	6→3
2→5	7→4
3→6	8→8
4→10	9→1
5→7	10→2

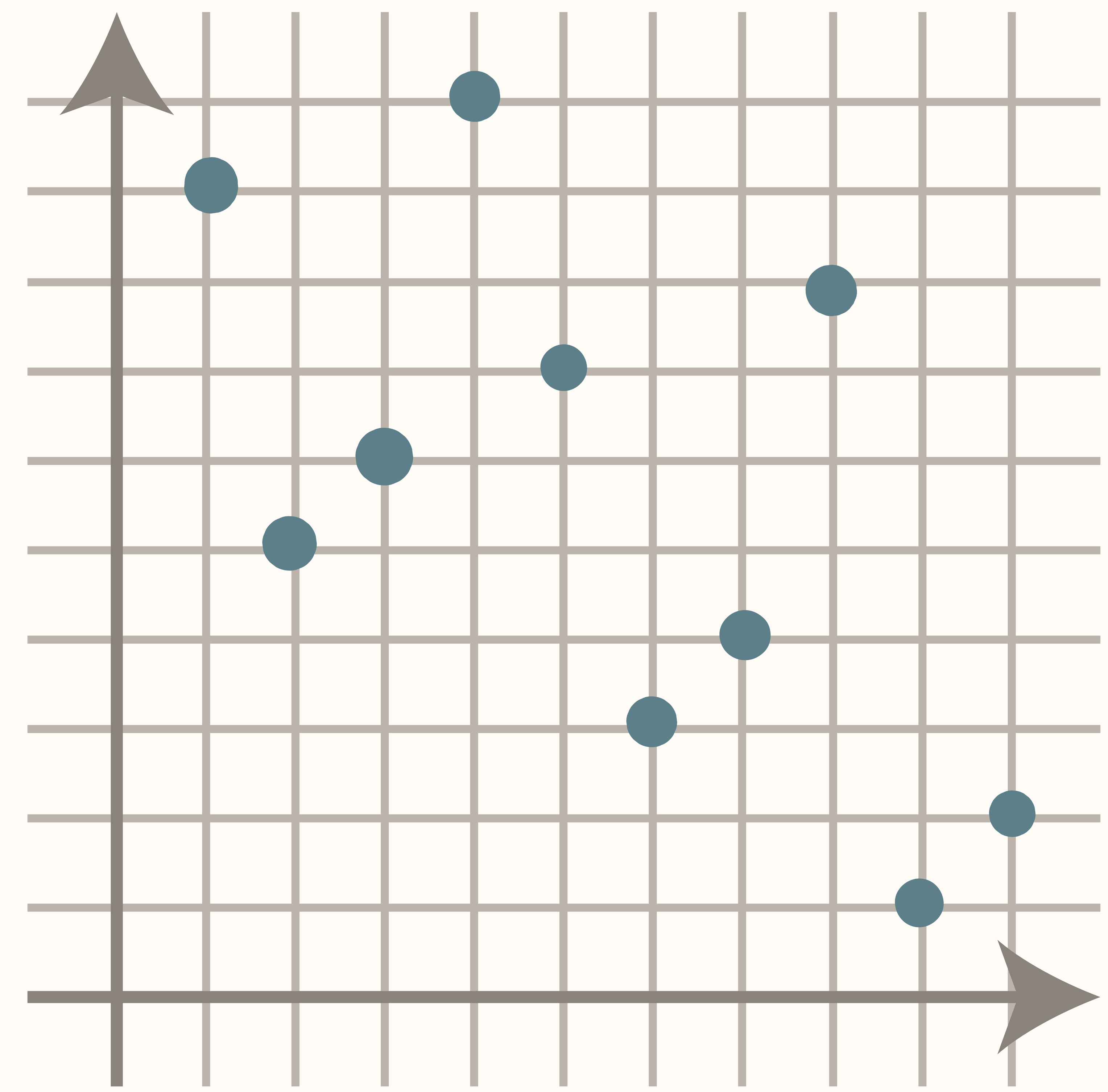
Link with plane permutations

Poset ⟶ *Plane permutation*





≫ *Area requirement and symmetry display of planar upward drawings,* G. Di Battista, R. Tamassia, and I. G. Tollis (1992)



π:

1→9	6→3
2→5	7→4
3→6	8→8
4→10	9→1
5→7	10→2

Summary

The KMSW bijection

1. Application to three map models

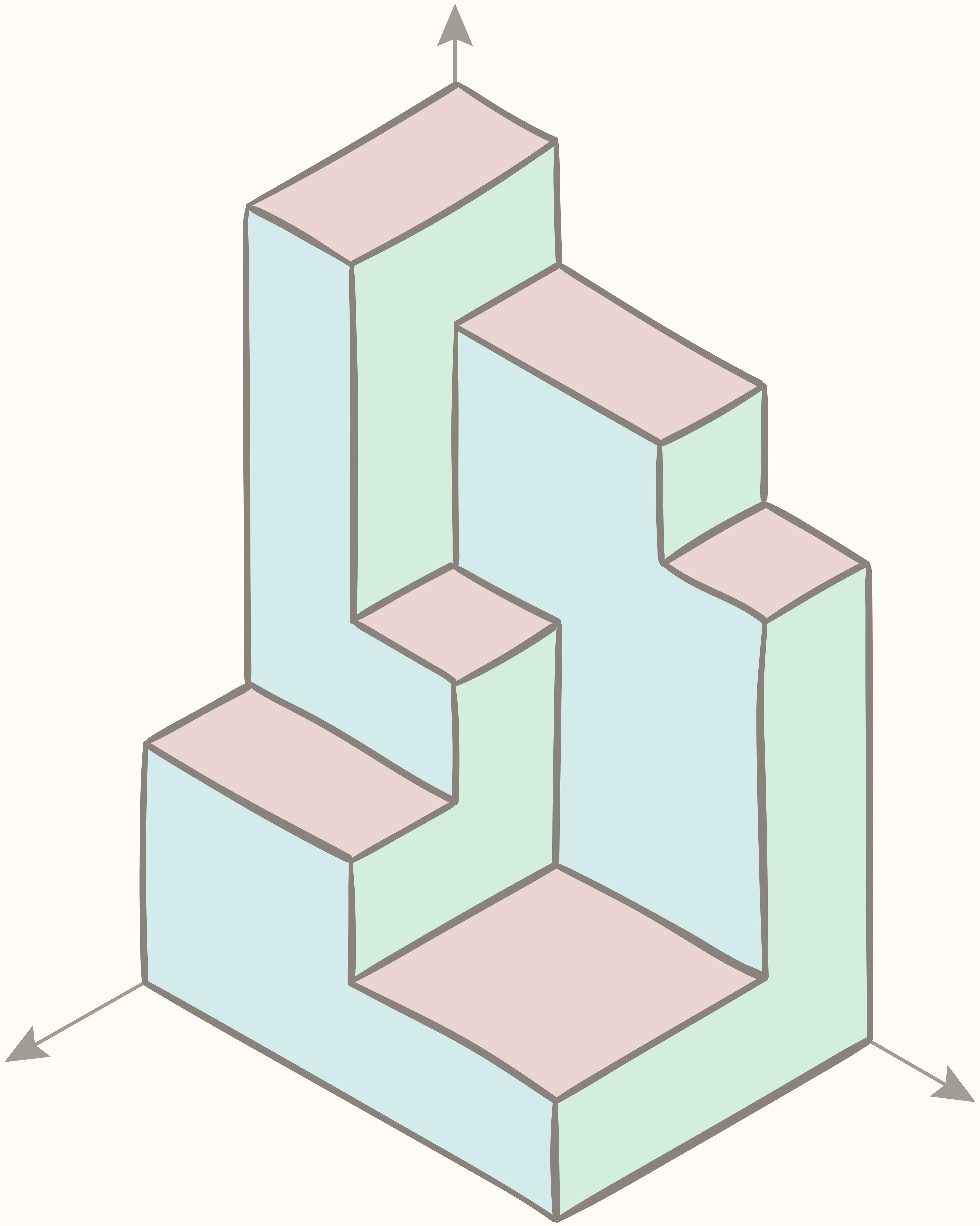
a. Plane bipolar posets
b. Plane bipolar posets by vertices
c. Transversal structures
d. Asymtotics

2. Interlude : plane permutations

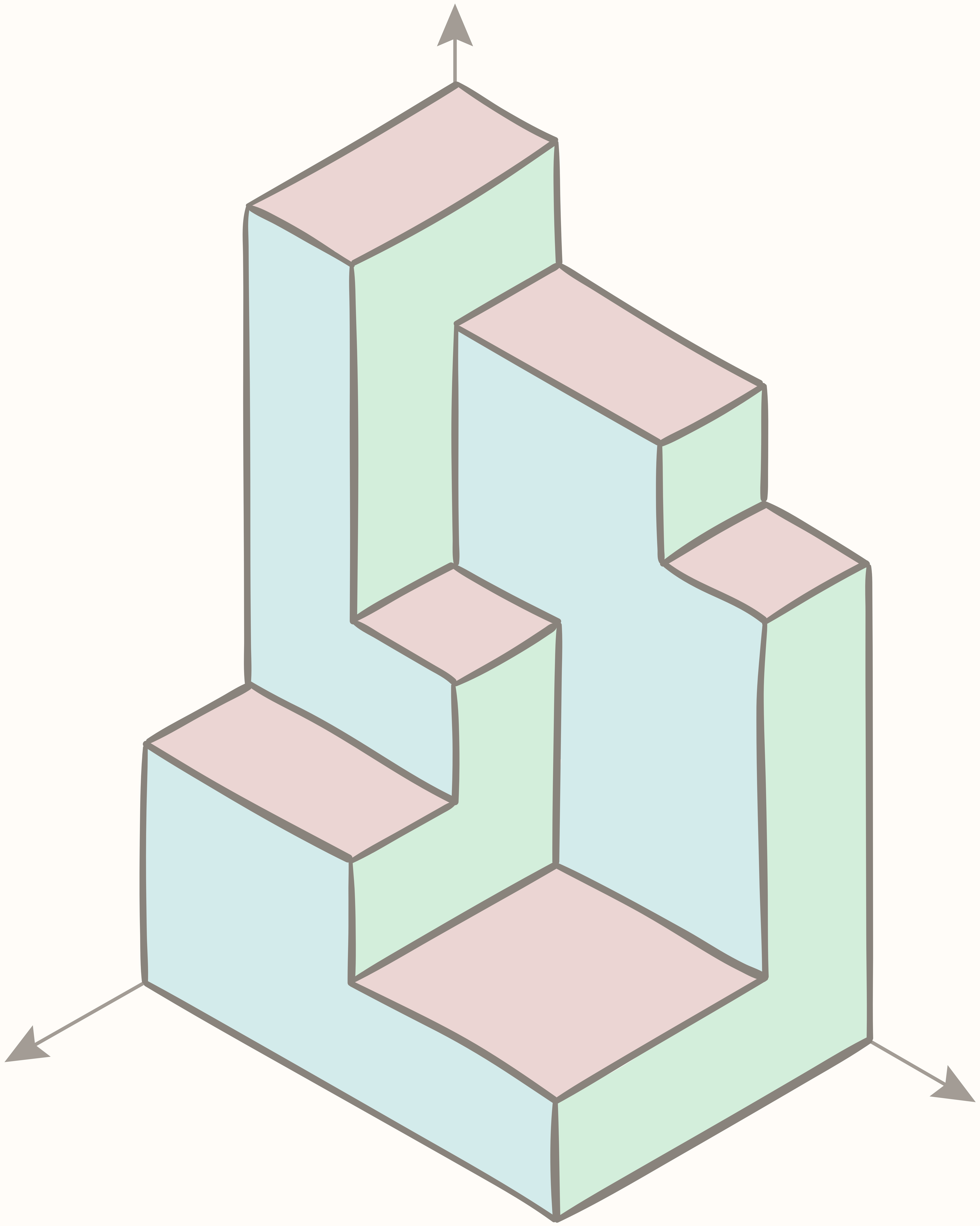
3. Application to corner polyhera

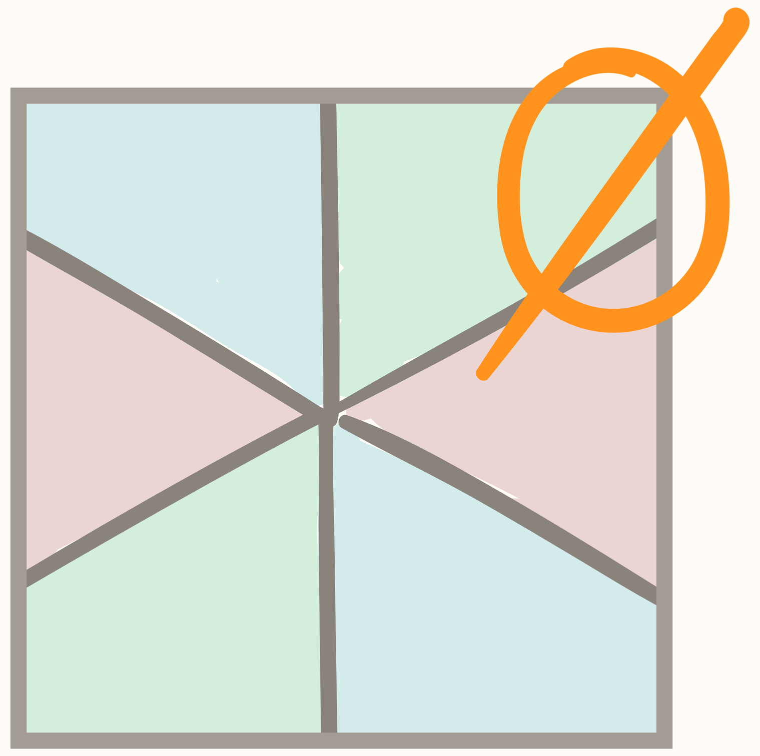
a. Via polyheral orientations
b. Via Schnyder colorings
c. Asymtotics

Corner polyhedra

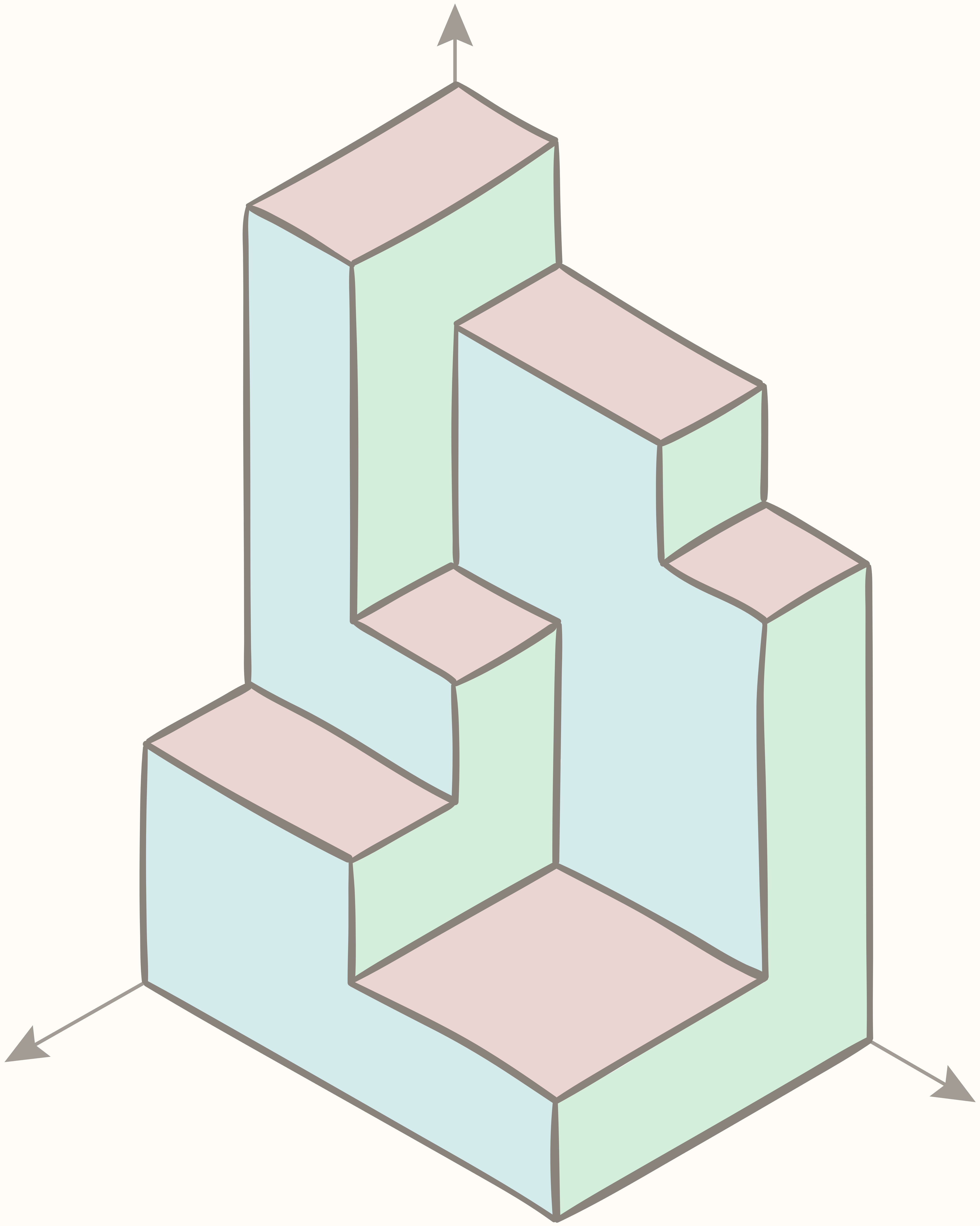


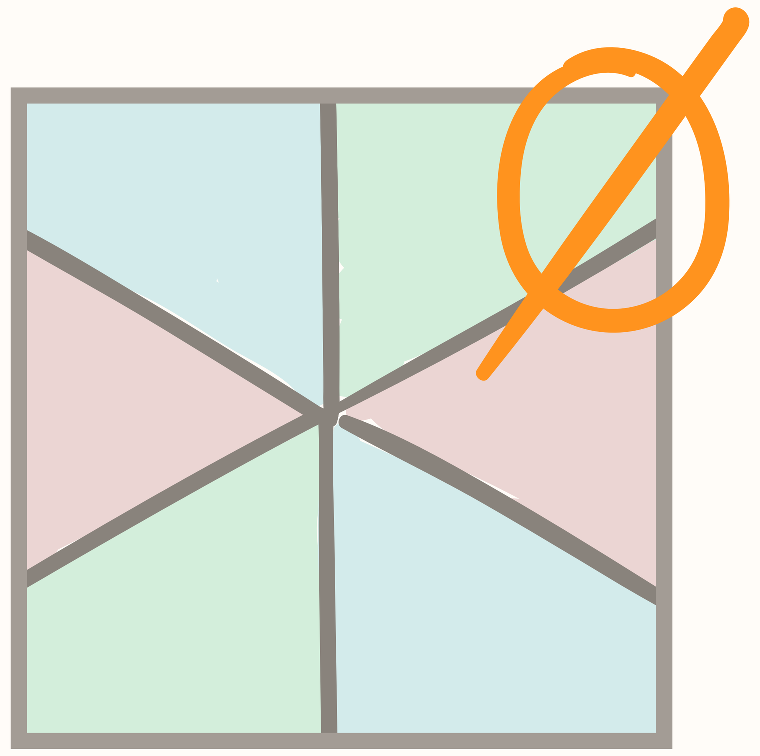
Corner polyhedra





Corner polyhedra

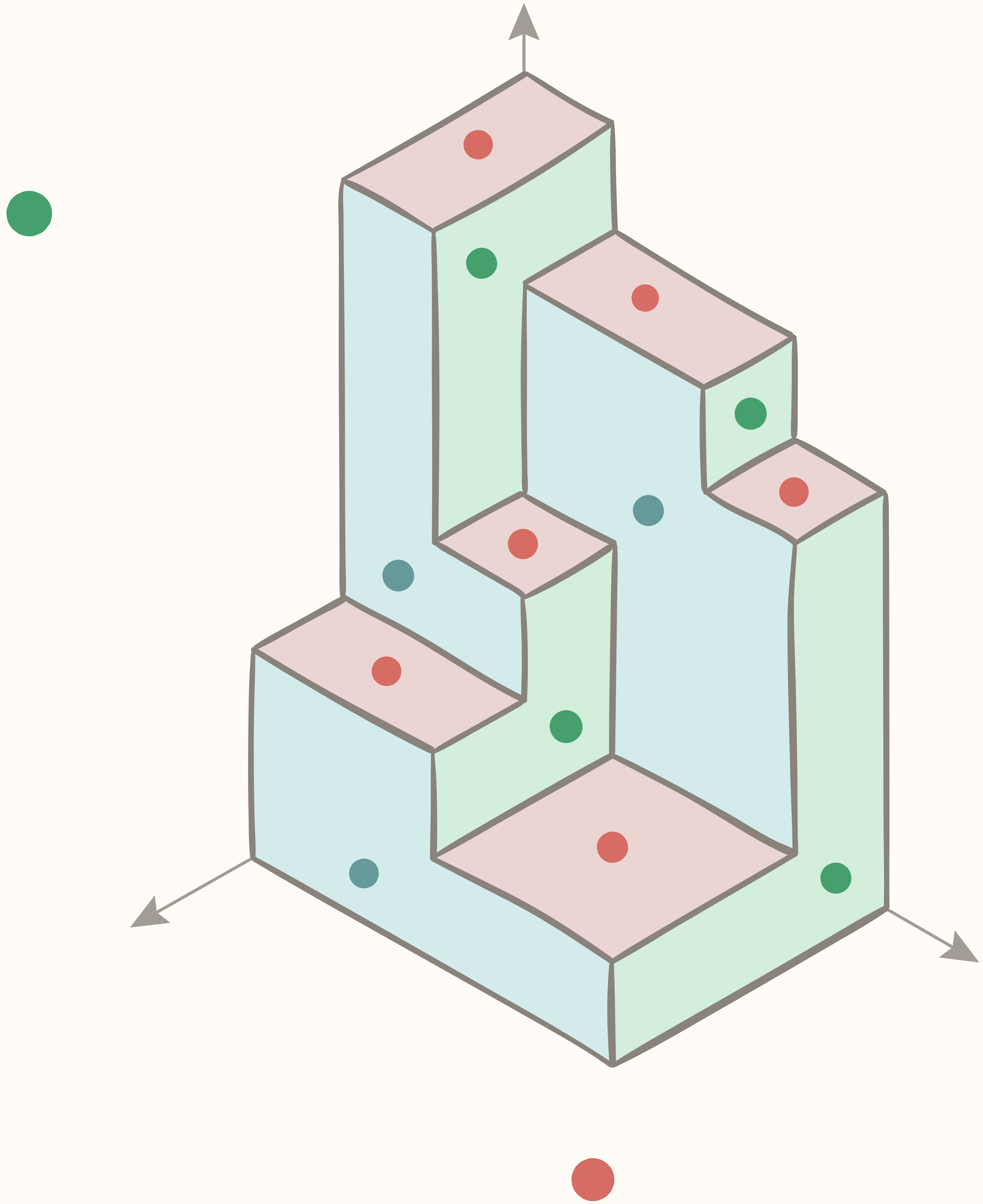


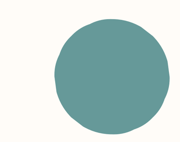


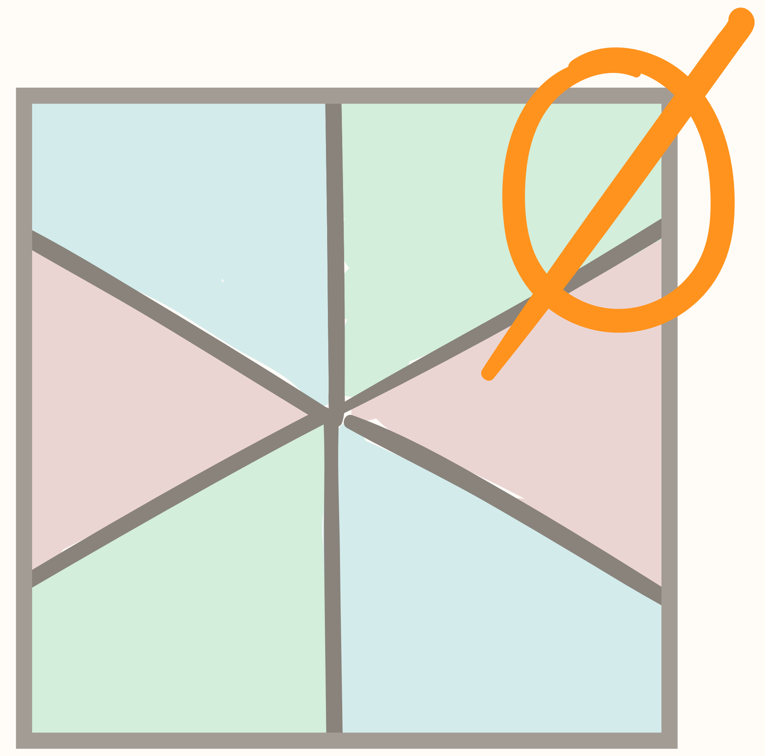
corner polyhedra ⟷ **polyhedral orientations**

↠ *Steinitz theorems for orthogonal polyhera,*
D. Eppstein & E. Mumford (2010)

Corner polyhedra







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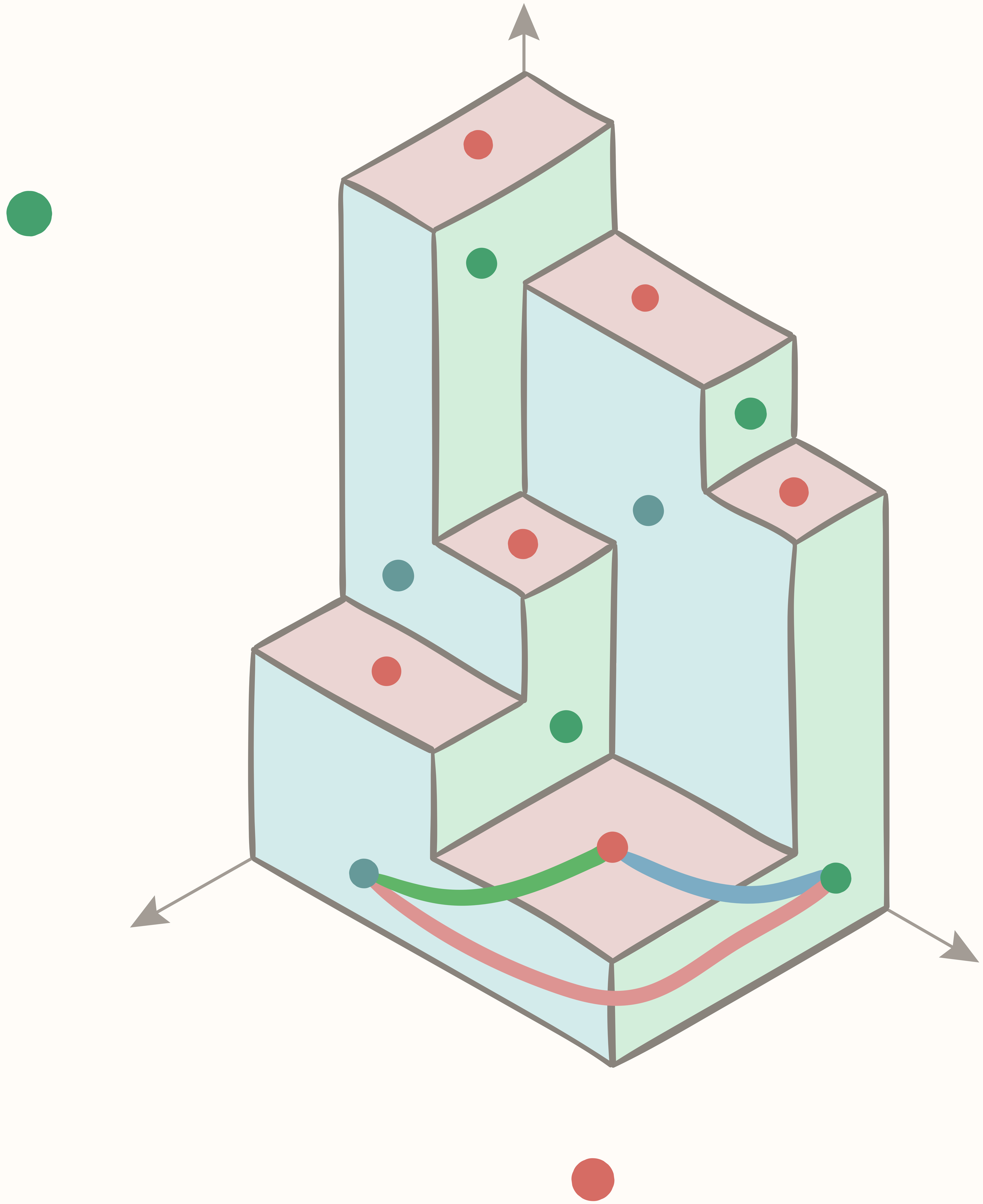

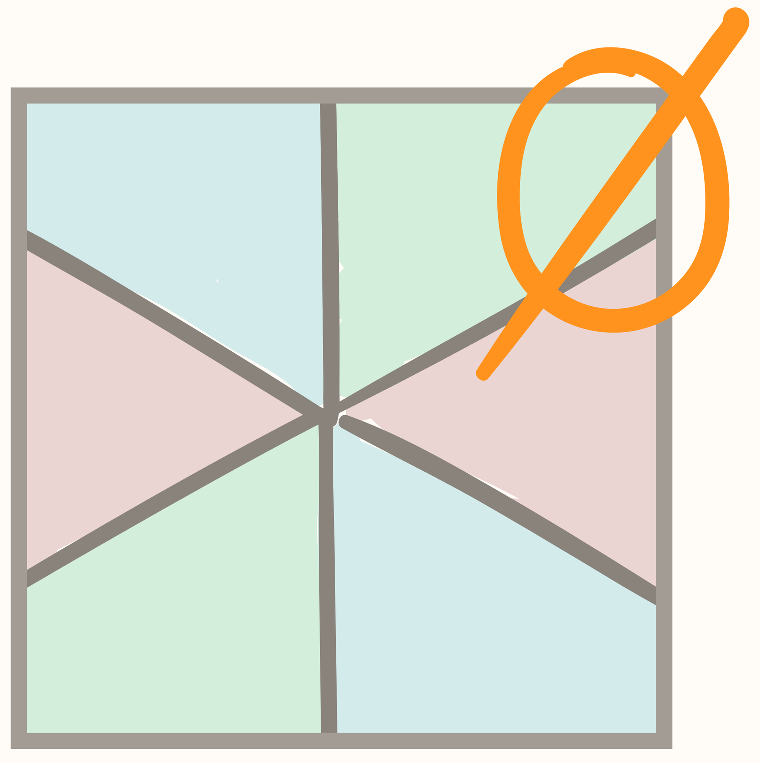

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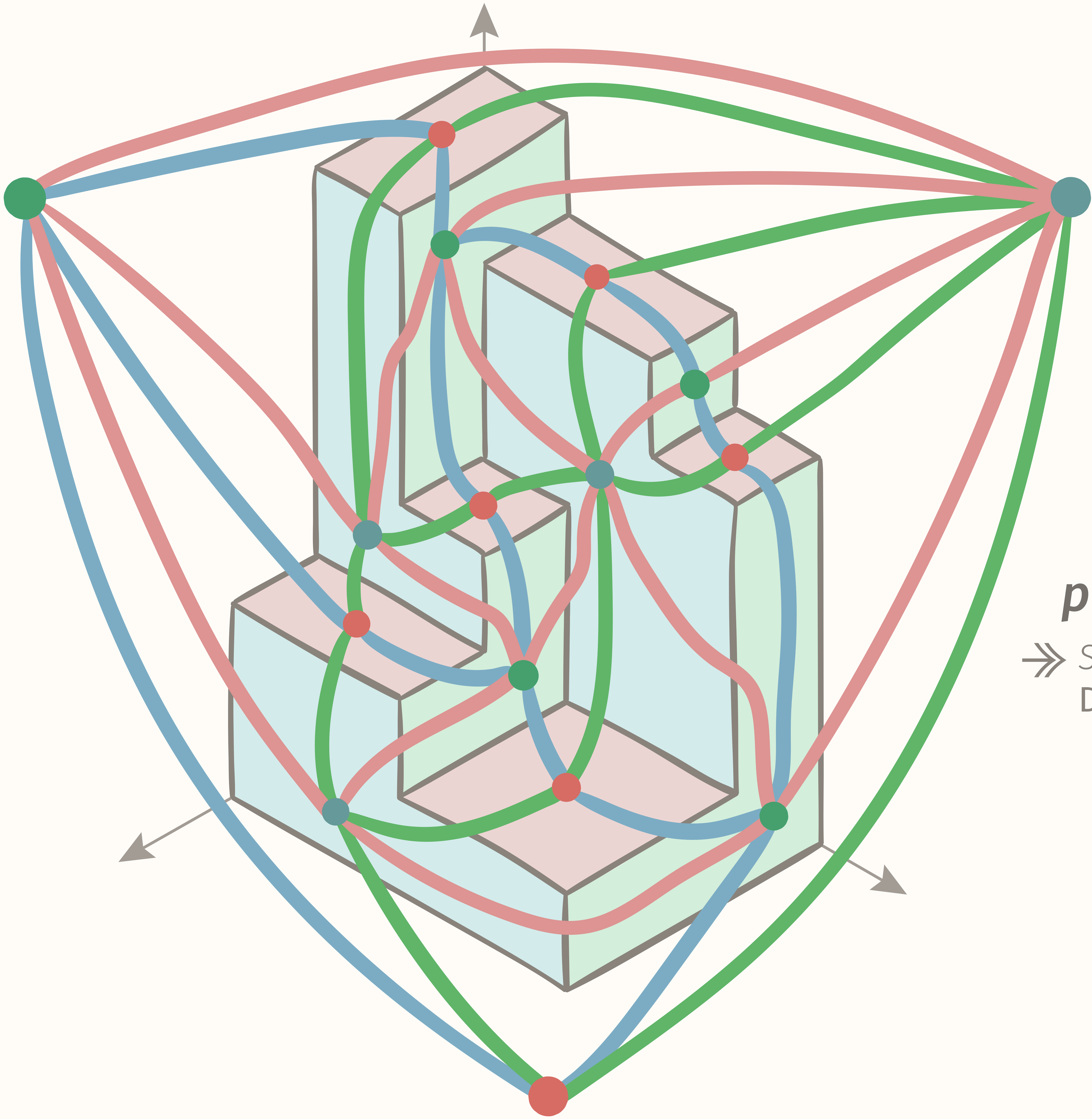

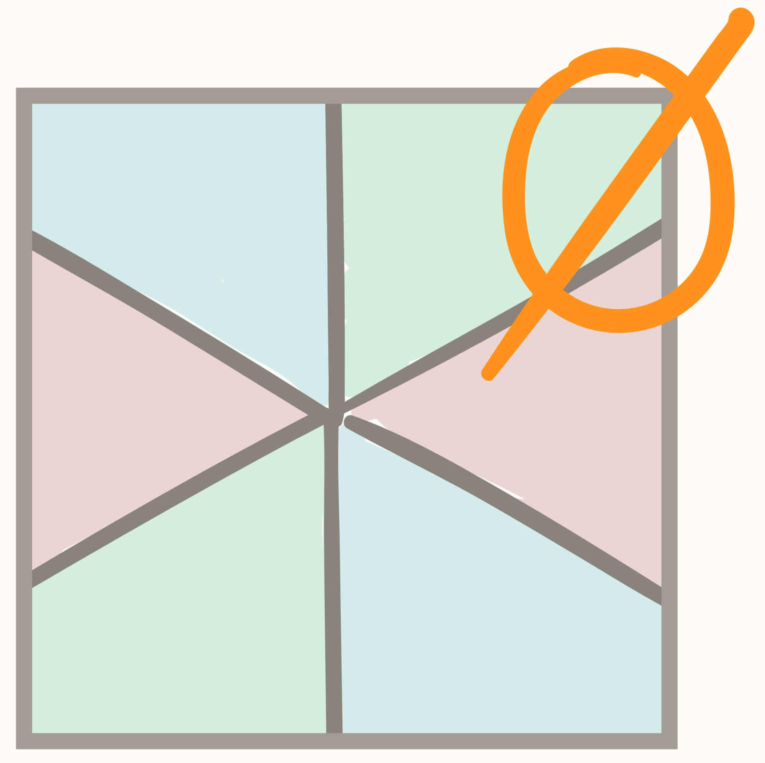

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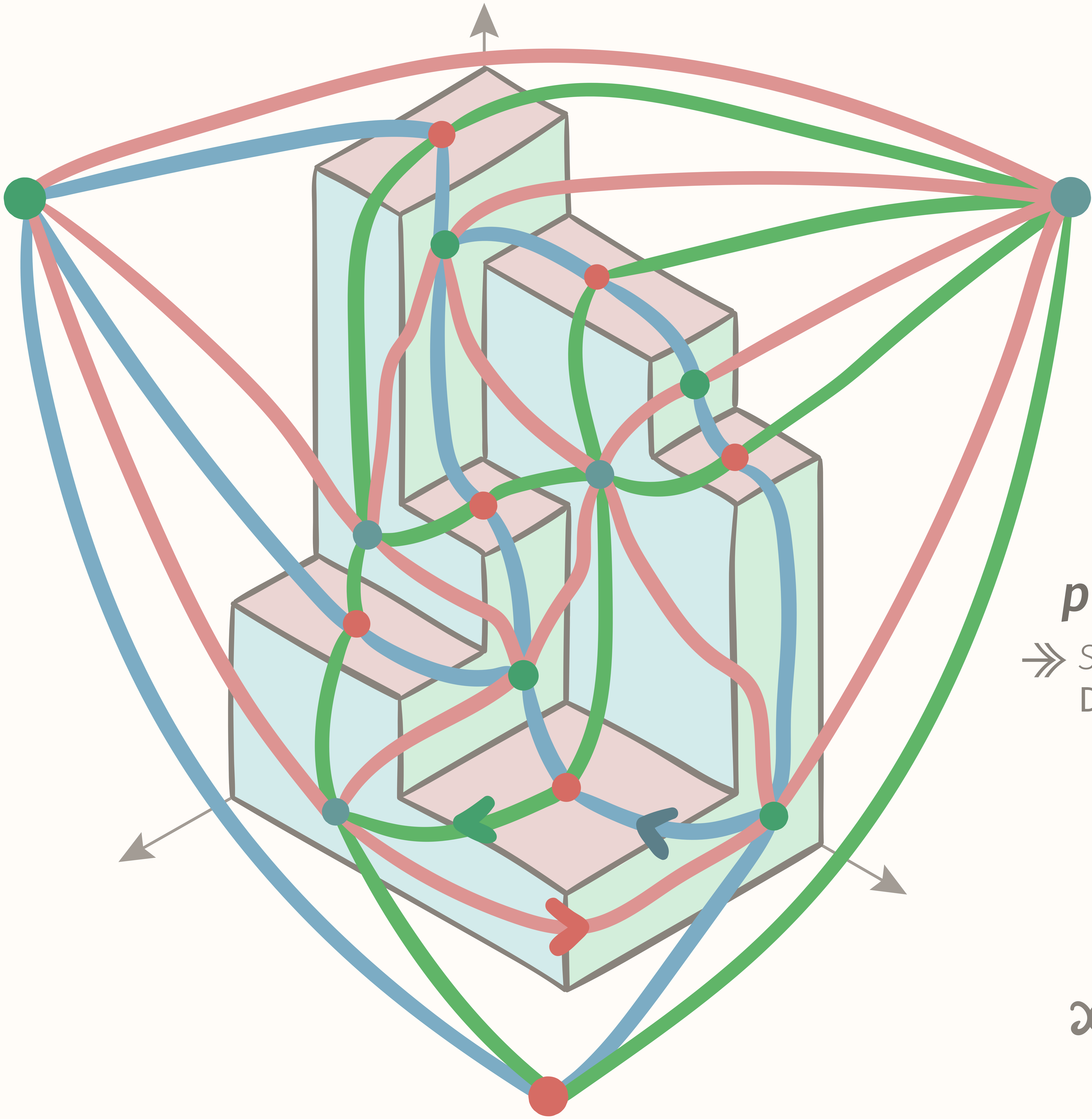

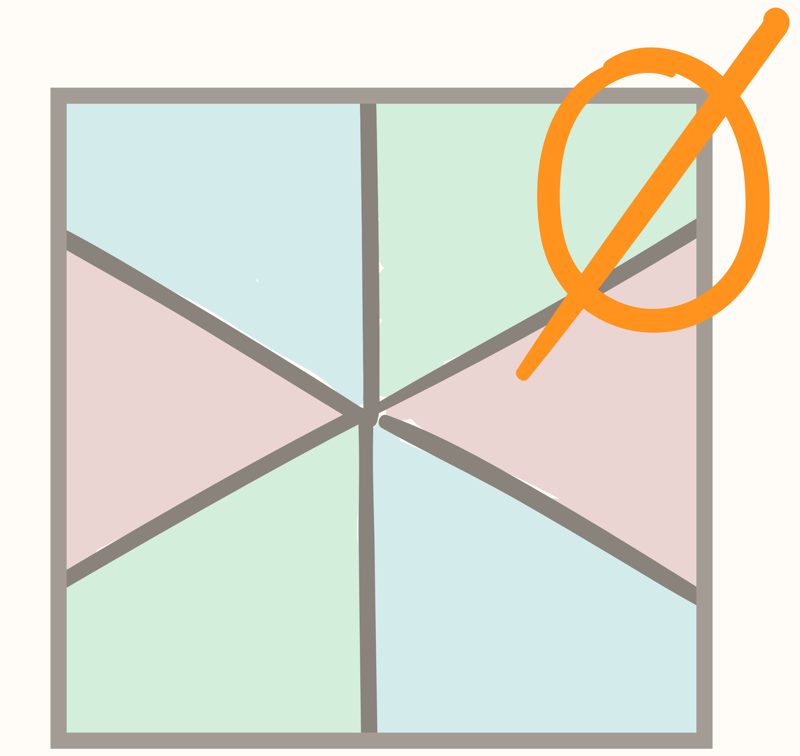

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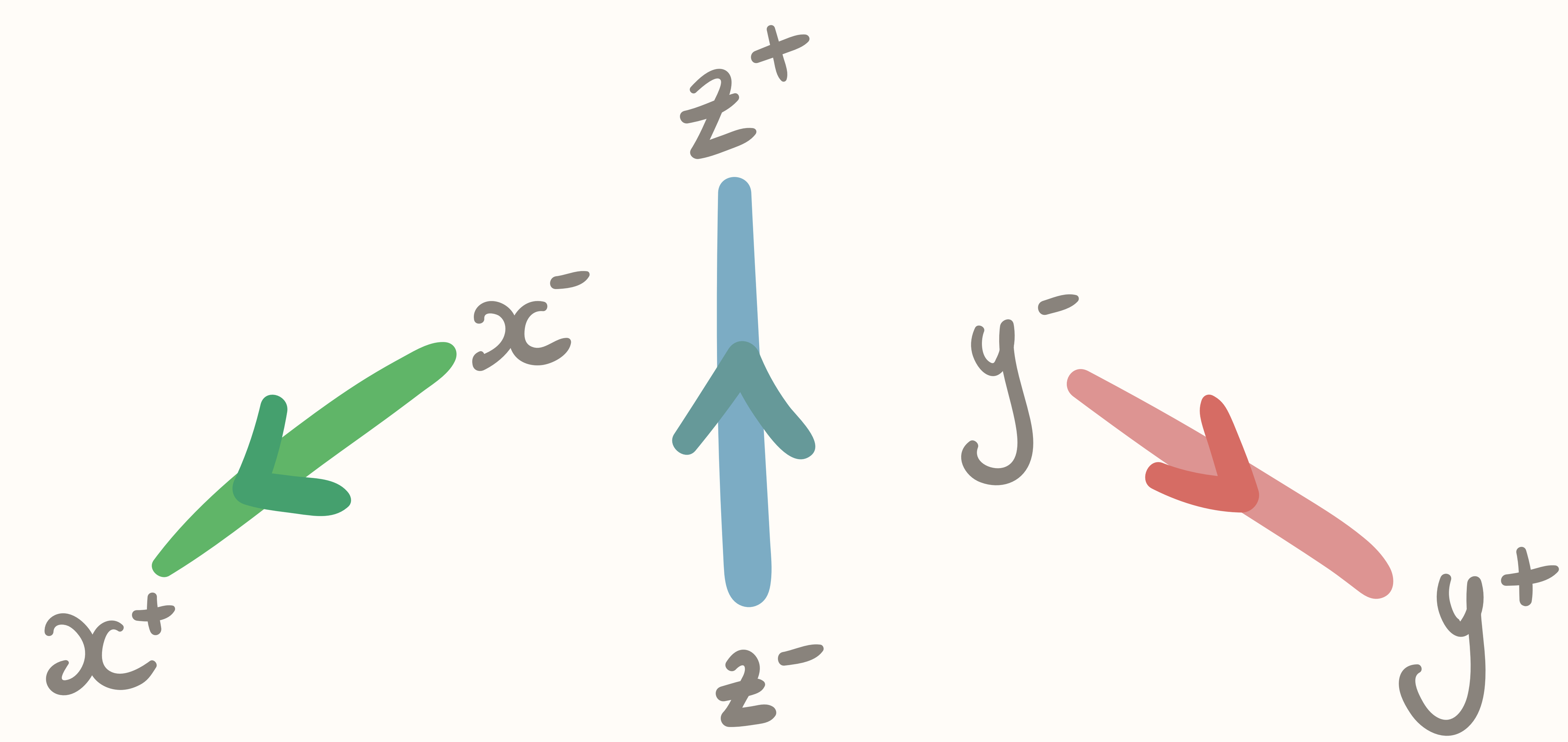
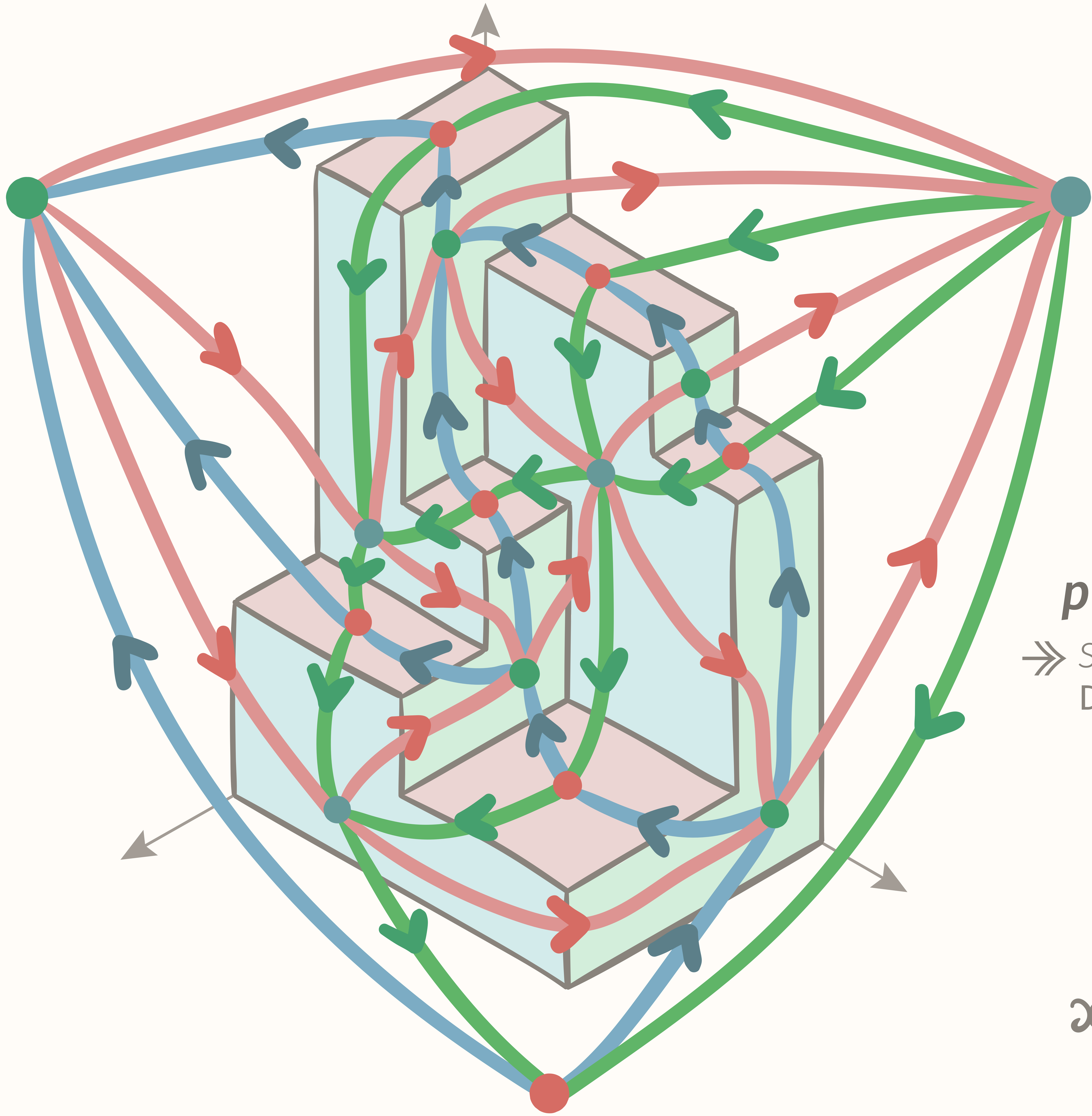
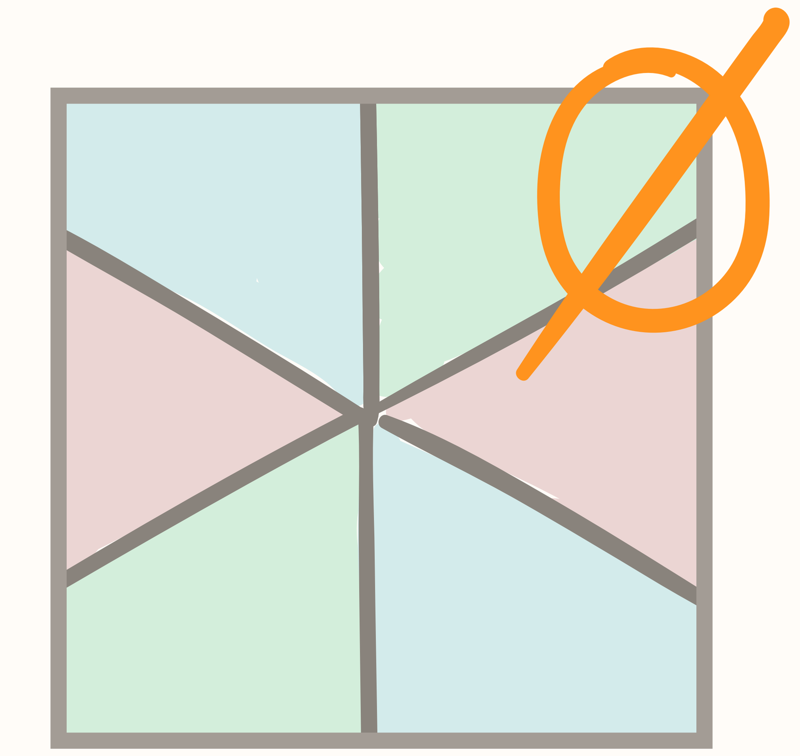


Corner polyhedra





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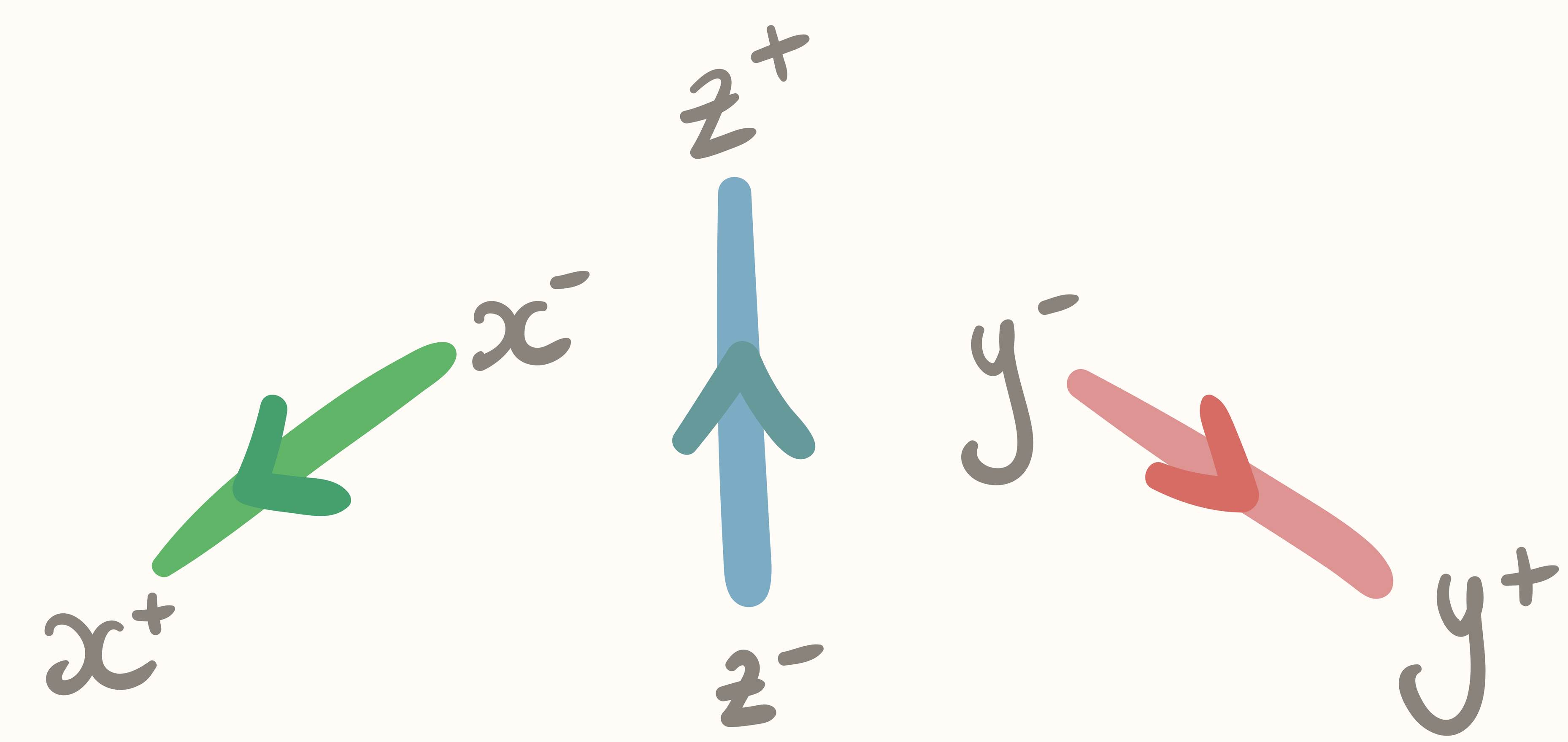
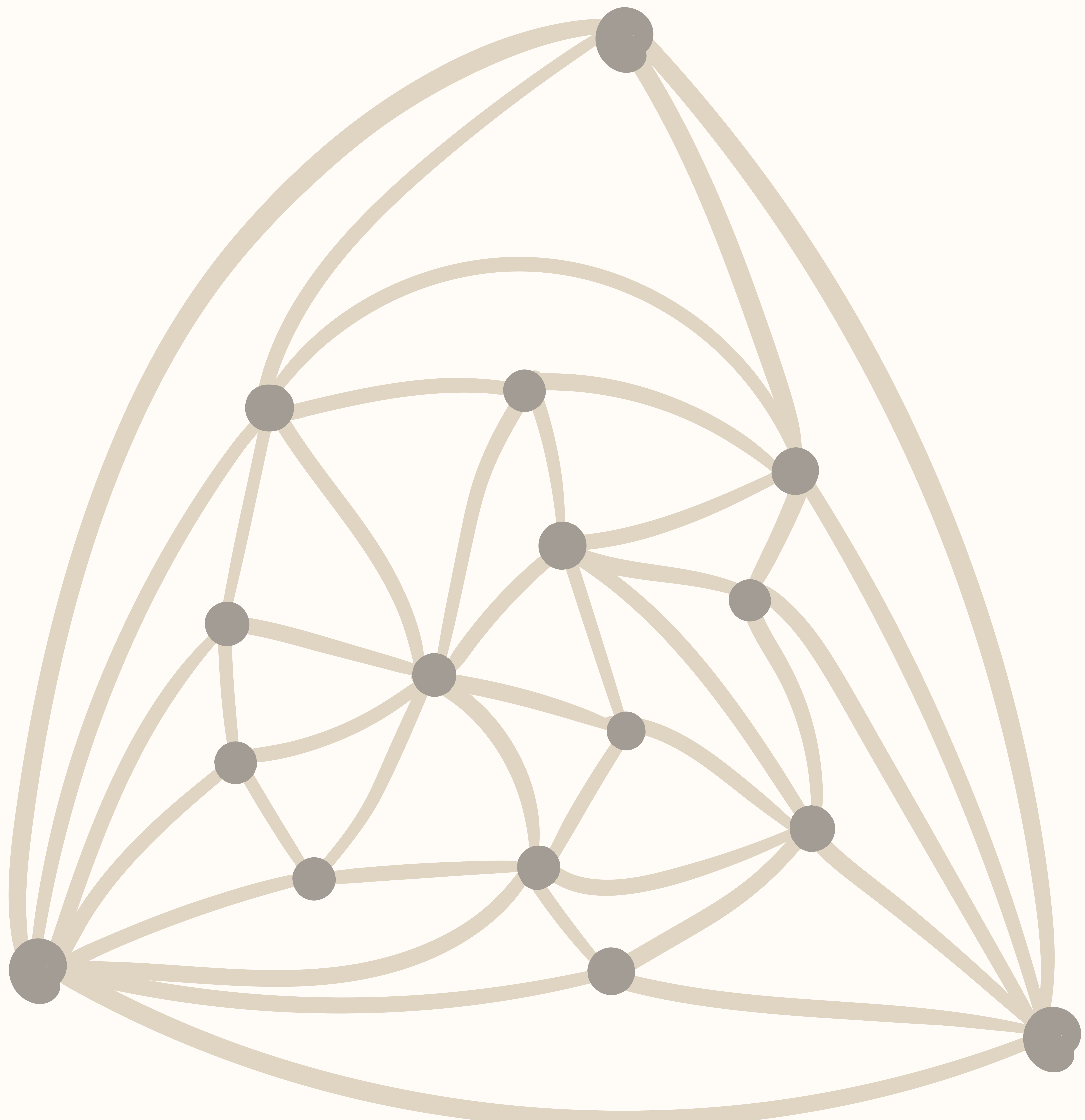
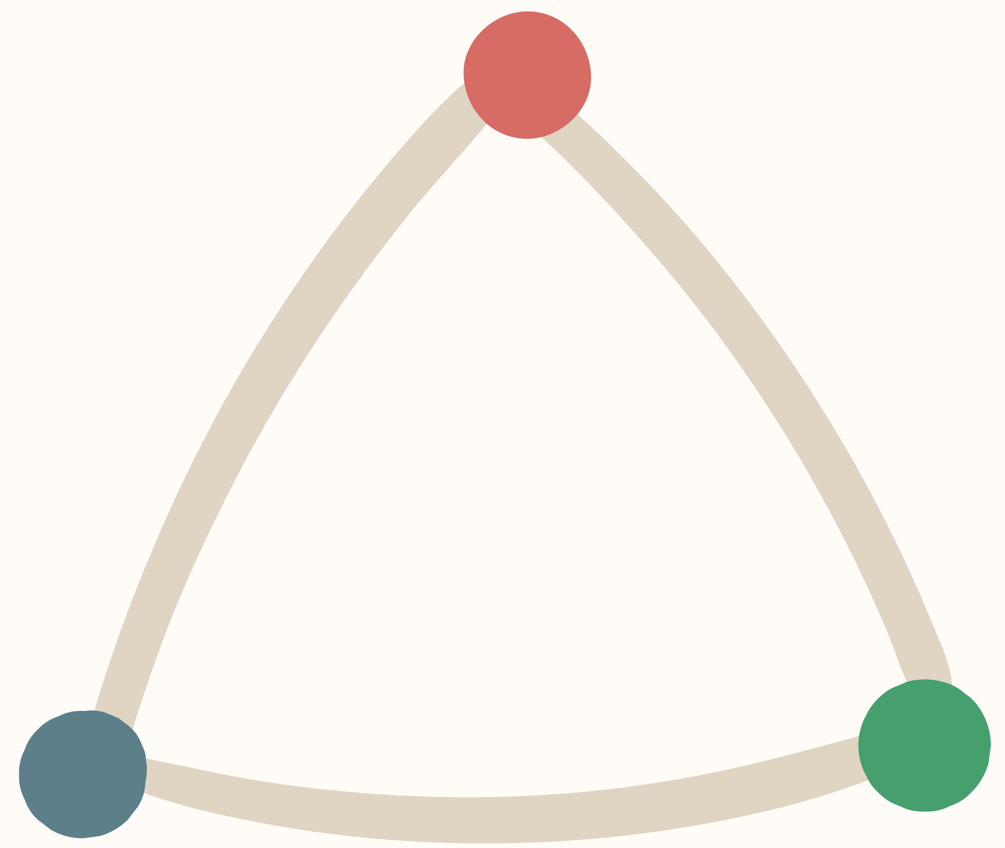
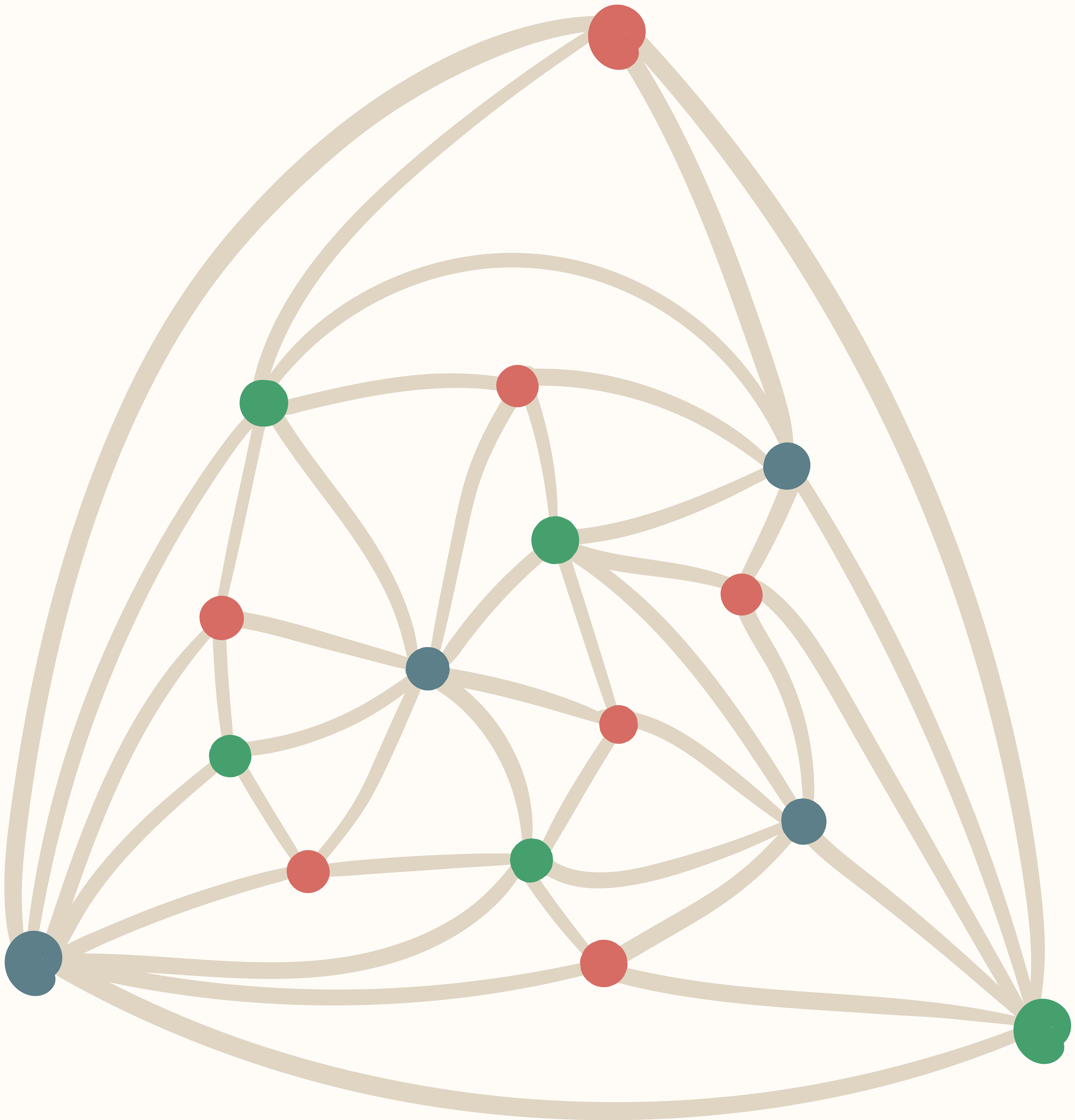


Polyhedral orientations

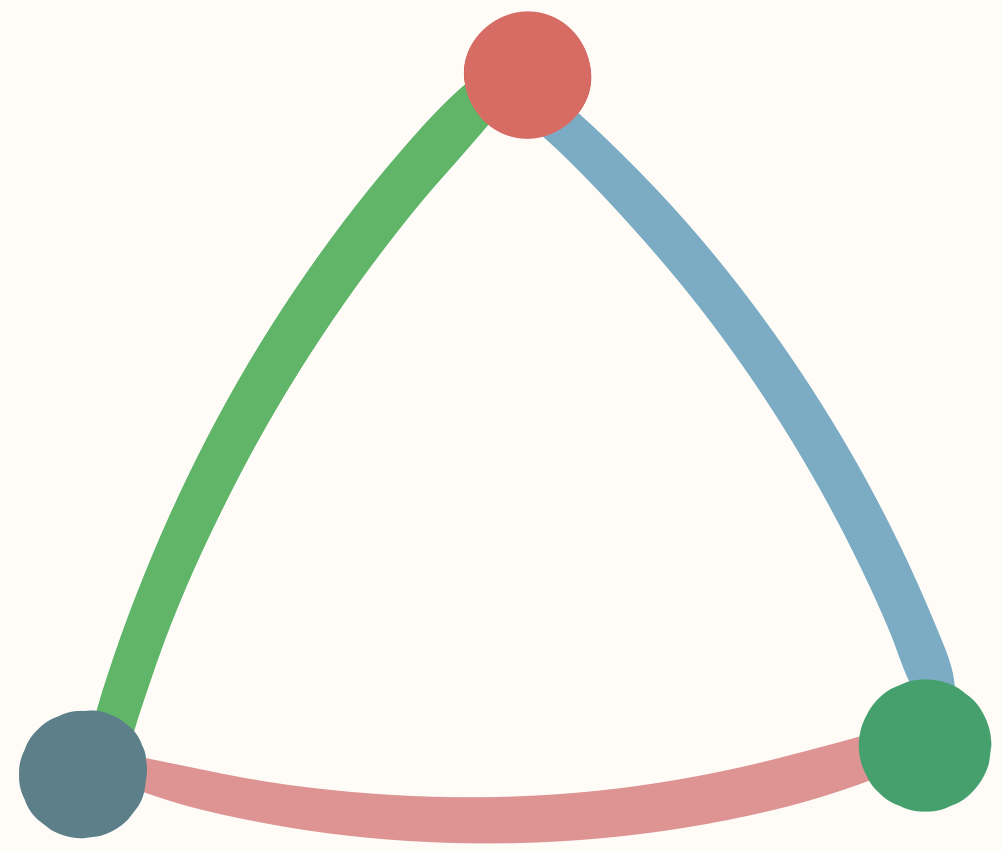


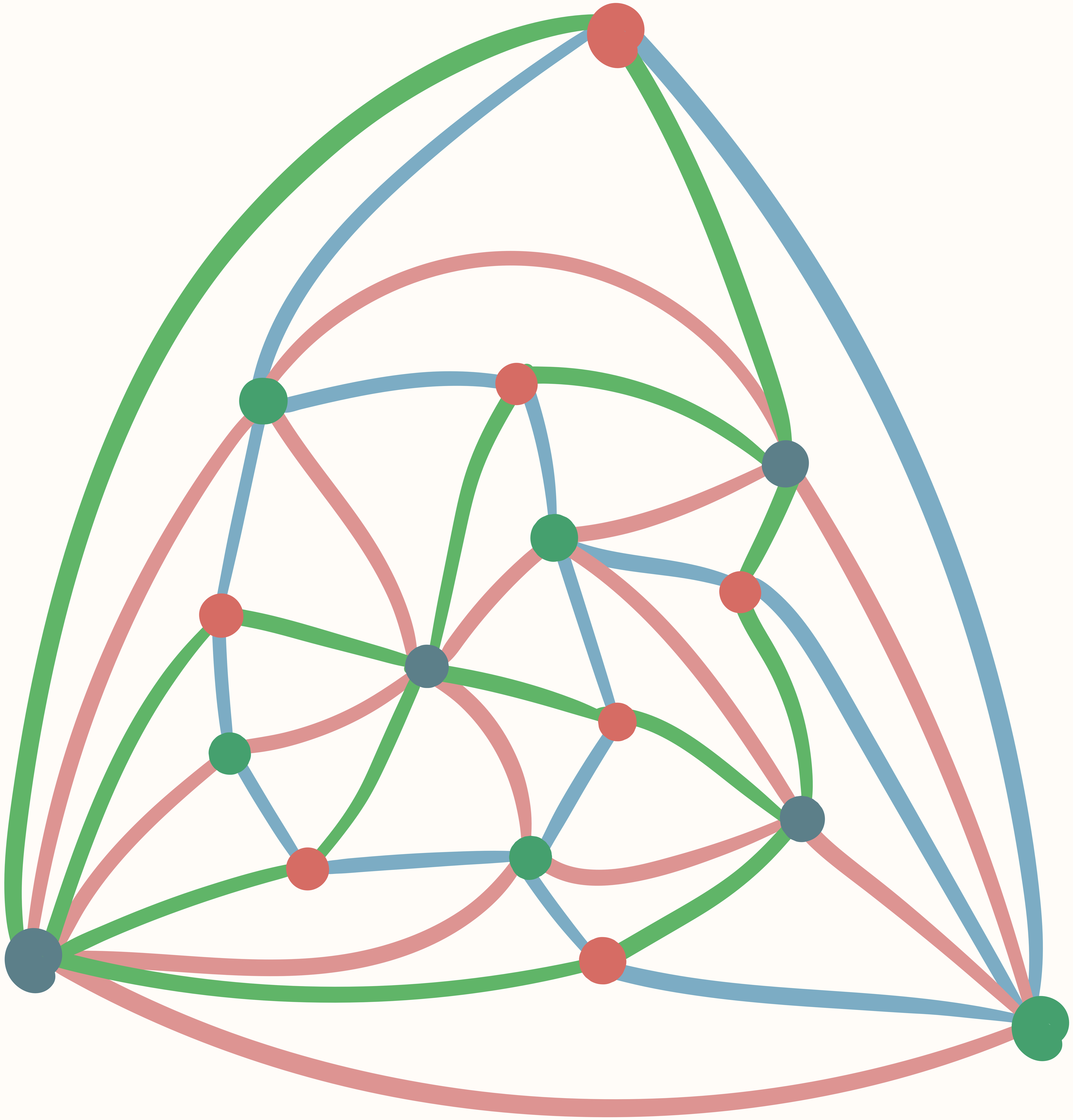
Polyhedral orientations



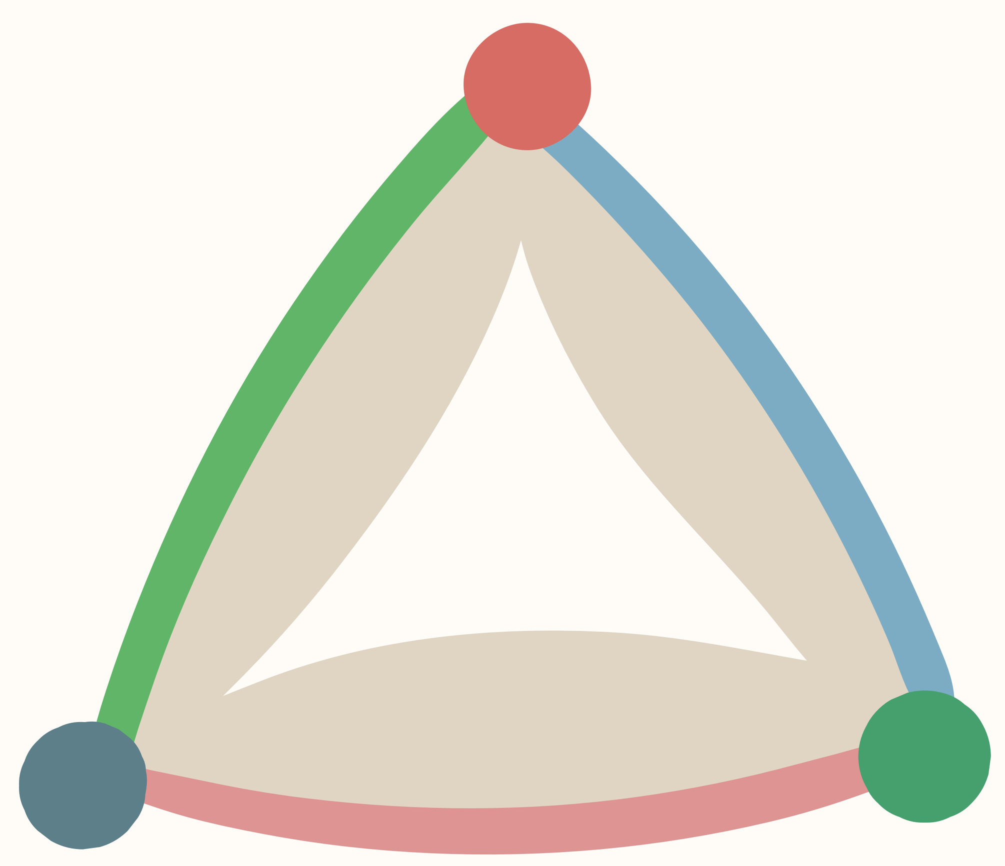


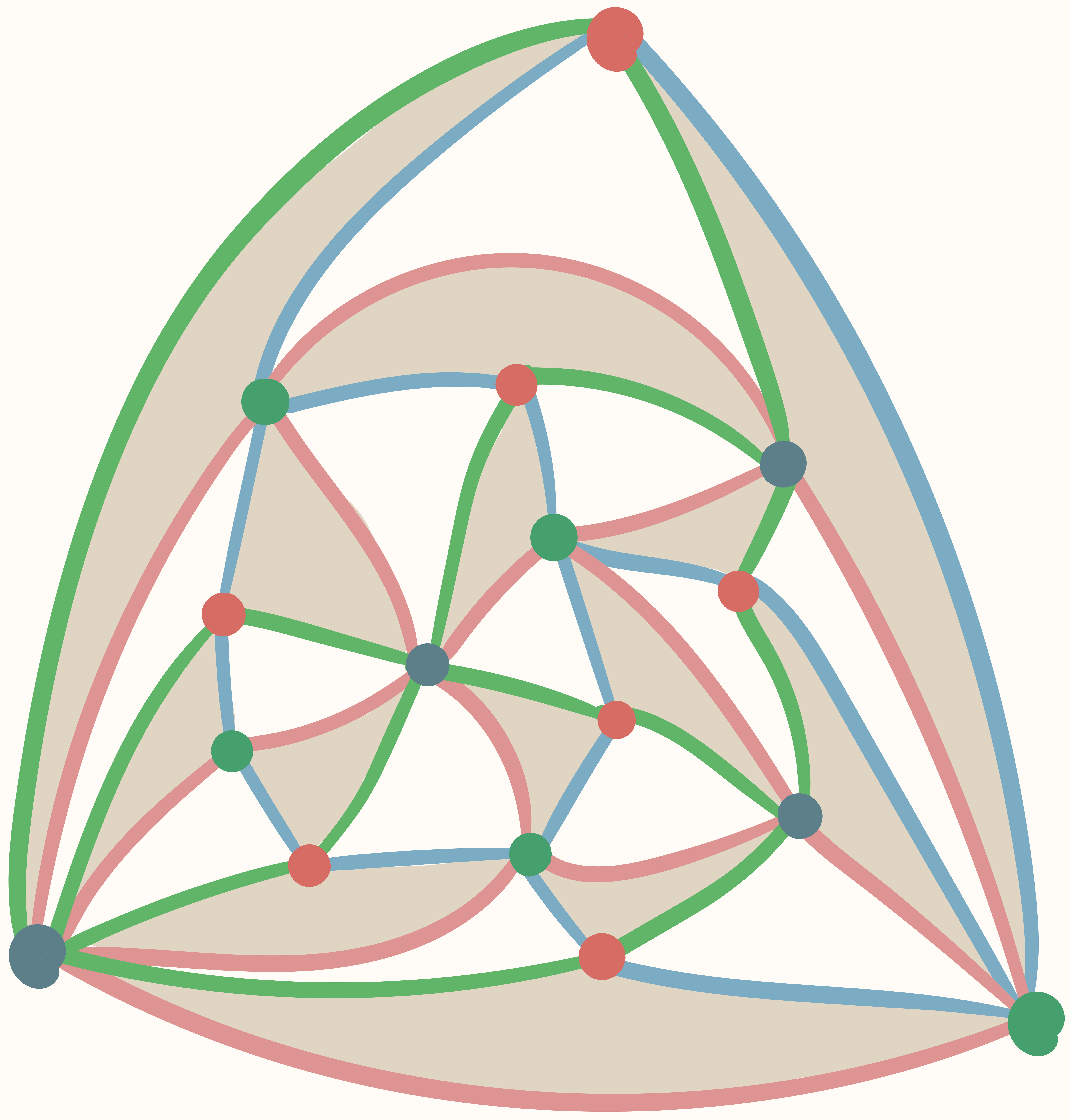
Polyhedral orientations





Polyhedral orientations





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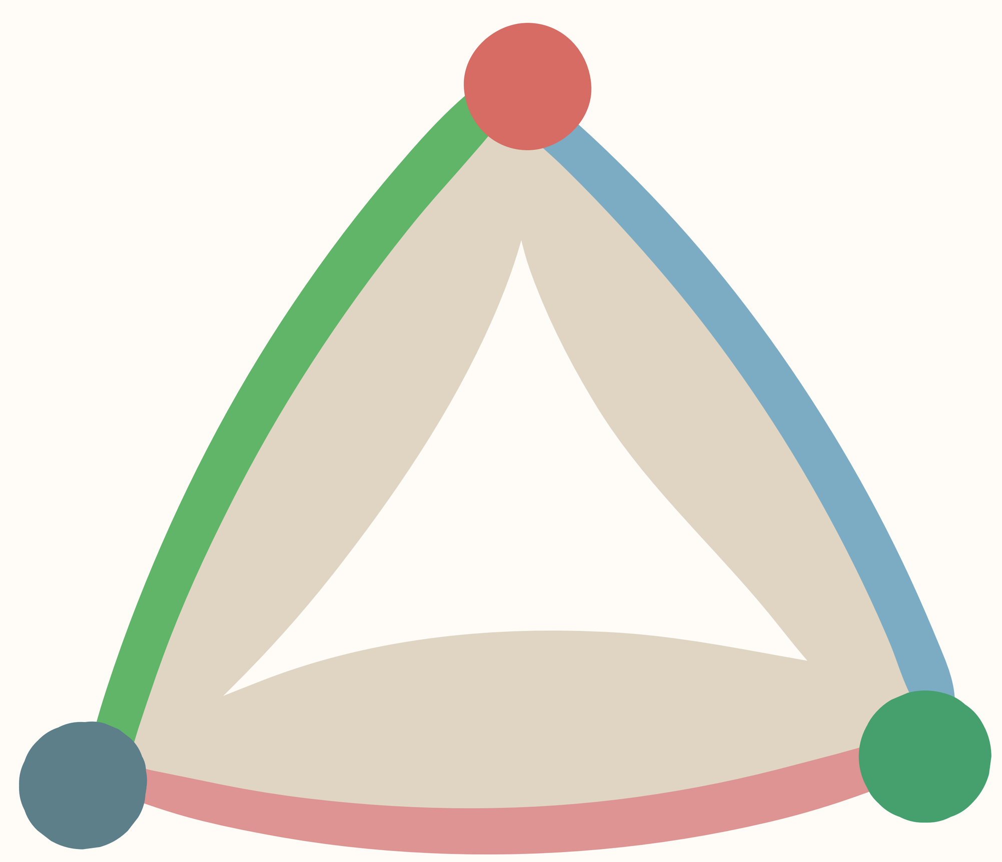
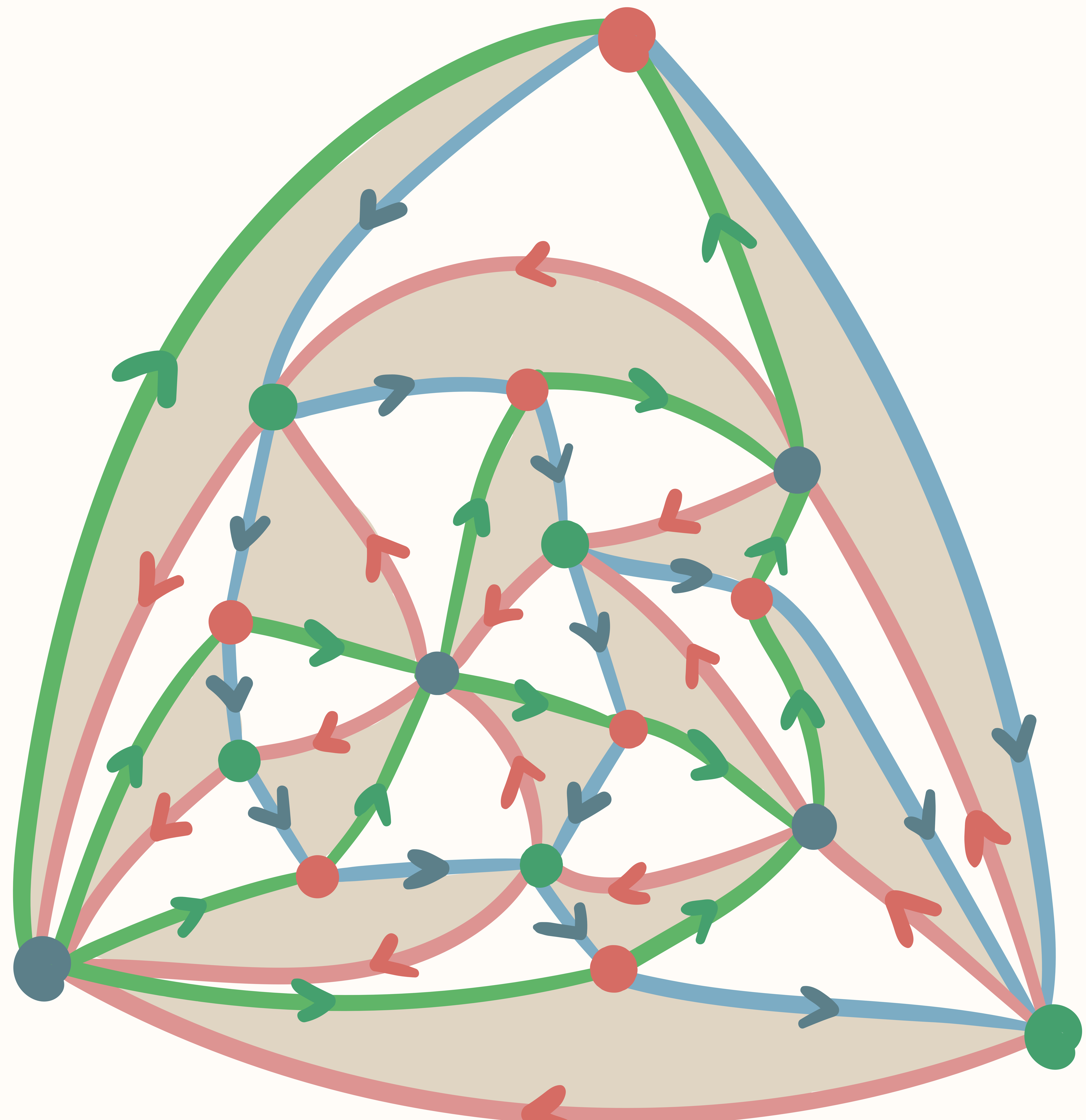
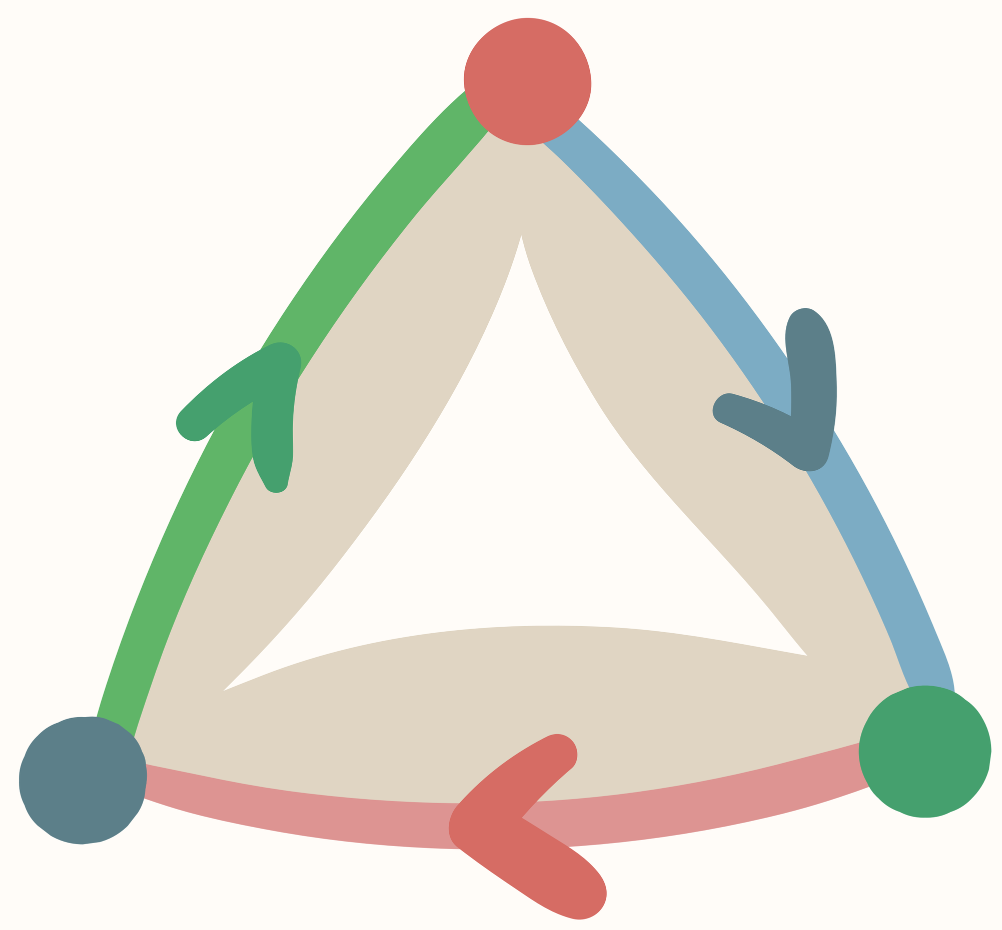
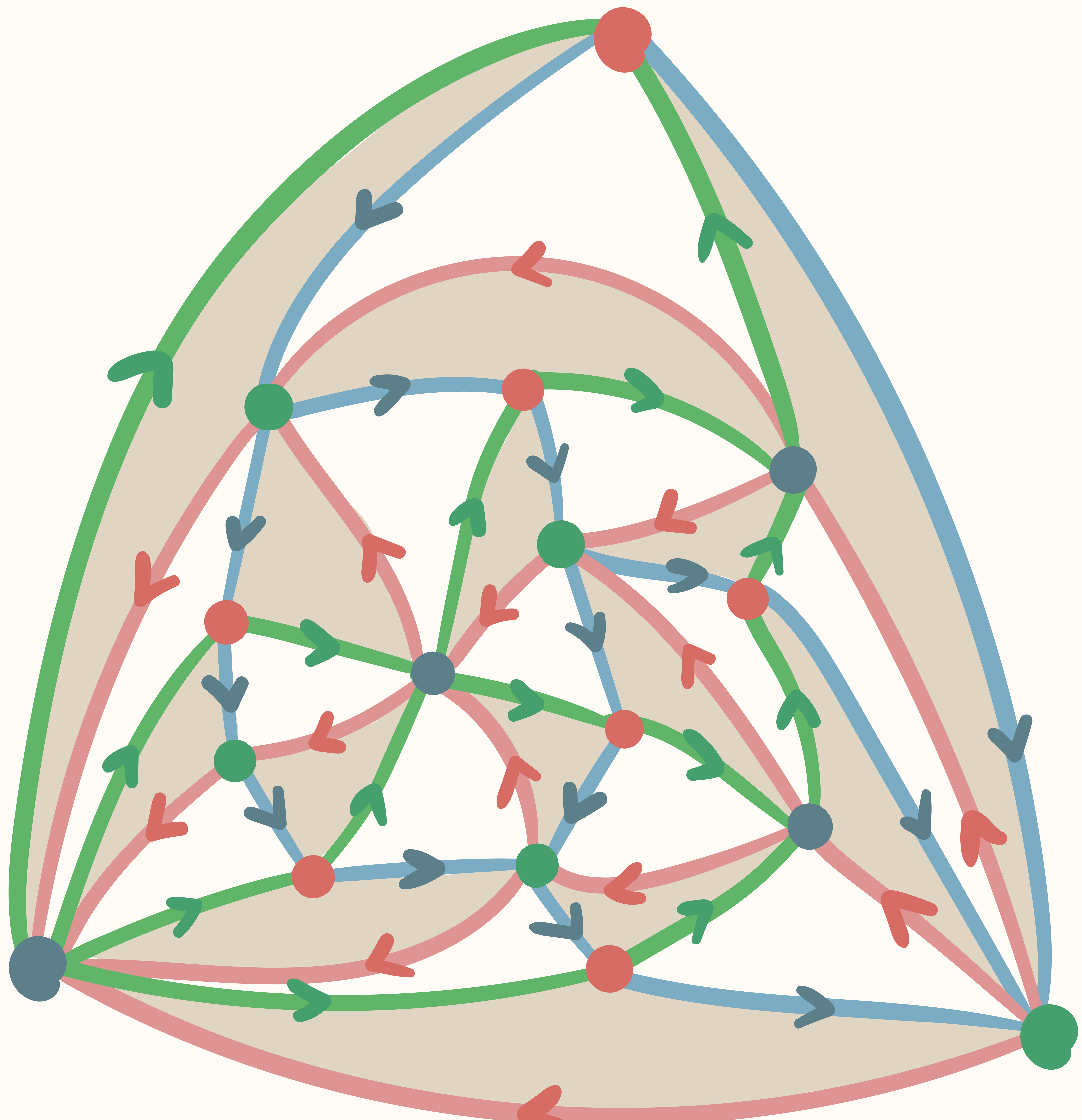


