Enumération de cartes décorées et marches dans le quart de plan

soutenance de thèse

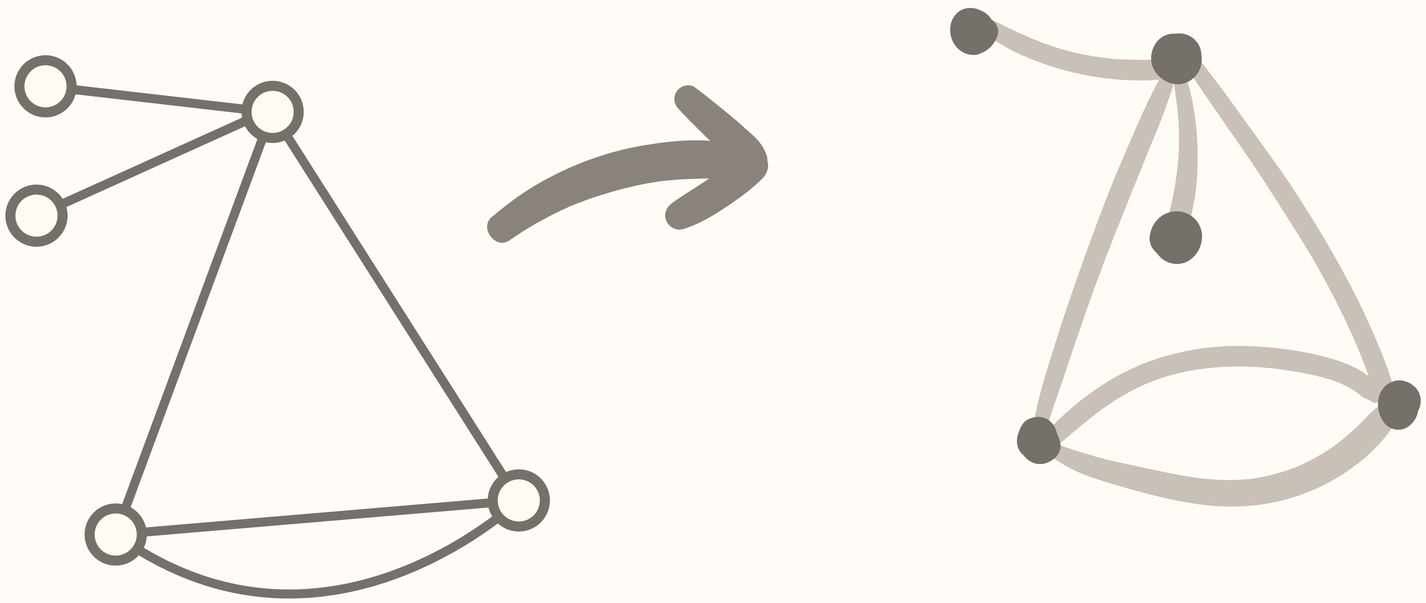
23 septembre 2022 - Erkan Narmanli

ÉCOLE
POLYTECHNIQUE

Introduction

Maps

graph embedding on the plane



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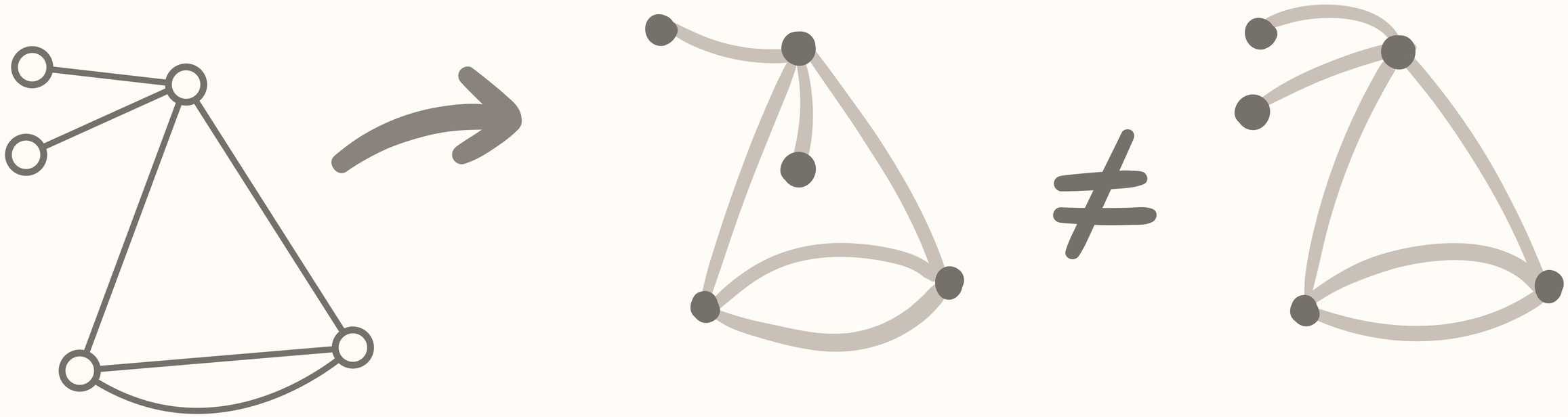

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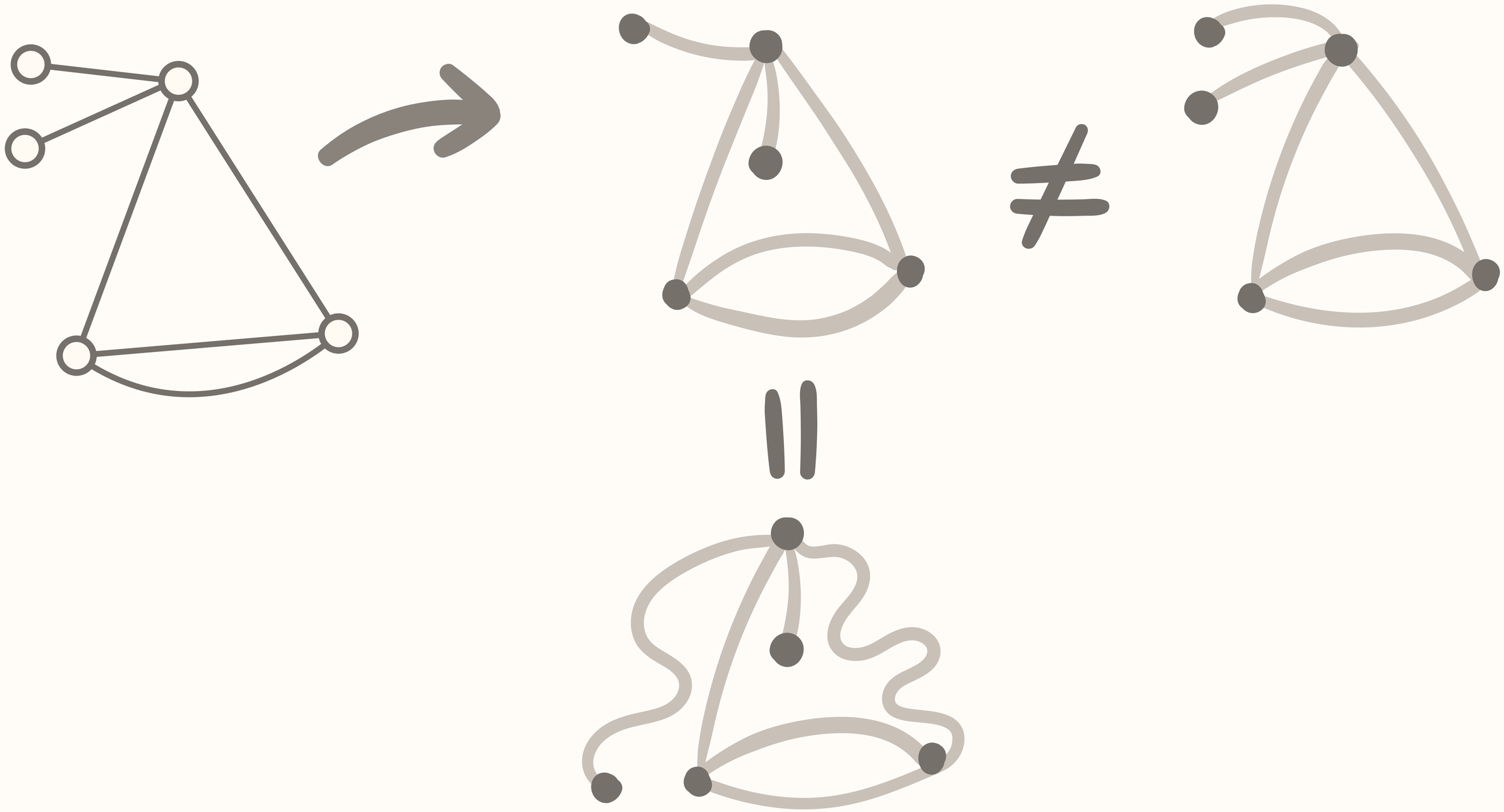

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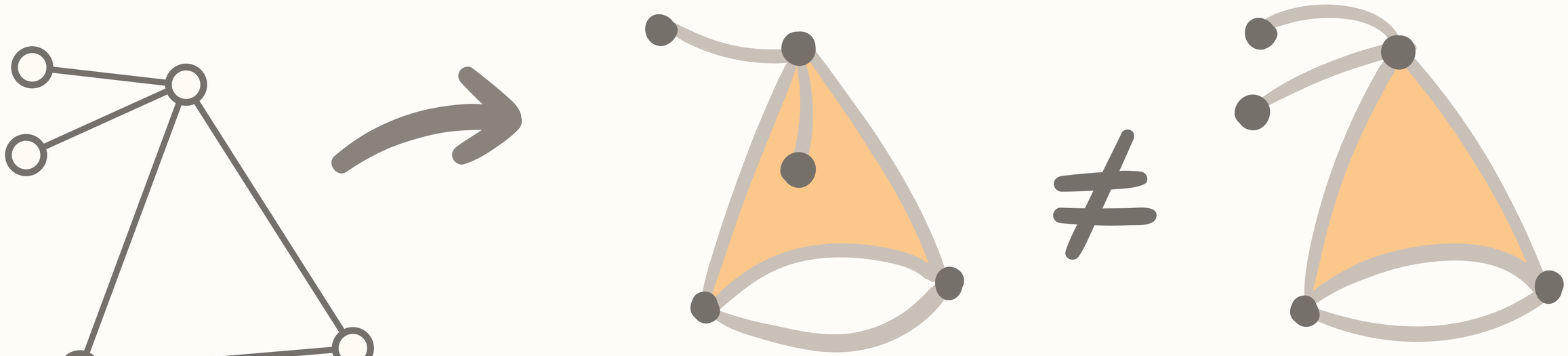

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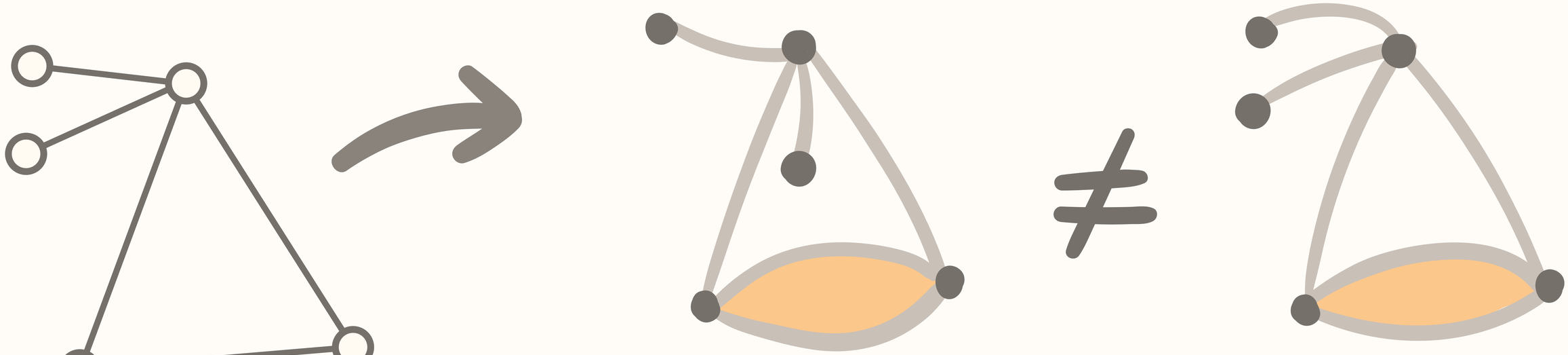

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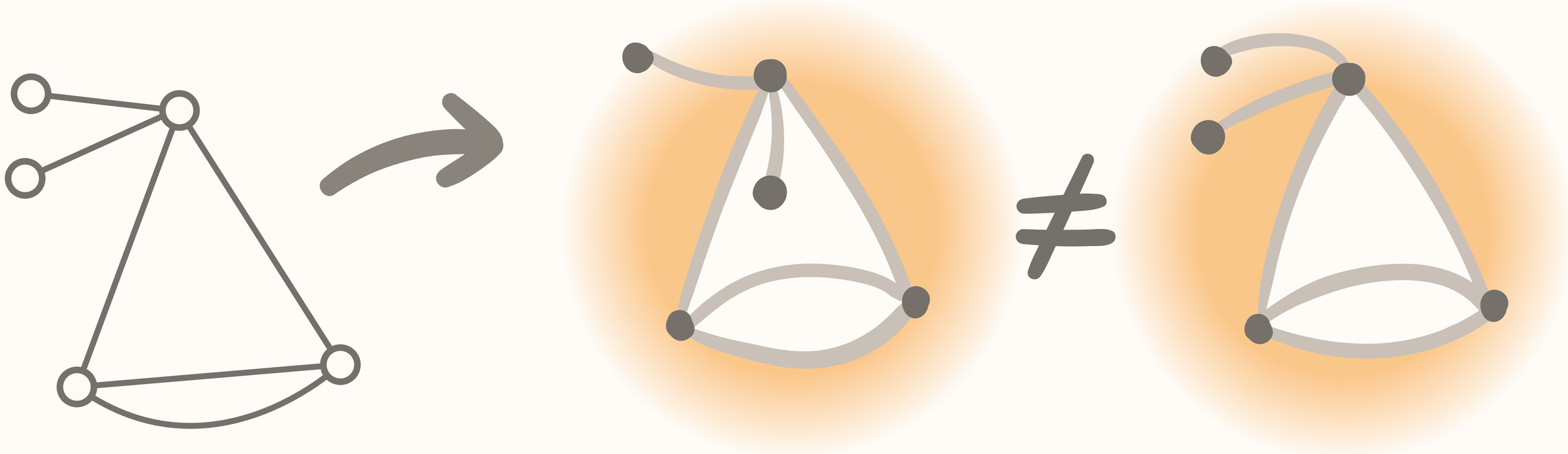

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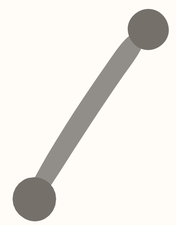

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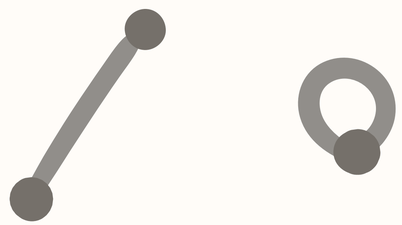

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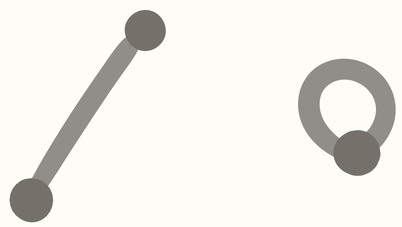

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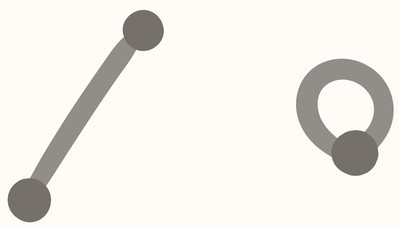

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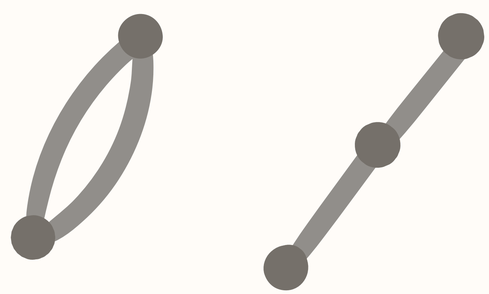

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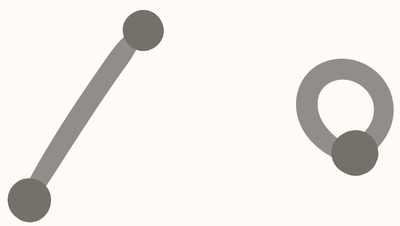

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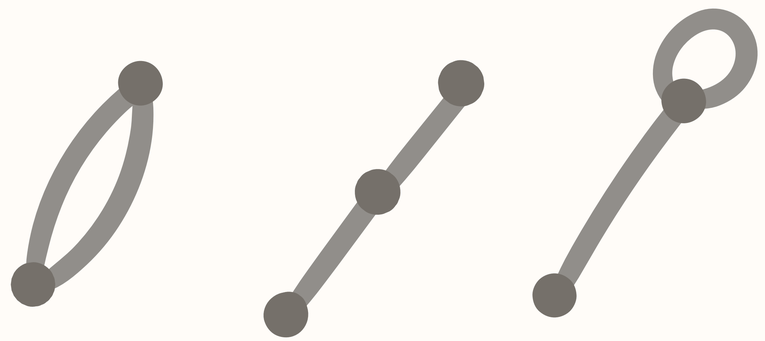

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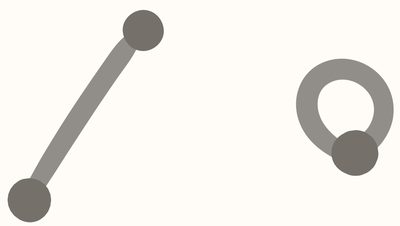

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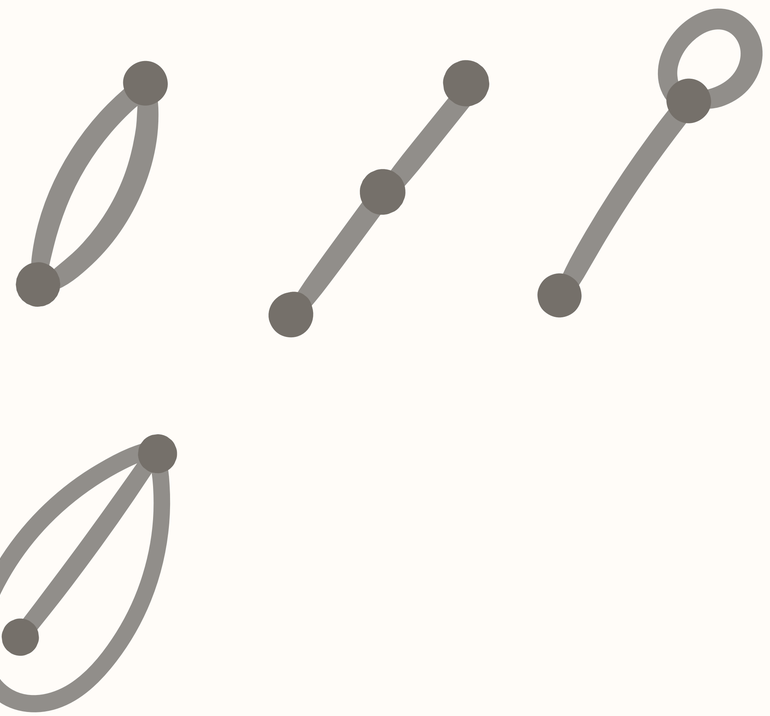

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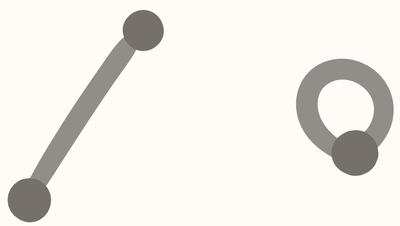

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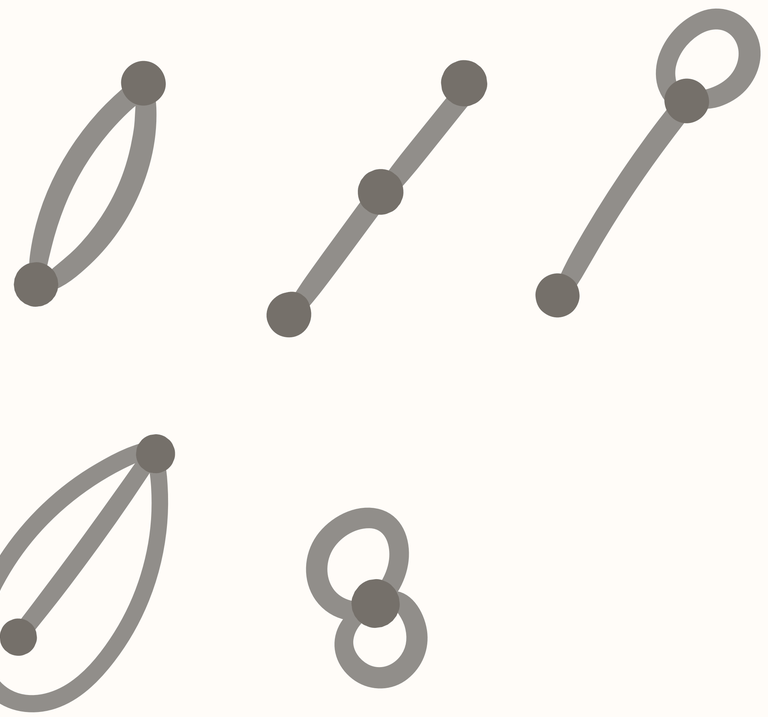

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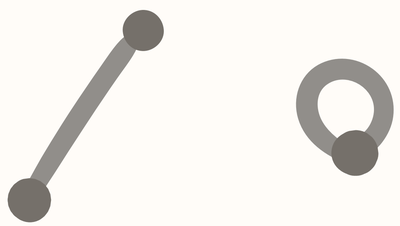

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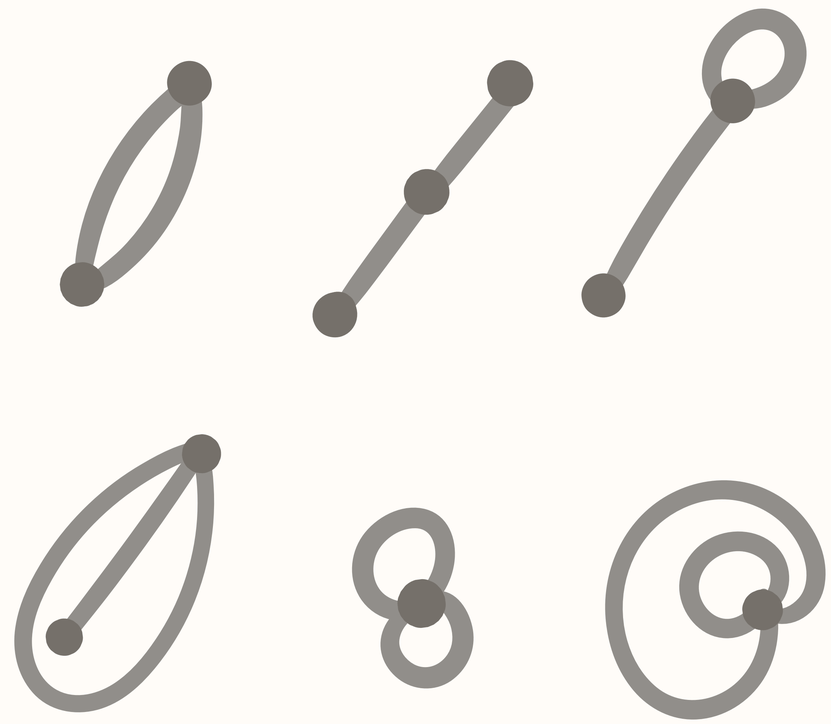