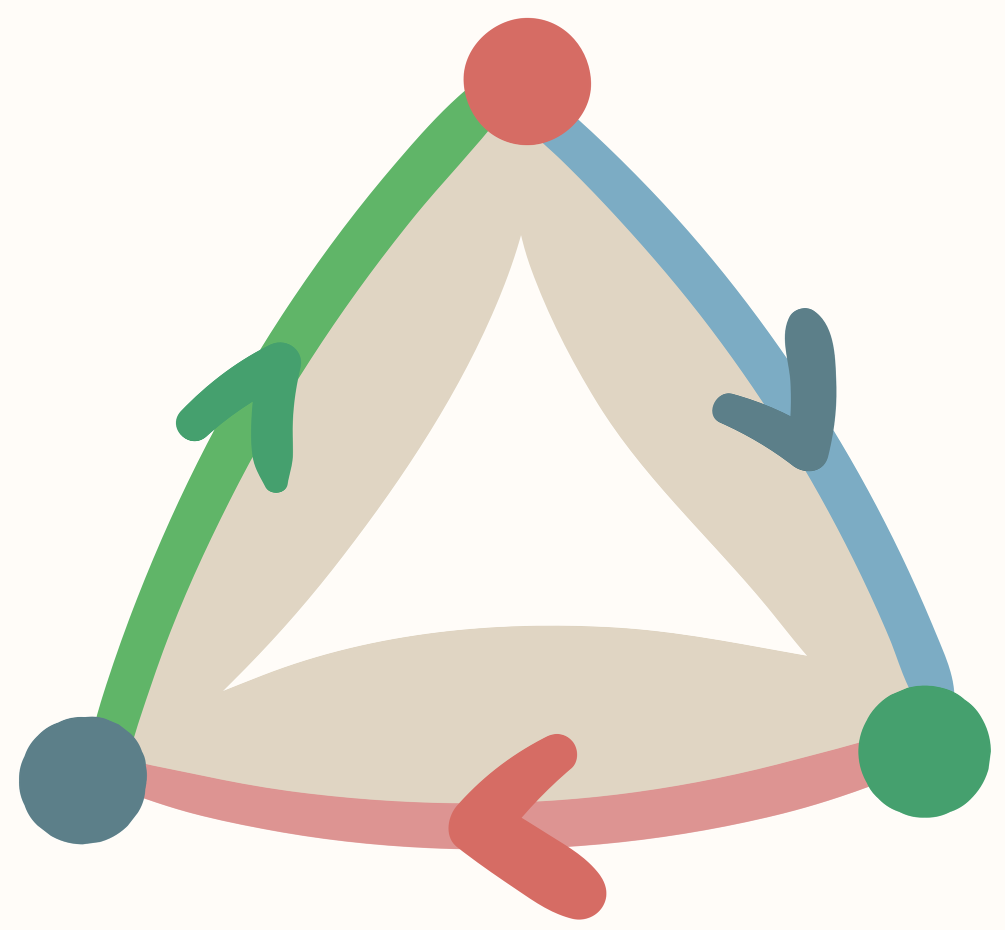


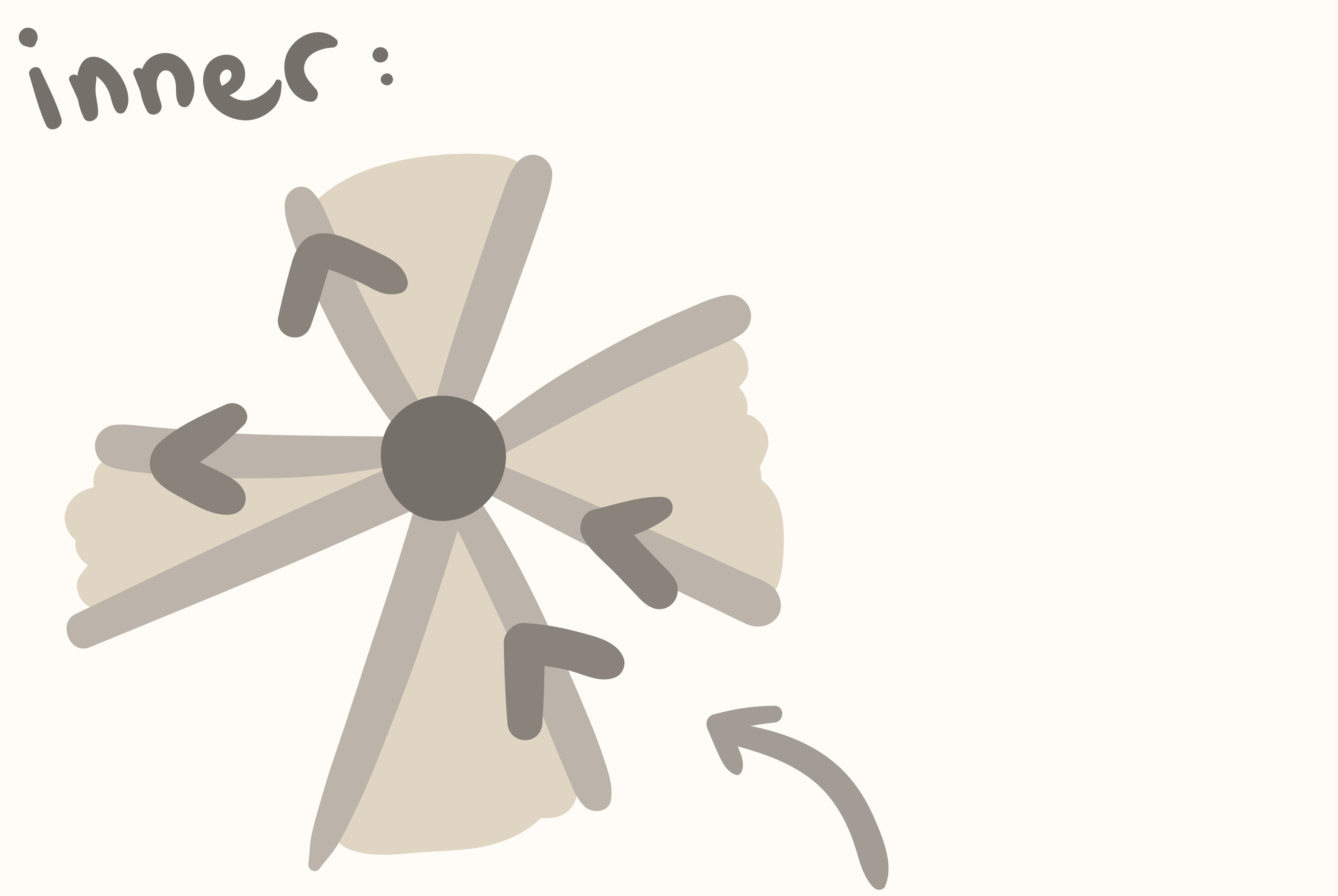
Polyhedral orientations

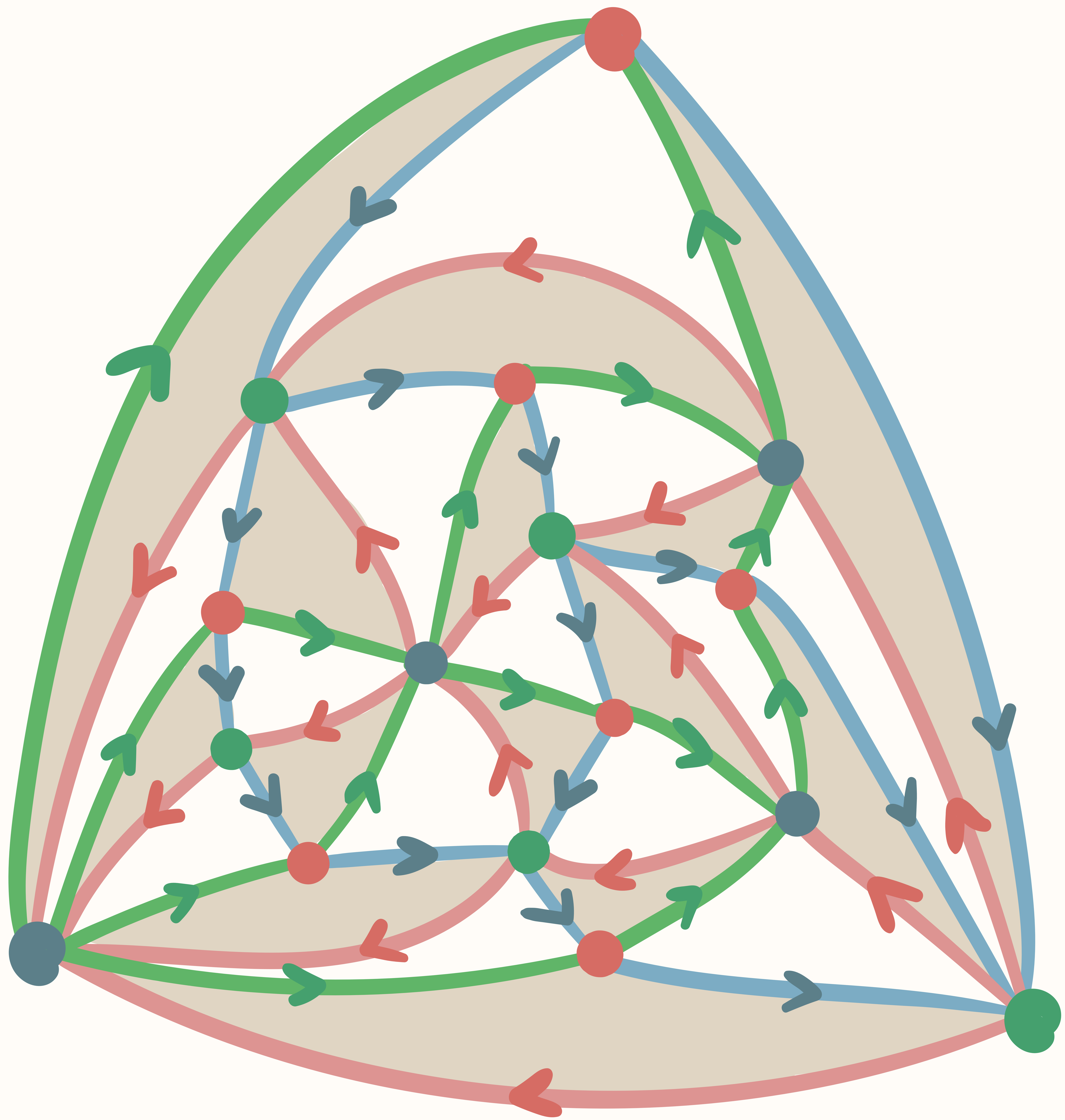




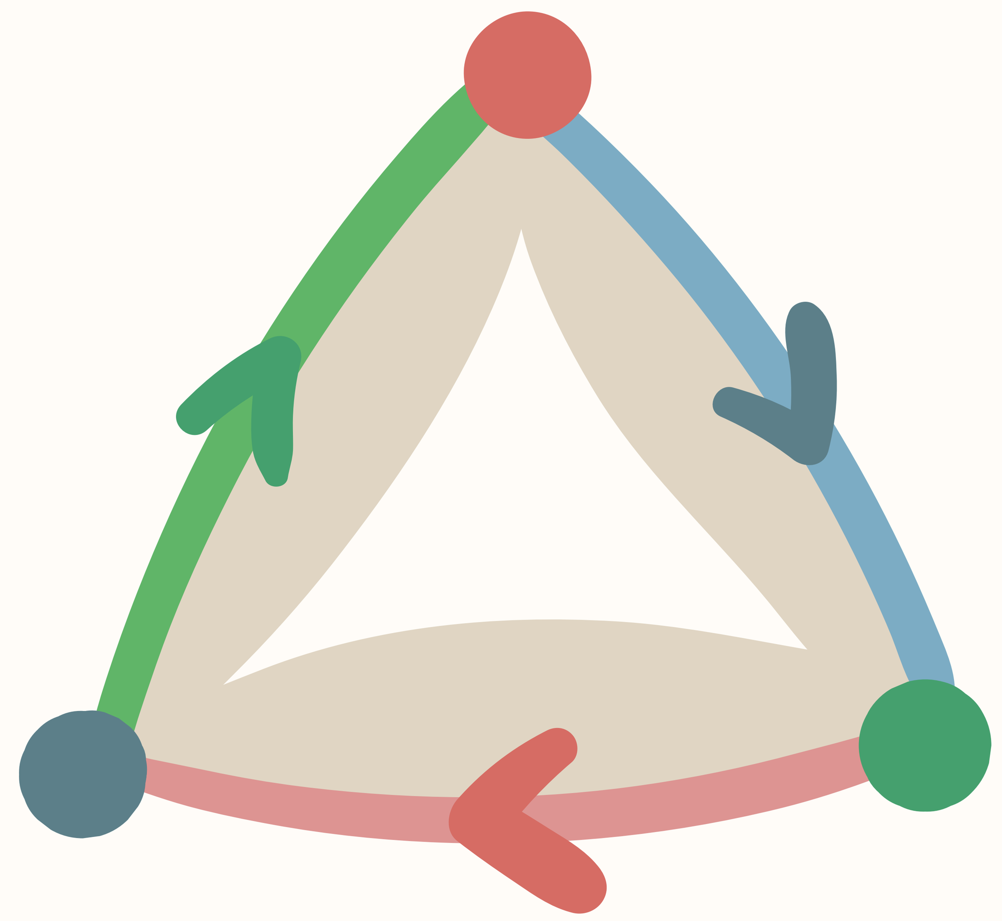
Polyhedral orientations

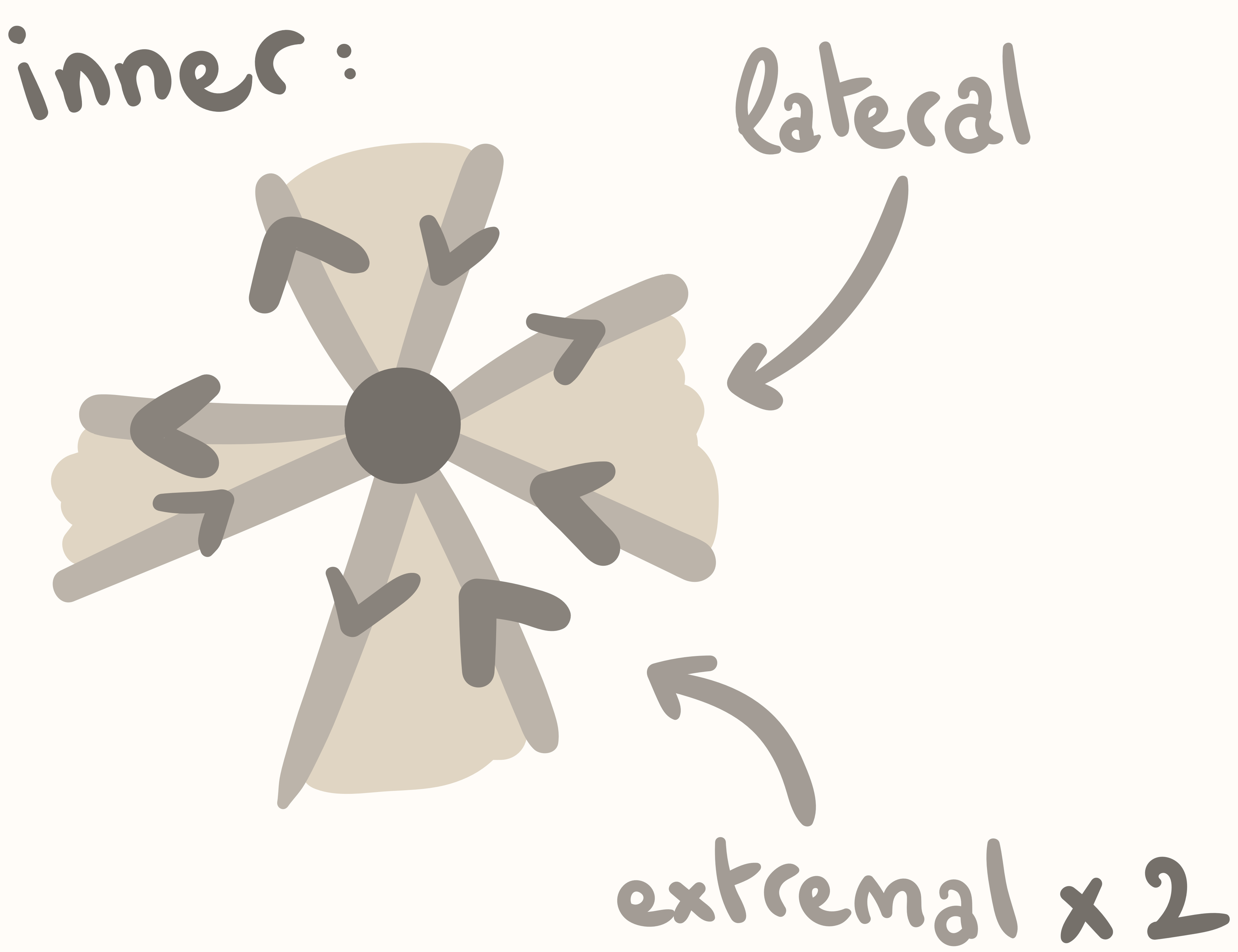


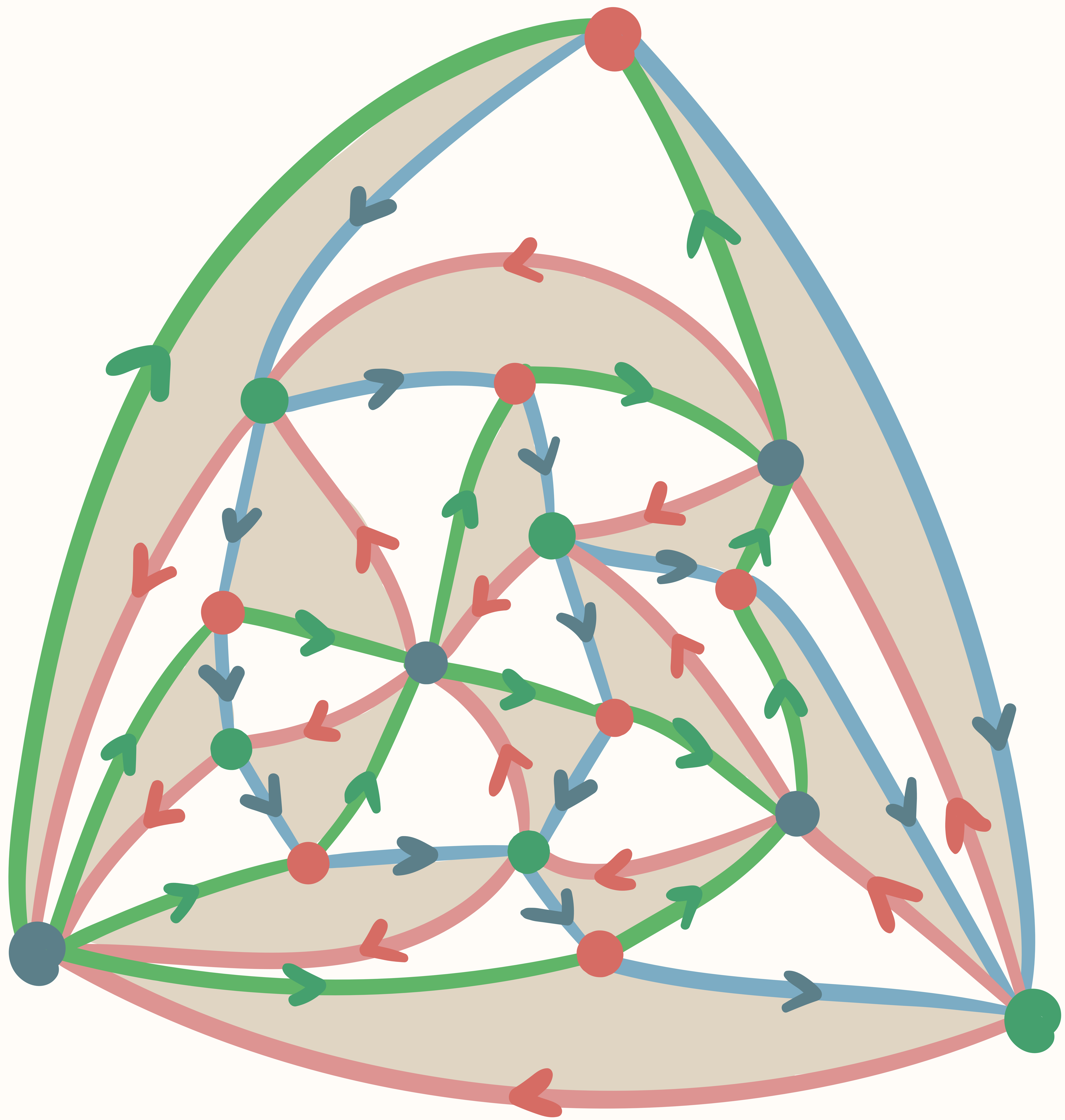




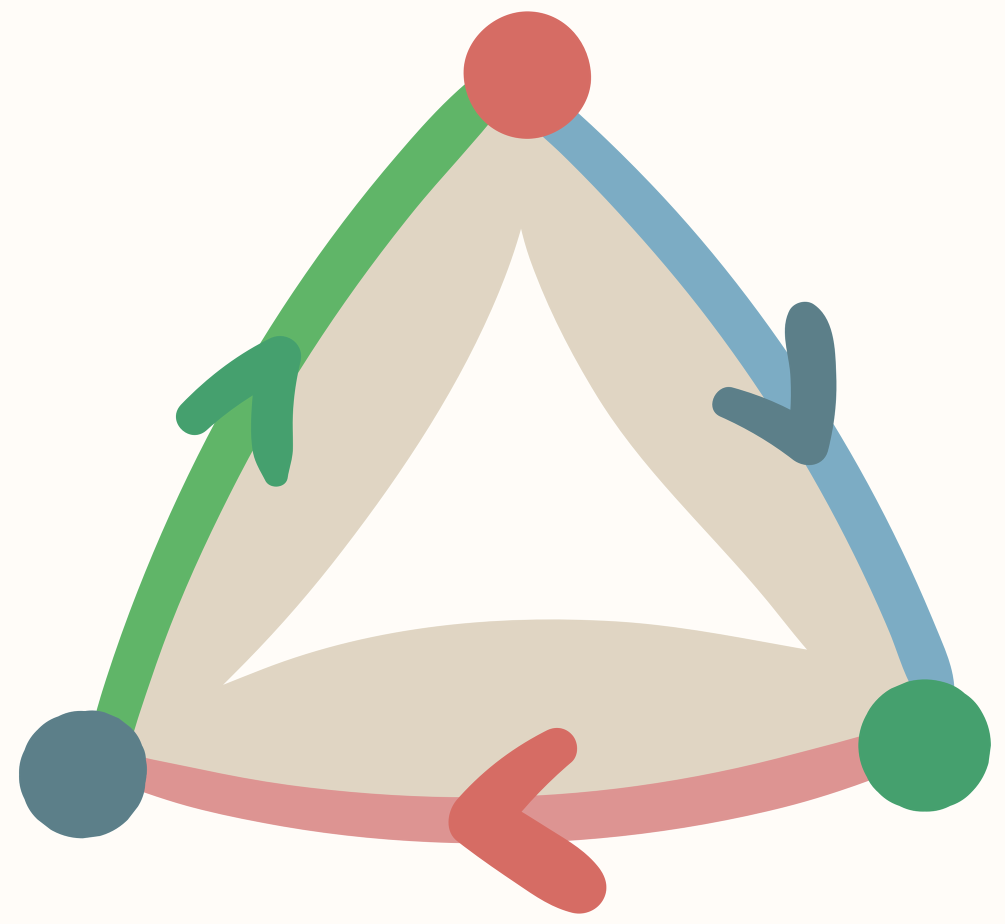
Polyhedral orientations

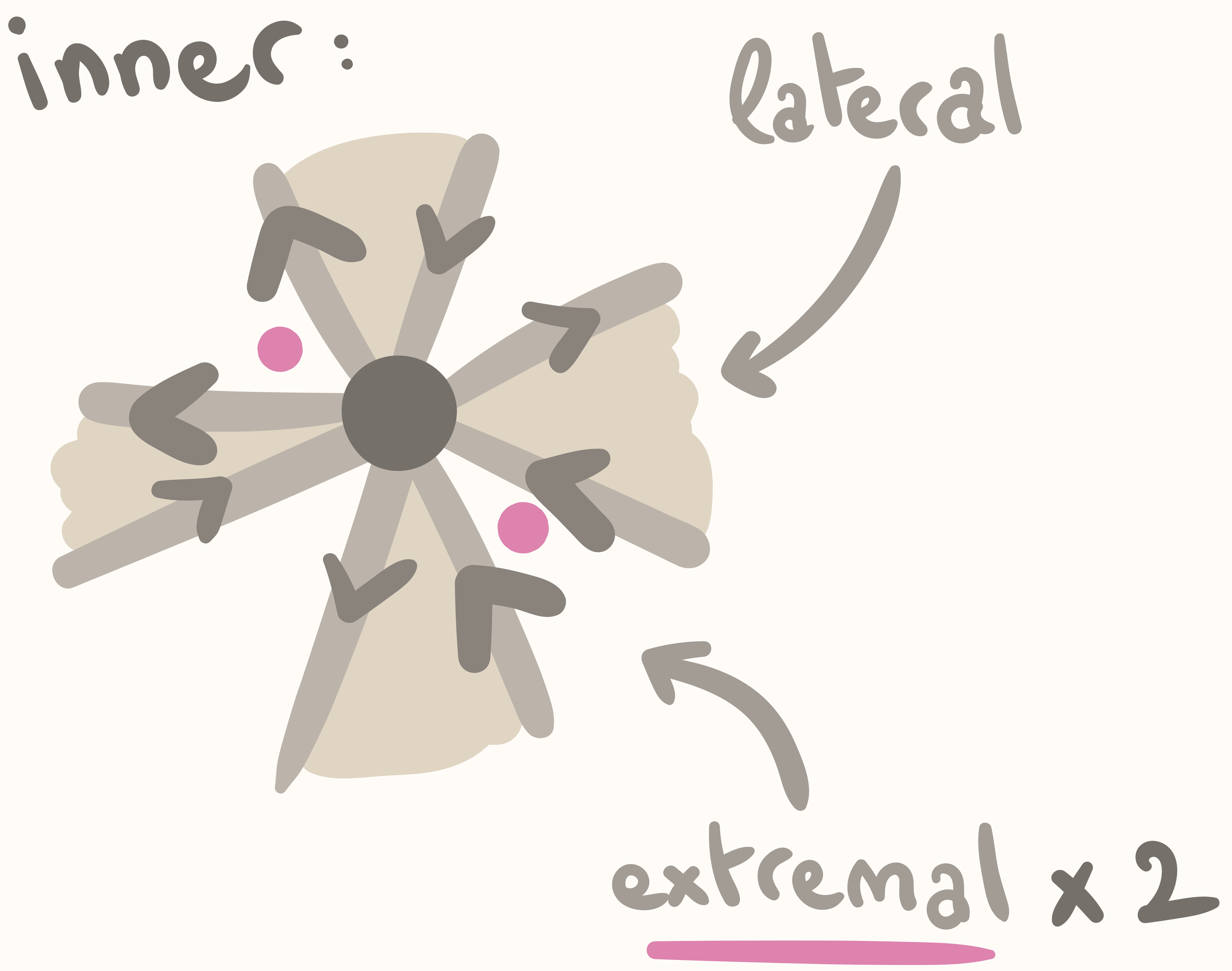


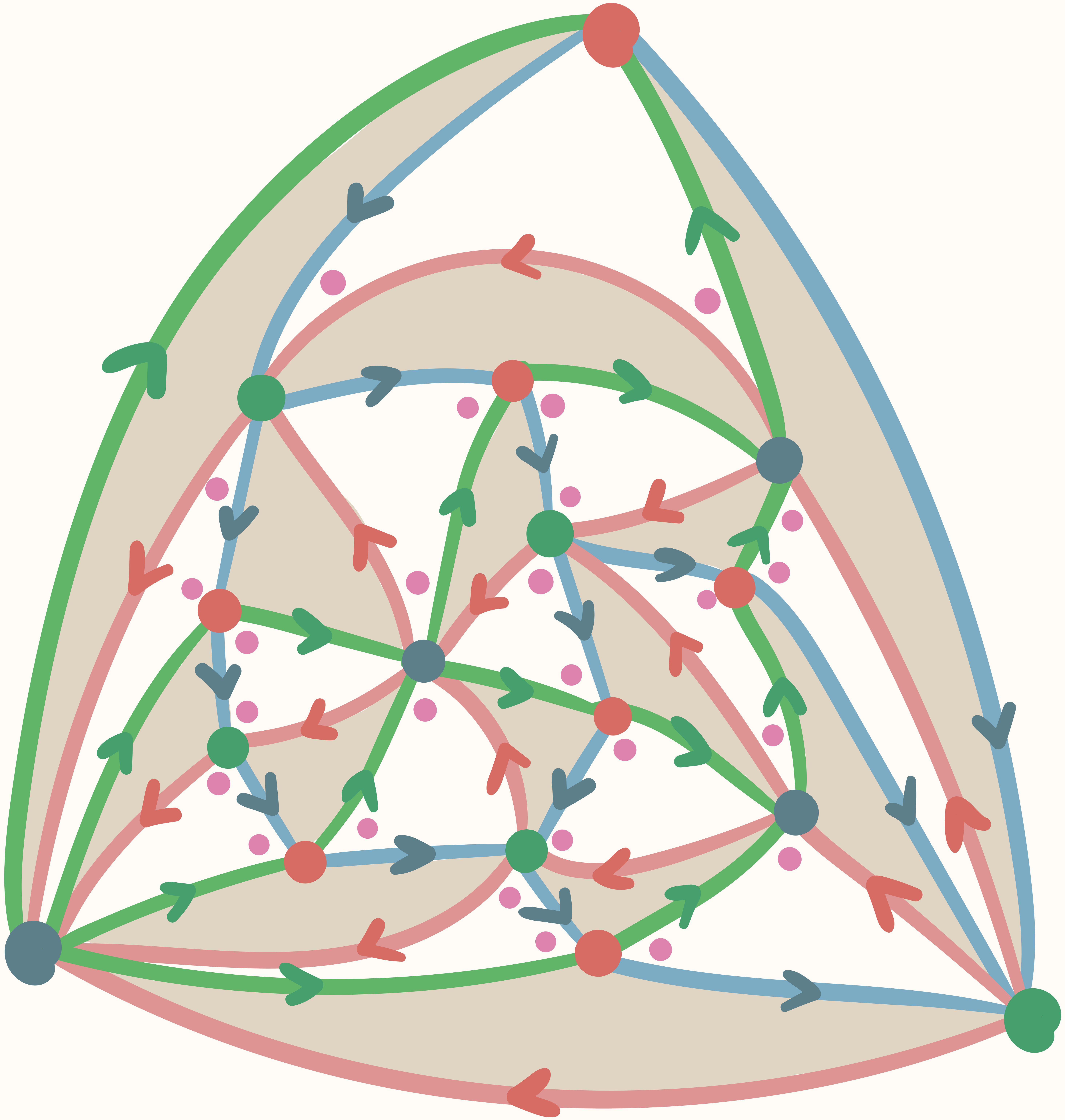




Polyhedral orientations







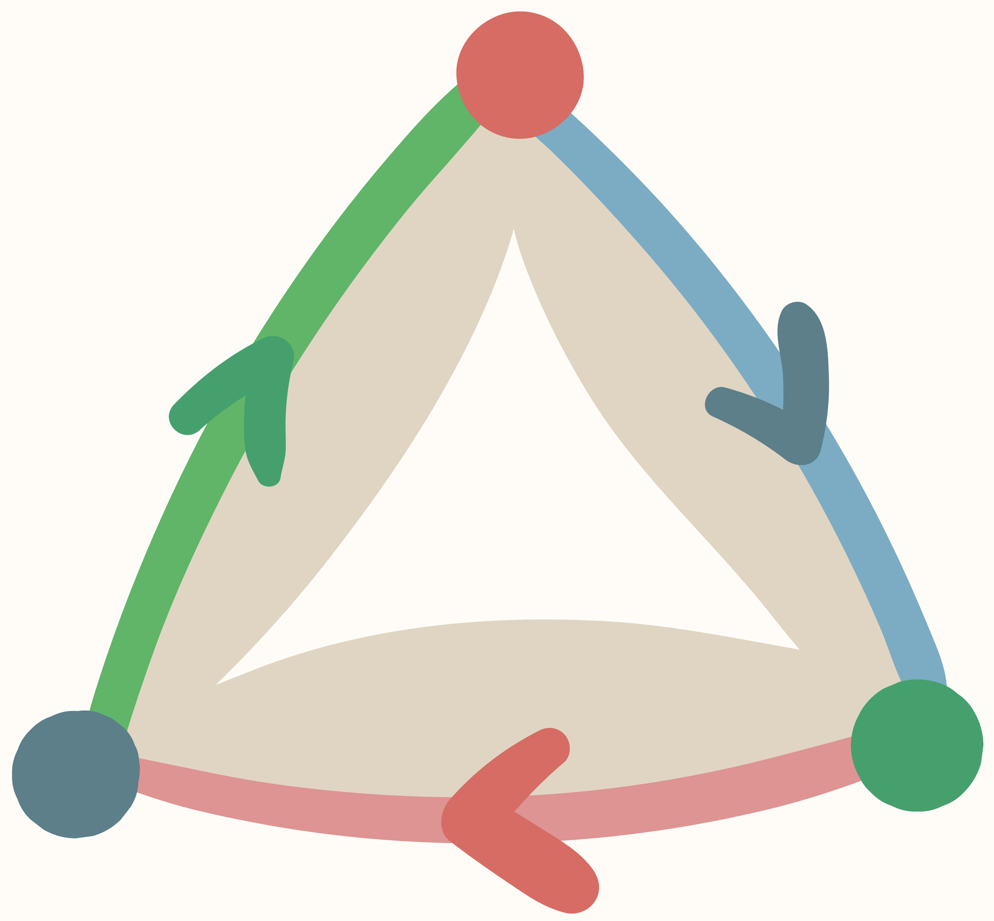
Polyhedral orient

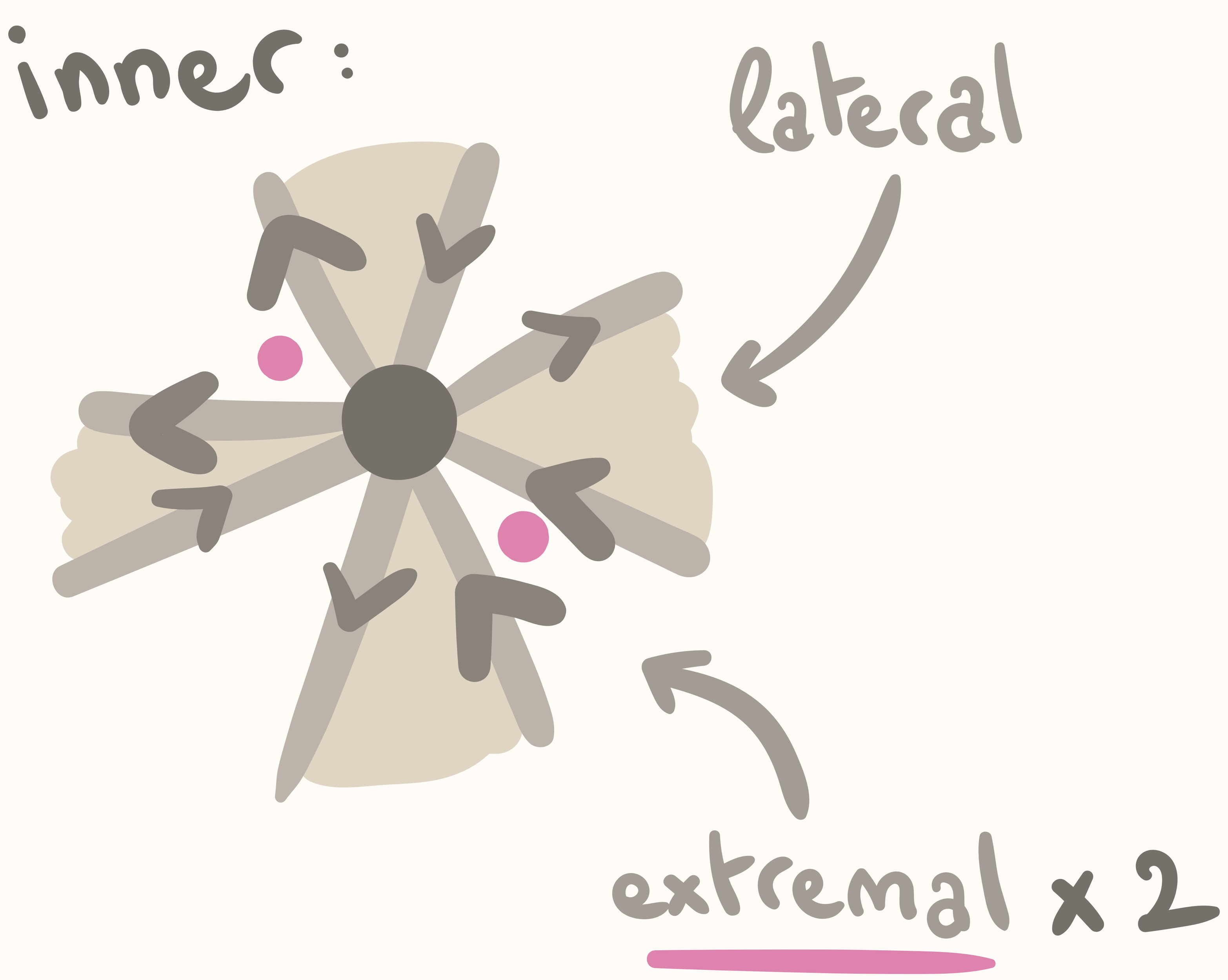
Polyhedral orient

Polyhedral orient

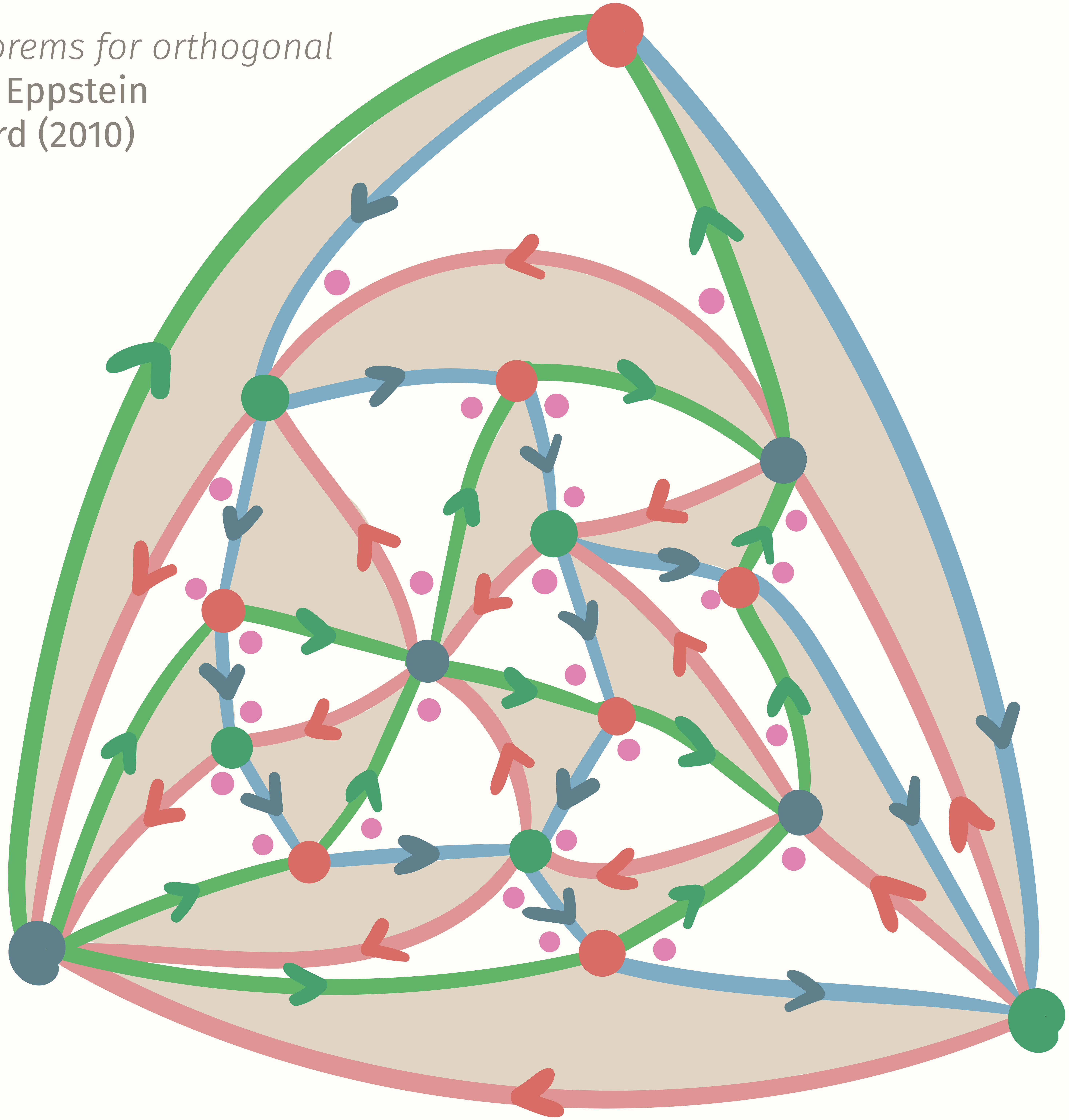
Polyhedral orient

Polyhedral orientations

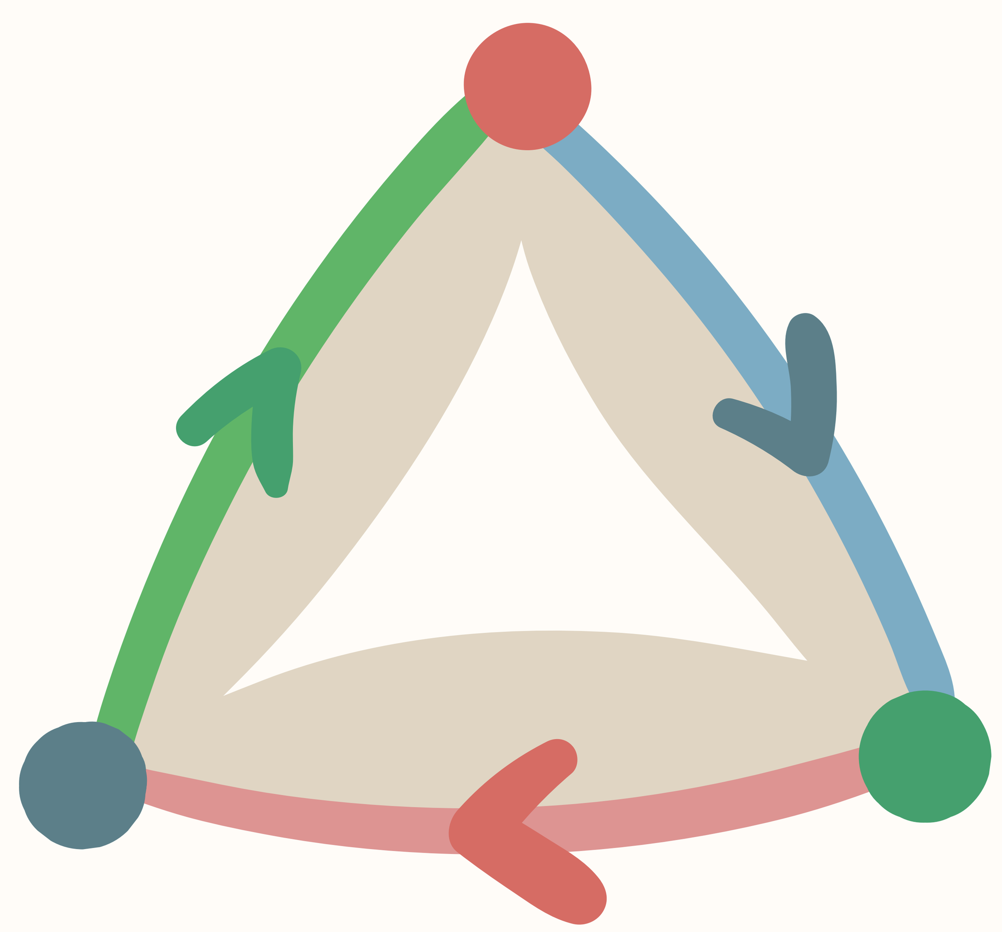




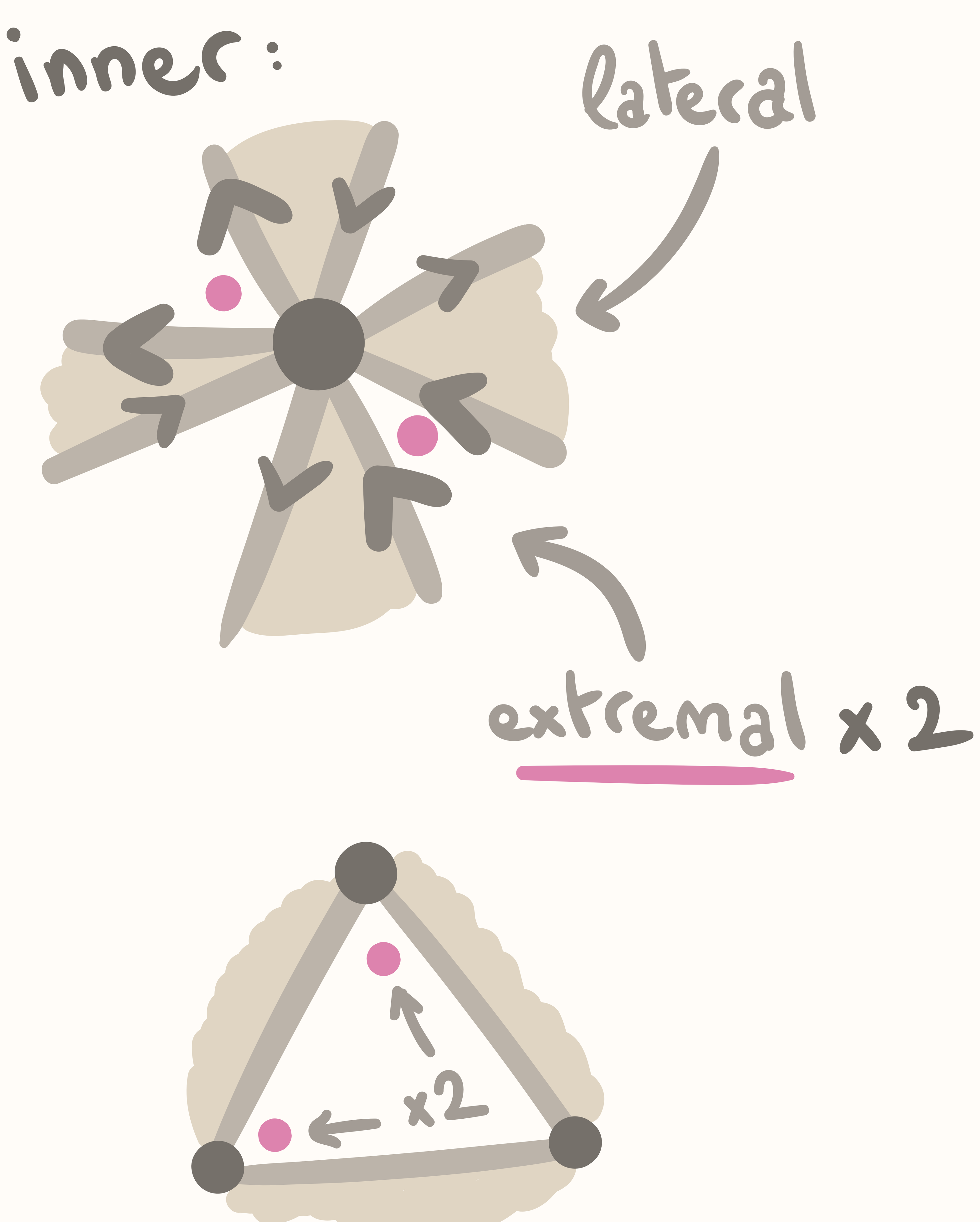
↠ *Steinitz theorems for orthogonal polyhera,* D. Eppstein & E. Mumford (2010)

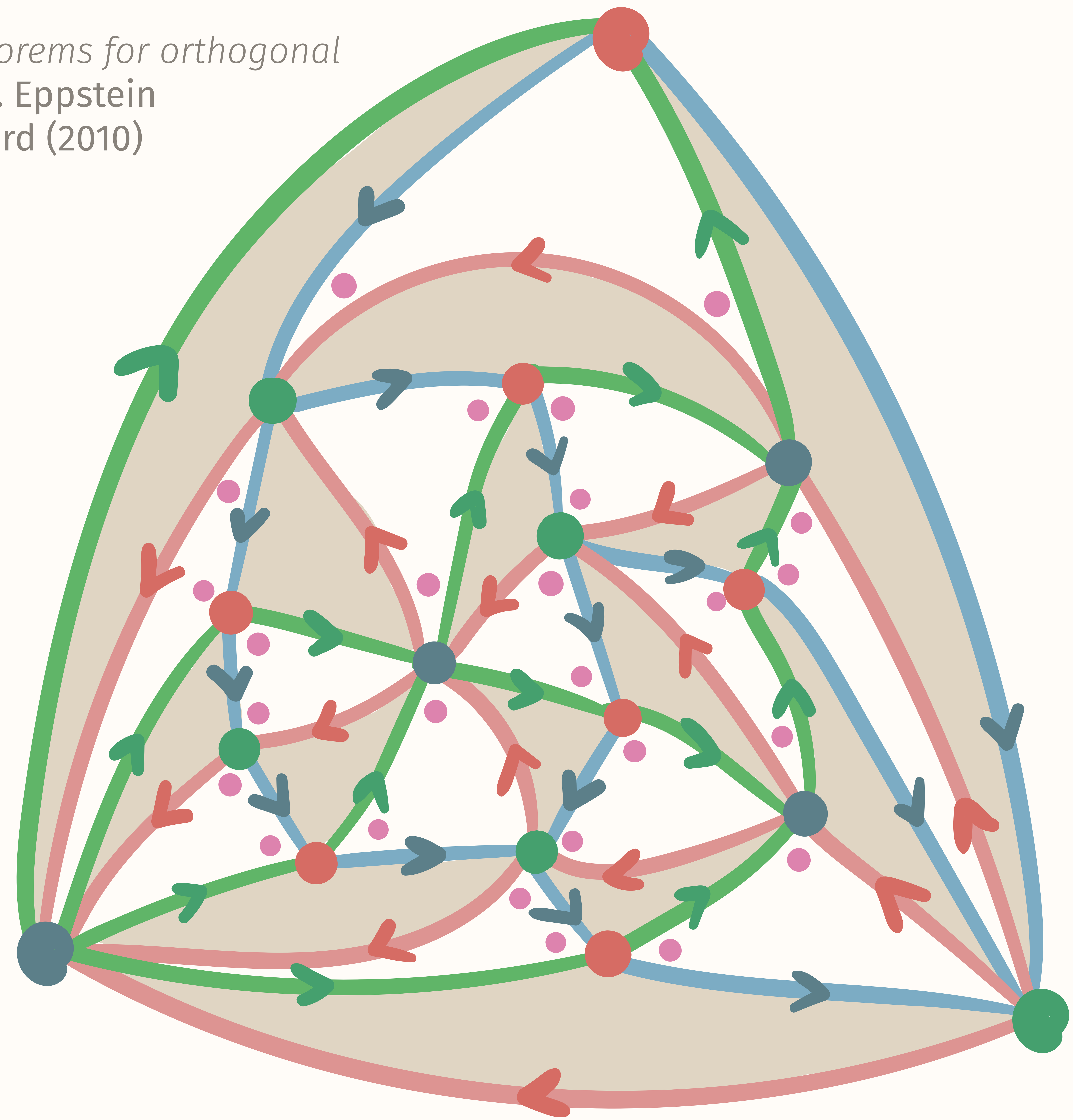


Polyhedral orientations

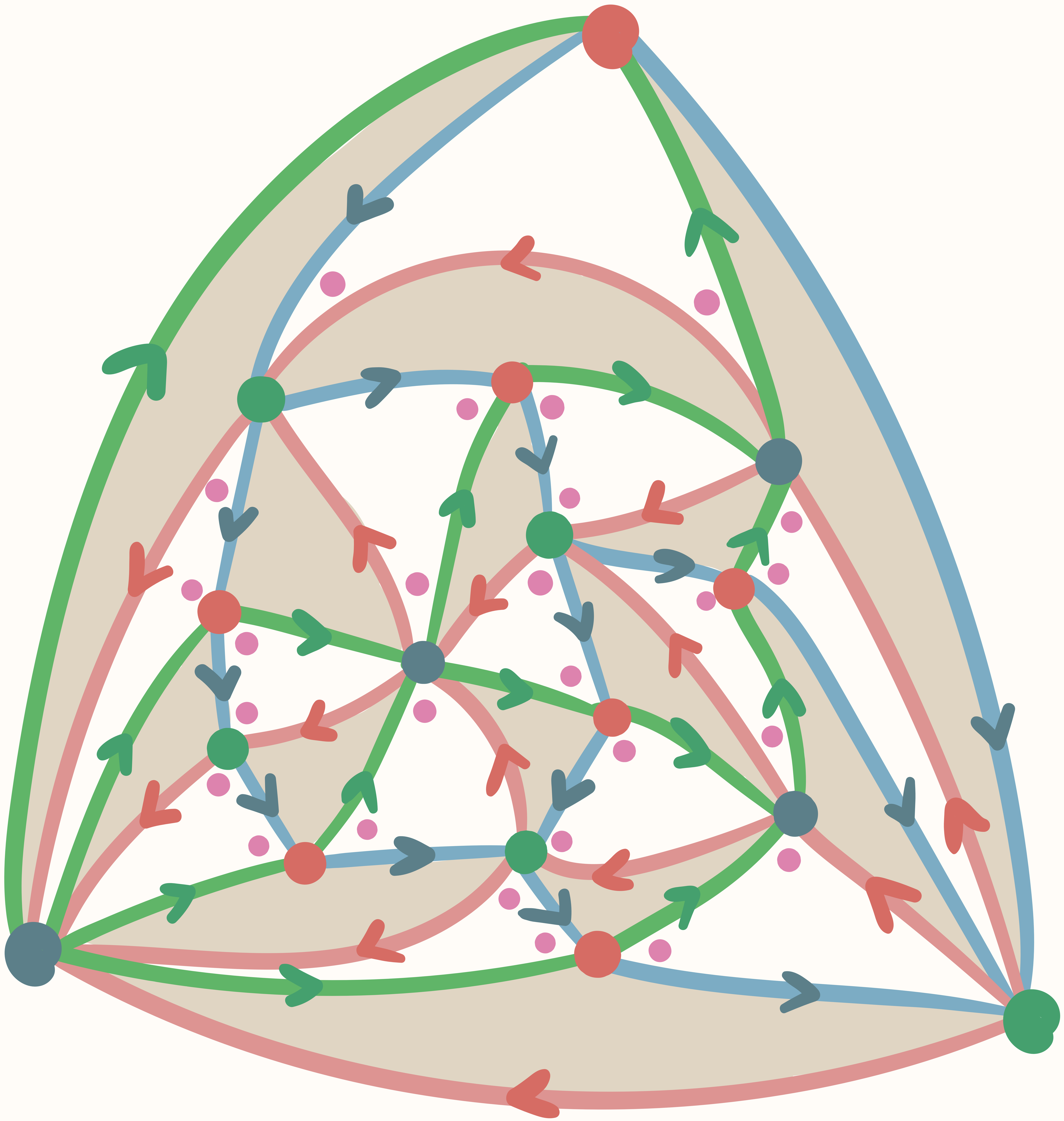


→ *Steinitz theorems for orthogonal polyhera,* D. Eppstein & E. Mumford (2010)

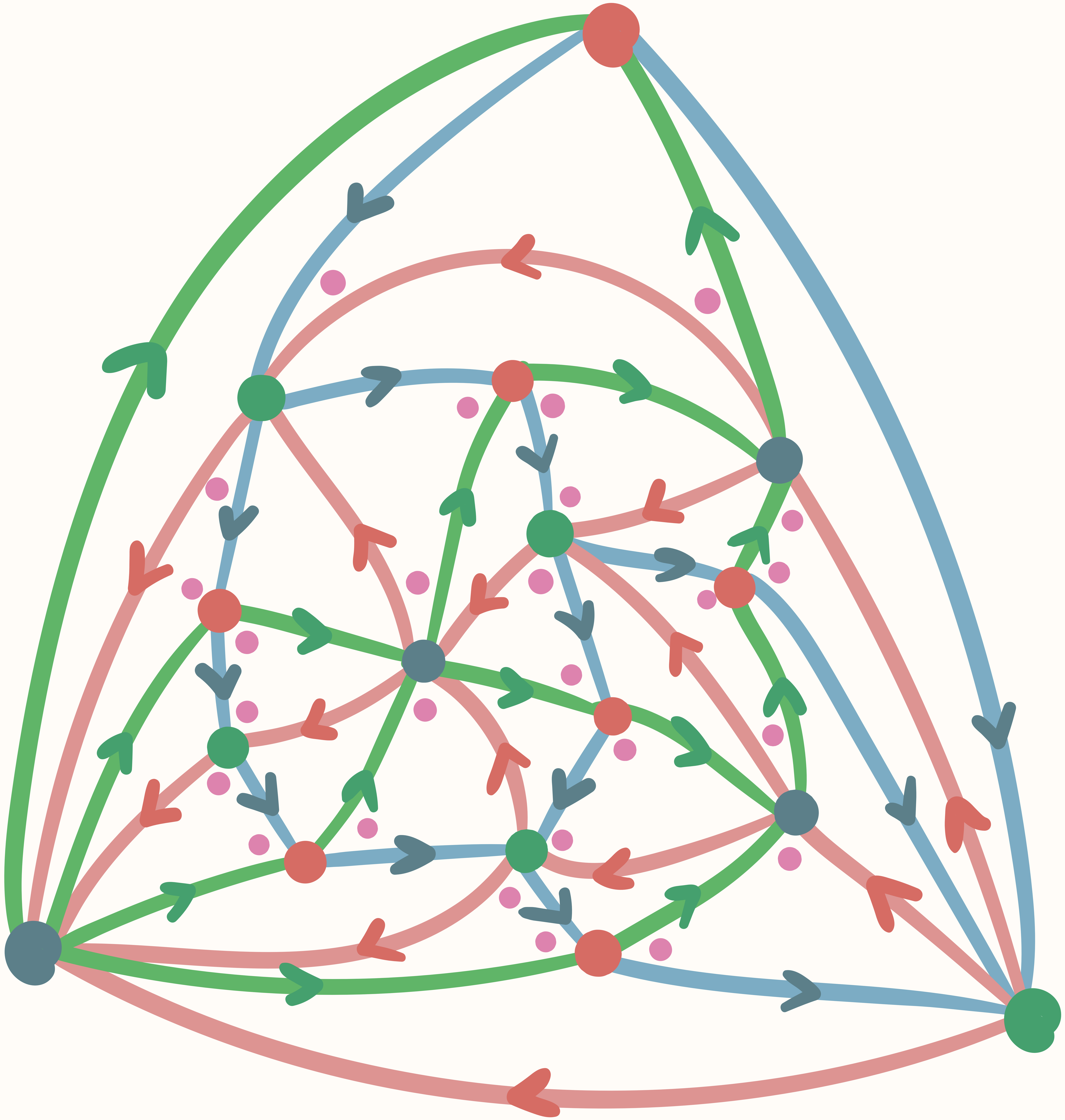




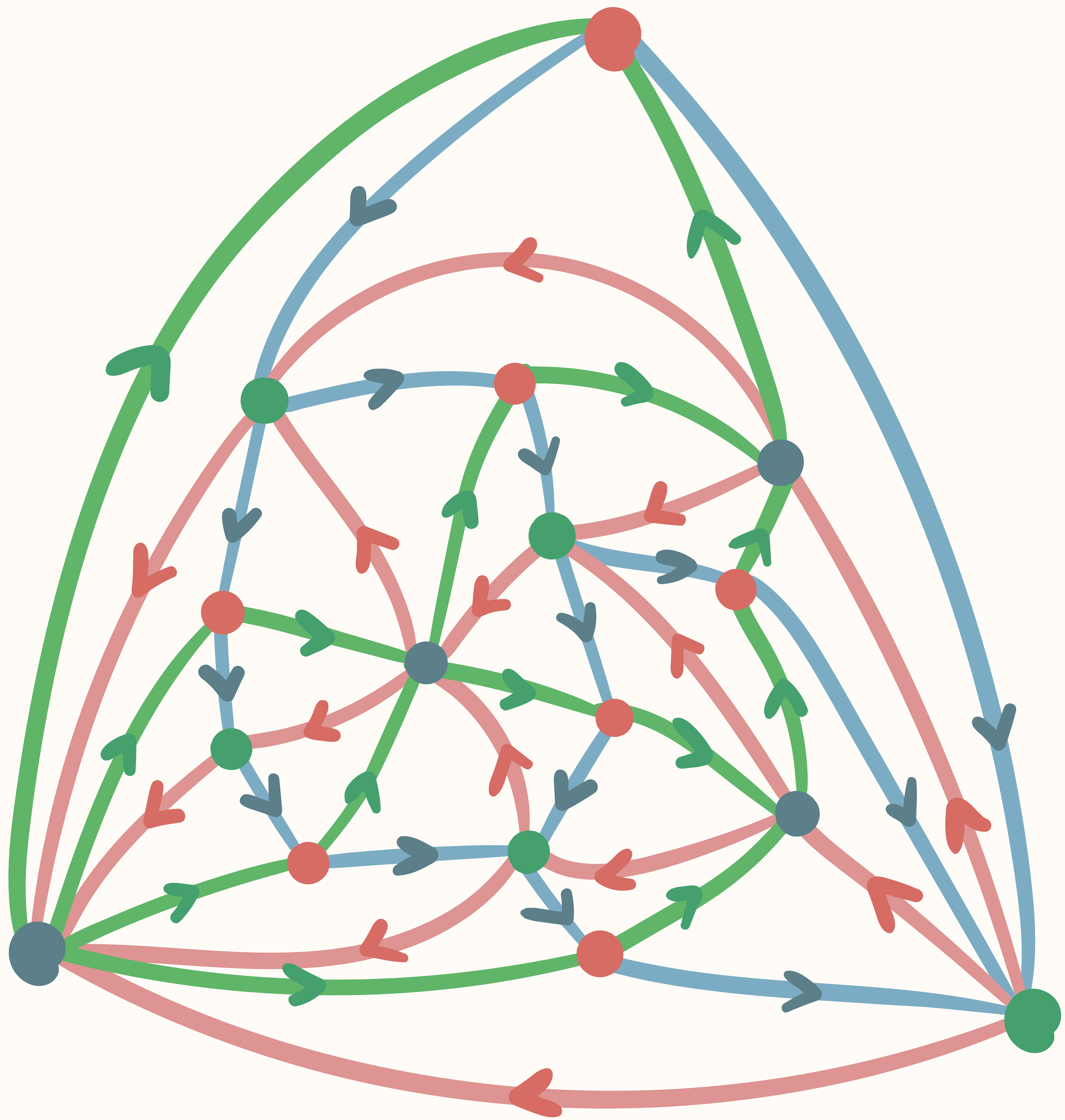
Polyhedral orientations



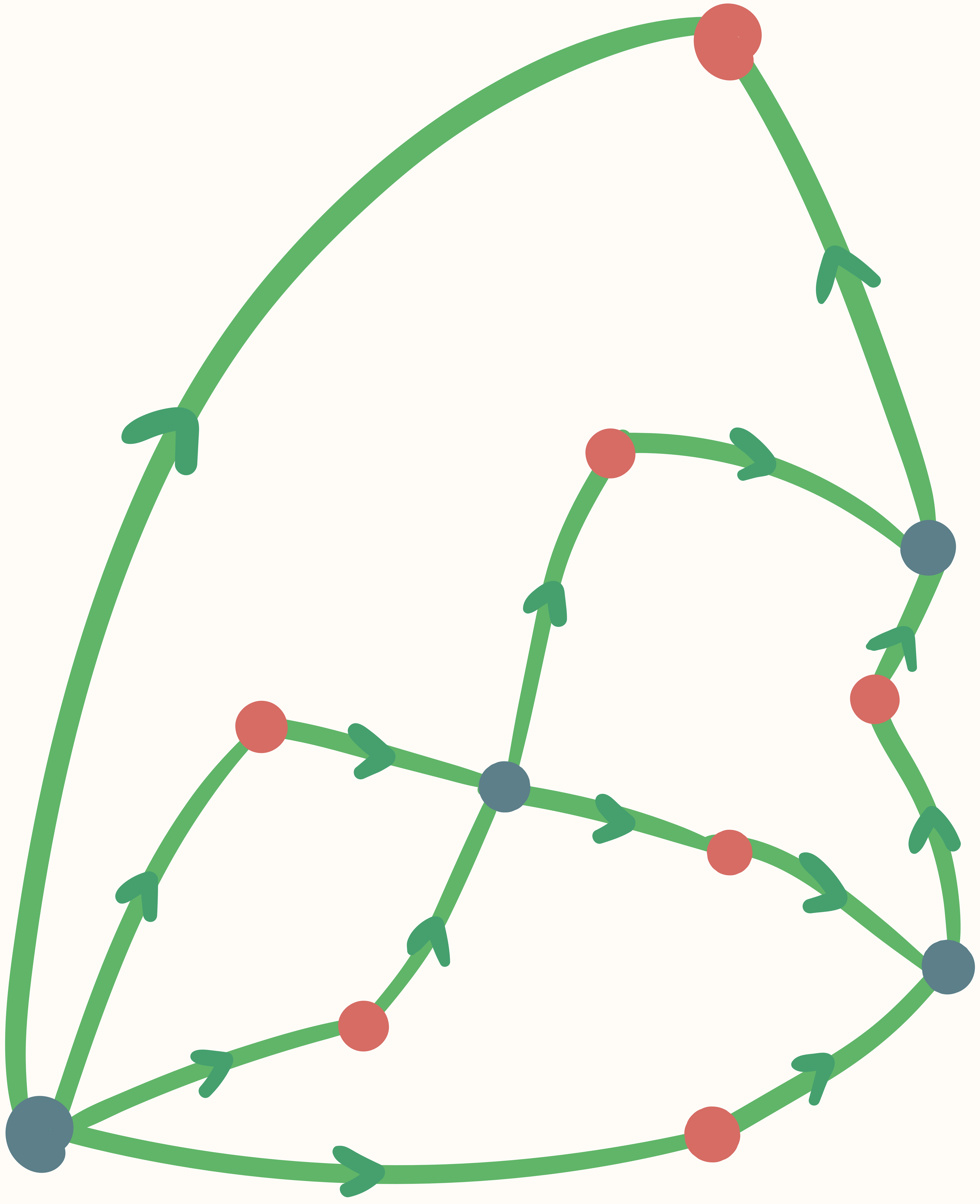
Polyhedral orientations



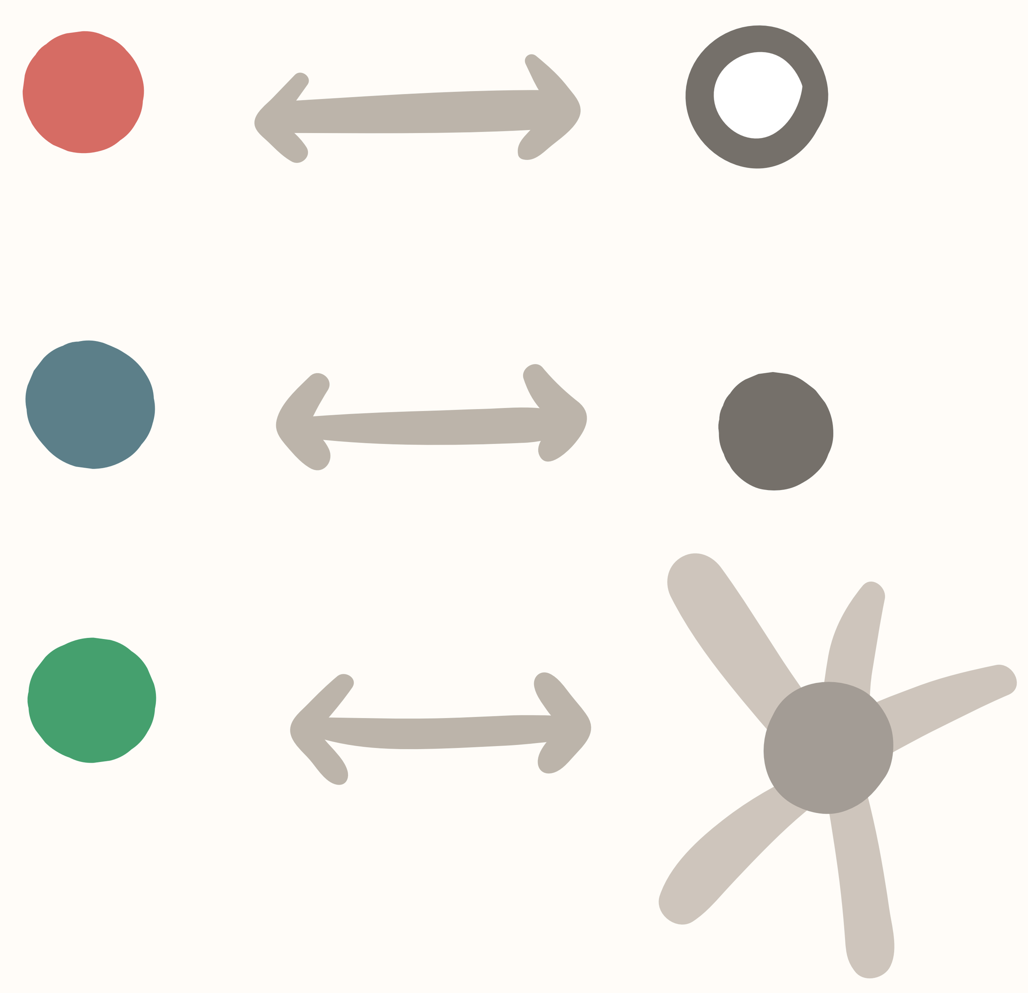
Polyhedral orientations

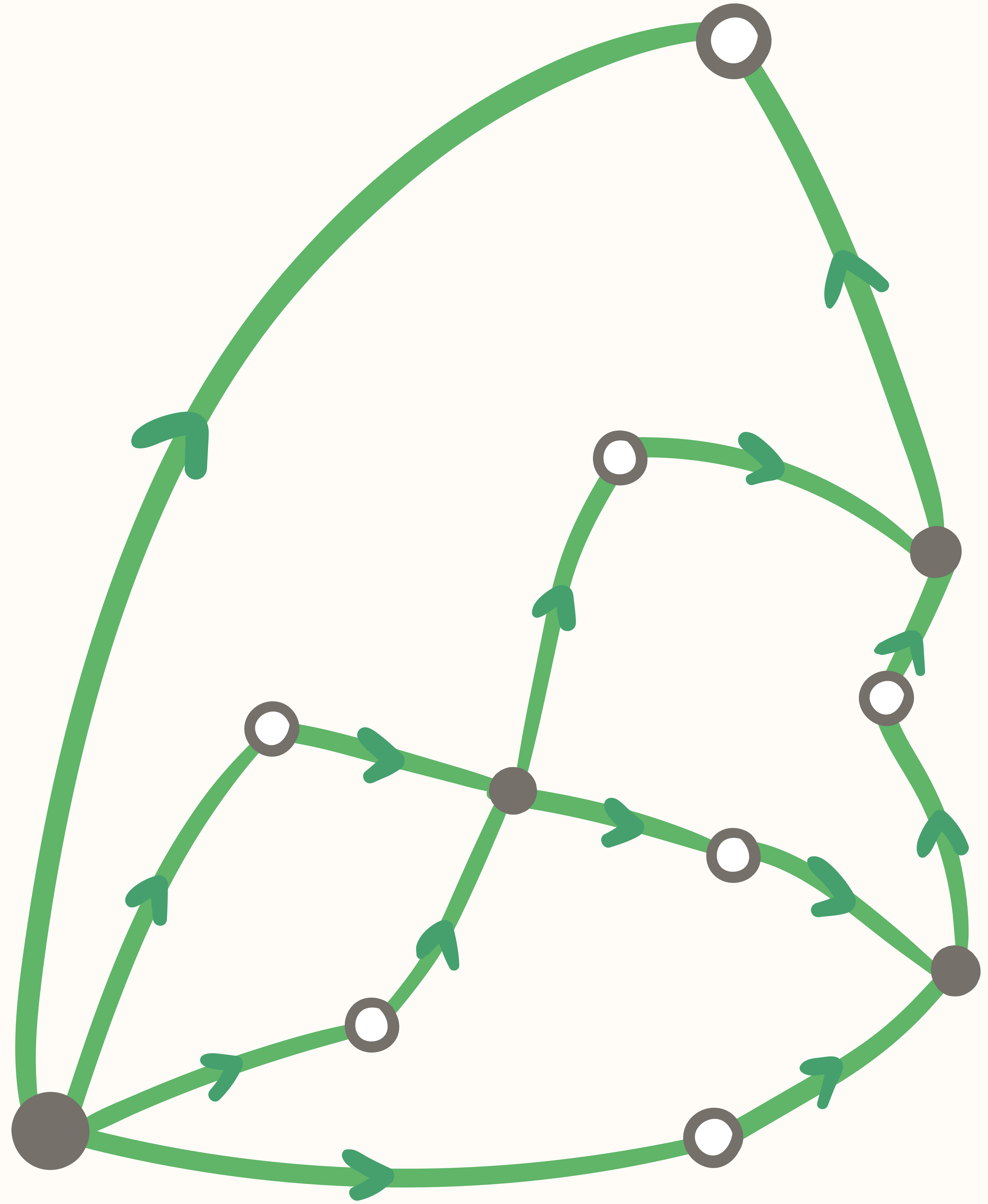


Polyhedral orientations

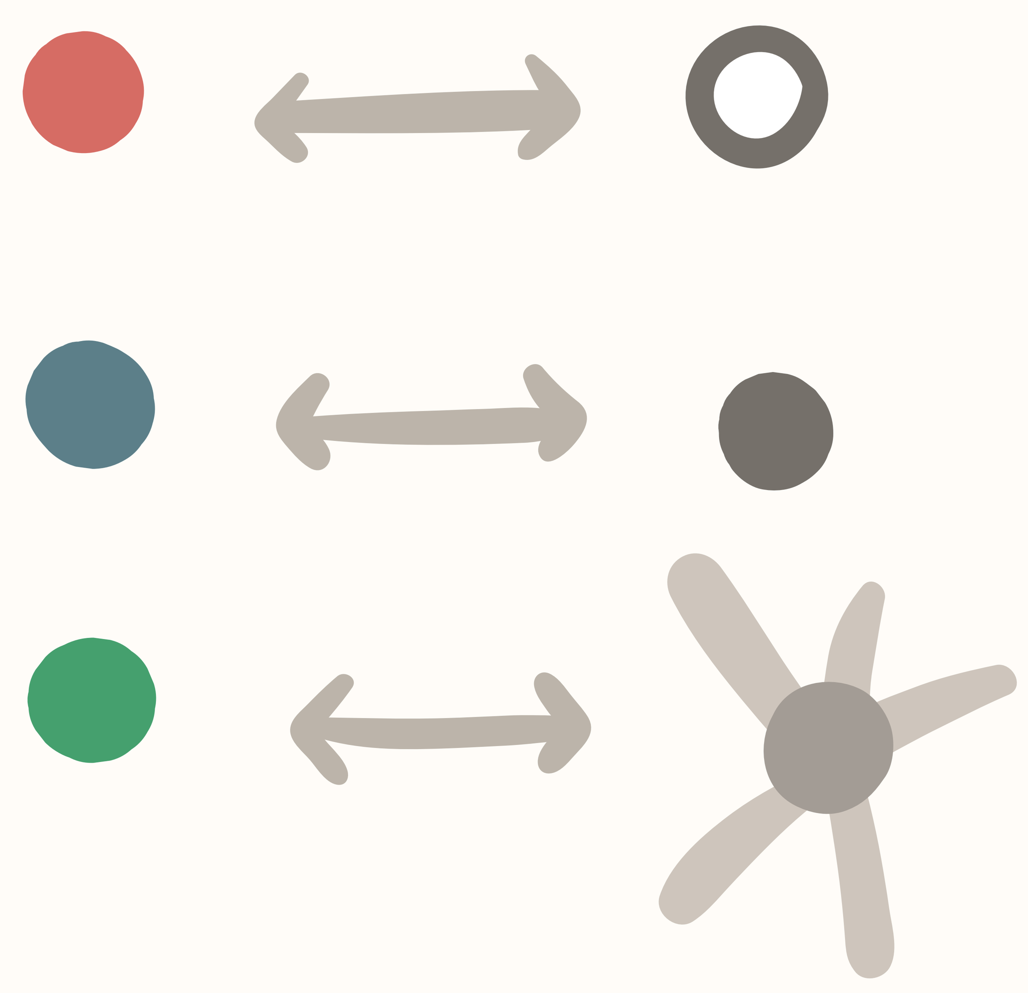


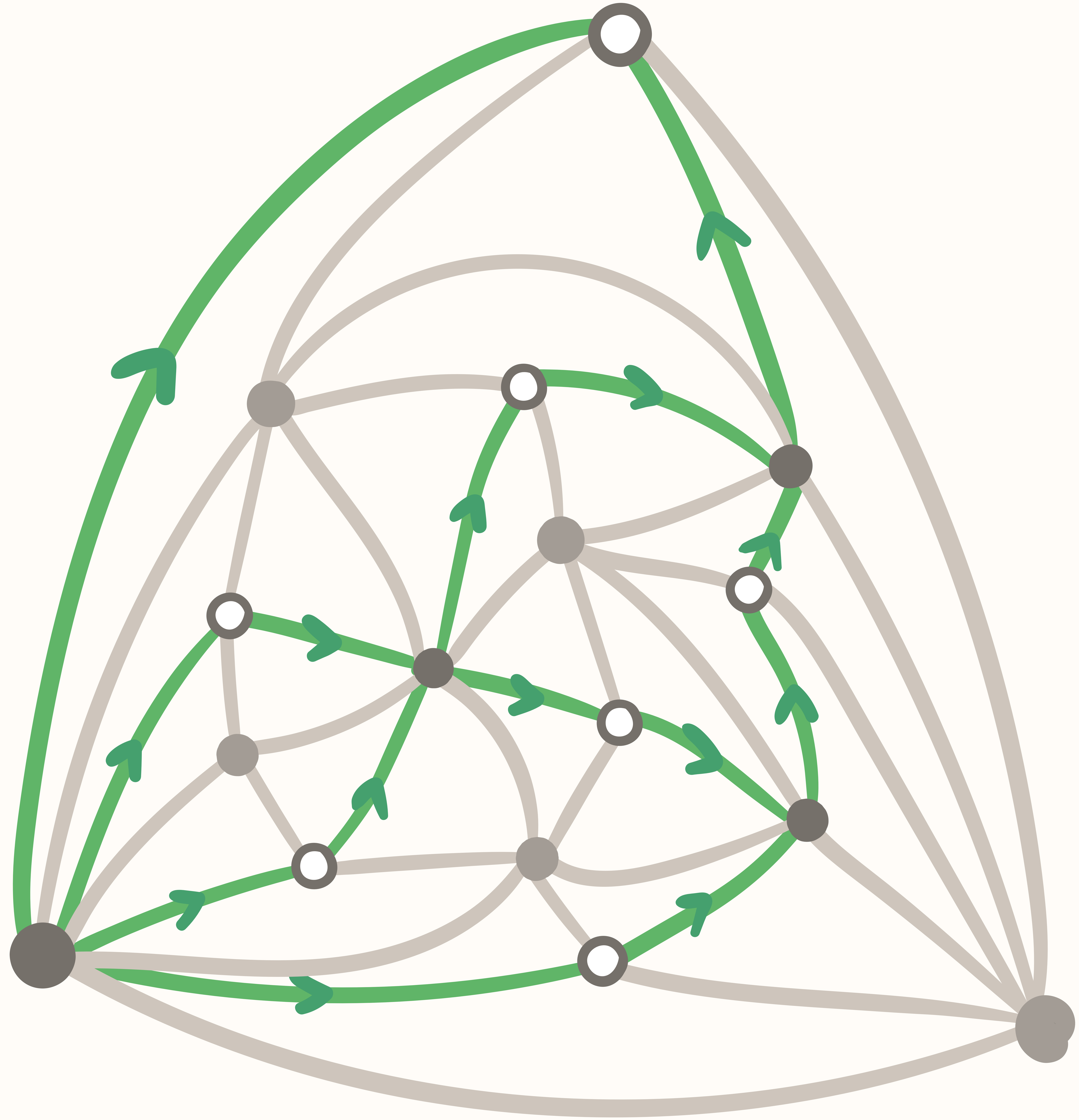
Polyhedral orientations



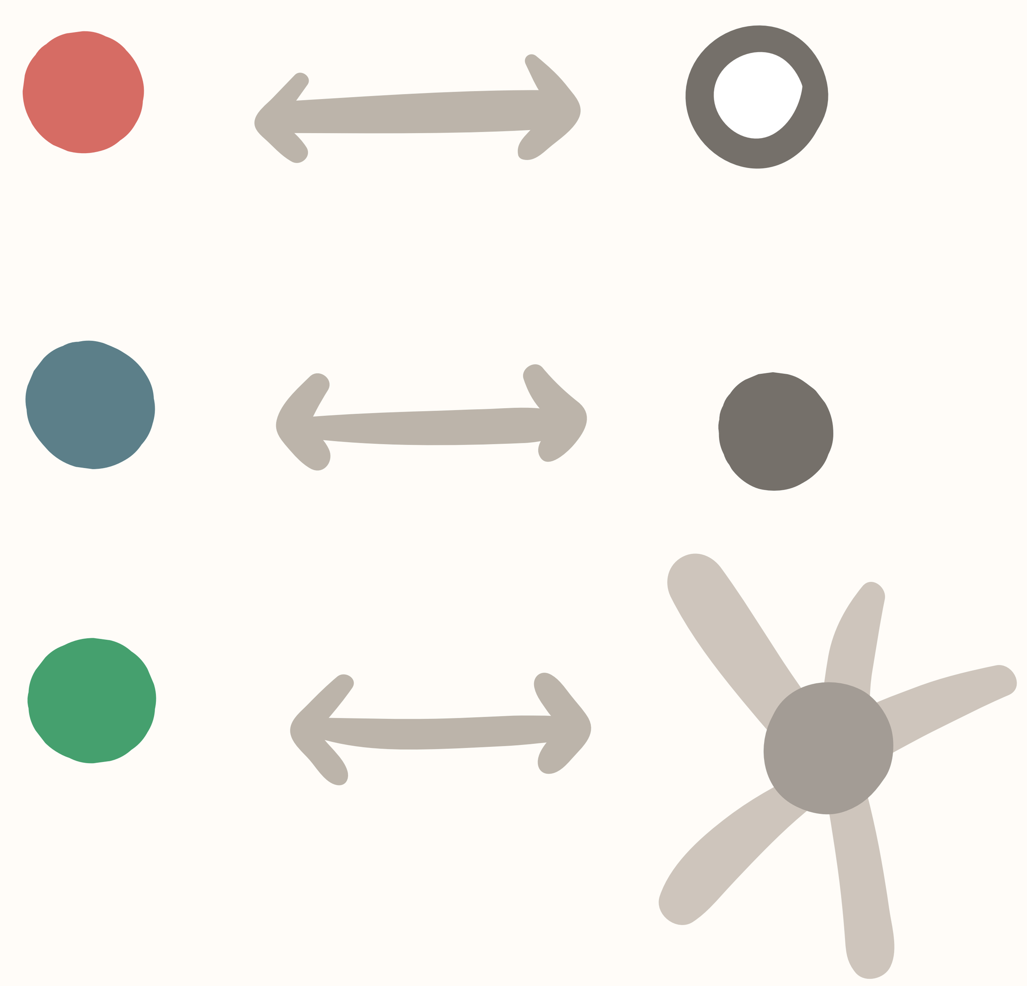


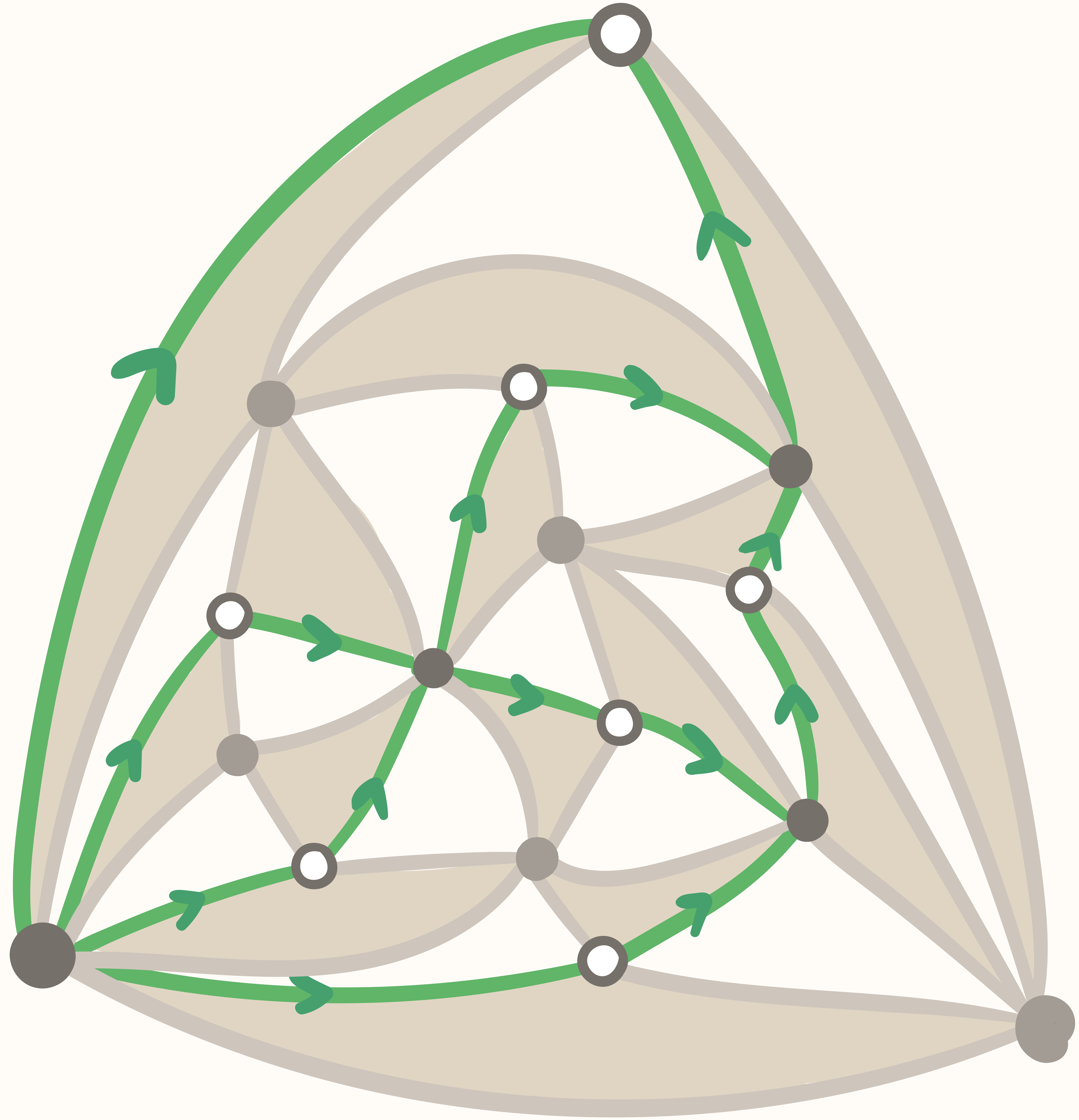
Polyhedral orientations



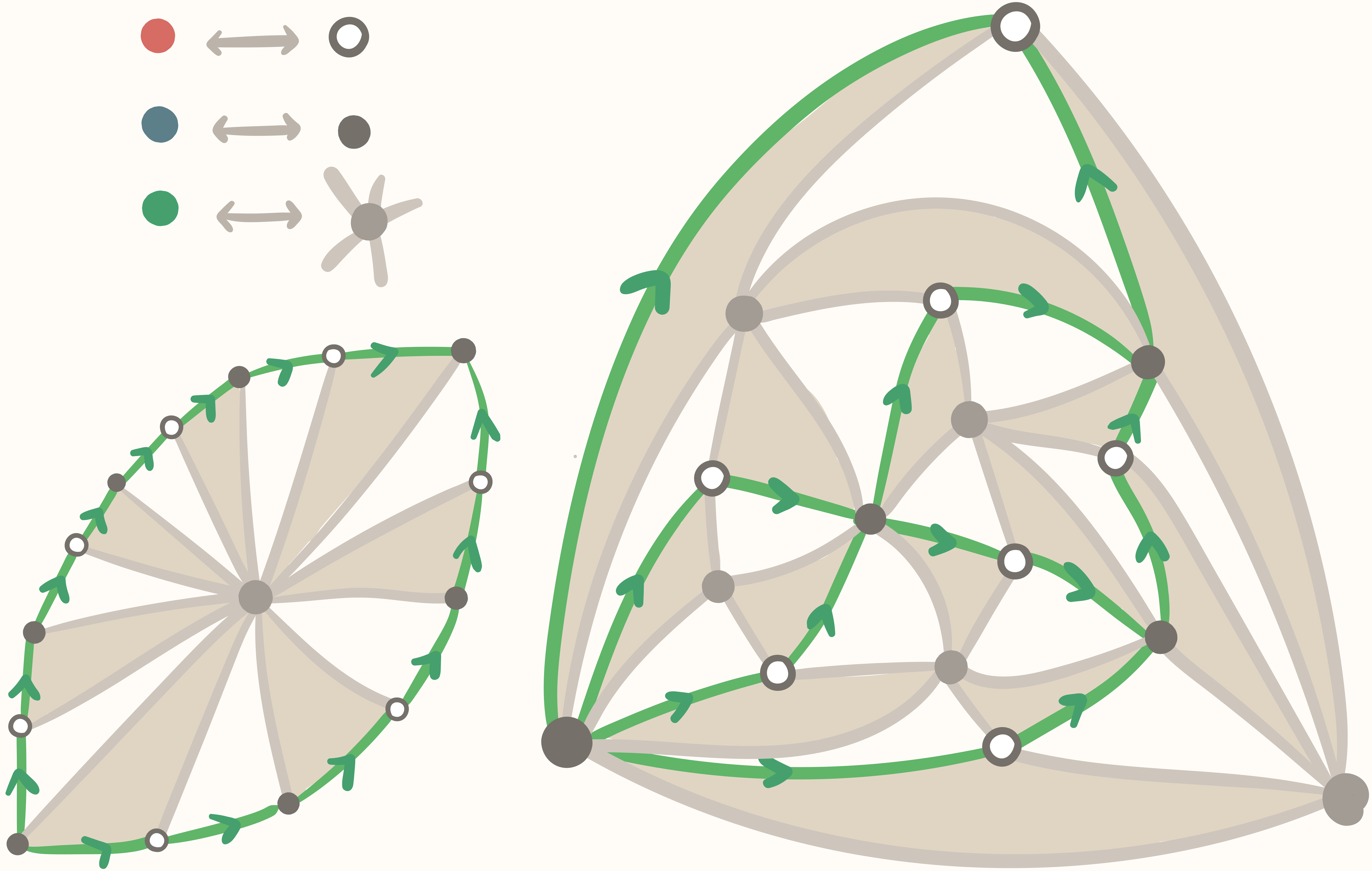


Polyhedral orientations

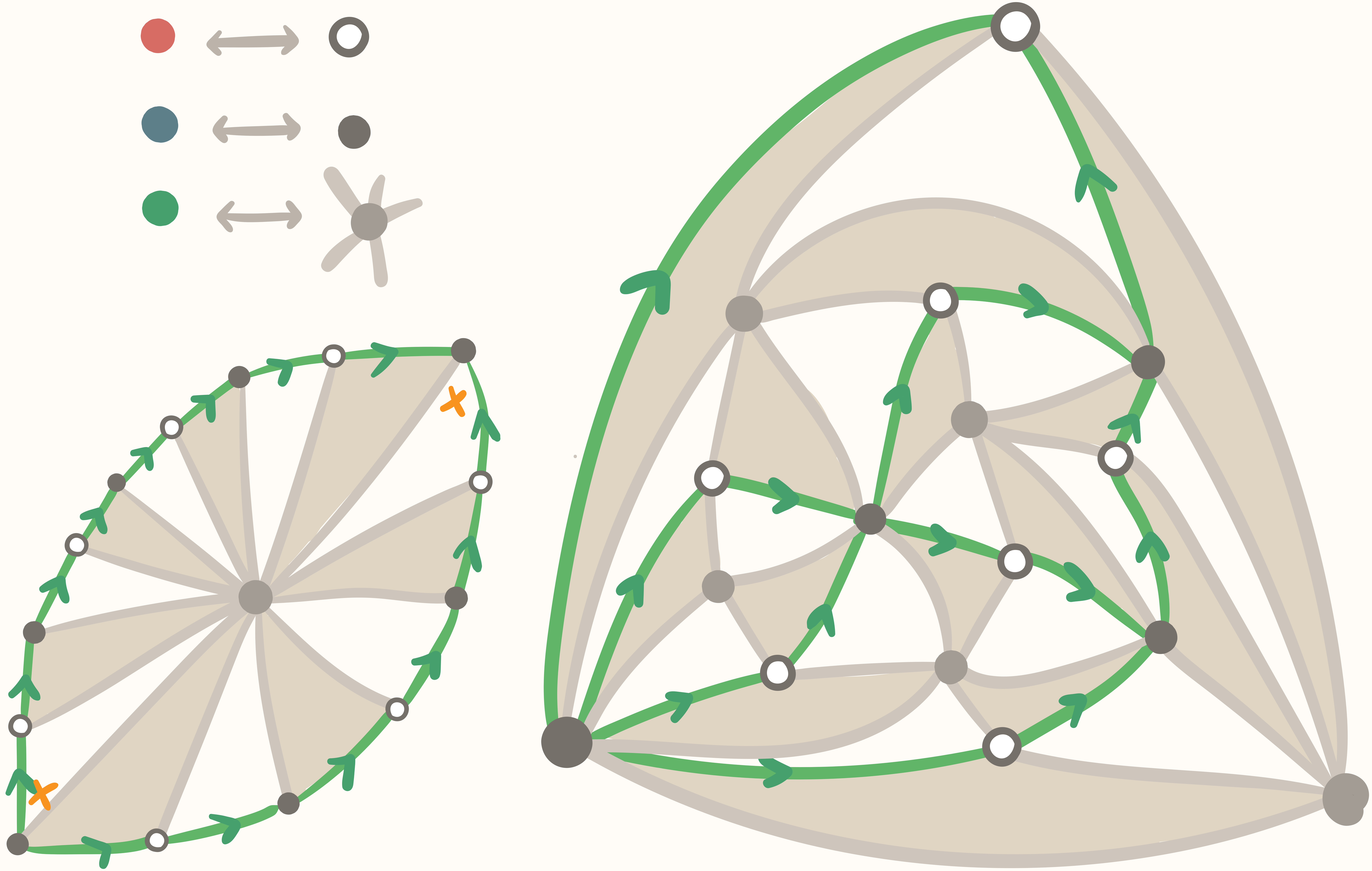




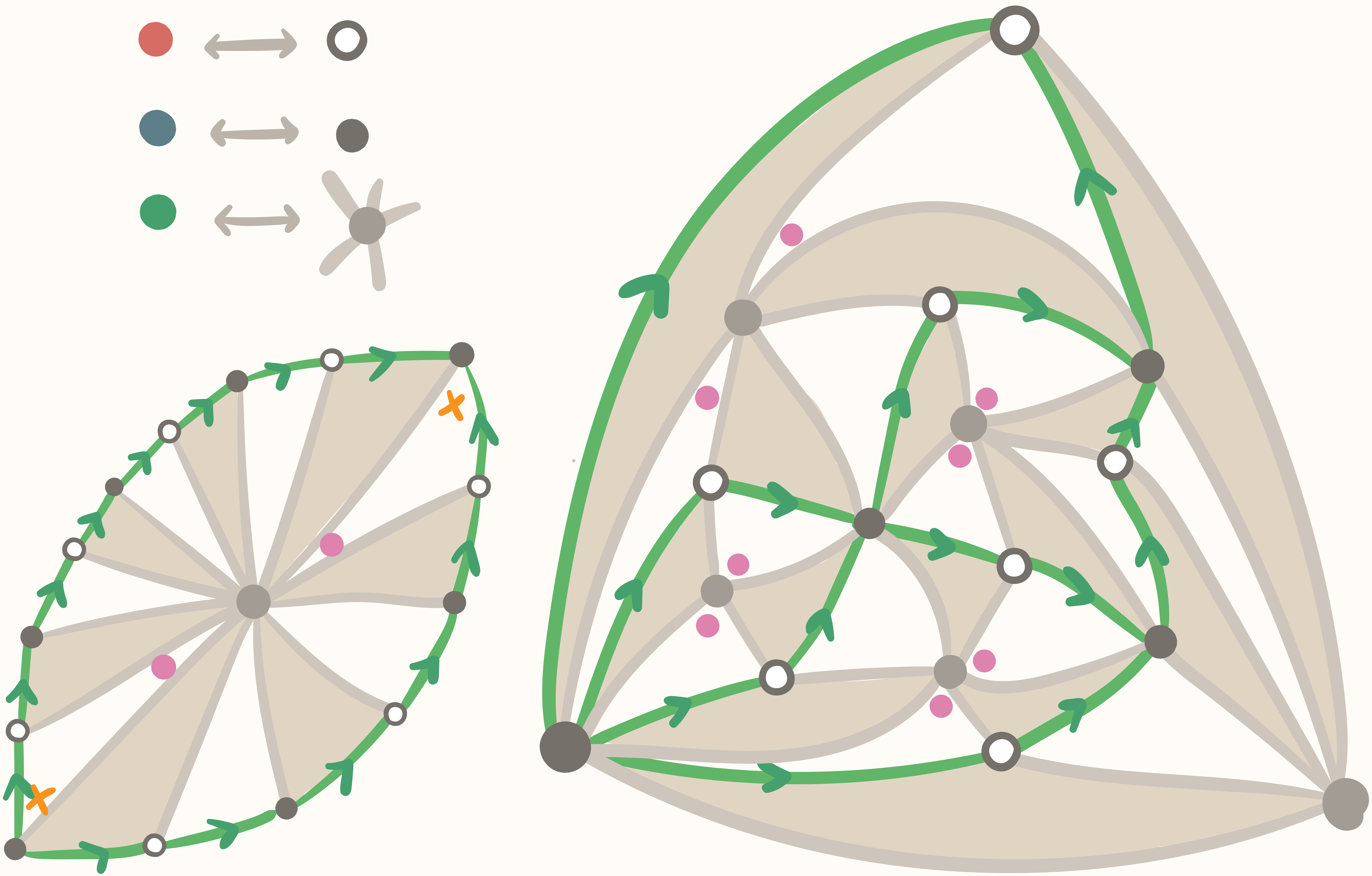
Polyhedral orientations



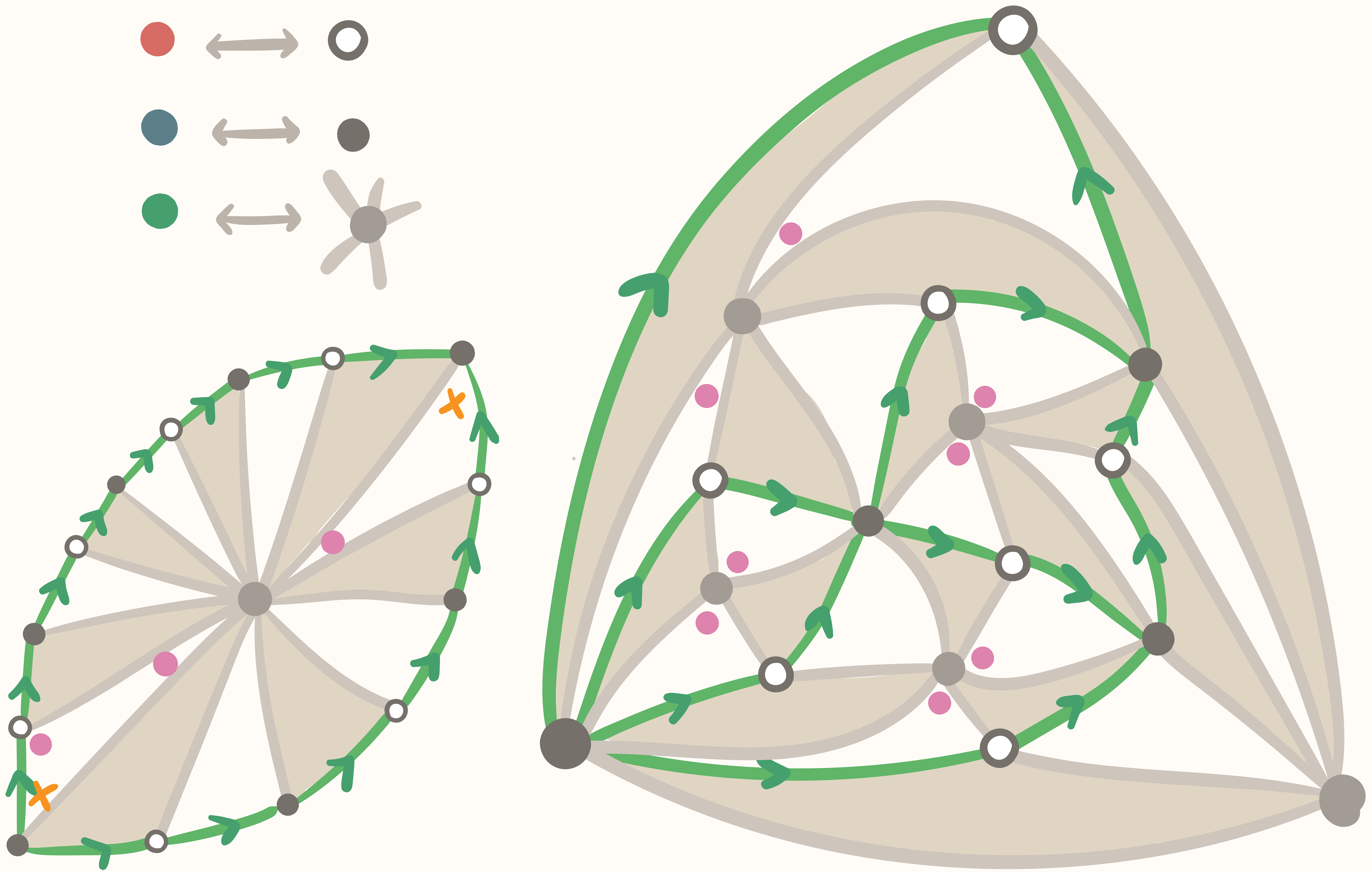
Polyhedral orientations



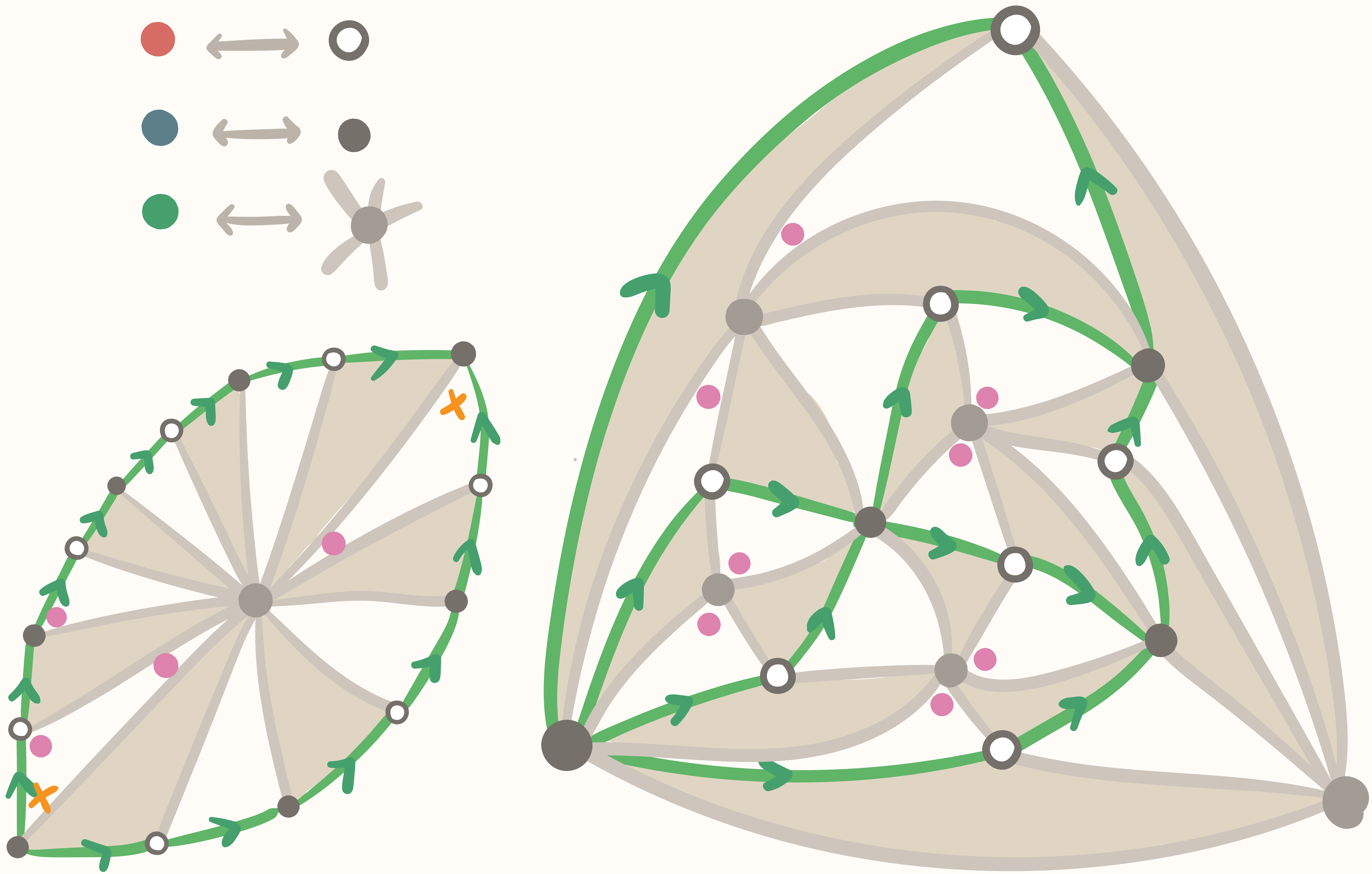
Polyhedral orientations



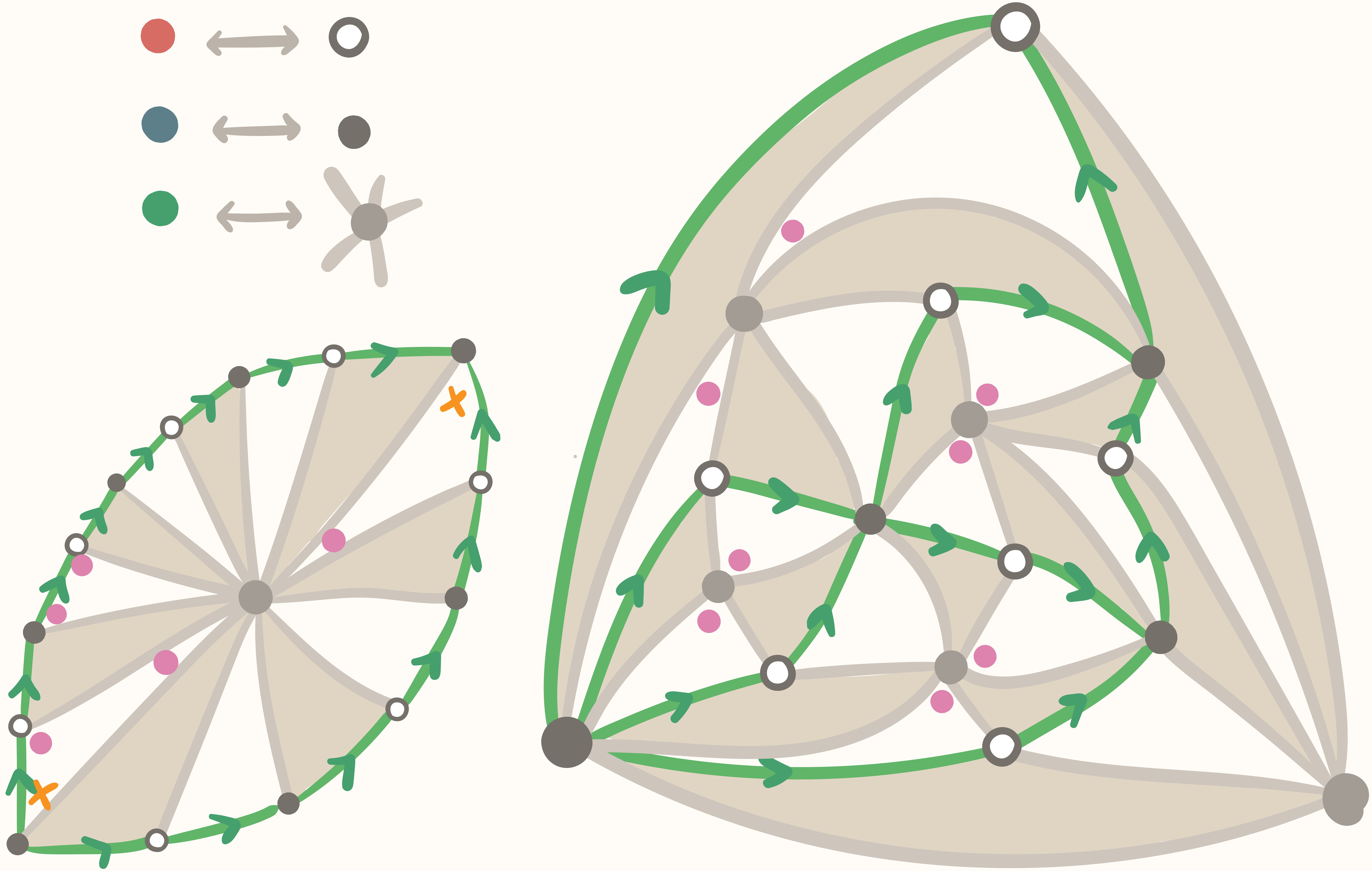
Polyhedral orientations



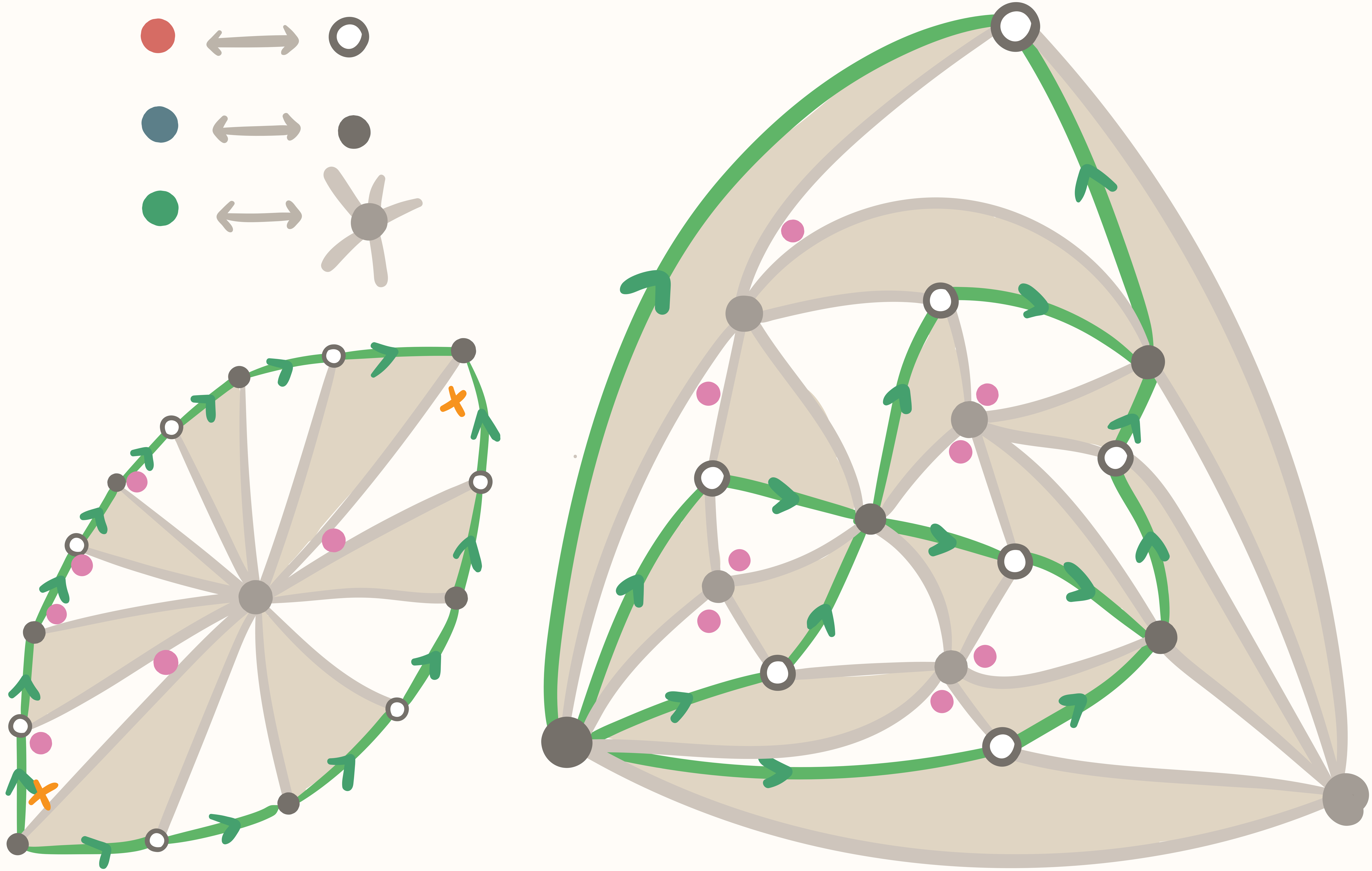
Polyhedral orientations



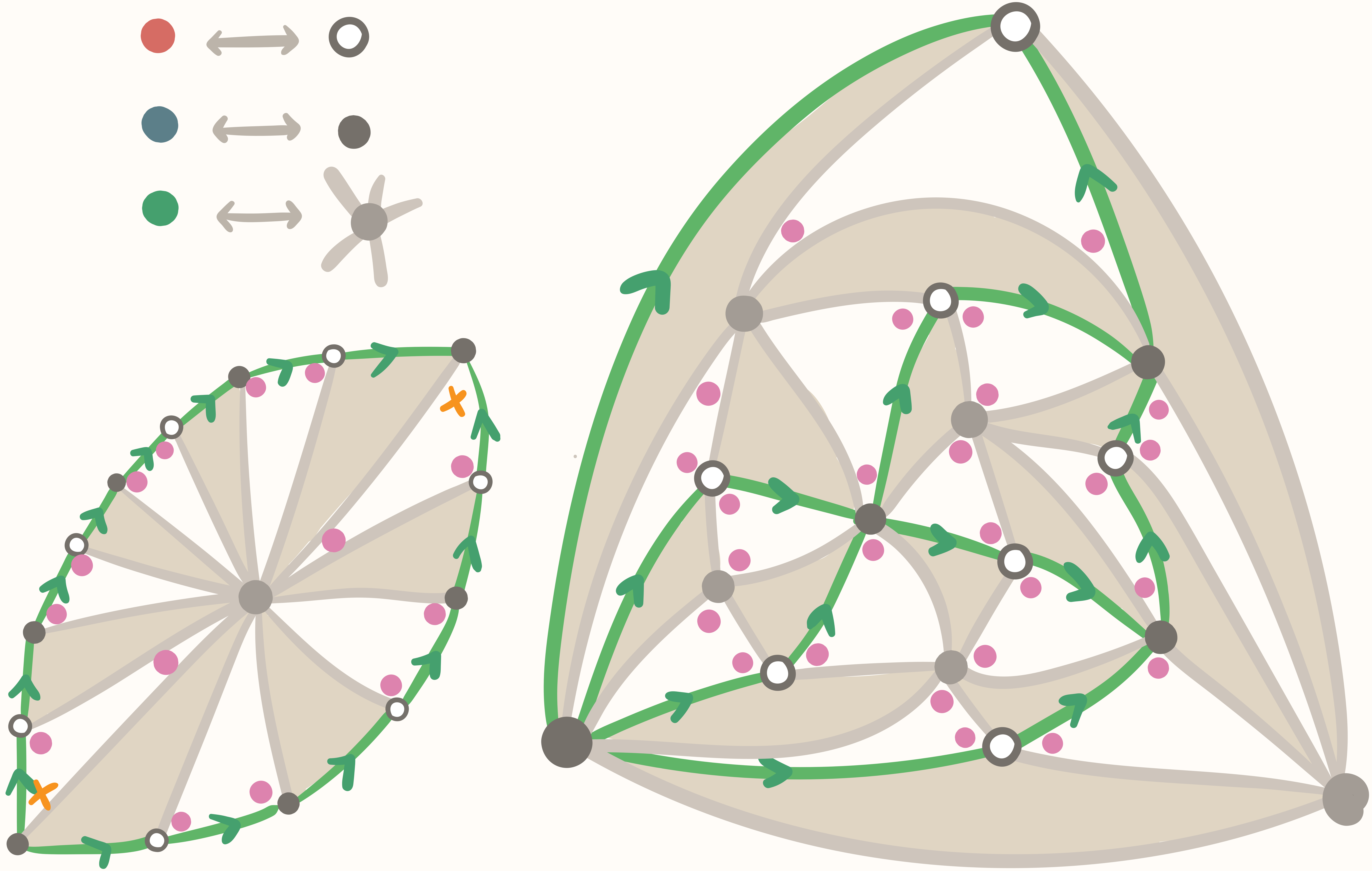
Polyhedral orientations



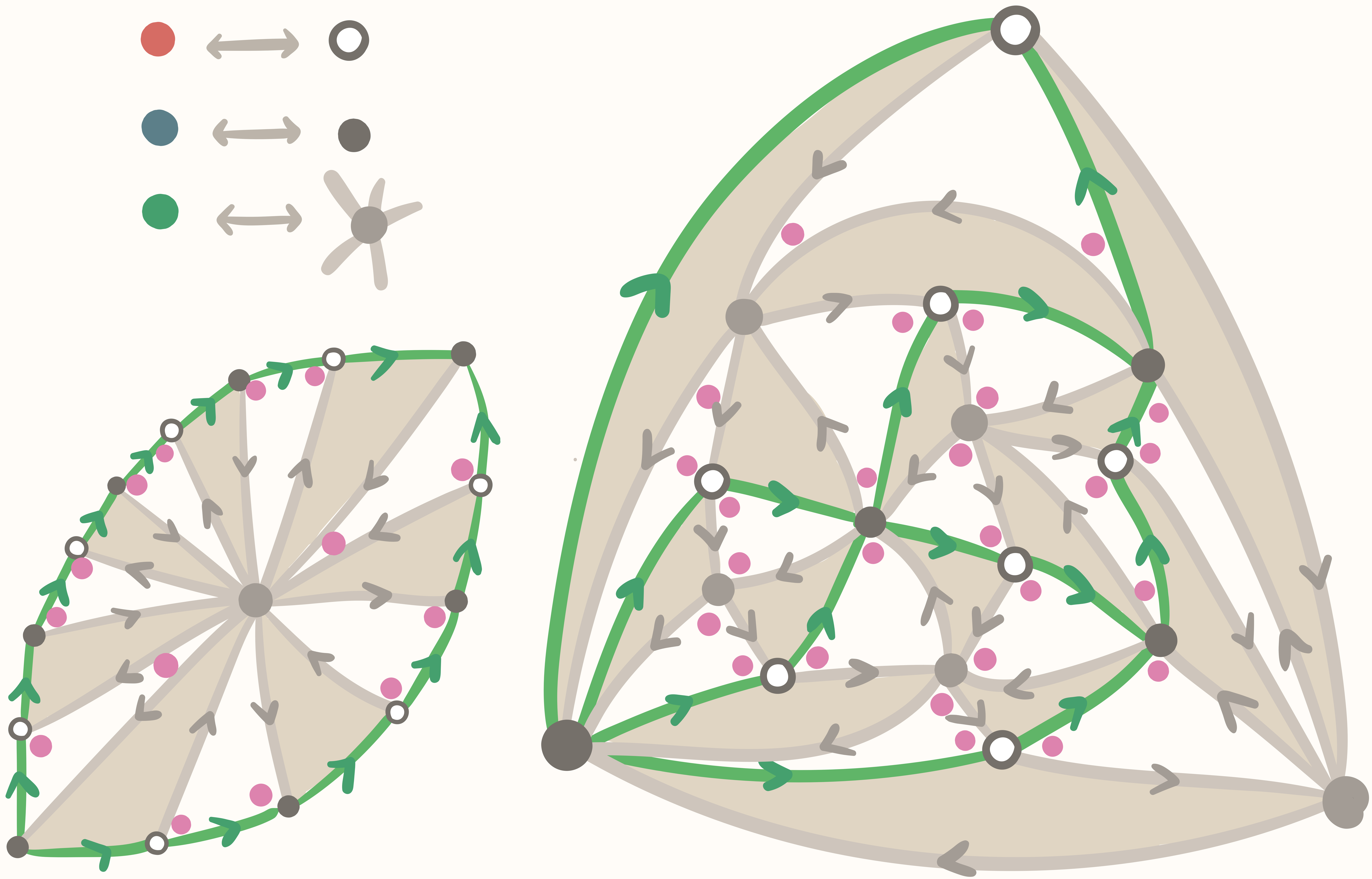
Polyhedral orientations



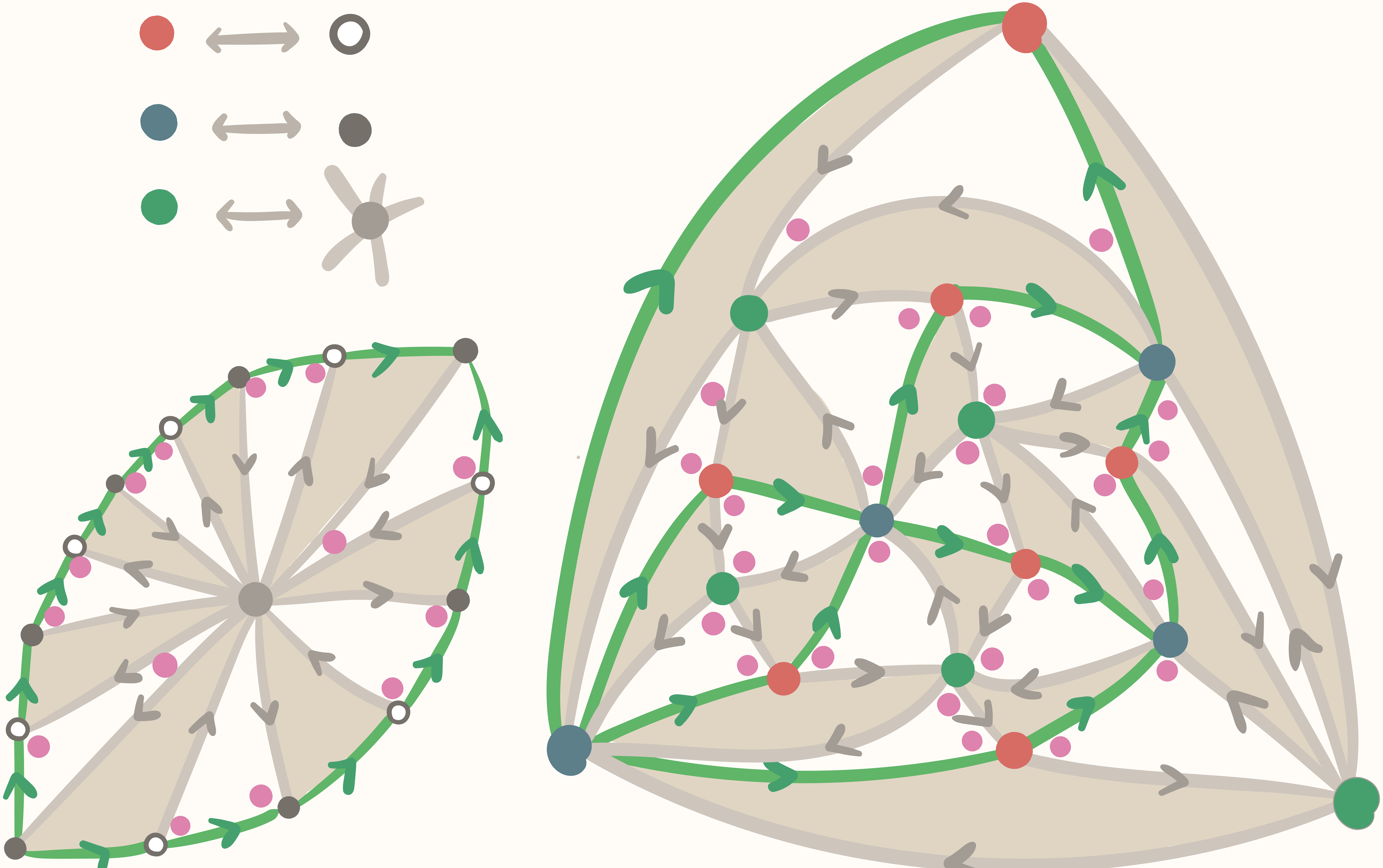
Polyhedral orientations



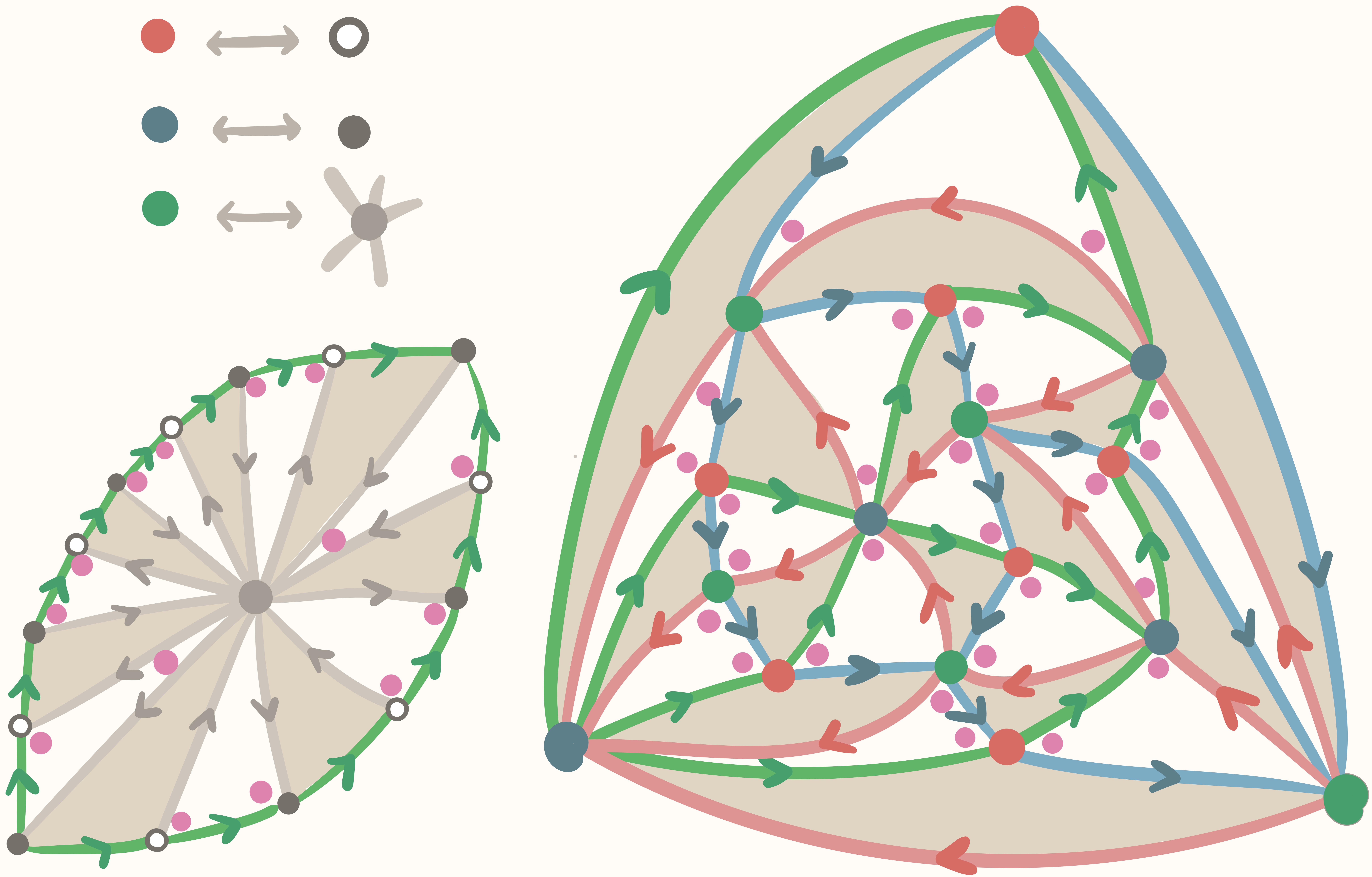
Polyhedral orientations



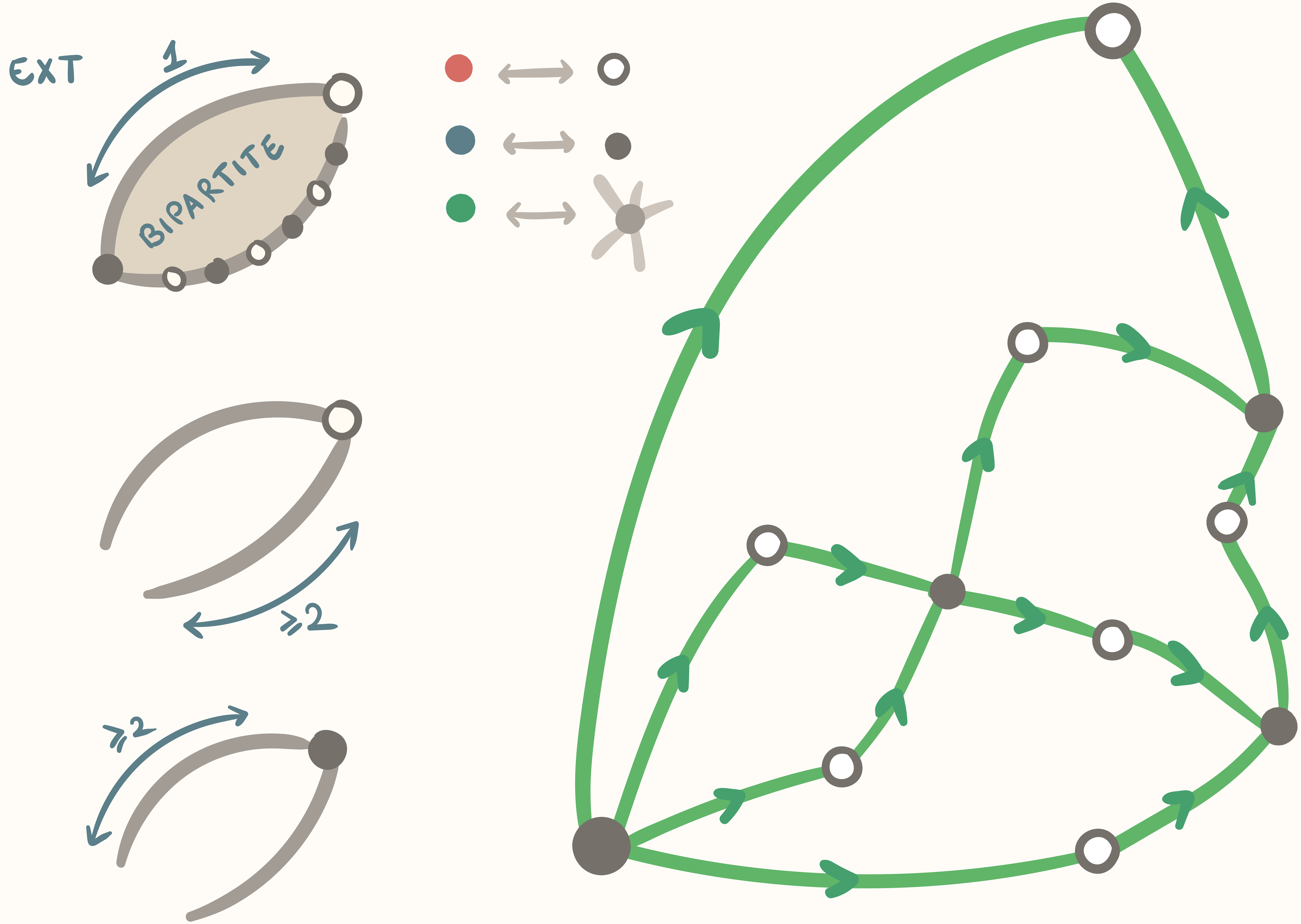
Polyhedral orientations



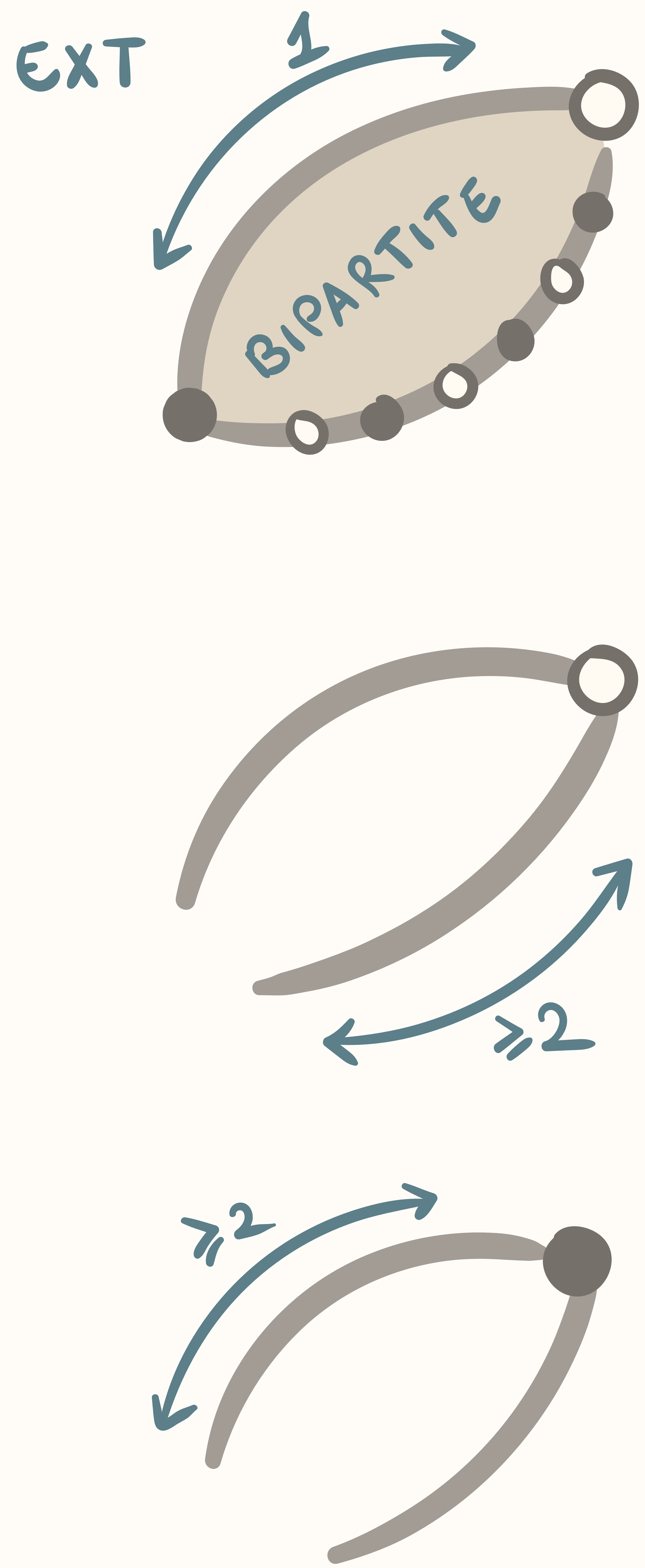
Polyhedral orientations



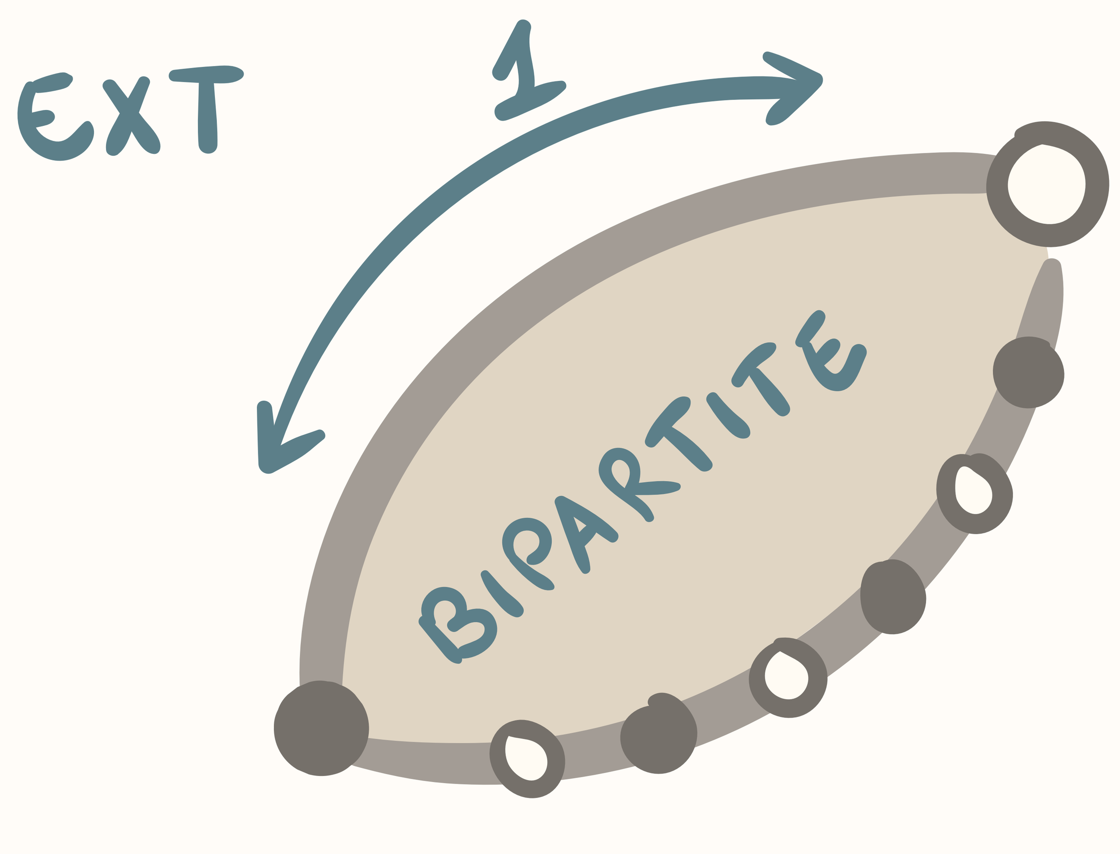
Polyhedral orientations

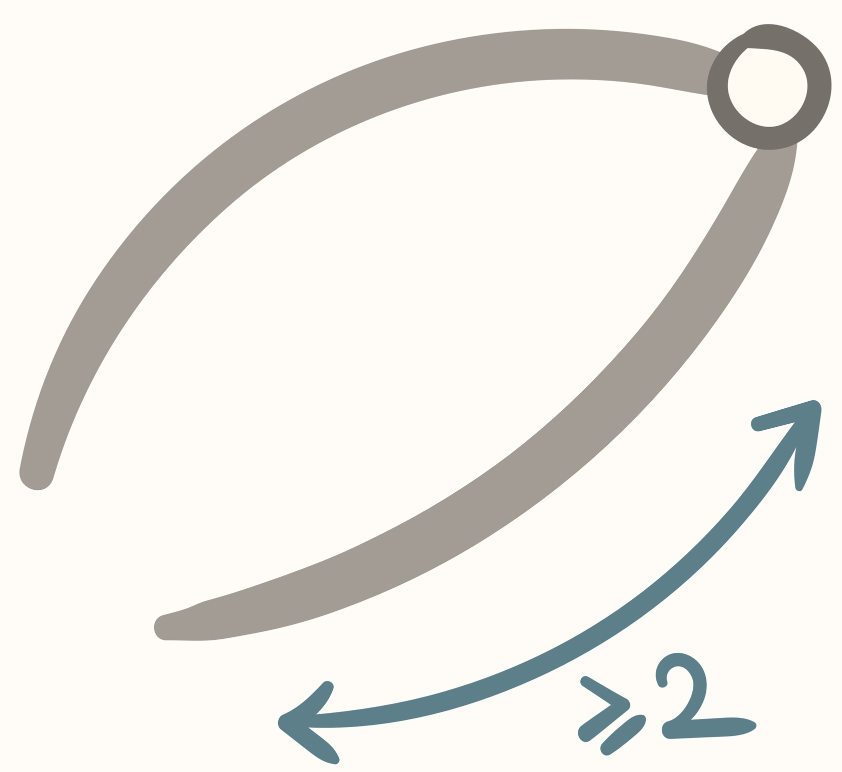


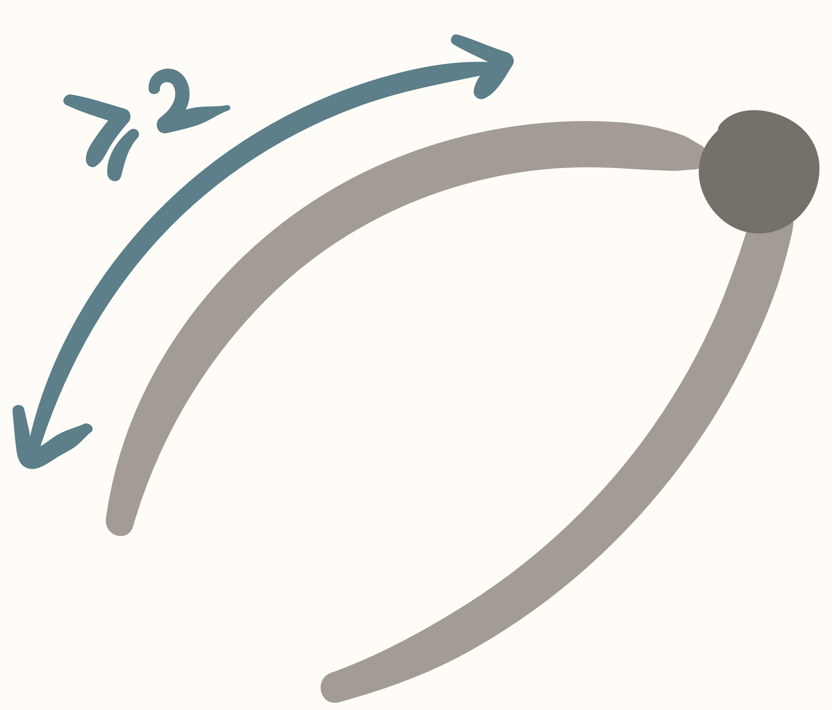
Polyhedral orientations via KMSW

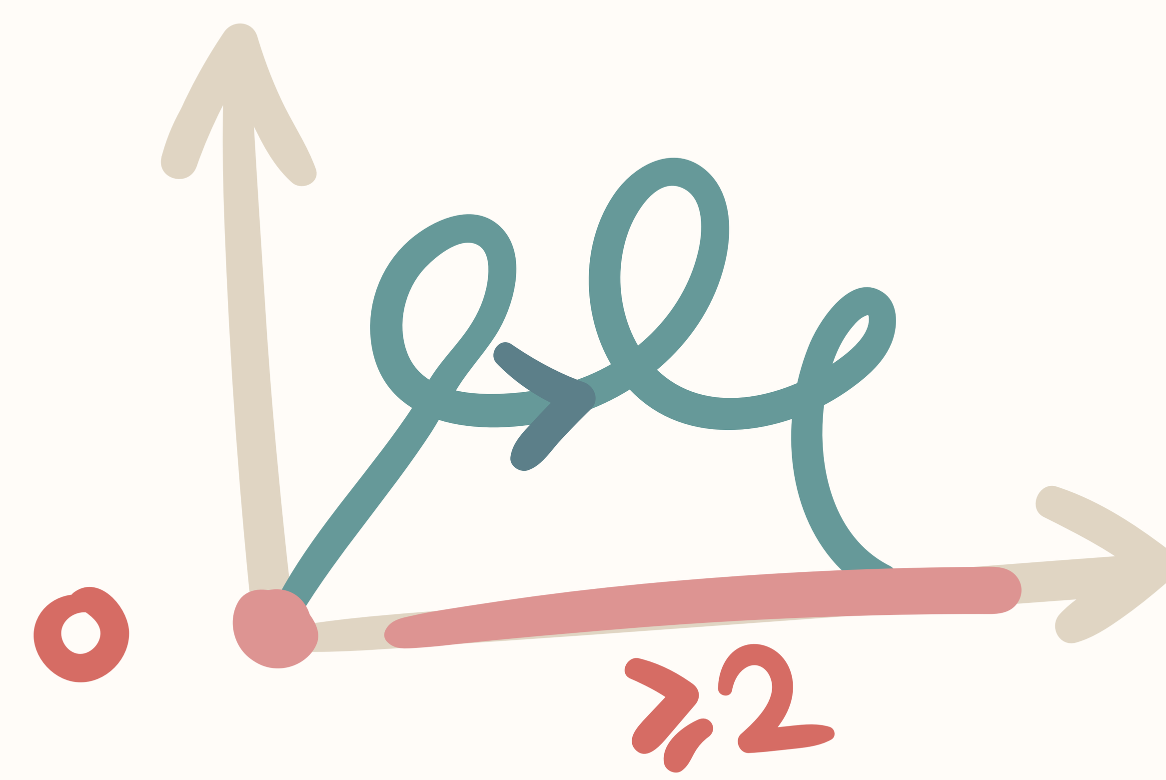


Polyhedral orientations via KMSW

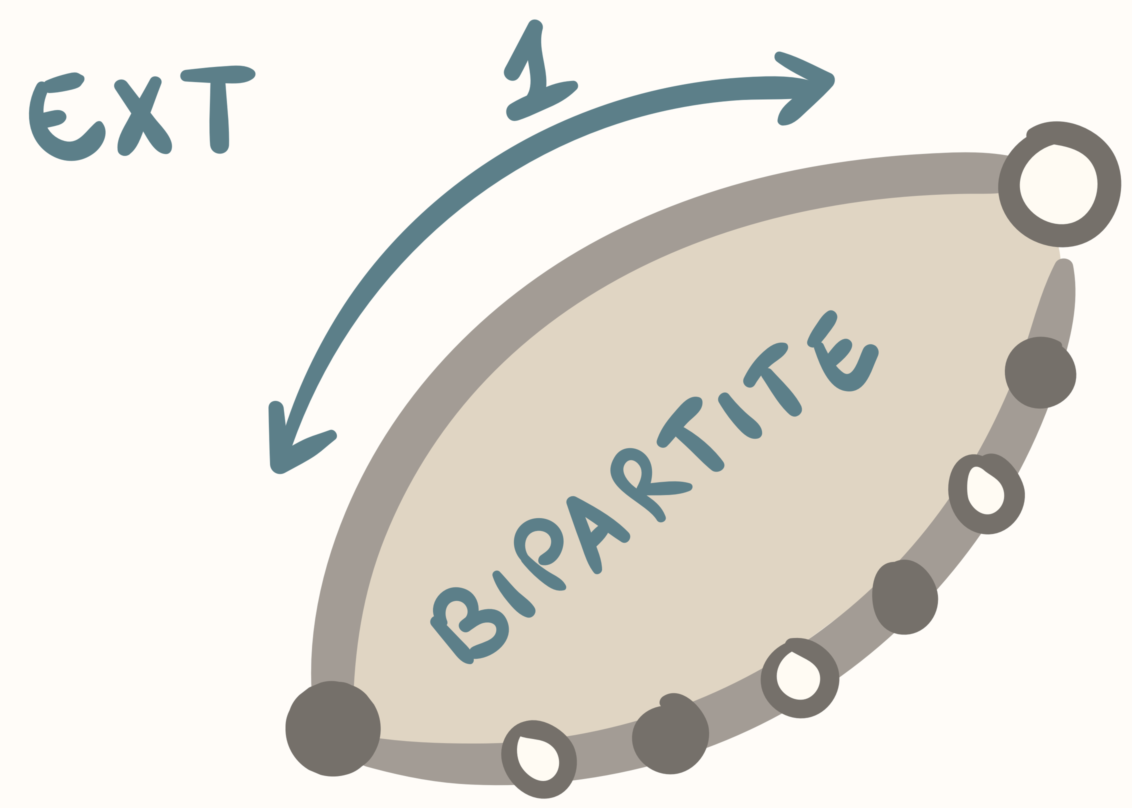


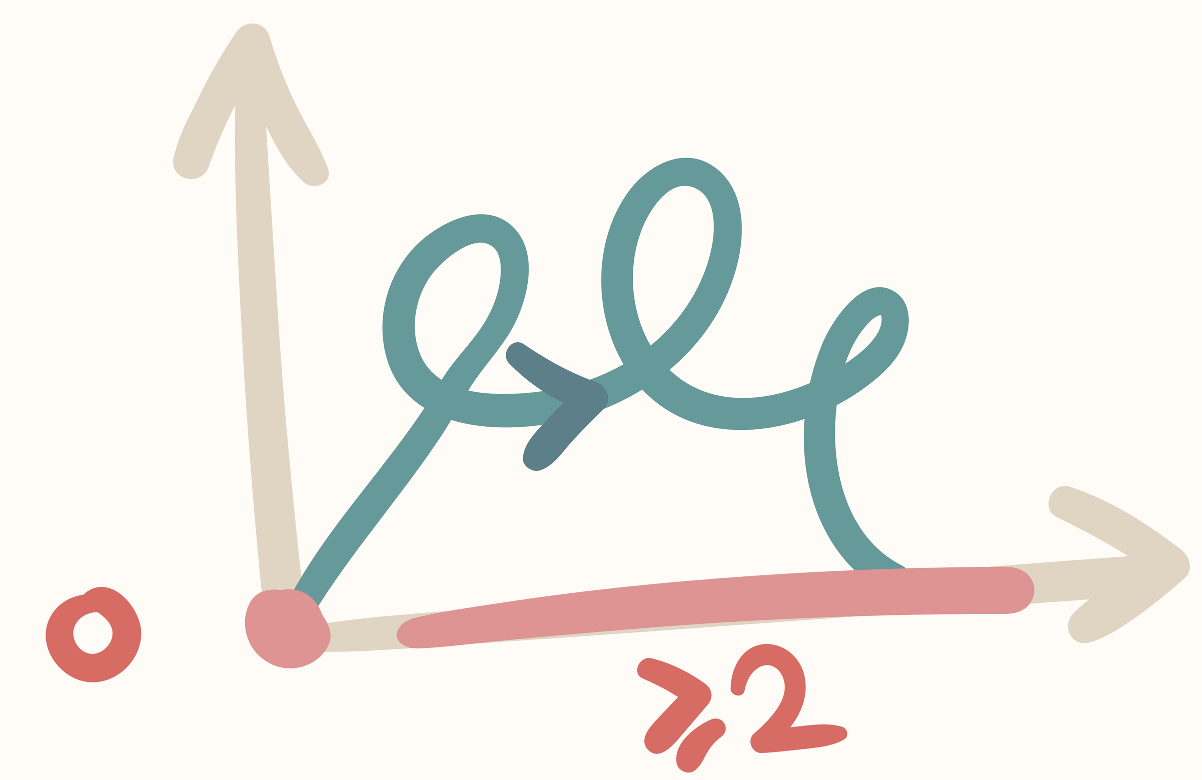


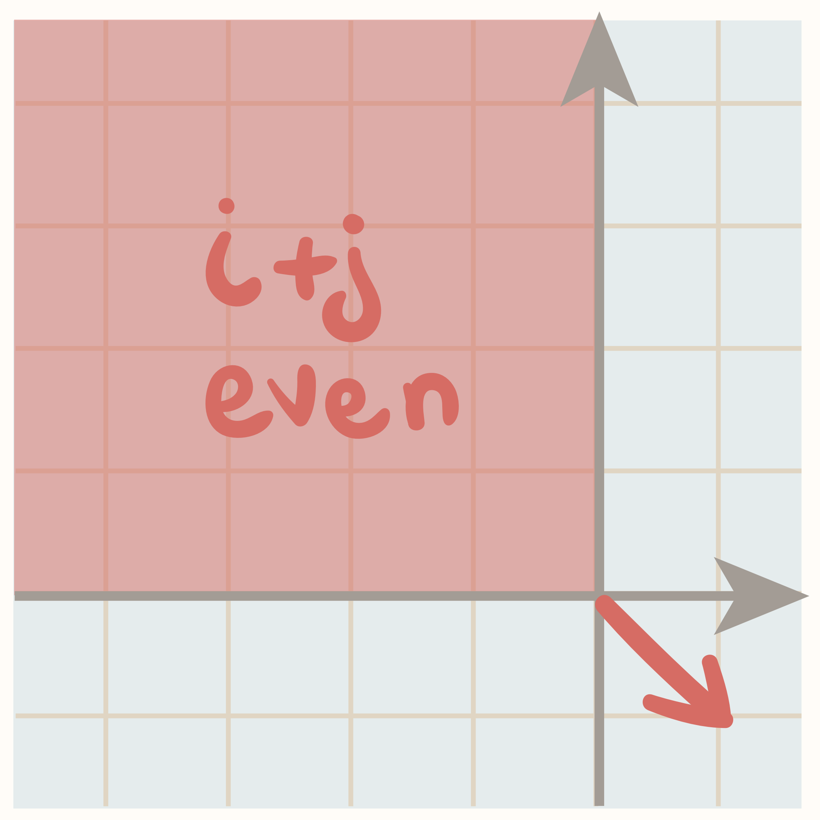


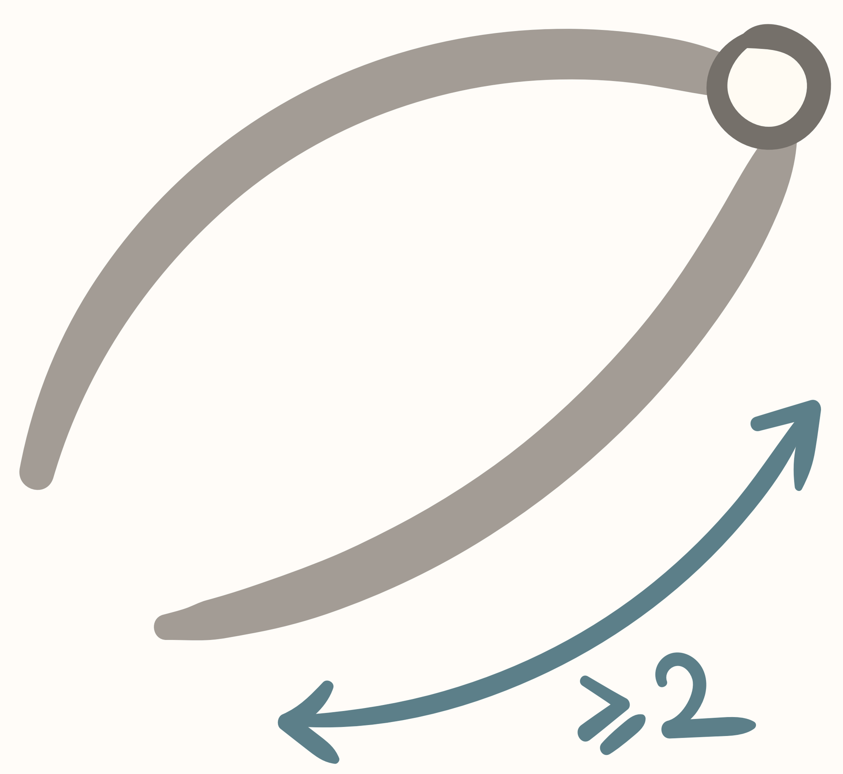


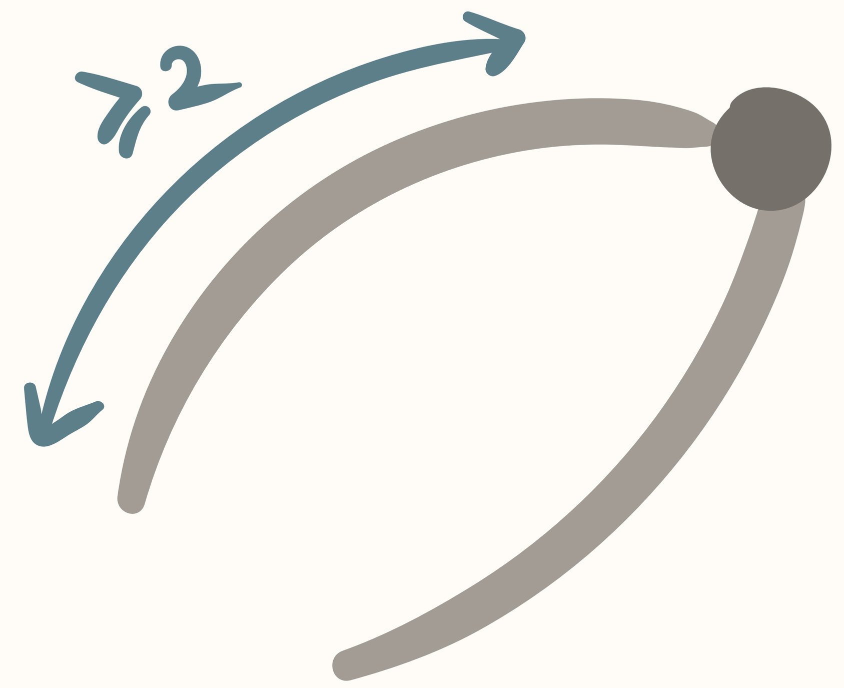
Polyhedral orientations via KMSW



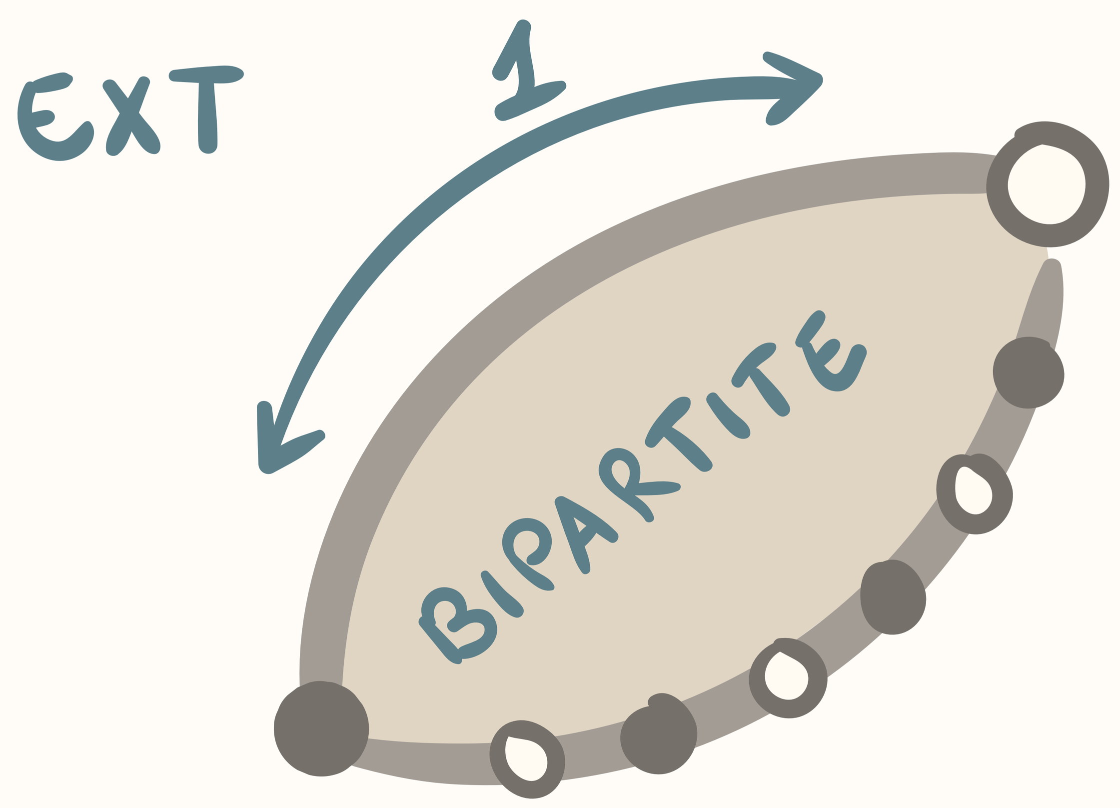


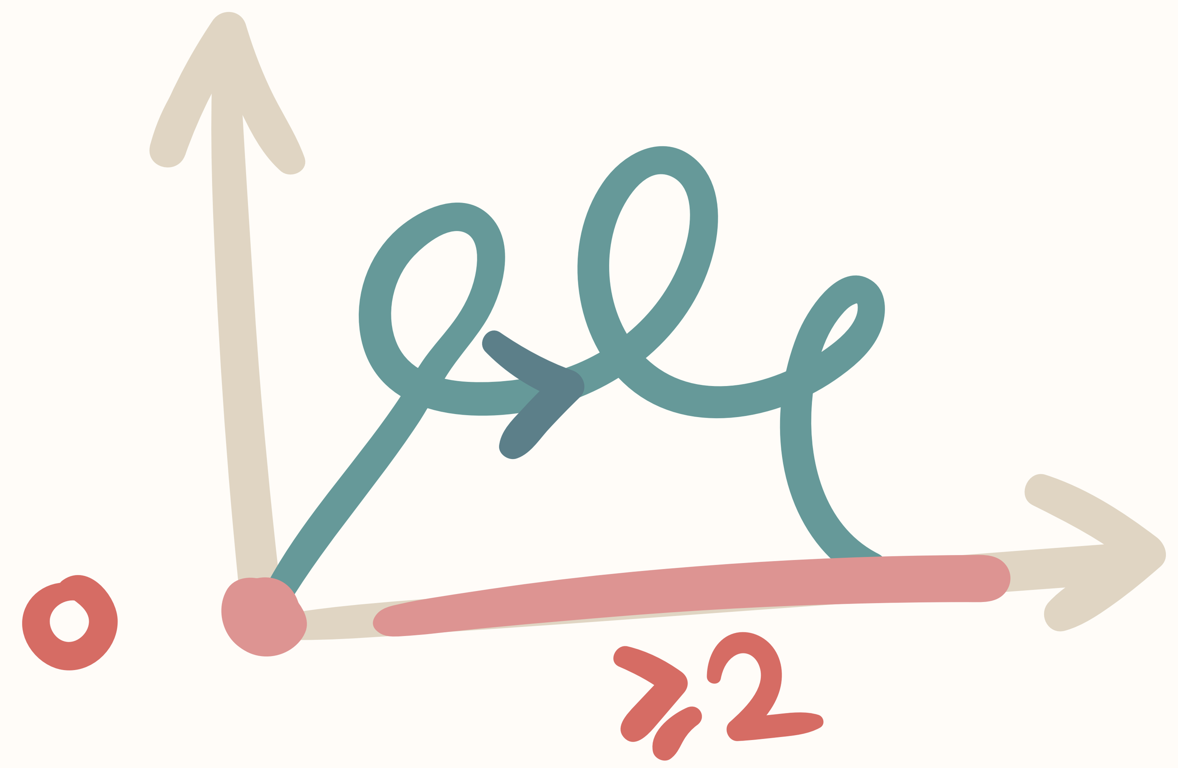


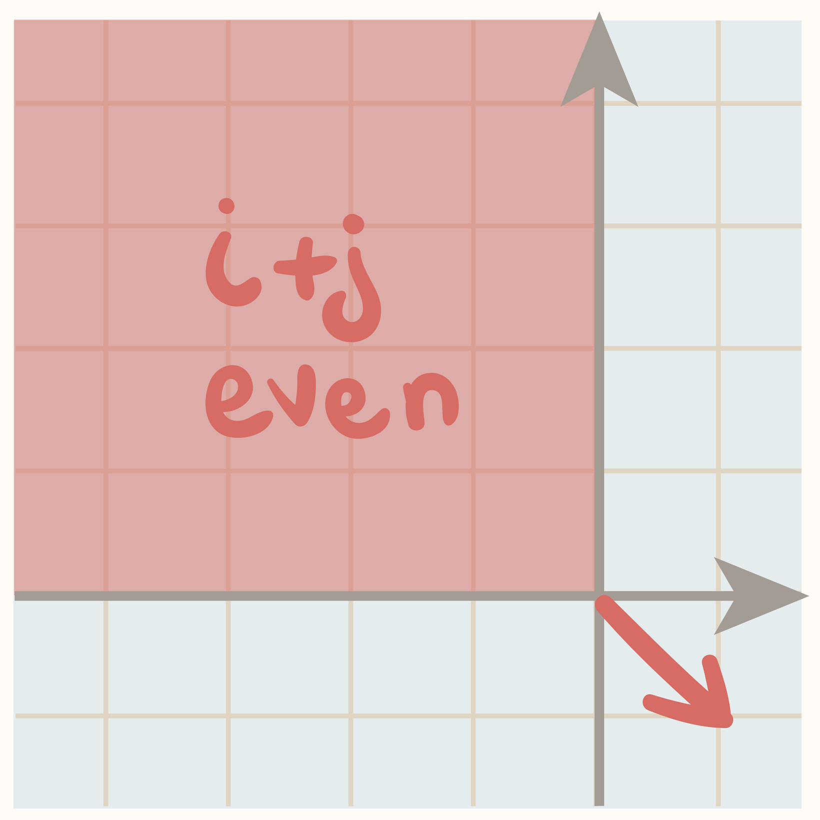


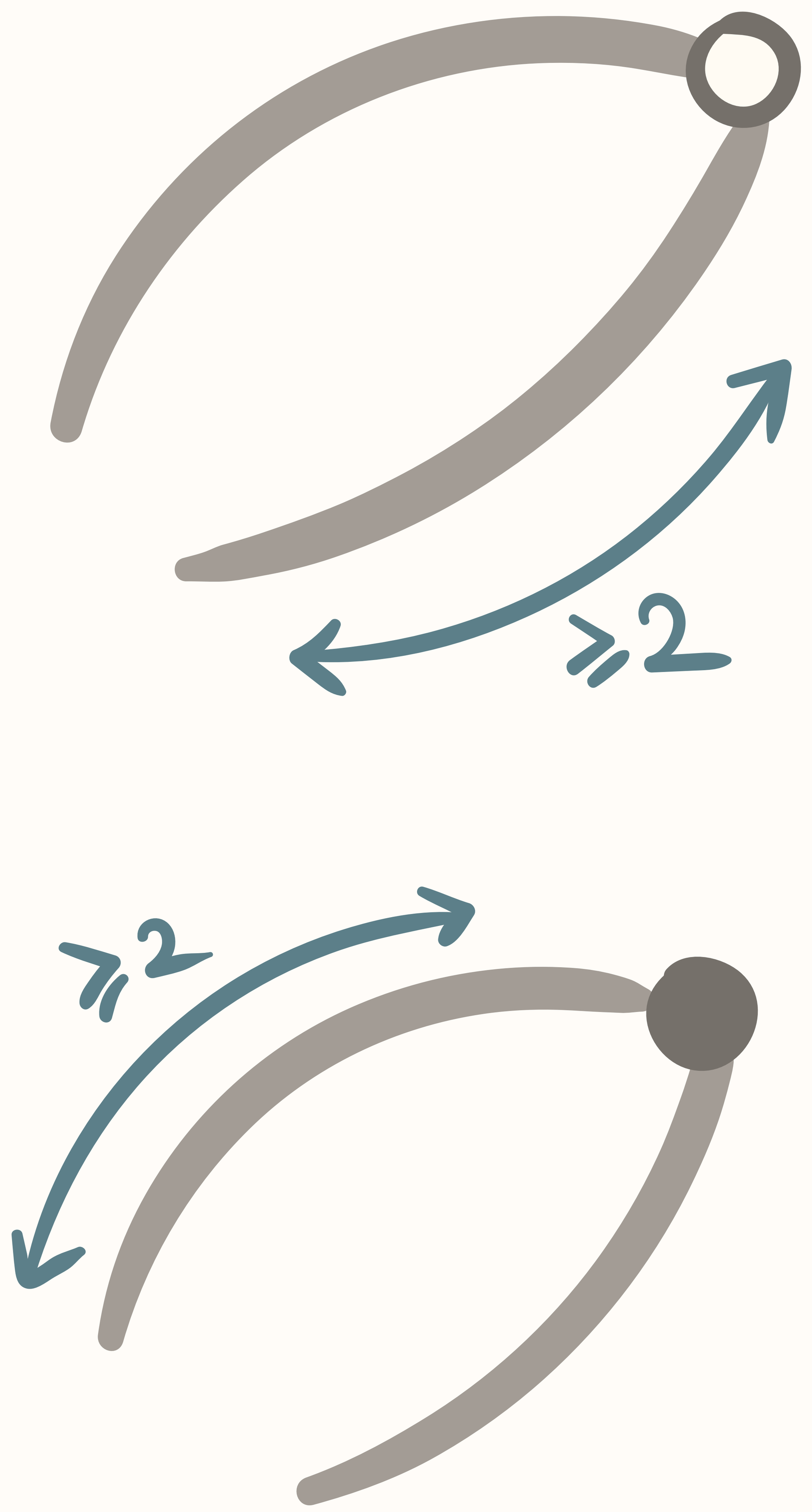


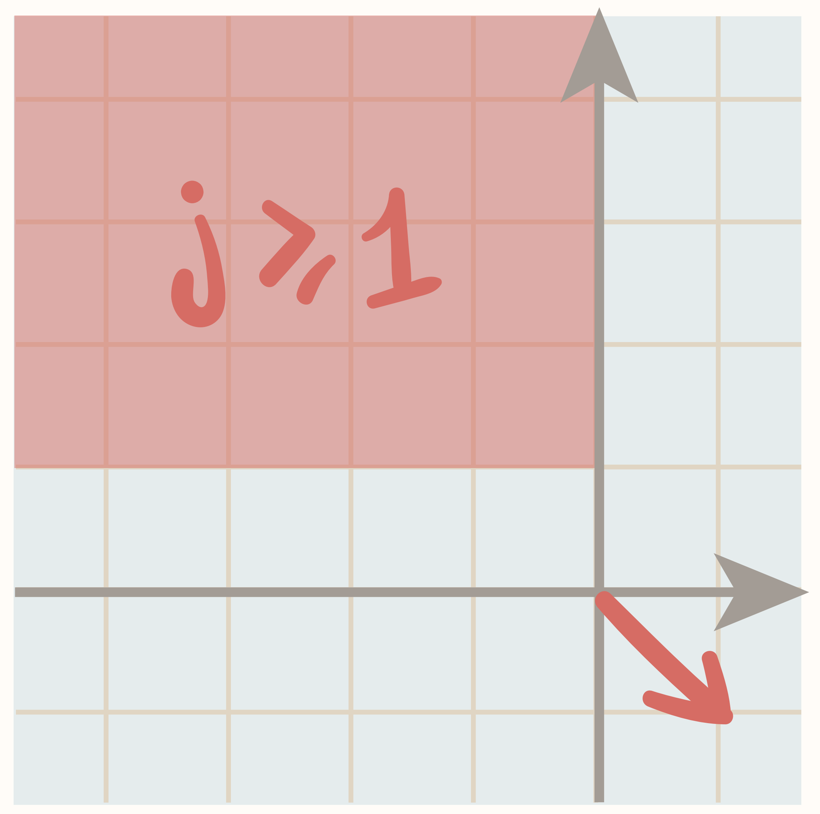
Polyhedral orientations via KMSW





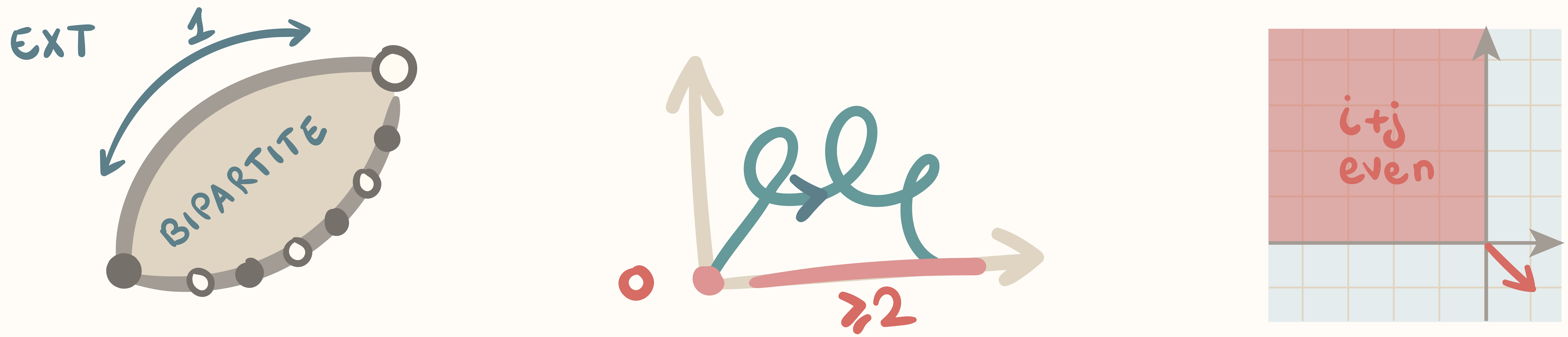


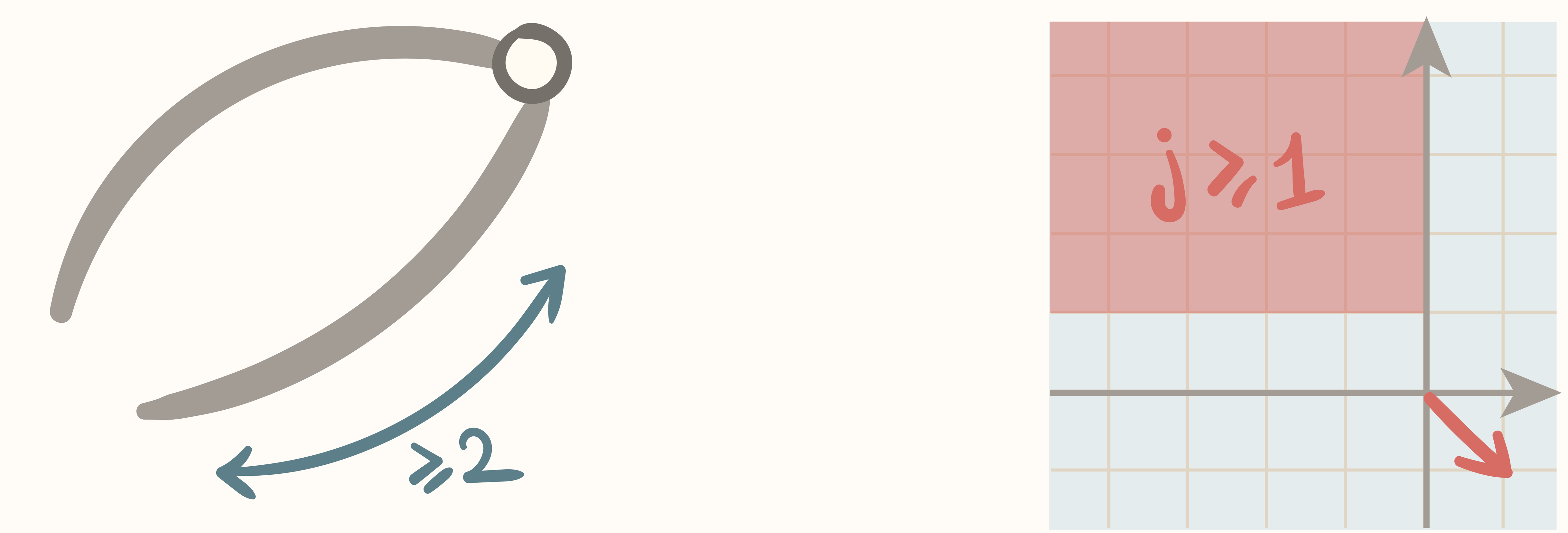




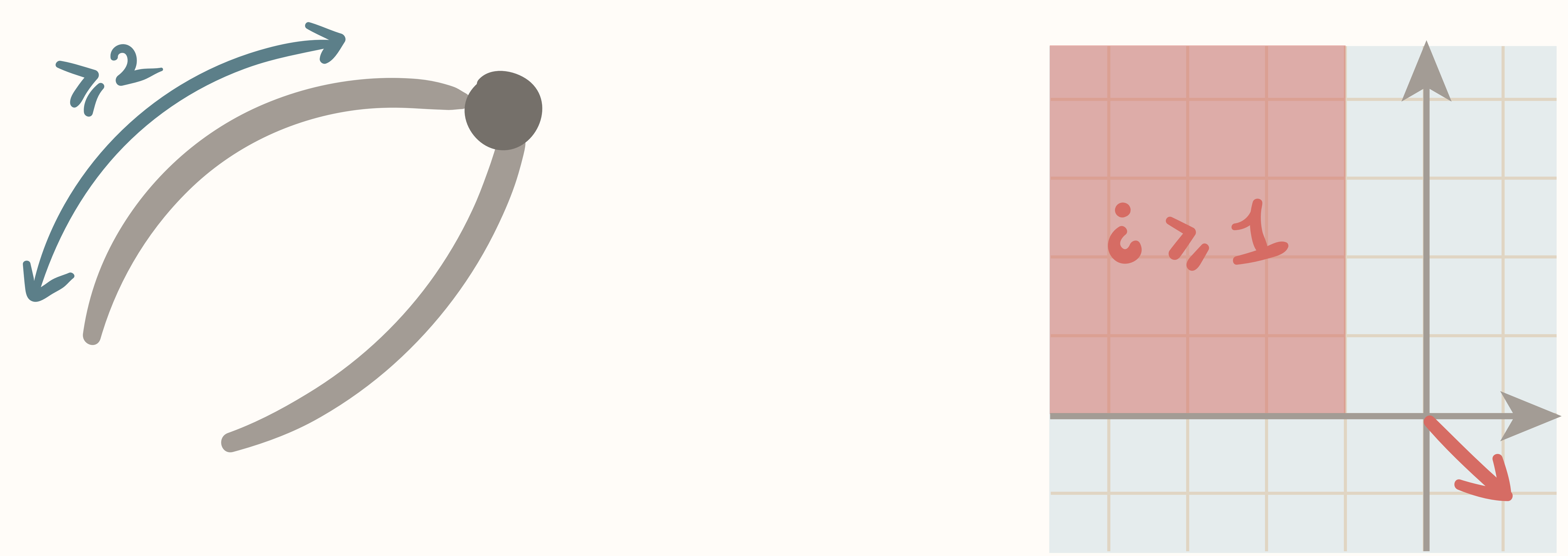
when x,y are even

Polyhedral orientations via KMSW



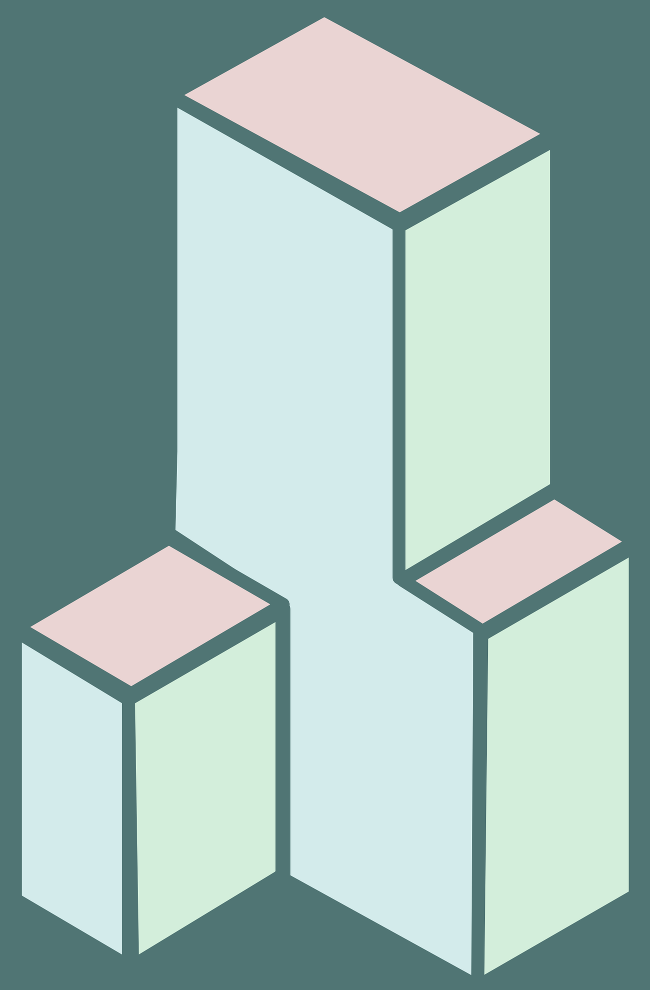


when x,y are even

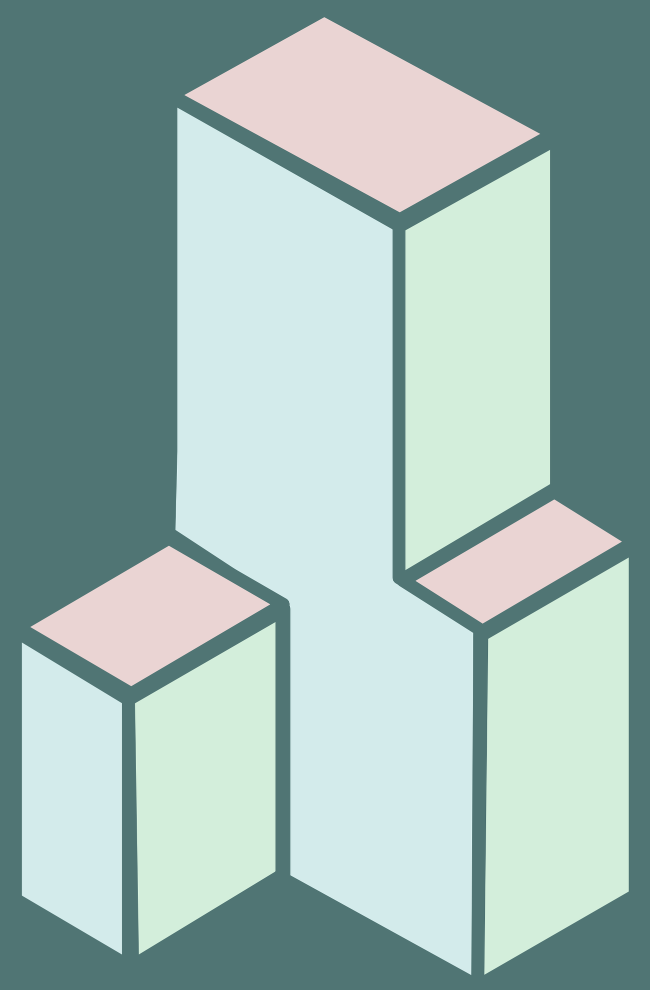


when x,y are odd

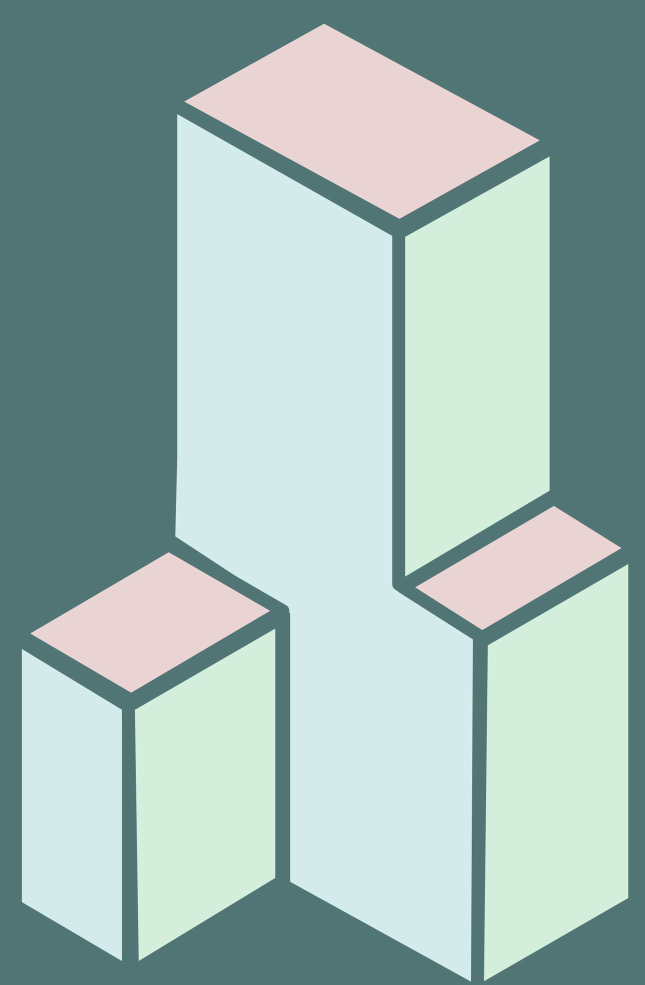
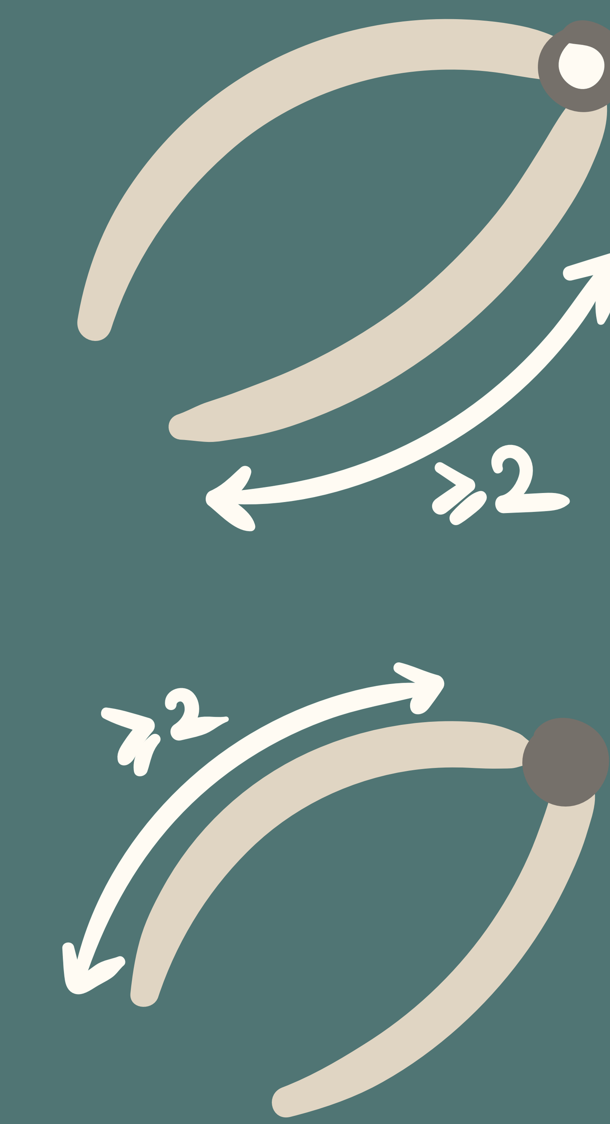
Bijections summary

Model	Combinatorial type	Bipolar orient./Tandem walk
 *n flats*		

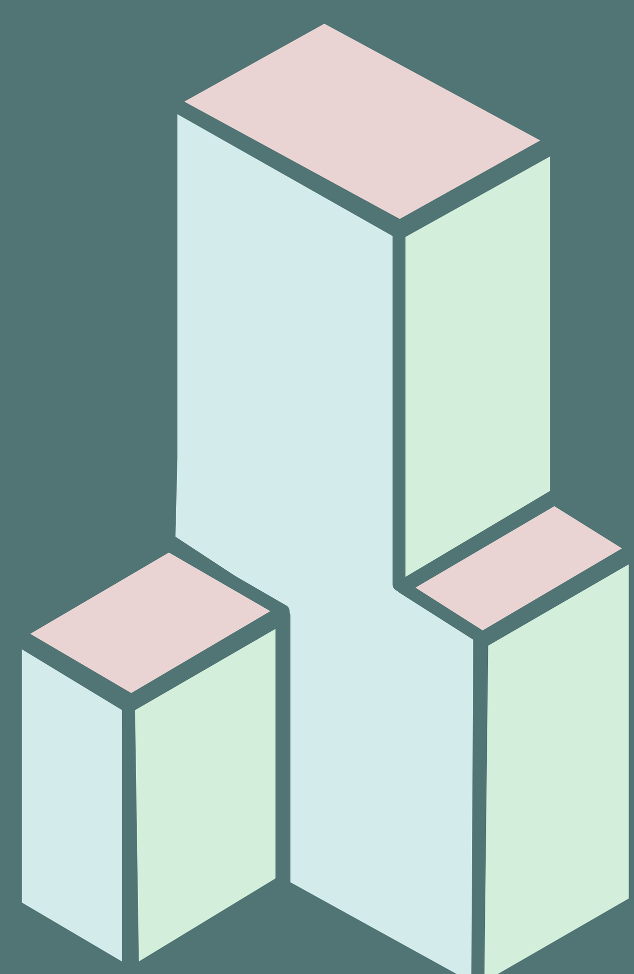
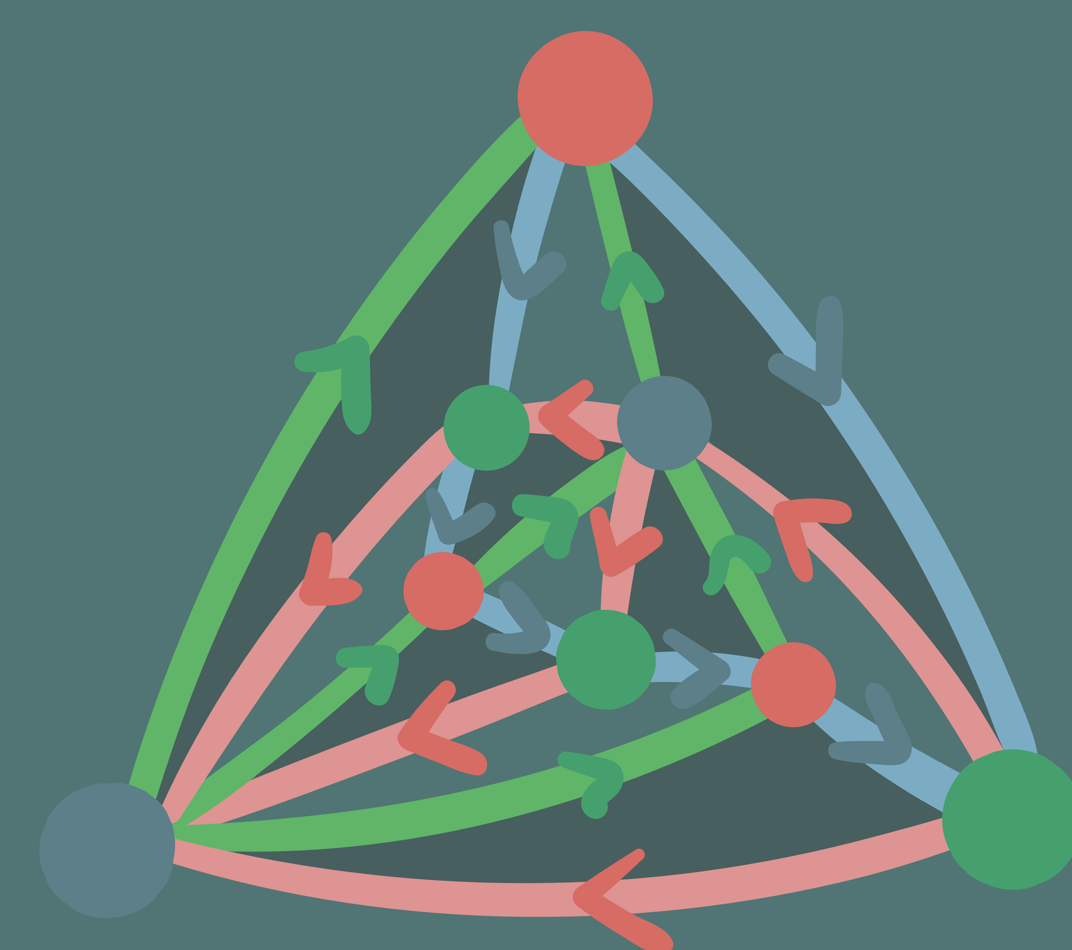
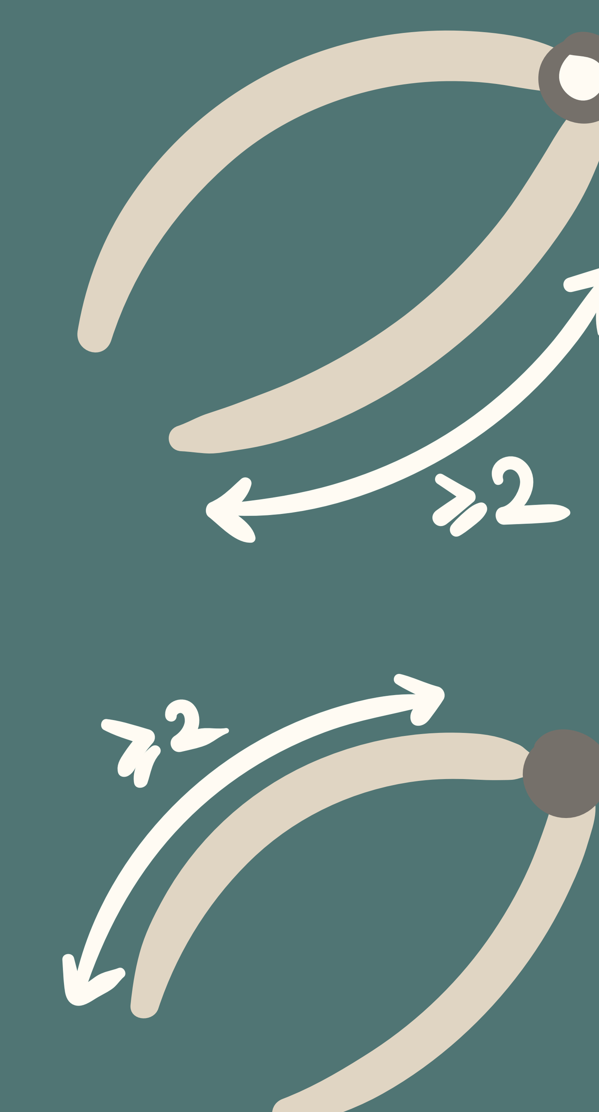
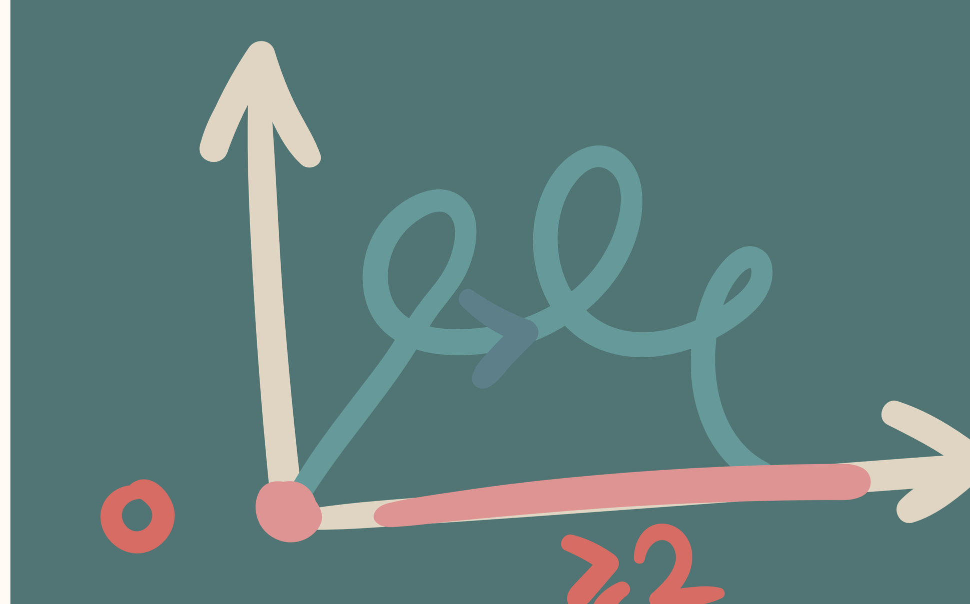
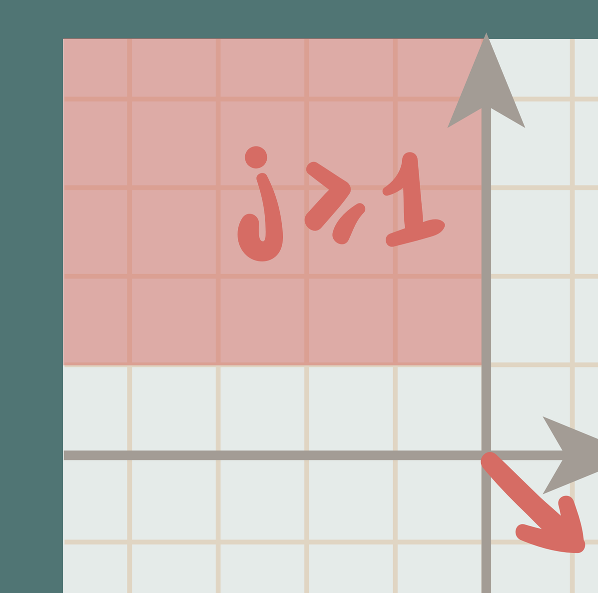
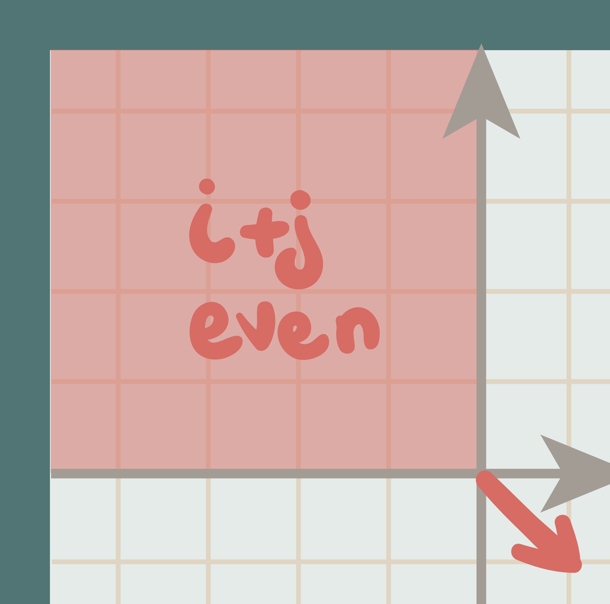
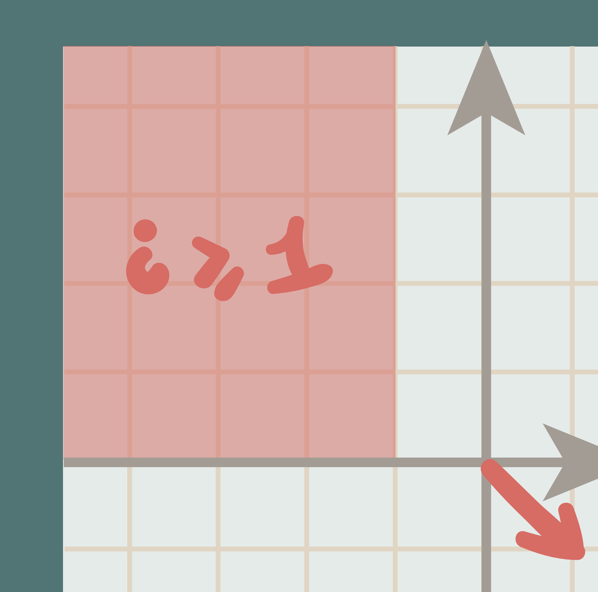
Bijections summary

Model	*Combinatorial type*	*Bipolar orient./Tandem walk*
 *n flats*	 *n inner vertices*	

Bijections summary

Model	*Combinatorial type*	*Bipolar orient./Tandem walk*
 *n flats*	 *n inner vertices*	  *n+1 edges*

Bijections summary

Model	Combinatorial type	Bipolar orient./Tandem walk
 *n flats*	 *n inner vertices*	  *n+1 edges*      

Summary

The KMSW bijection

1. Application to three map models

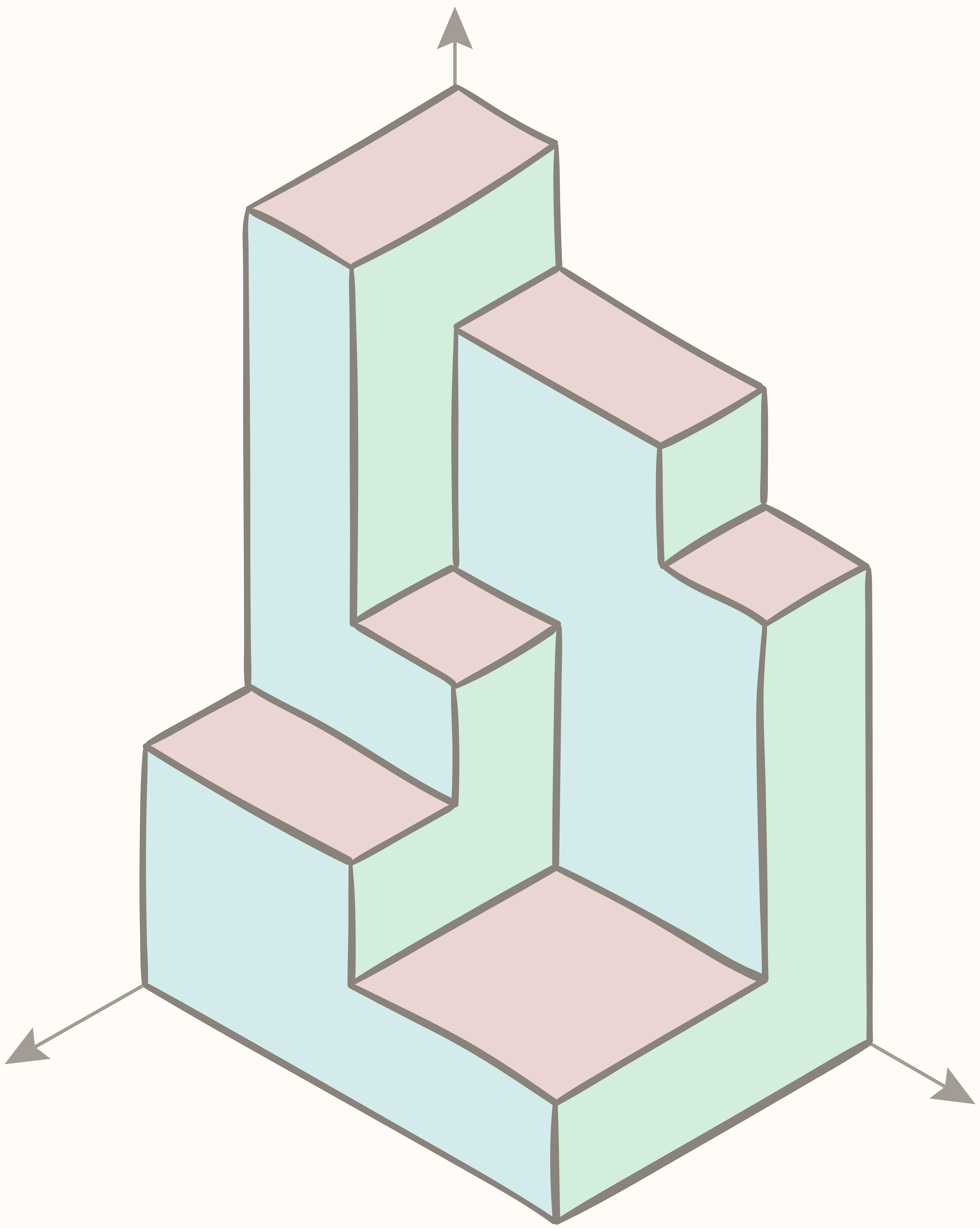
a. Plane bipolar posets
b. Plane bipolar posets by vertices
c. Transversal structures
d. Asymtotics

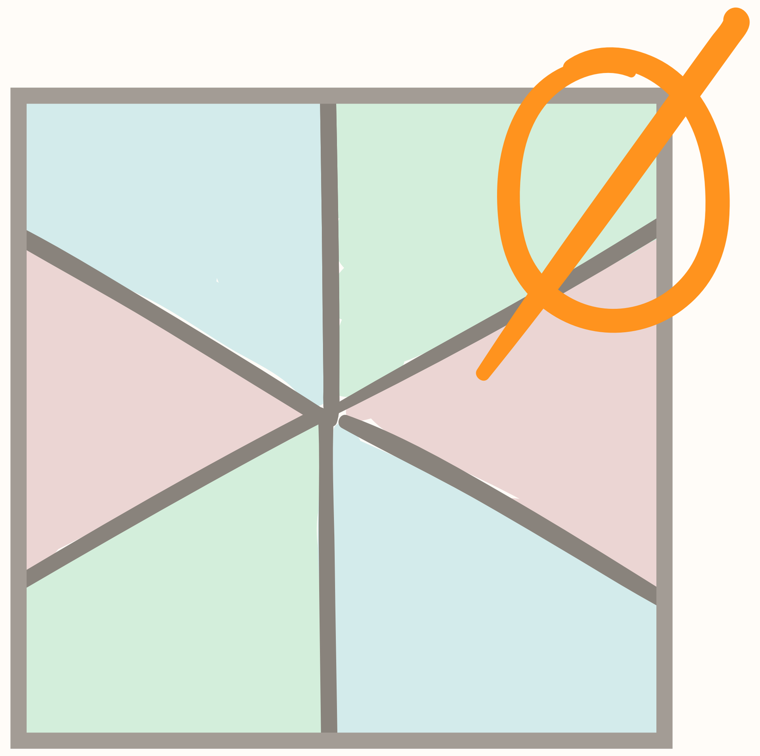
2. Interlude : plane permutations

3. Application to corner polyhera

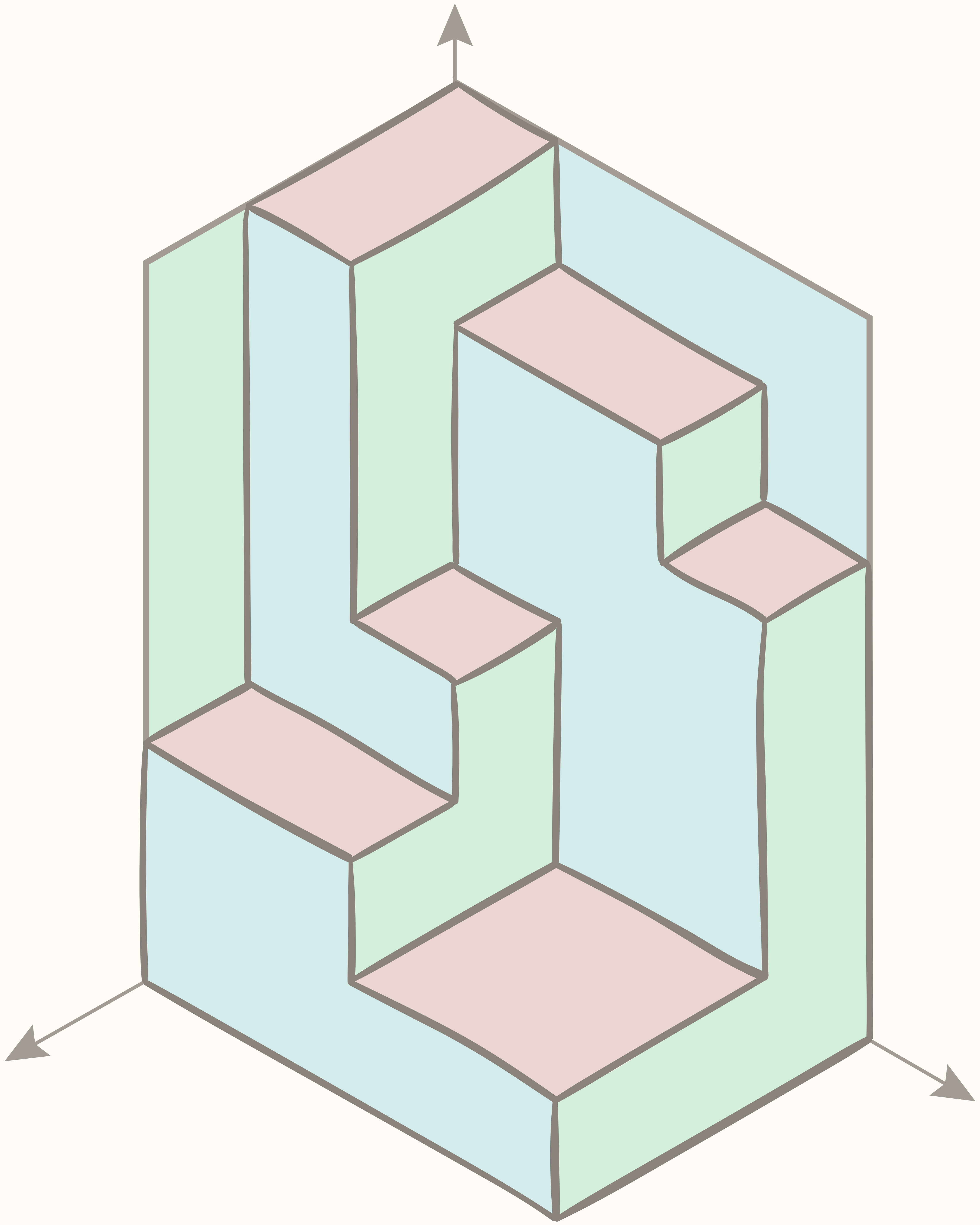
a. Via polyheral orientations
b. Via Schnyder colorings
c. Asymtotics