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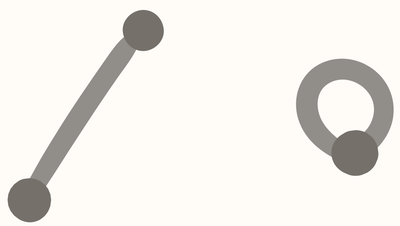

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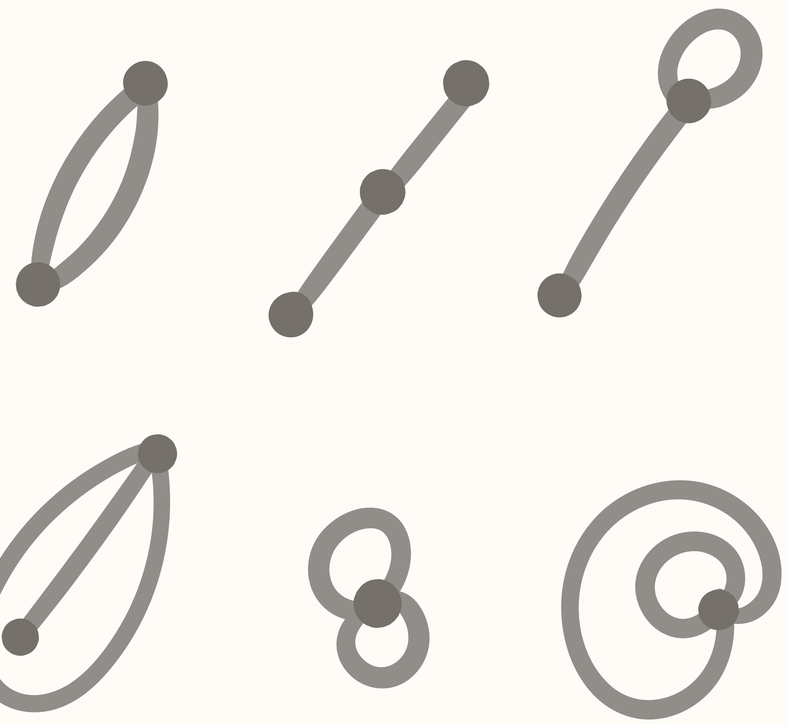

2, 6, . . .

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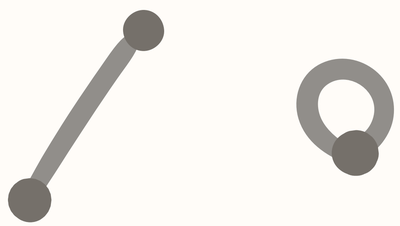

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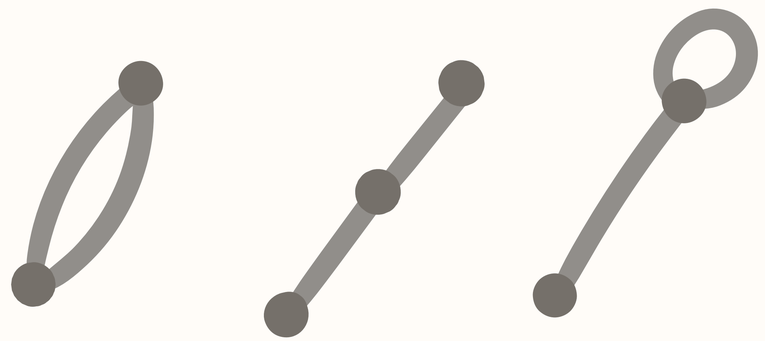

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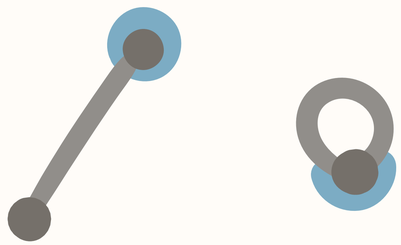

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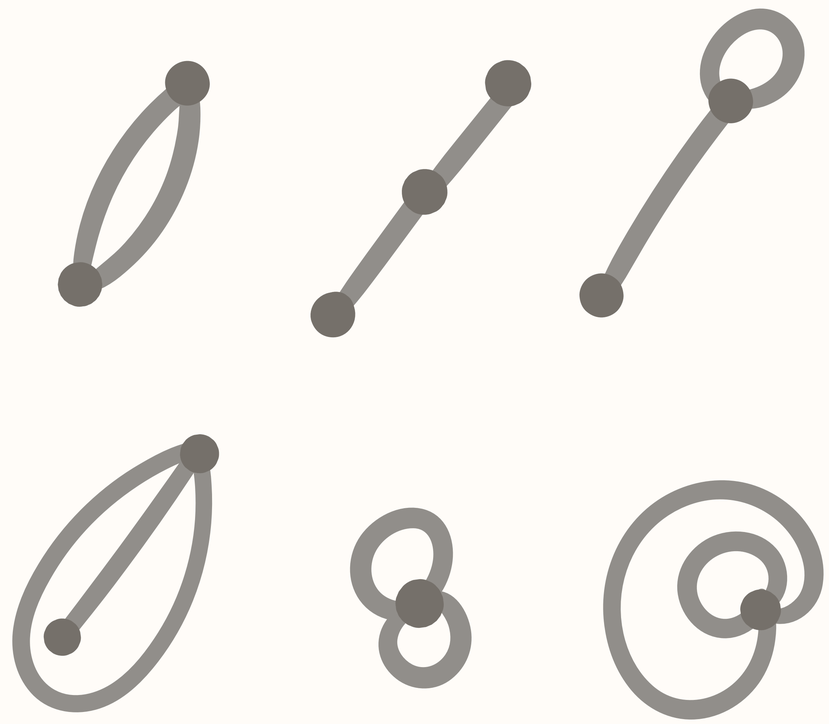

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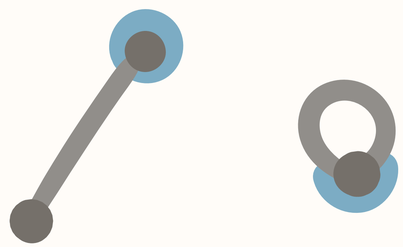

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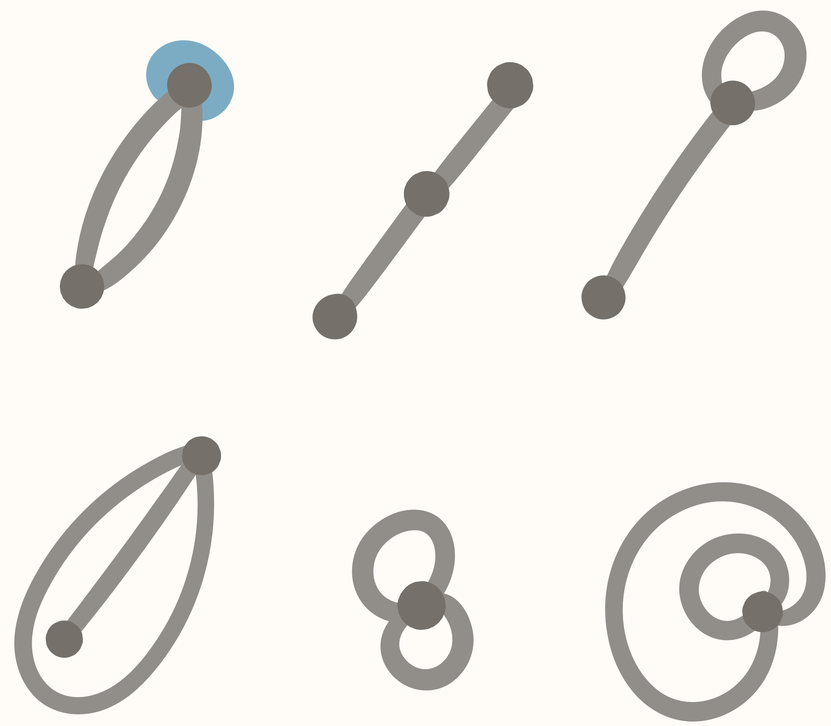

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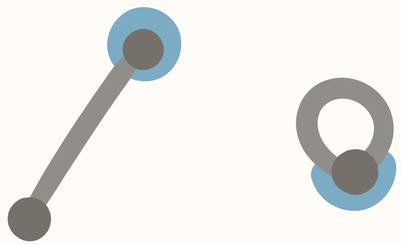

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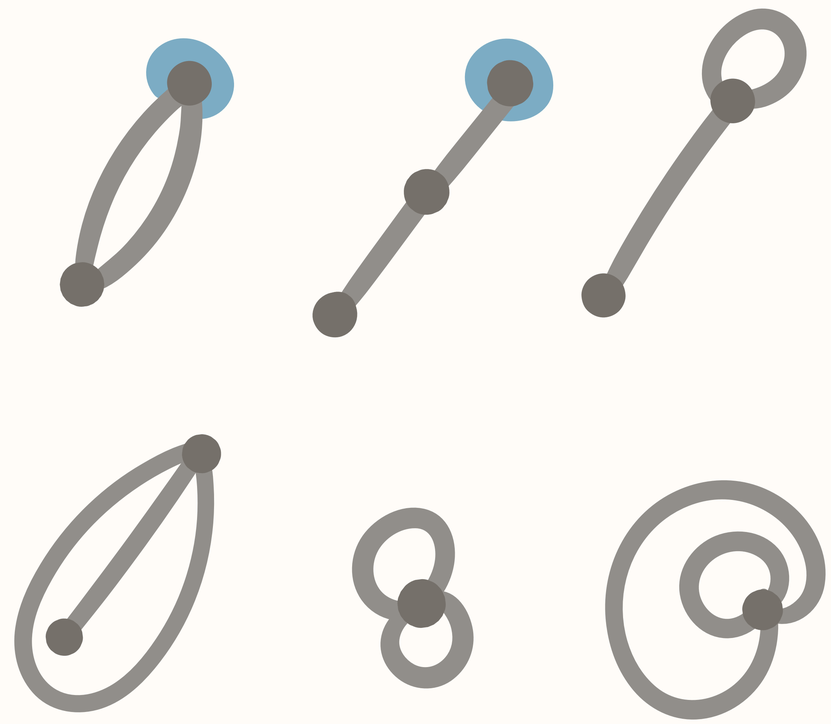

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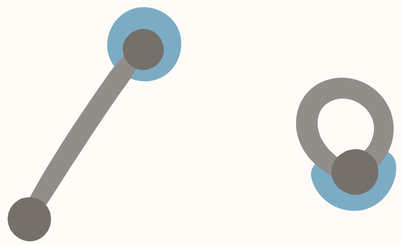