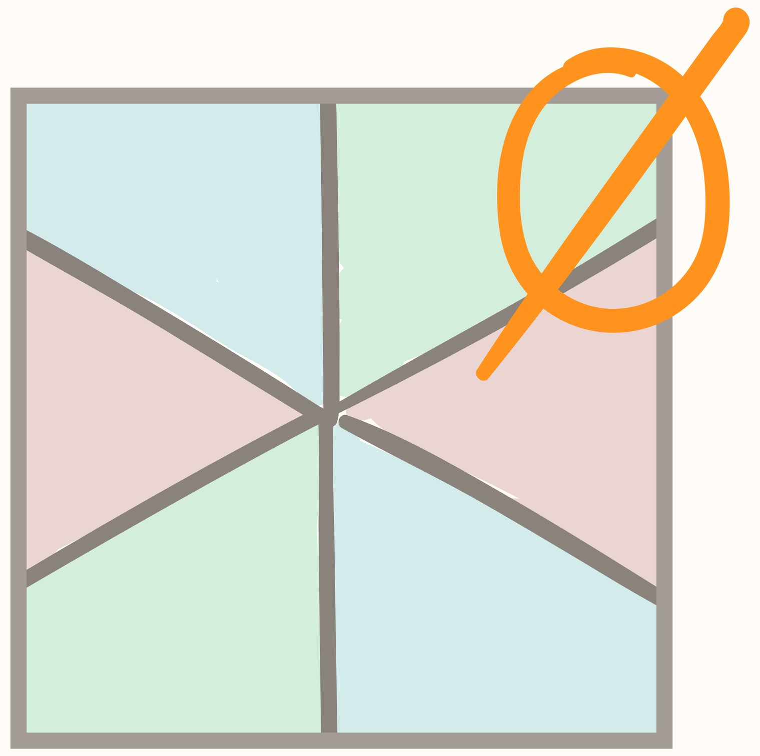
Rigid corner polyhedra



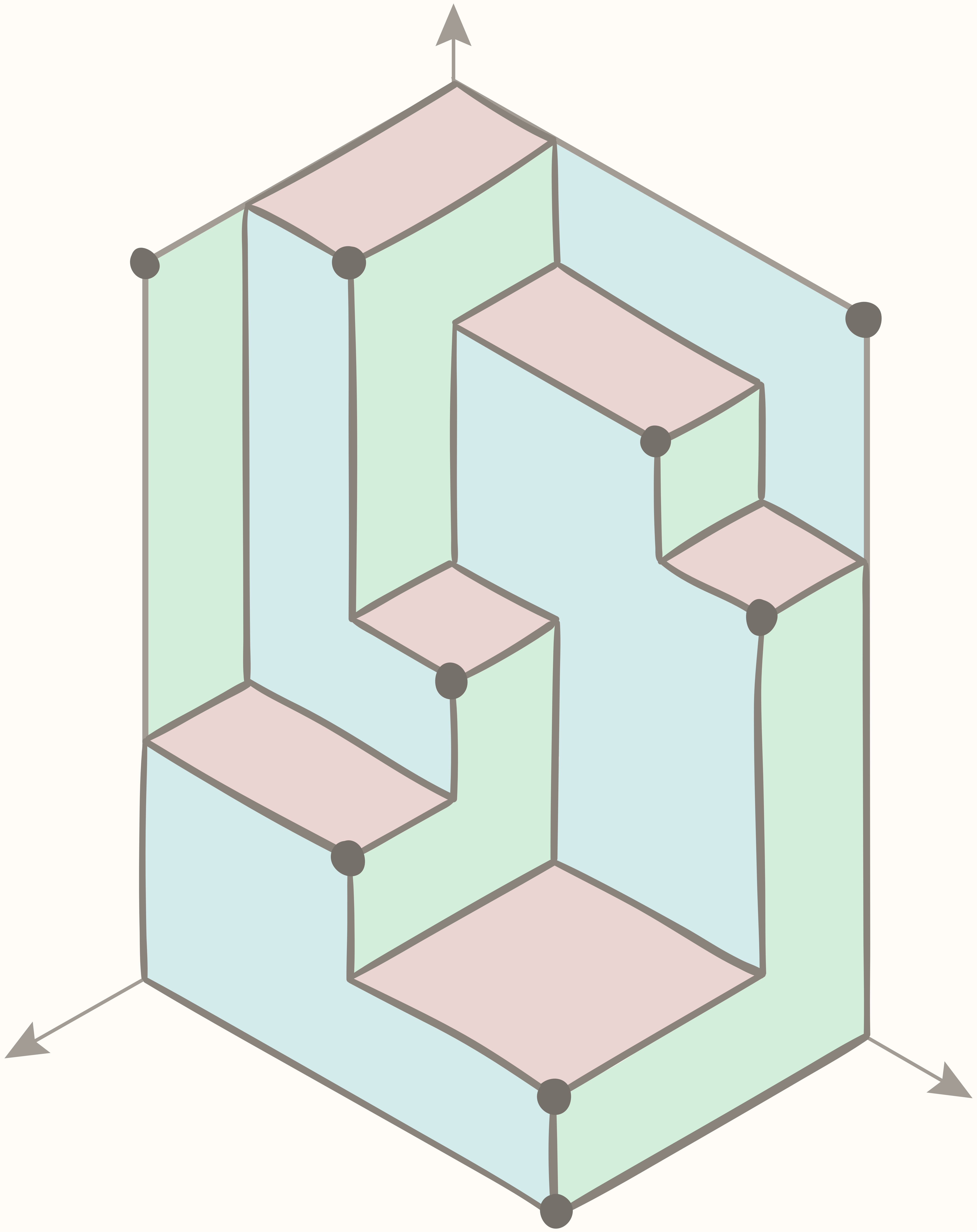


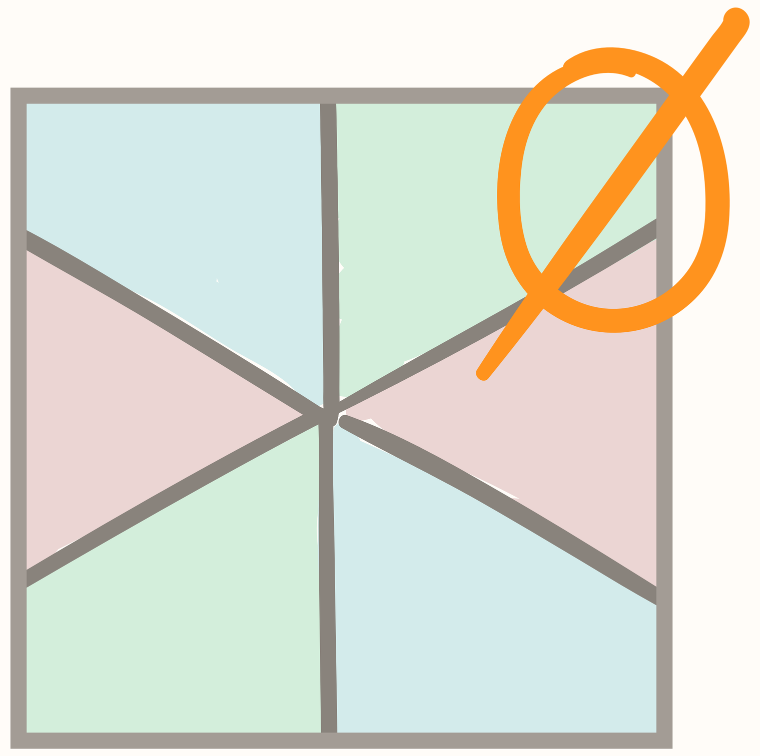
Rigid corner polyhedra



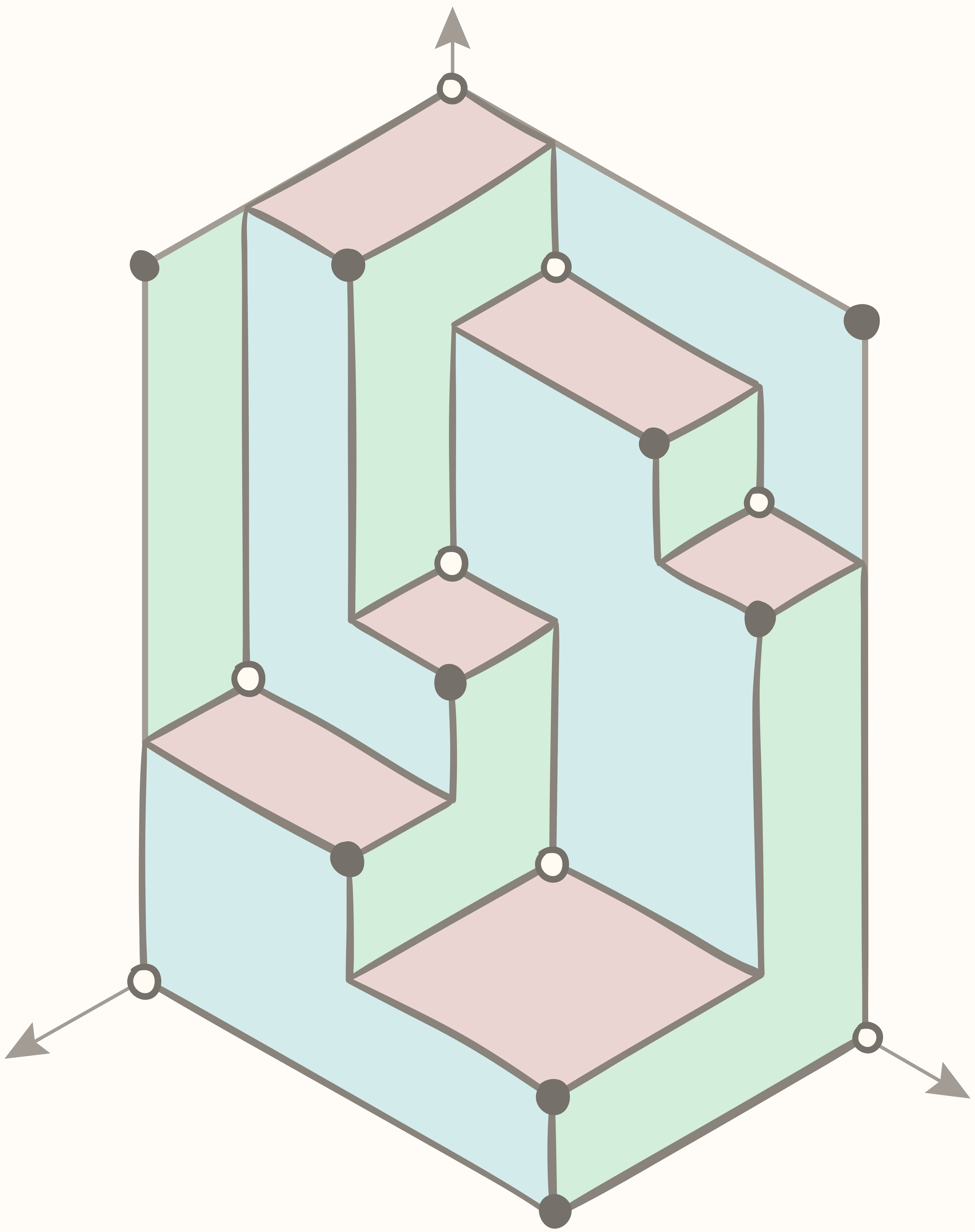


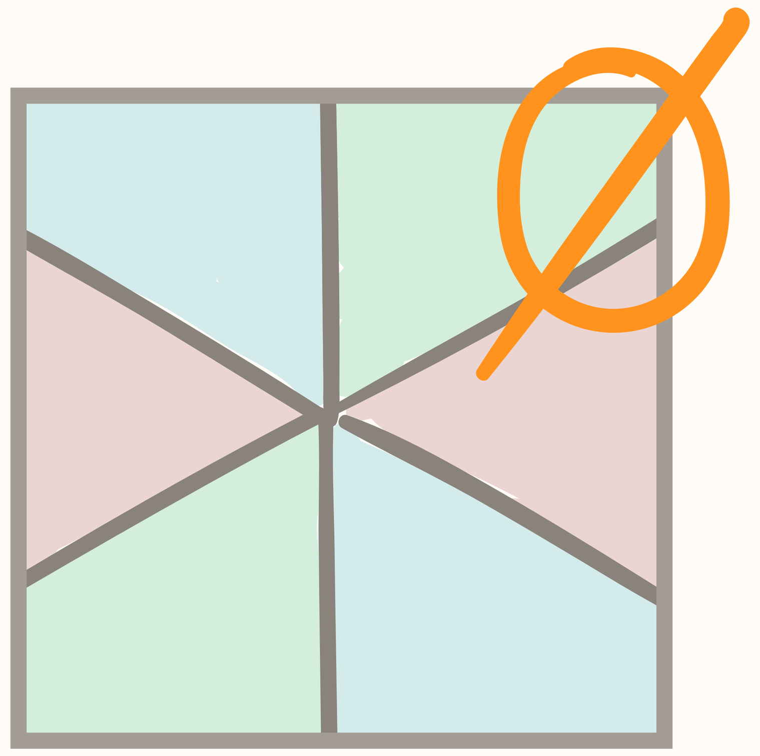
Rigid corner polyhedra



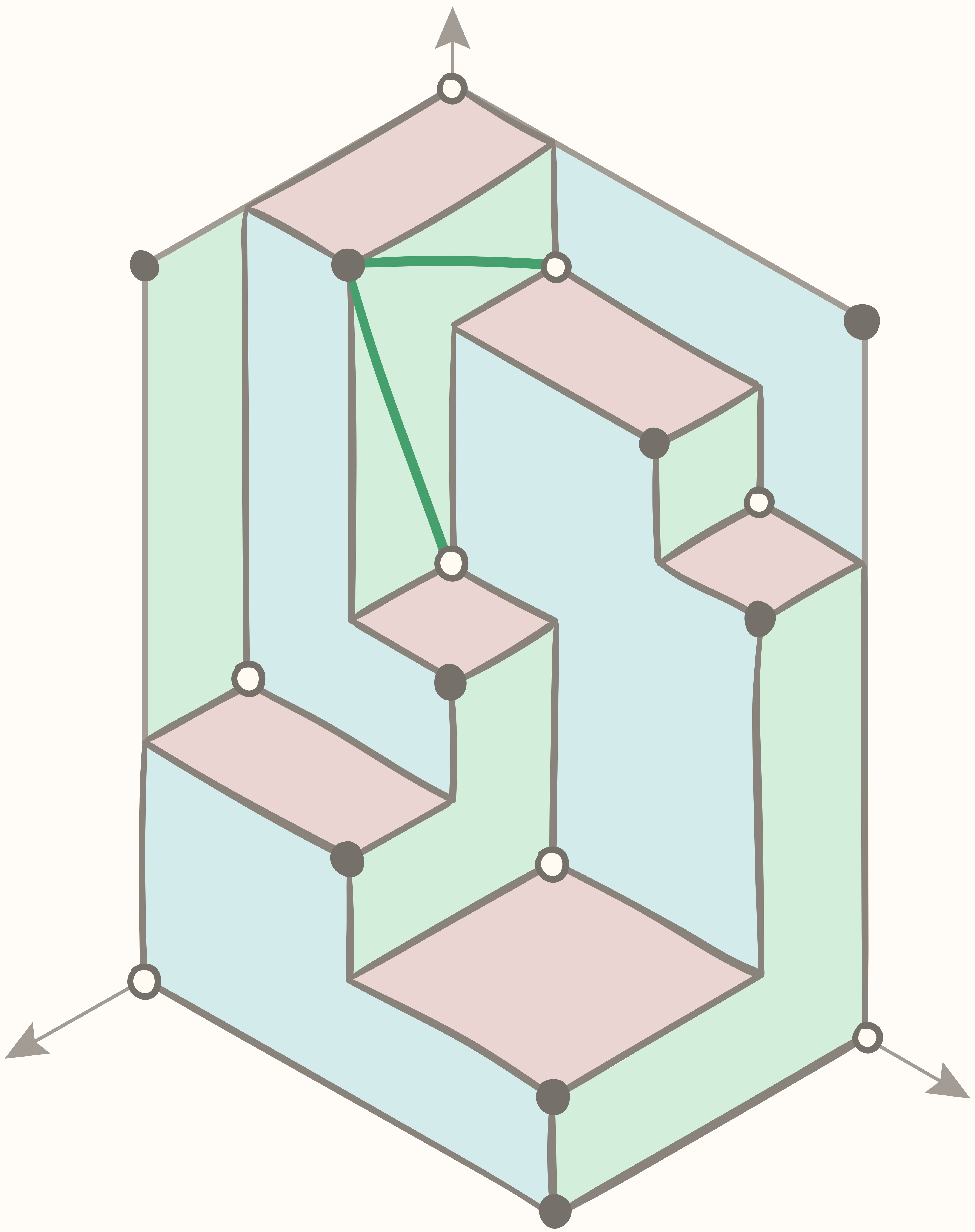


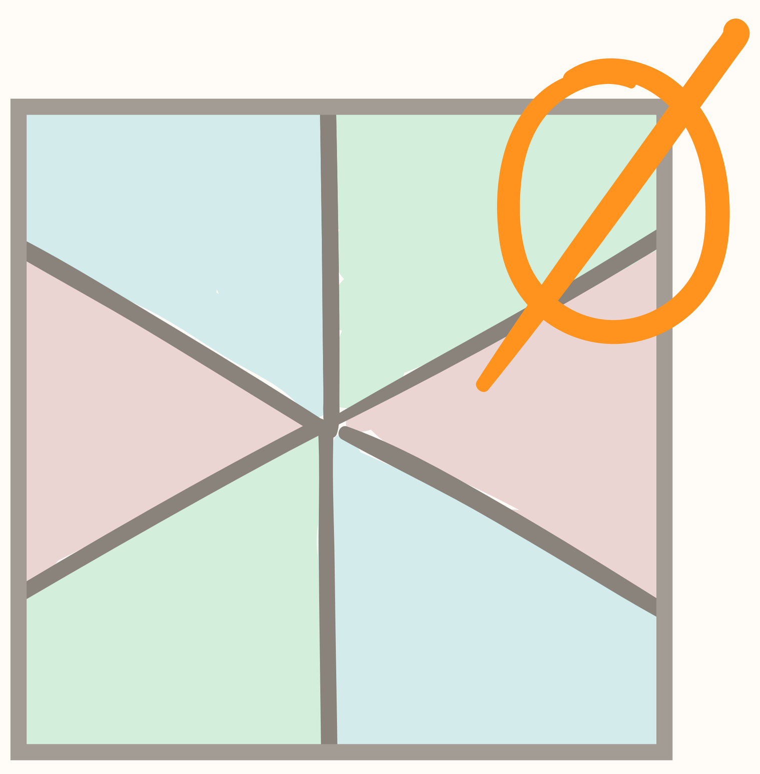
Rigid corner polyhedra



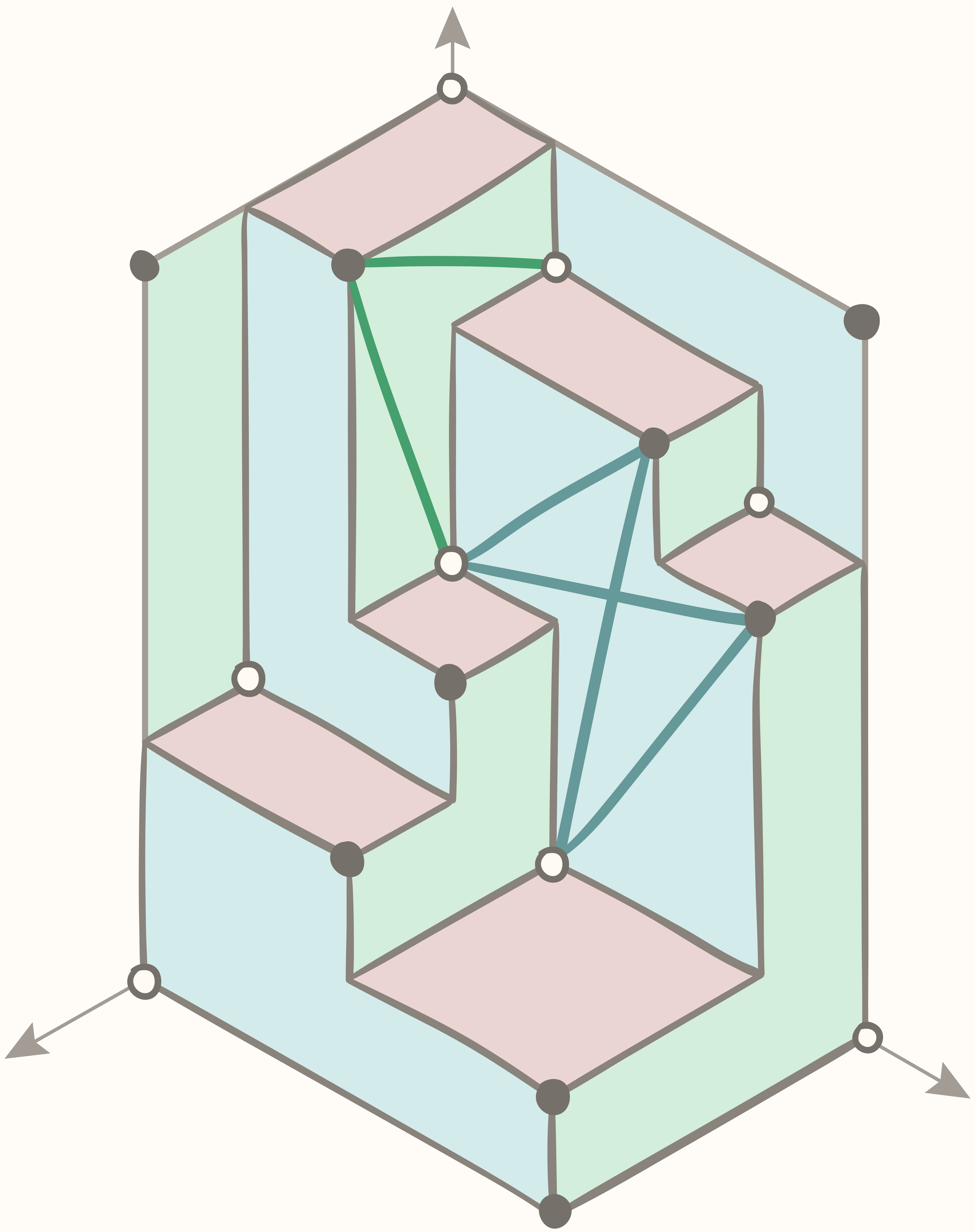


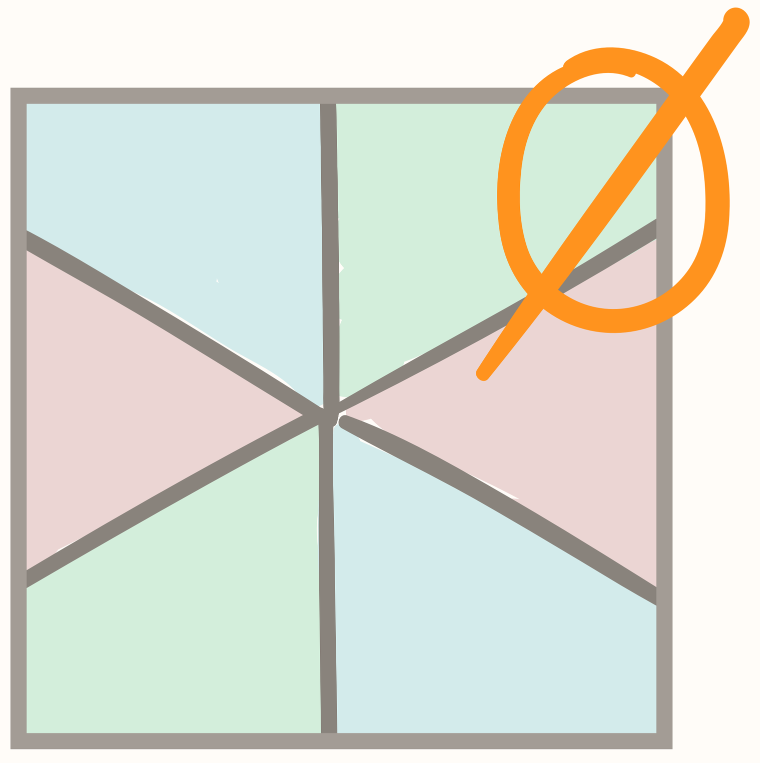
Rigid corner polyhedra



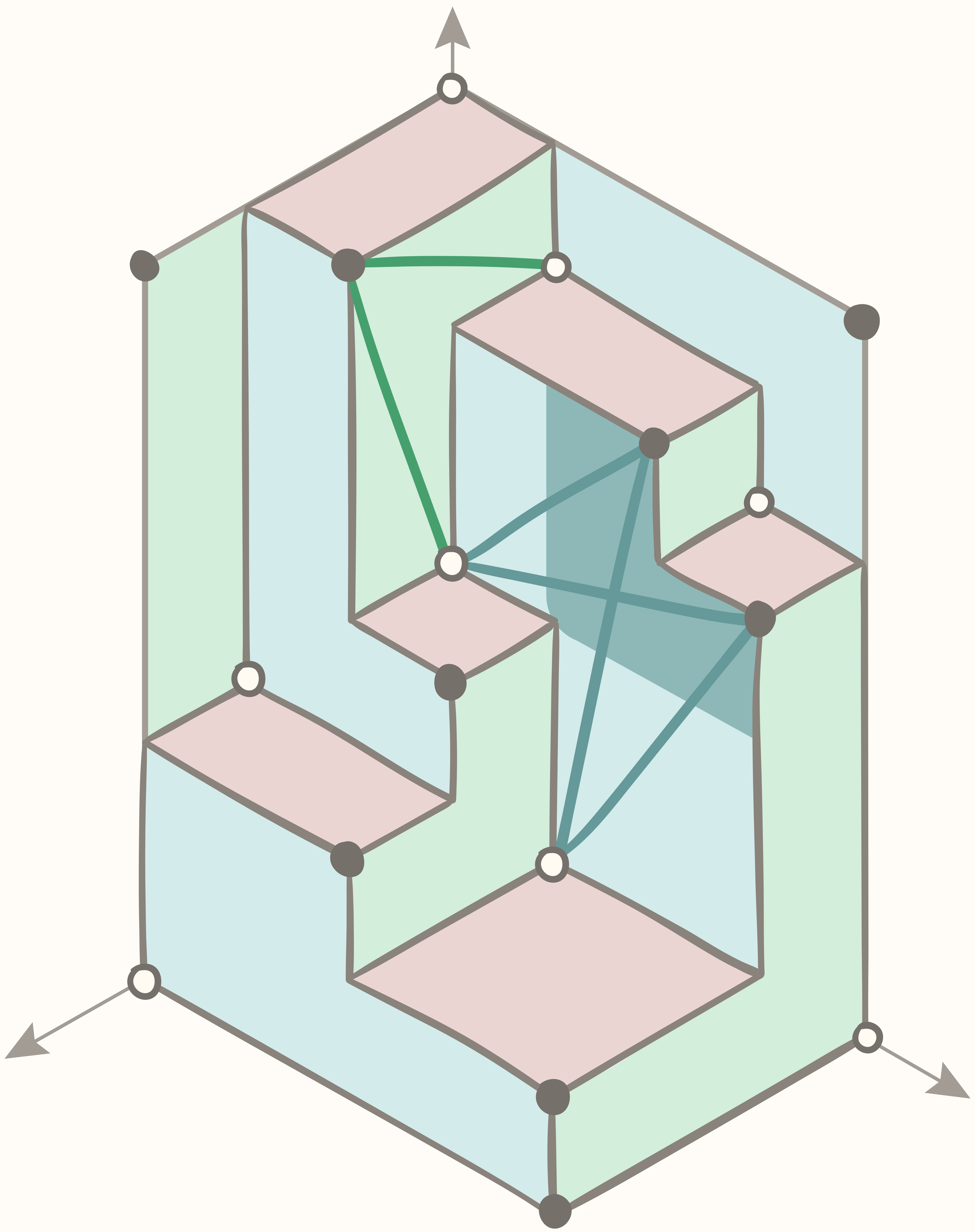


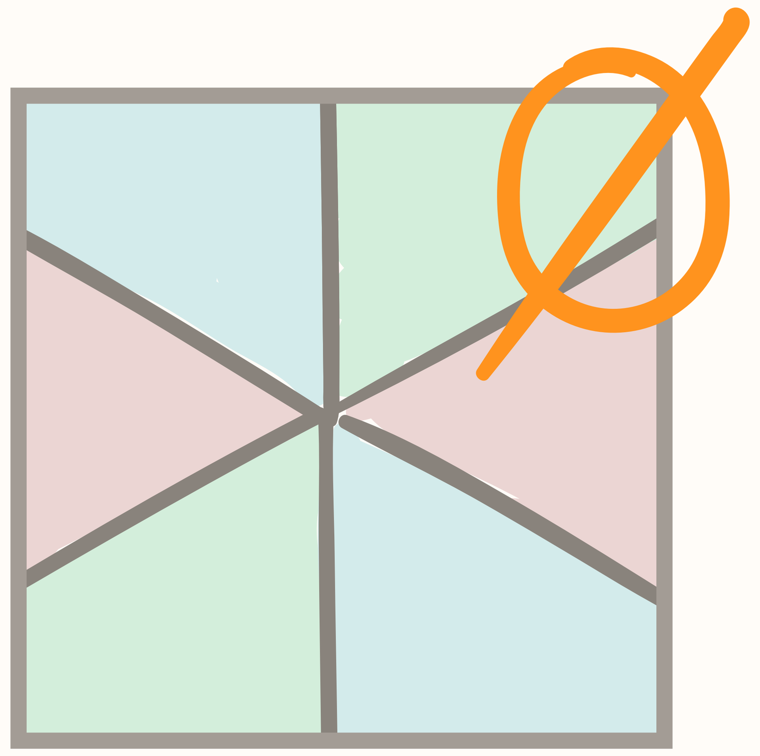
Rigid corner polyhedra





Rigid corner polyhedra





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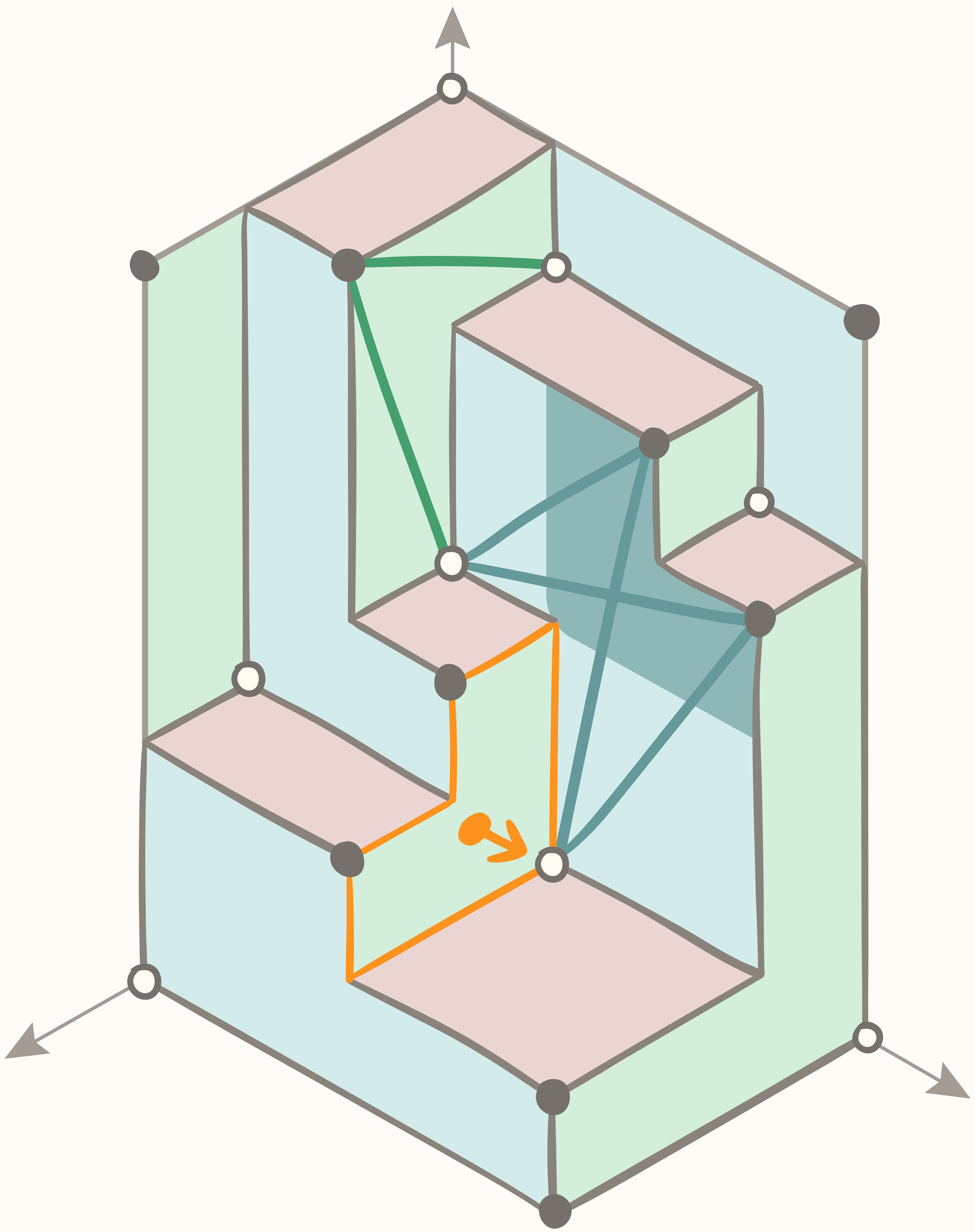

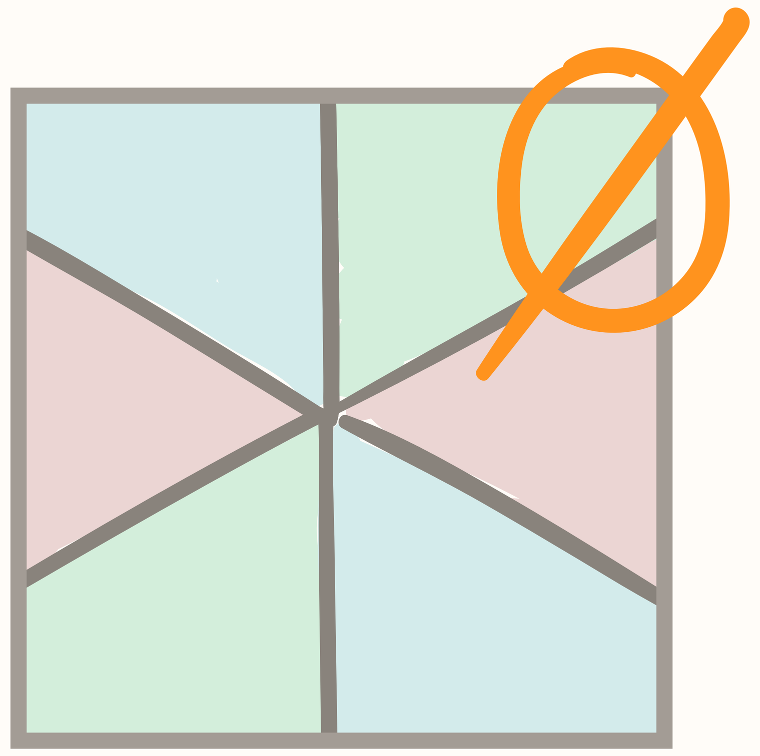

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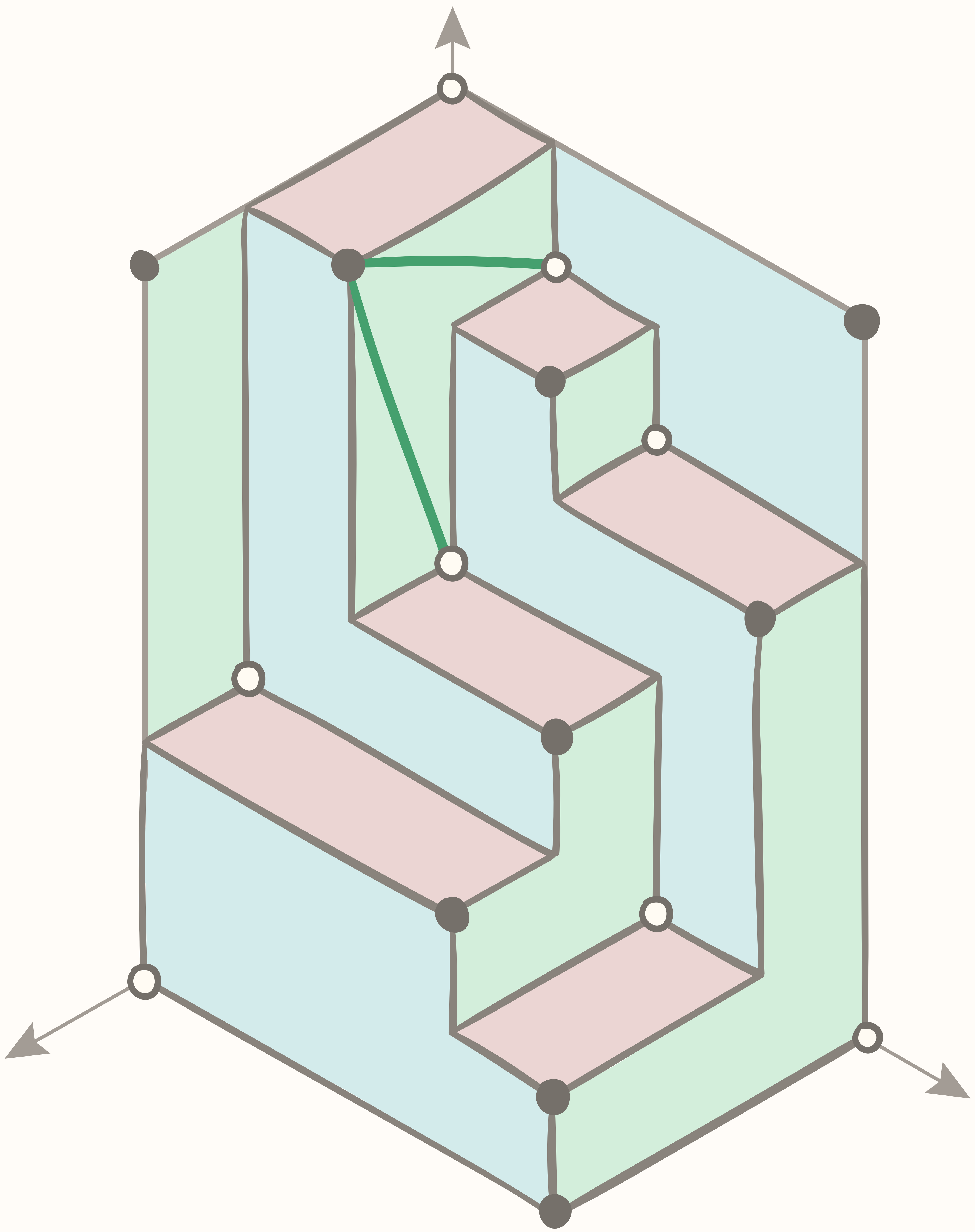

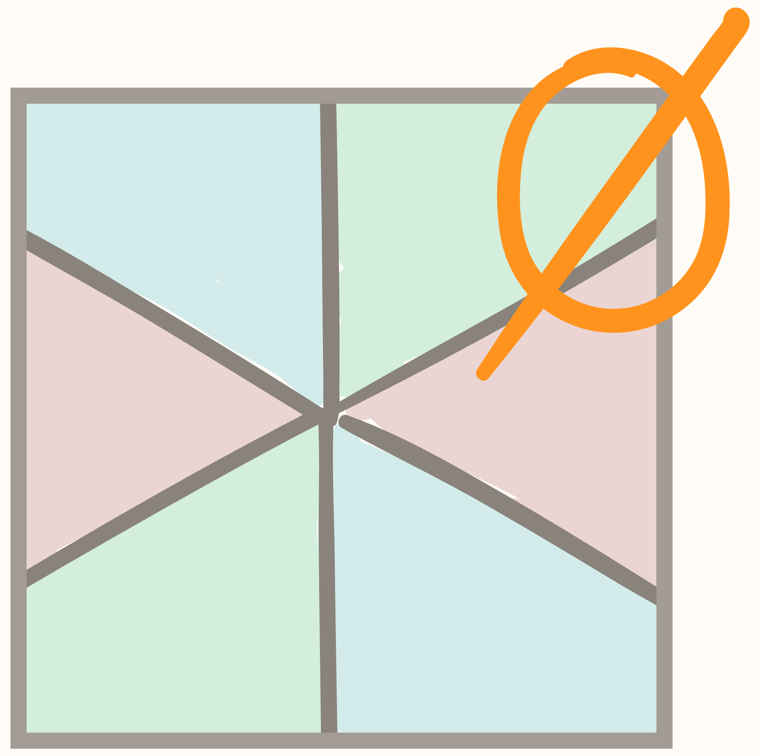

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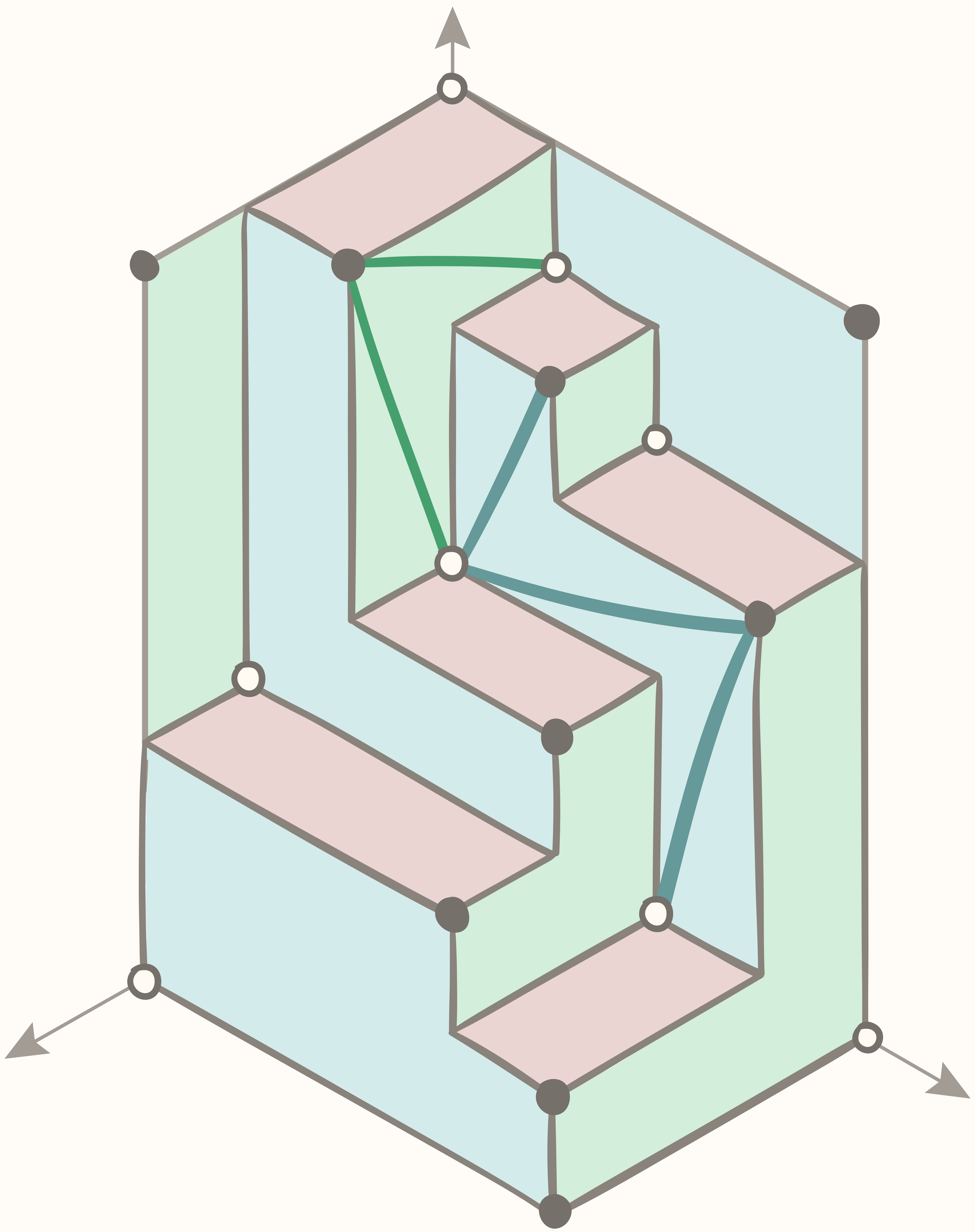

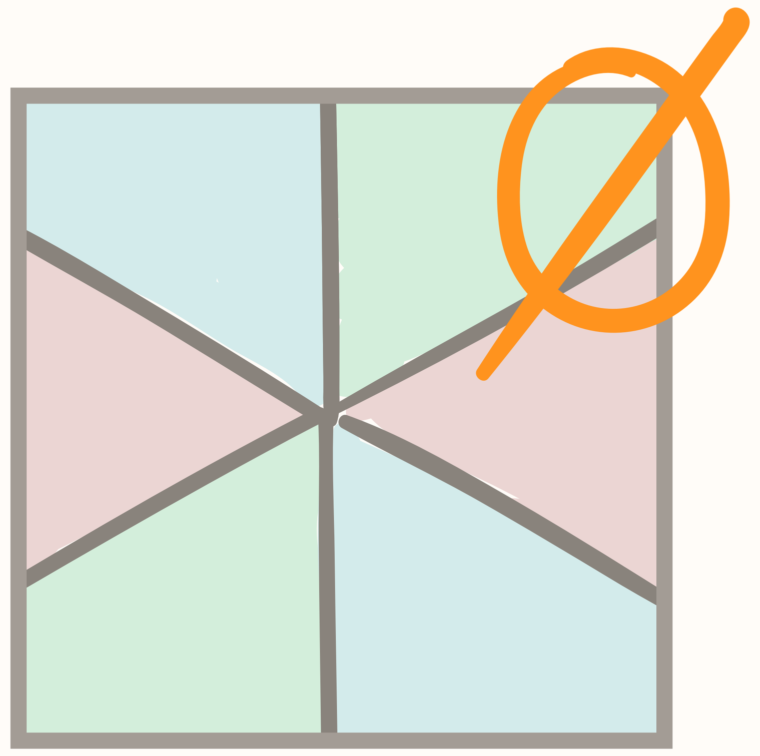

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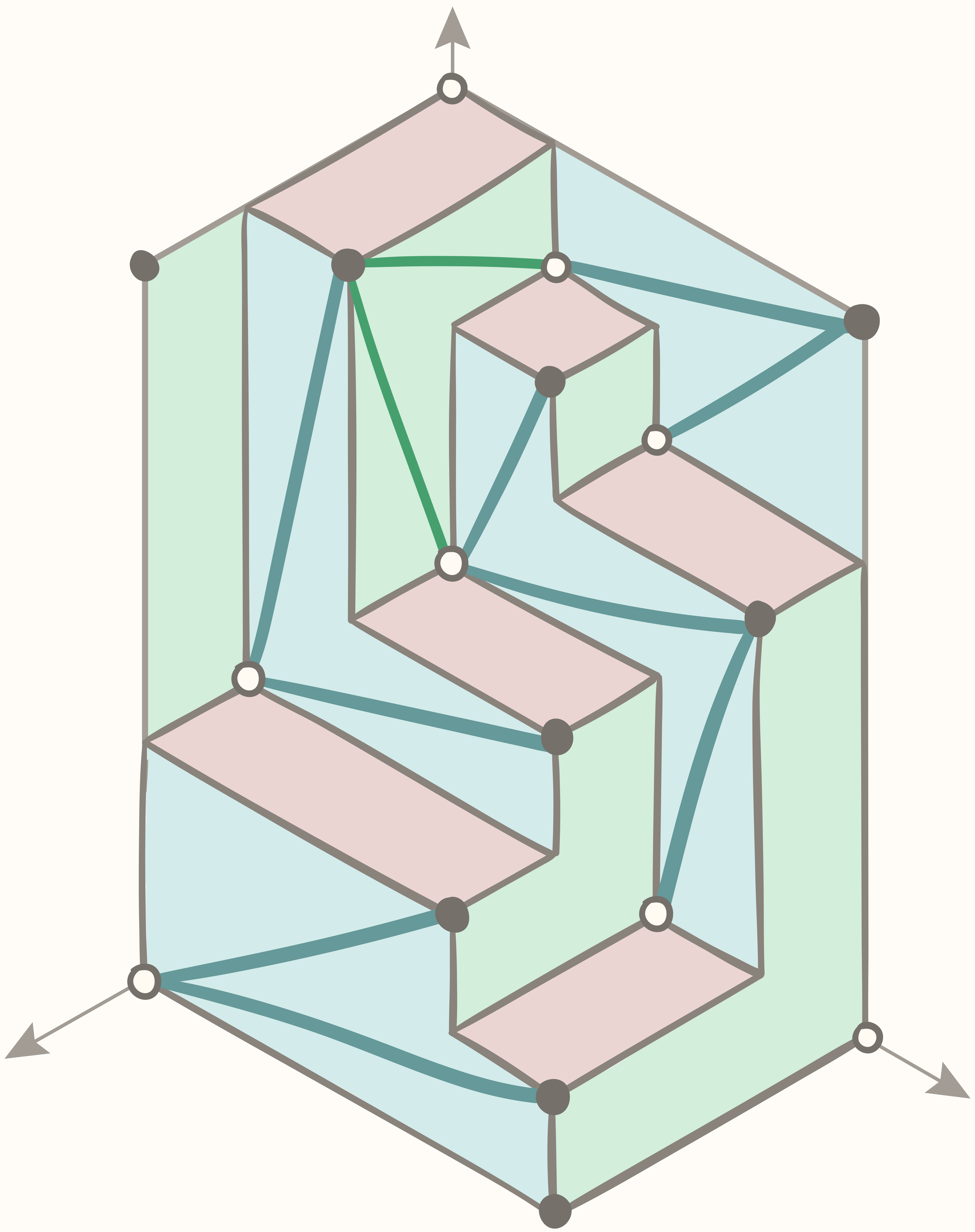

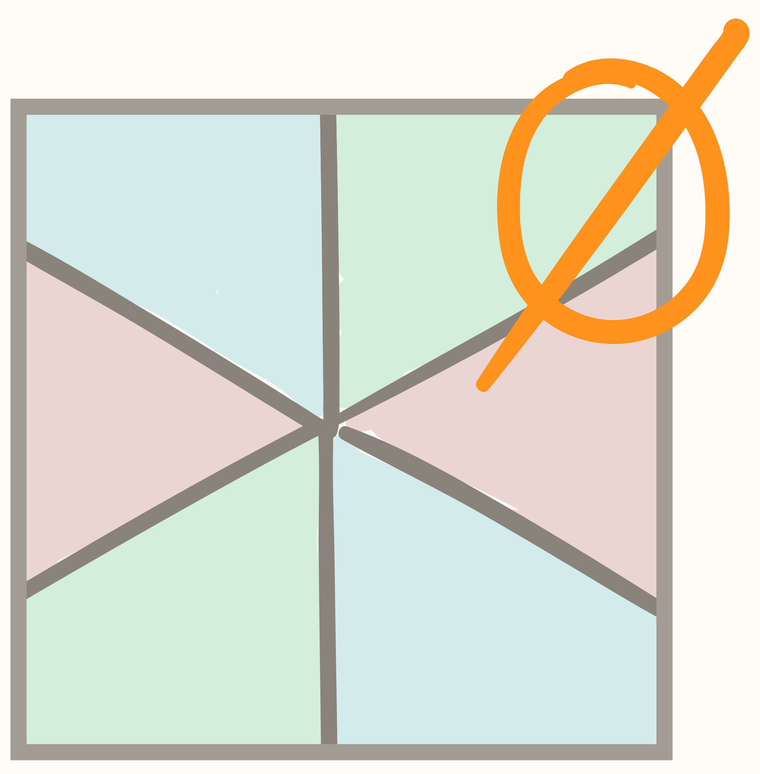

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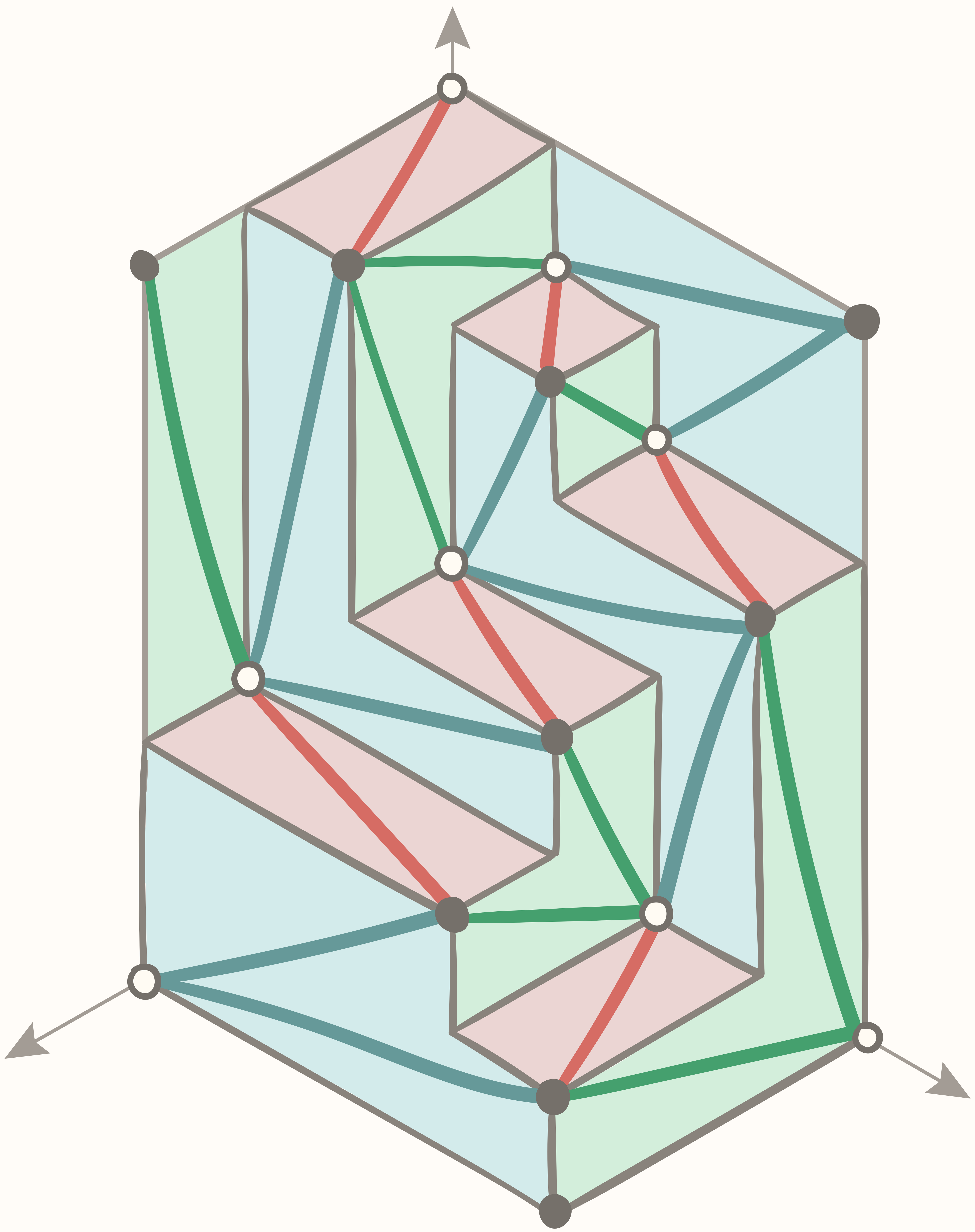

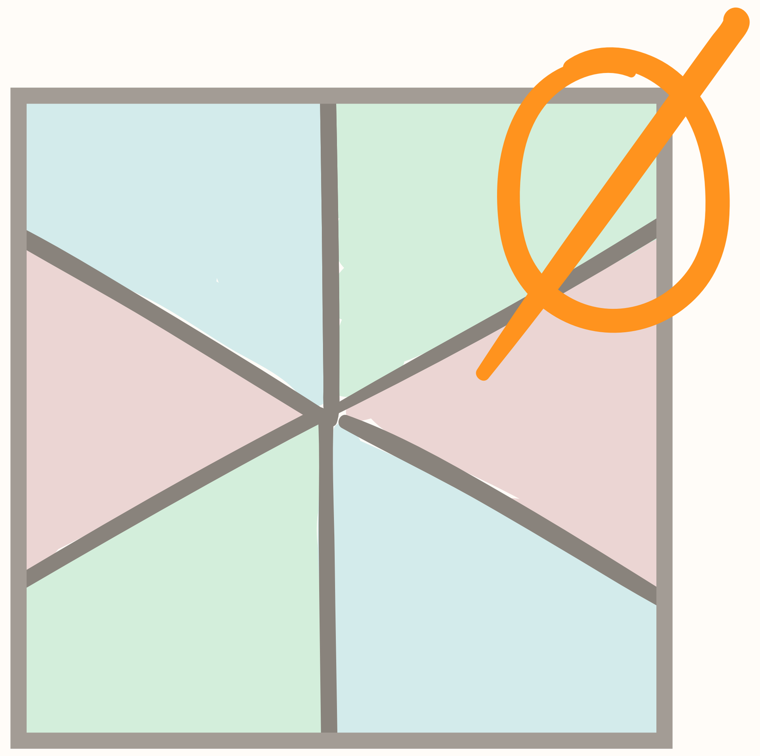

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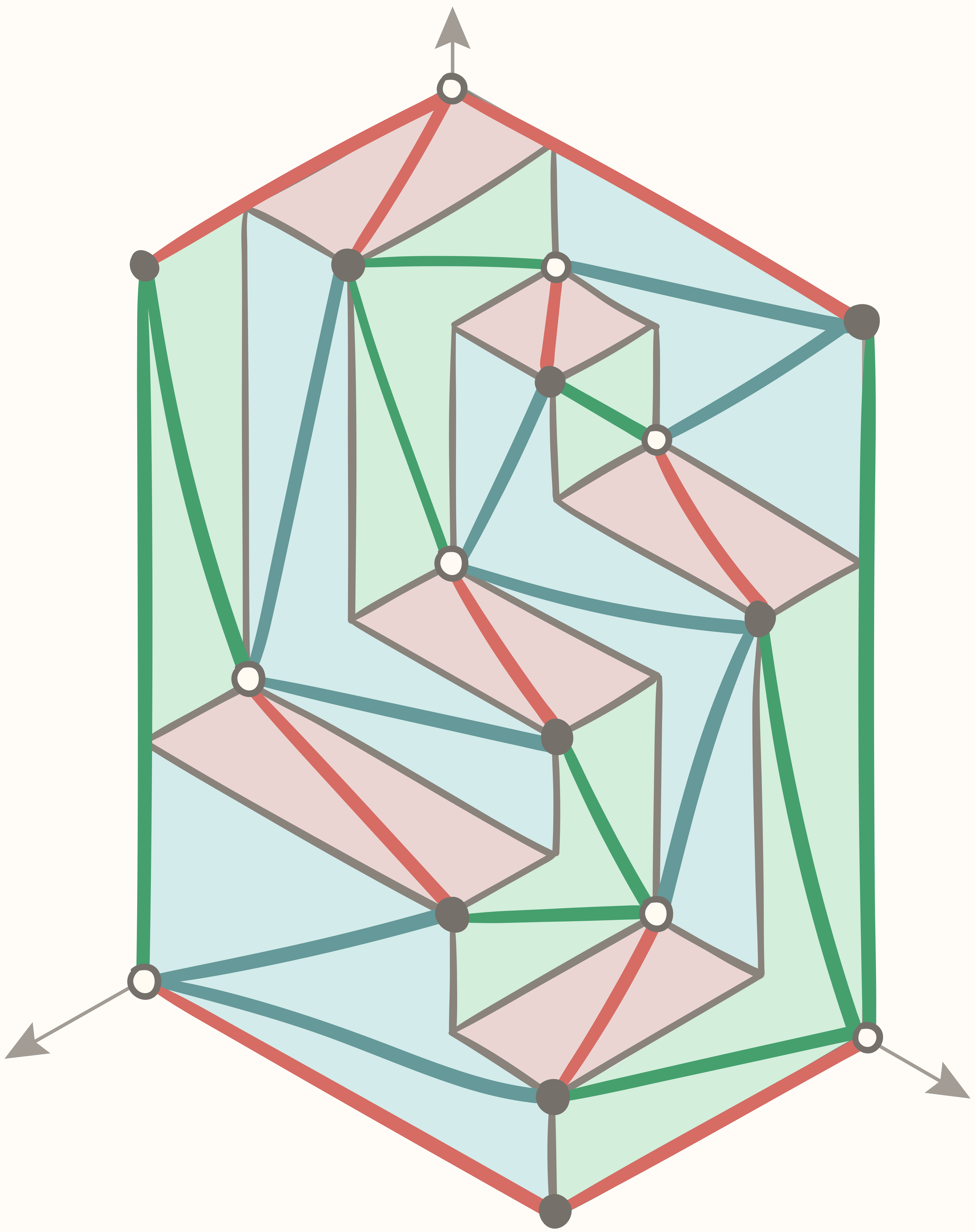

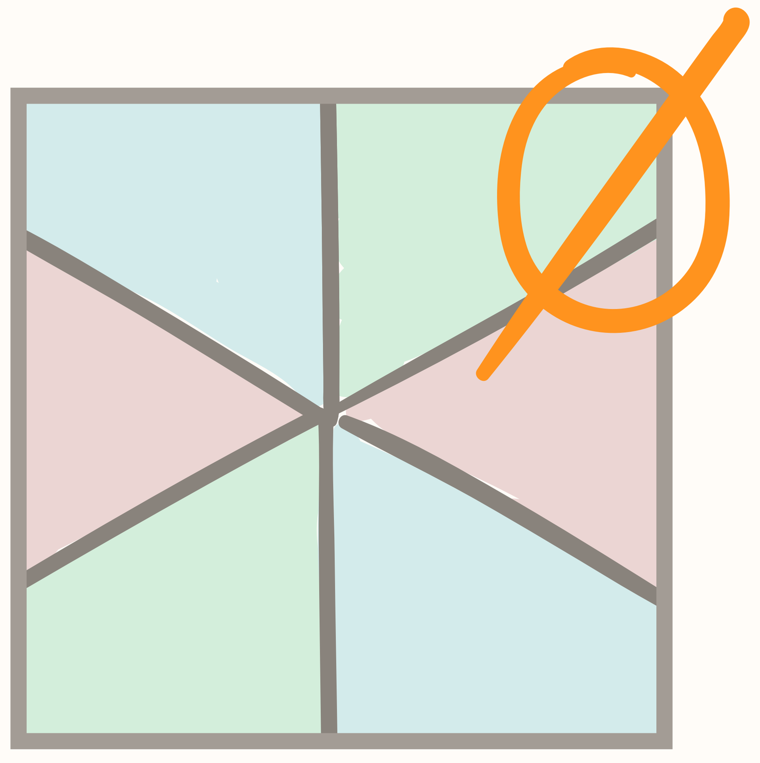

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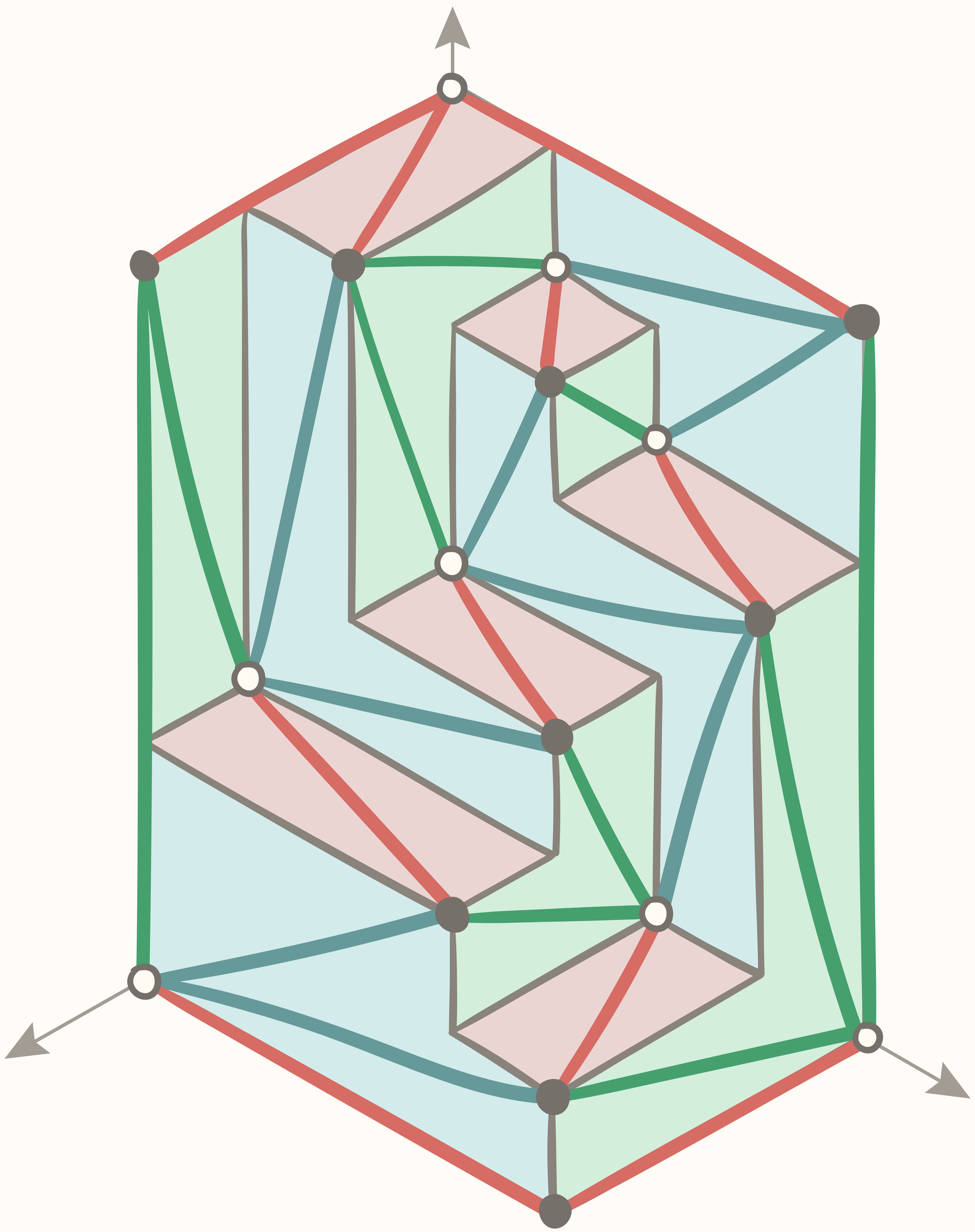
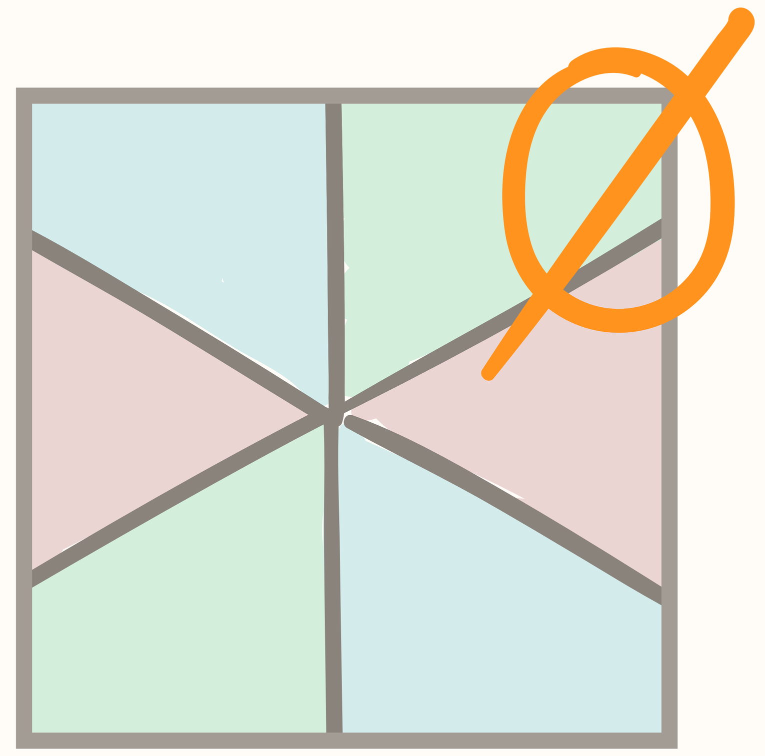
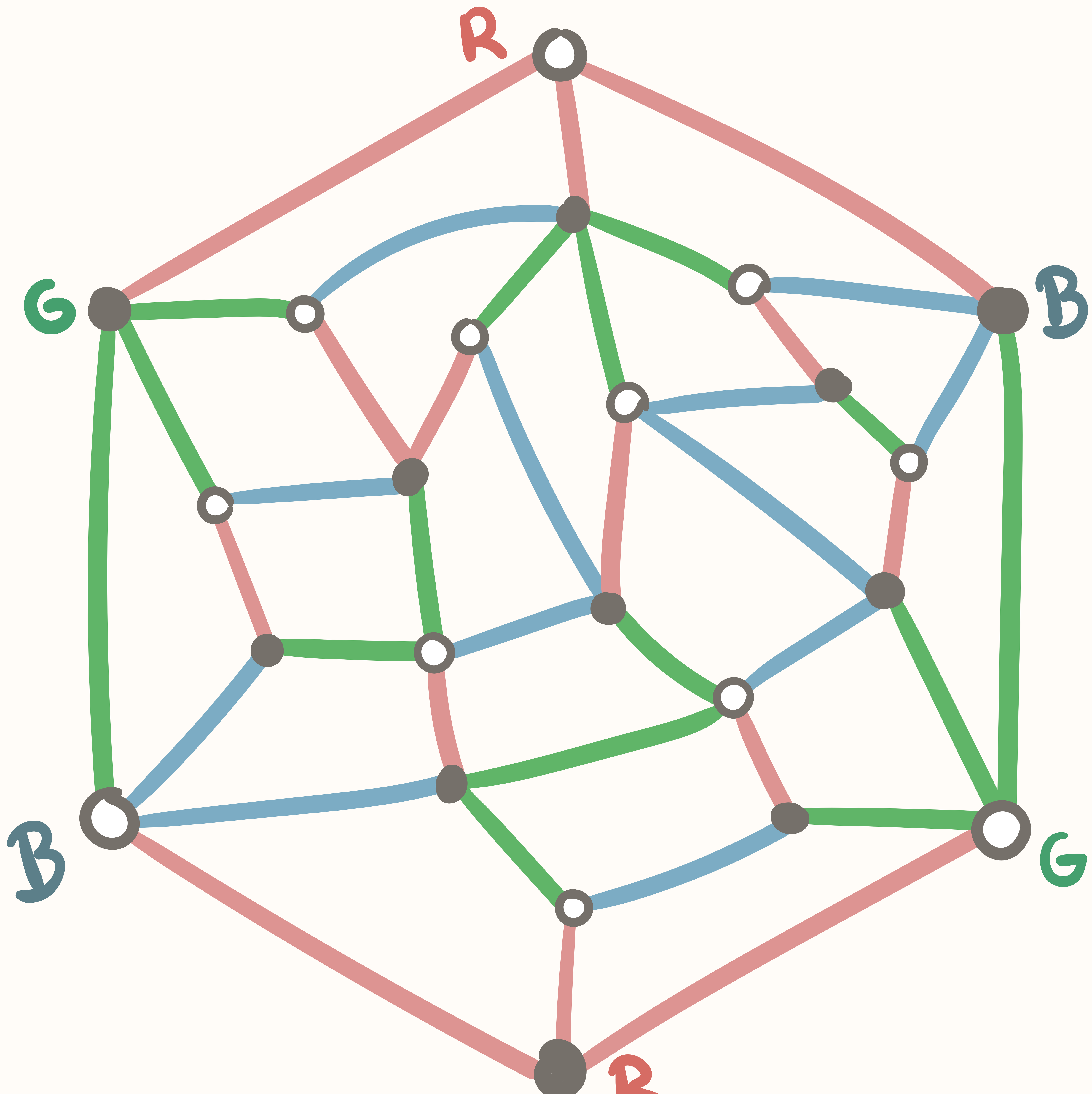




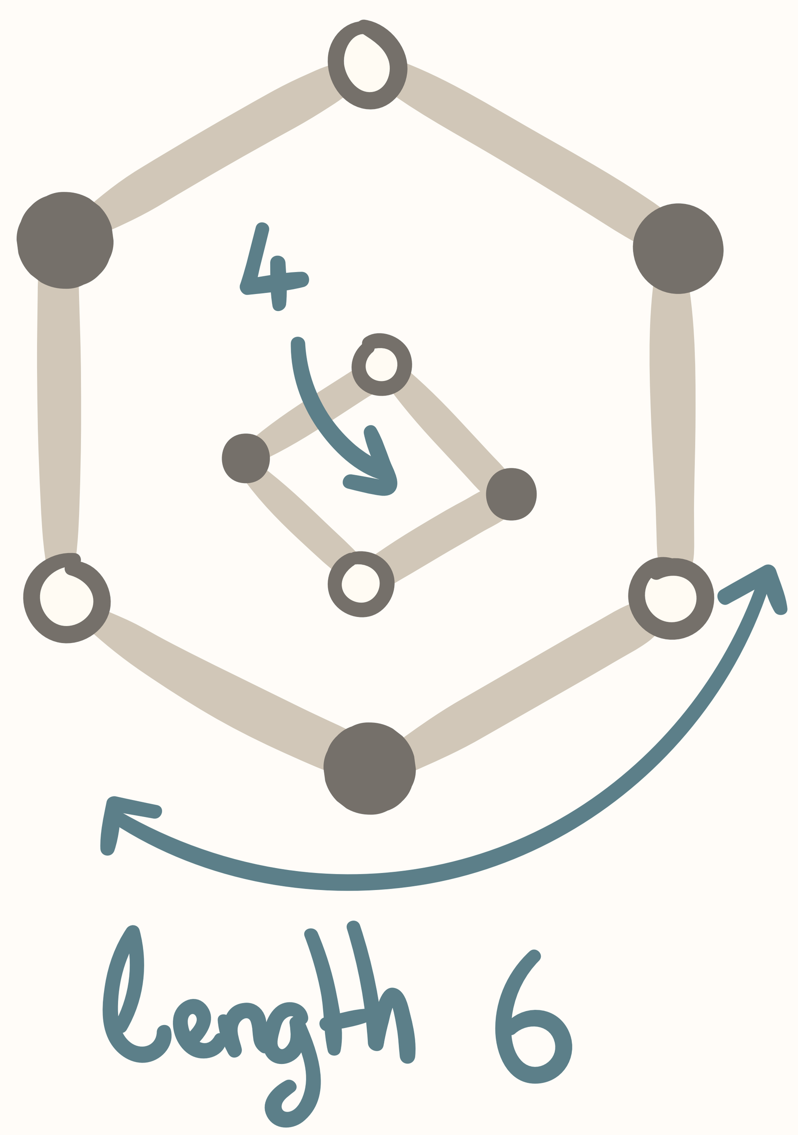
rigid corner polyhedra ⟷ ***Schnyder labellings***

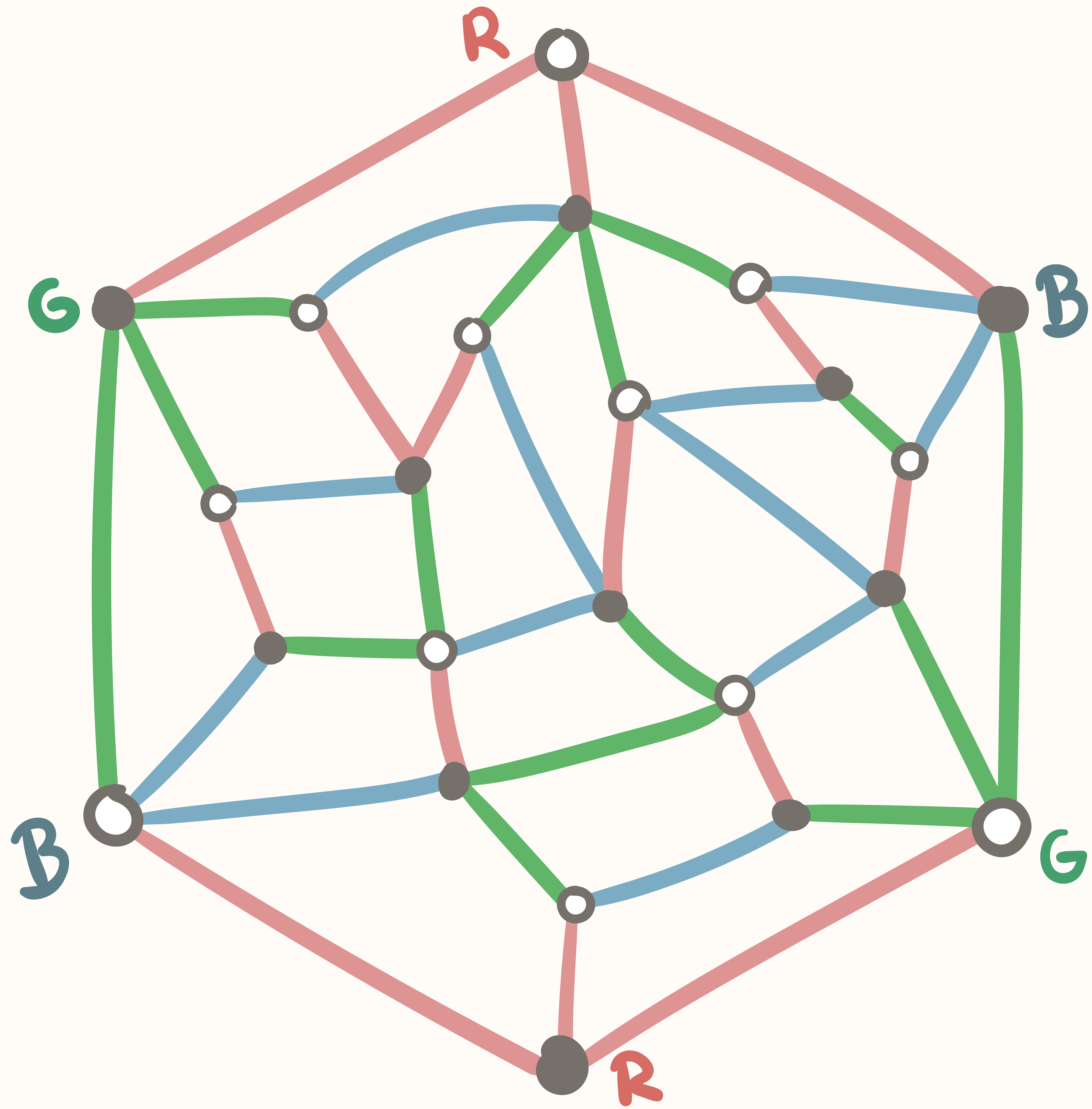
↠ *Geodesic embeddings and planar graphs,* S. Felsner (2003)

Schnyder labellings



Schnyder labellings





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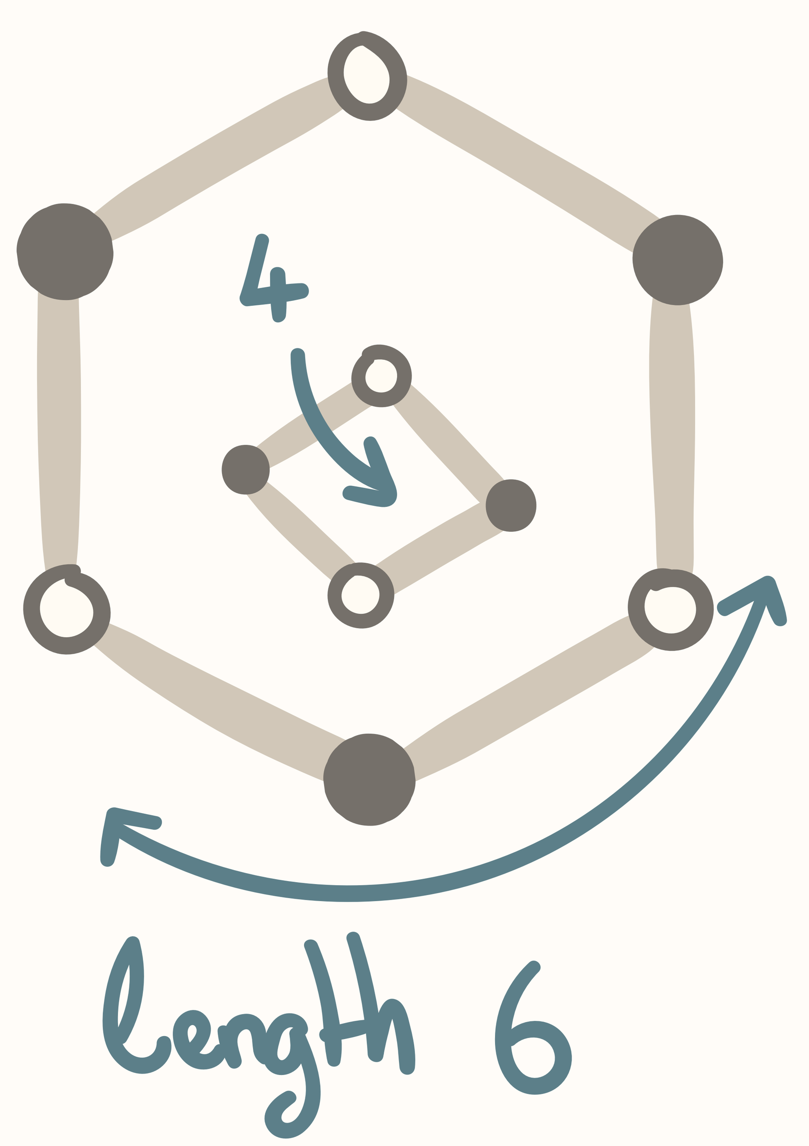

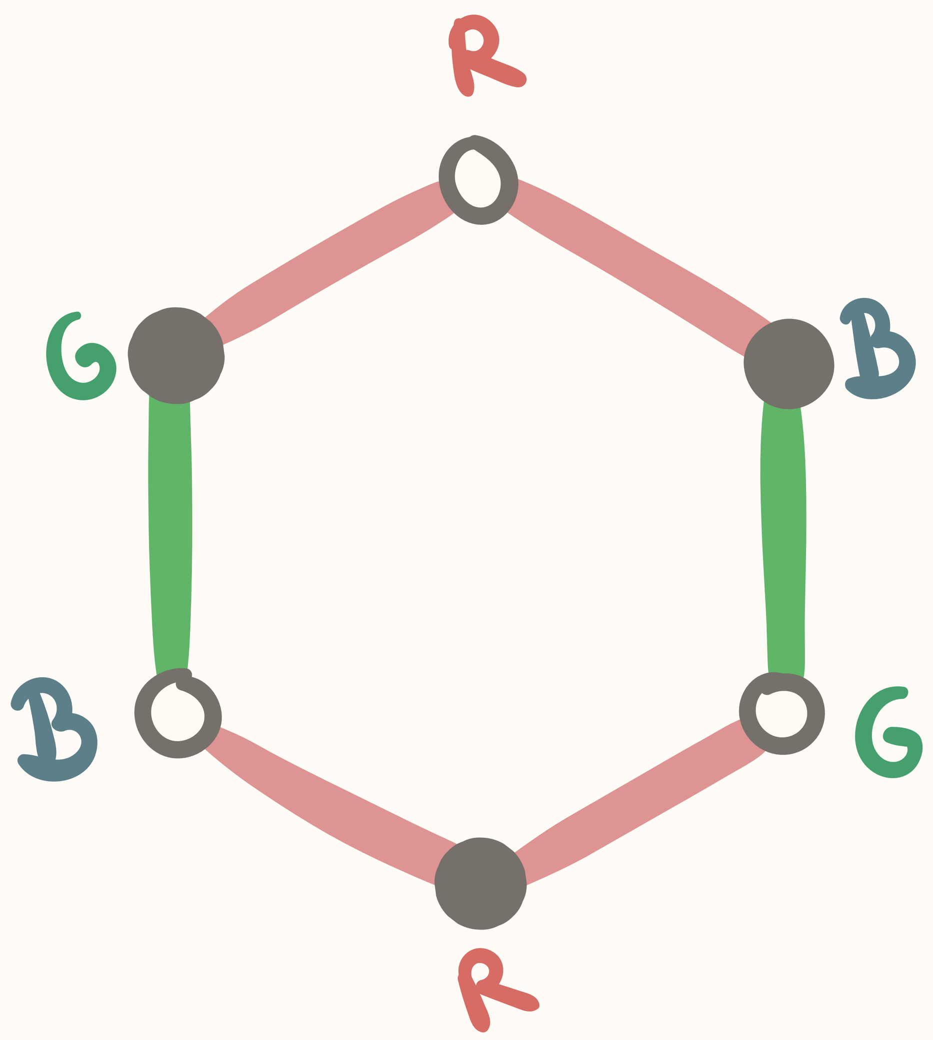

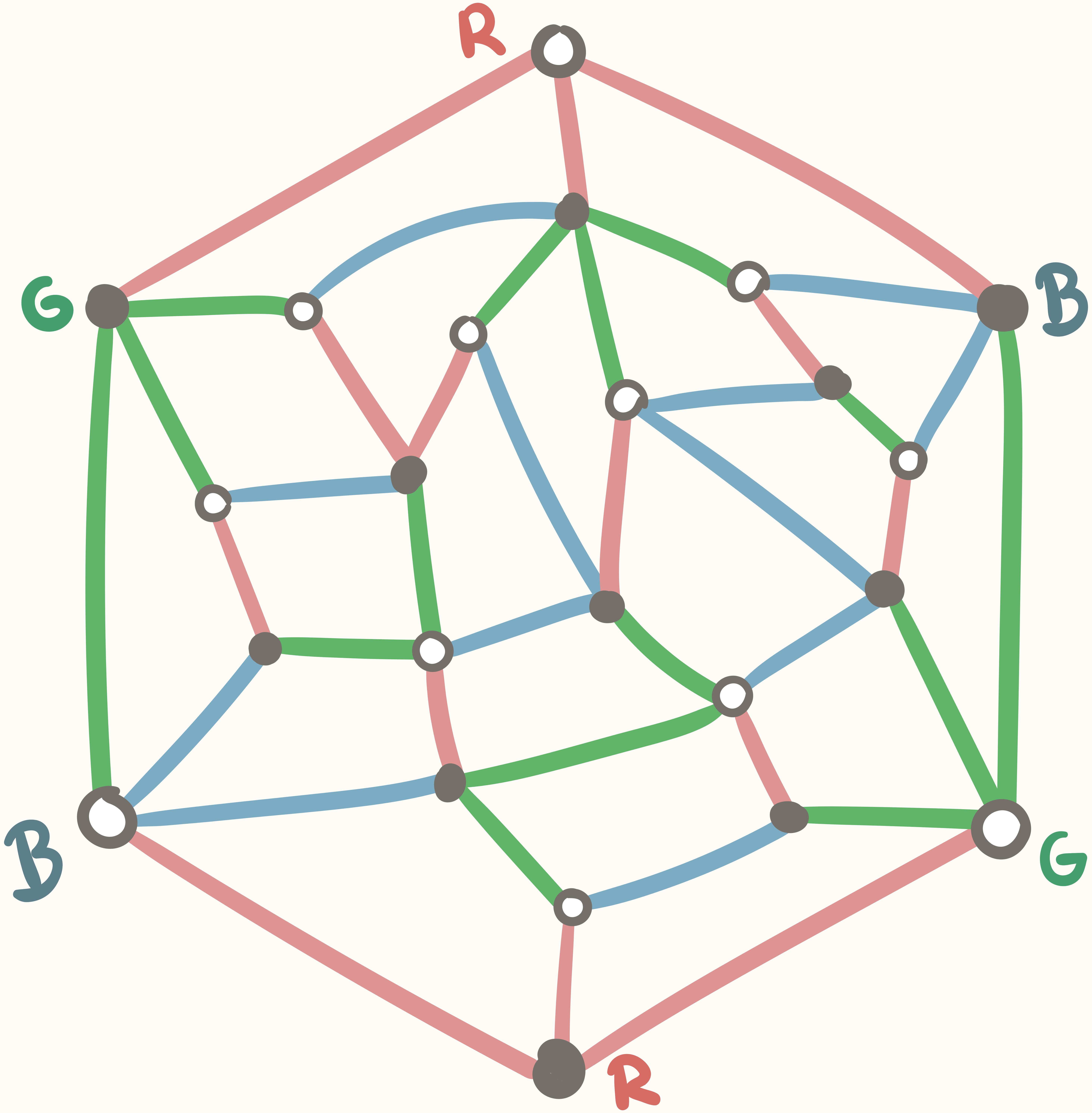

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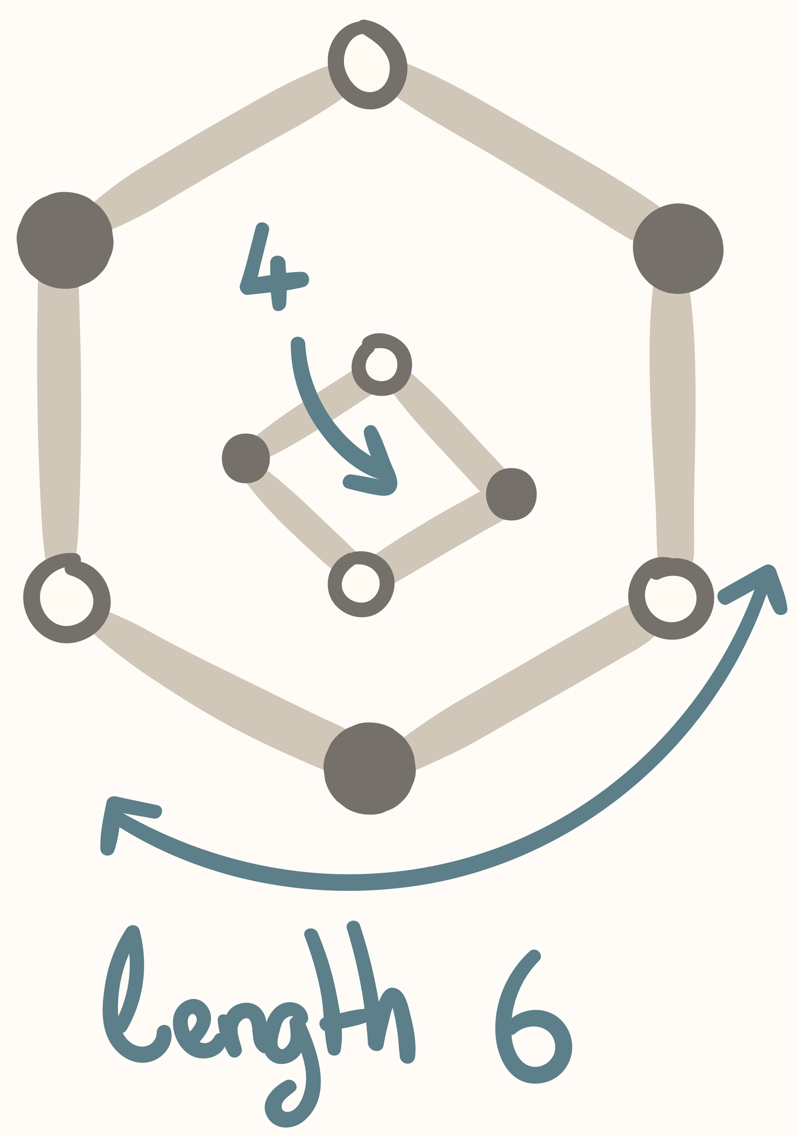