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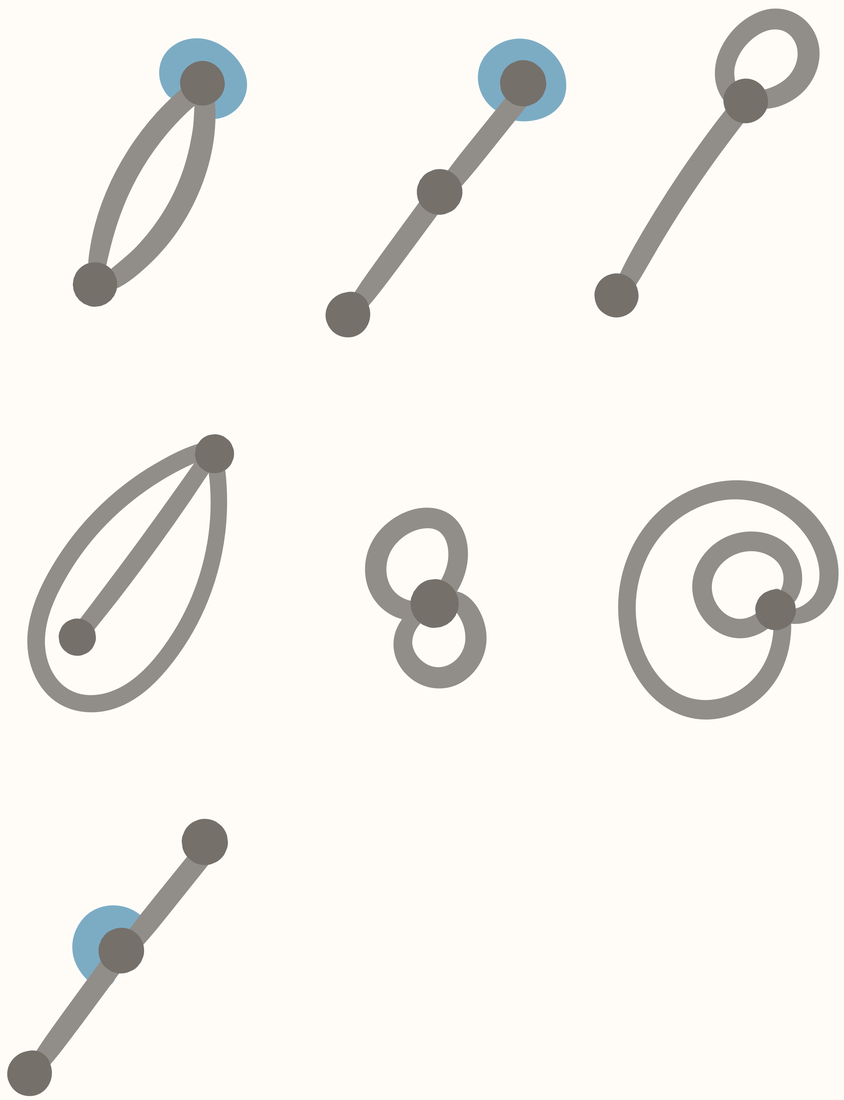

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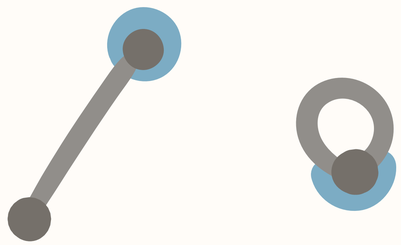

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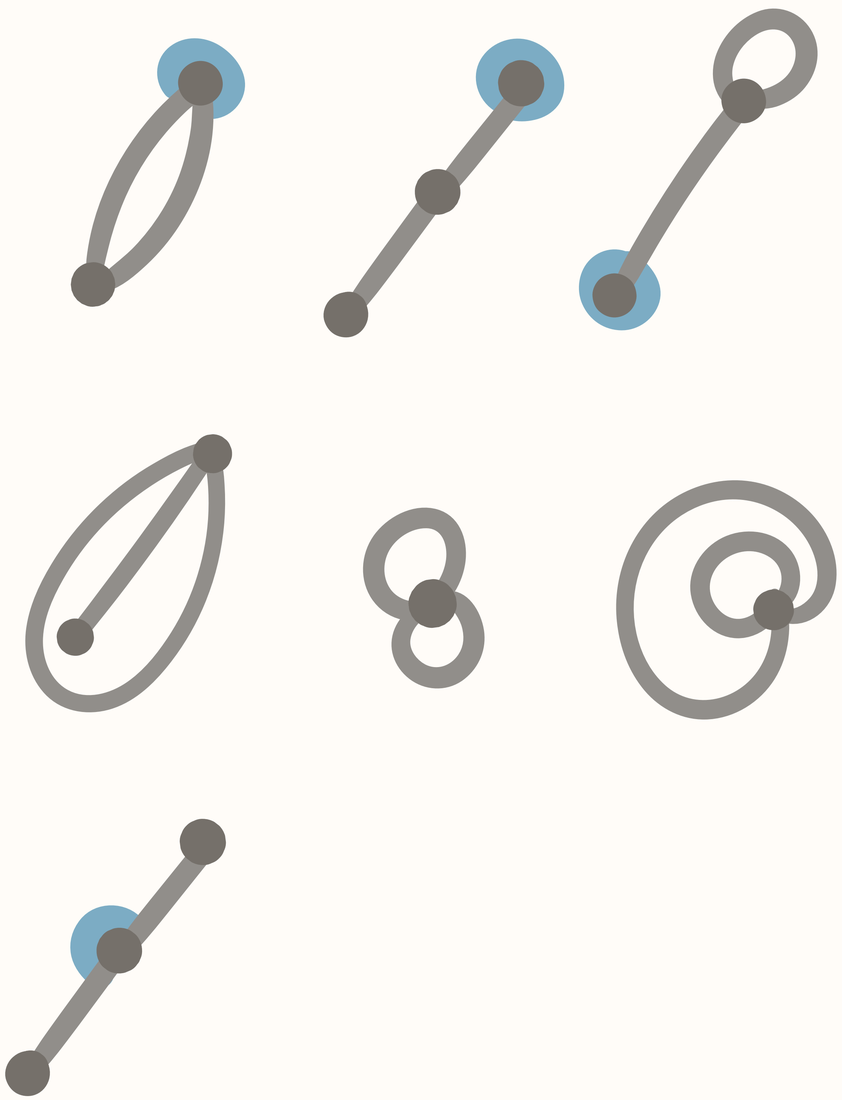

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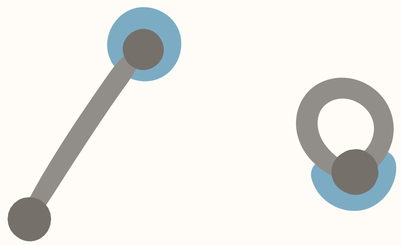

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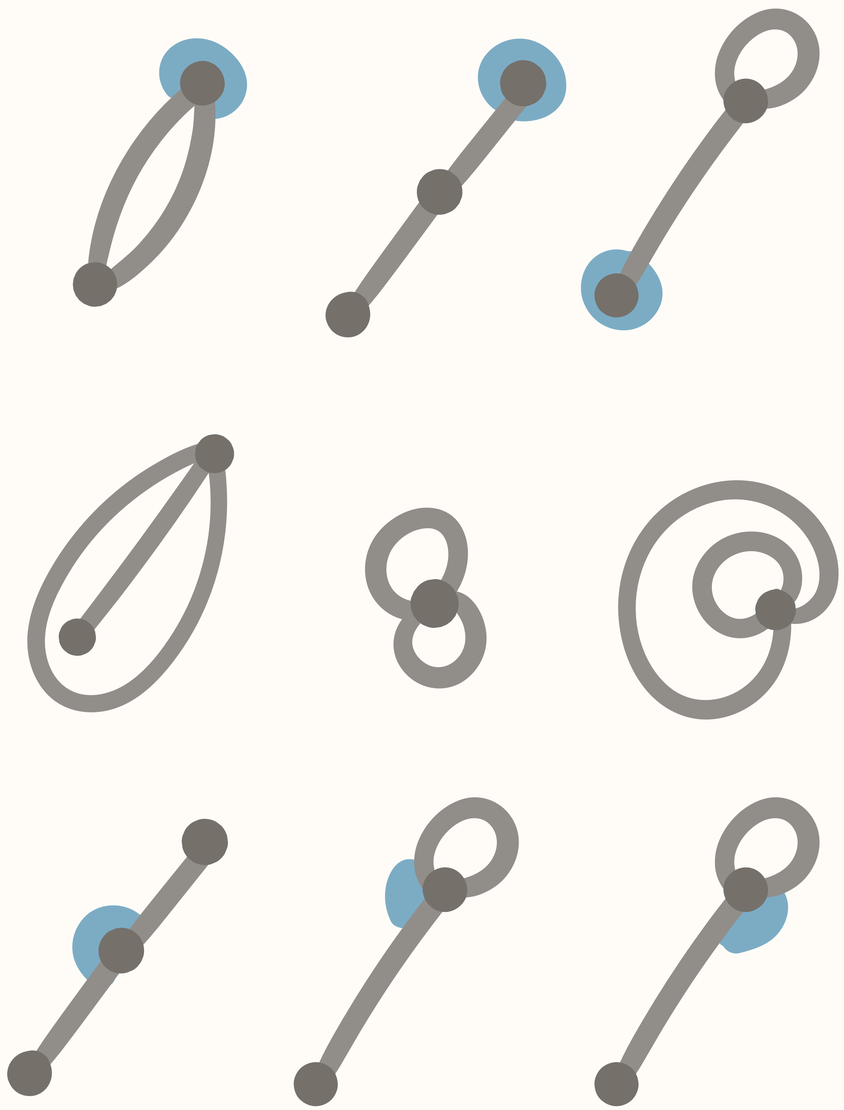

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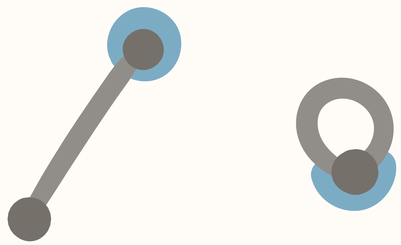

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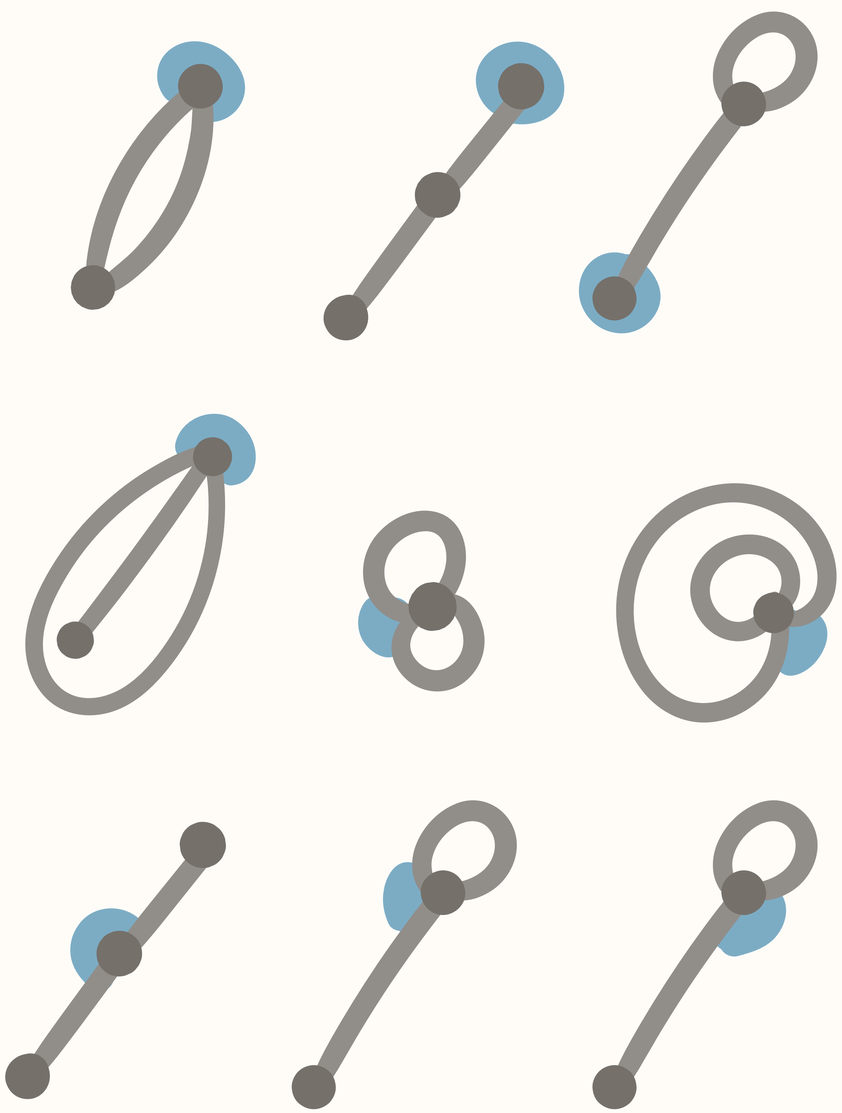

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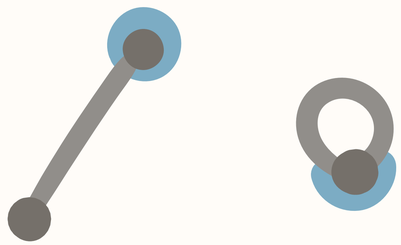

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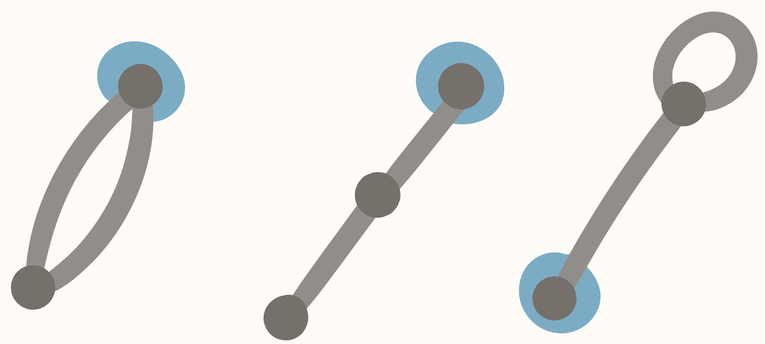
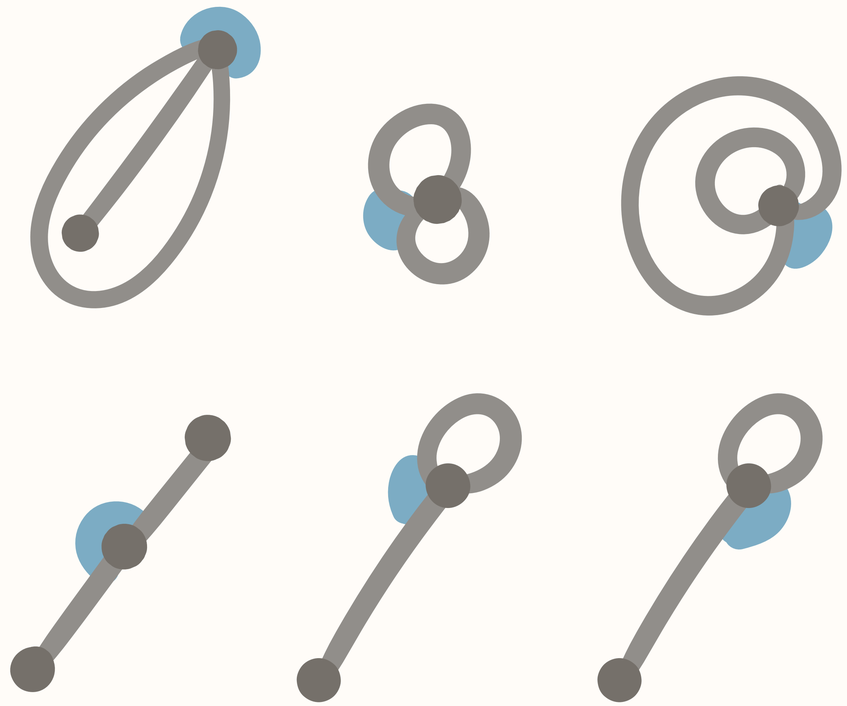


 # maps with n edges

$$2 \cdot 3^n \frac{(2n)!}{n!(n+2)!}$$

$2, 9, 54, \ldots$



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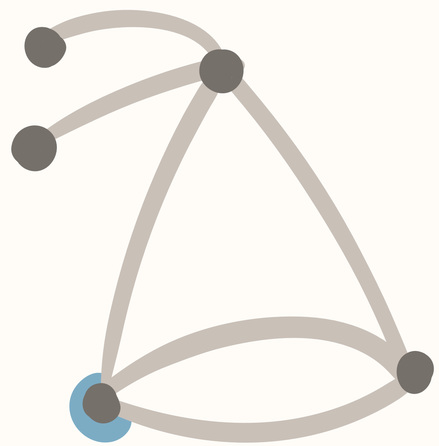

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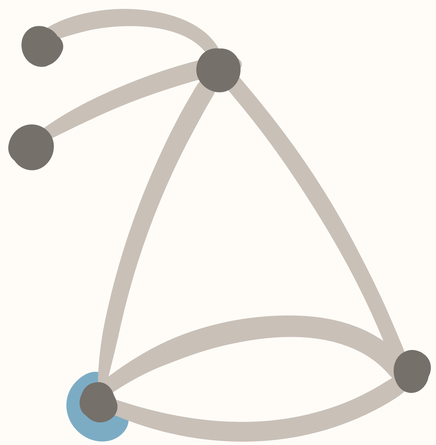
$$2 \cdot 3^n \frac{(2n)!}{n!(n+2)!}$$

$$\sim (2/\sqrt{\pi}) \cdot 12^n \, n^{-5/2}$$

$$2, 9, 54, \ldots$$



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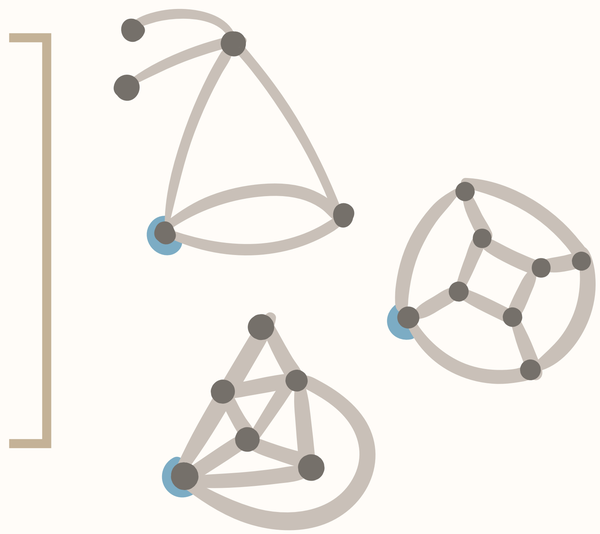
universal critical exponent

$$\varkappa \cdot \gamma^n n^{-5/2}$$



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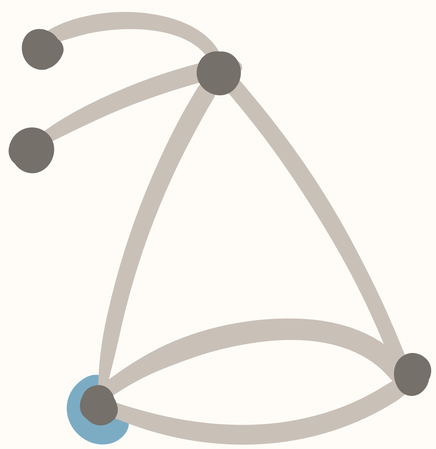

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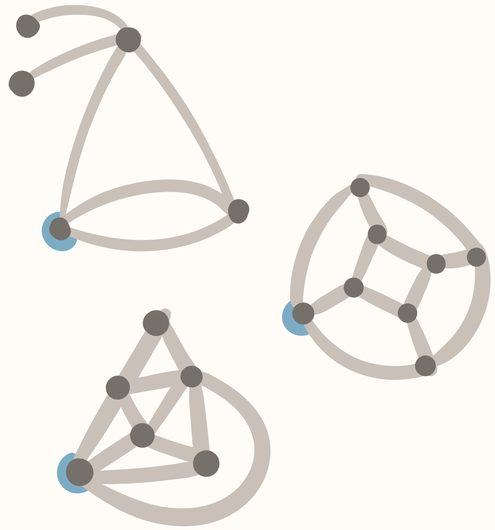
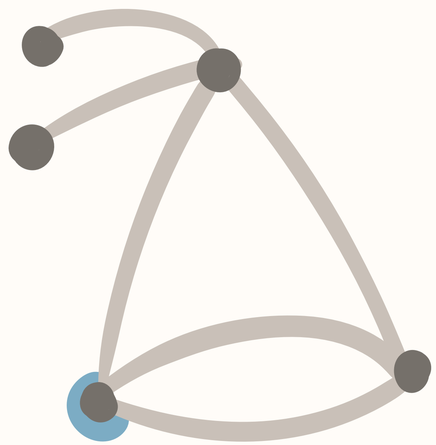
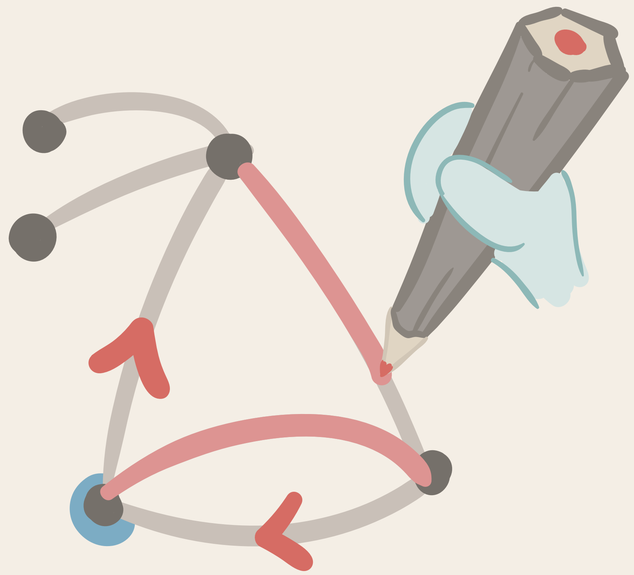