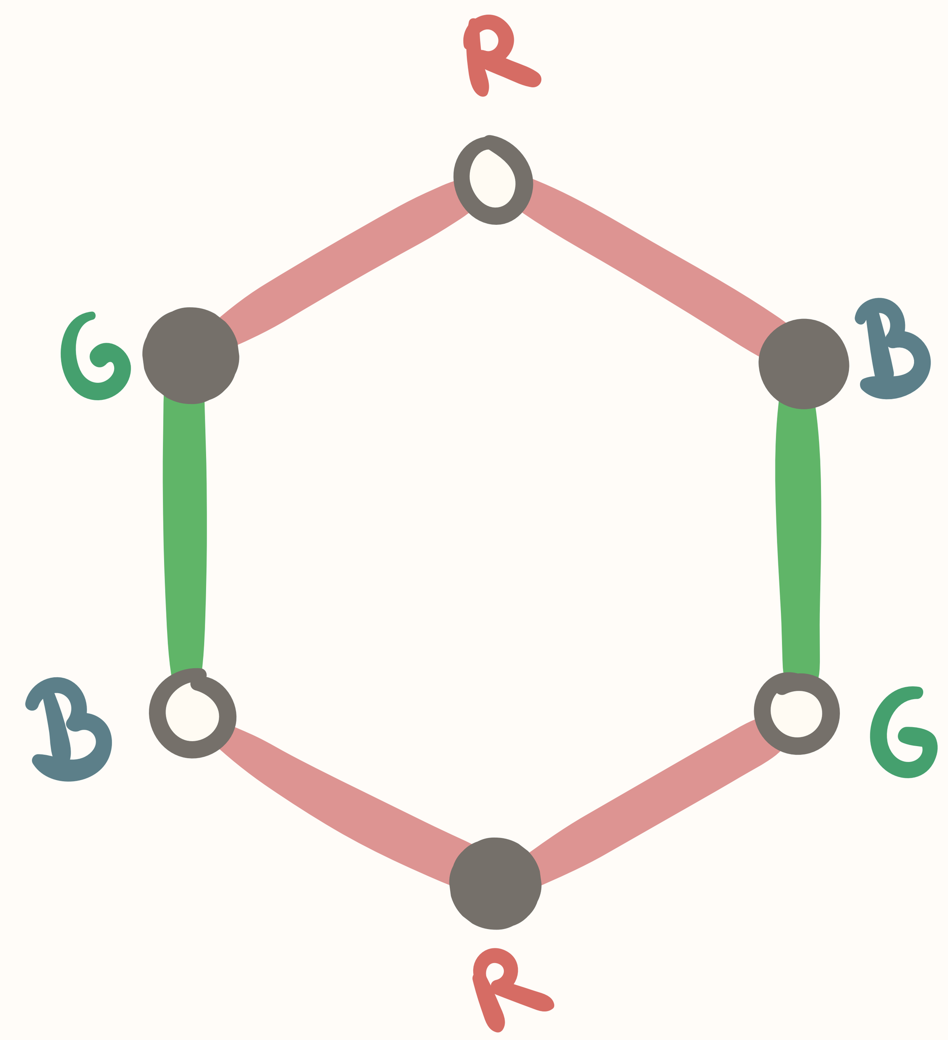

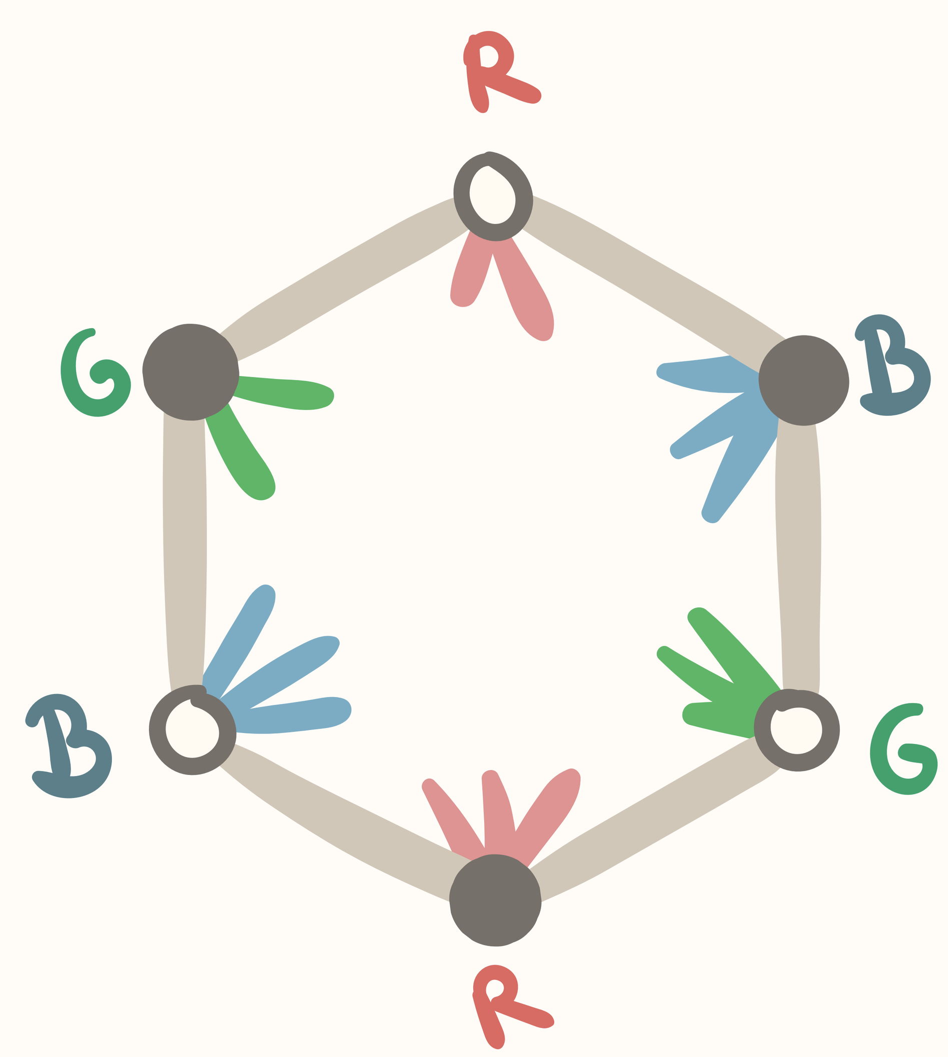

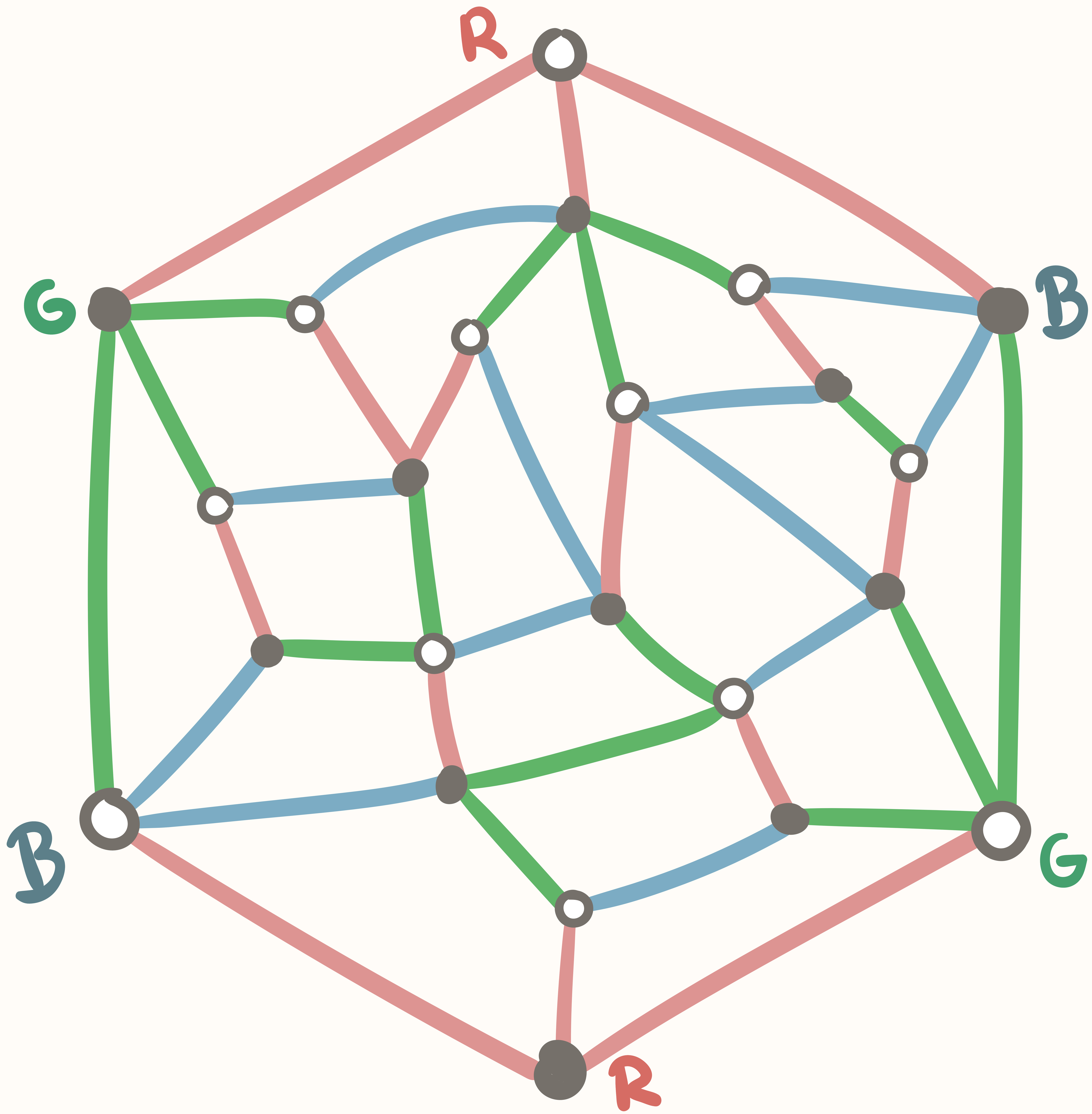

Schnyder labellings

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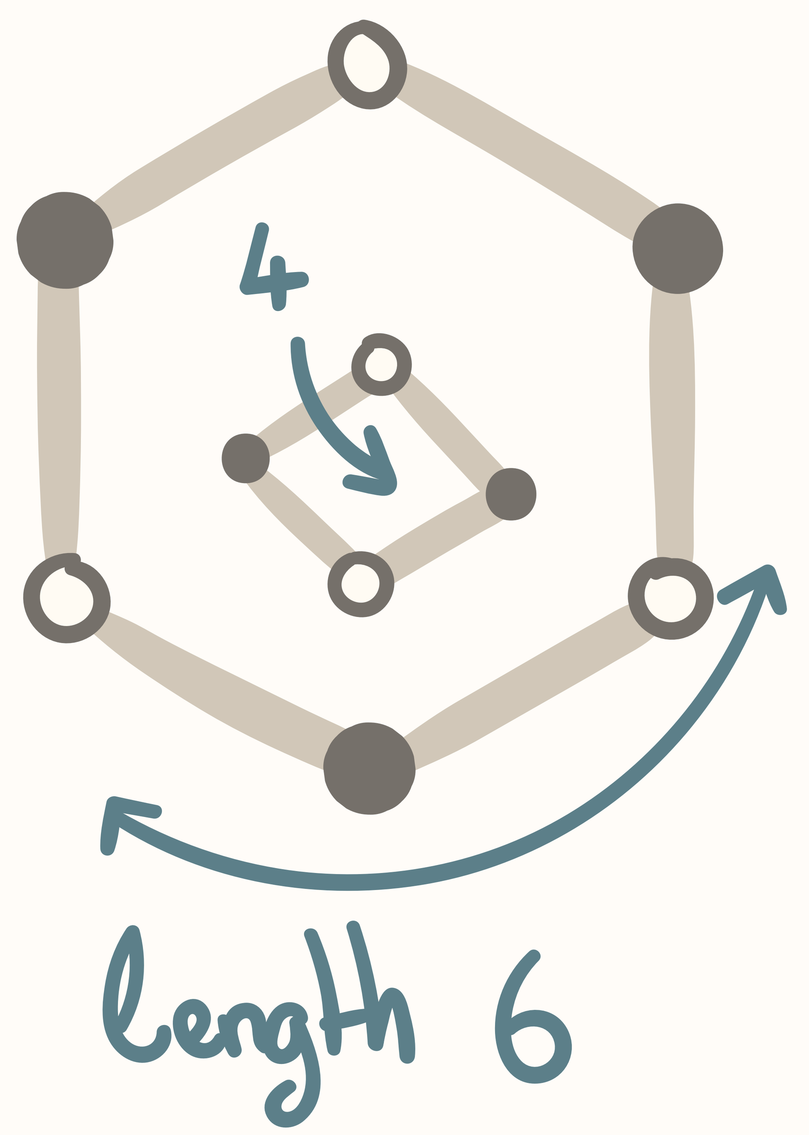
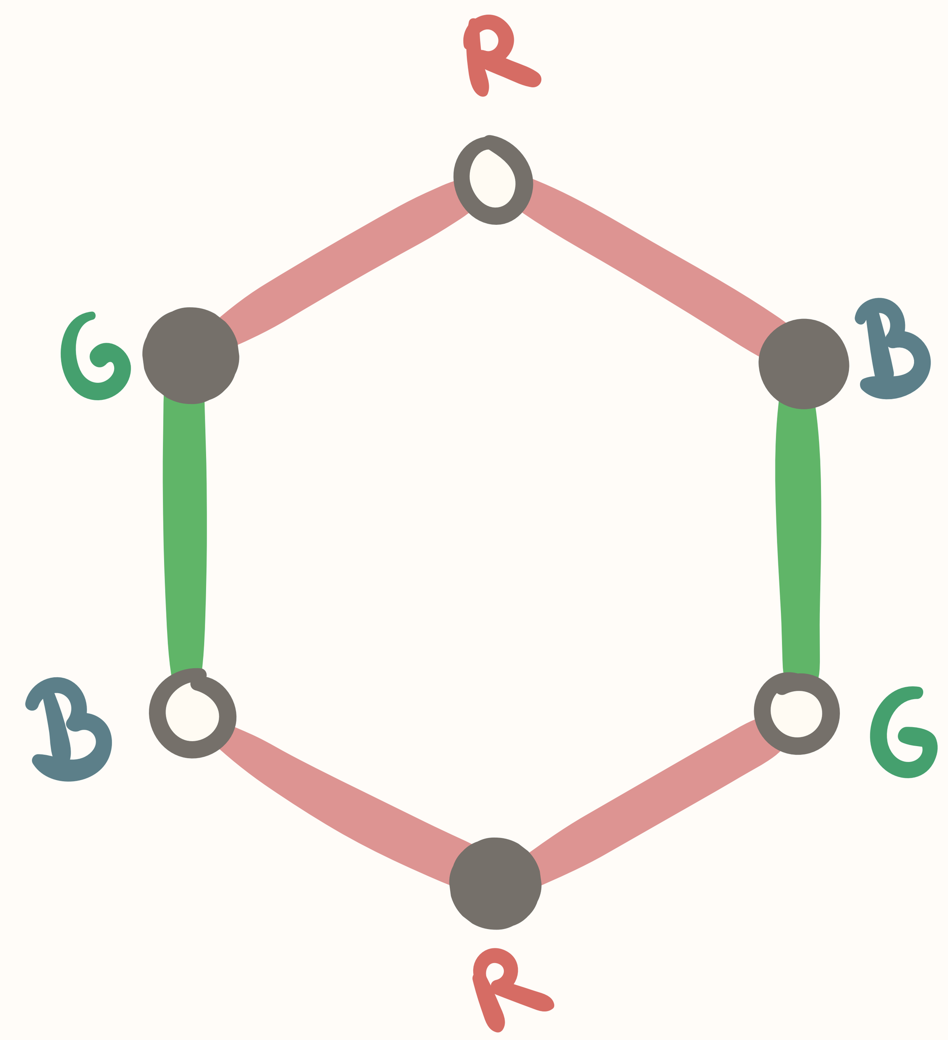
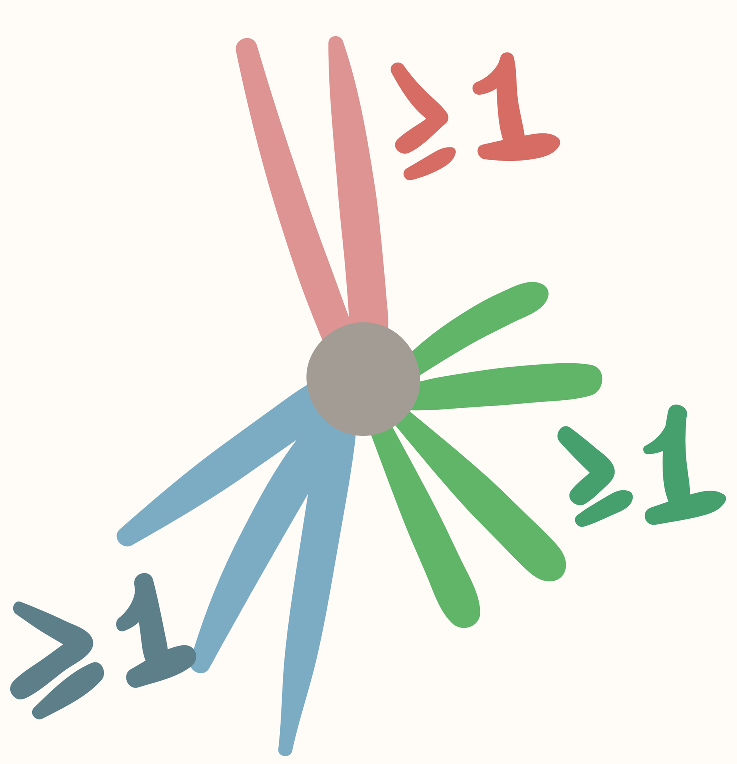
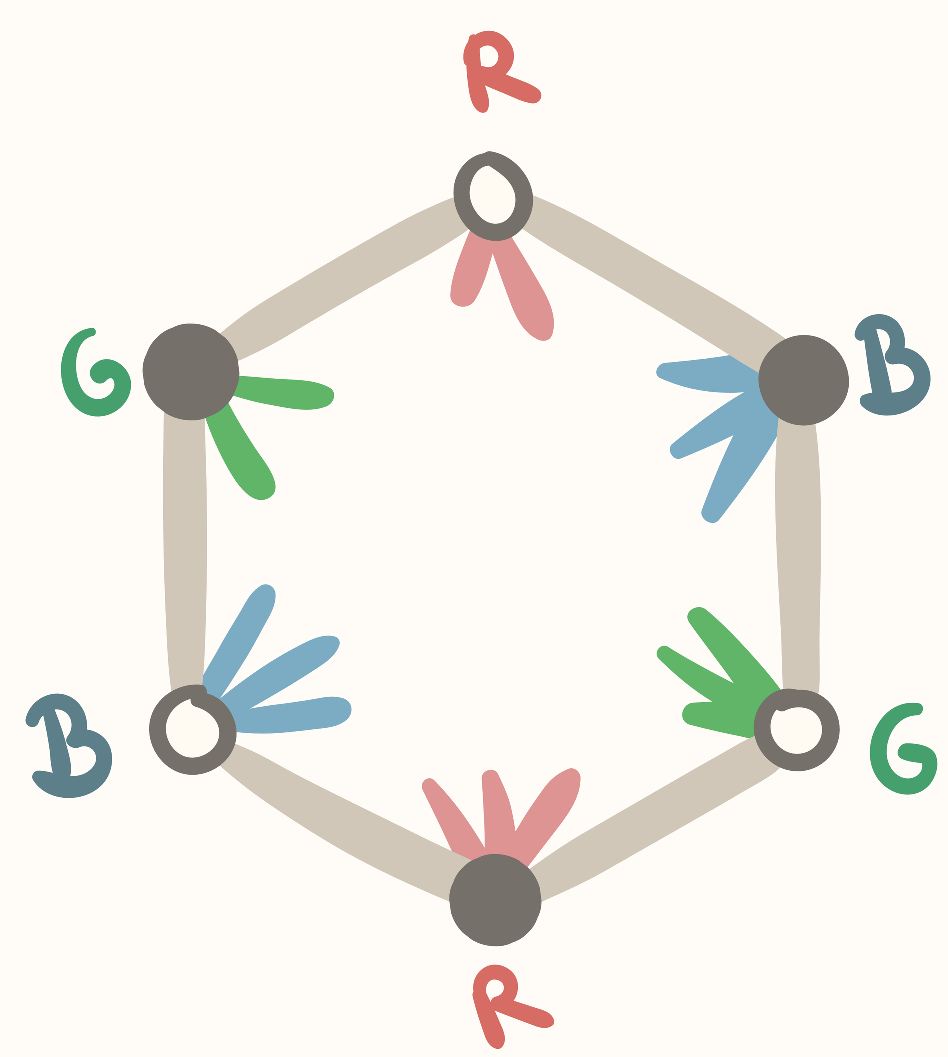
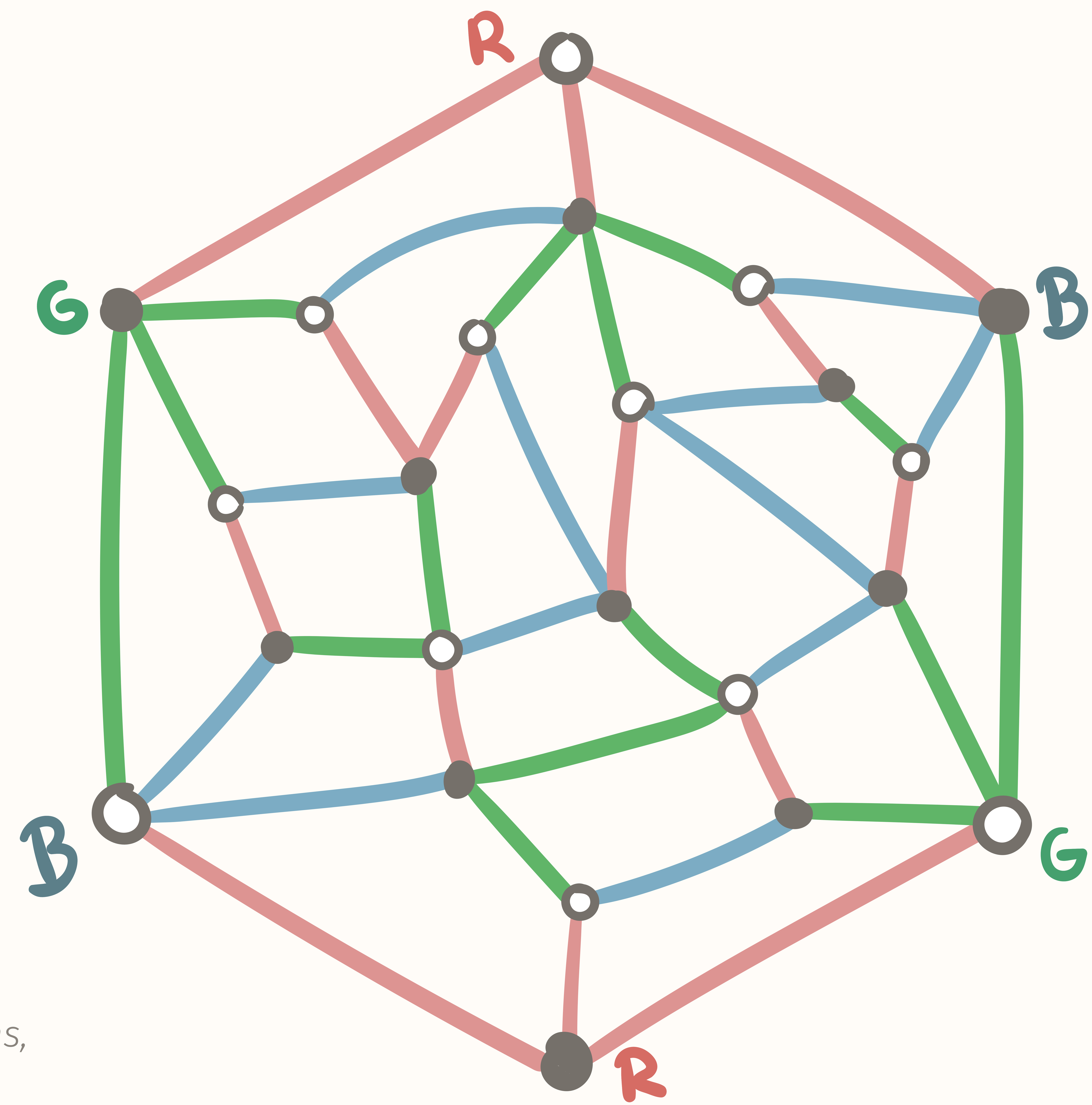






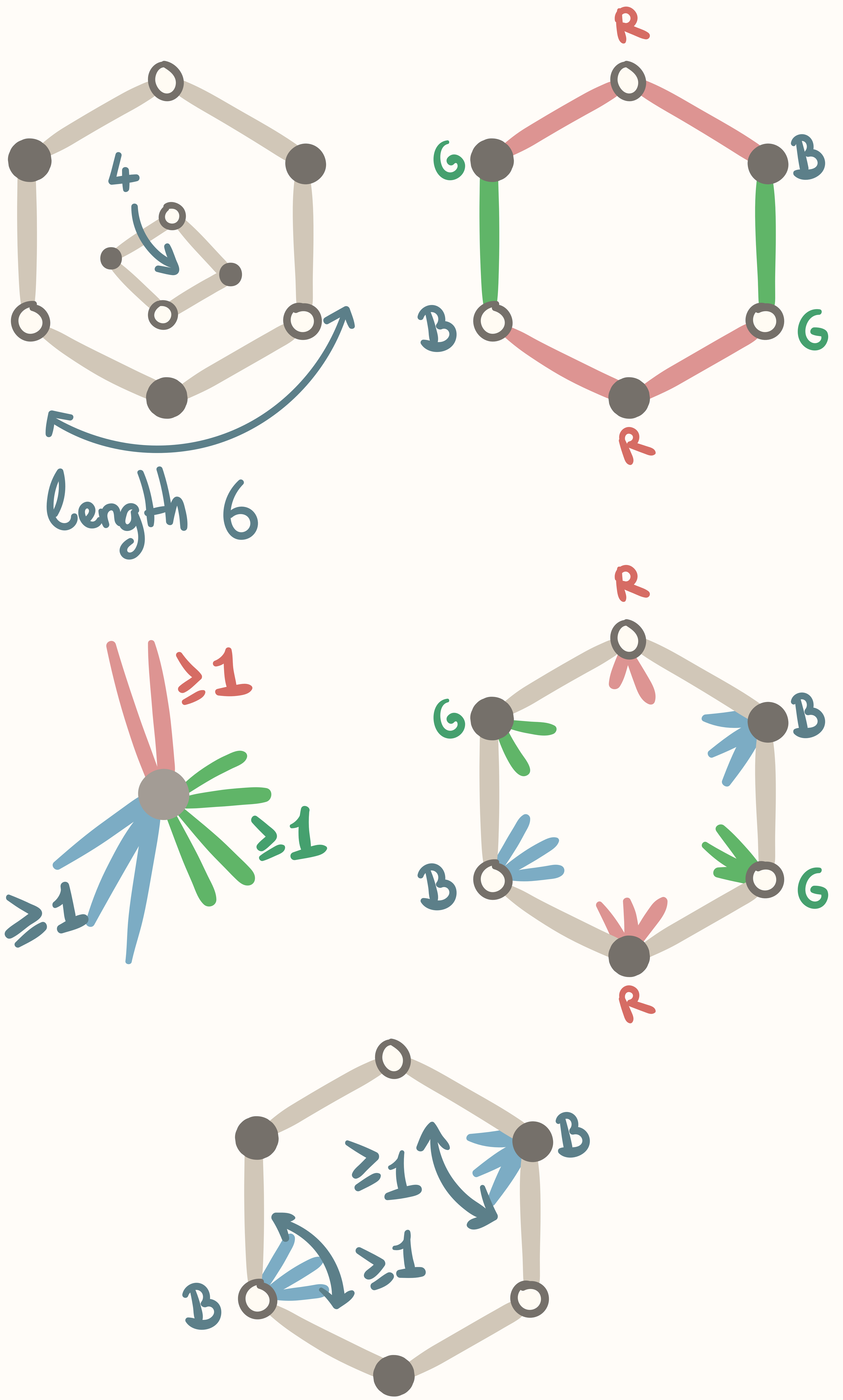


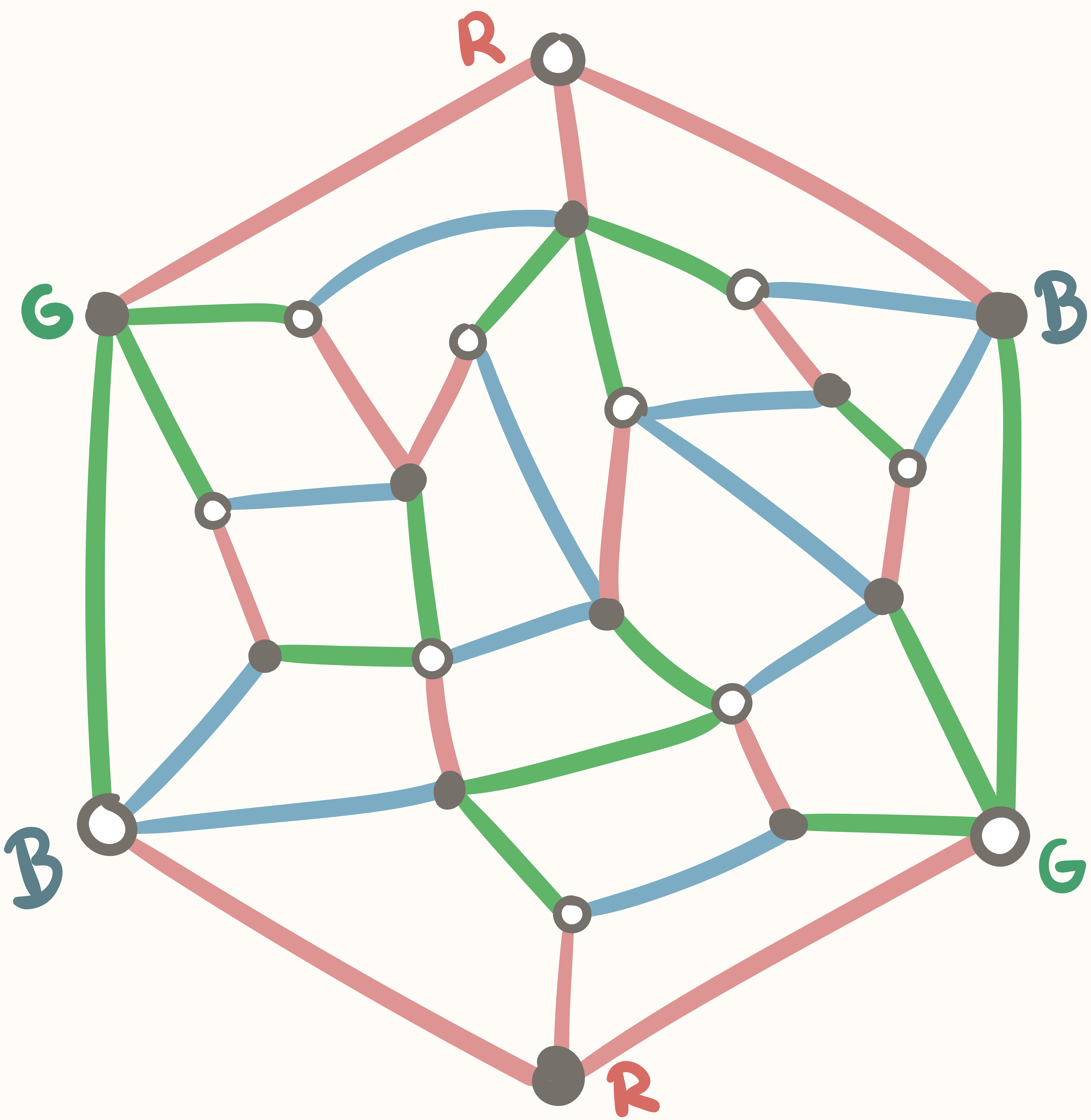


rigid corner polyhedra ⟷ ***Schnyder labellings***

↠ *Geodesic embeddings and planar graphs*, S. Felsner (2003)

Schnyder labellings



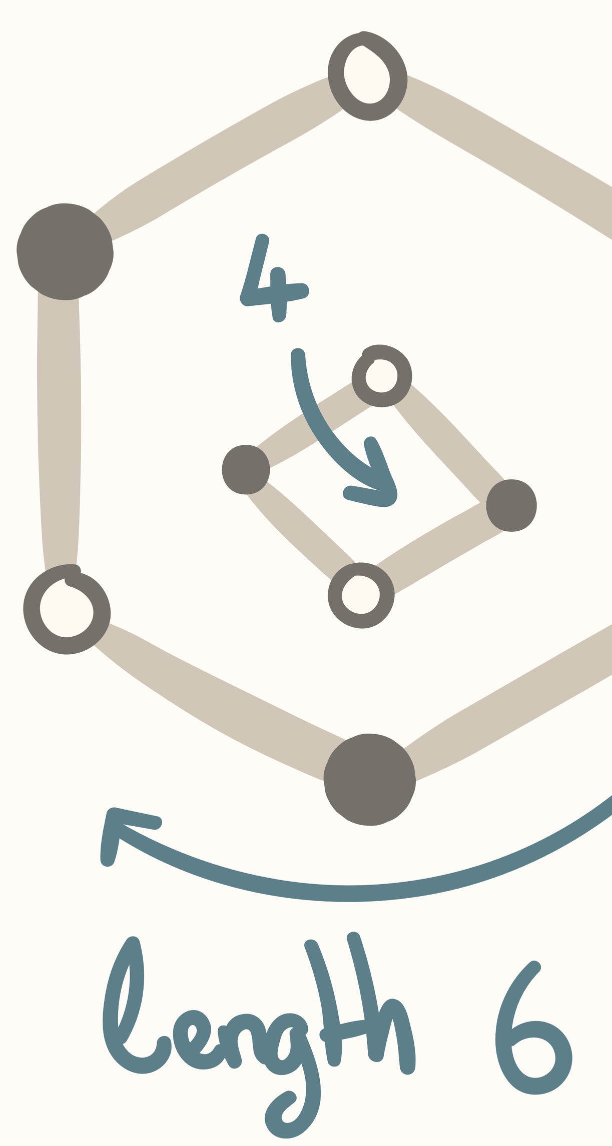


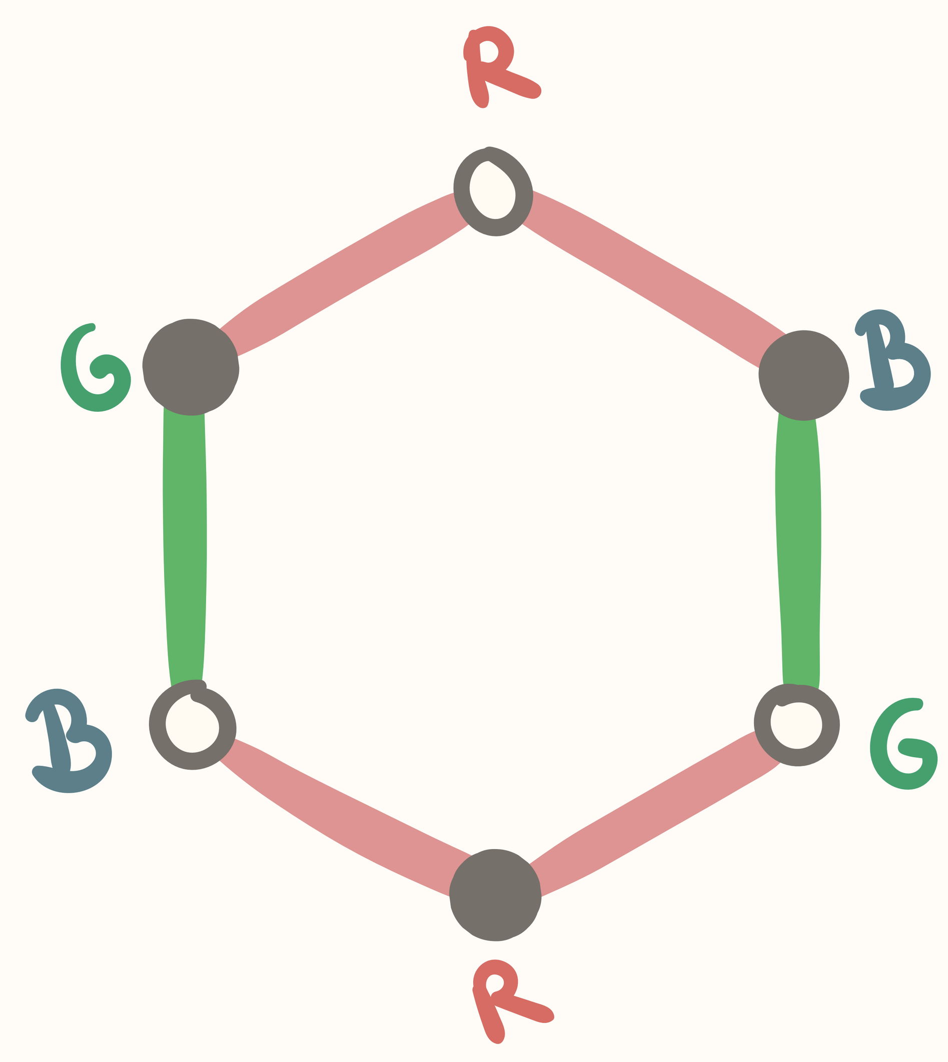
Schnyder labellin

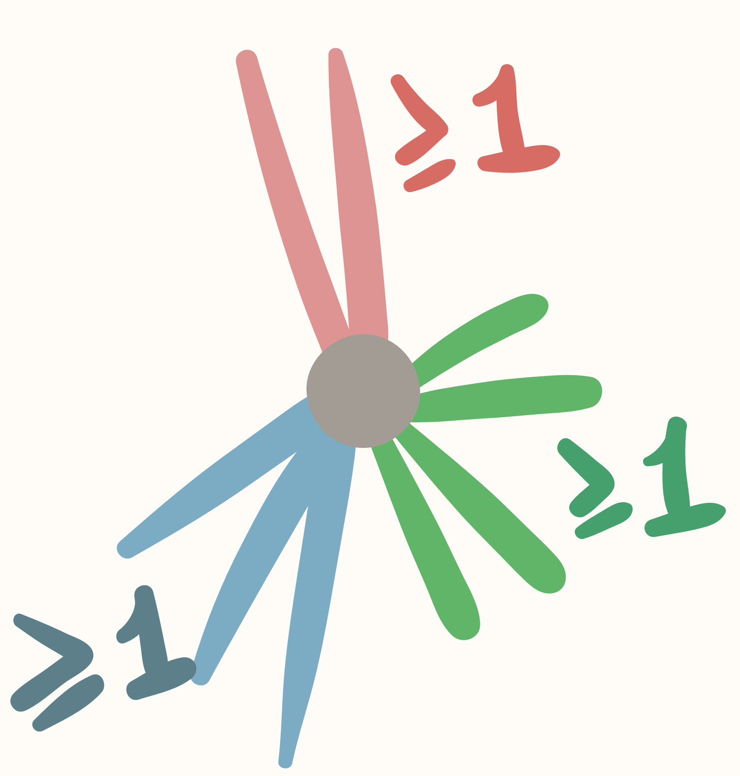
Schnyder labelling

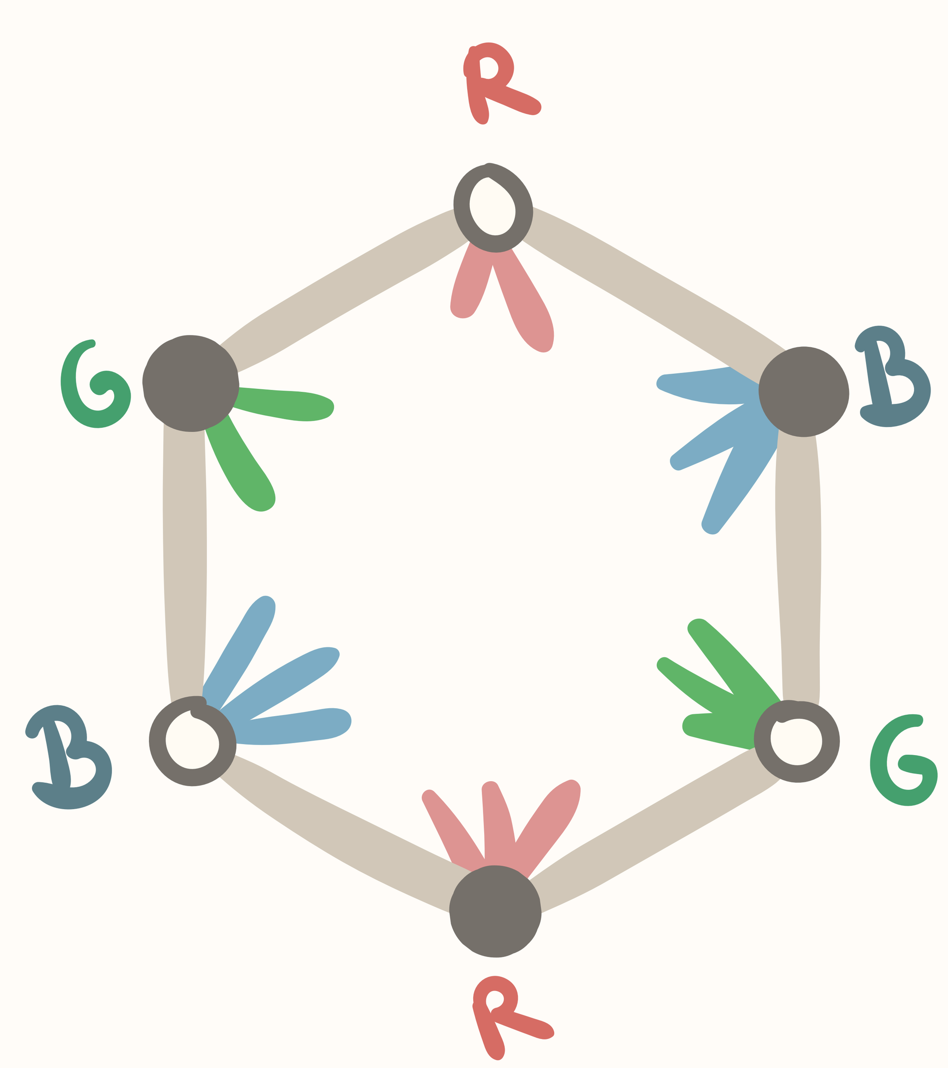
Schnyder labellin

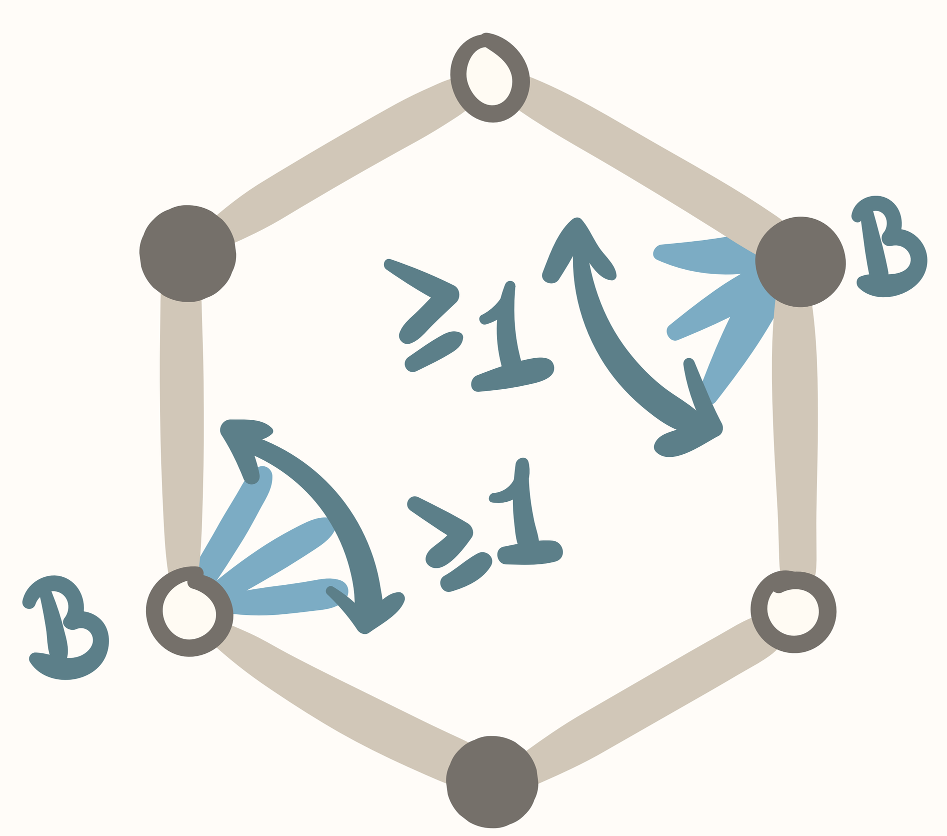
Schnyder labellings

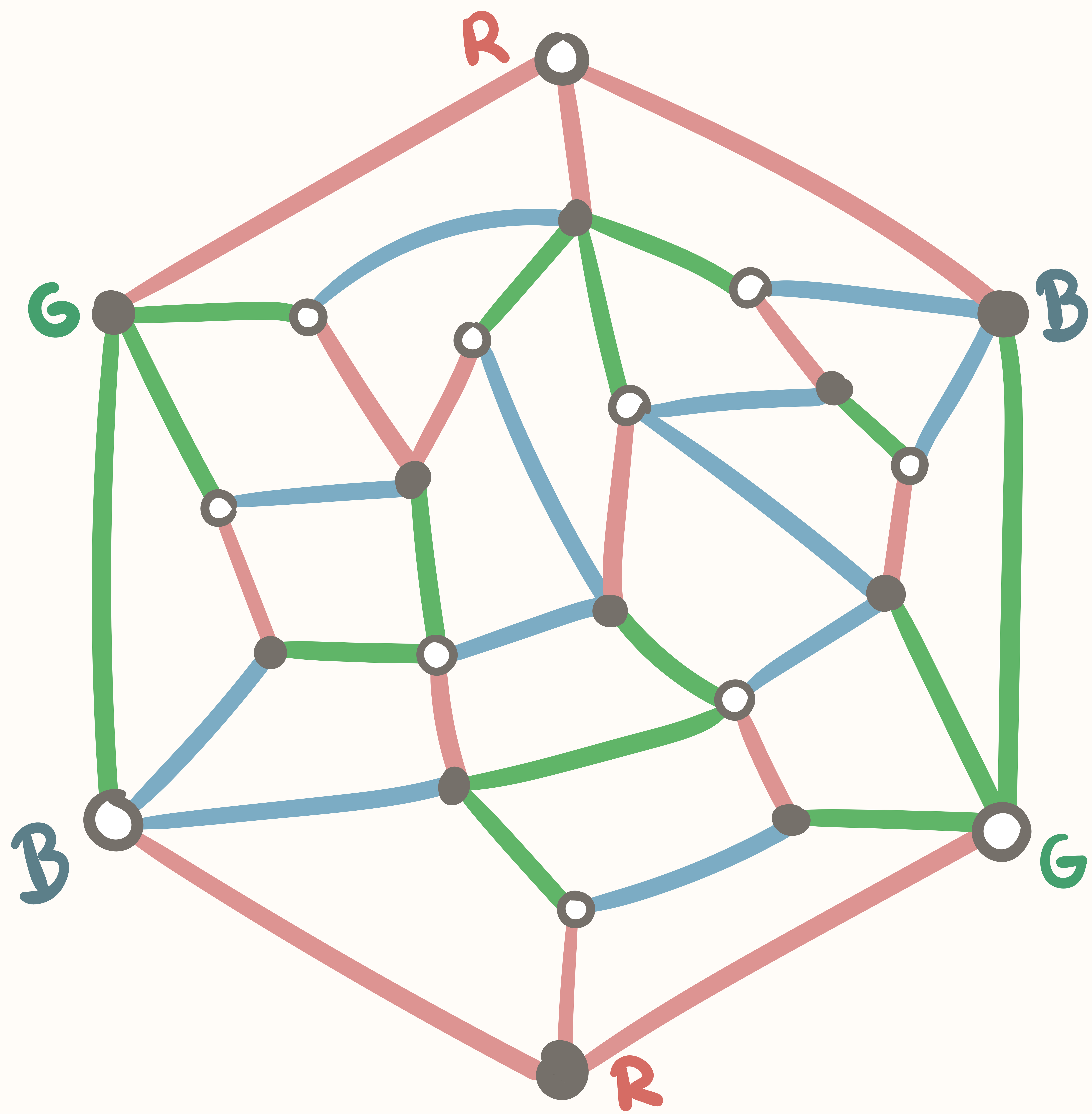












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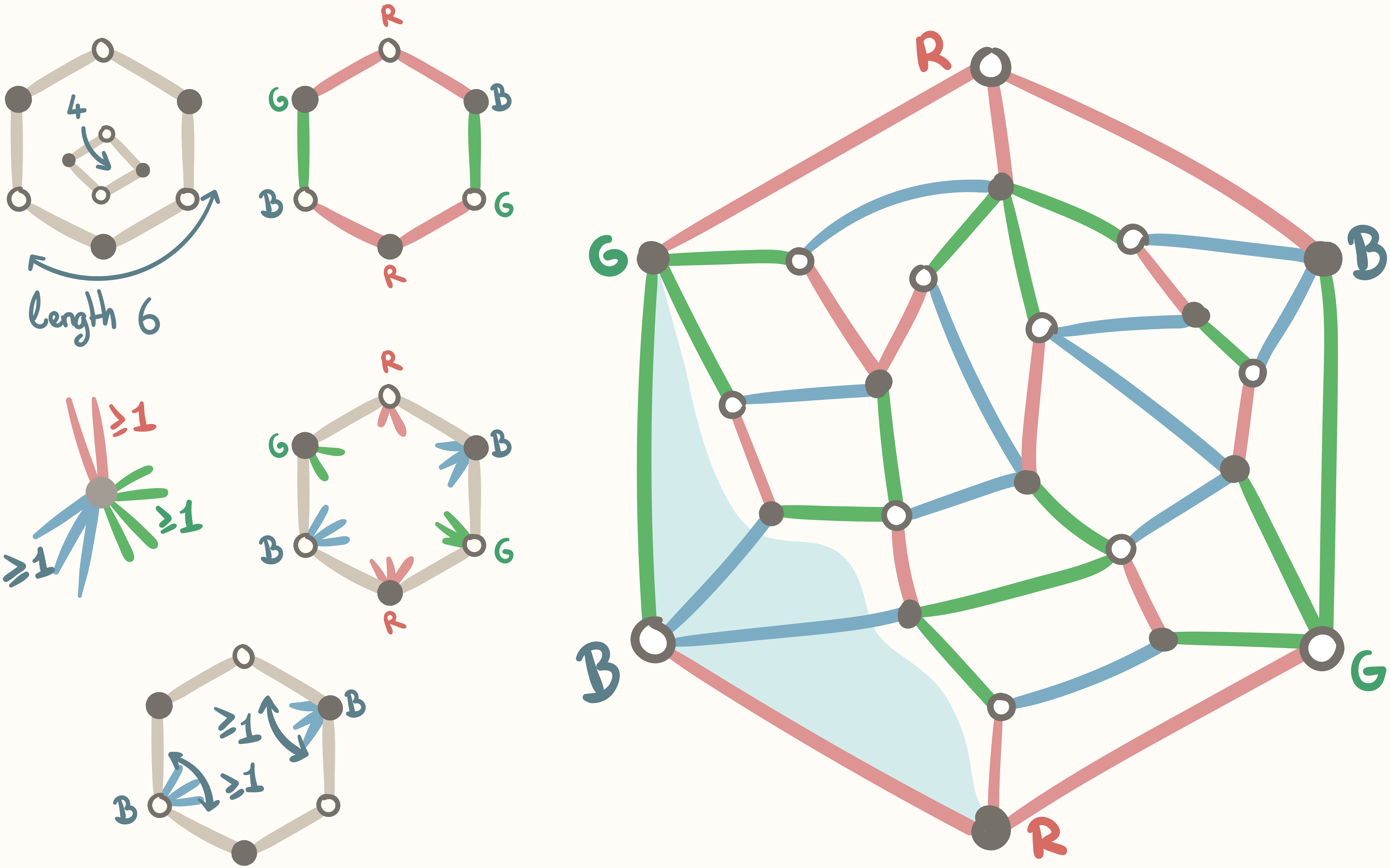

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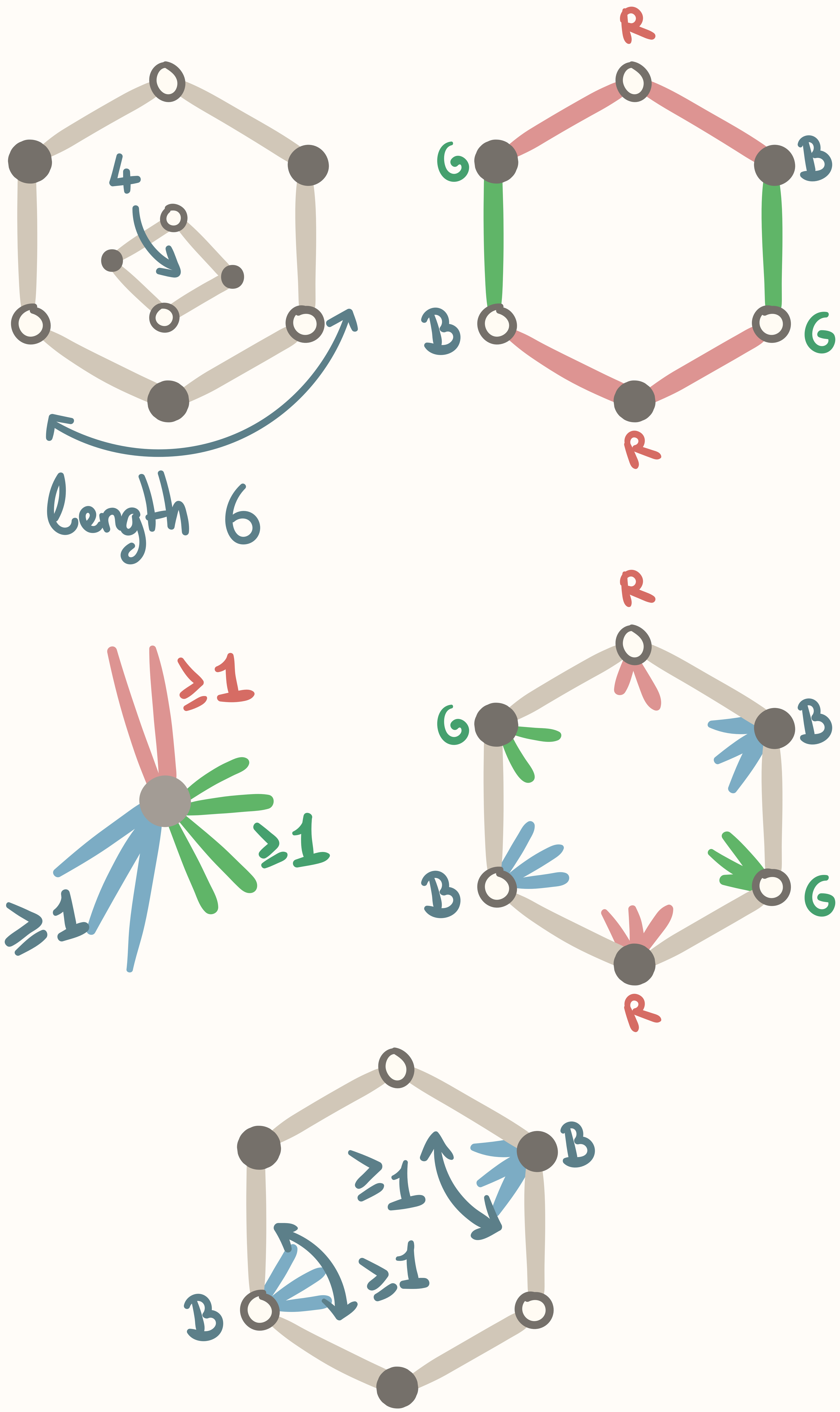

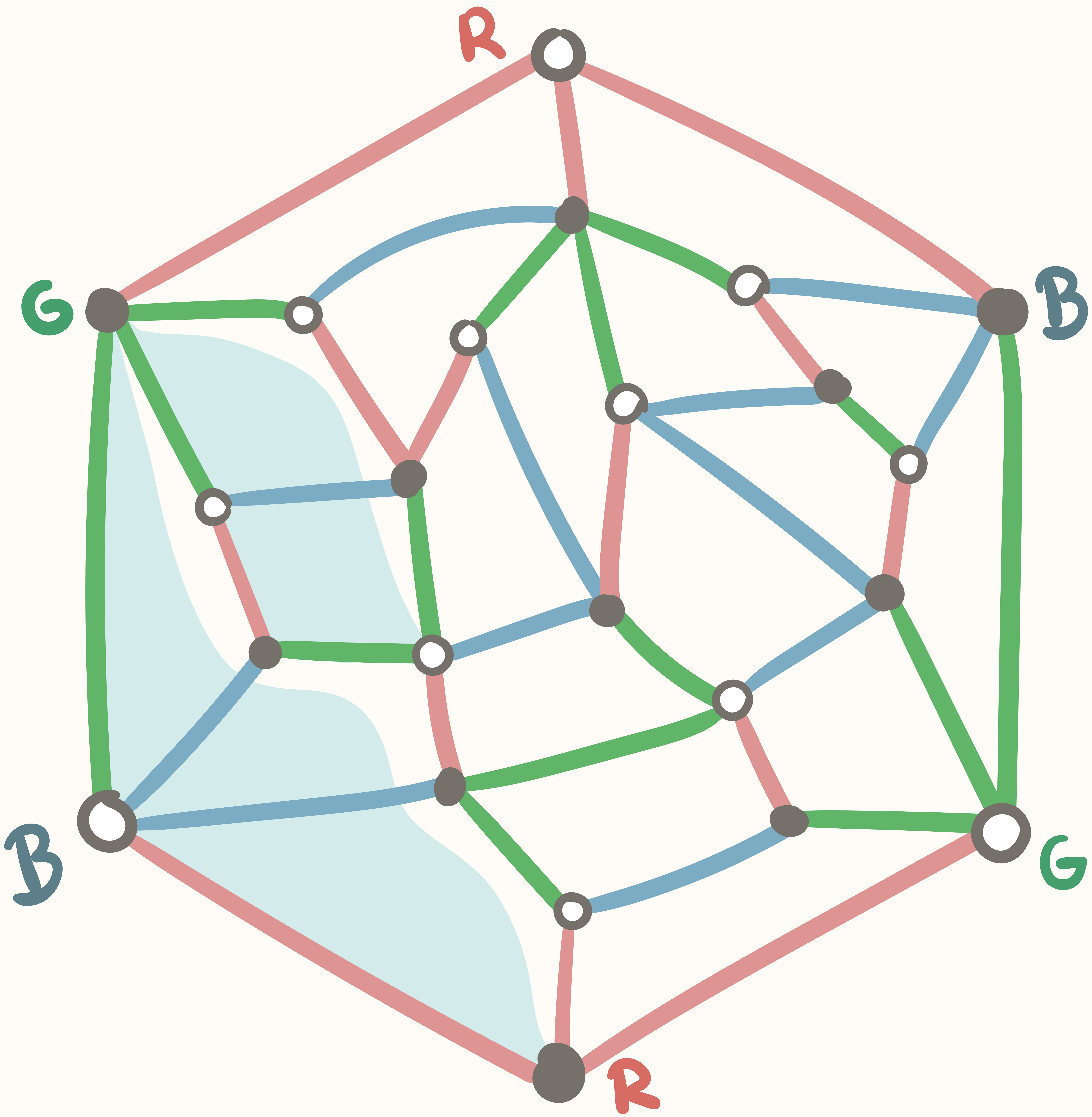

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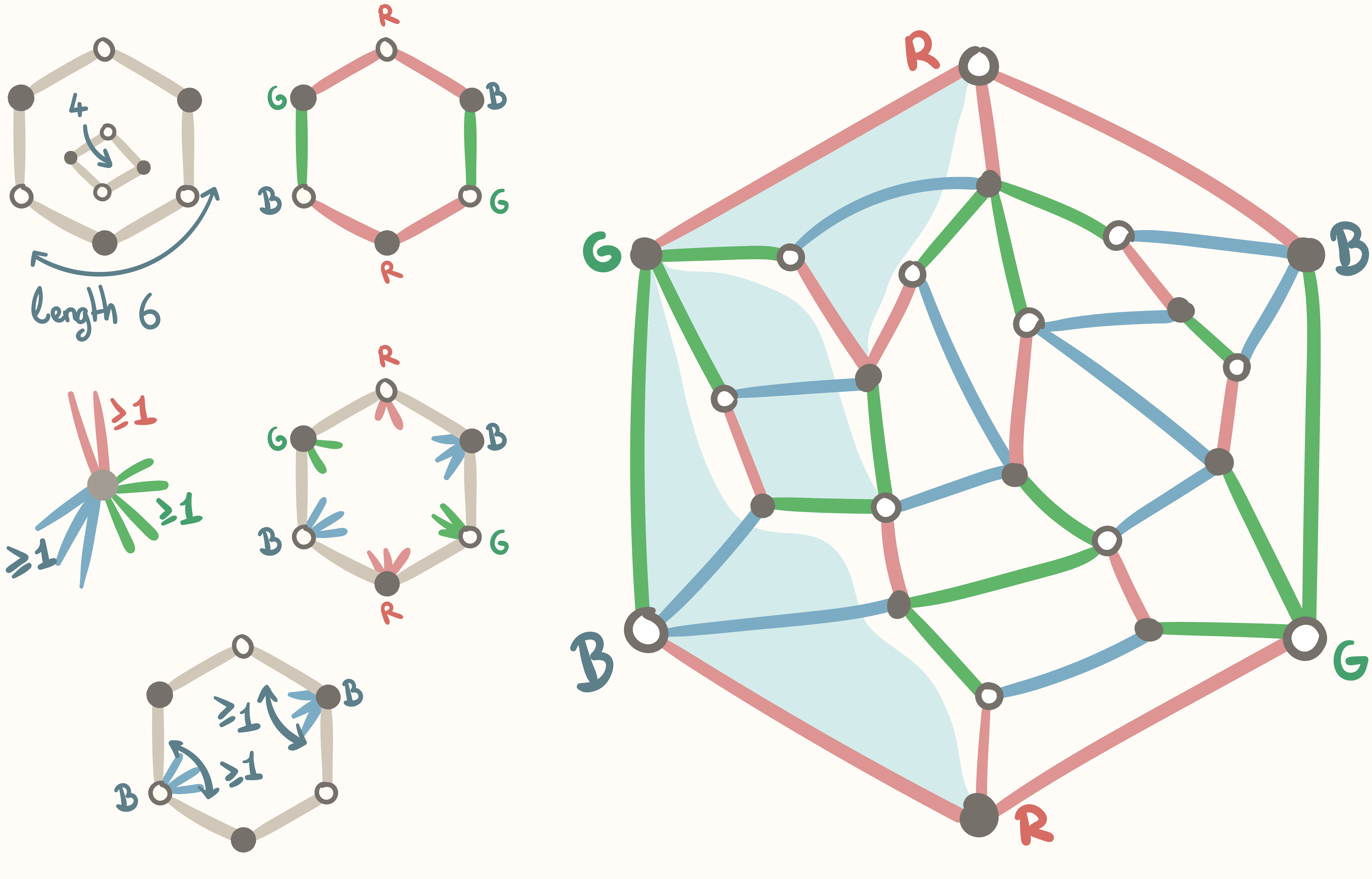

Schnyder labellings

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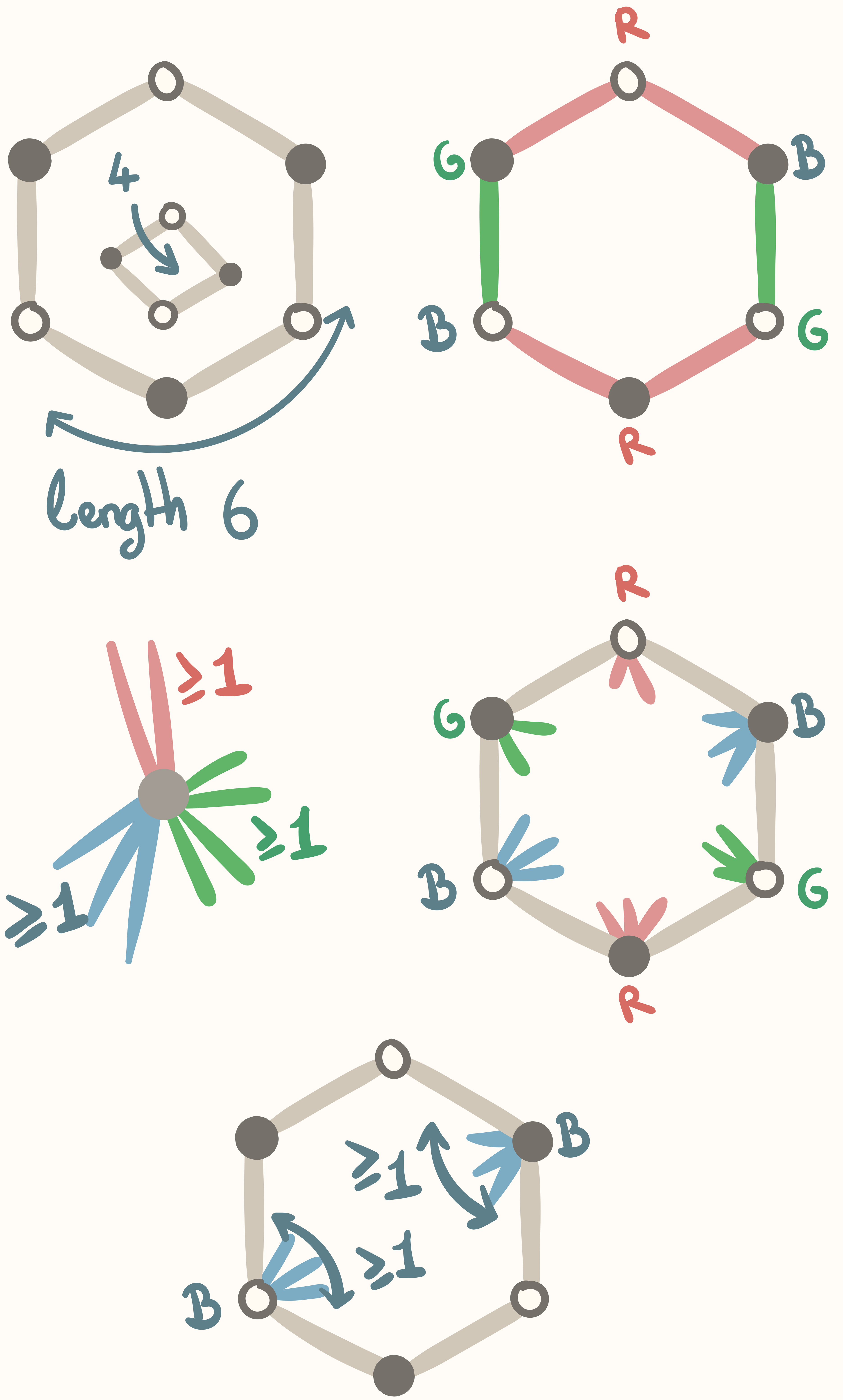

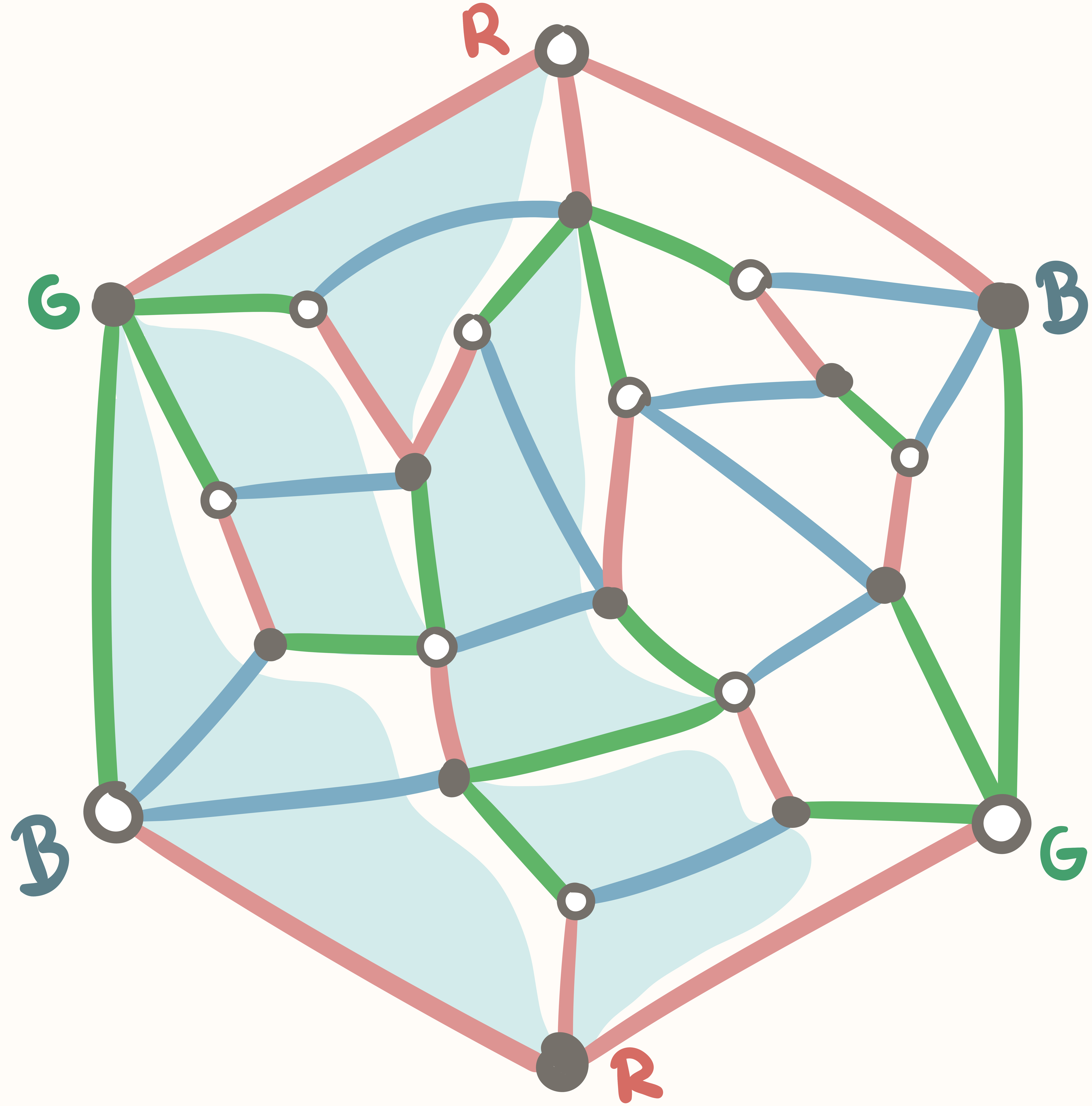

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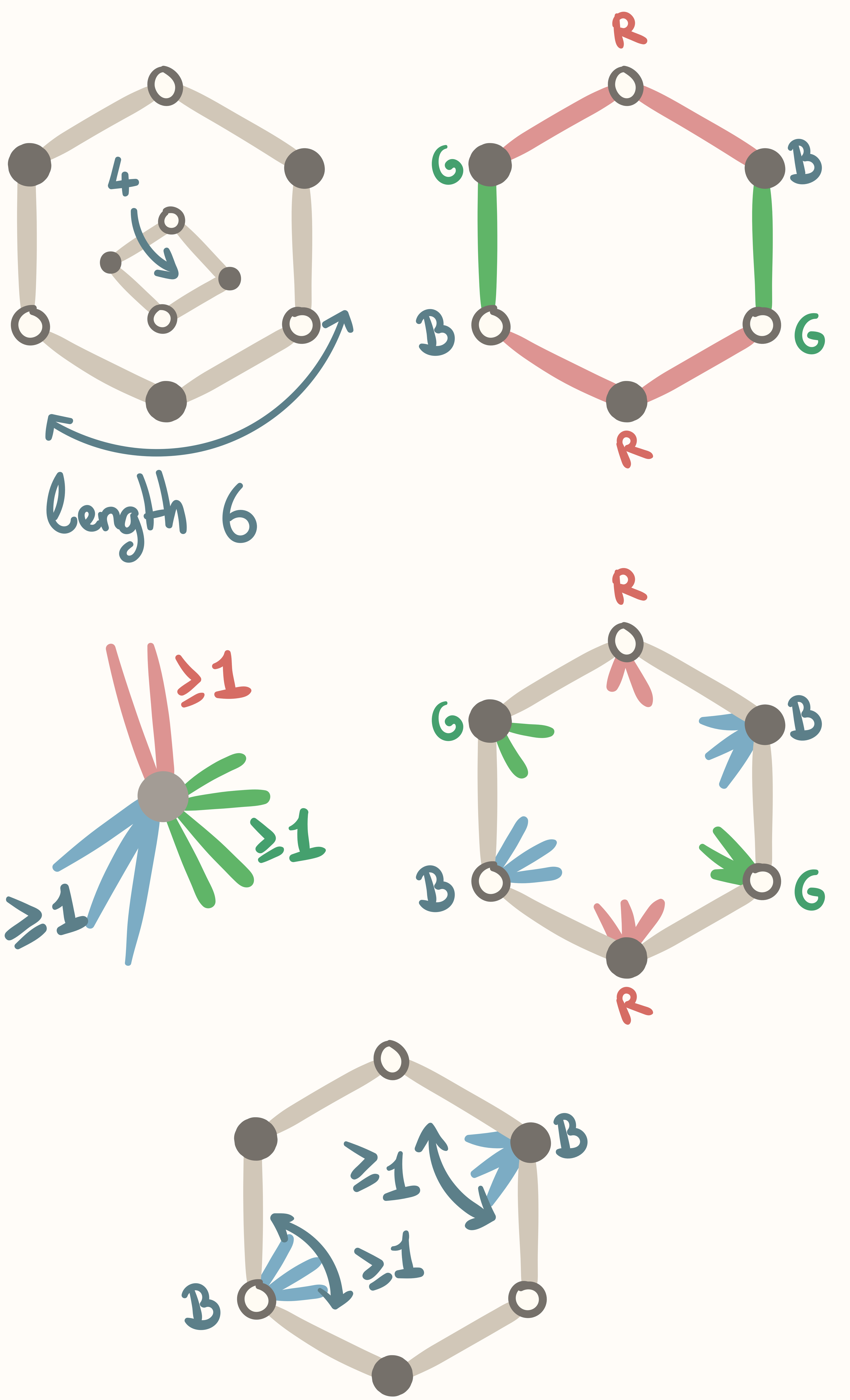

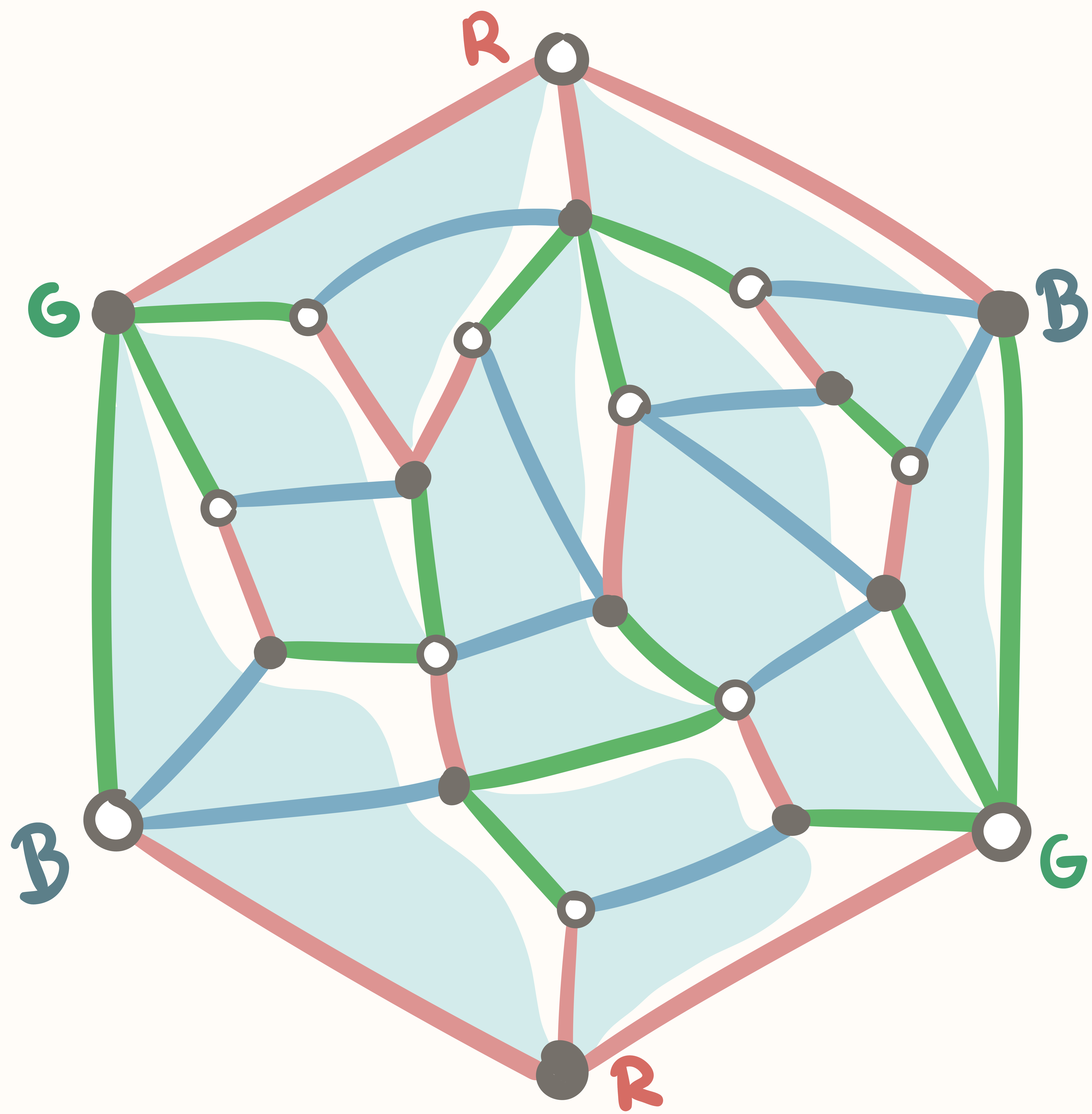

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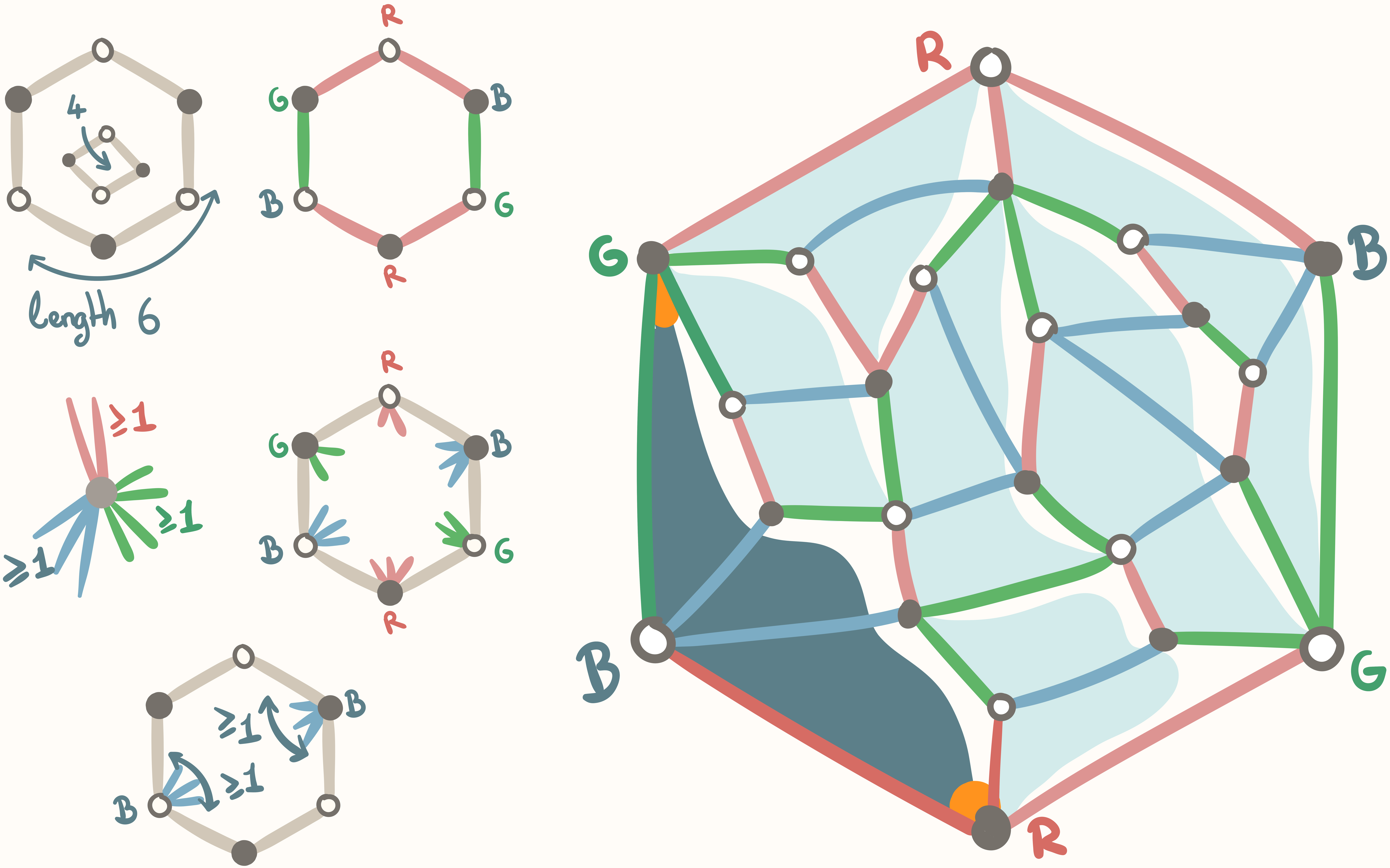

Schnyder labellings

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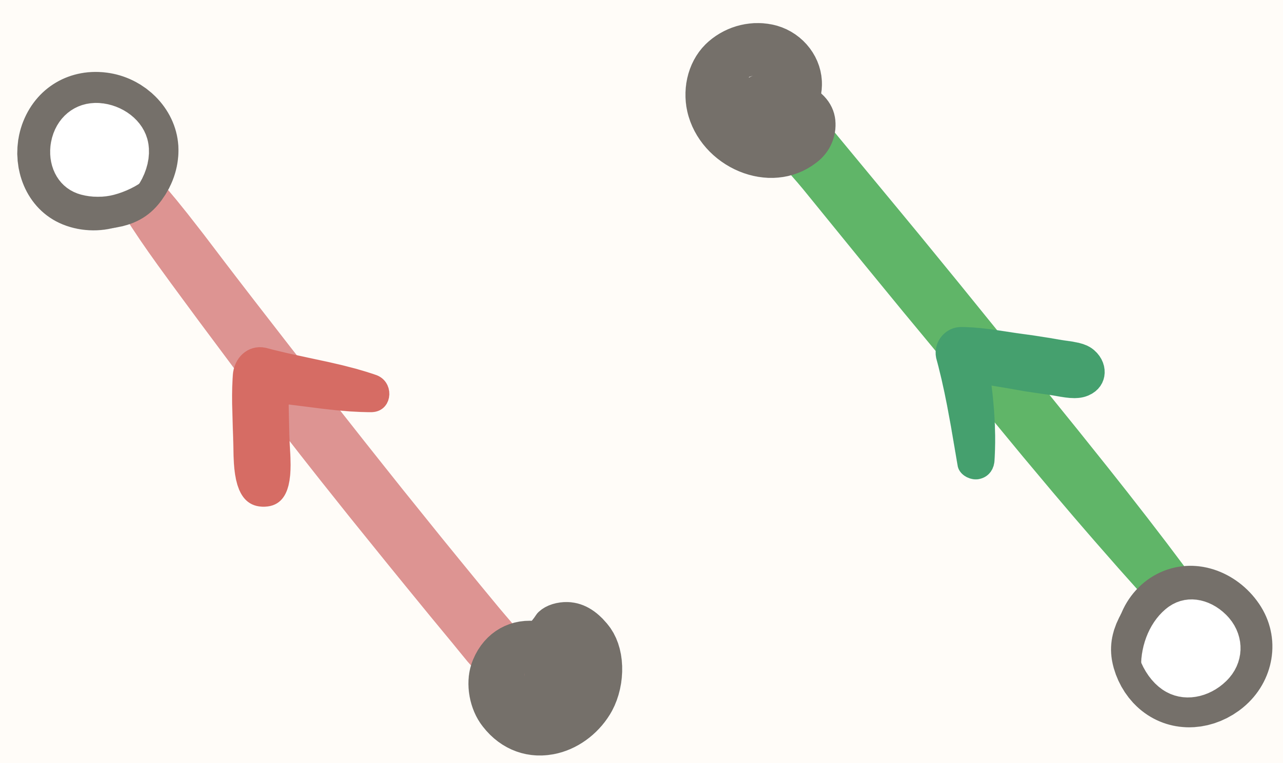

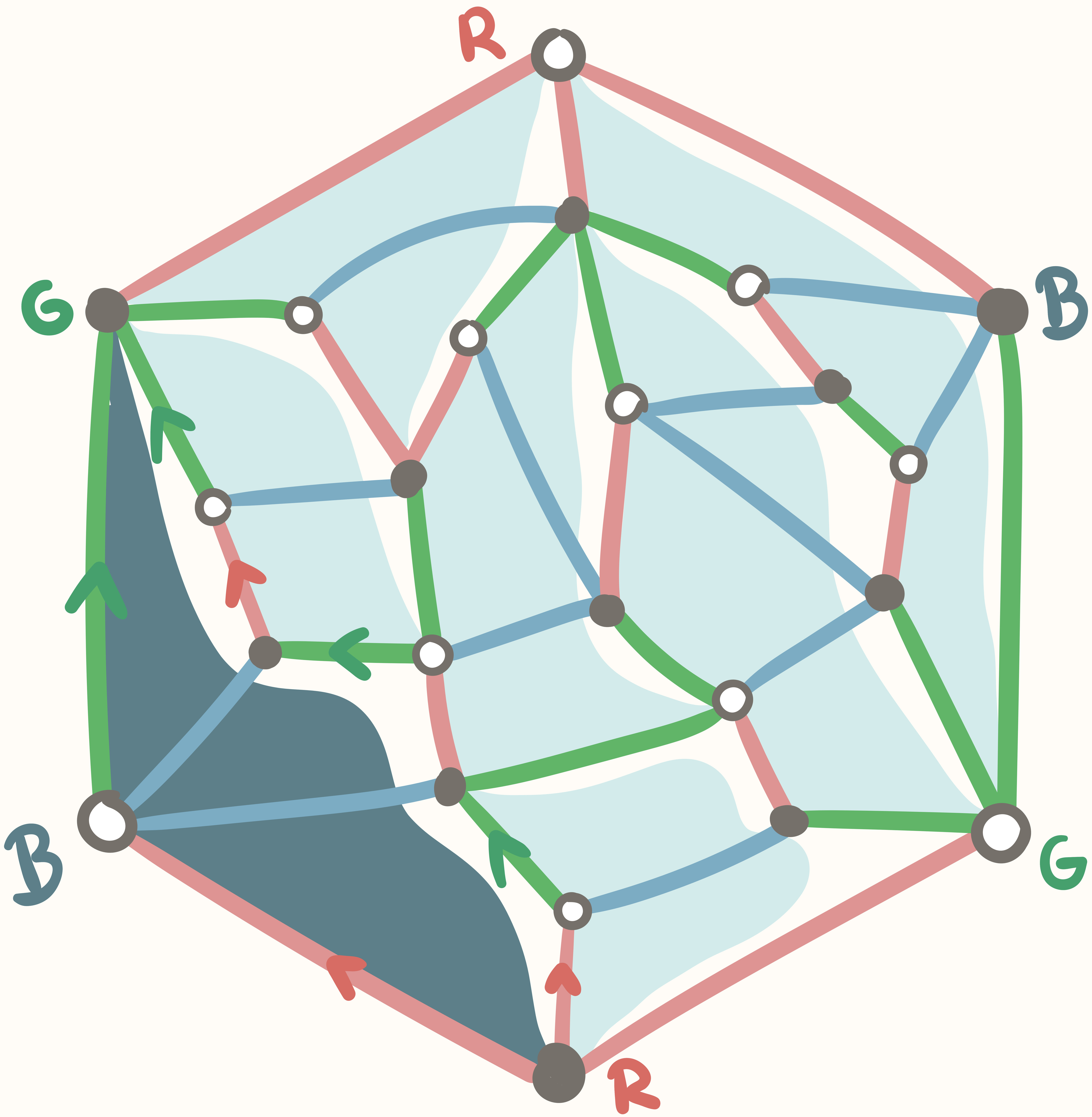

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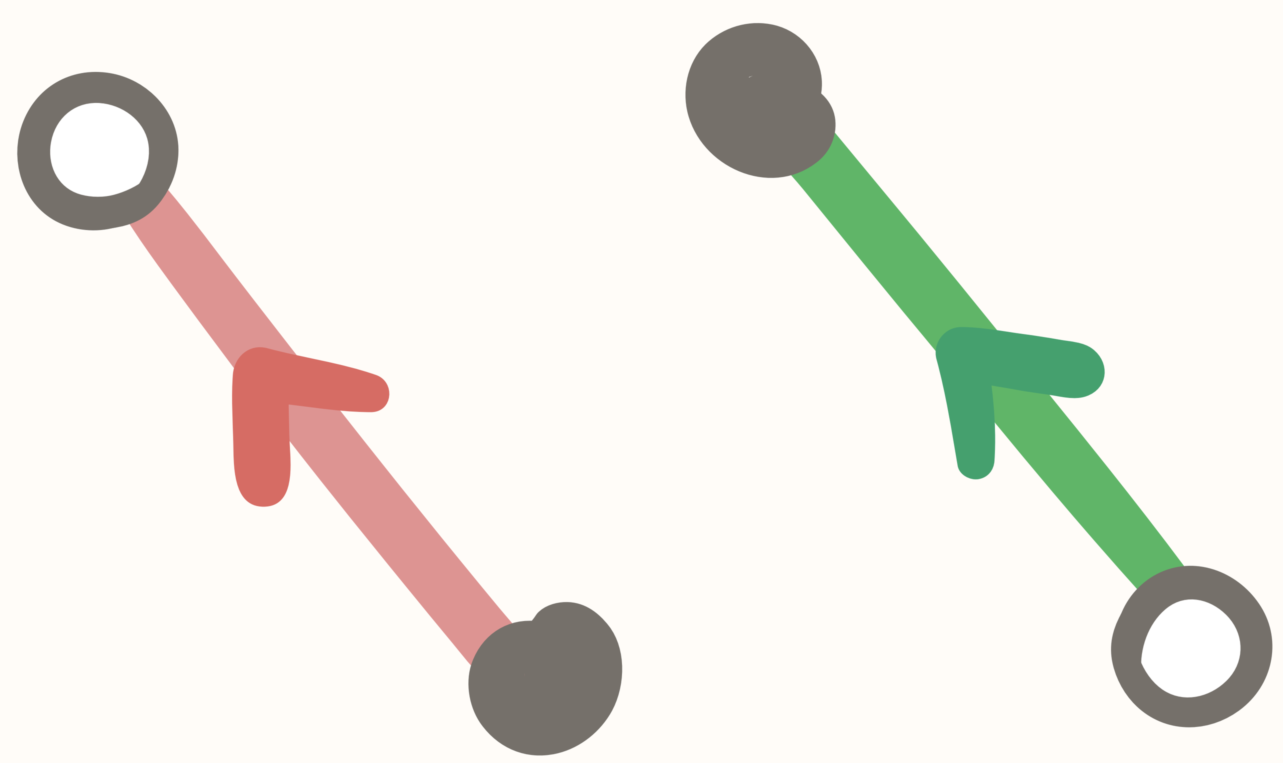

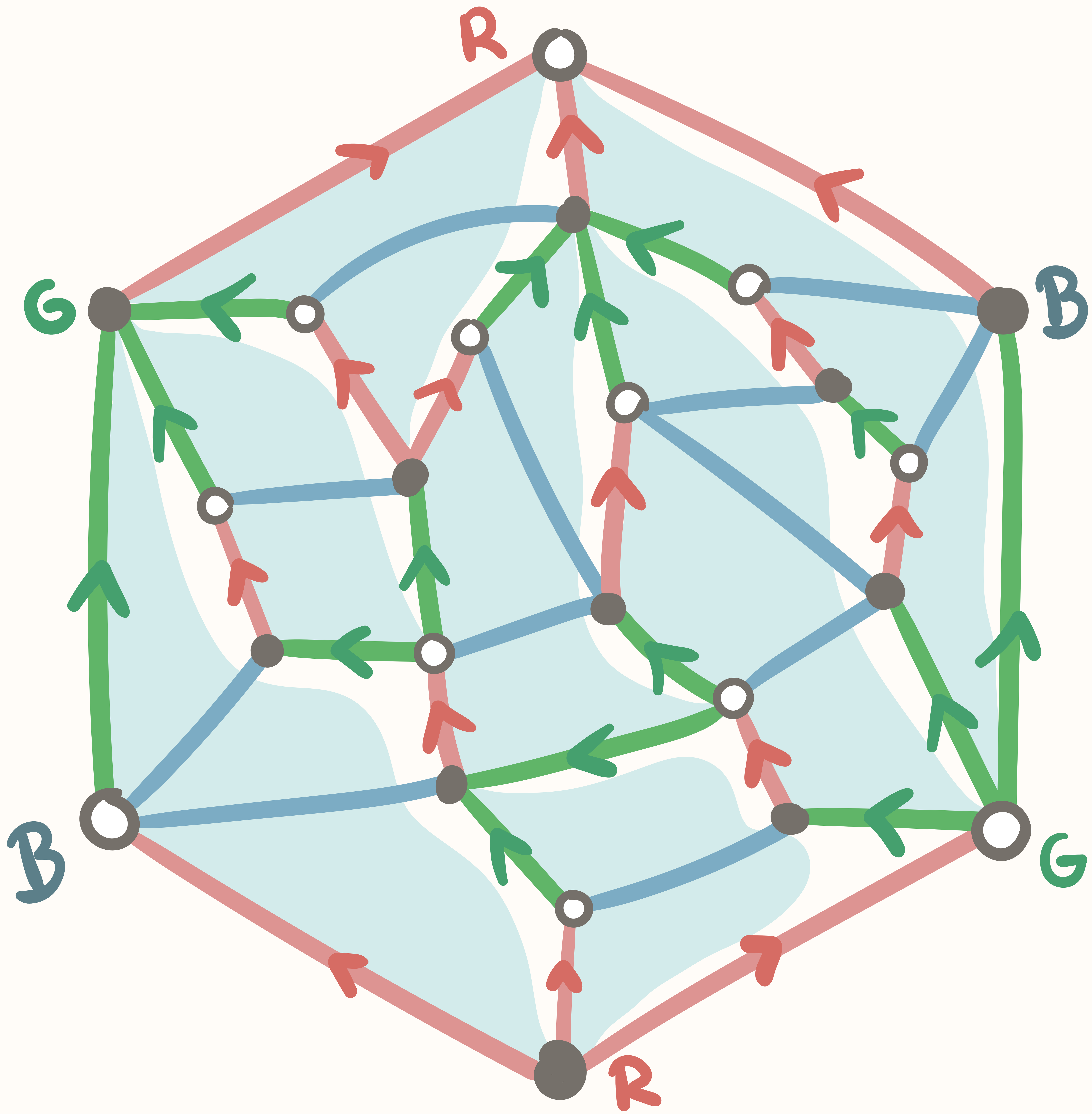

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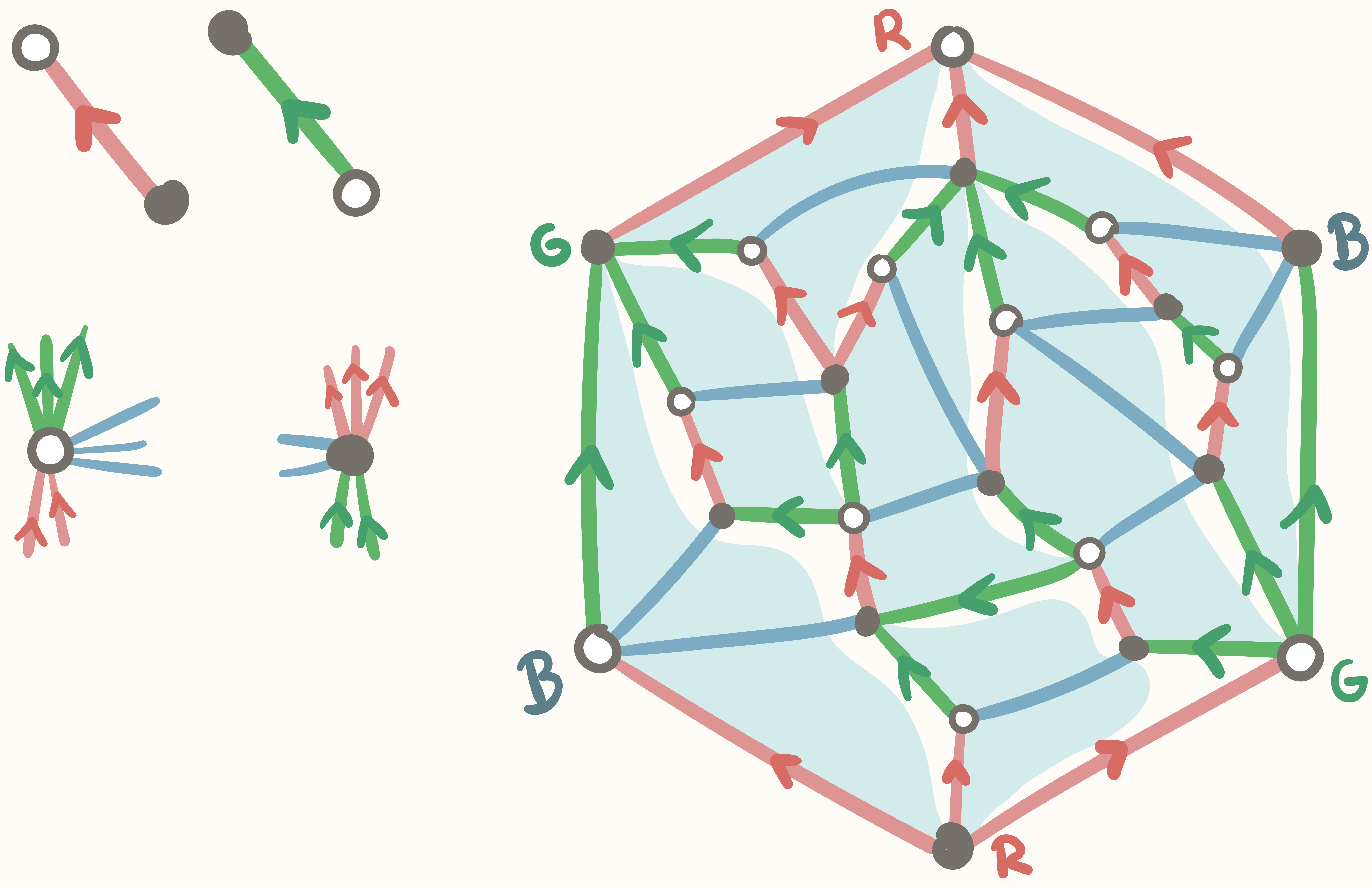

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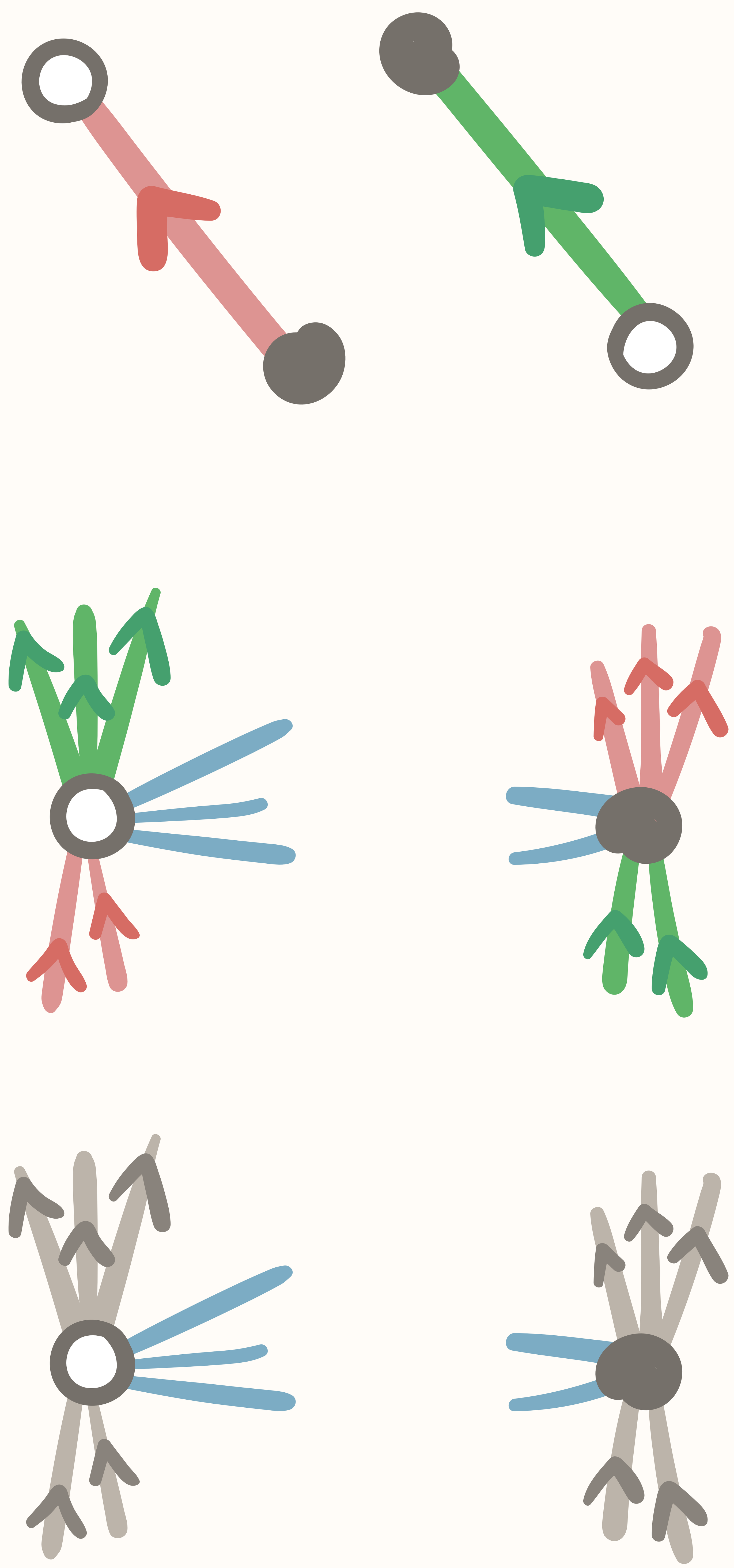
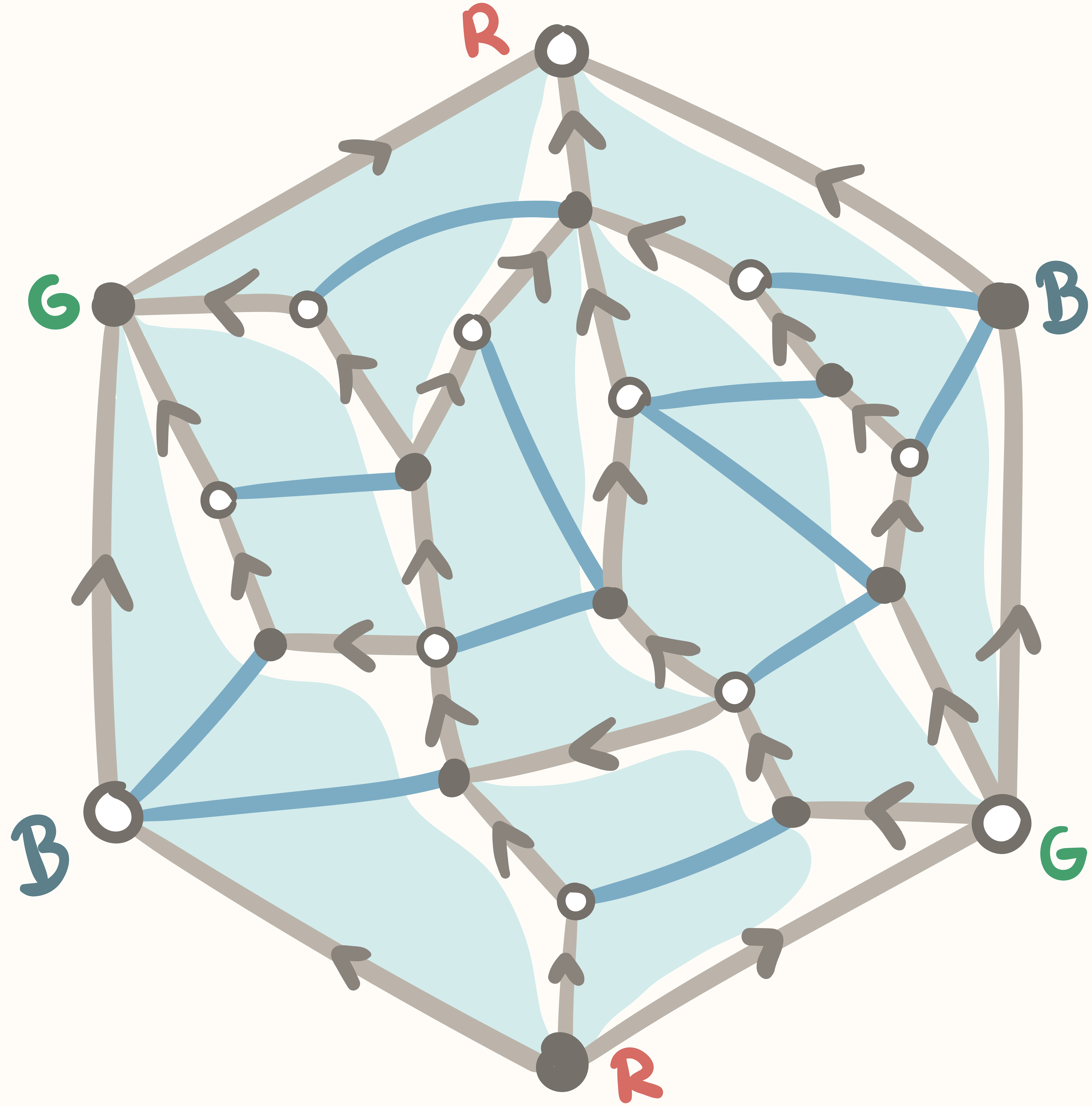


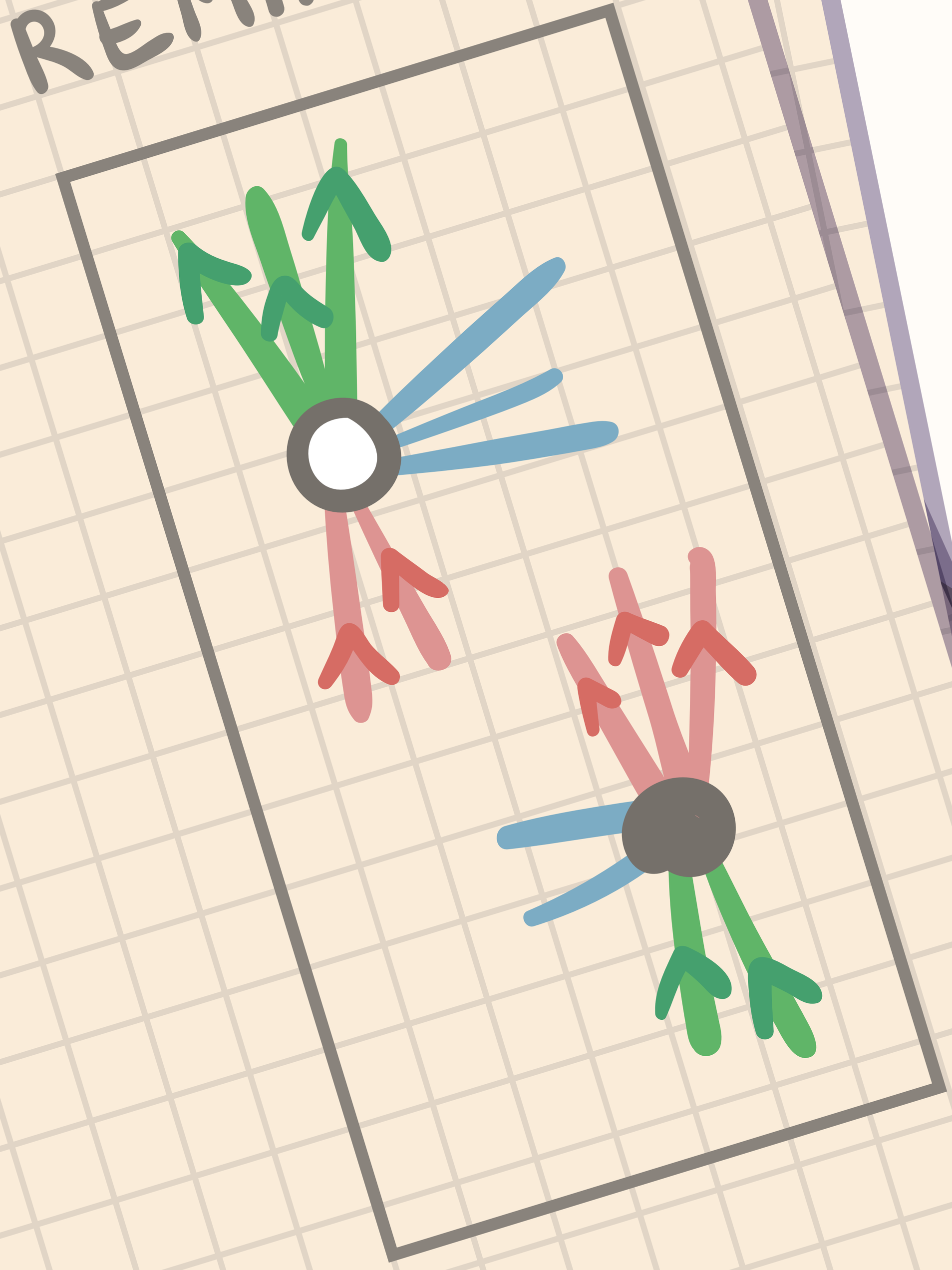


Schnyder labellin

IT IS A BIPOLAR ORIENTATION :

1) ACYCLIC

REMINDER



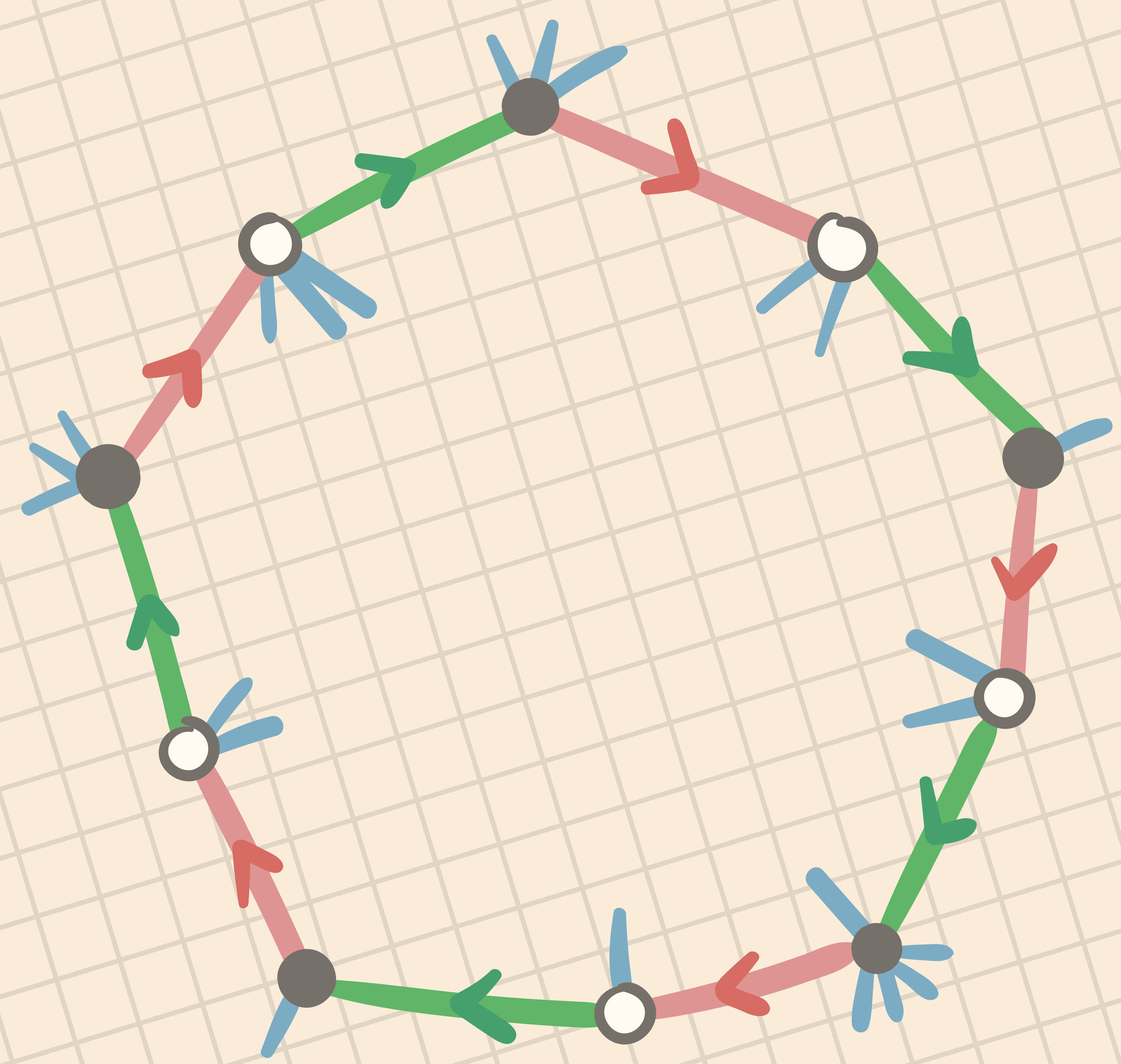
Schnyder labellin

Schnyder labellin

IT IS A BIPOLAR ORIENTATION :