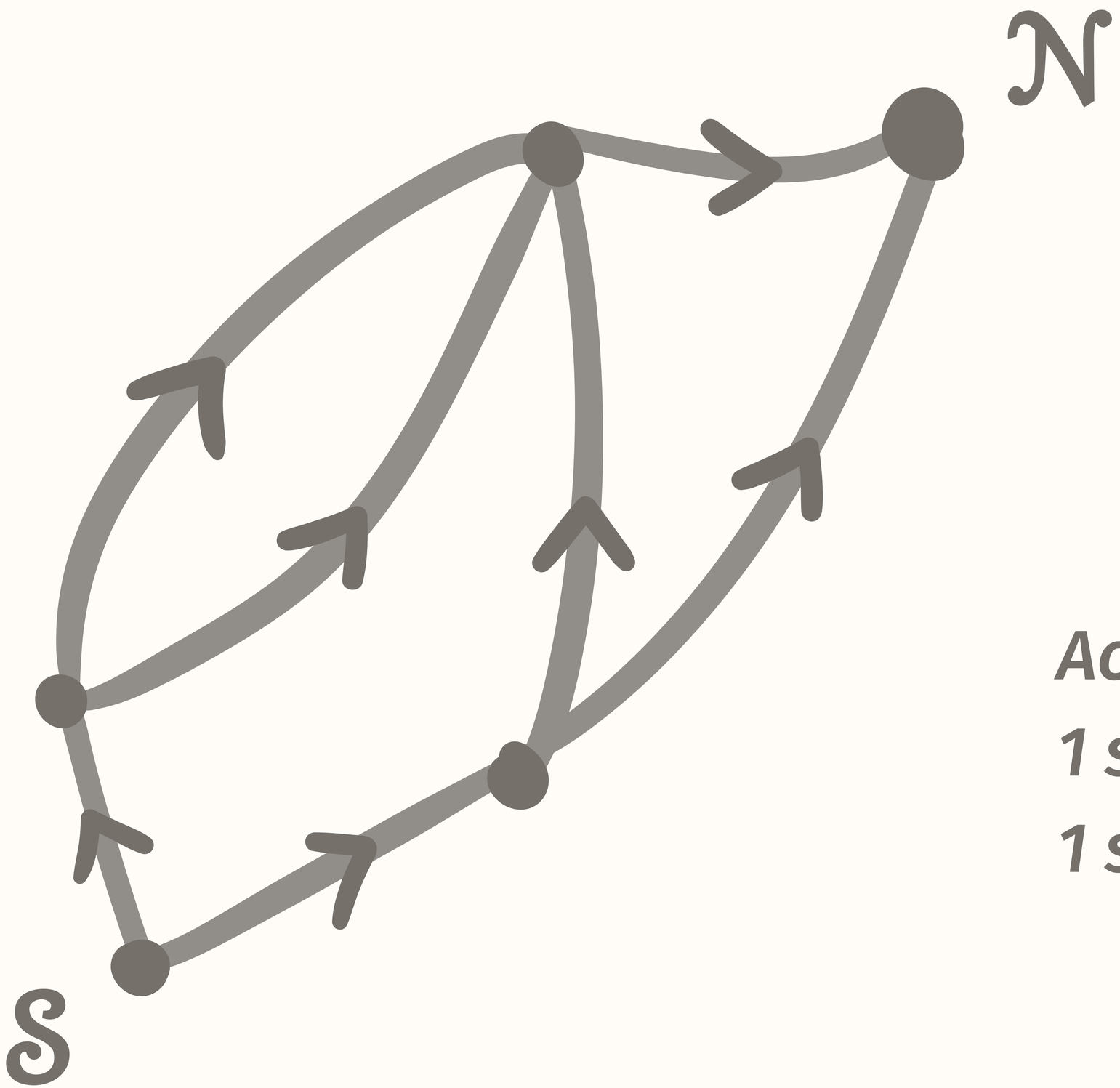
Decorated maps

orientation, coloration, etc.



Plane bipolar orientation

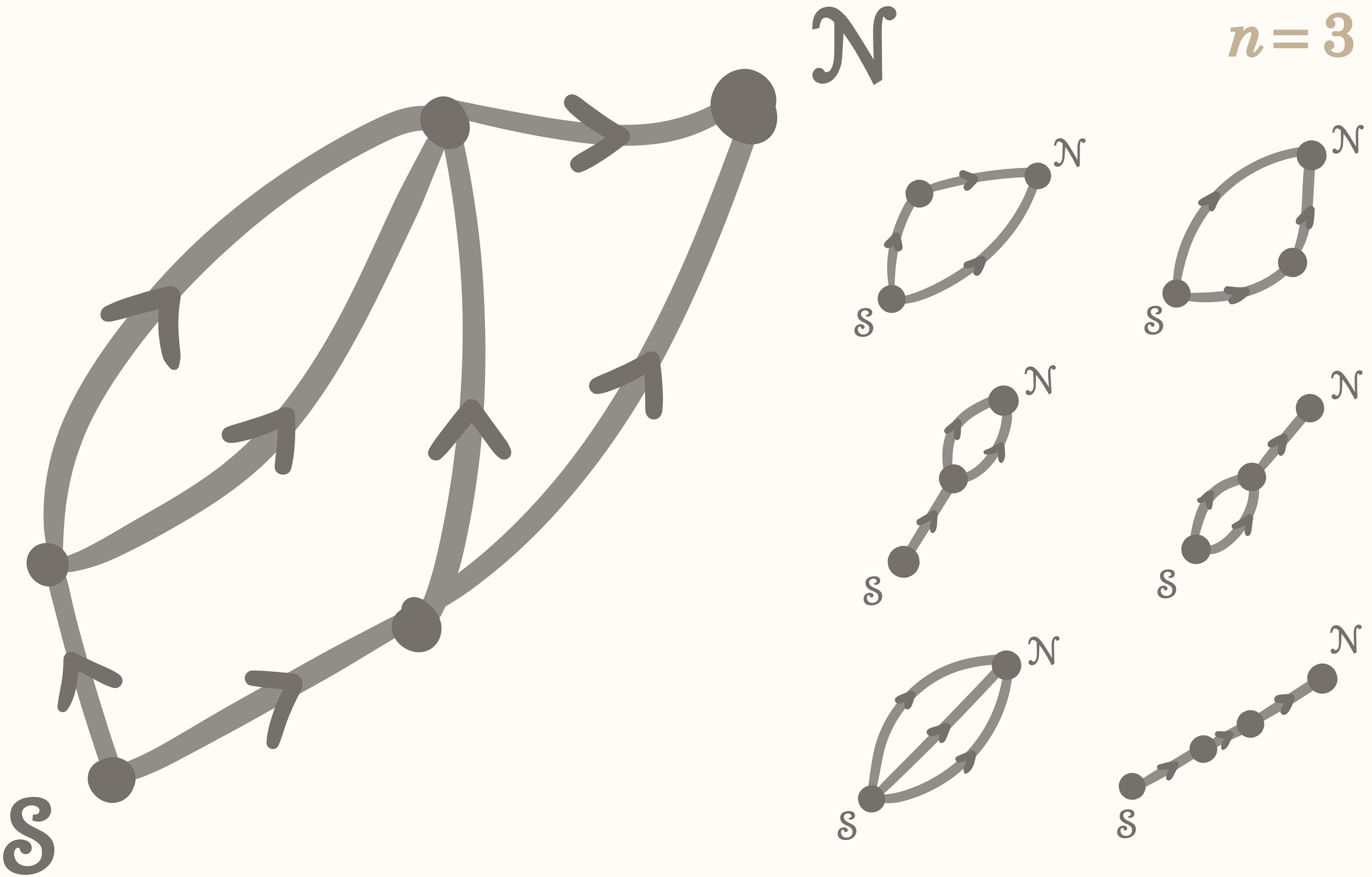
an example of decorated map



Acyclic
1 single source $\mathcal{S}$
1 single sink $\mathcal{N}$

Plane bipolar orientation

an example of decorated map



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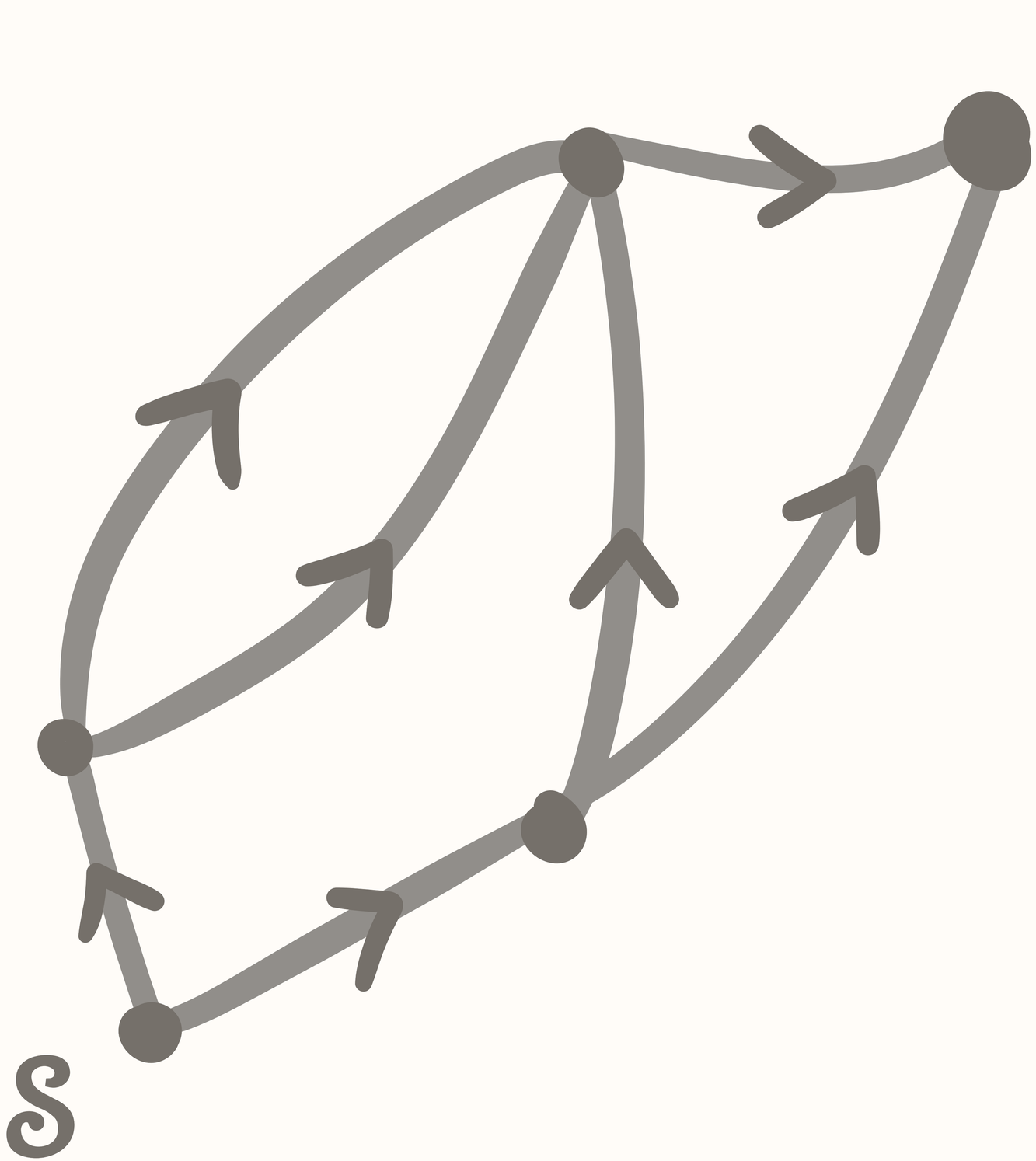
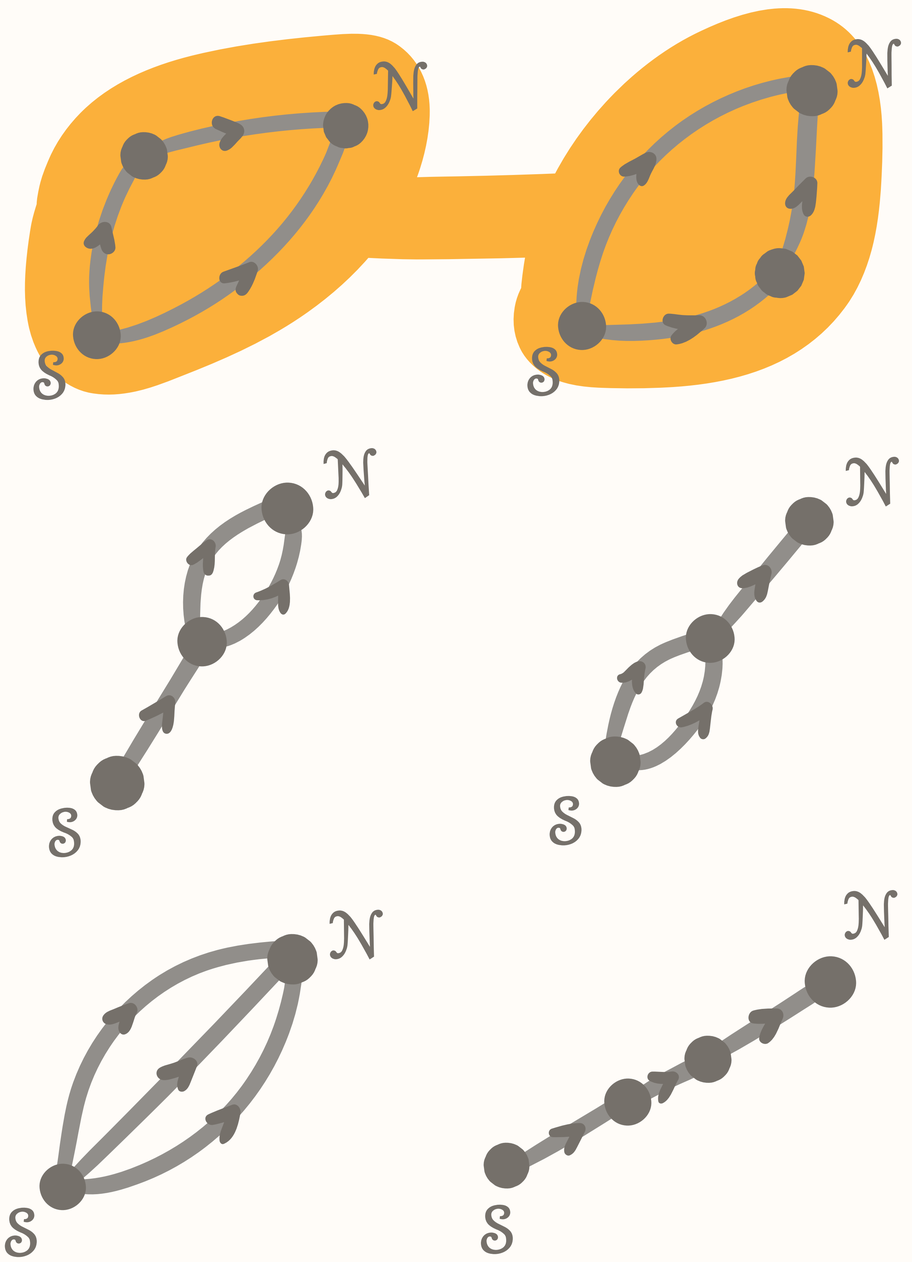
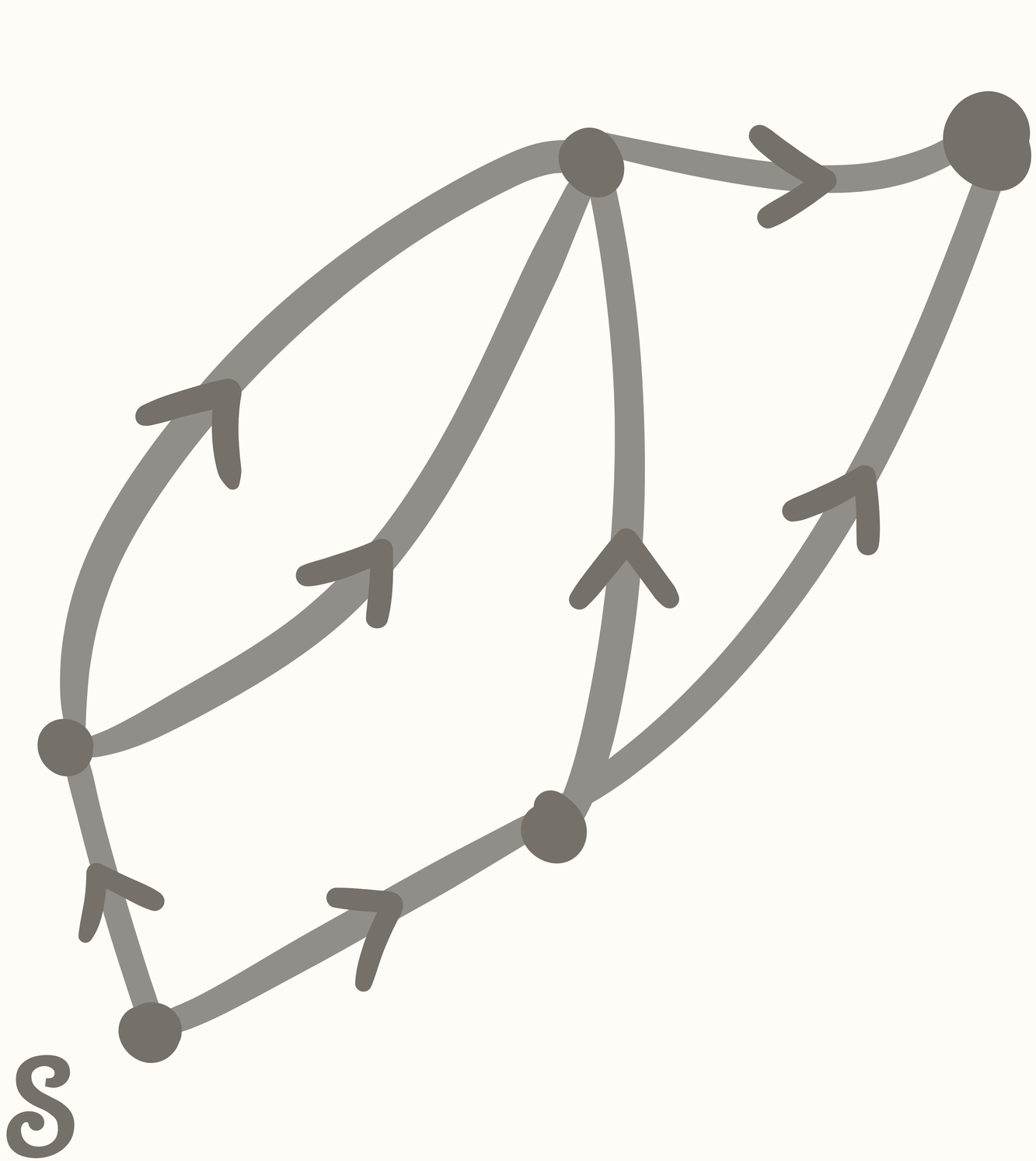


$n = 3$



Plane bipolar orientation

an example of decorated map



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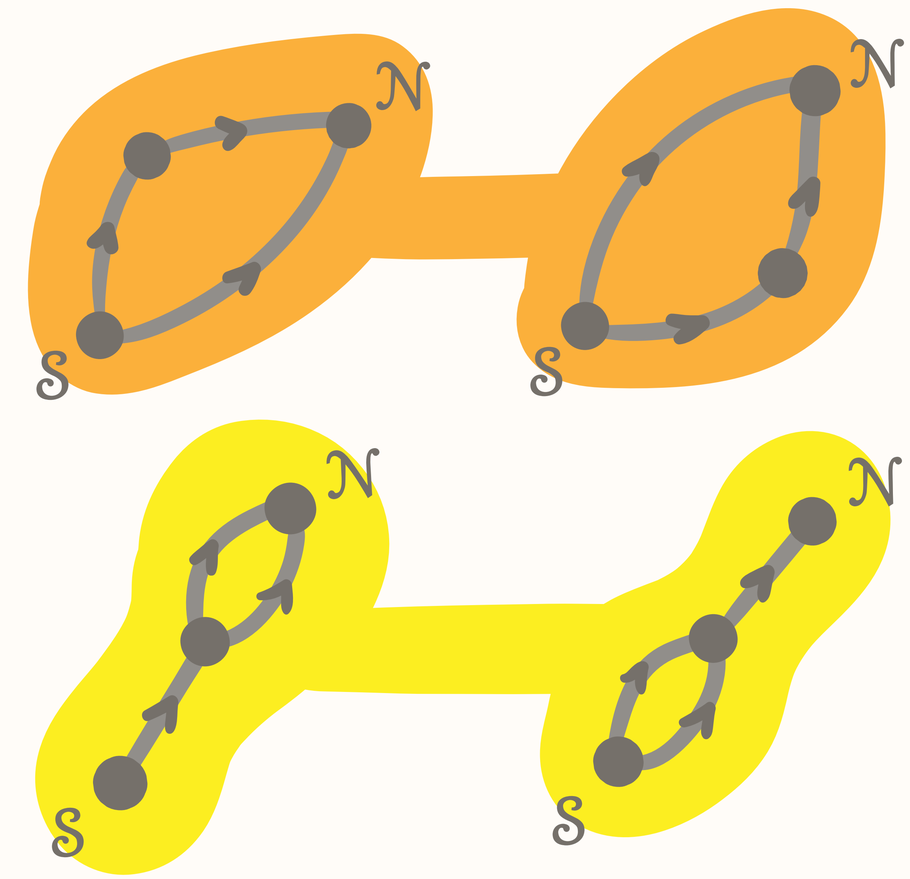
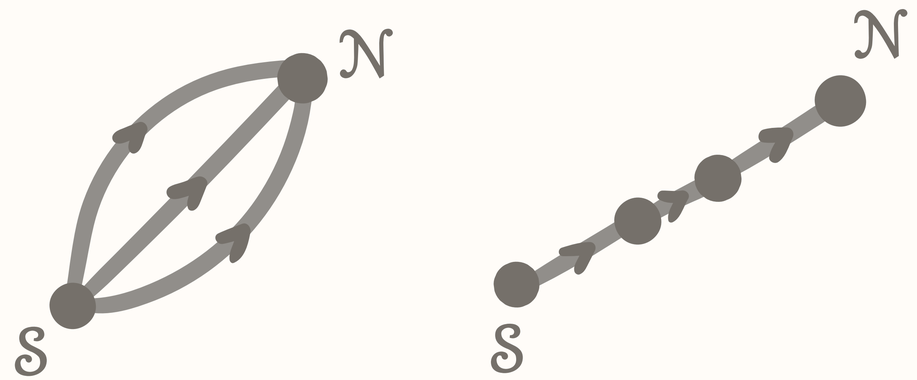
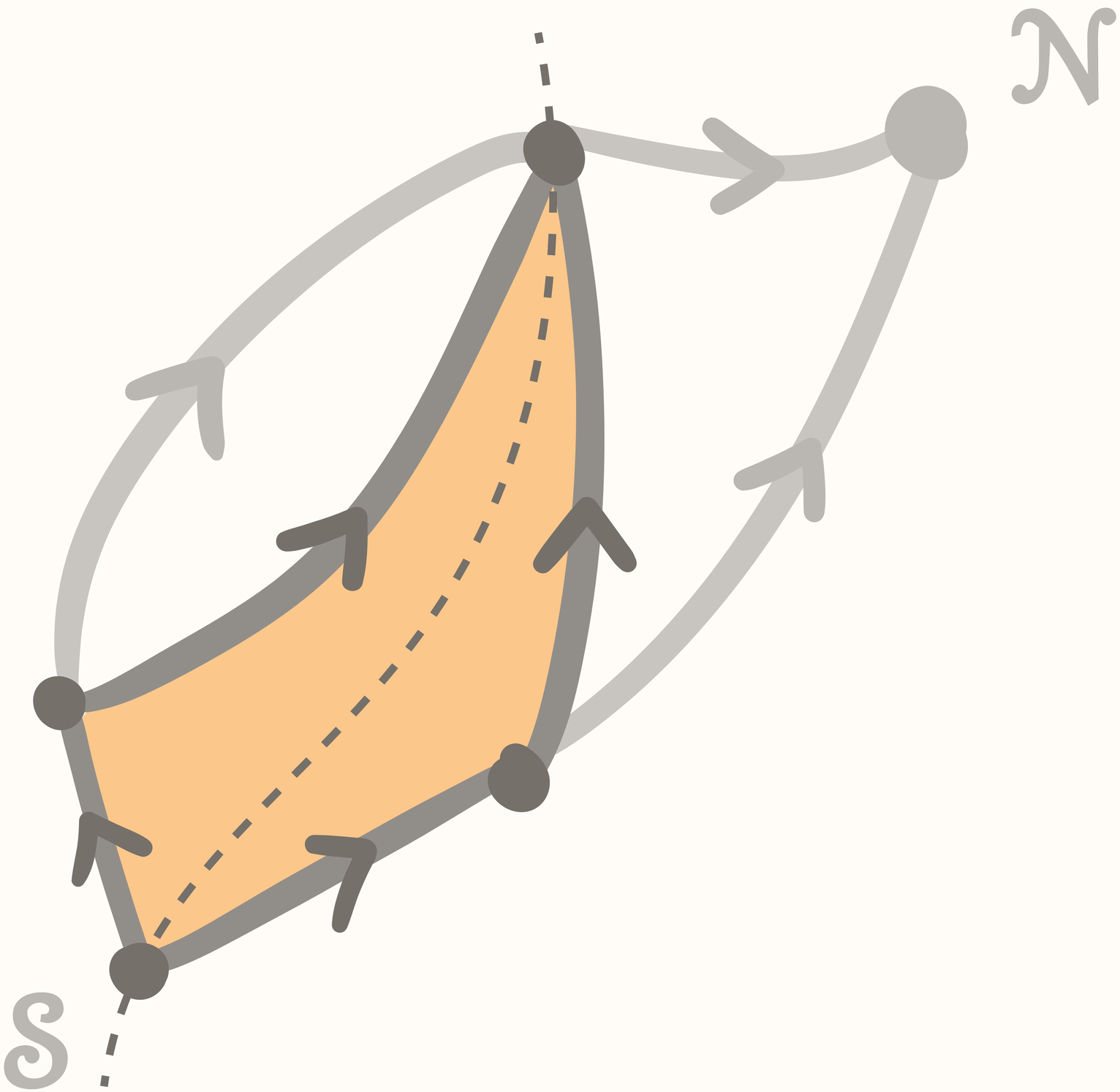