1) ACYCLIC

ASSUME ∃ CYCLE, TAKEN MINIMAL



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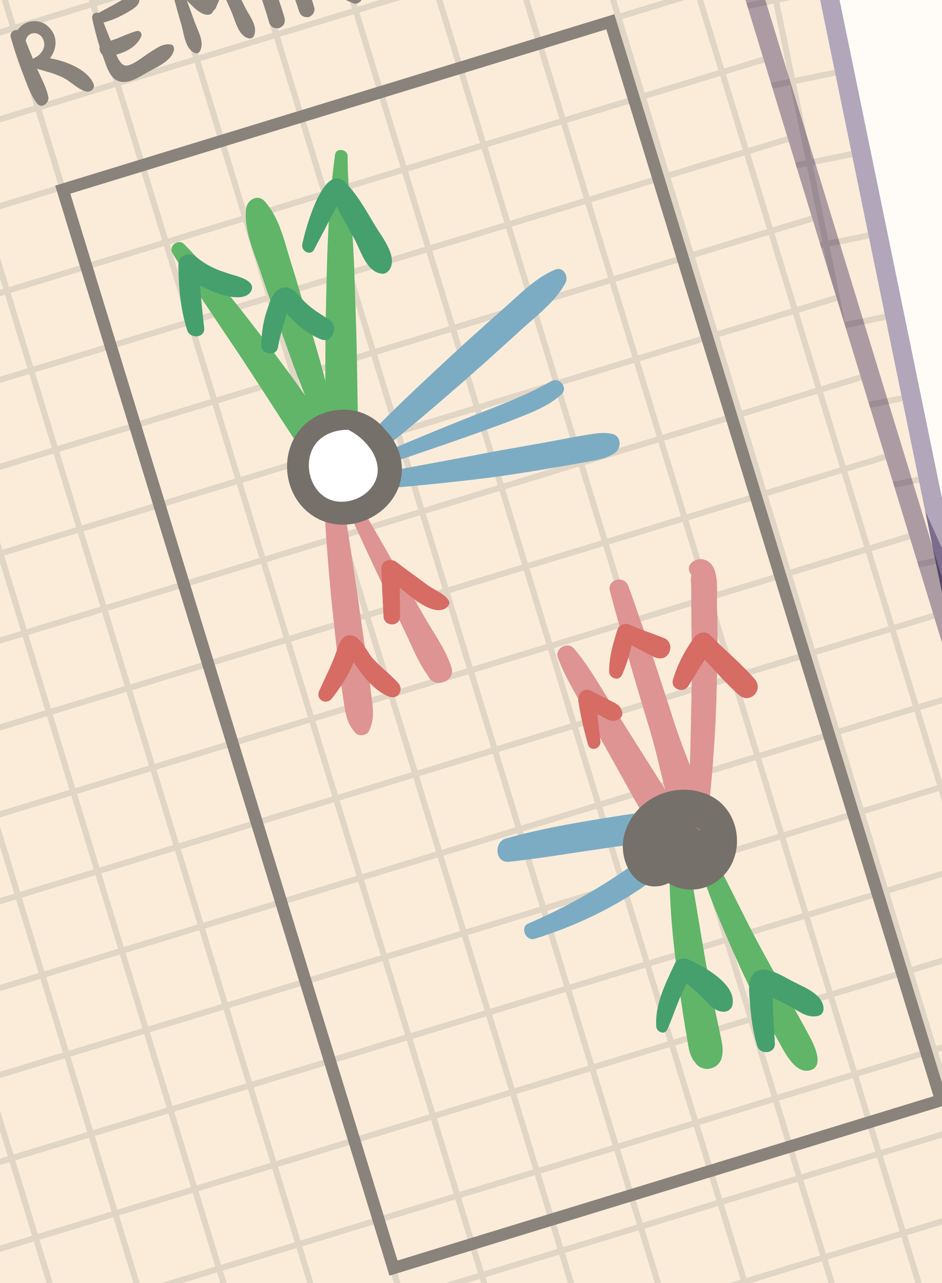

B

G

R

Schnyder labellin

Schnyder labellin

Schnyder labellin

Schnyder labelling

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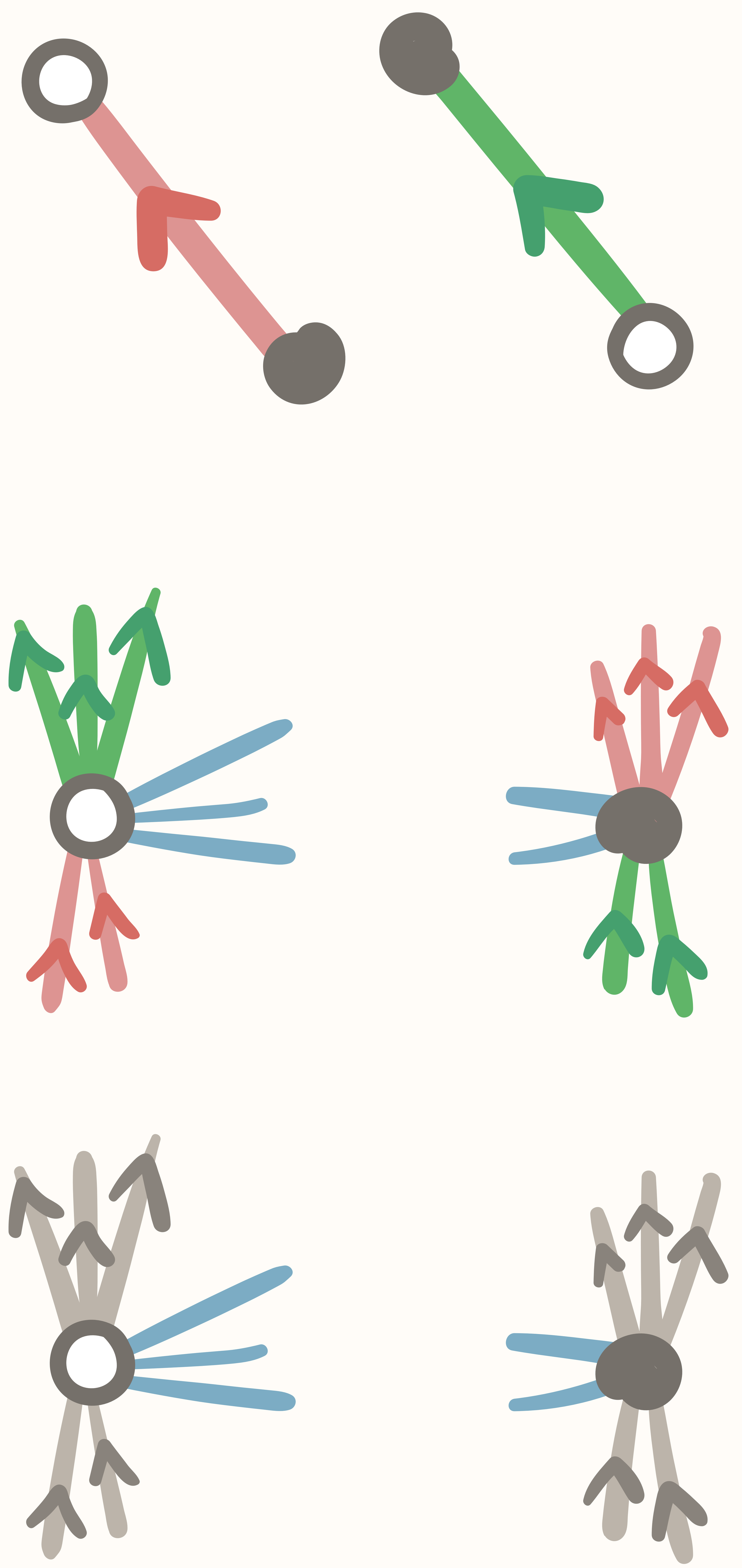
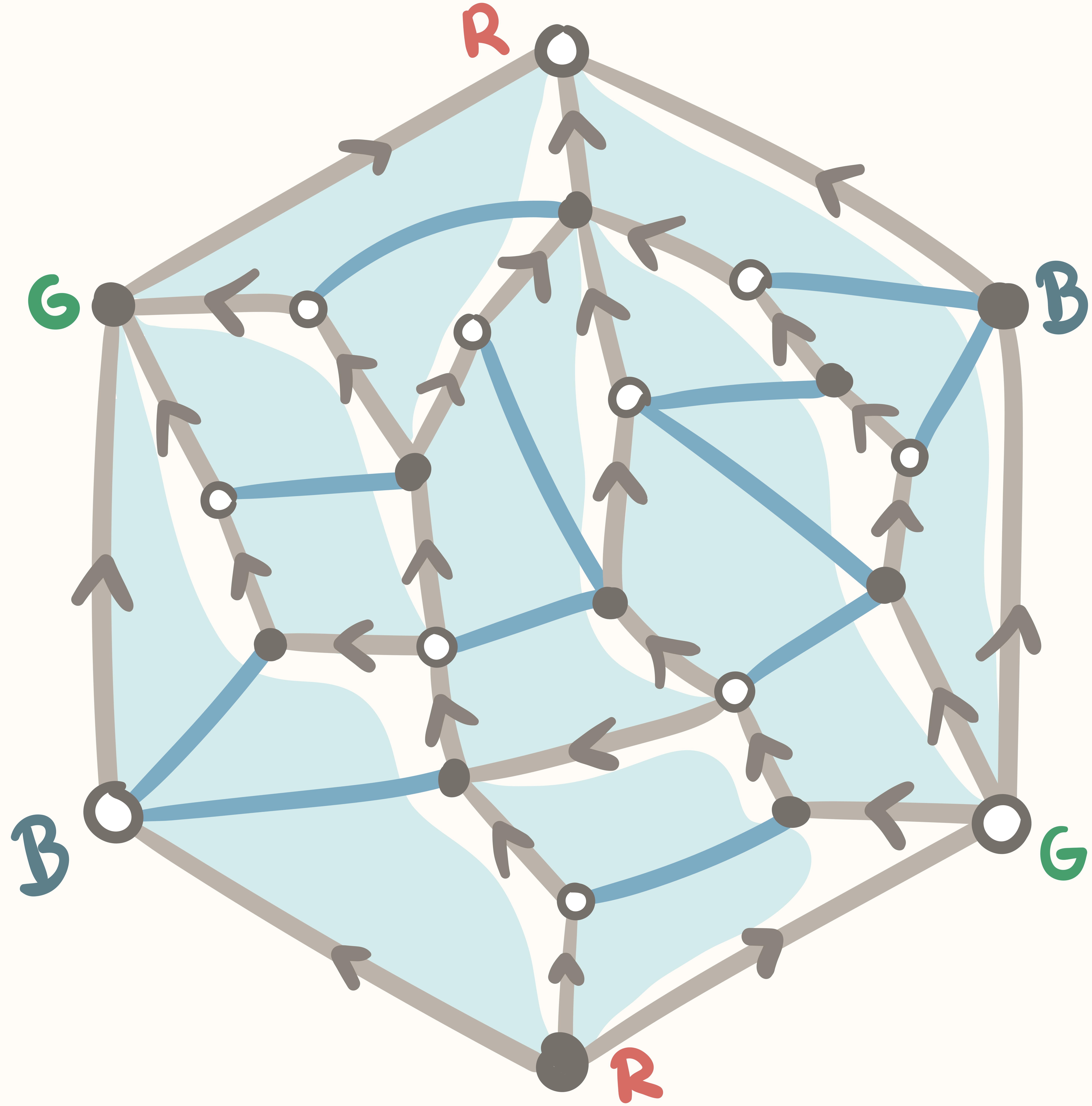
Schnyder labelling

Schnyder labellin

Schnyder labelling

Schnyder labellings





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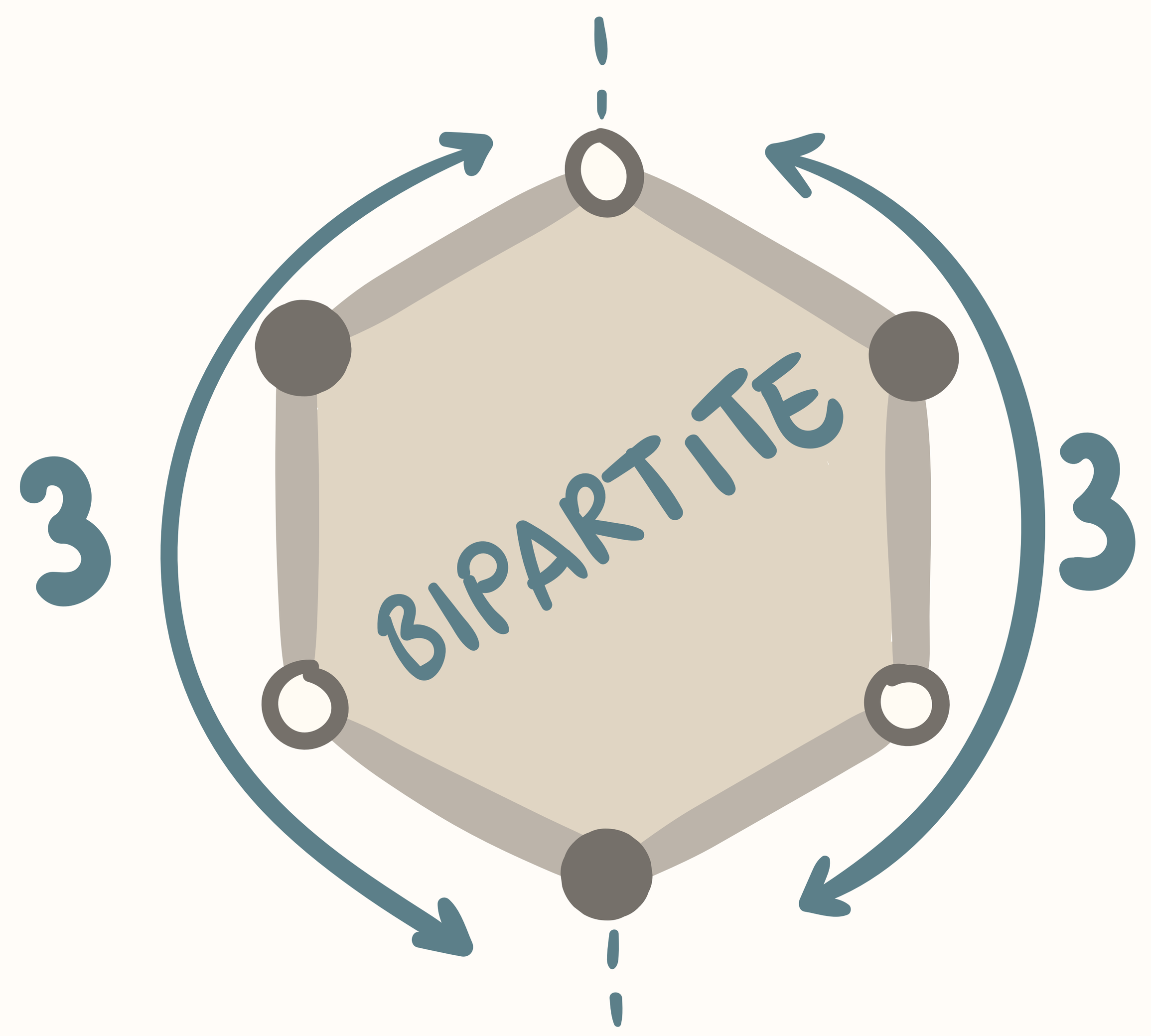

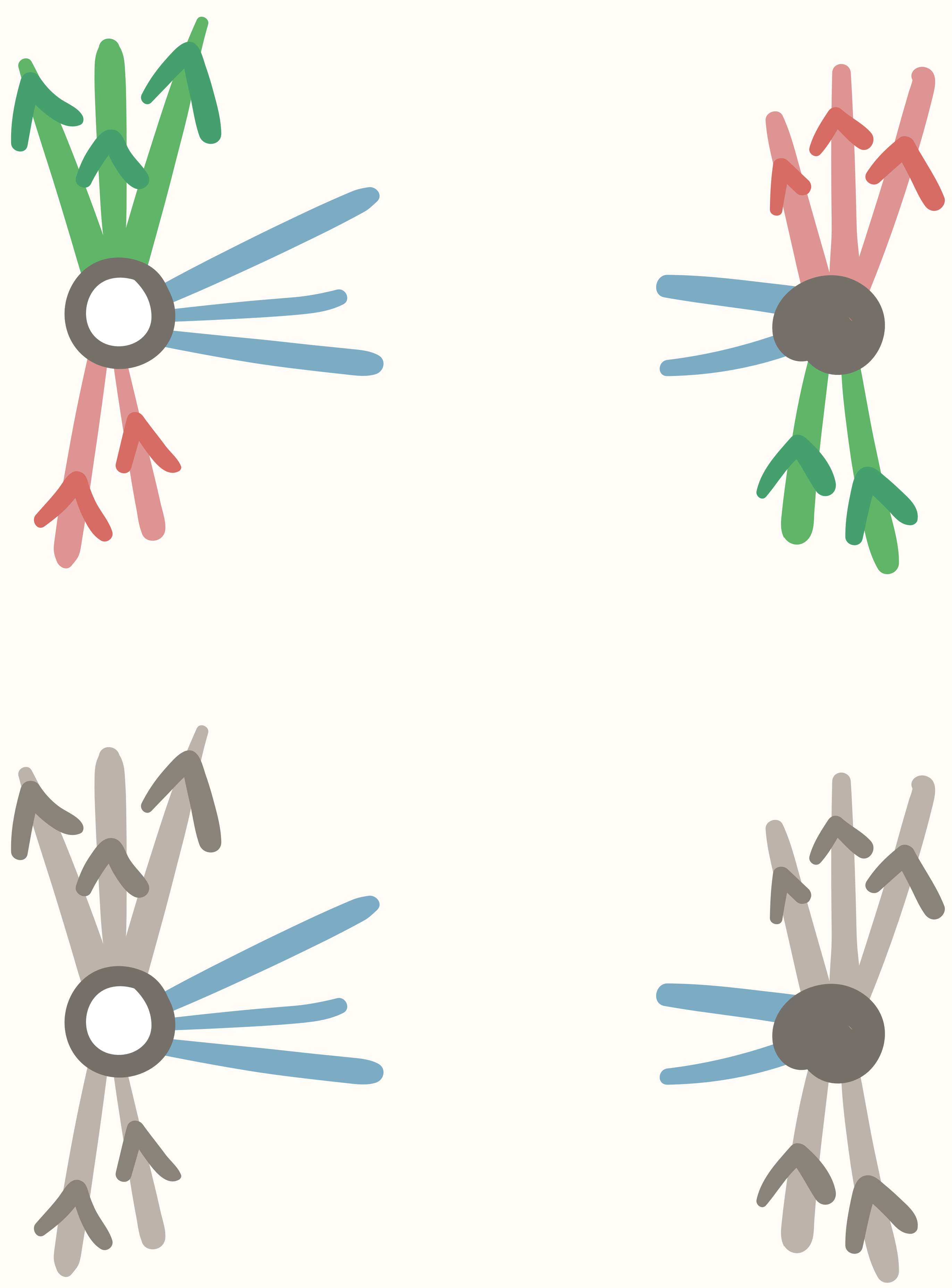

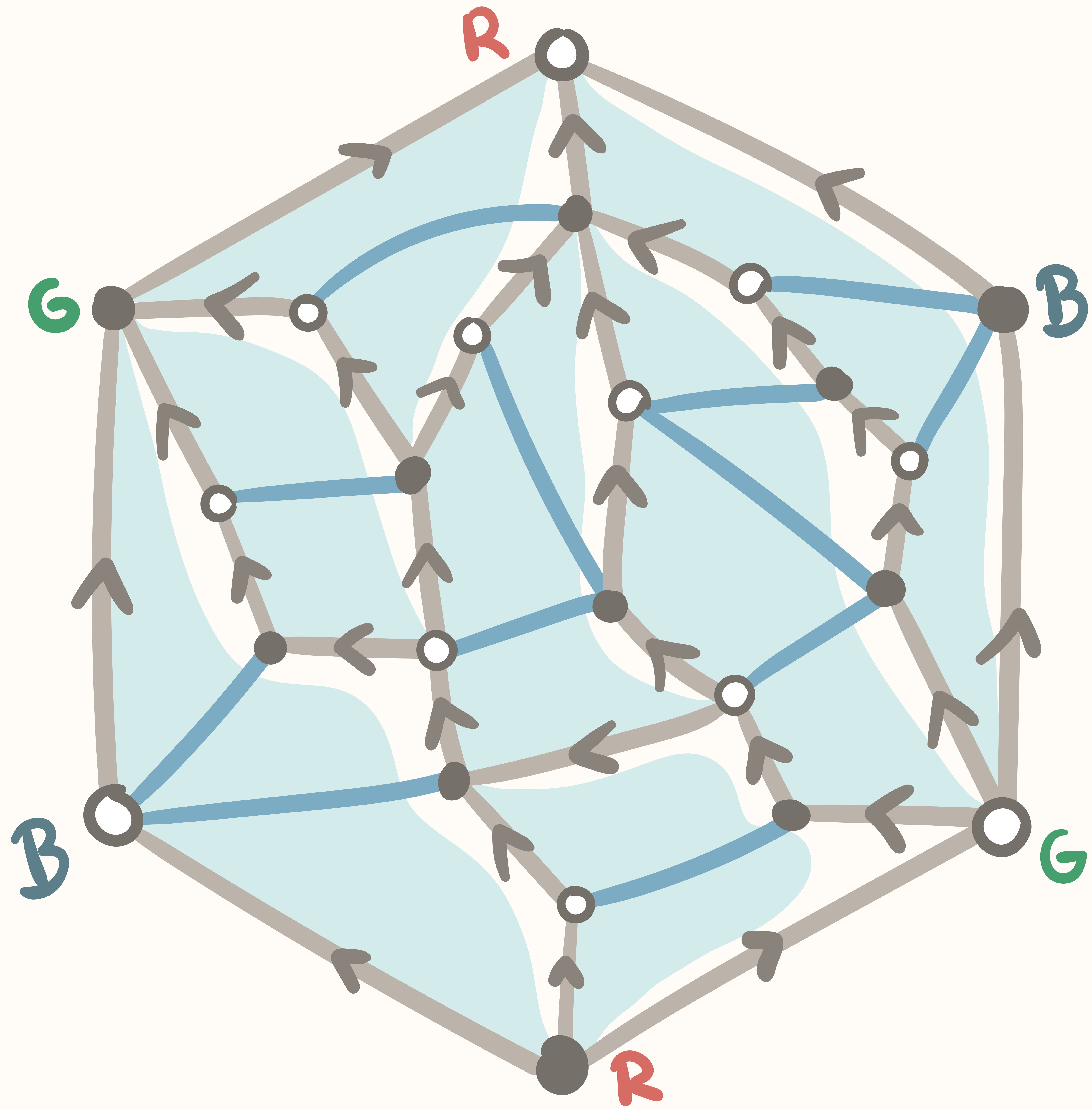

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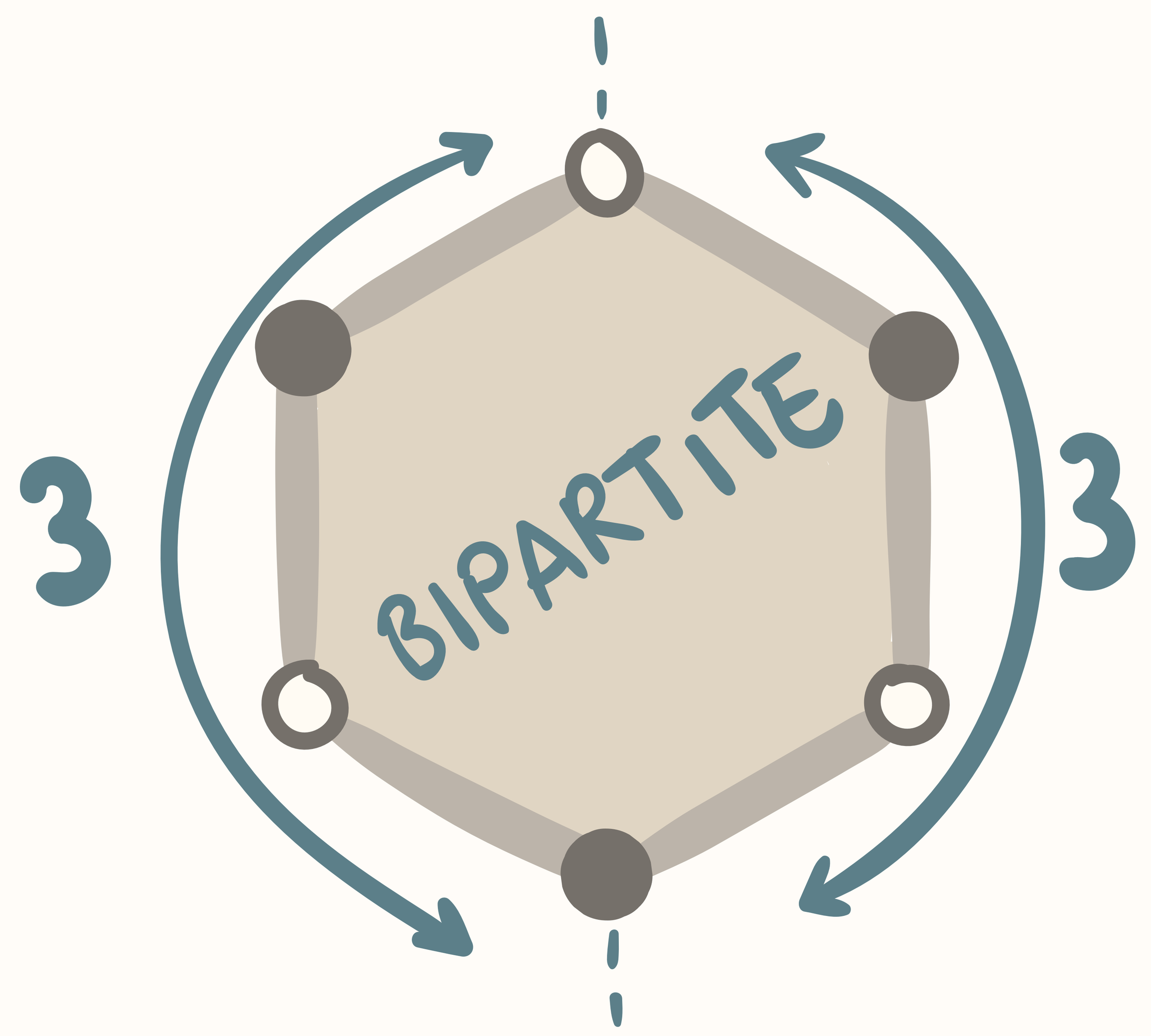

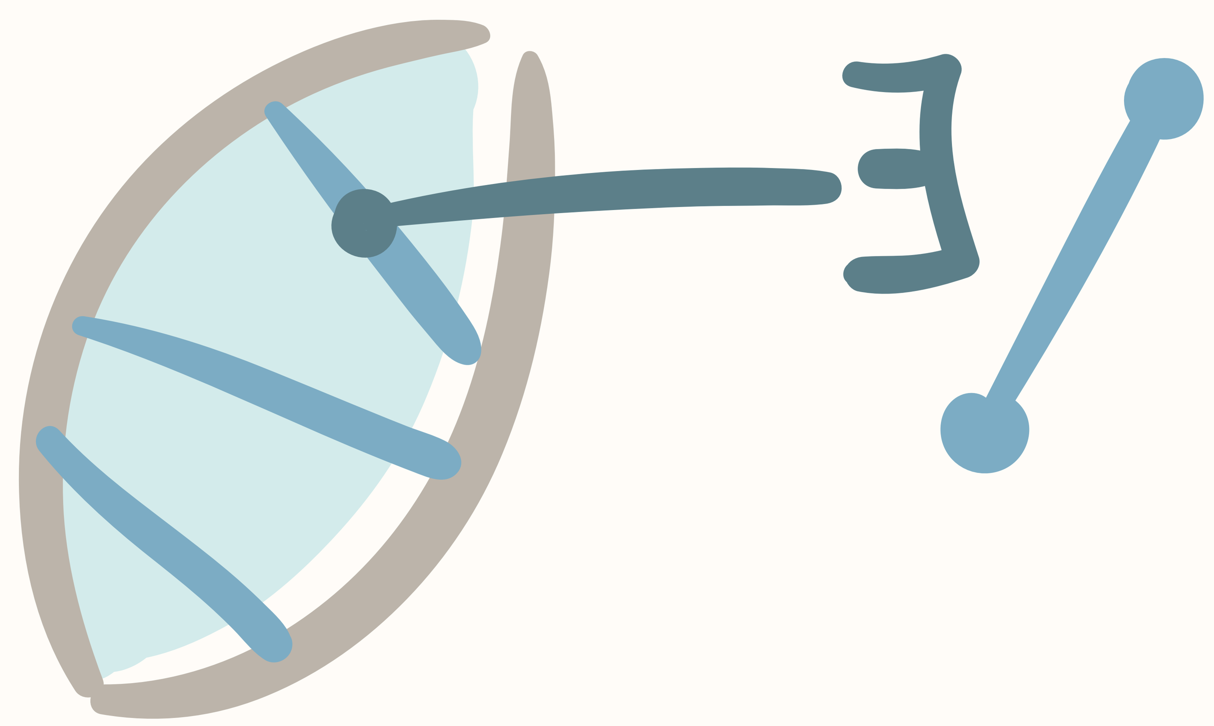

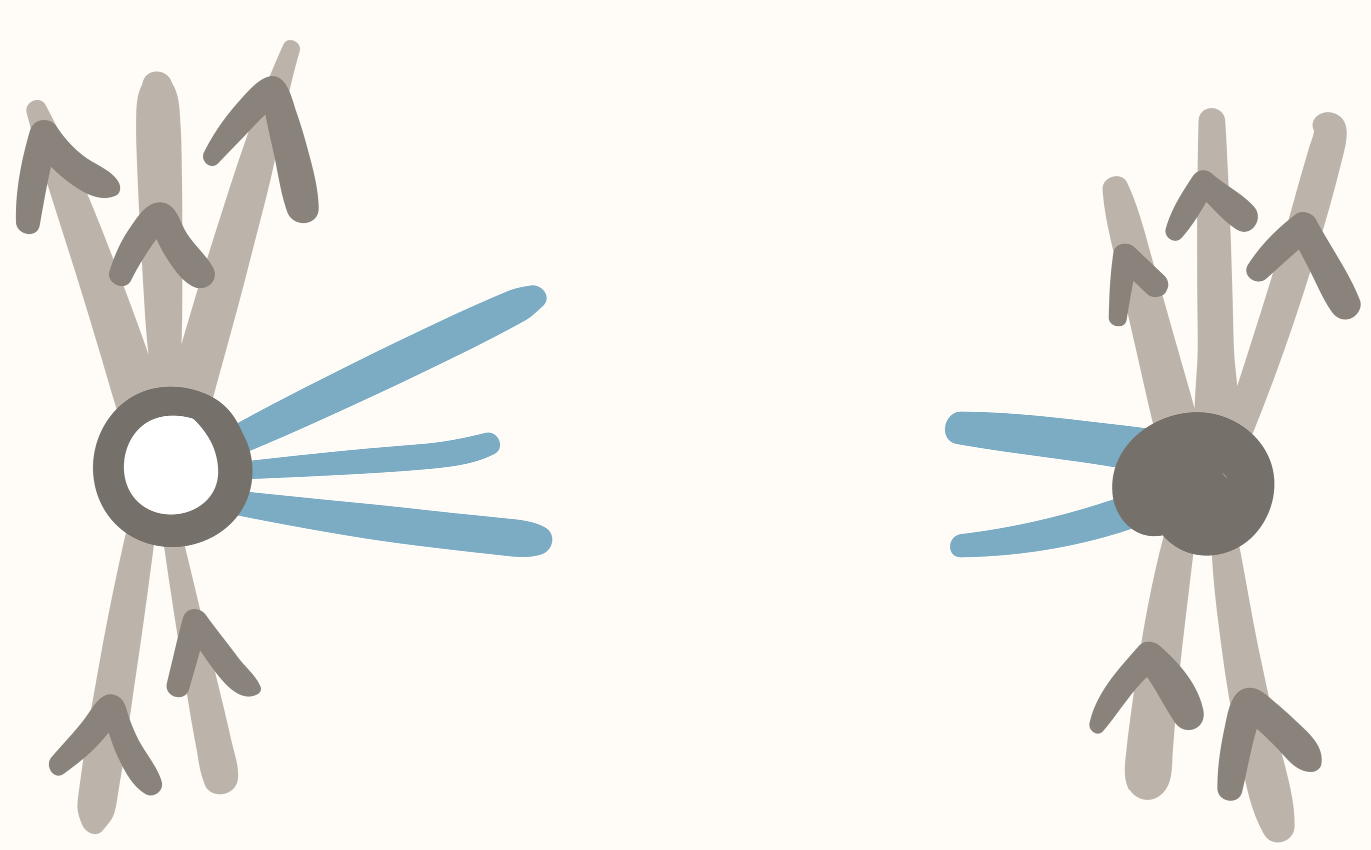

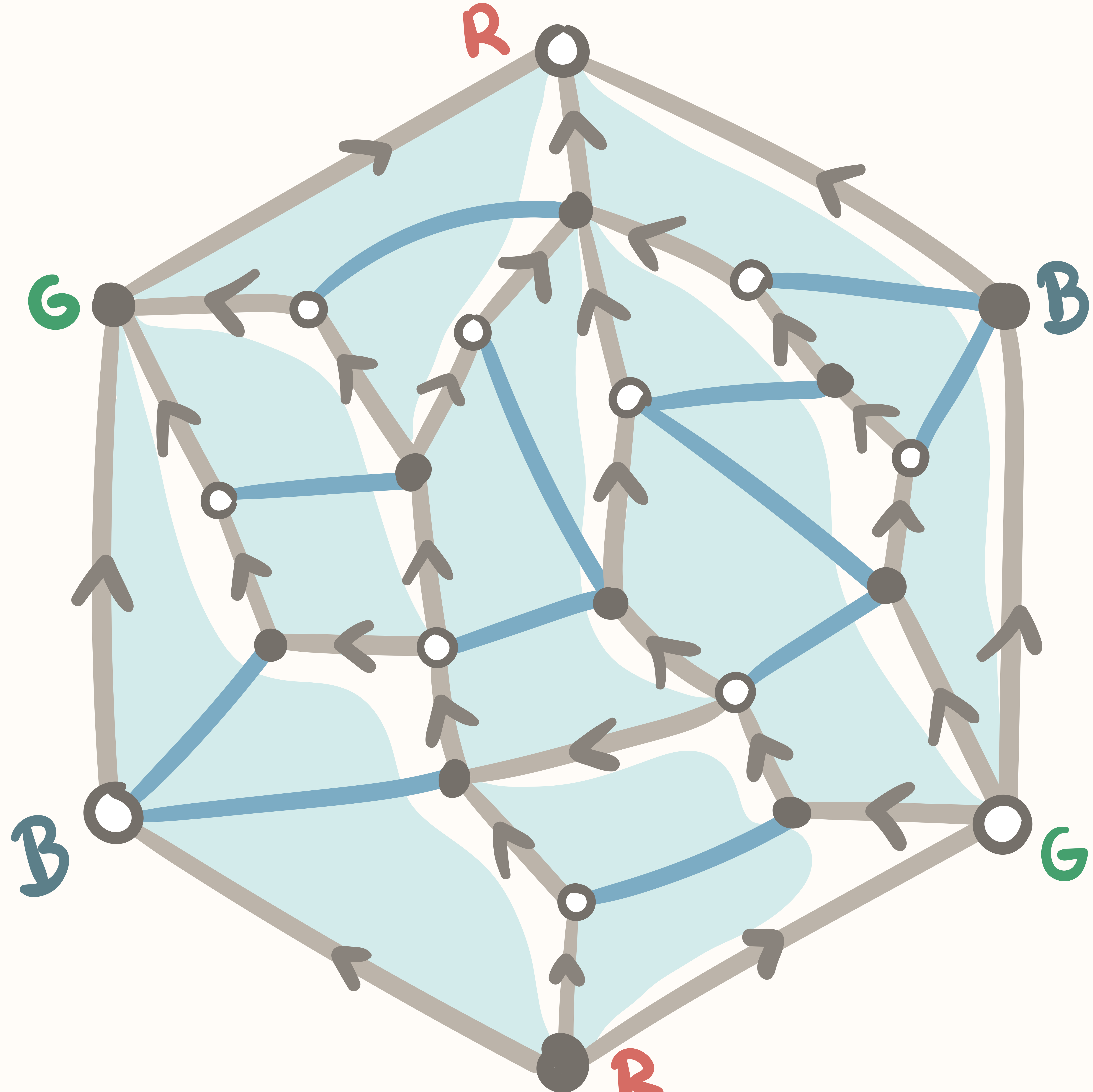

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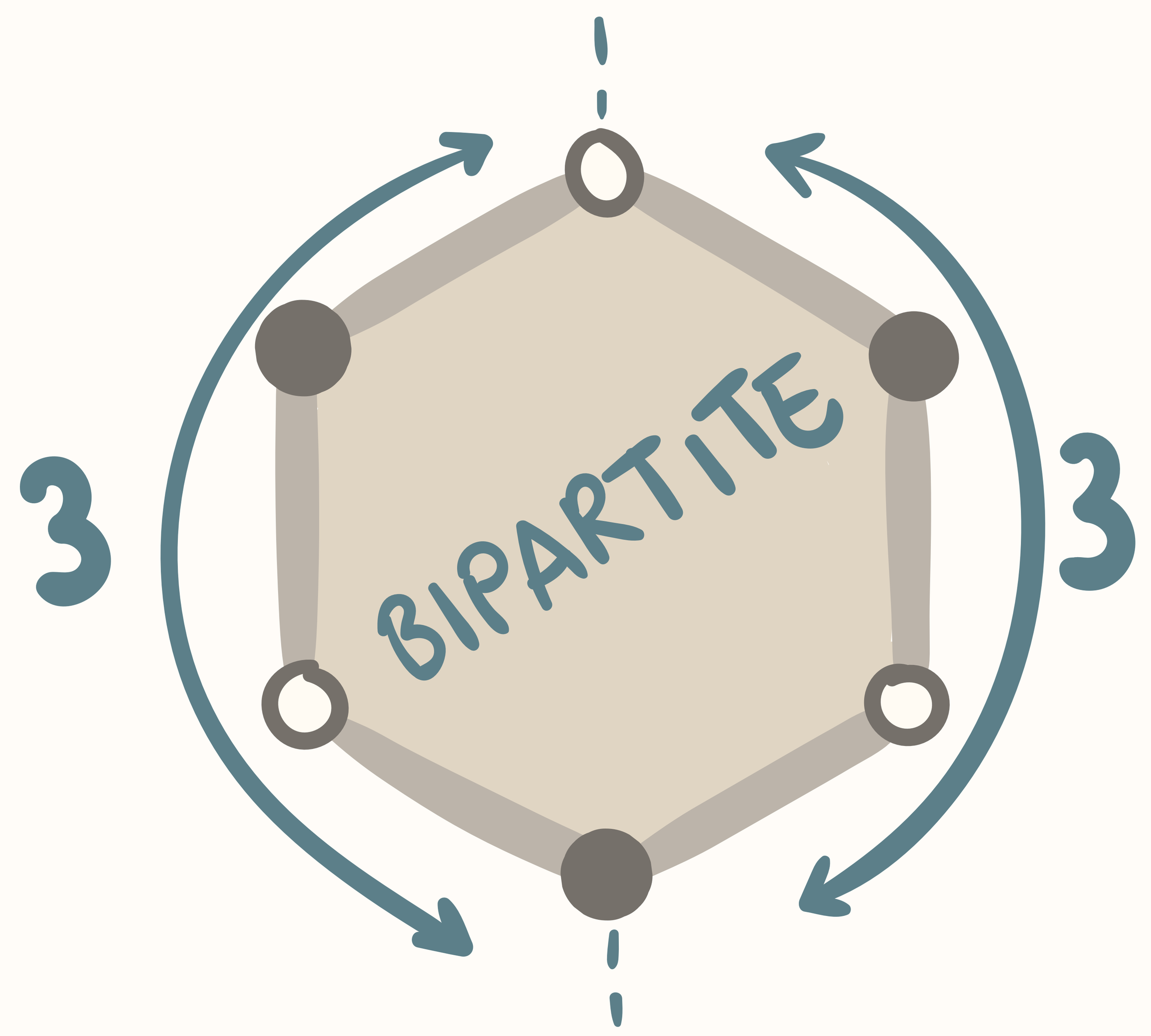
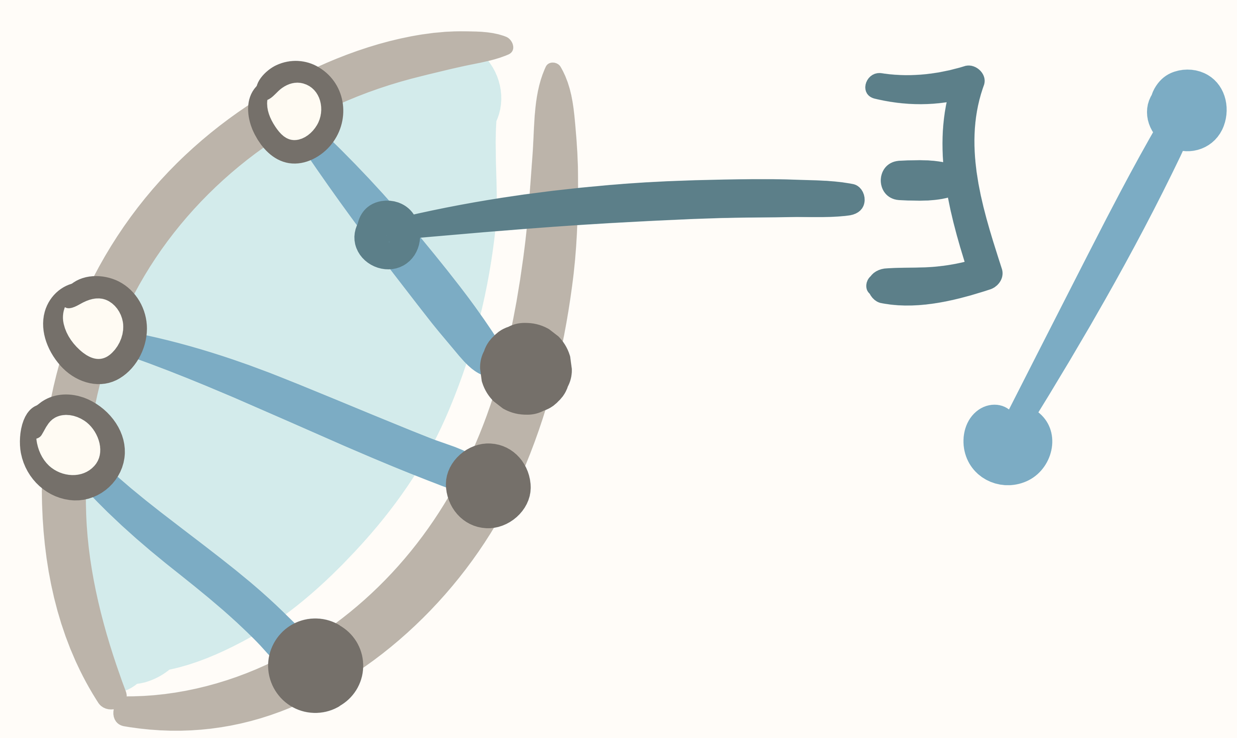
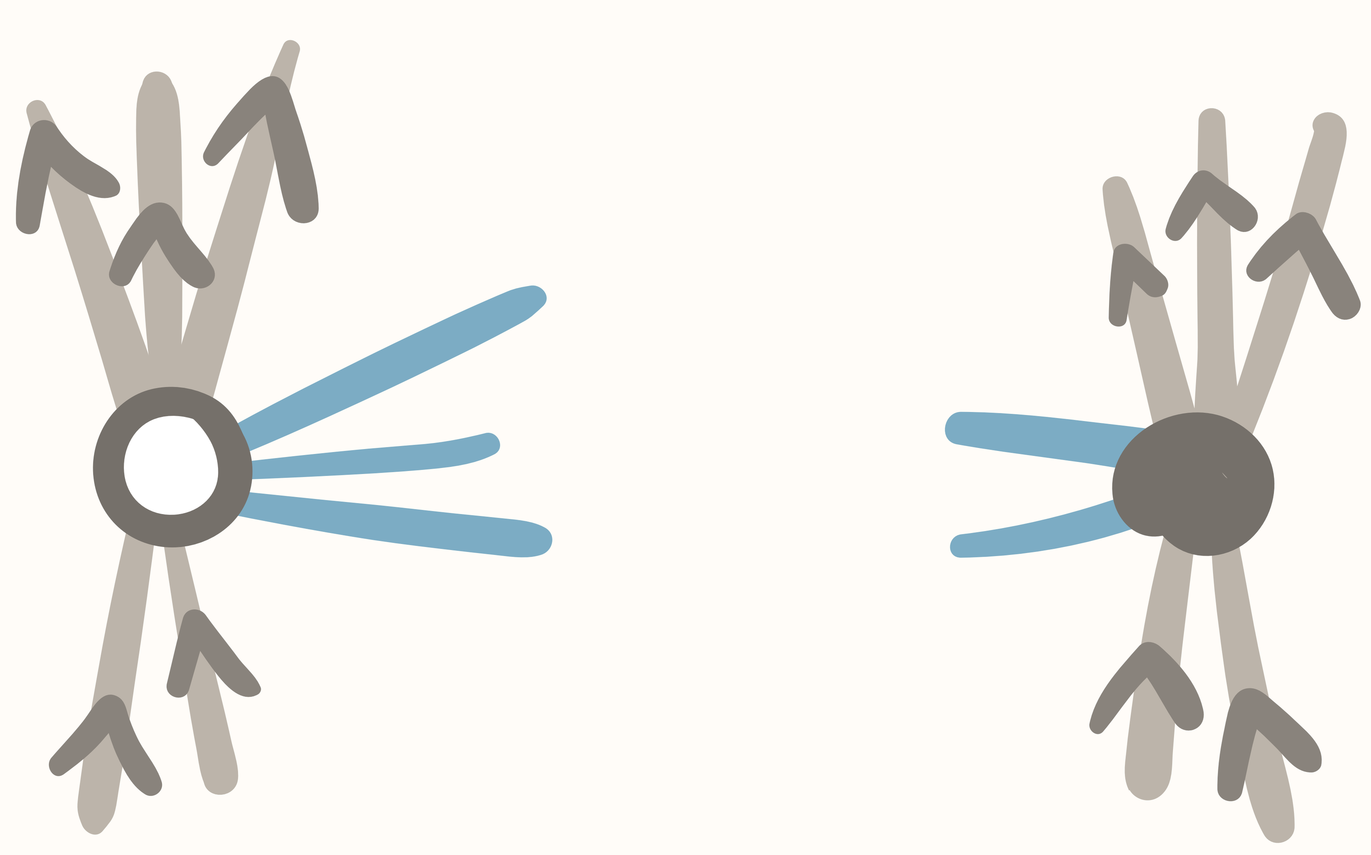
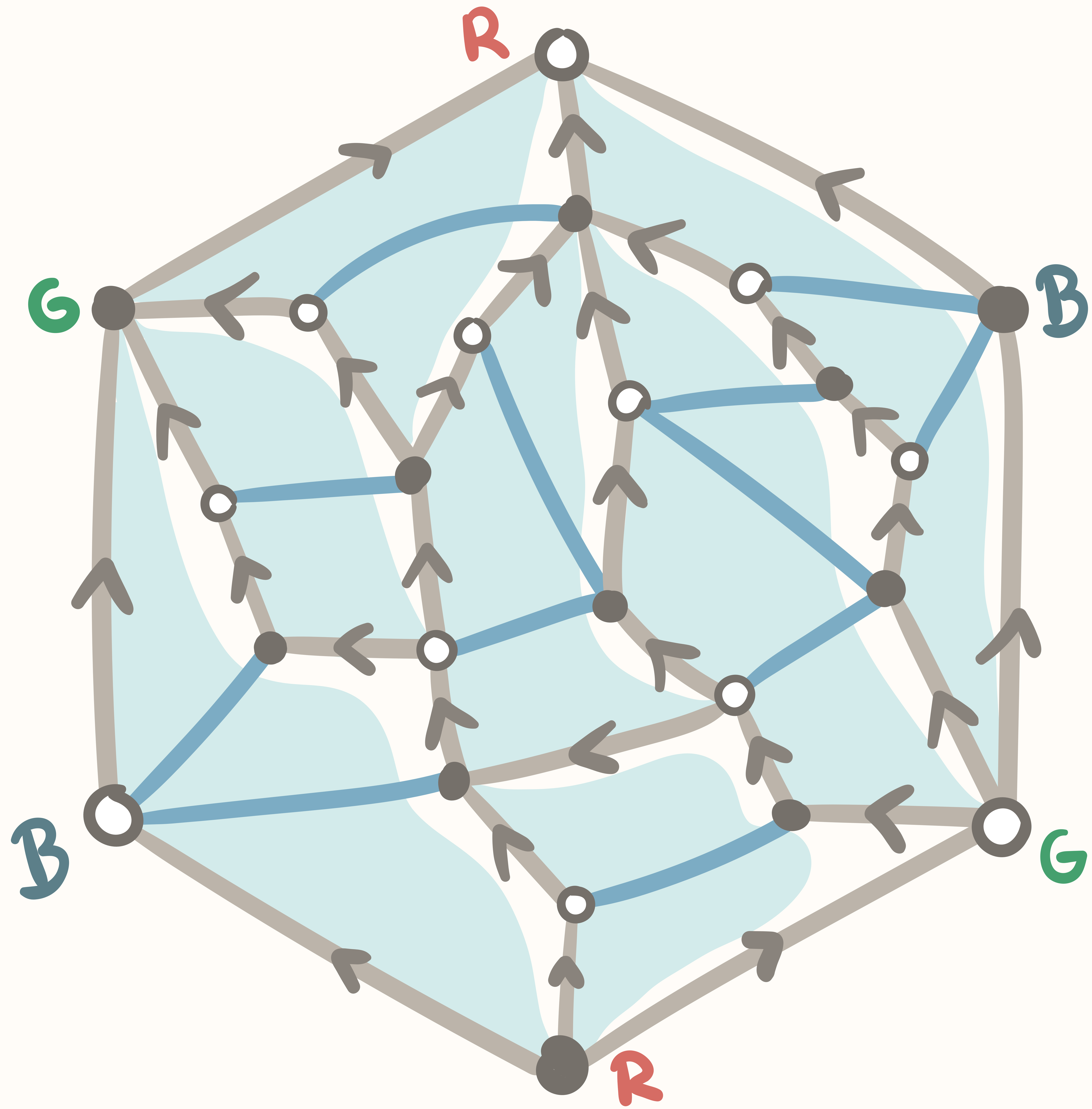
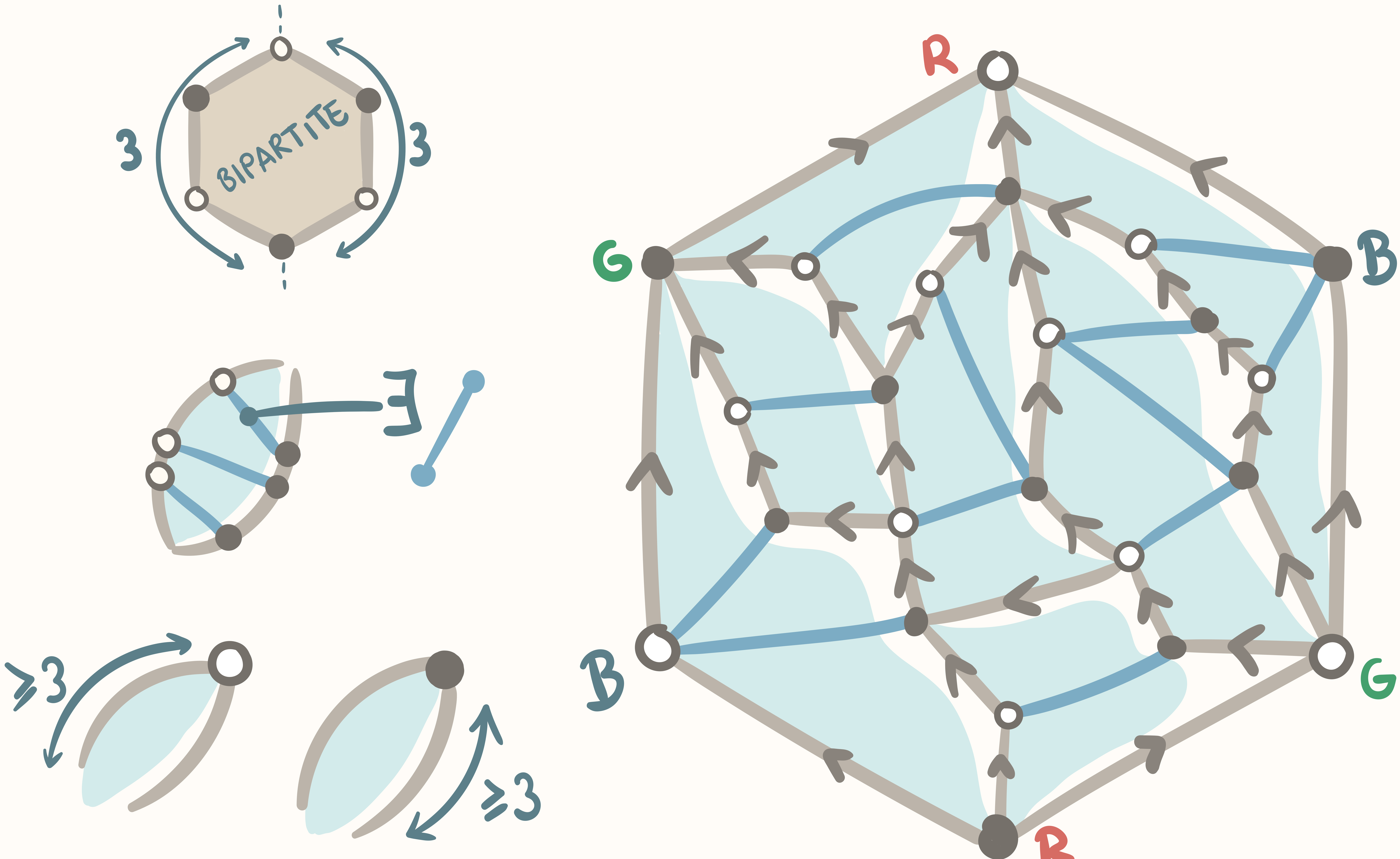




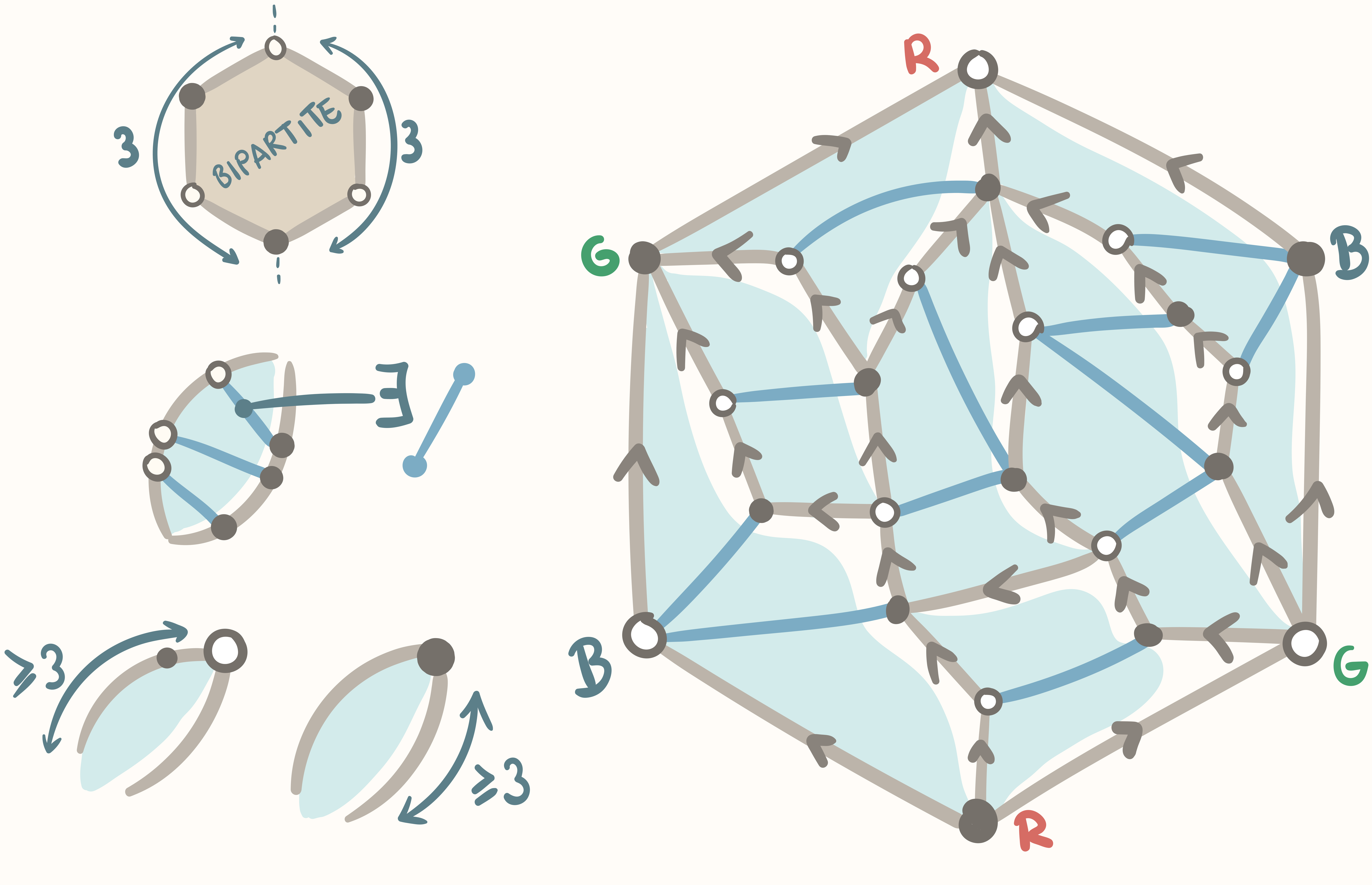




Schnyder labellings



Schnyder labellings



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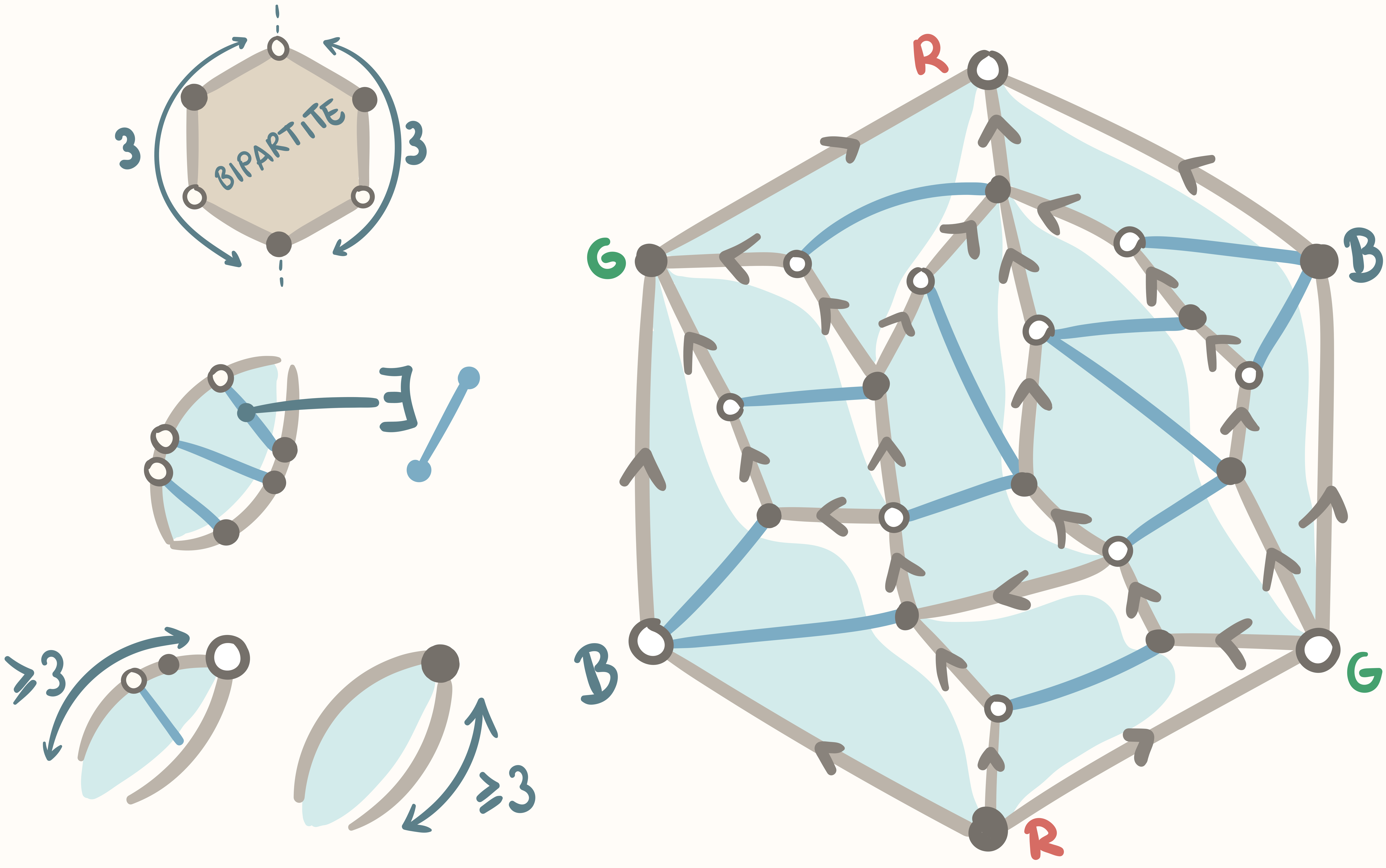
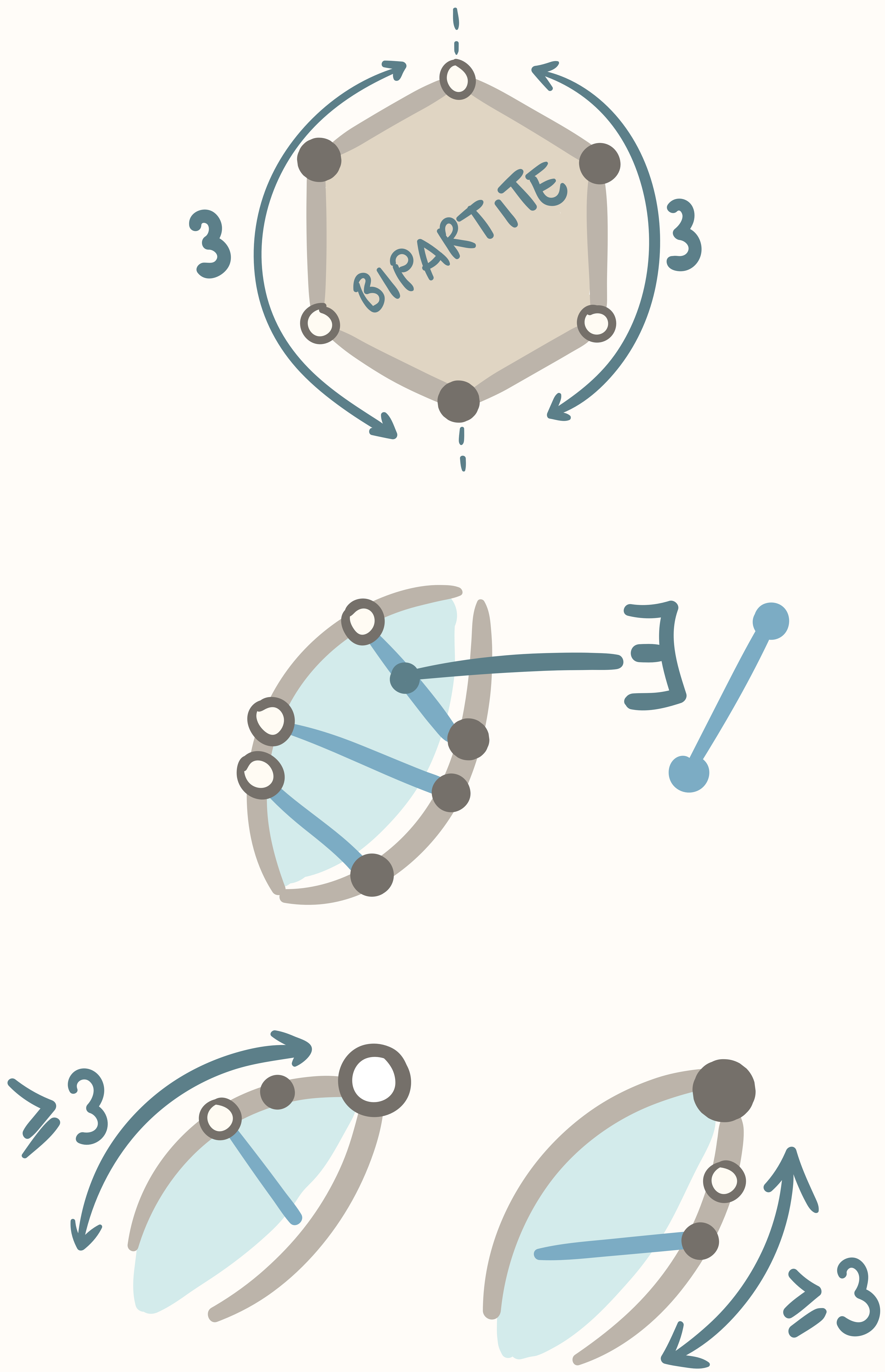
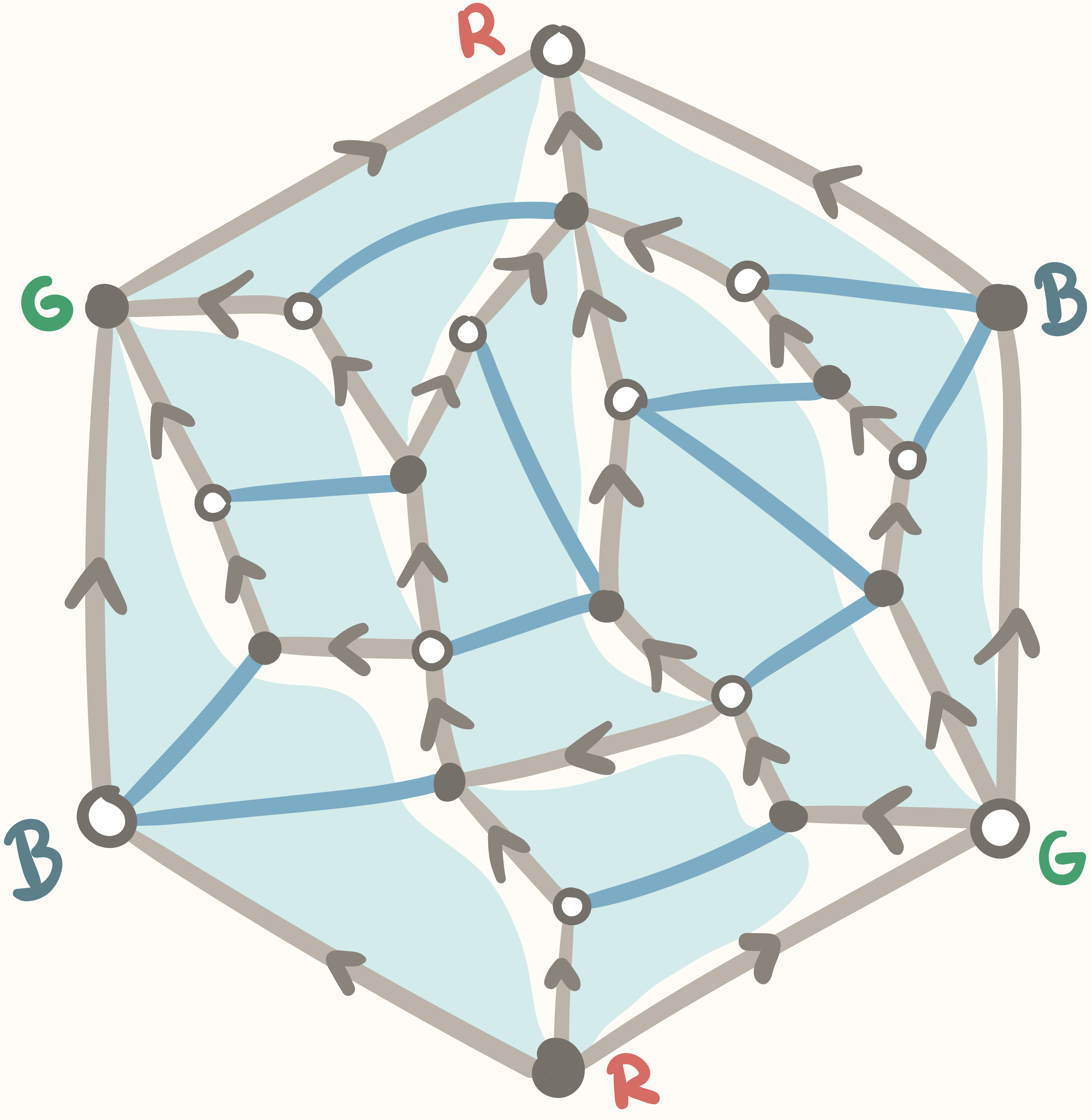


Schnyder labellings





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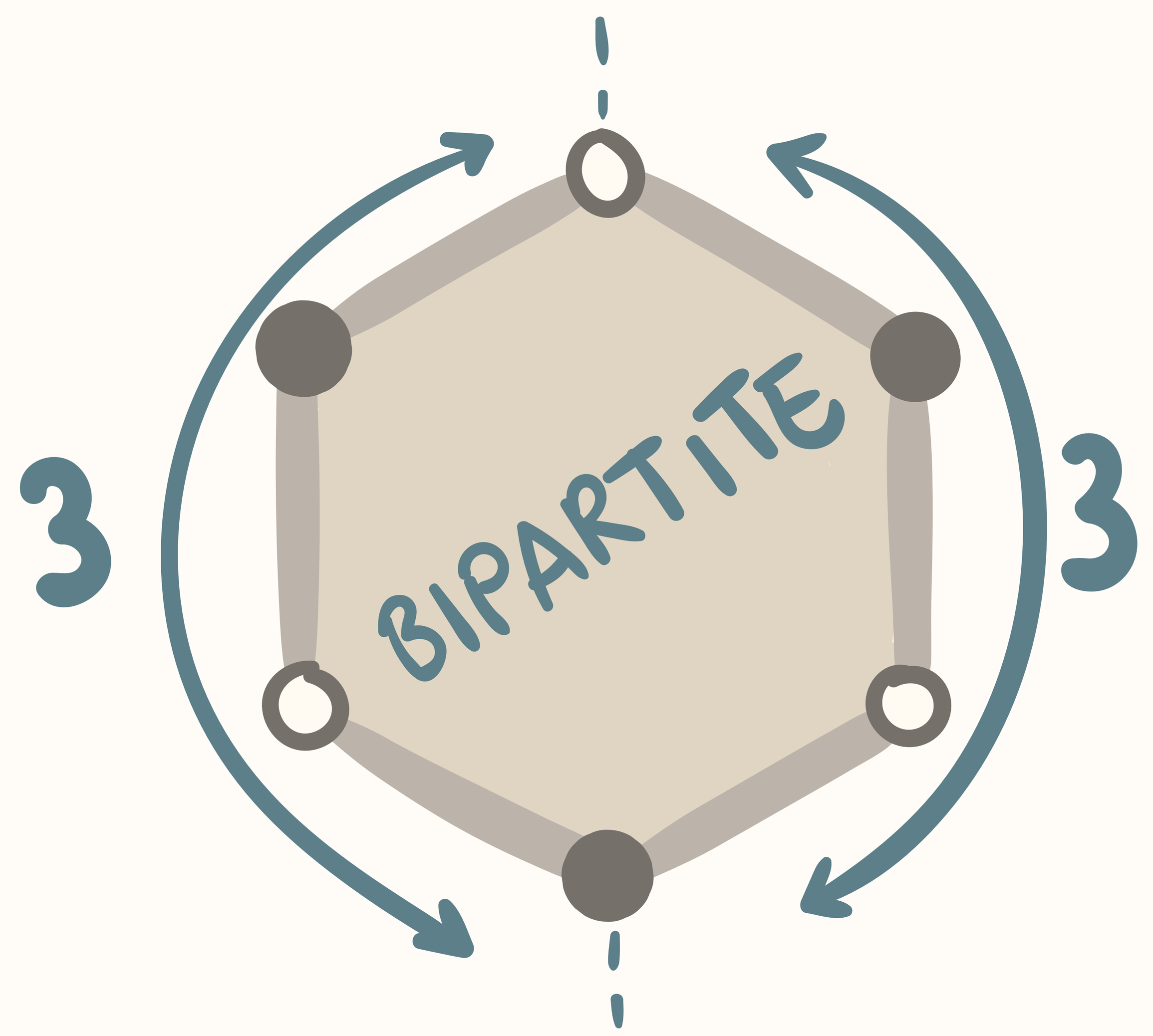
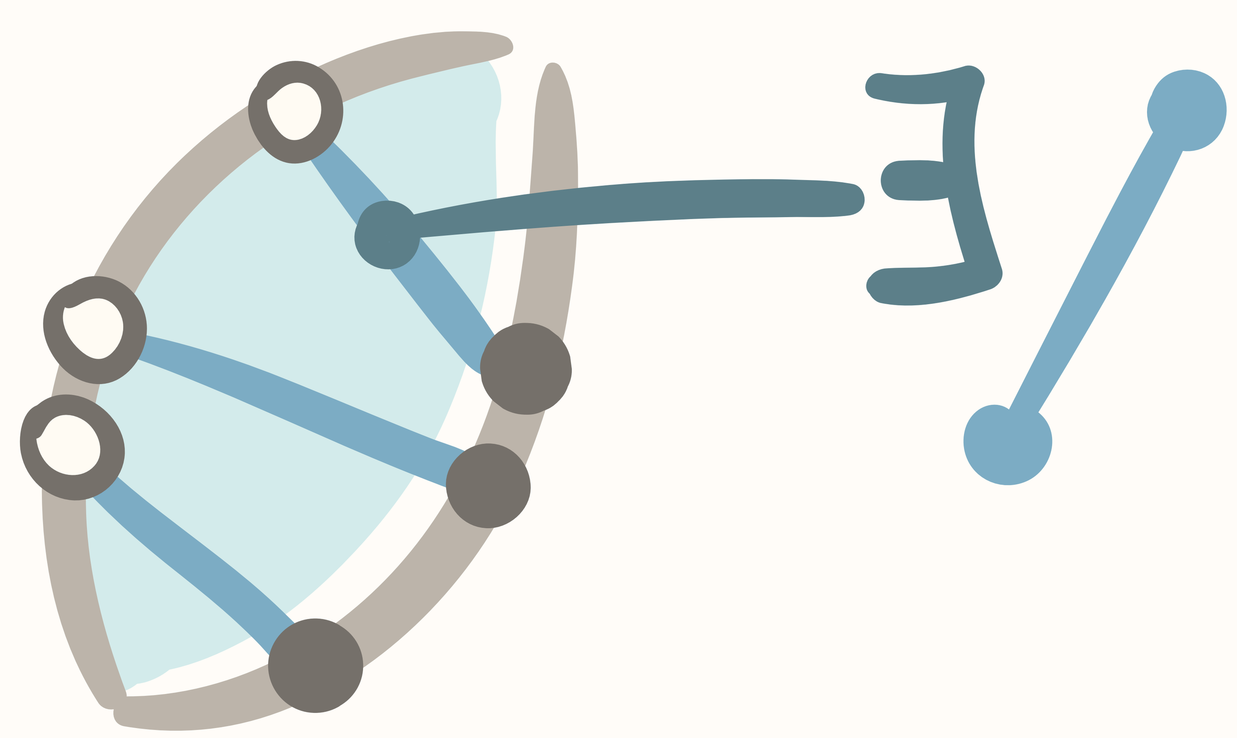
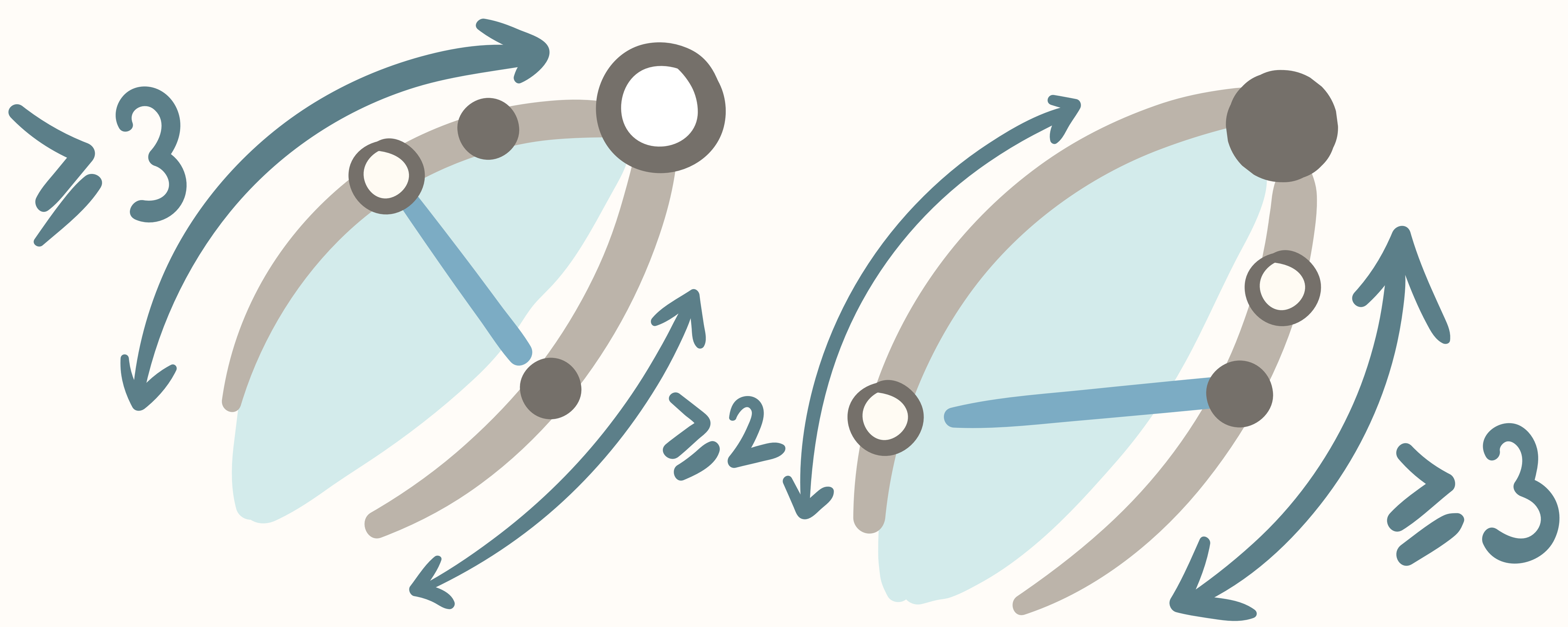
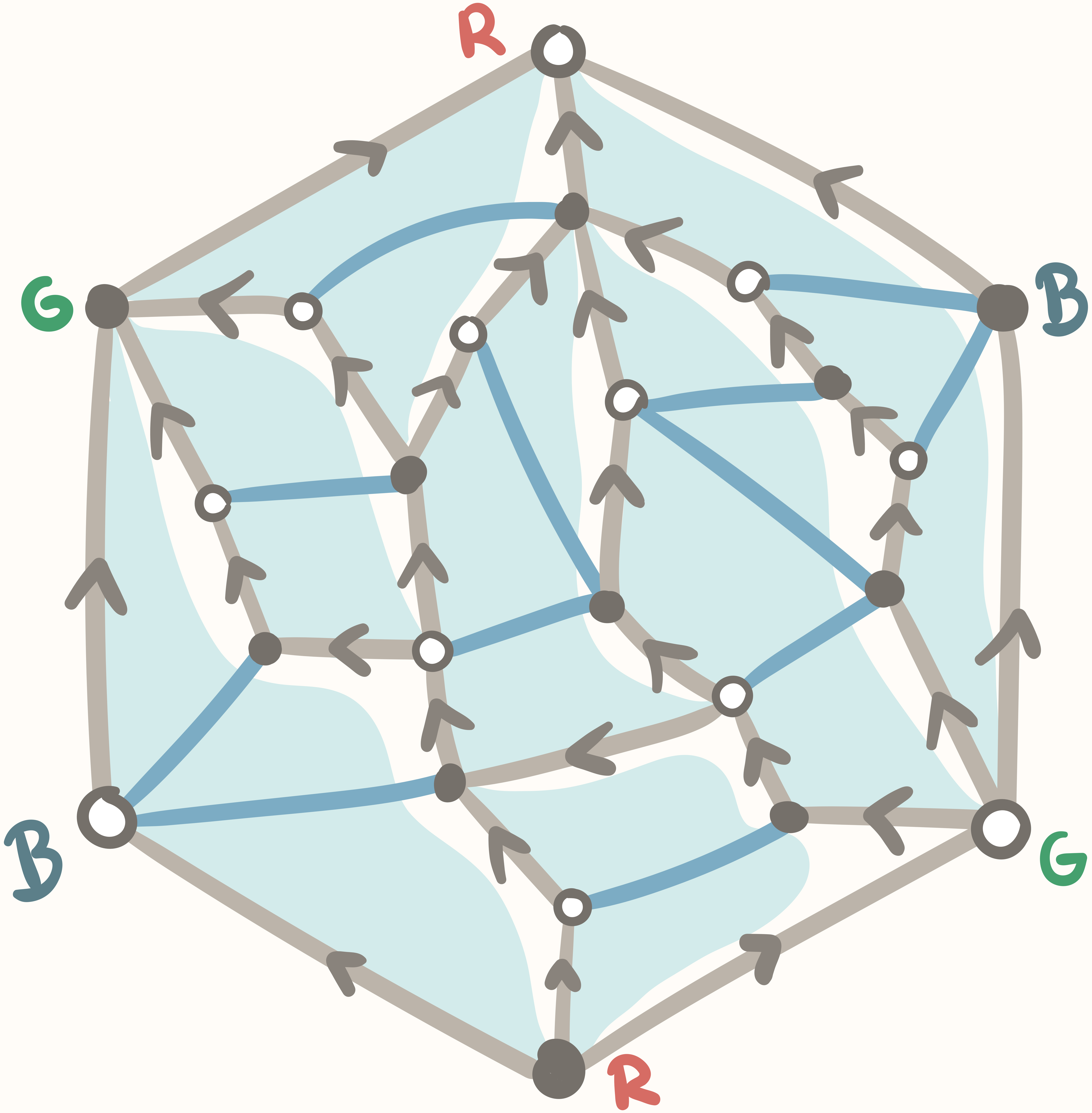
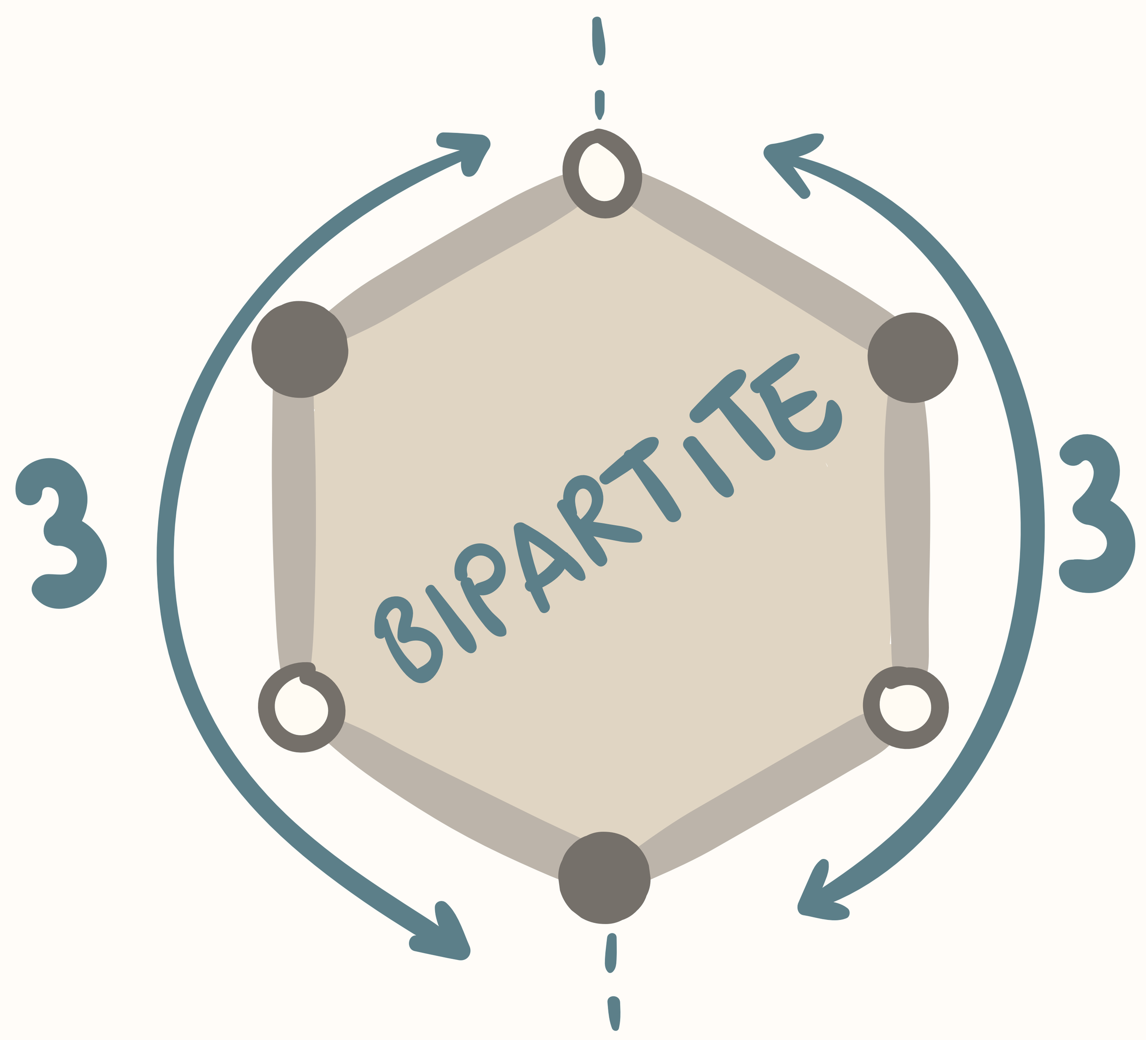


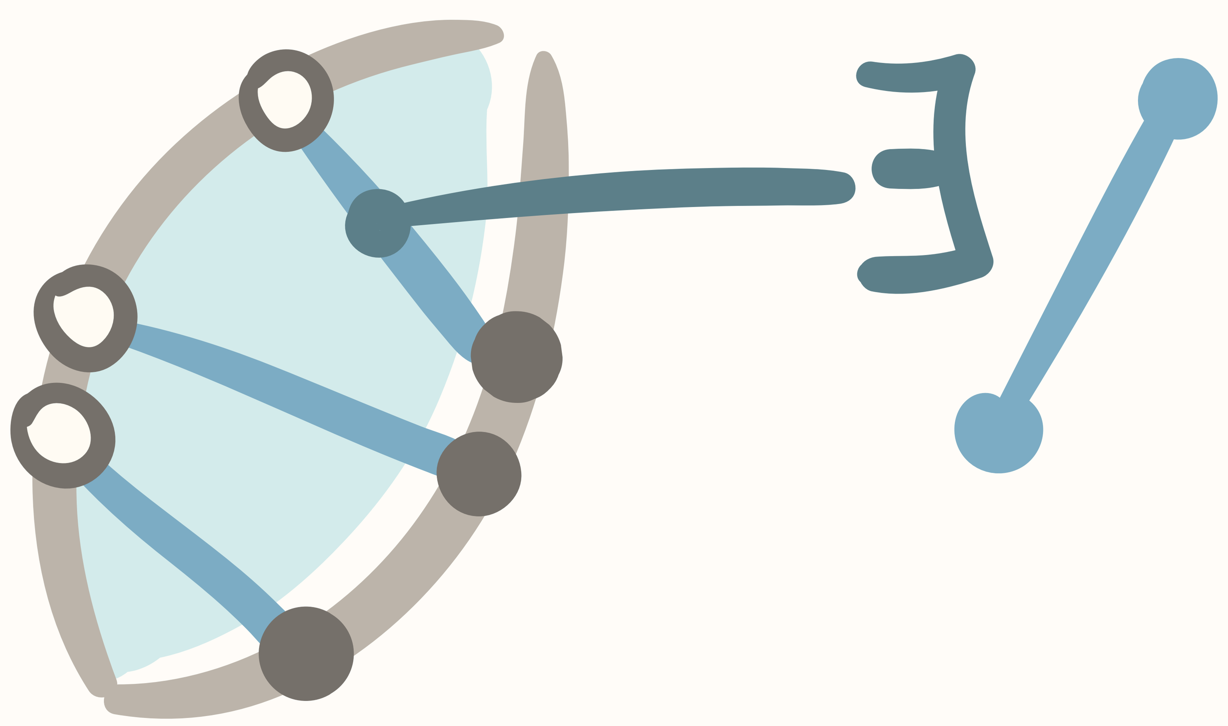


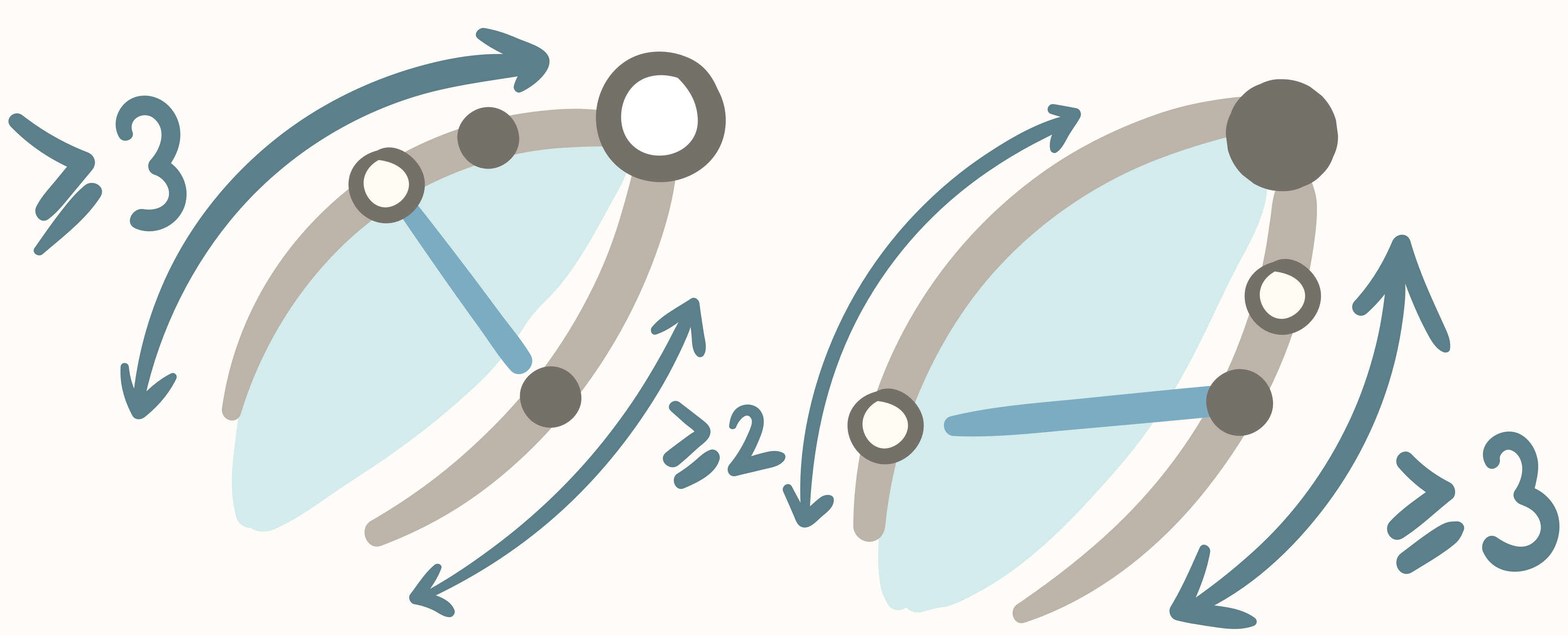




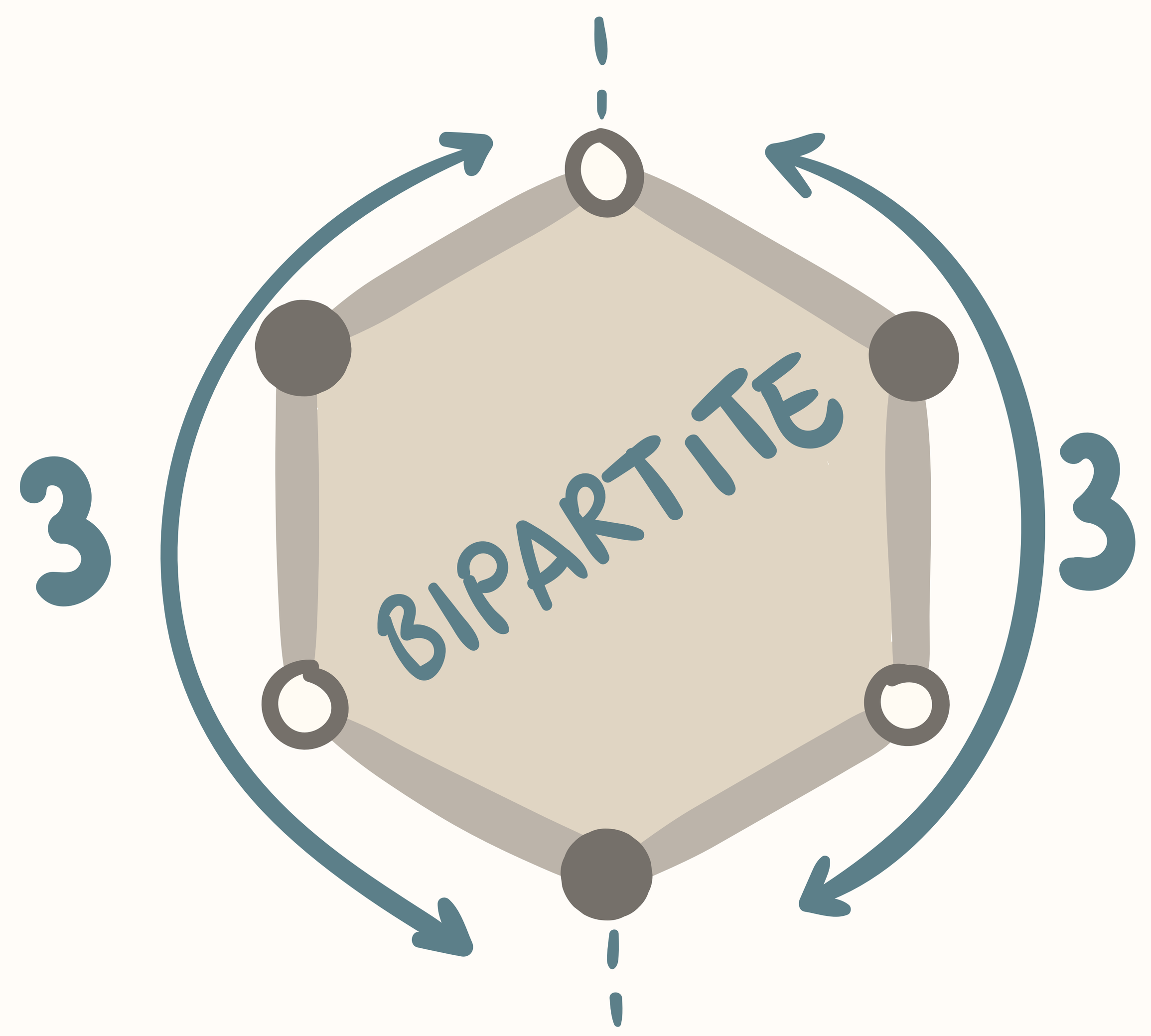
Schnyder labellings KMSW

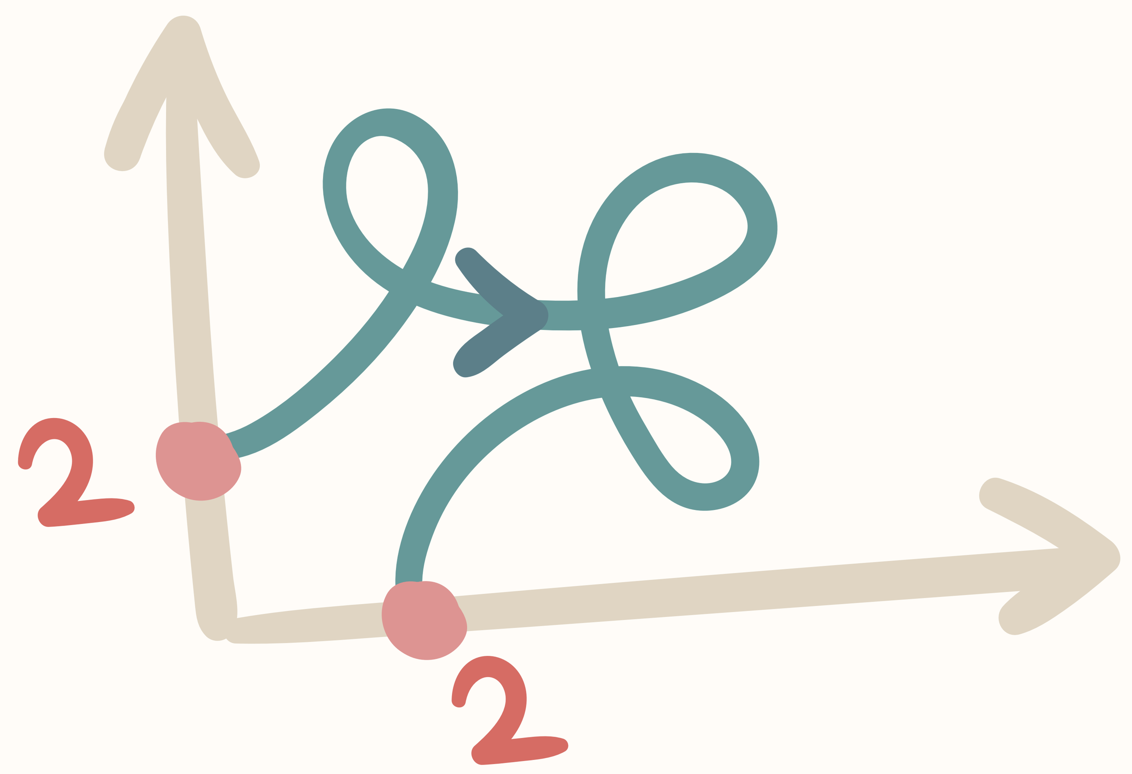


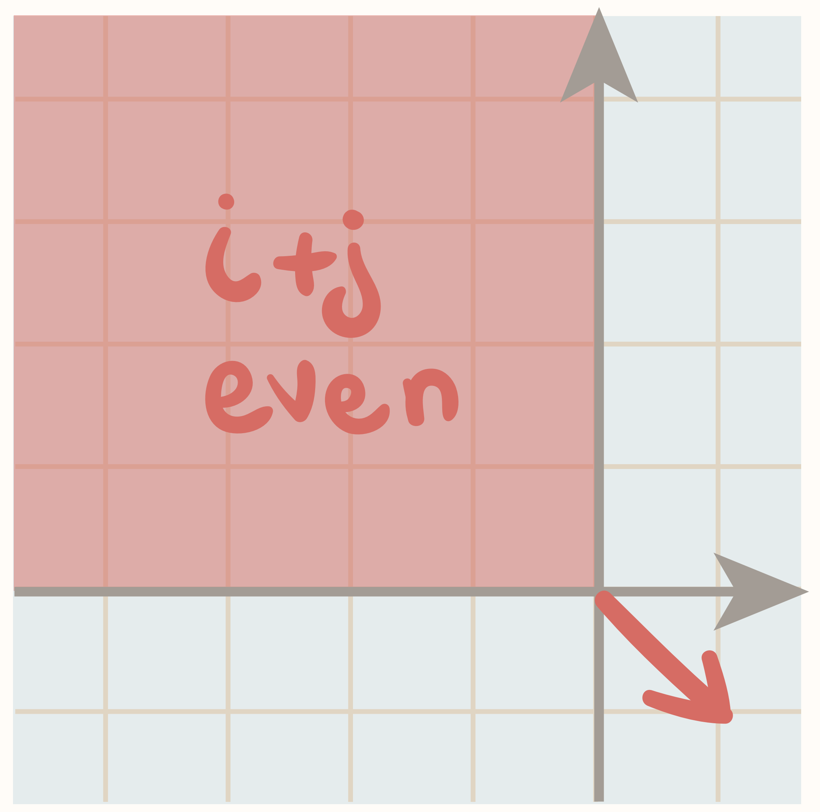


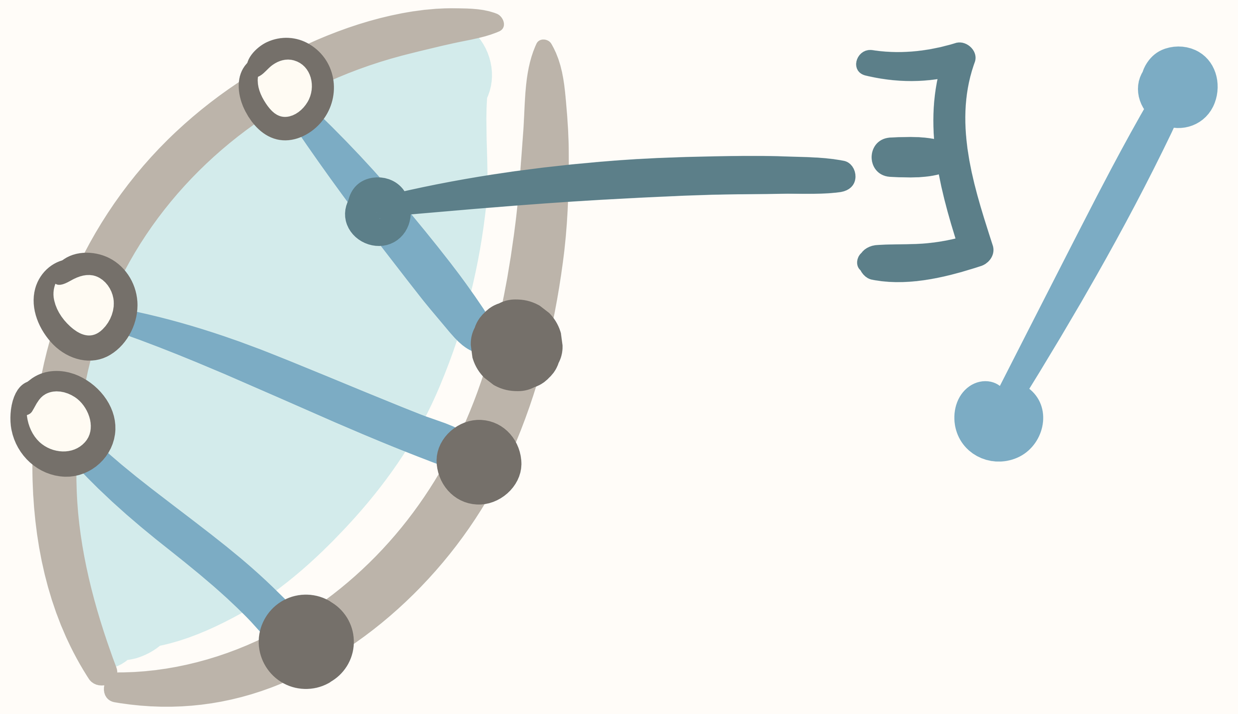


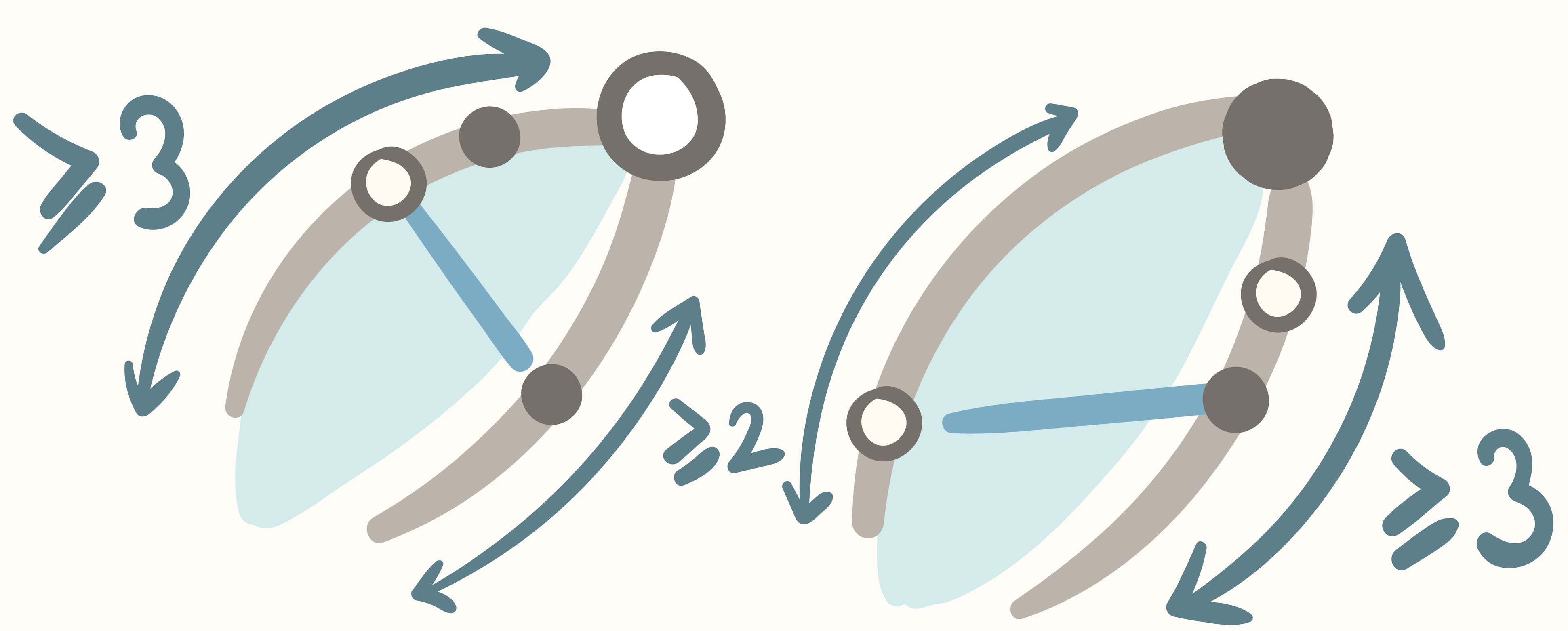
Schnyder labellings KMSW



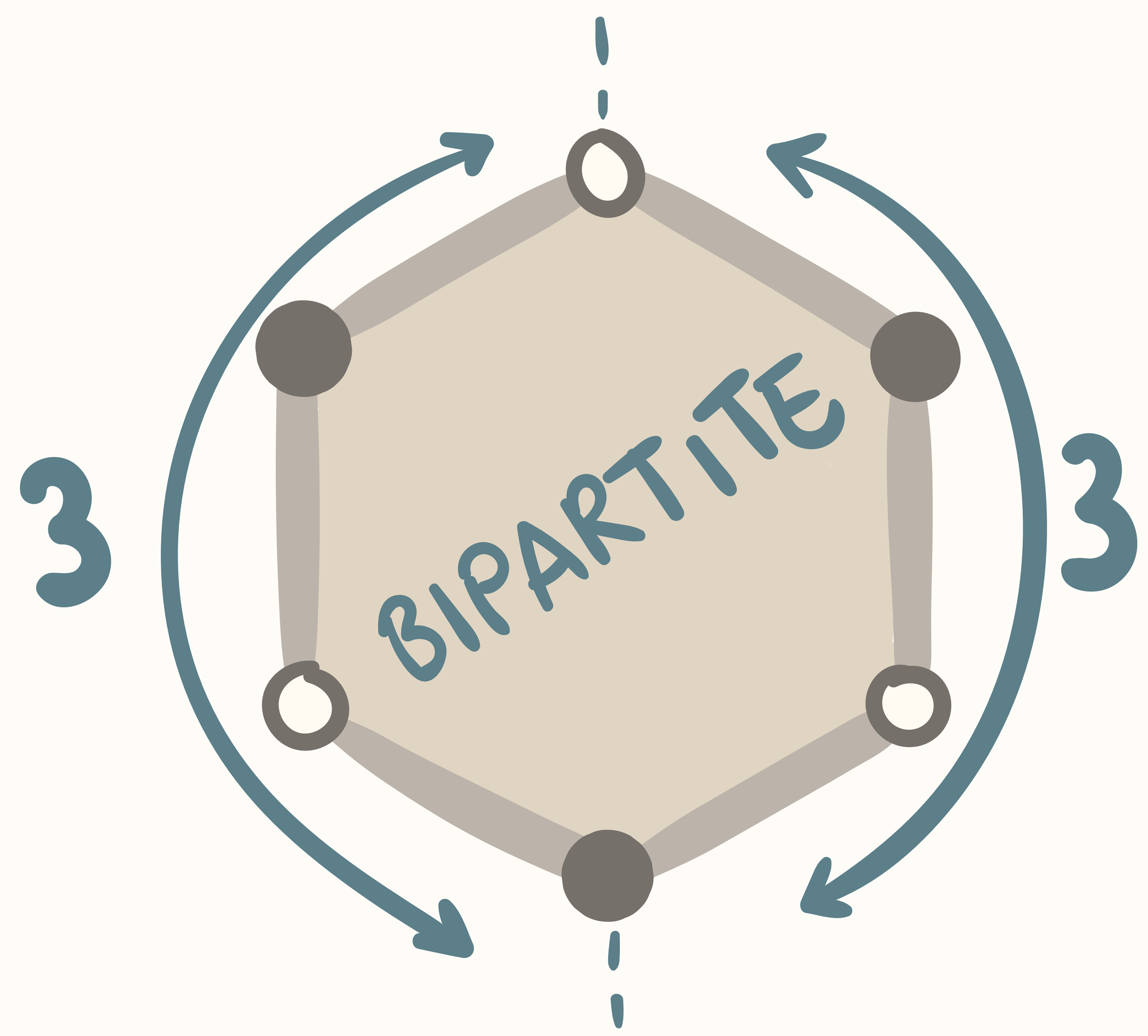


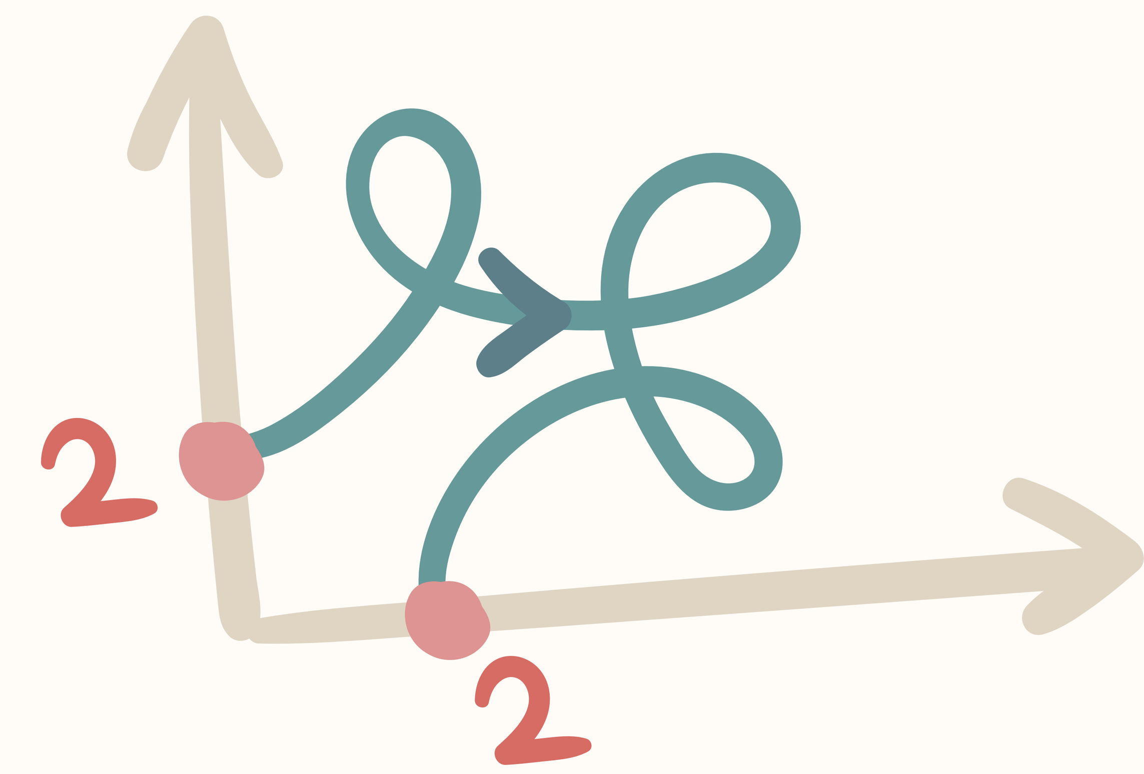


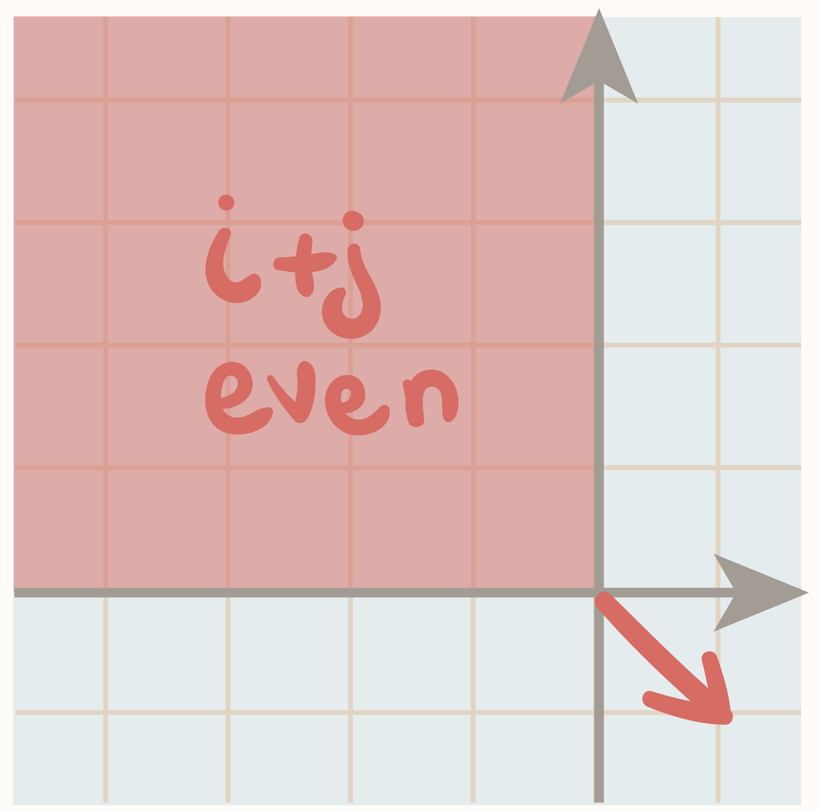


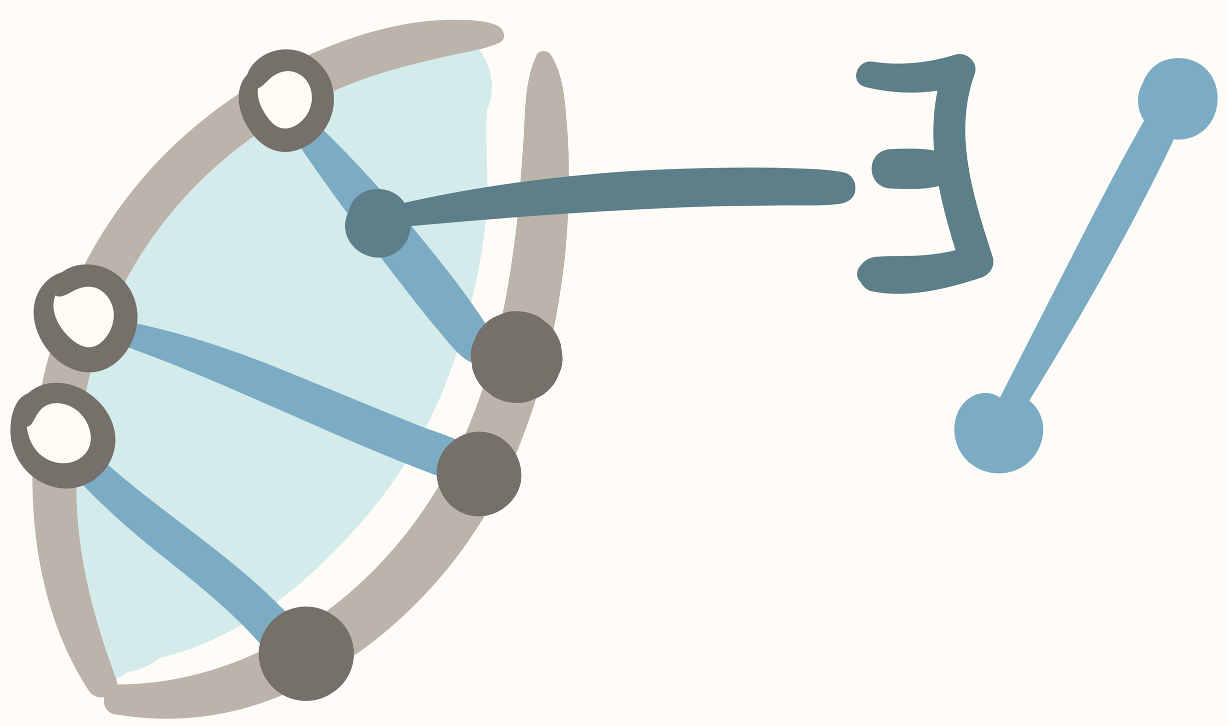


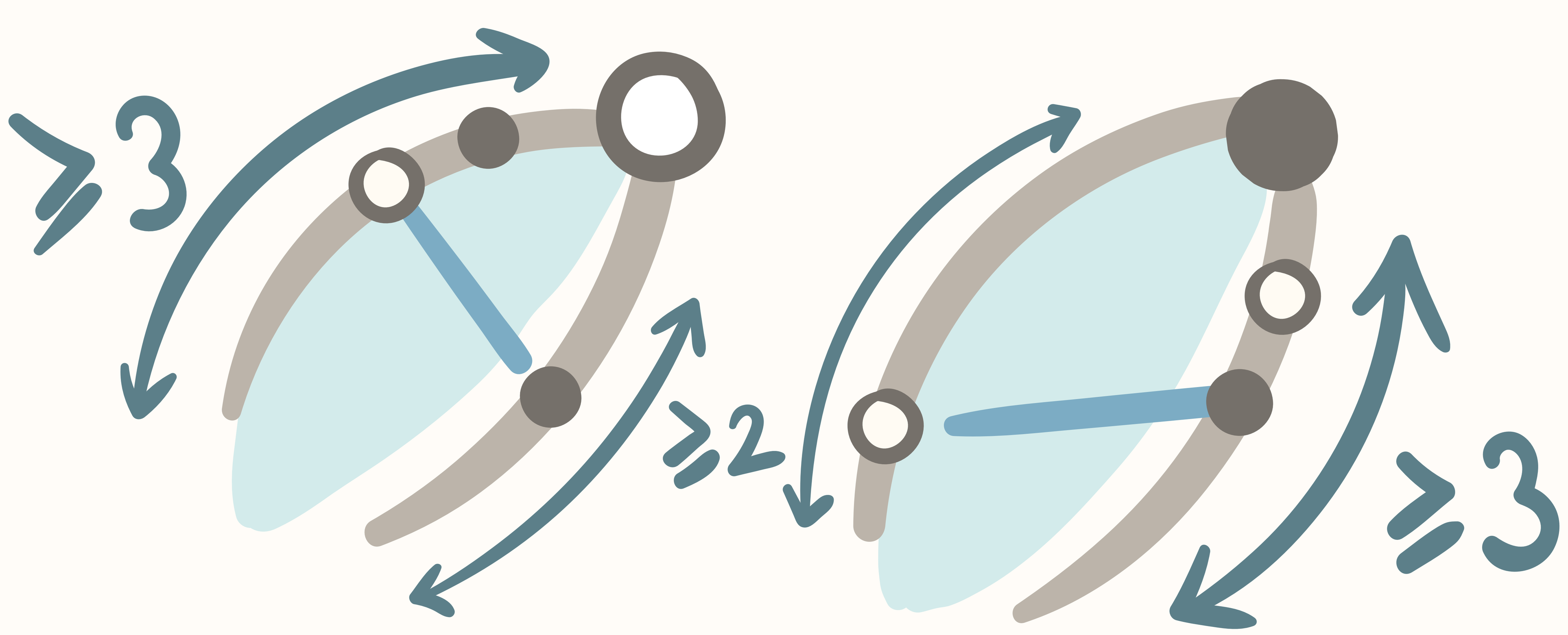
Schnyder labellings KMSW

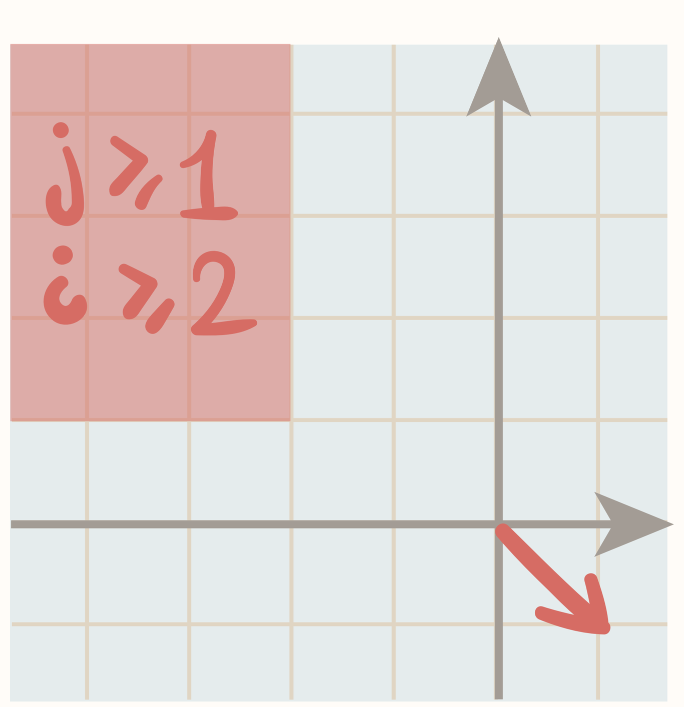




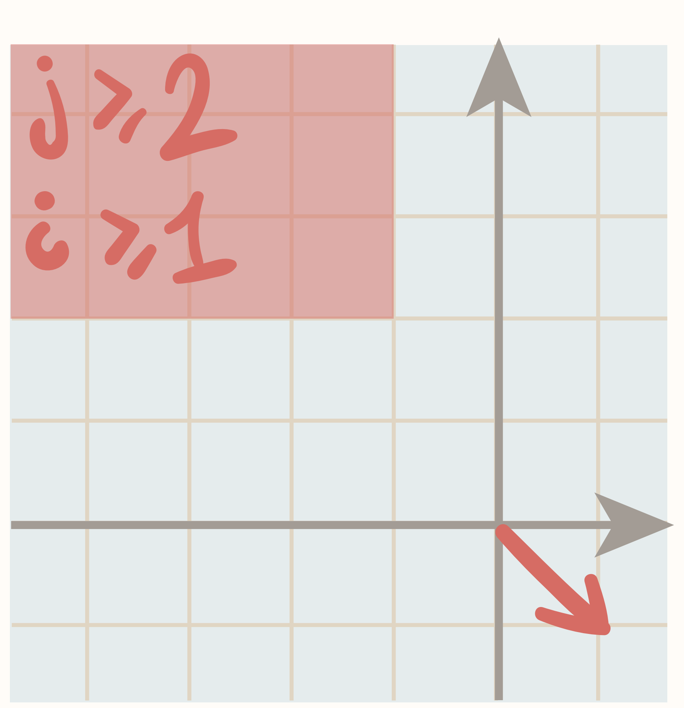






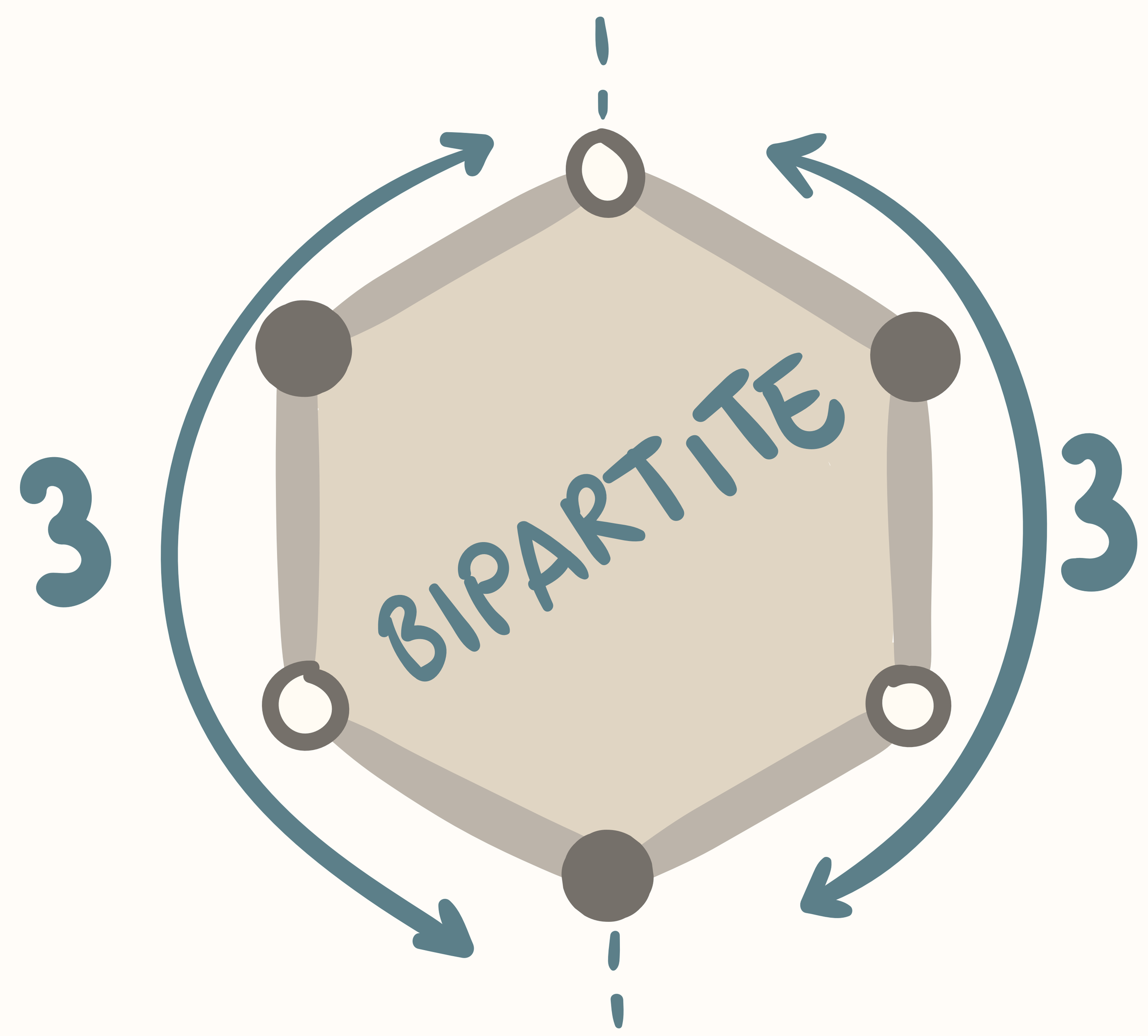


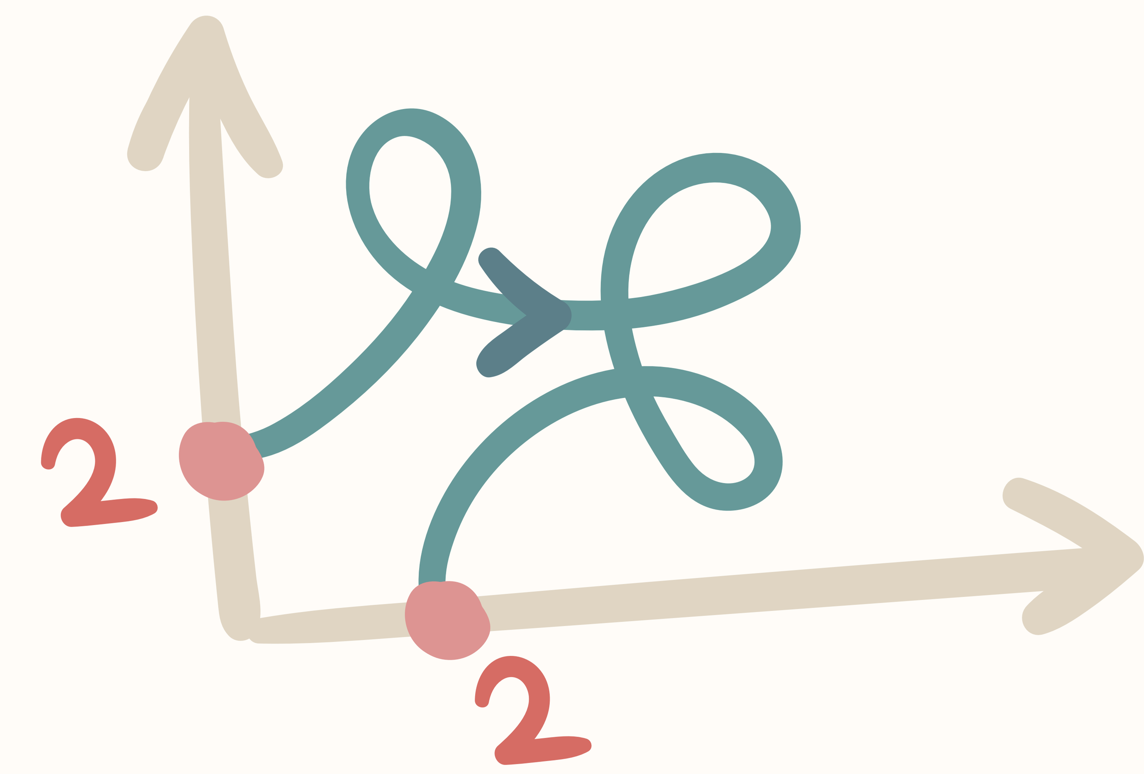
when x,y are even

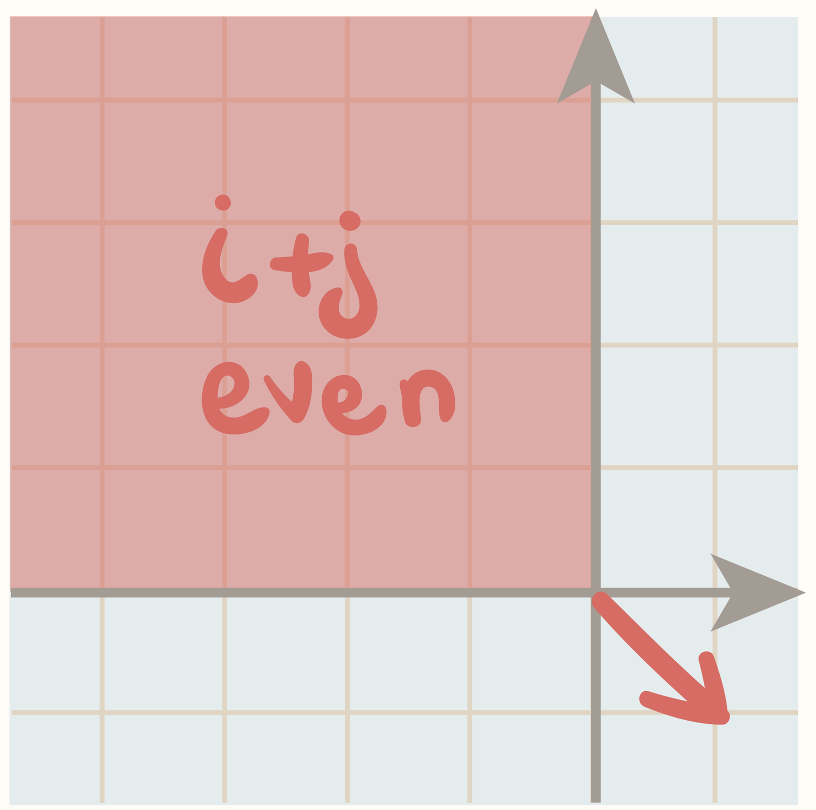


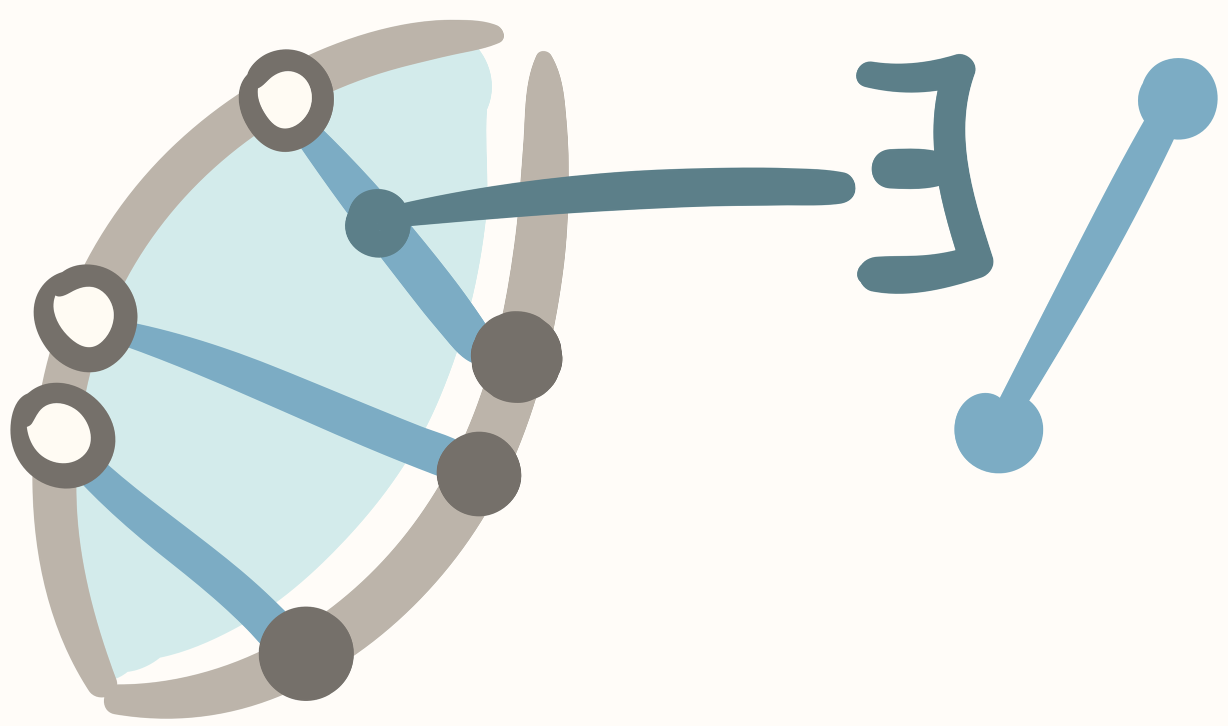
when x,y are odd

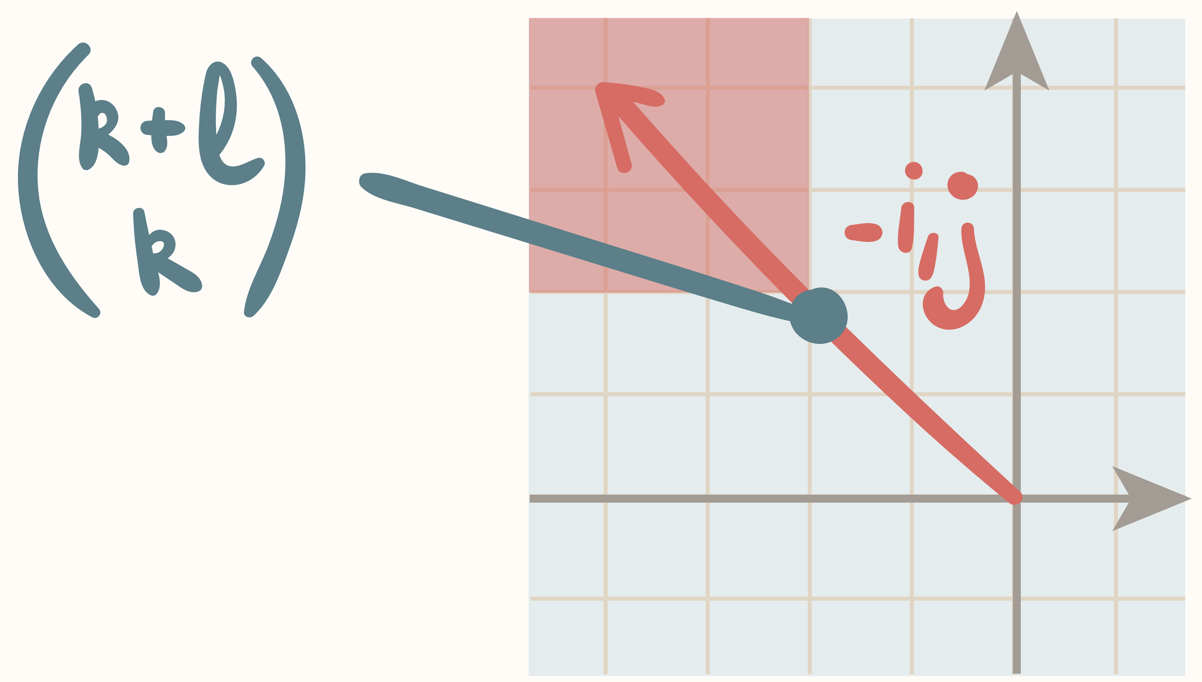
Schnyder labellings KMSW





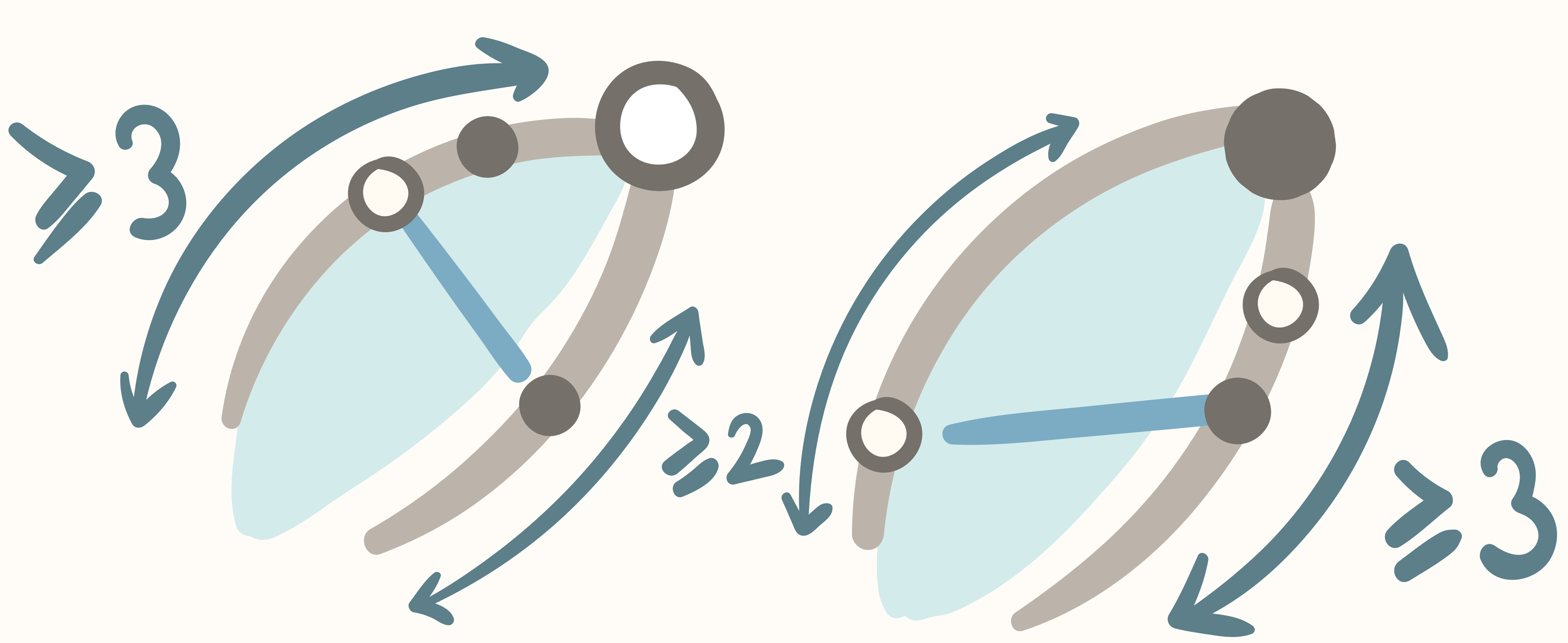


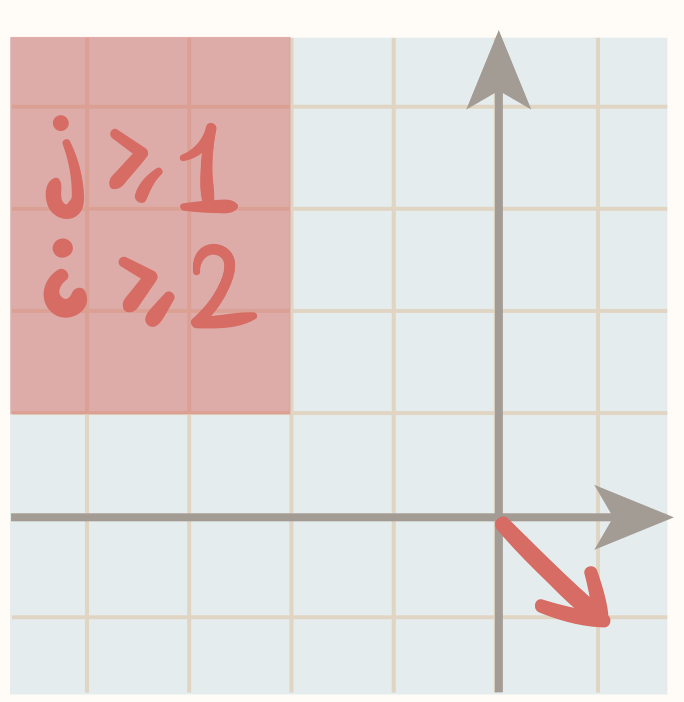




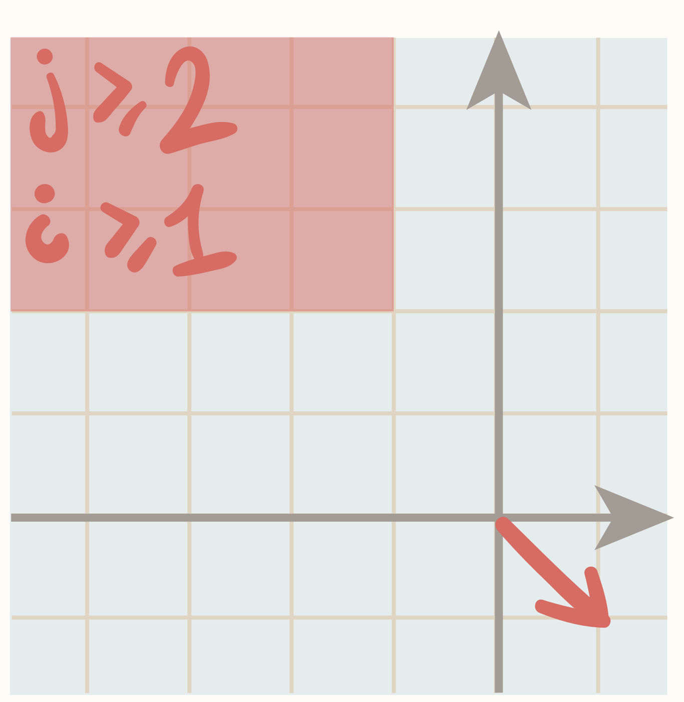
regarding
x,y parity

with
$i = 2k+2$
$j = 2l+2$



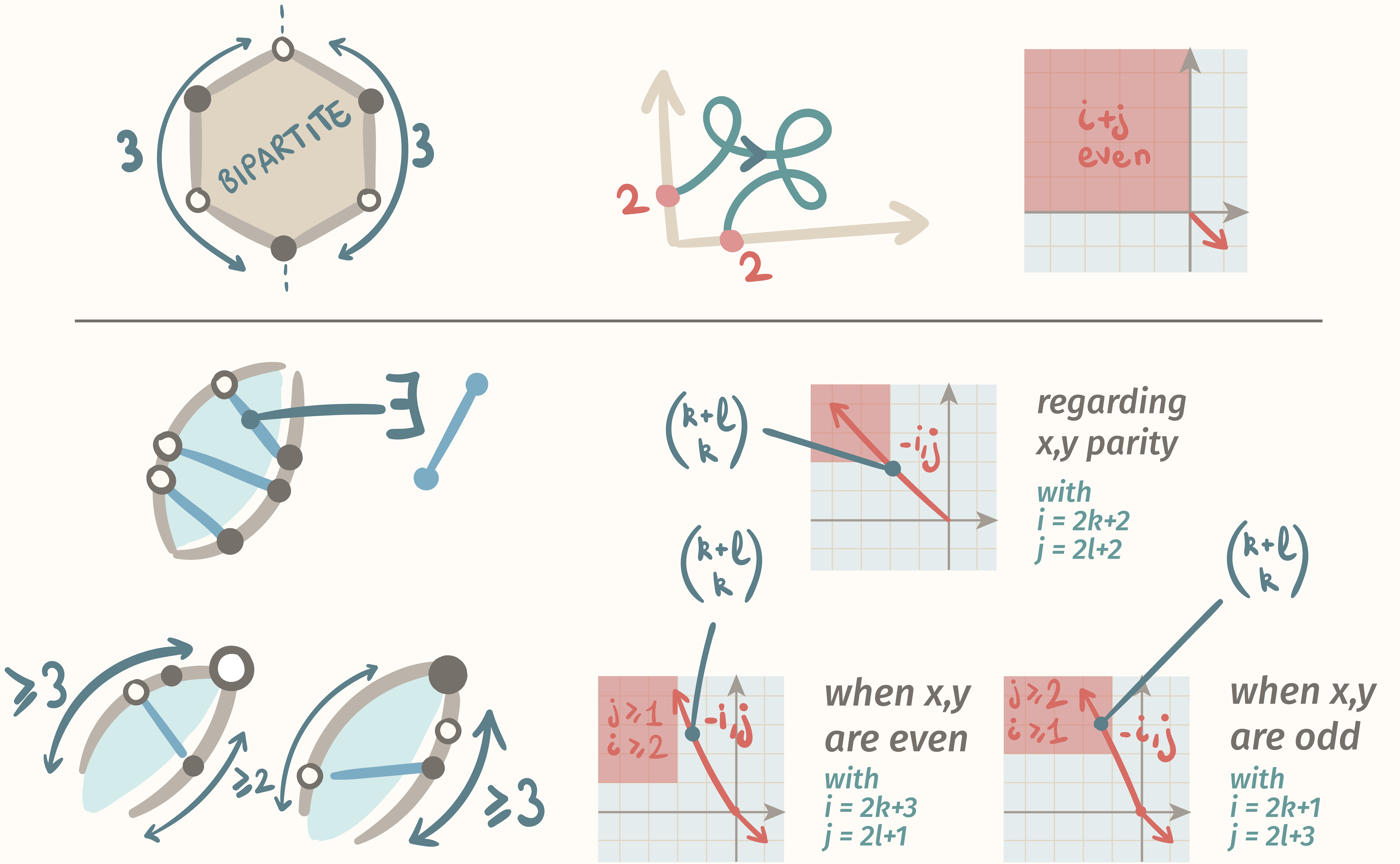


when x,y
are even

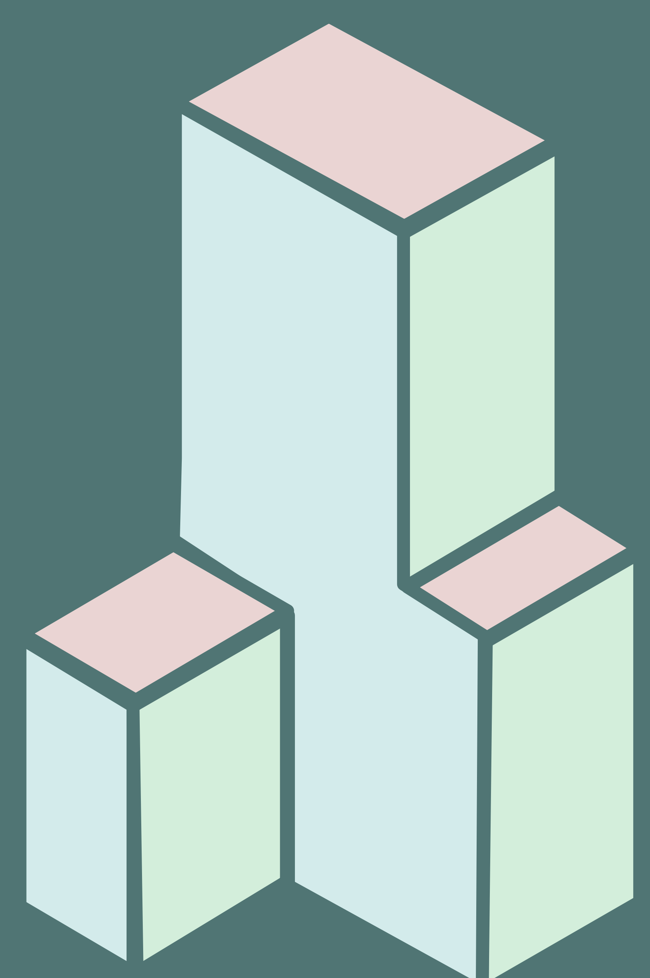
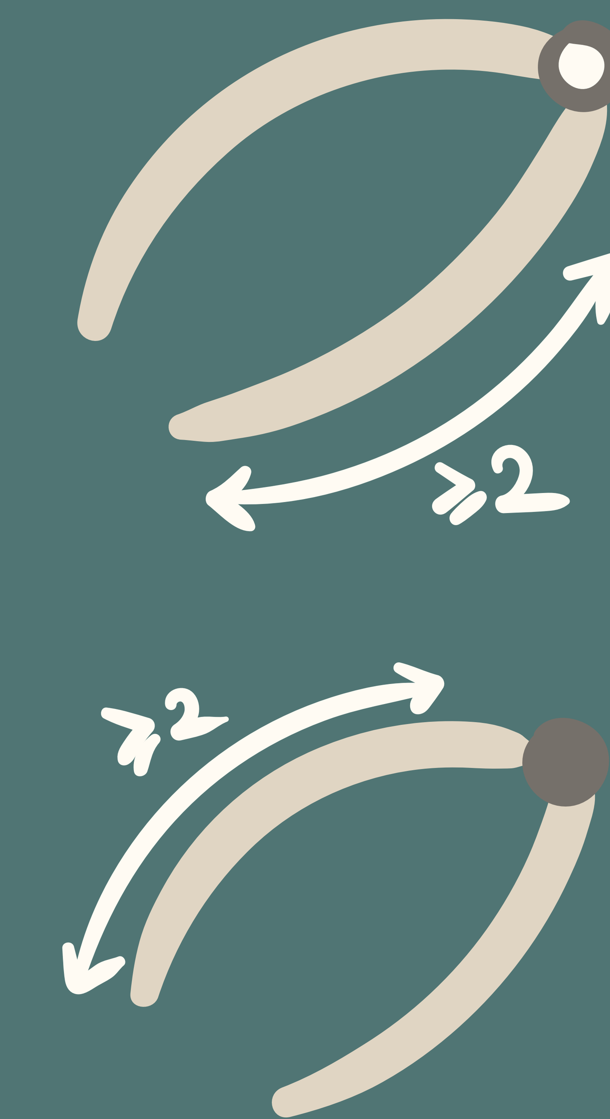
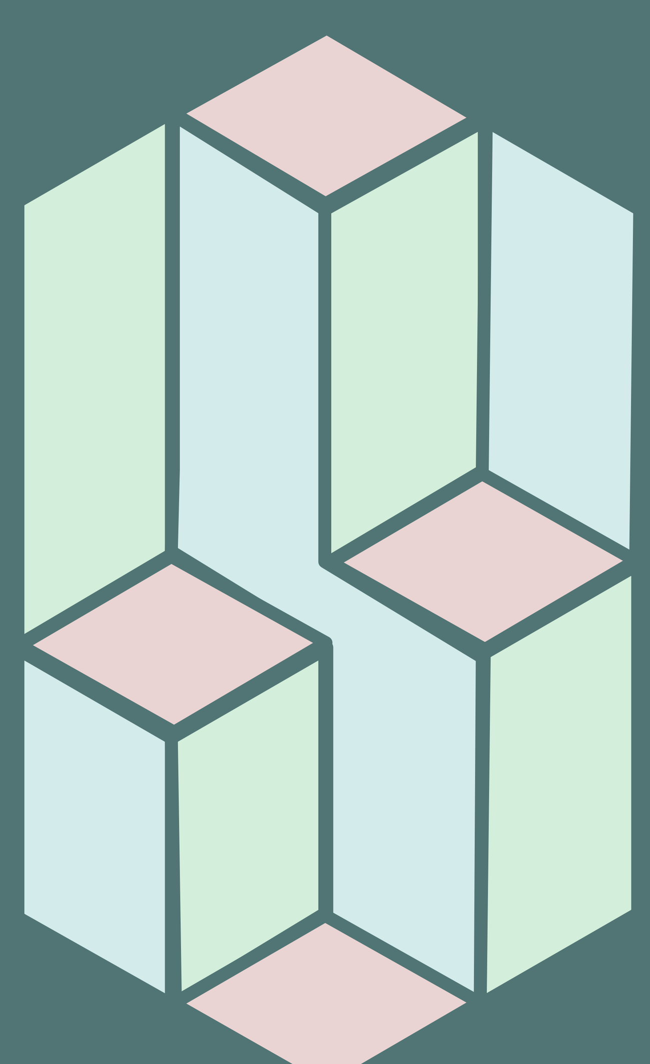


when x,y
are odd

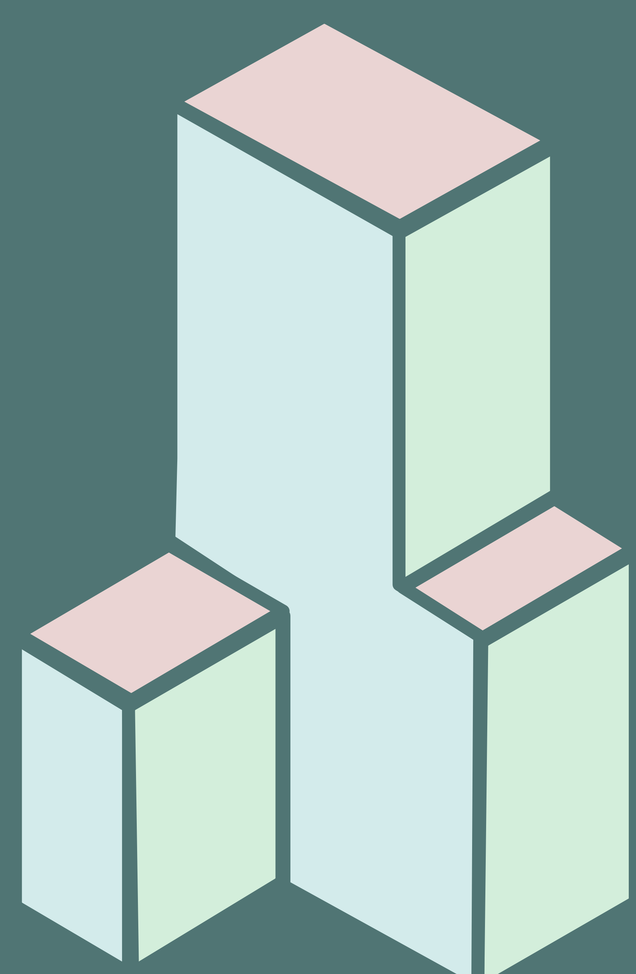
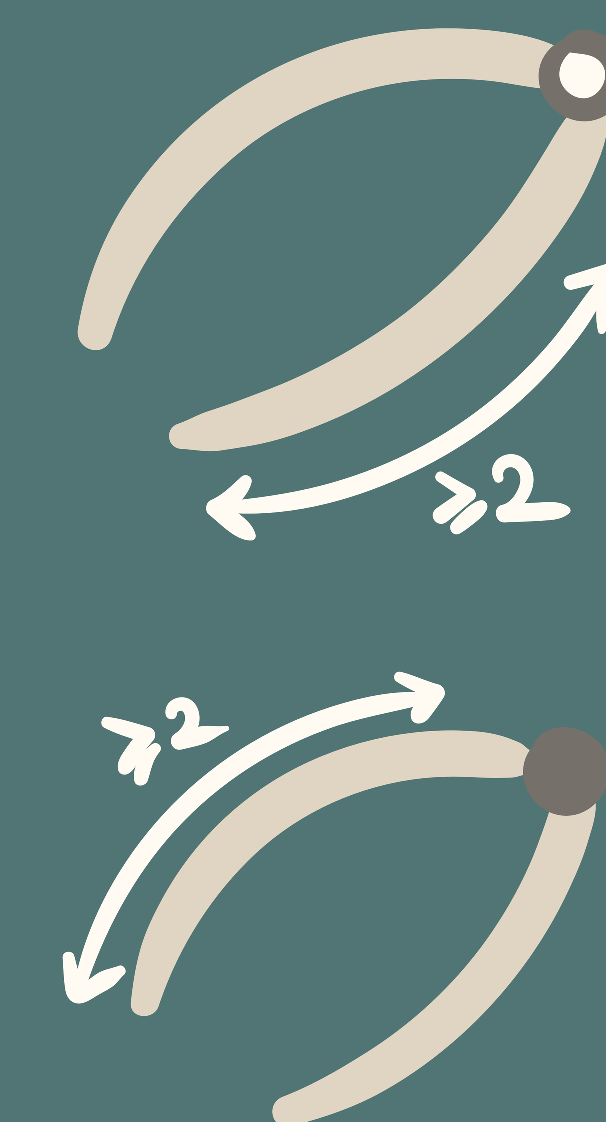
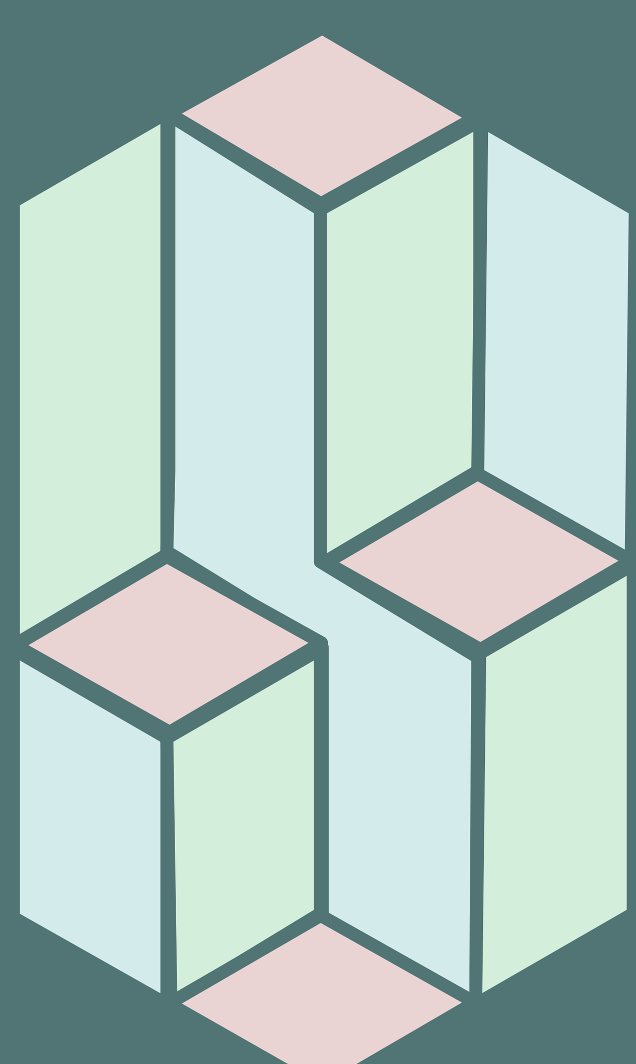
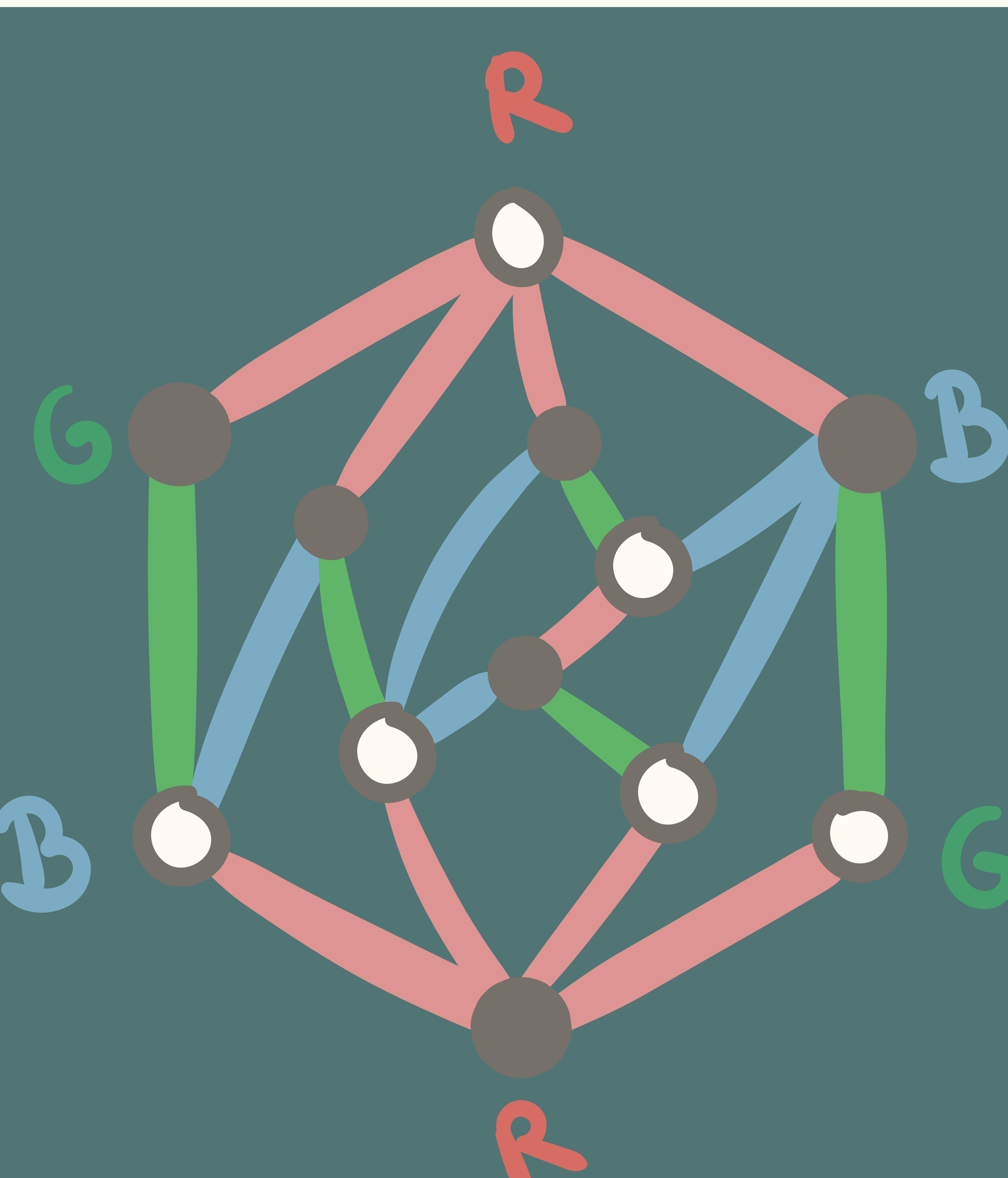
Schnyder labellings KMSW



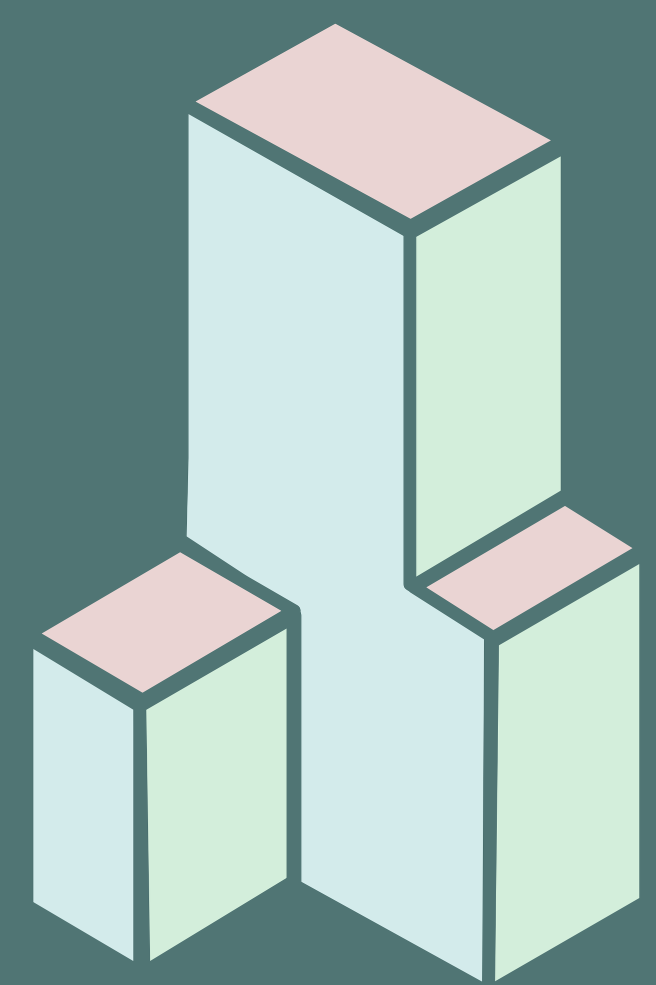
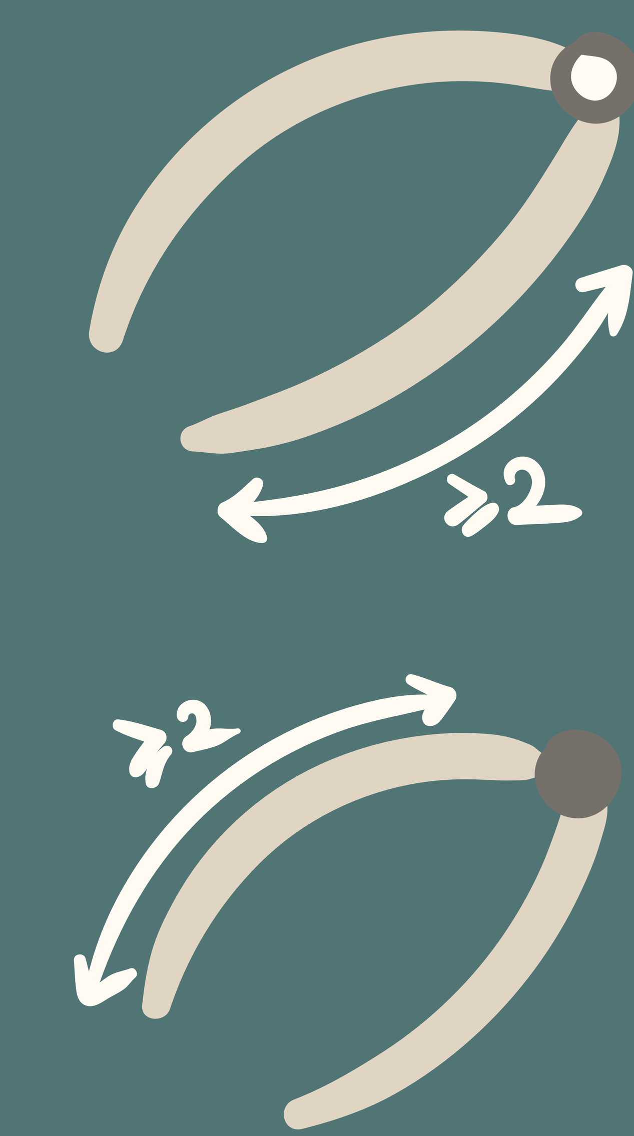
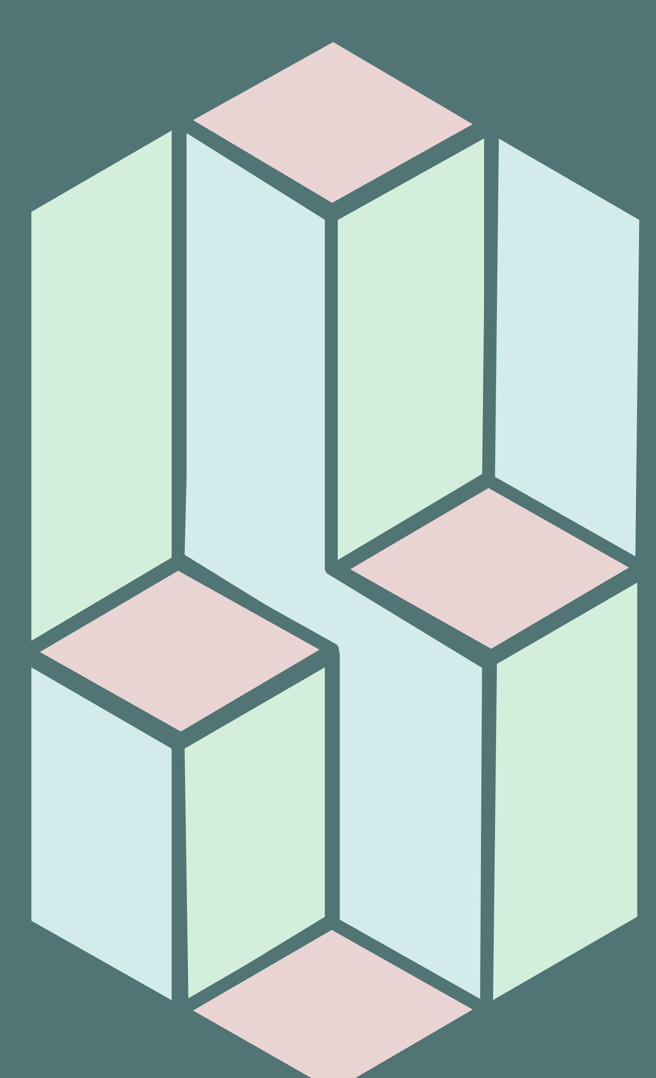
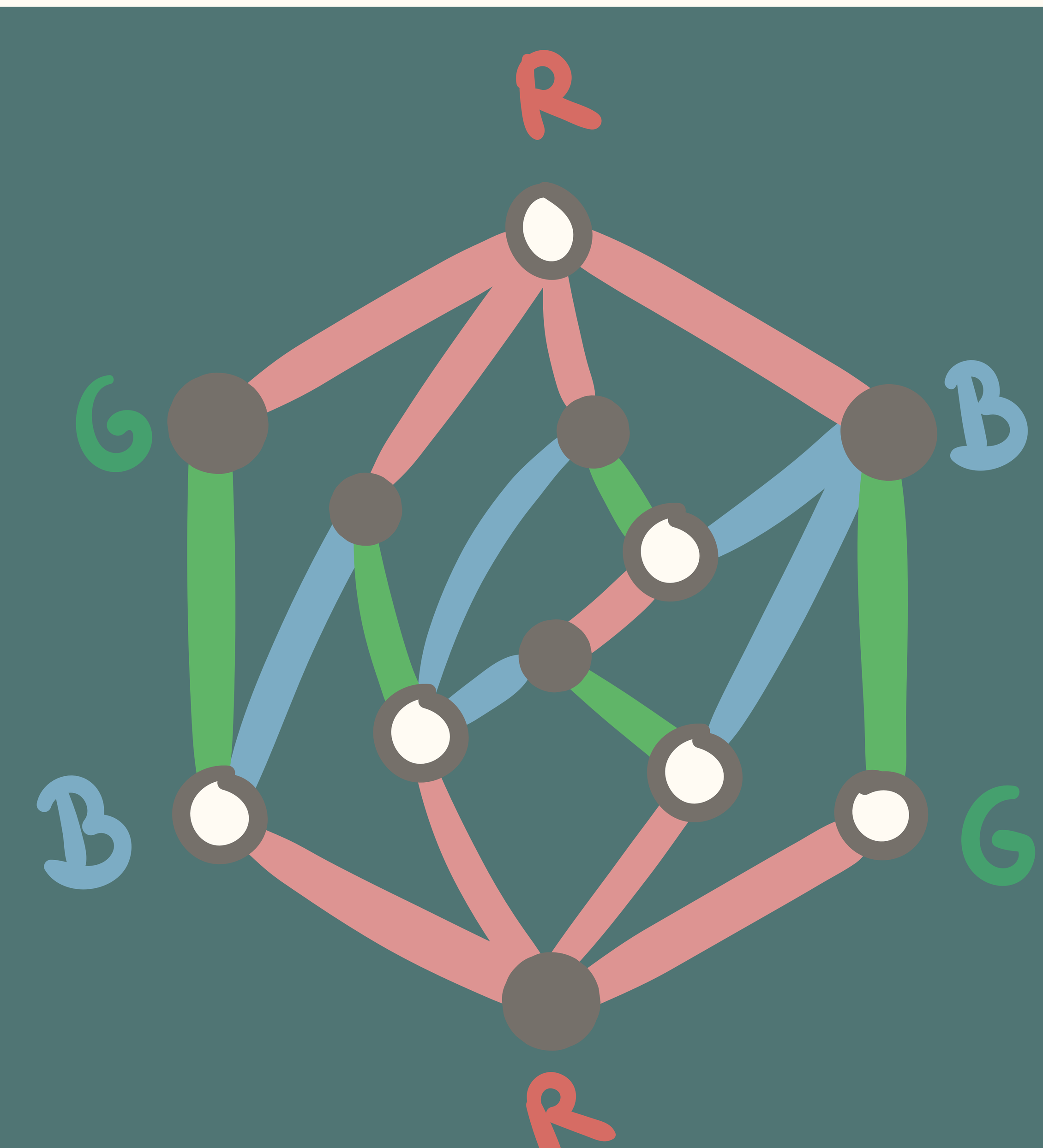
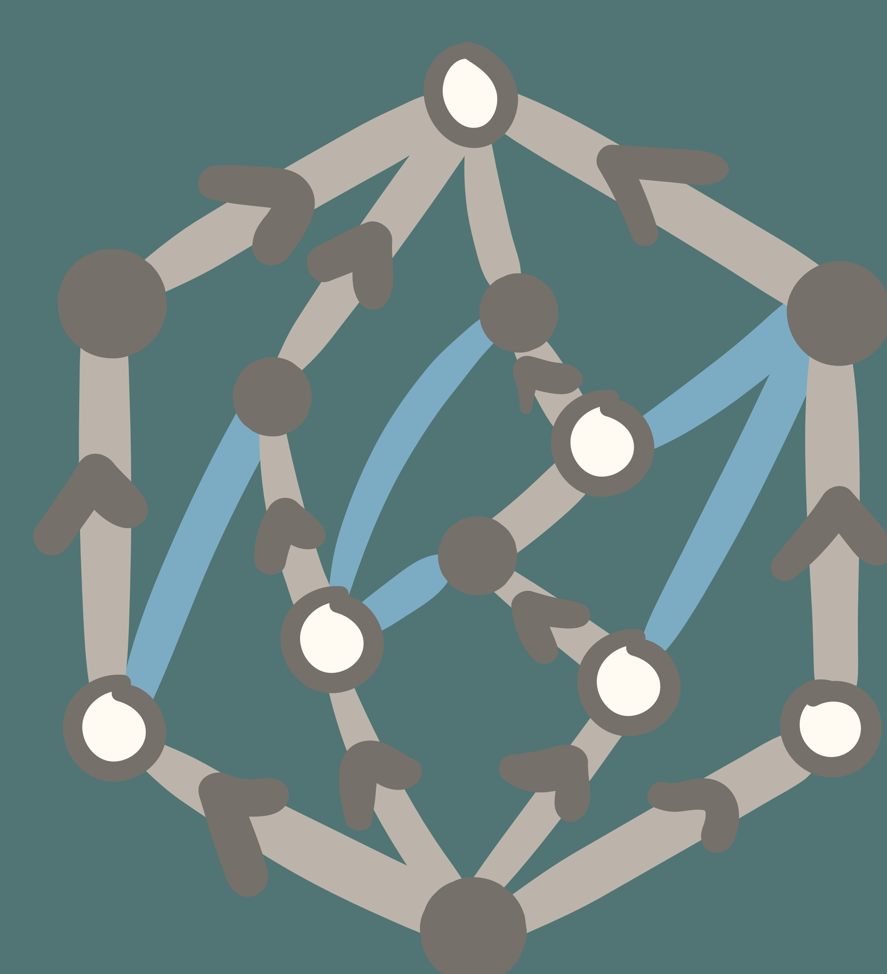
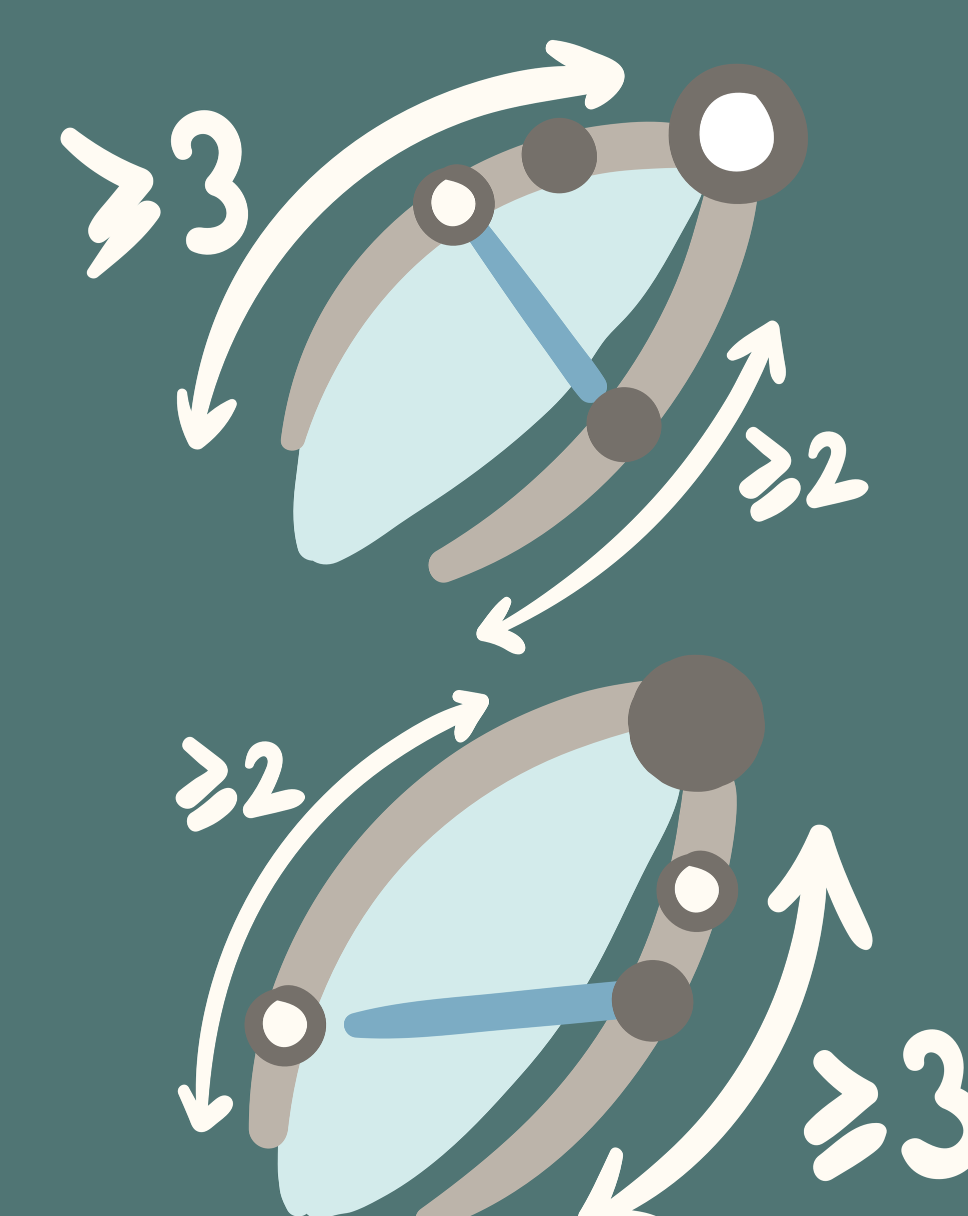
Bijections summary

Model	Combinatorial type	Bipolar orient./Tandem walk
 *n flats*	 *n inner vertices*	  *n+1 edges*
 *n flats*		

Bijections summary

Model	Combinatorial type	Bipolar orient./Tandem walk
 n flats	 n inner vertices	  n+1 edges
 n flats	 n inner faces	

Bijections summary

Model	Combinatorial type	Bipolar orient./Tandem walk	
 n flats	 n inner vertices	 n+1 edges	
 n flats	 n inner faces	 n+4 vertices	

Summary

The KMSW bijection

1. Application to three map models

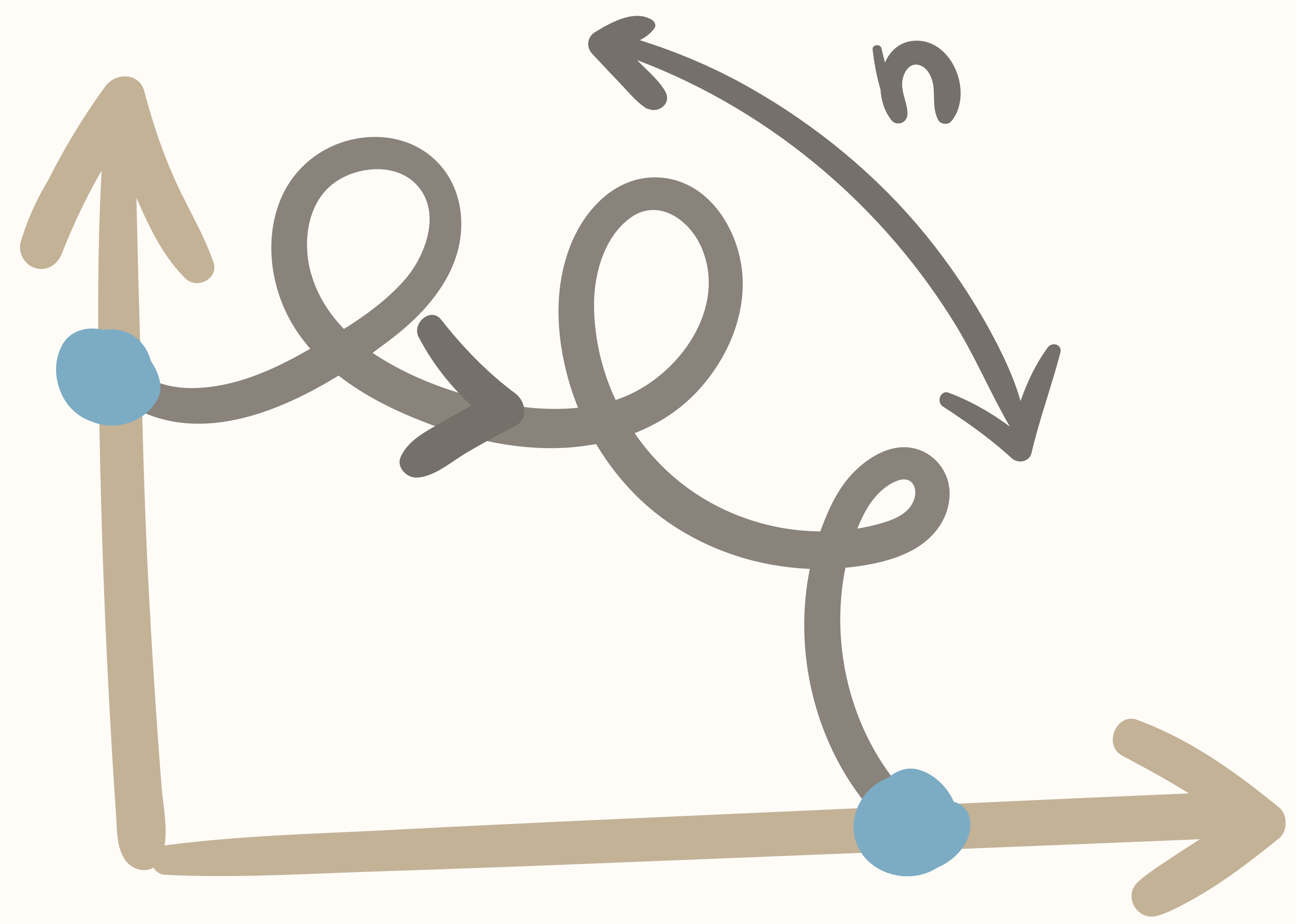
a. Plane bipolar posets
b. Plane bipolar posets by vertices
c. Transversal structures
d. Asymtotics

2. Interlude : plane permutations

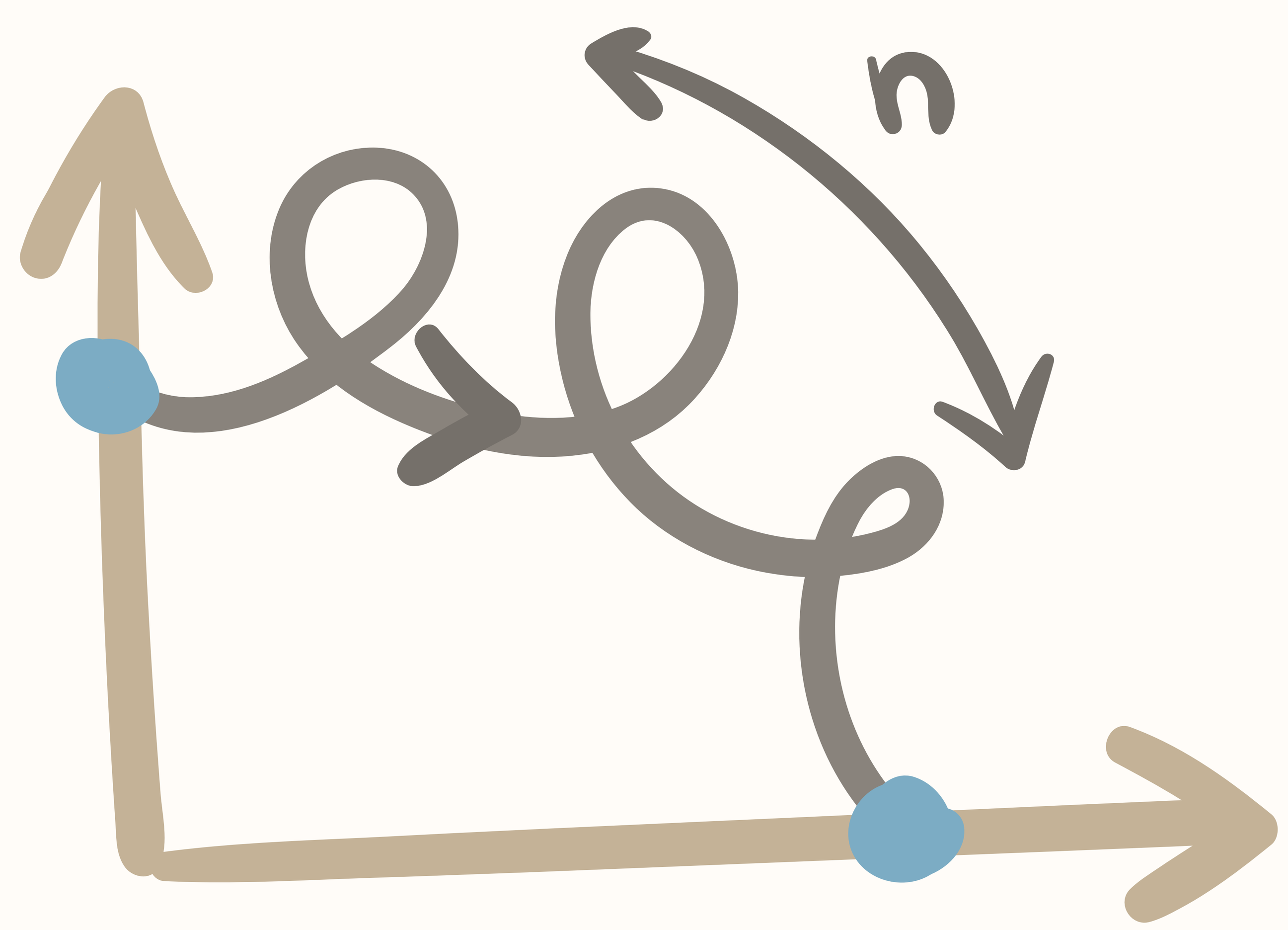
3. Application to corner polyhera

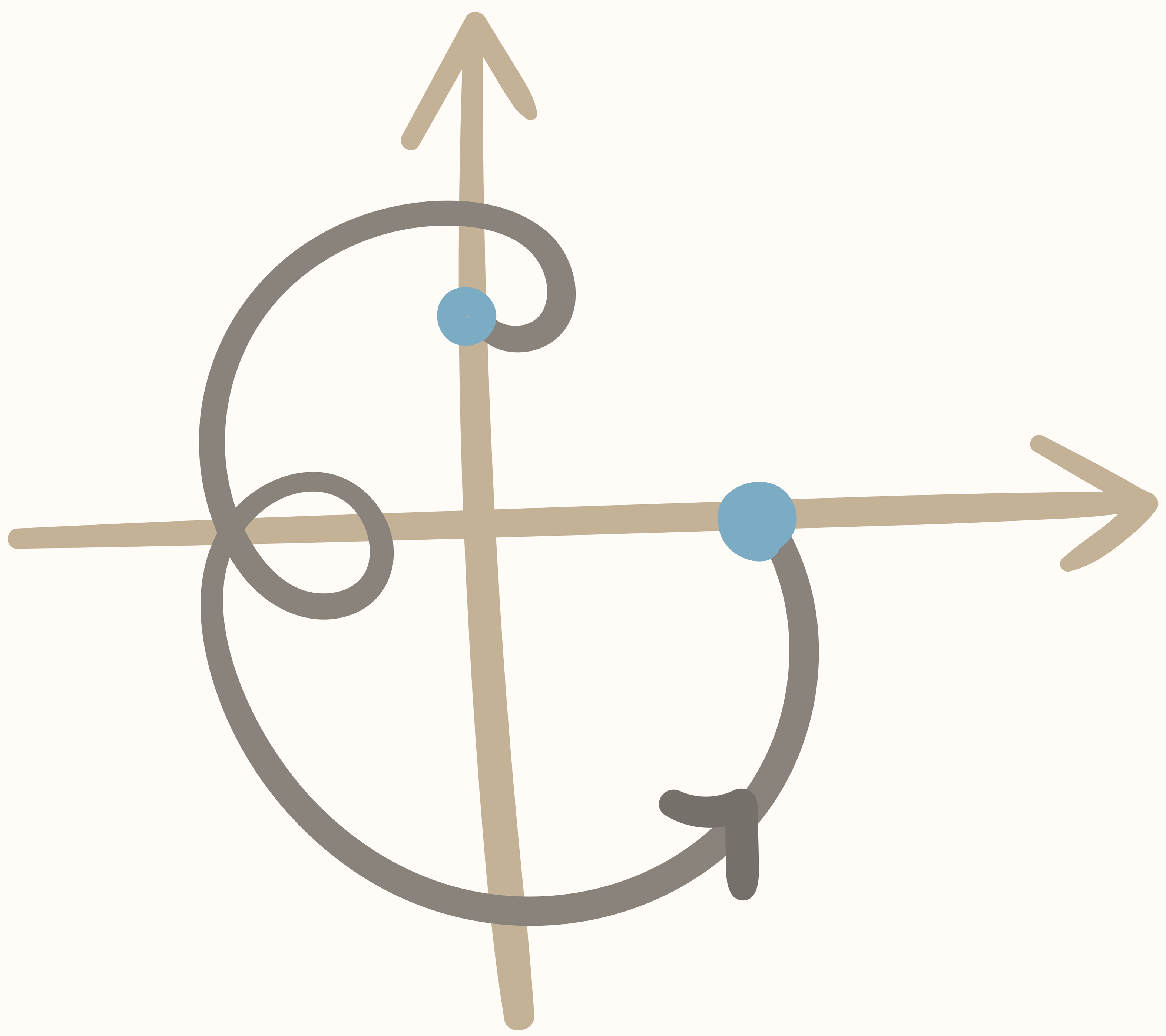
a. Via polyheral orientations
b. Via Schnyder colorings
c. Asymtotics

Asymptotic counting results

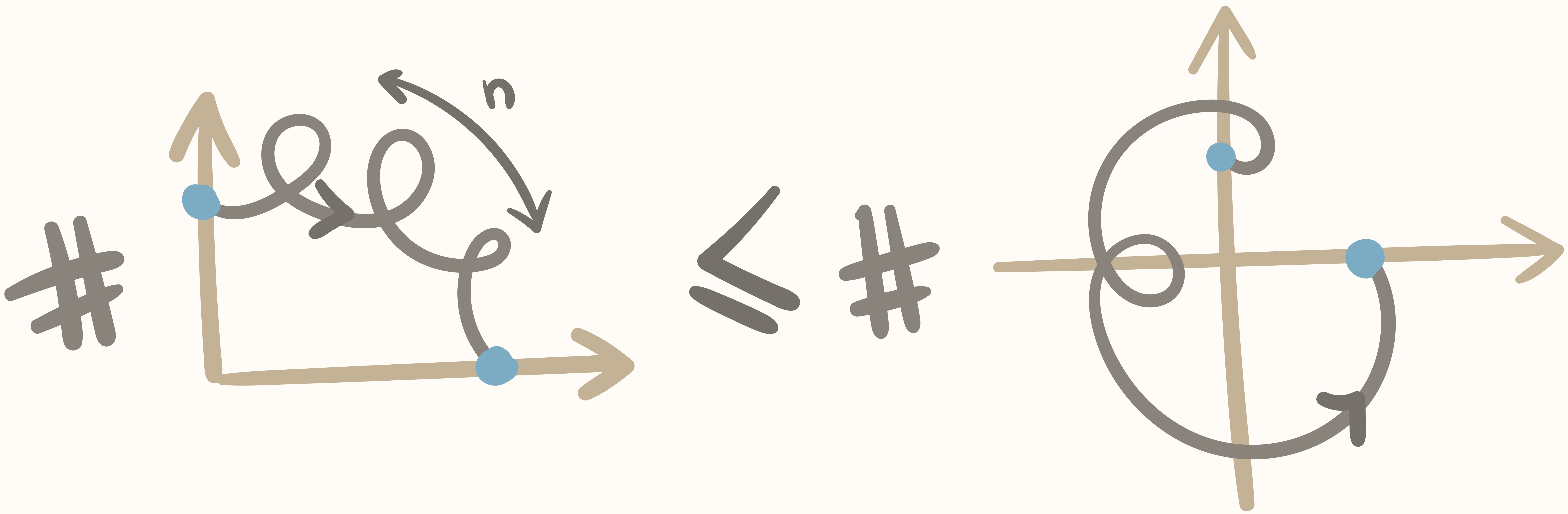


Asymptotic counting results

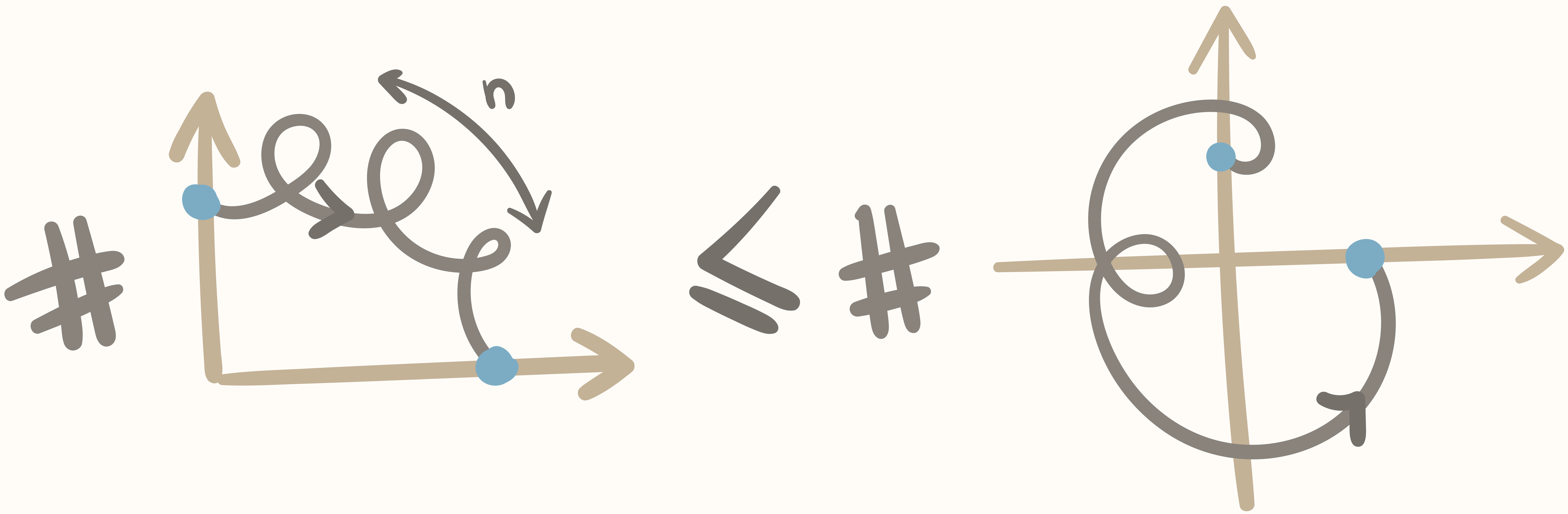


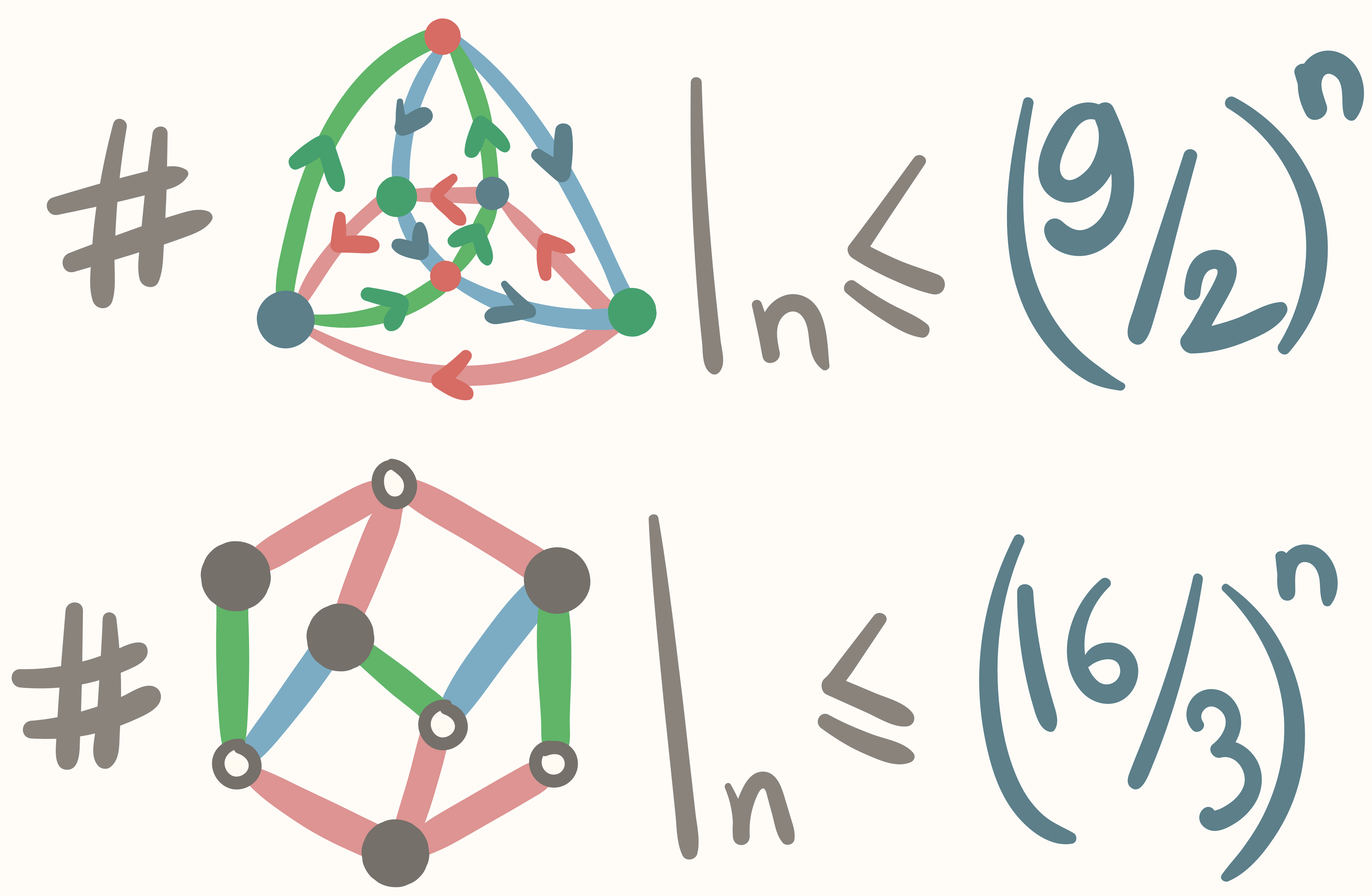


Asymptotic counting results

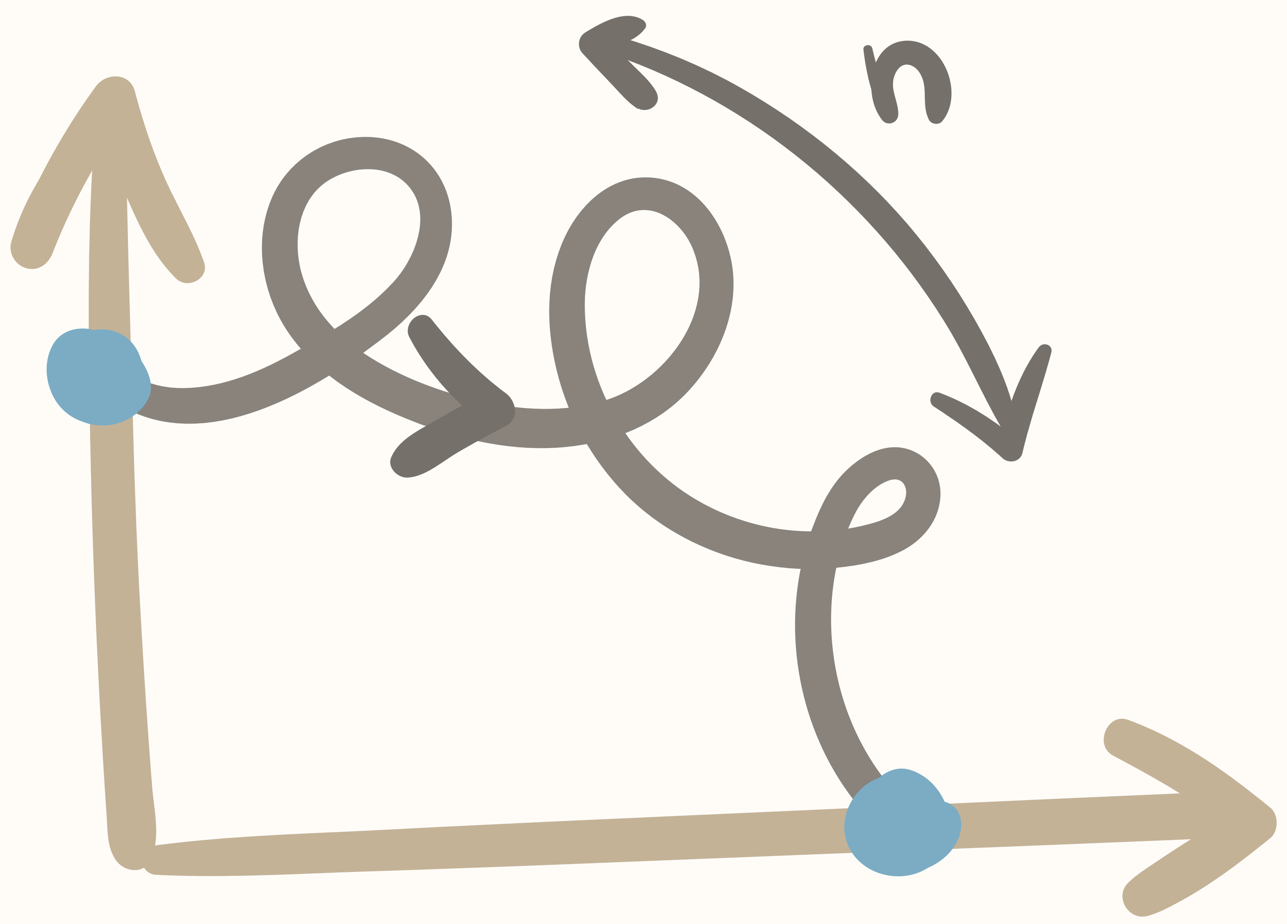


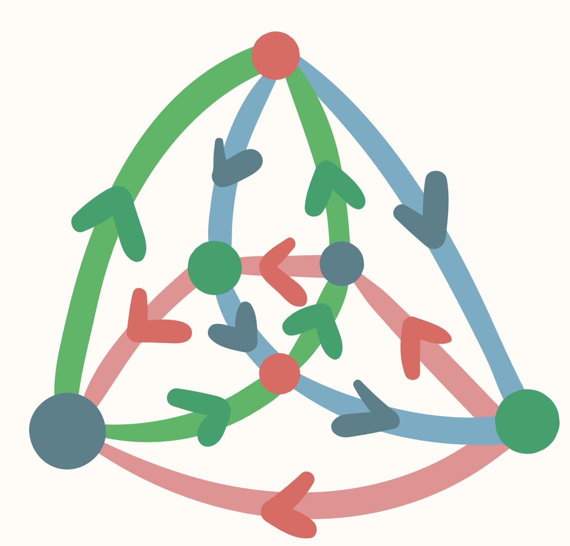
Asymptotic counting results

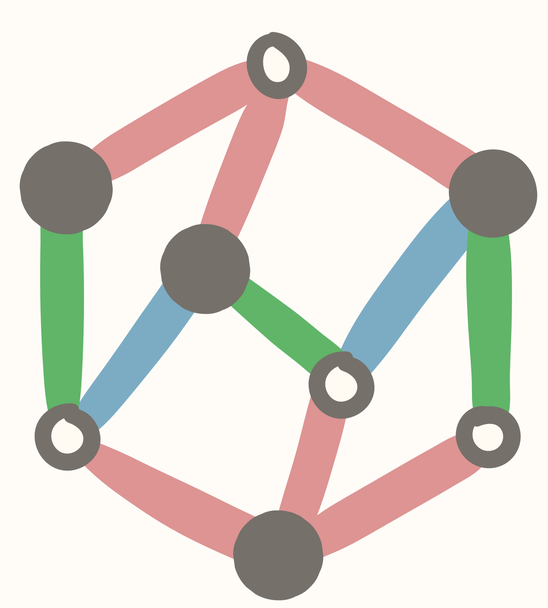




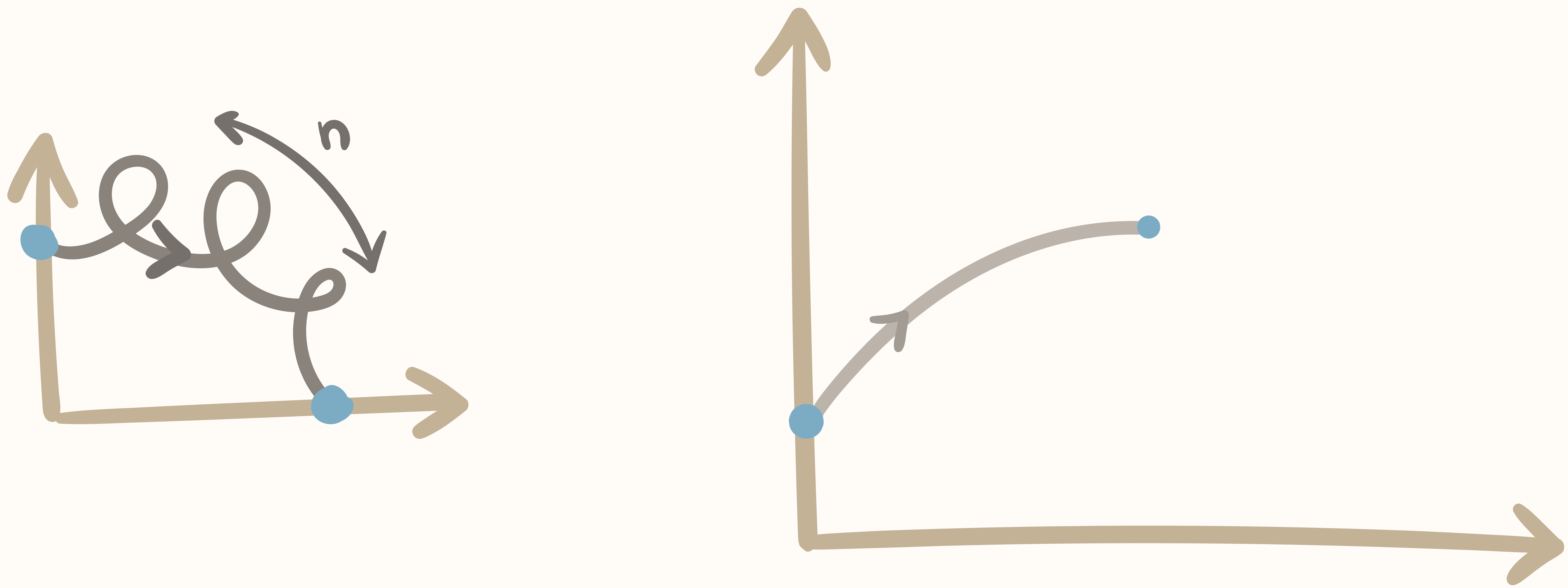
Asymptotic counting results

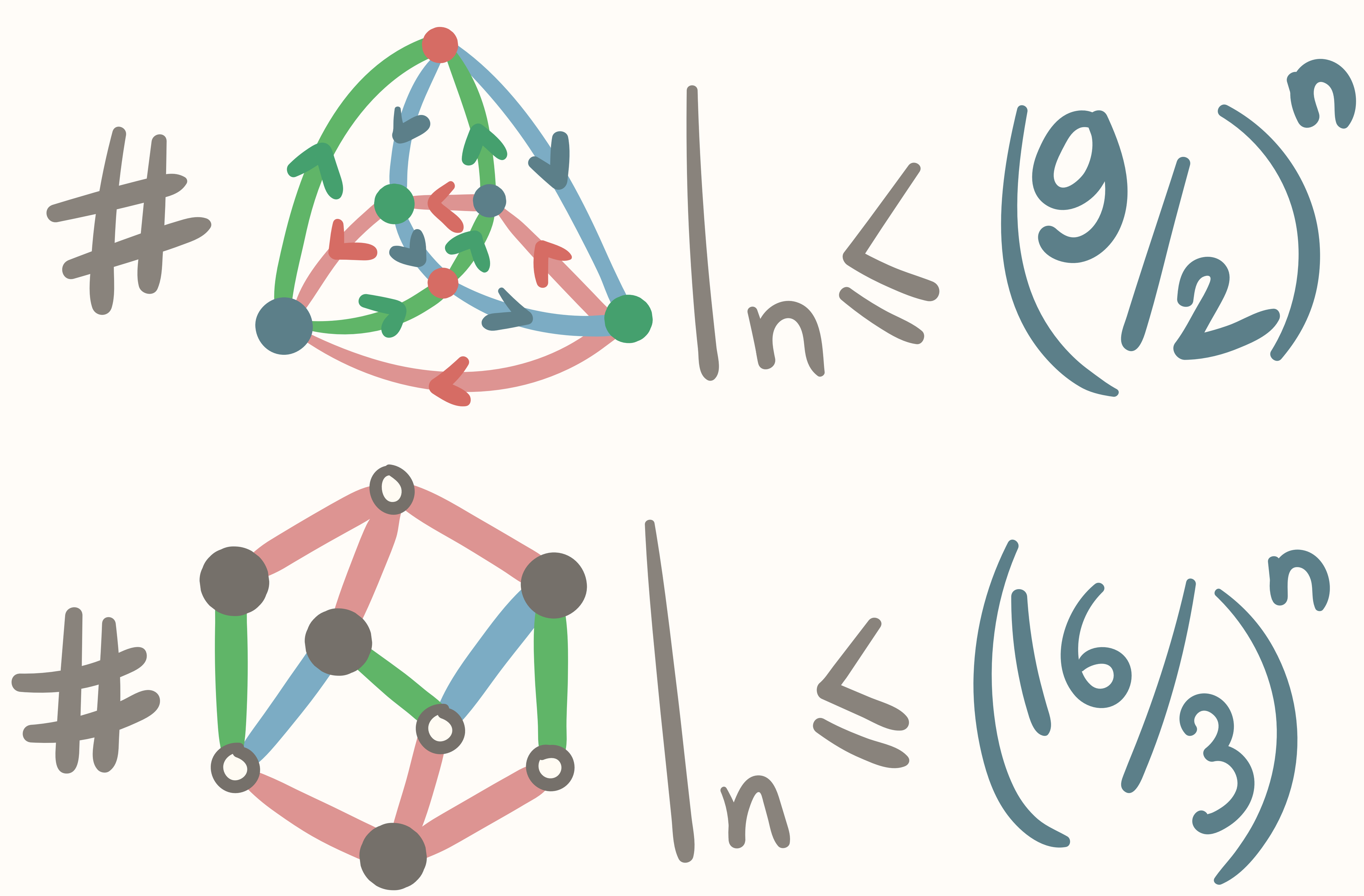


$\#$  $\big|_n \leq (9/2)^n$

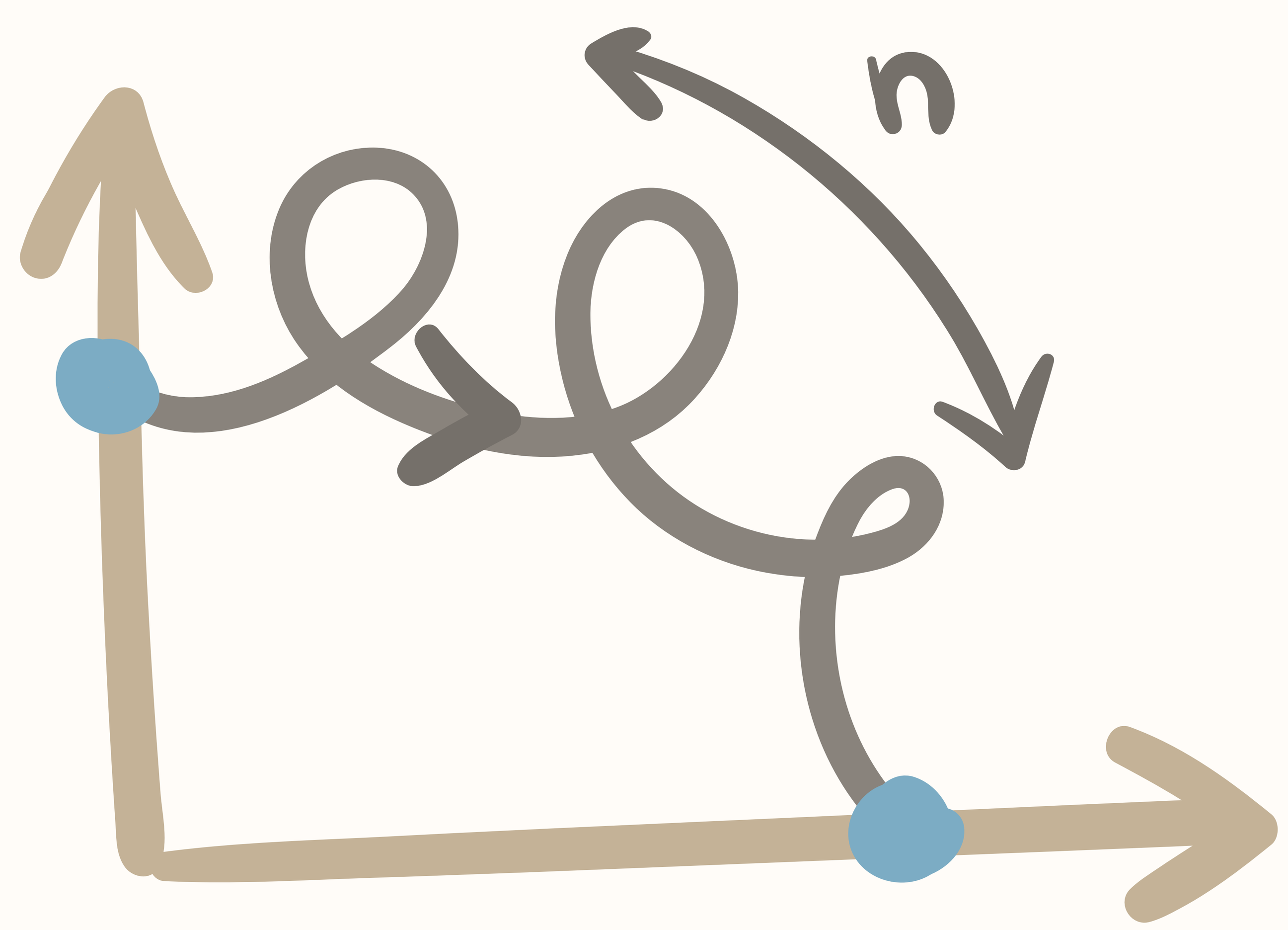
$\#$  $\big|_n \leq (16/3)^n$

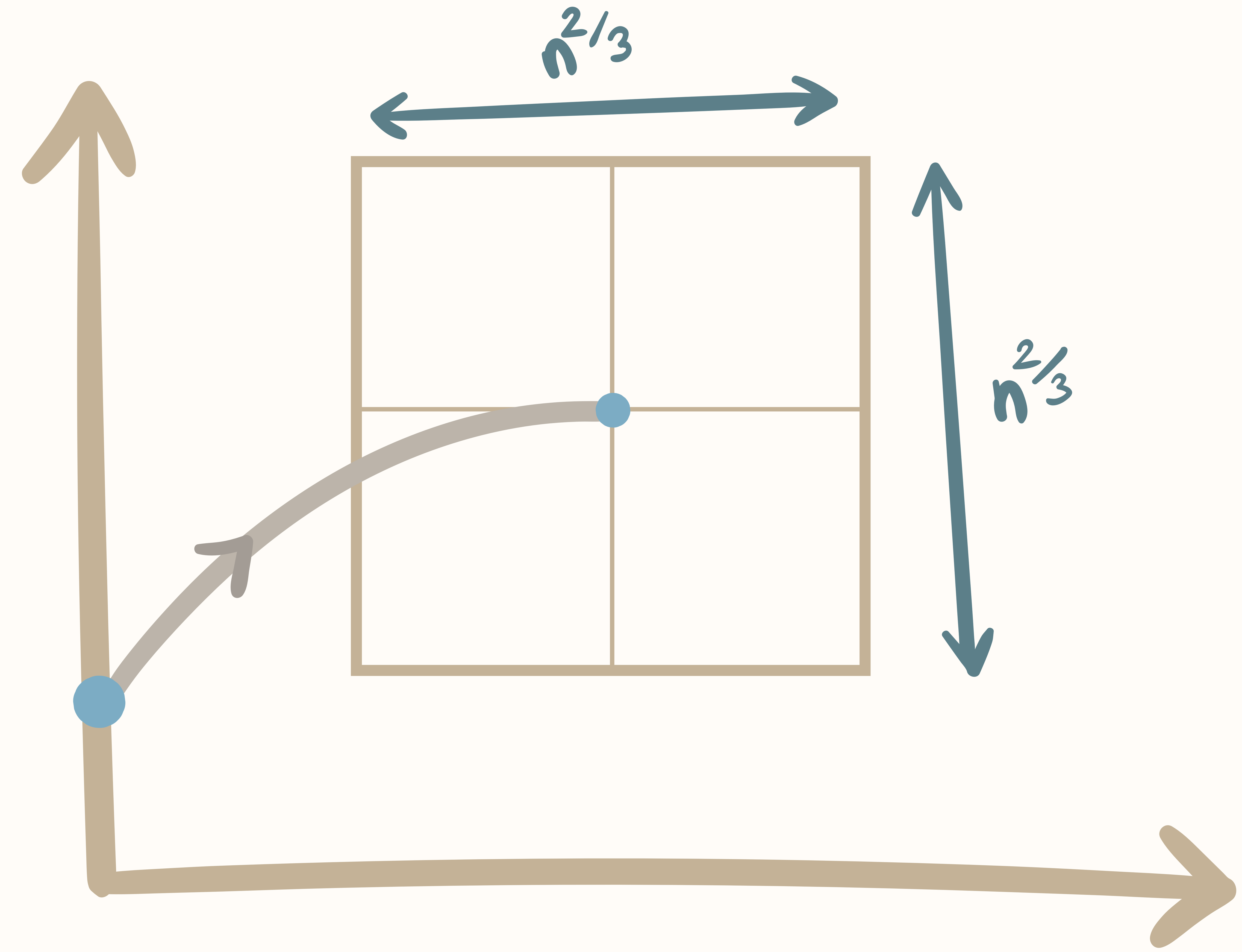
Asymptotic counting results

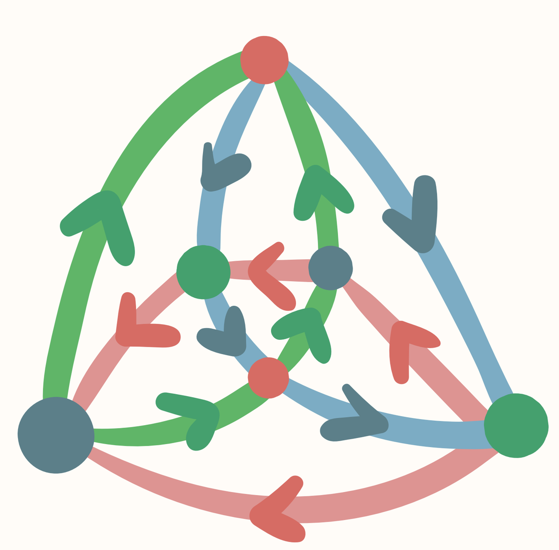


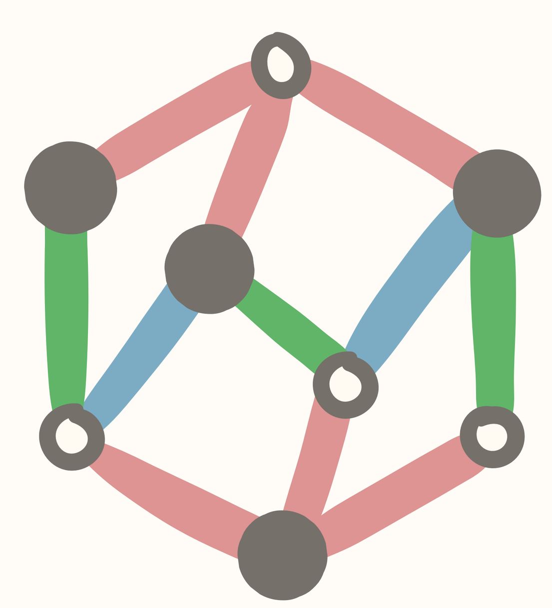


Asymptotic counting results

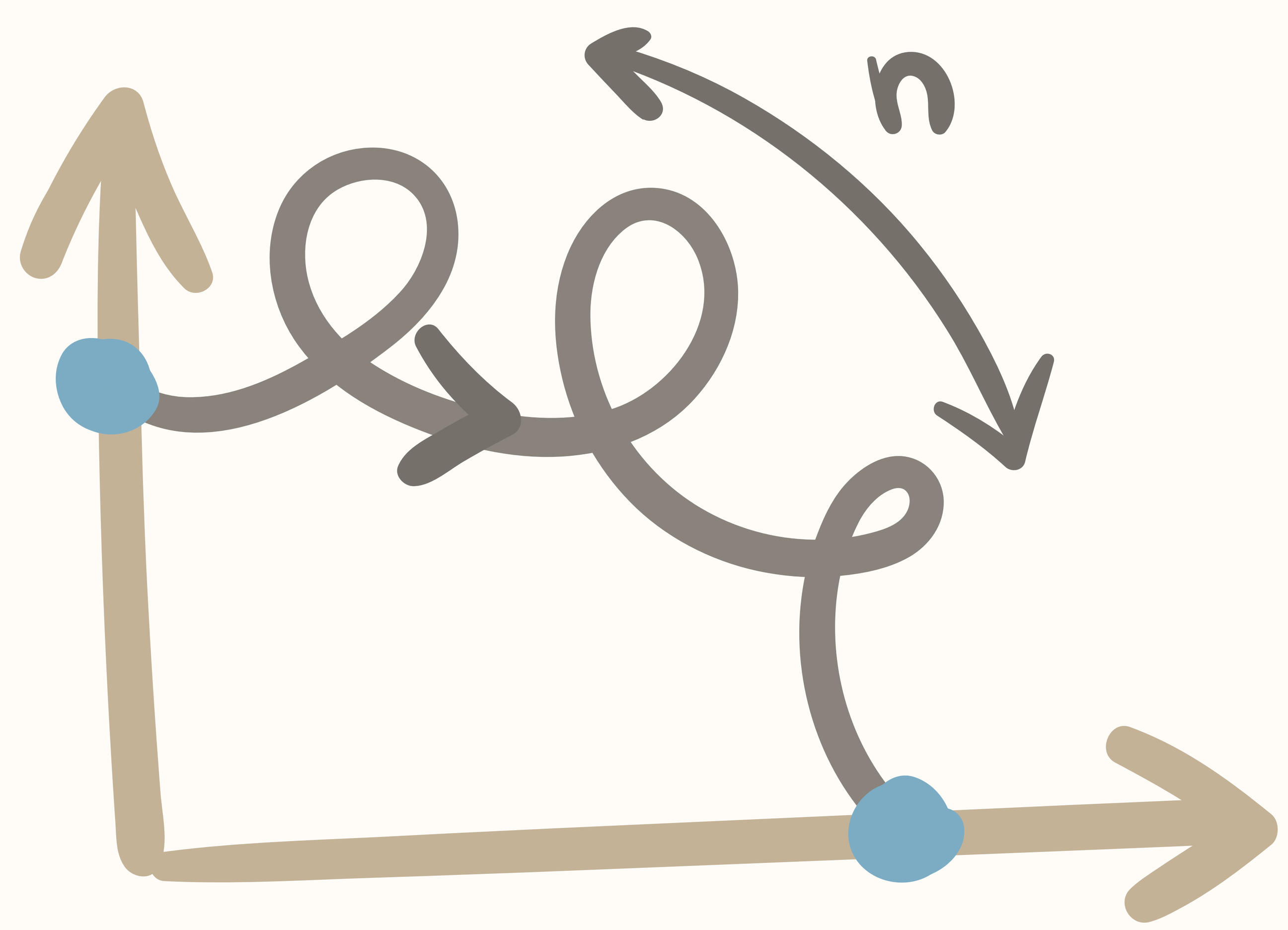


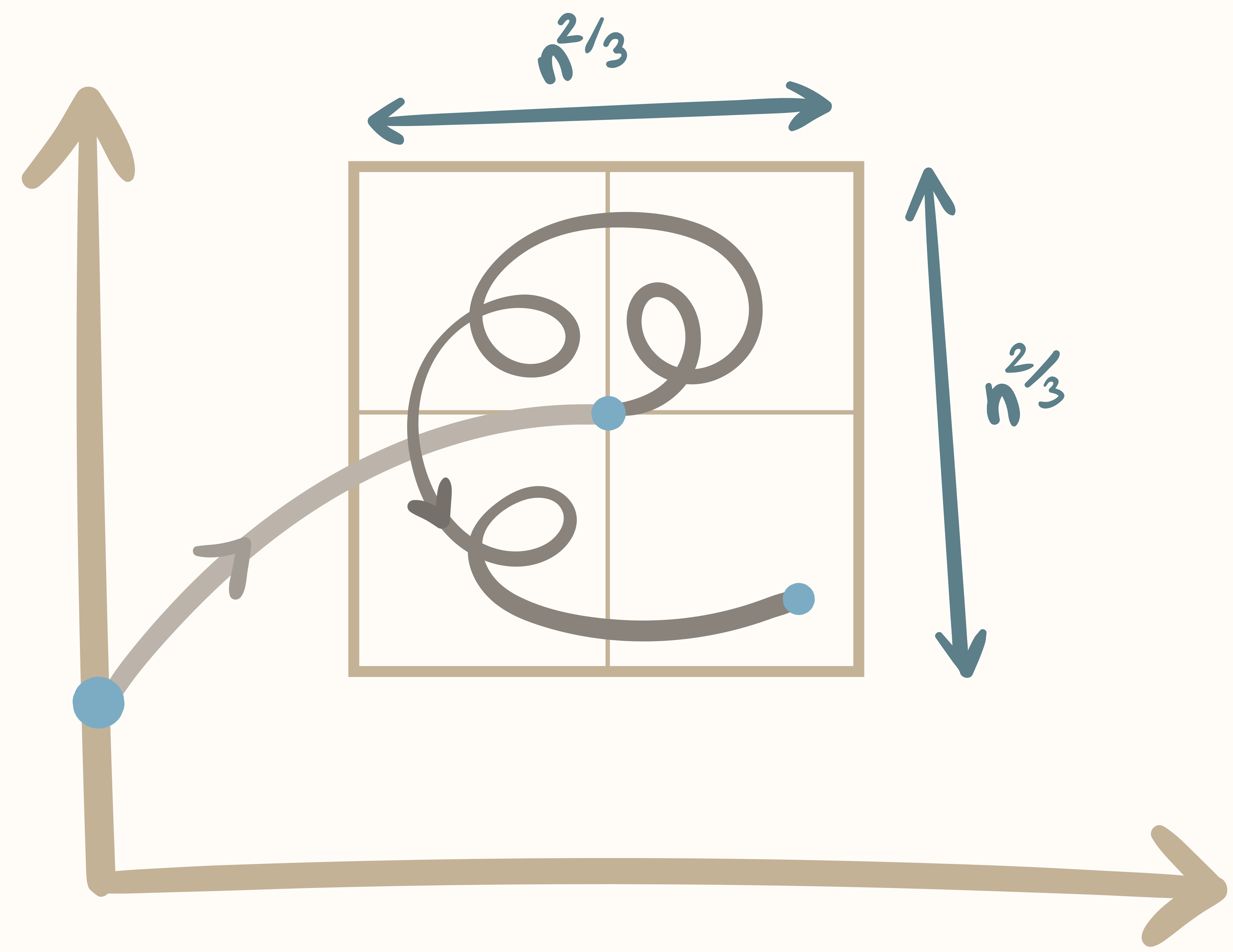


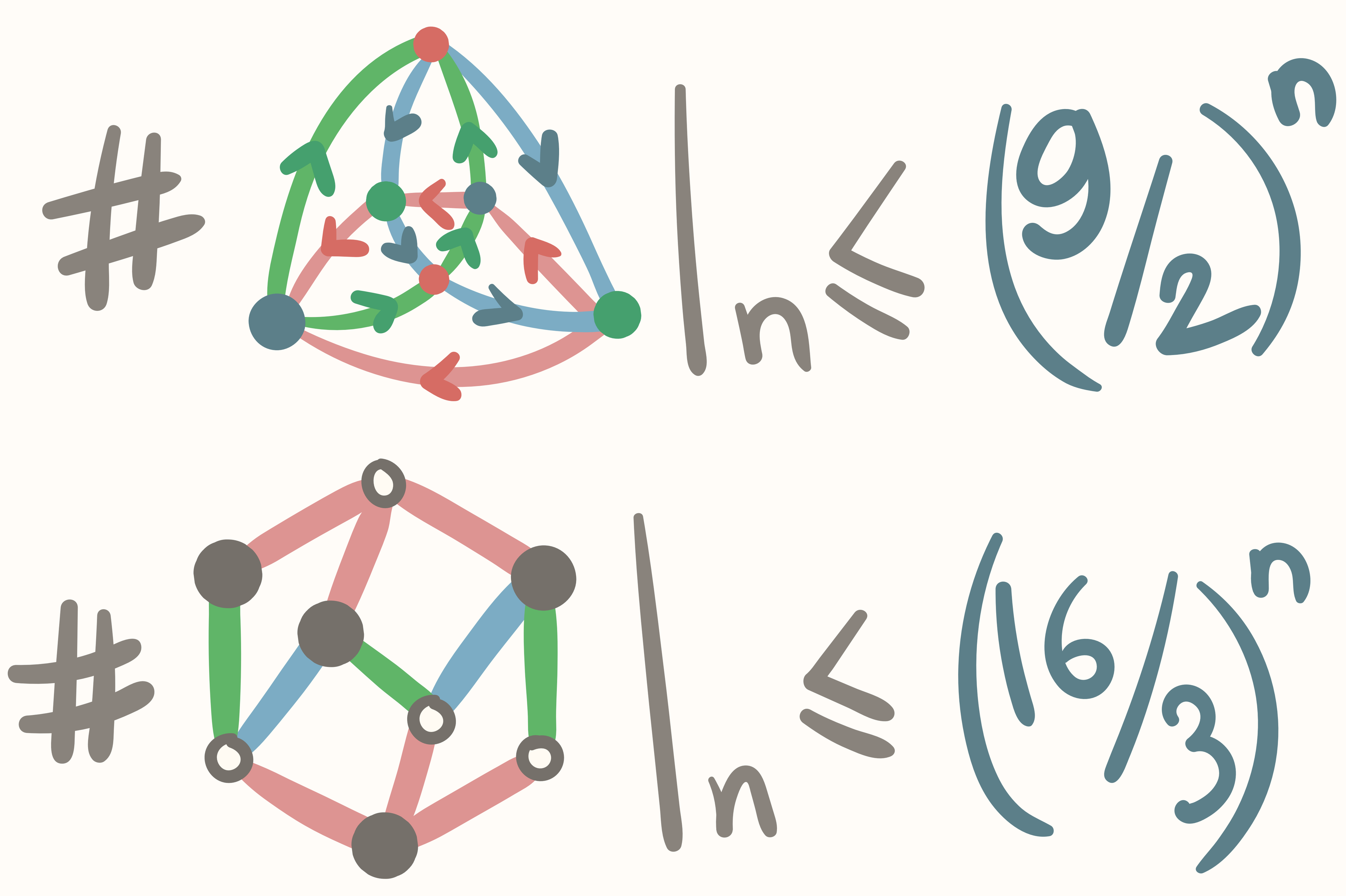
\#  $|_n \leq (9/2)^n$

\#  $|_n \leq (16/3)^n$

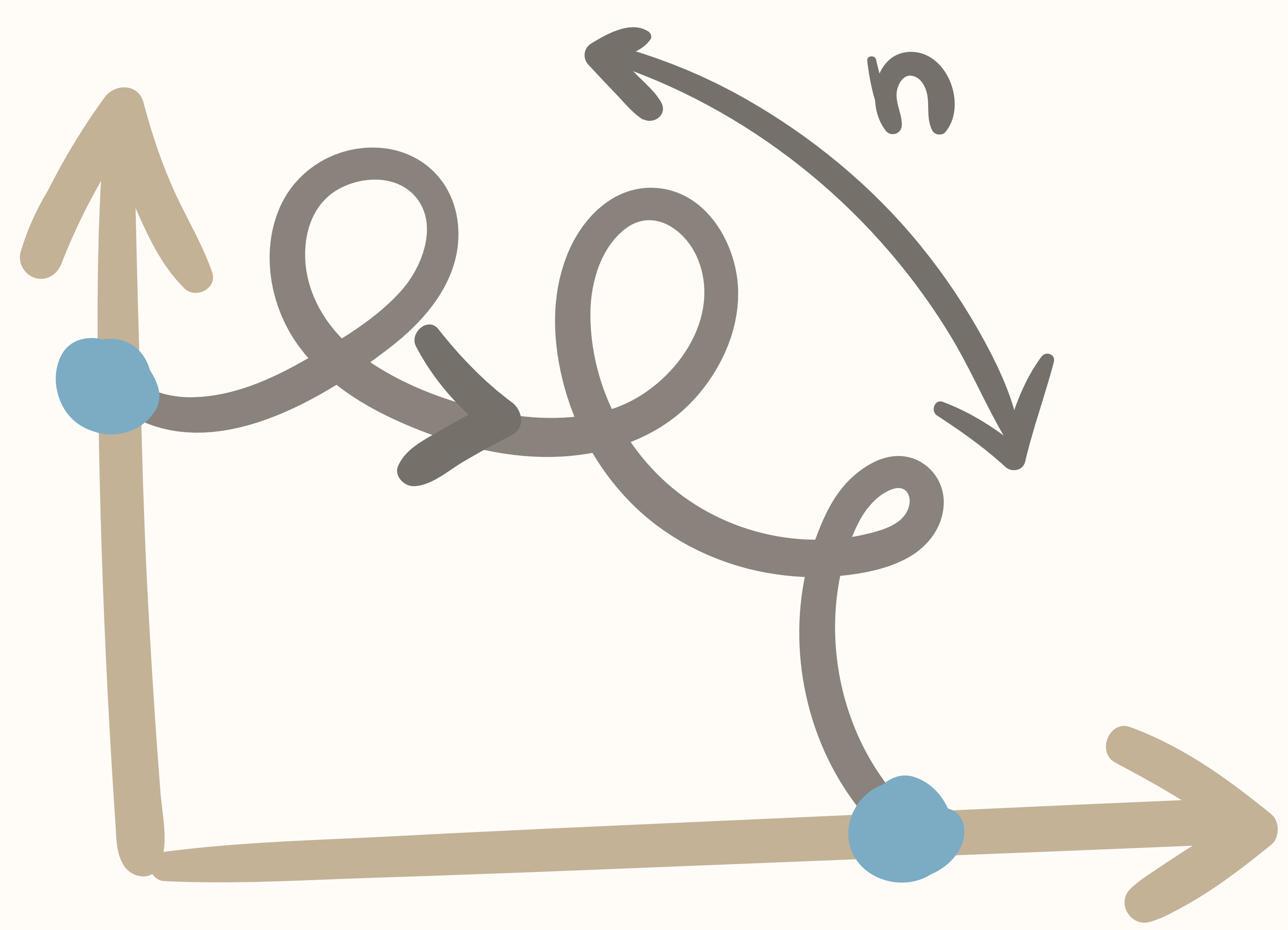
Asymptotic counting results

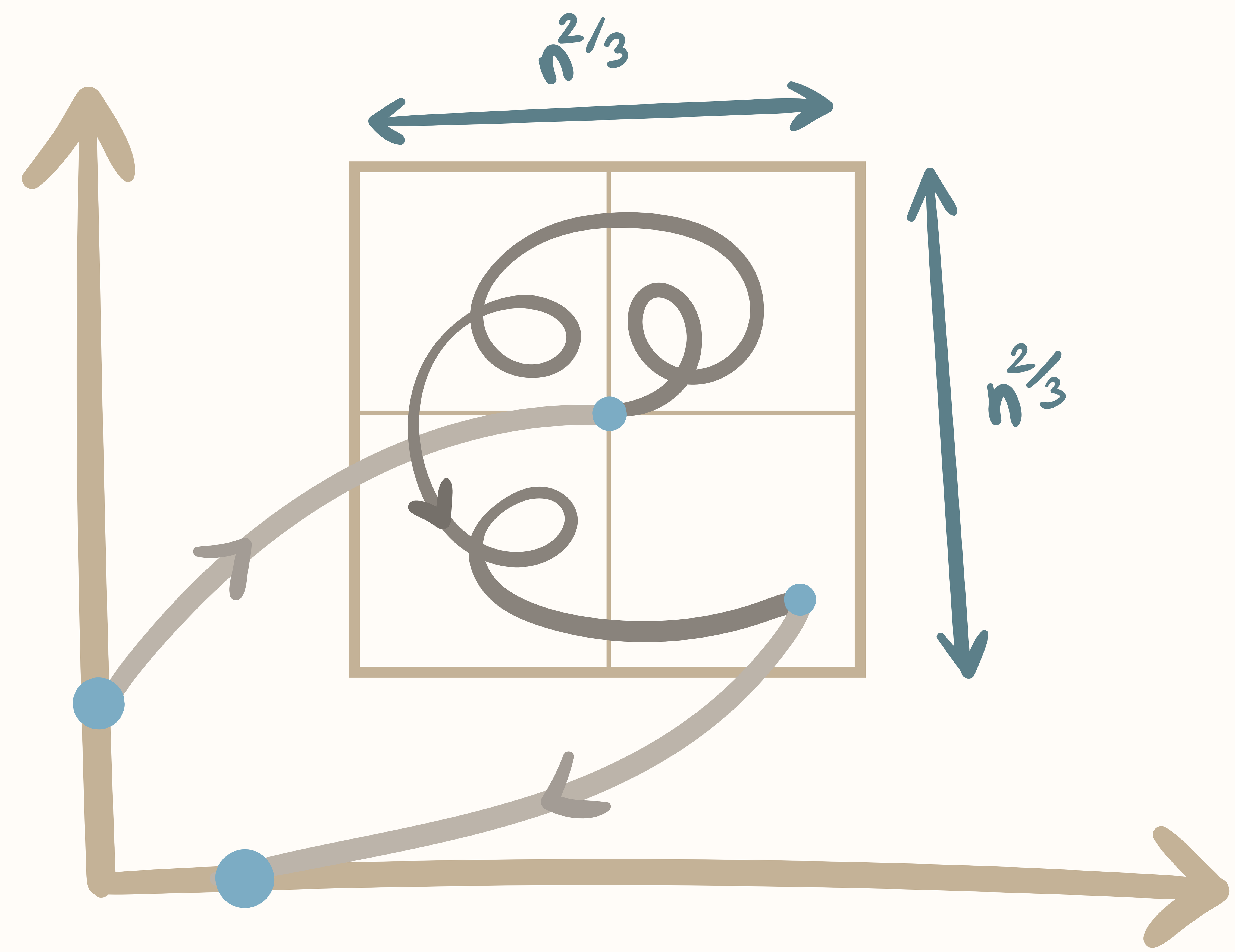


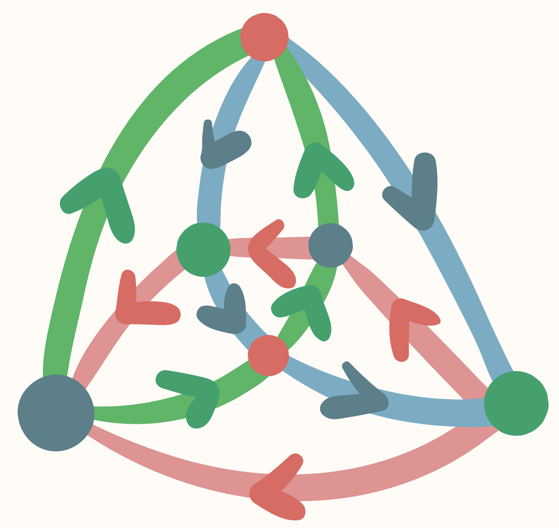


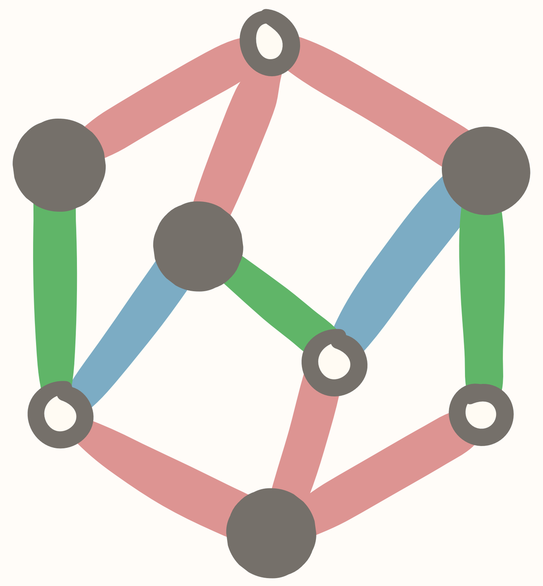


Asymptotic counting results

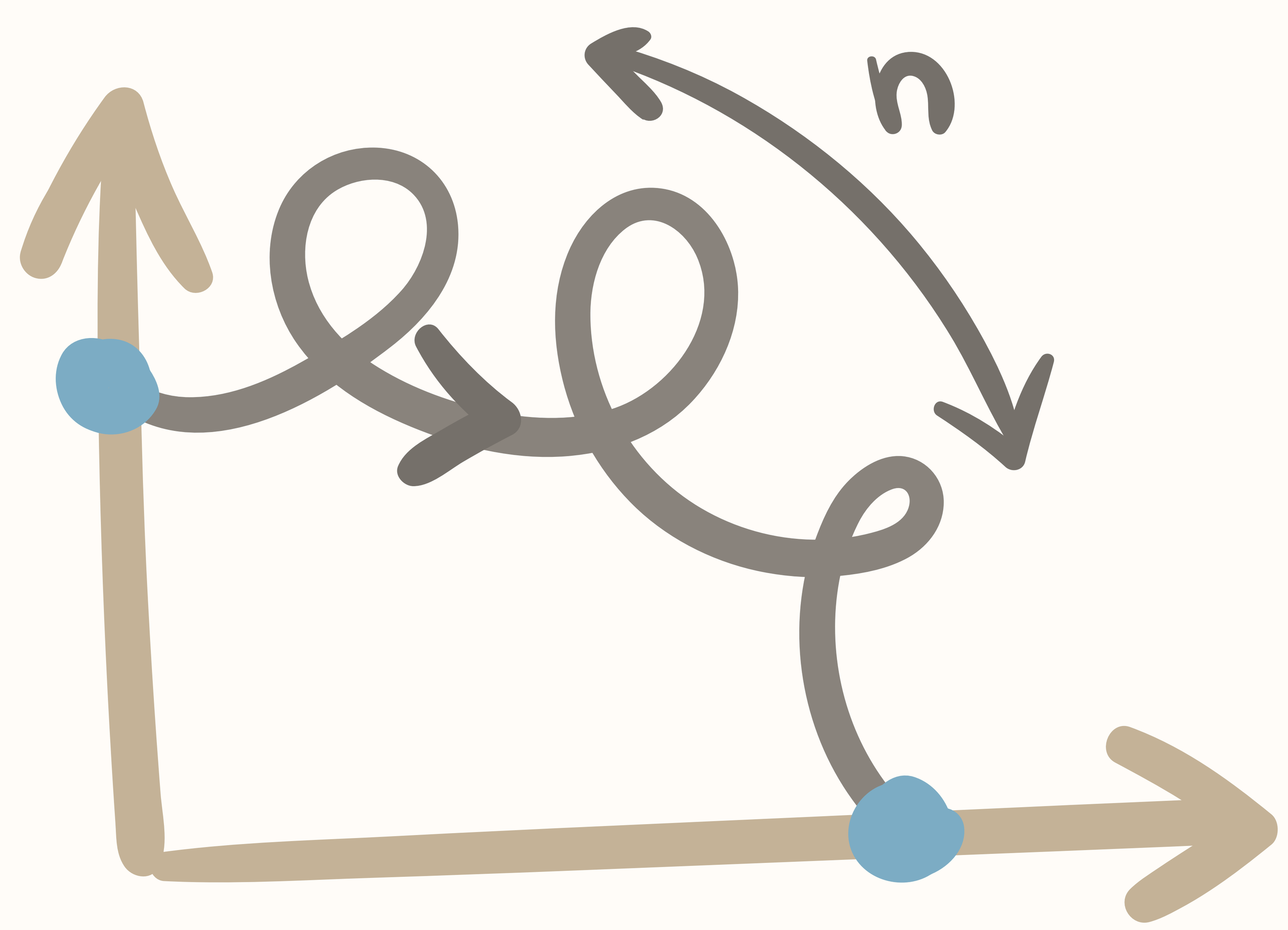


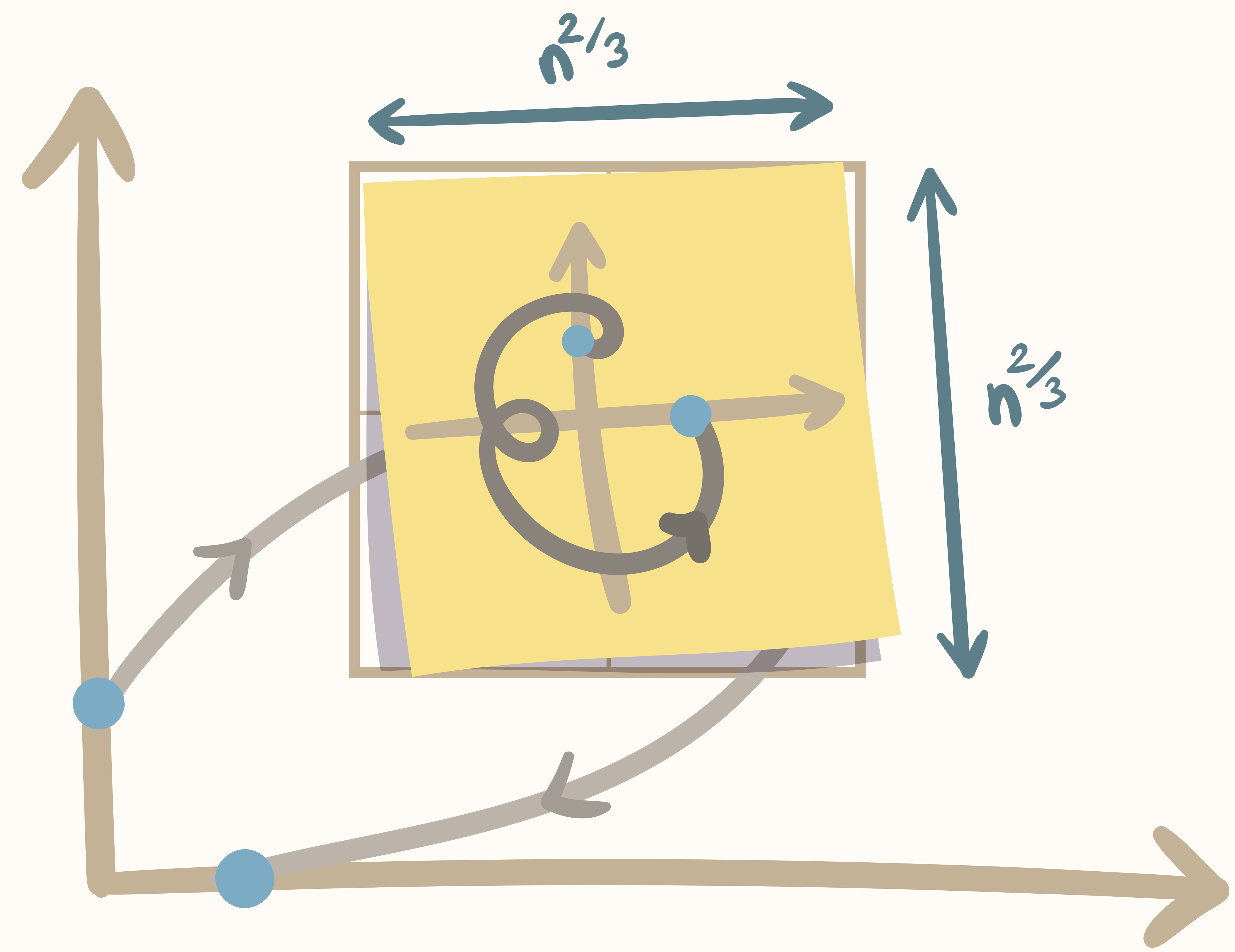


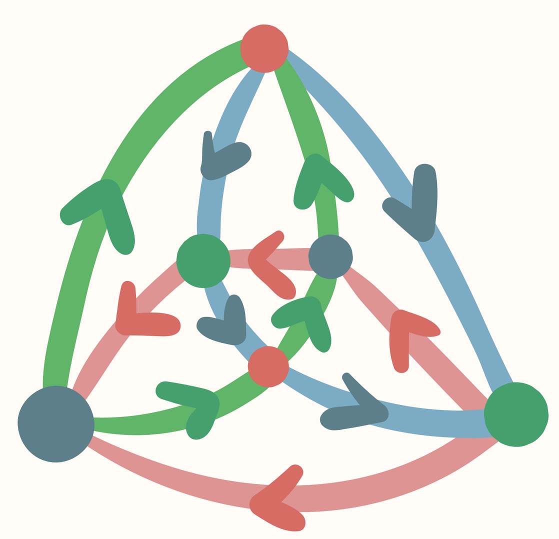
 $|_n \leq (9/2)^n$

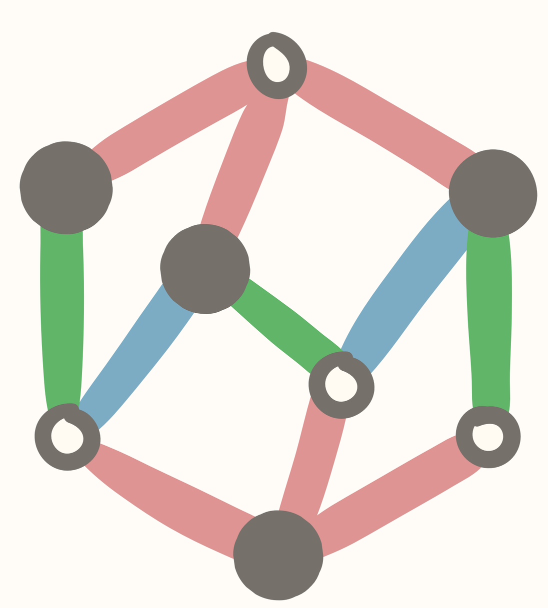
 $|_n \leq (16/3)^n$

Asymptotic counting results

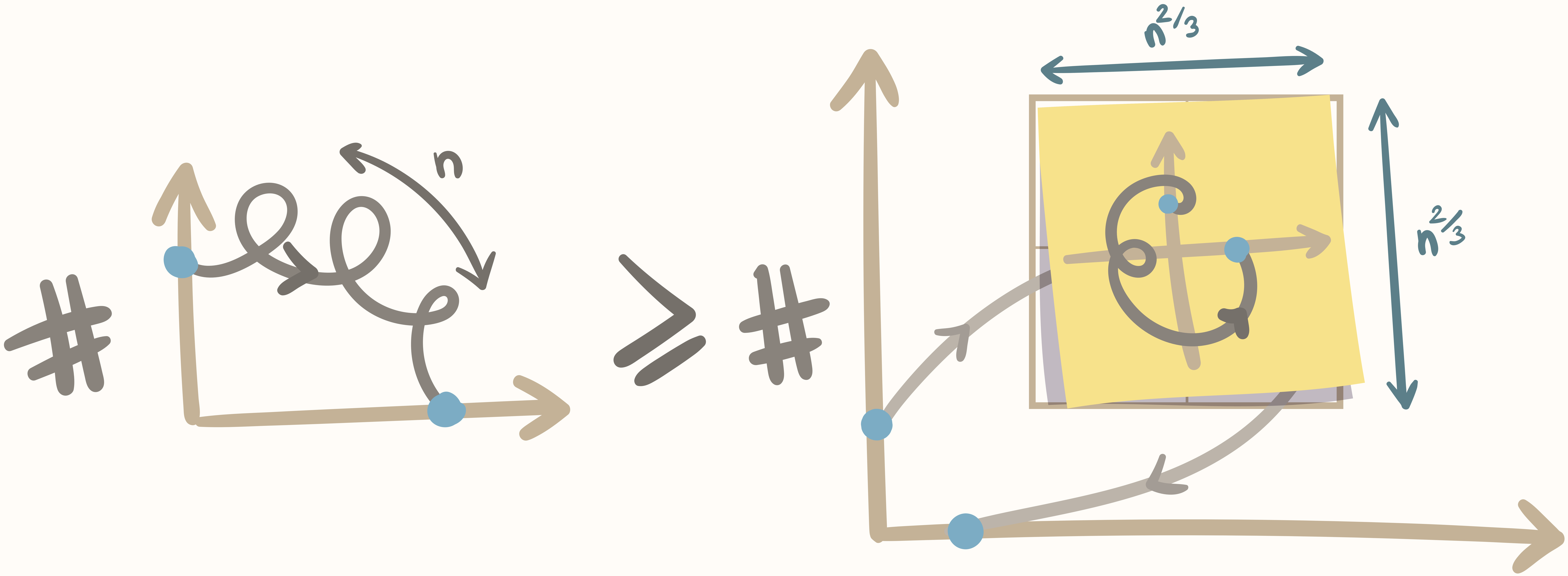


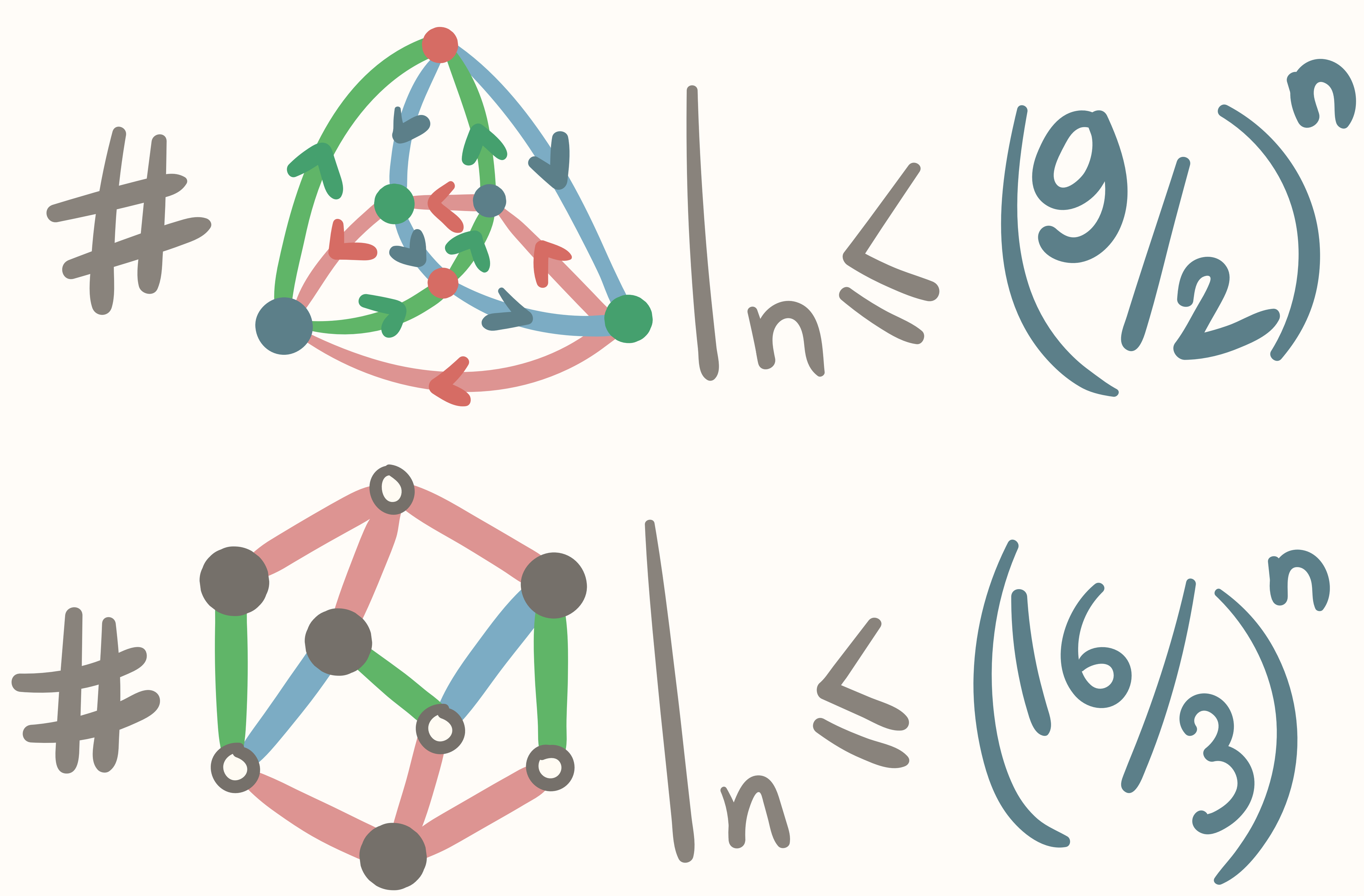


 $\Big|_n \leq \left(9/2\right)^n$

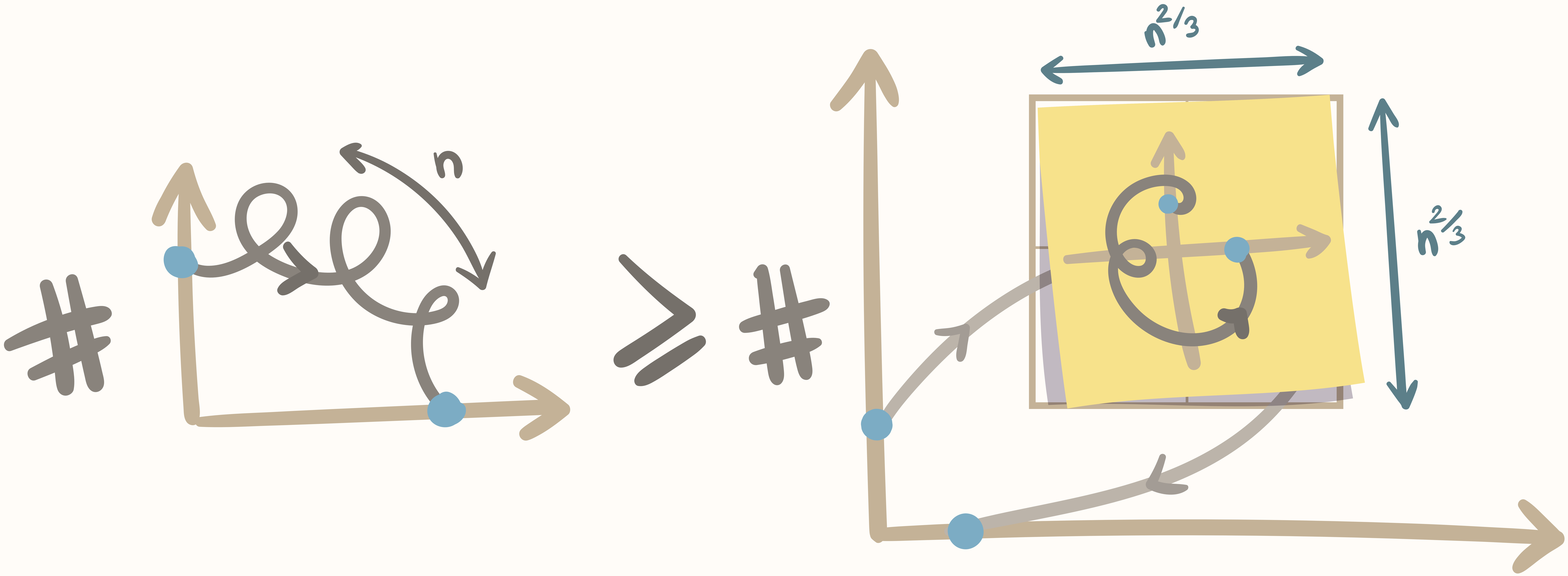
 $\Big|_n \leq \left(16/3\right)^n$

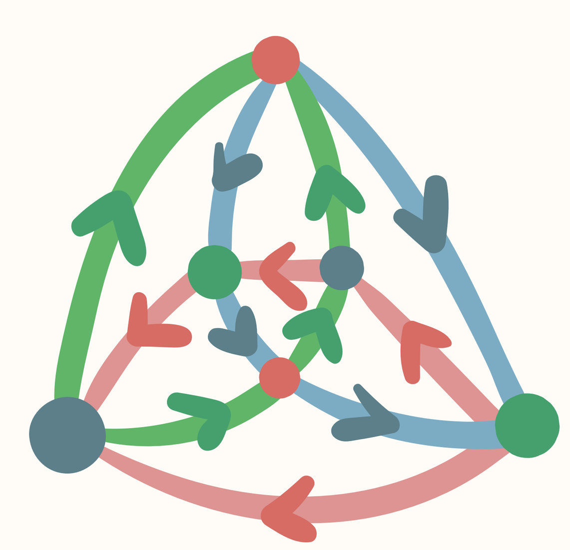
Asymptotic counting results

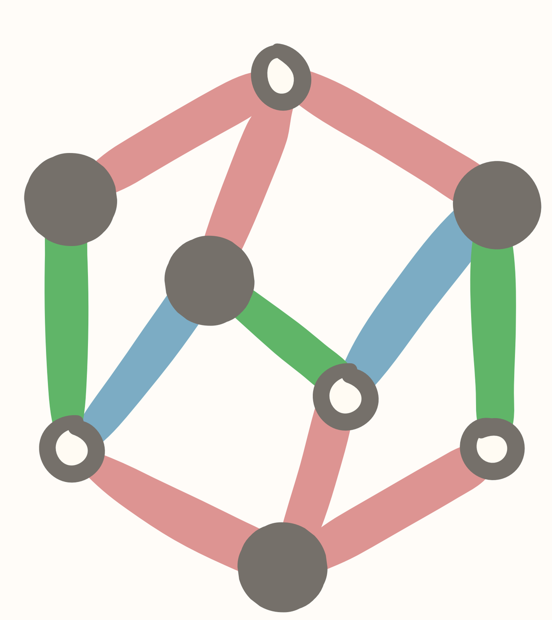




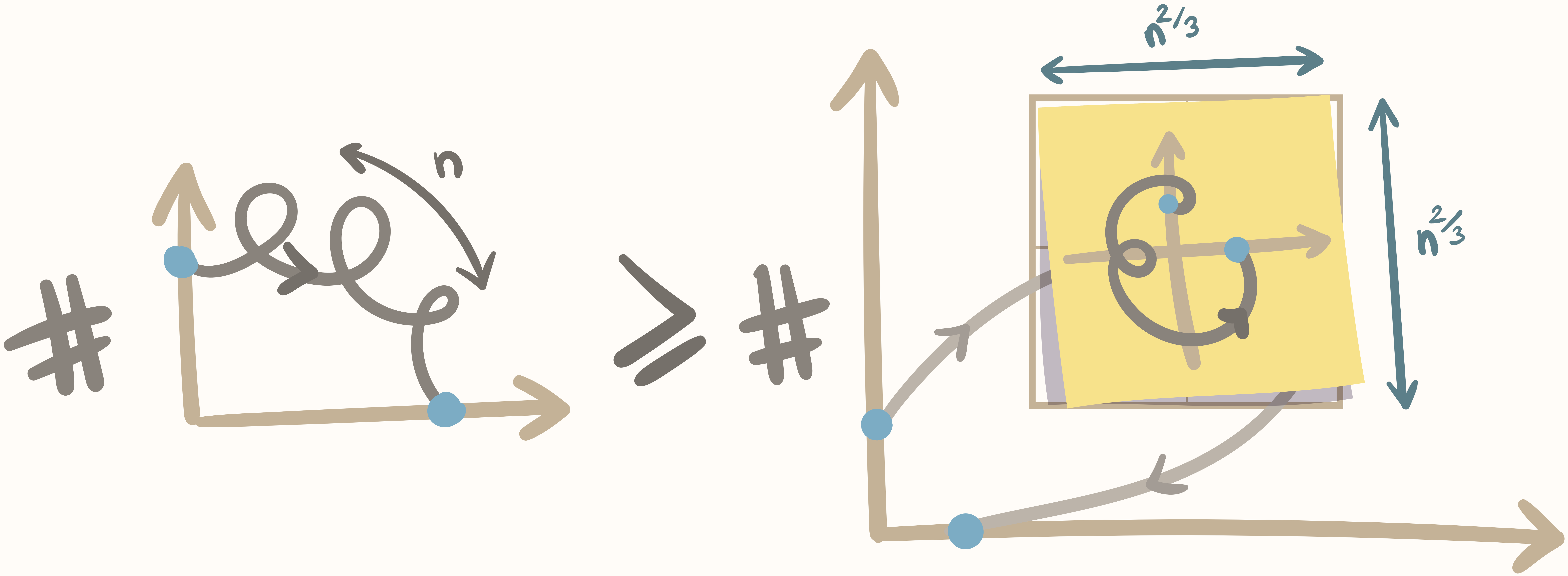
Asymptotic counting results

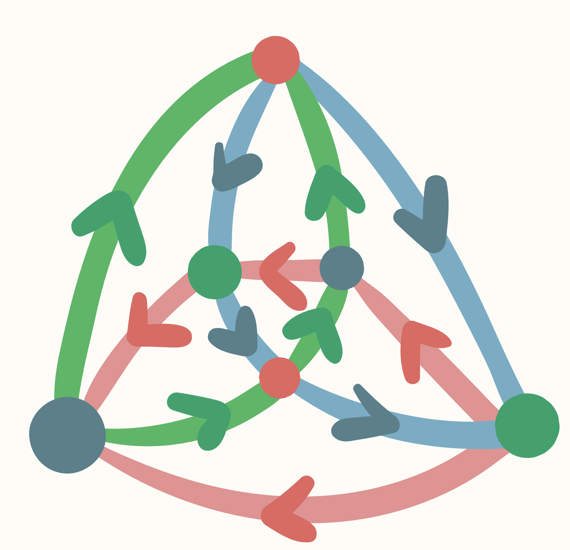


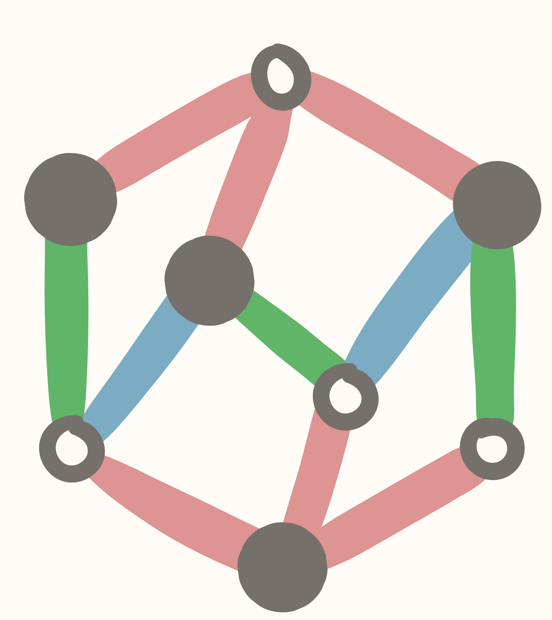
$\left(\frac{9}{2}\right)^{n+o(n)} \leq \#$  $\Big|_n \leq \left(\frac{9}{2}\right)^n$

$\left(\frac{16}{3}\right)^{n+o(n)} \leq \#$  $\Big|_n \leq \left(\frac{16}{3}\right)^n$

Asymptotic counting results



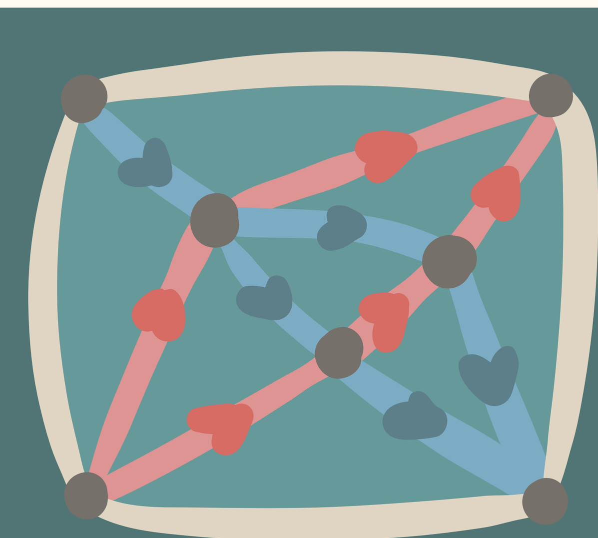
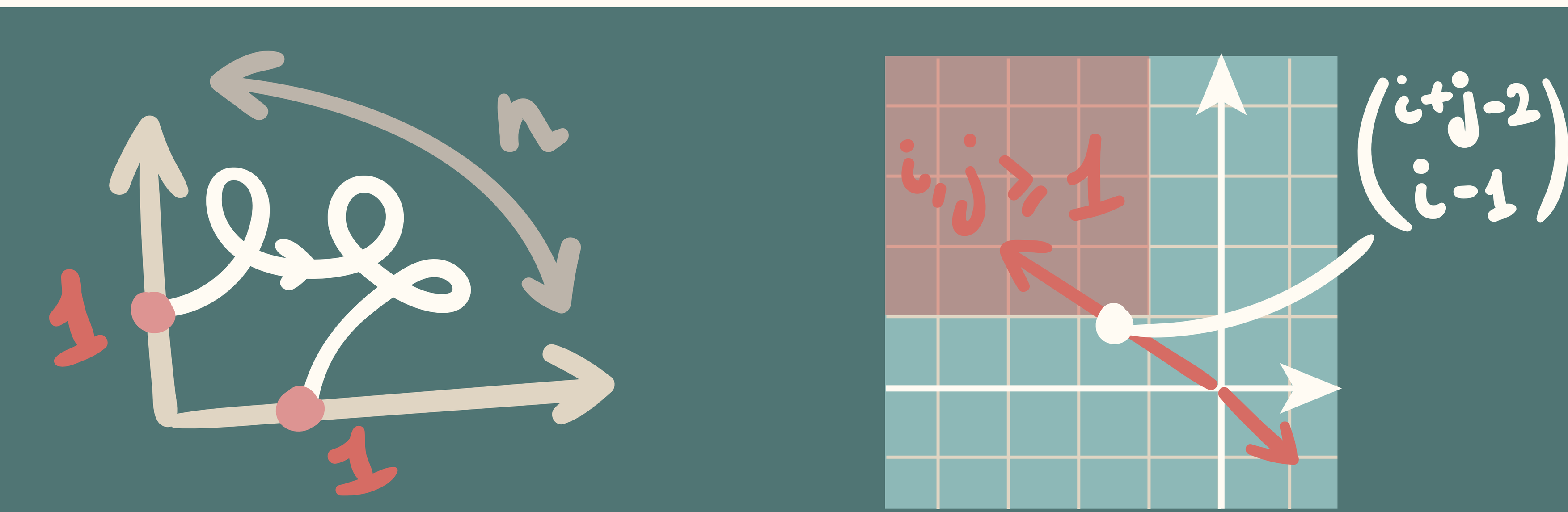
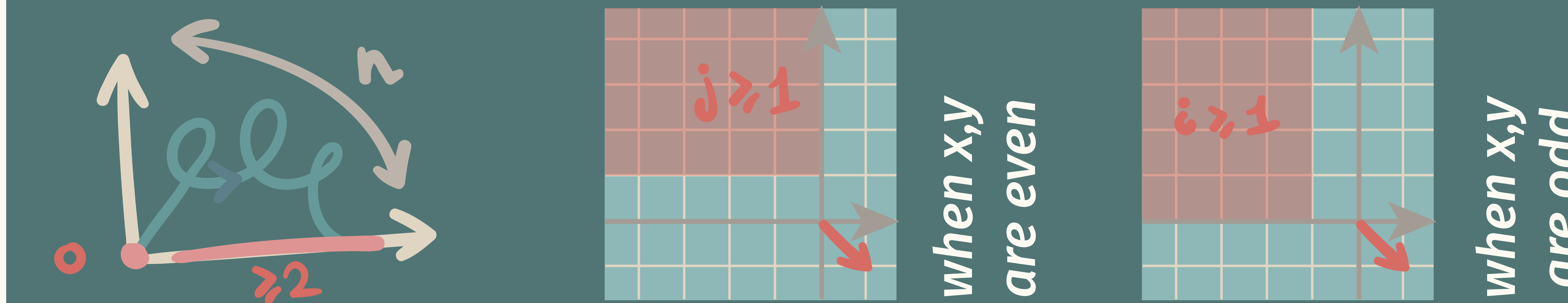
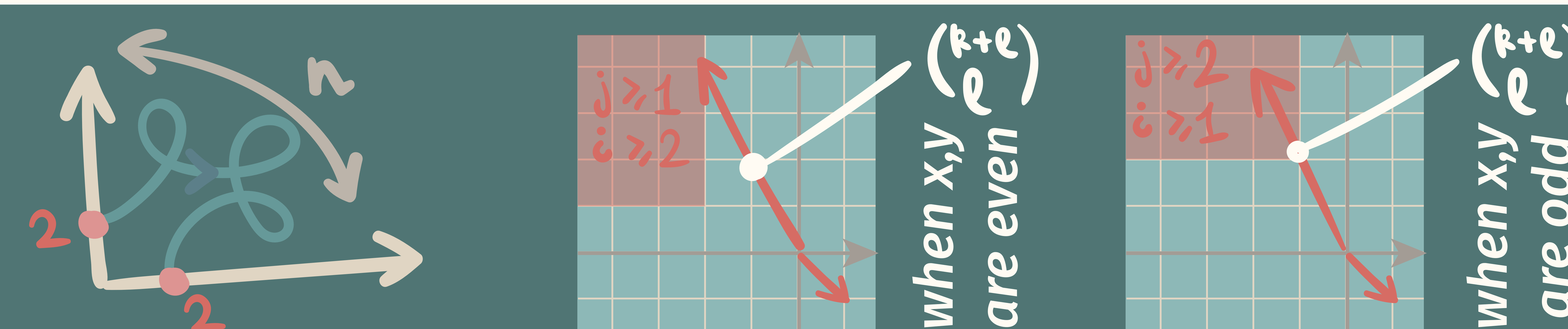
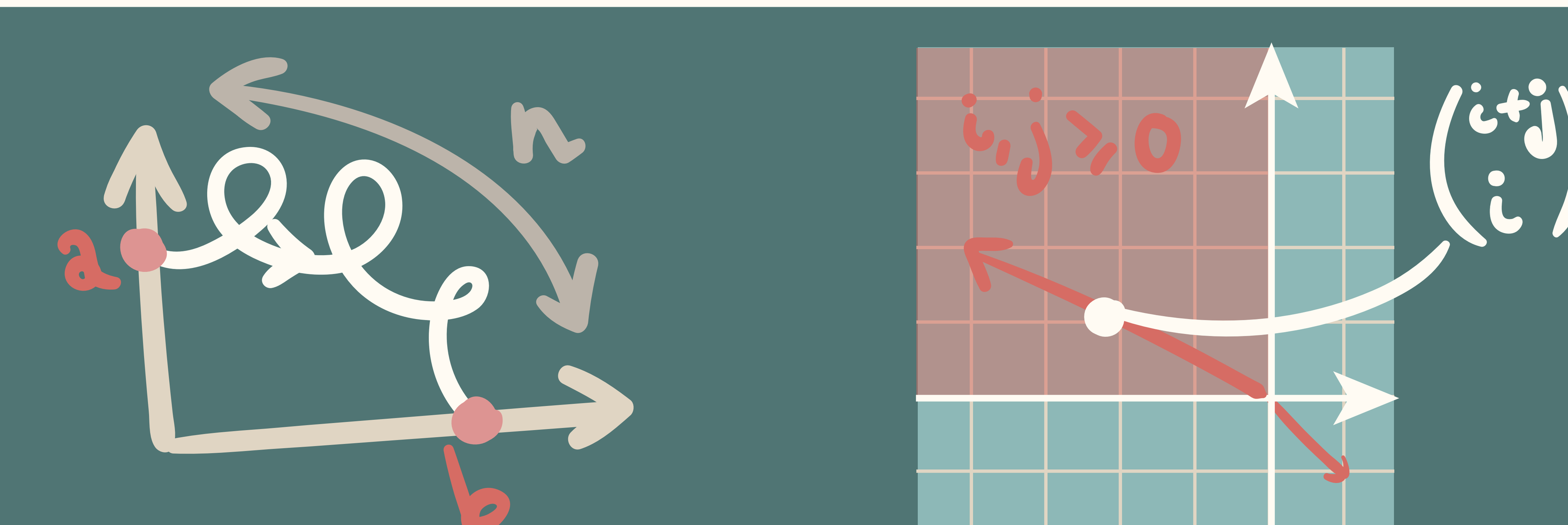
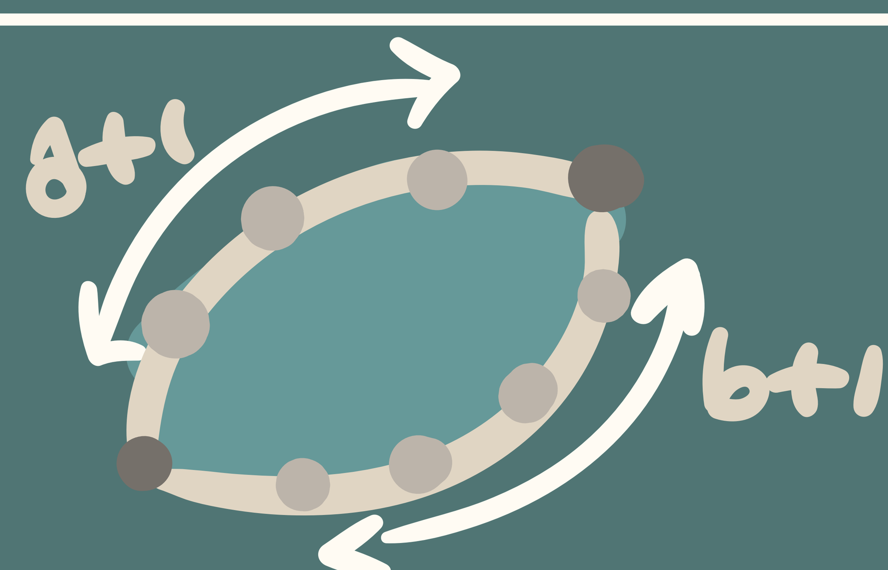
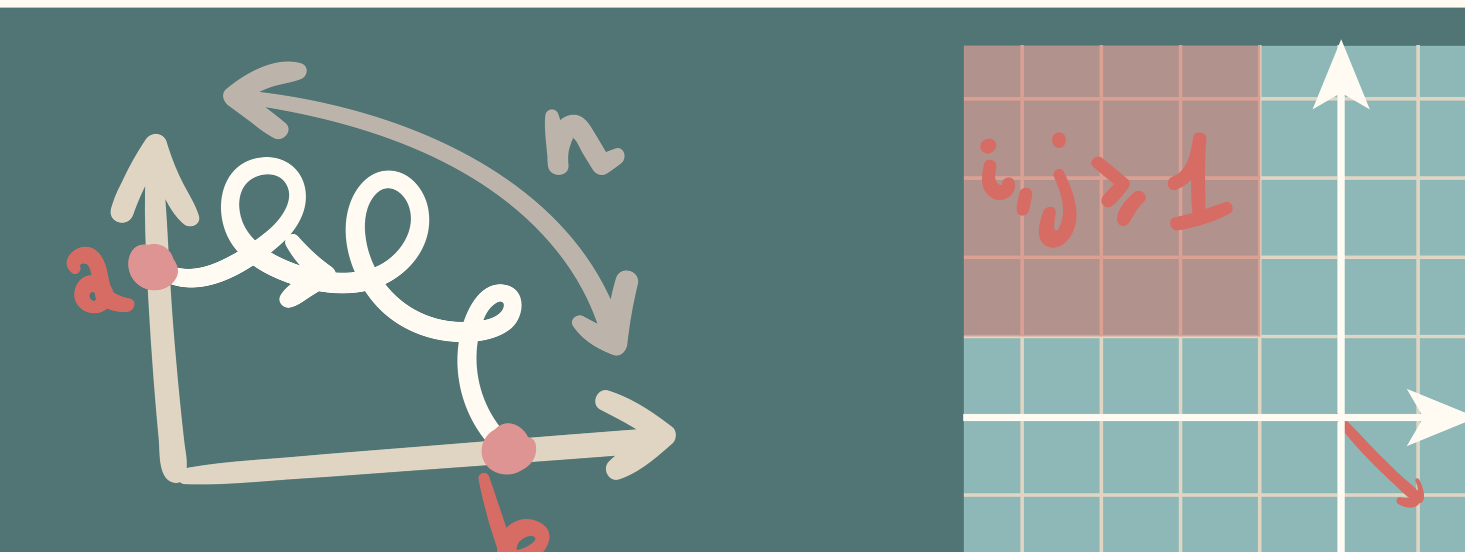
$\left(9/2\right)^{n+o(n)} \leq \#$  $\Big|_n \leq \left(9/2\right)^{n}$

$\left(16/3\right)^{n+o(n)} \leq \#$  $\Big|_n \leq \left(16/3\right)^{n}$

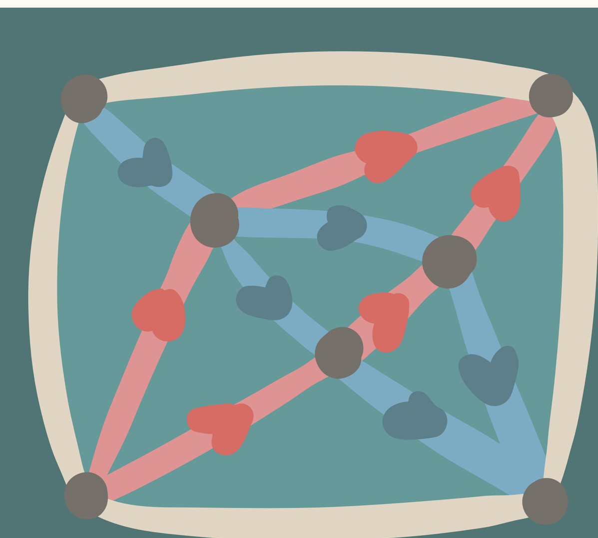
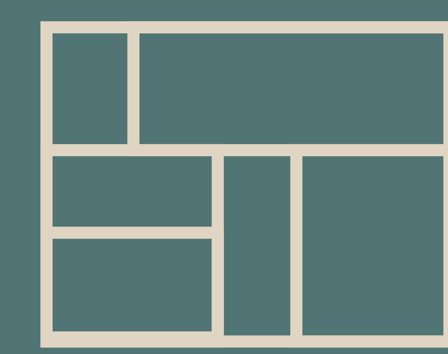
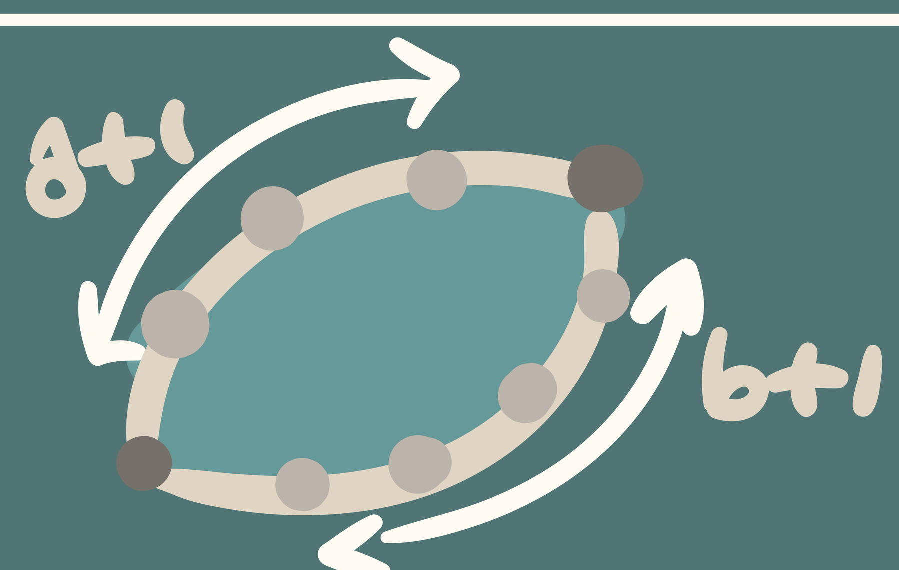
$$\lim_{n\to\infty} p_n^{1/n} = 9/2$$

$$\lim_{n\to\infty} s_n^{1/n} = 16/3$$

Summary

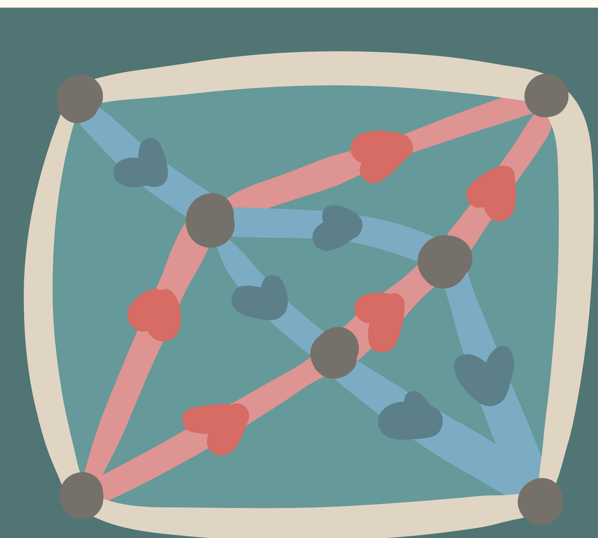
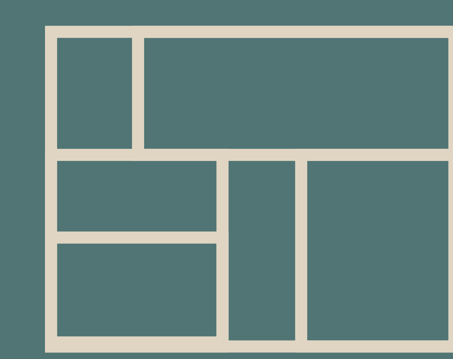
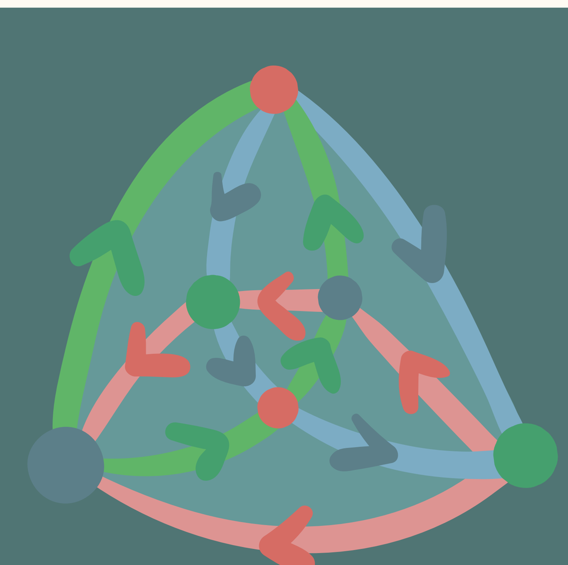
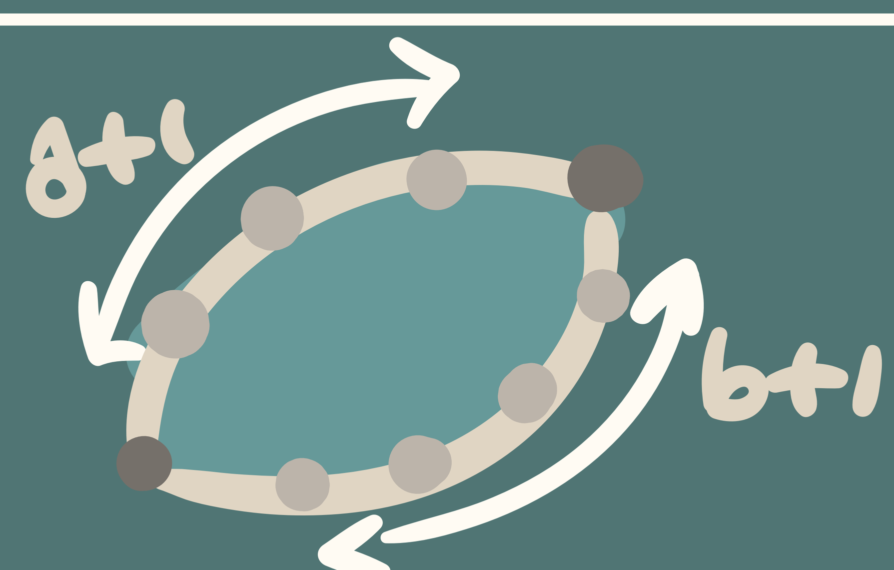
Model		Tandem Walks
Transversal structures *n blue edges*		
Polyhedral orientations *n inner faces*		
Schnyder colorings *n inner faces*		
Posets *n vertices*	 	
Posets *n+2 edges*	 	

Summary

Model		*Asymtotics*	
Transversal structures *n blue edges* 			
Polyhedral orientations *n inner faces* 			
Schnyder colorings *n inner faces* 			
Posets *n vertices* 			
Posets *n+2 edges* 			

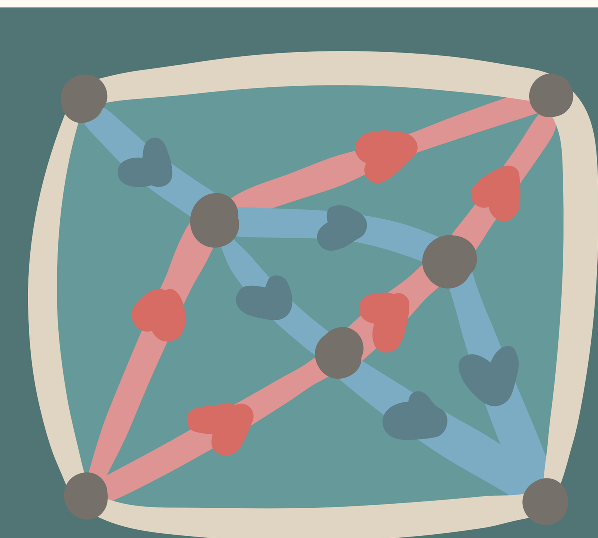
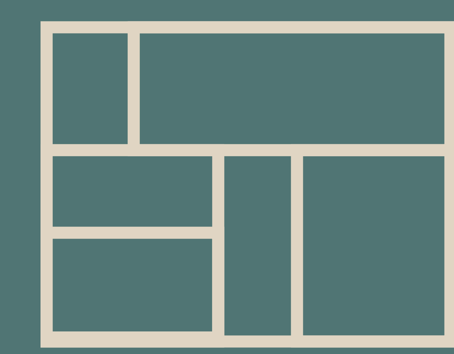
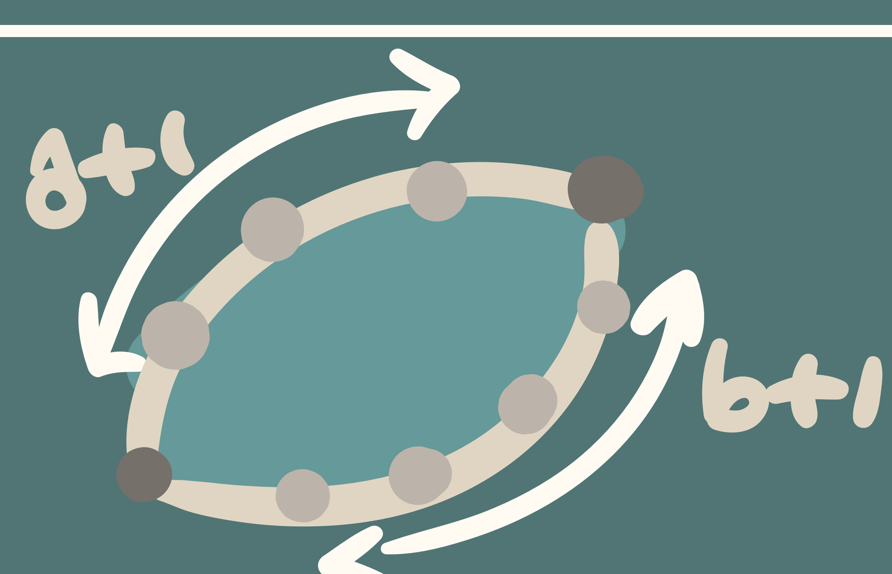
Summary

$a_n \sim \varkappa \cdot \gamma^n n^\alpha$ | $\alpha = 1 + \pi / \arccos(\xi)$

Model		Asymptotics	
Transversal structures *n blue edges* 		$\gamma = 27/2$	non D-Finite
		$\alpha \approx 7.21$	$\xi = 7/8$
Polyhedral orientations *n inner faces* 			
Schnyder colorings *n inner faces* 			
Posets *n vertices* 		$\gamma = (11 + 5\sqrt{5})/2$	D-Finite
		$\alpha = 6$	$\xi = (1 + \sqrt{5})/4$
Posets *n+2 edges* 		$\gamma \approx 4.80$	non D-Finite
		$\alpha \approx 5.14$	$\xi \approx 0.73$

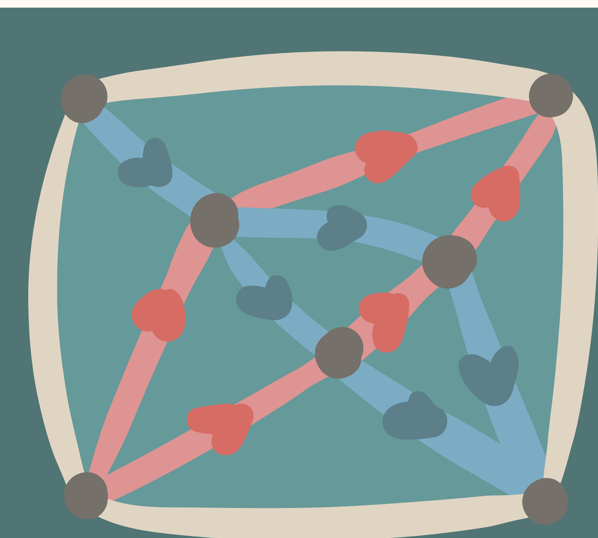
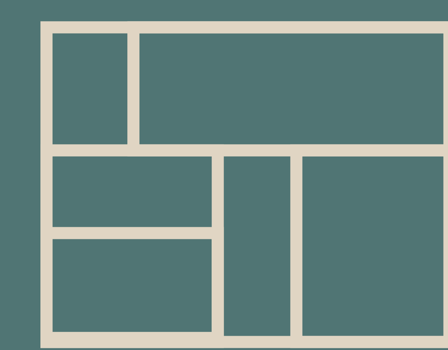
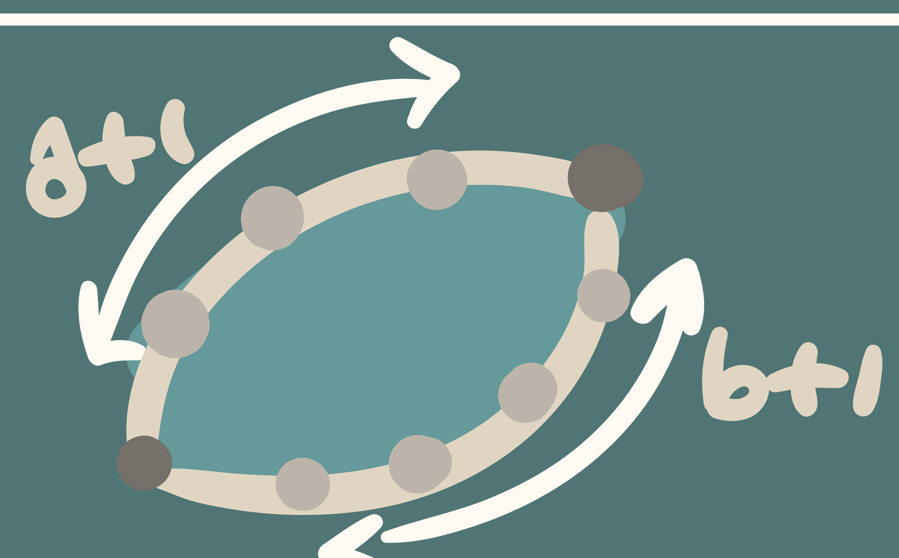
Summary

$\lim a_n^{1/n} = \gamma$ | $a_n \sim \varkappa \cdot \gamma^n \, n^\alpha$ | $\alpha = 1 + \pi / \arccos(\xi)$

Model		***Asymtotics***		
Transversal structures *n blue edges*			$\gamma = 27/2$	non D-Finite
			$\alpha \approx 7.21$	$\xi = 7/8$
Polyhedral orientations *n inner faces*			$\gamma = 9/2$	
Schnyder colorings *n inner faces*			$\gamma = 16/3$	
Posets *n vertices*			$\gamma = (11 + 5\sqrt{5})/2$	D-Finite
			$\alpha = 6$	$\xi = (1 + \sqrt{5})/4$
Posets *n+2 edges*			$\gamma \approx 4.80$	non D-Finite
			$\alpha \approx 5.14$	$\xi \approx 0.73$

Summary

$\lim a_n^{1/n} = \gamma$ | $a_n \sim \varkappa \cdot \gamma^n n^\alpha$ | $\alpha = 1 + \pi / \arccos(\xi)$

Model		***Asymtotics***	
Transversal structures *n blue edges* 		$\gamma = 27/2$	non D-Finite
		$\alpha \approx 7.21$	$\xi = 7/8$
Polyhedral orientations *n inner faces* 		$\gamma = 9/2$	(non D-Finite)
		$(\alpha \approx 4.23)$	$(\xi = 9/16)$
Schnyder colorings *n inner faces* 		$\gamma = 16/3$	(non D-Finite)
		$(\alpha \approx 6.08)$	$(\xi = 22/27)$
Posets *n vertices* 		$\gamma = (11 + 5\sqrt{5})/2$	D-Finite
		$\alpha = 6$	$\xi = (1 + \sqrt{5})/4$
Posets *n+2 edges* 		$\gamma \approx 4.80$	non D-Finite
		$\alpha \approx 5.14$	$\xi \approx 0.73$

Merci