Plane bipolar orientation

an example of decorated map



Plane bipolar orientation

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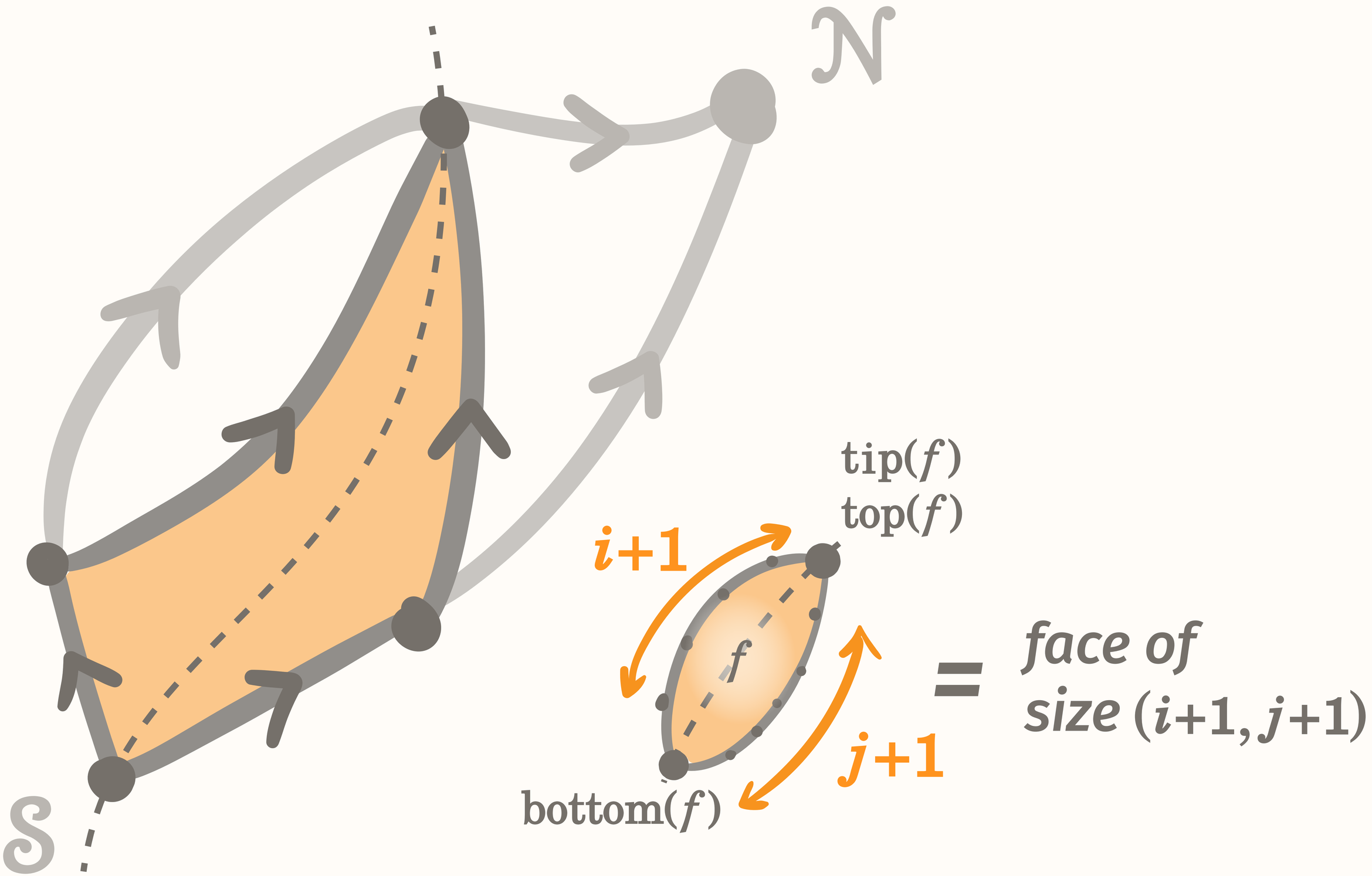

Summary

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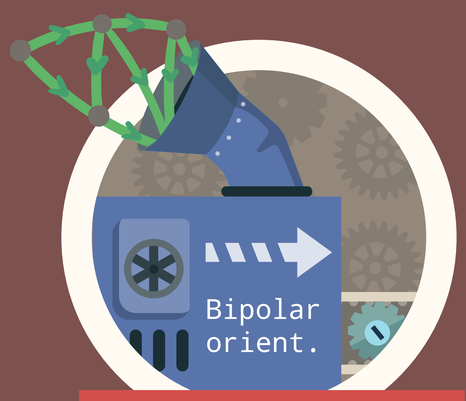
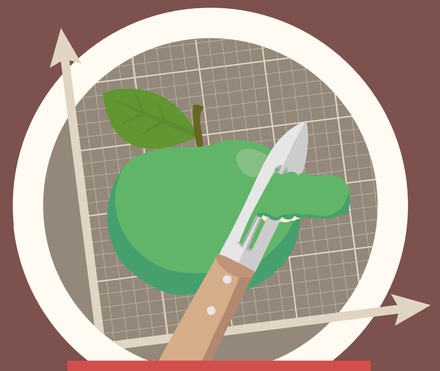


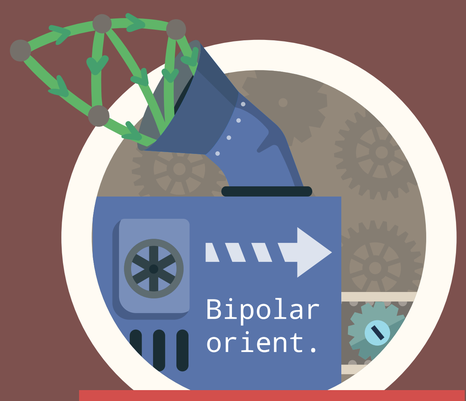
Decorated bipolar orientations

Summary



Quadrant tandem walks

Asymptotics



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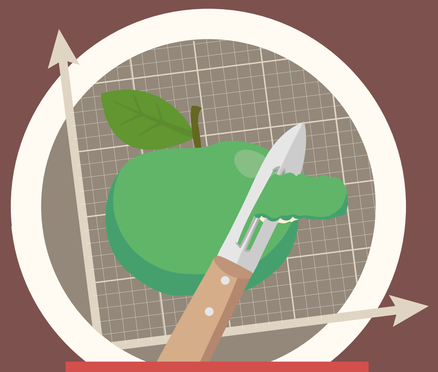

Quadrant tandem walks

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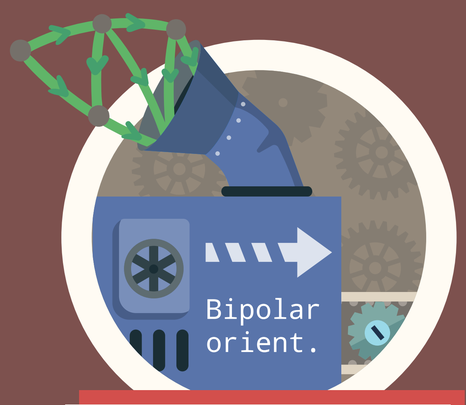
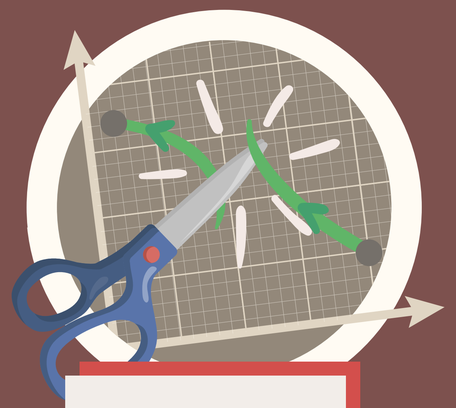


Decorated bipolar orientations



Small steps

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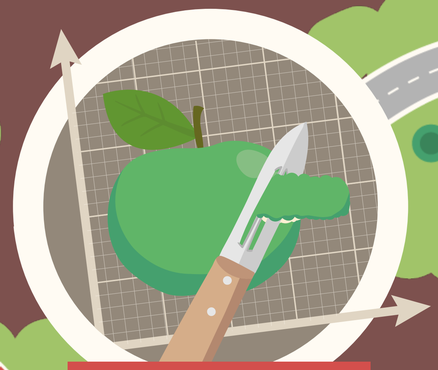
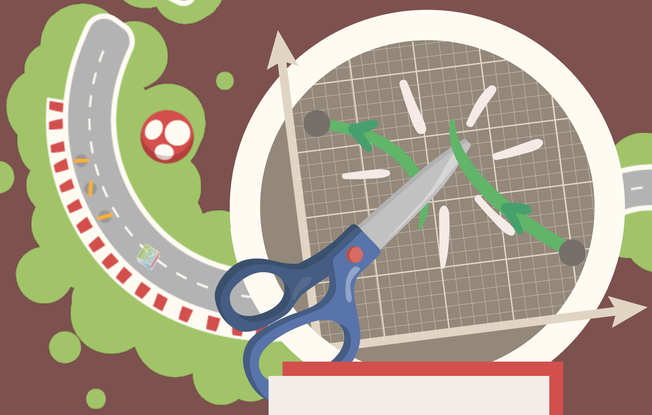
Decorated bipolar orientations



Quadrant tandem walks



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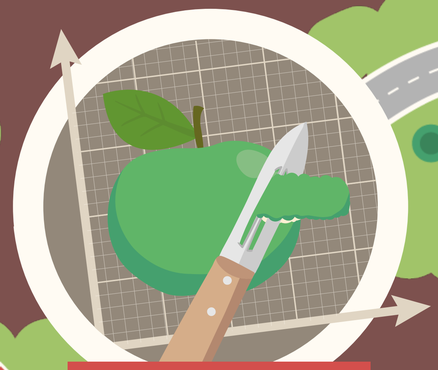

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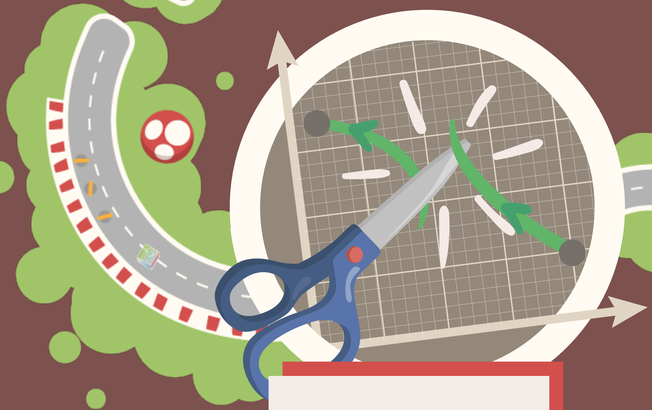

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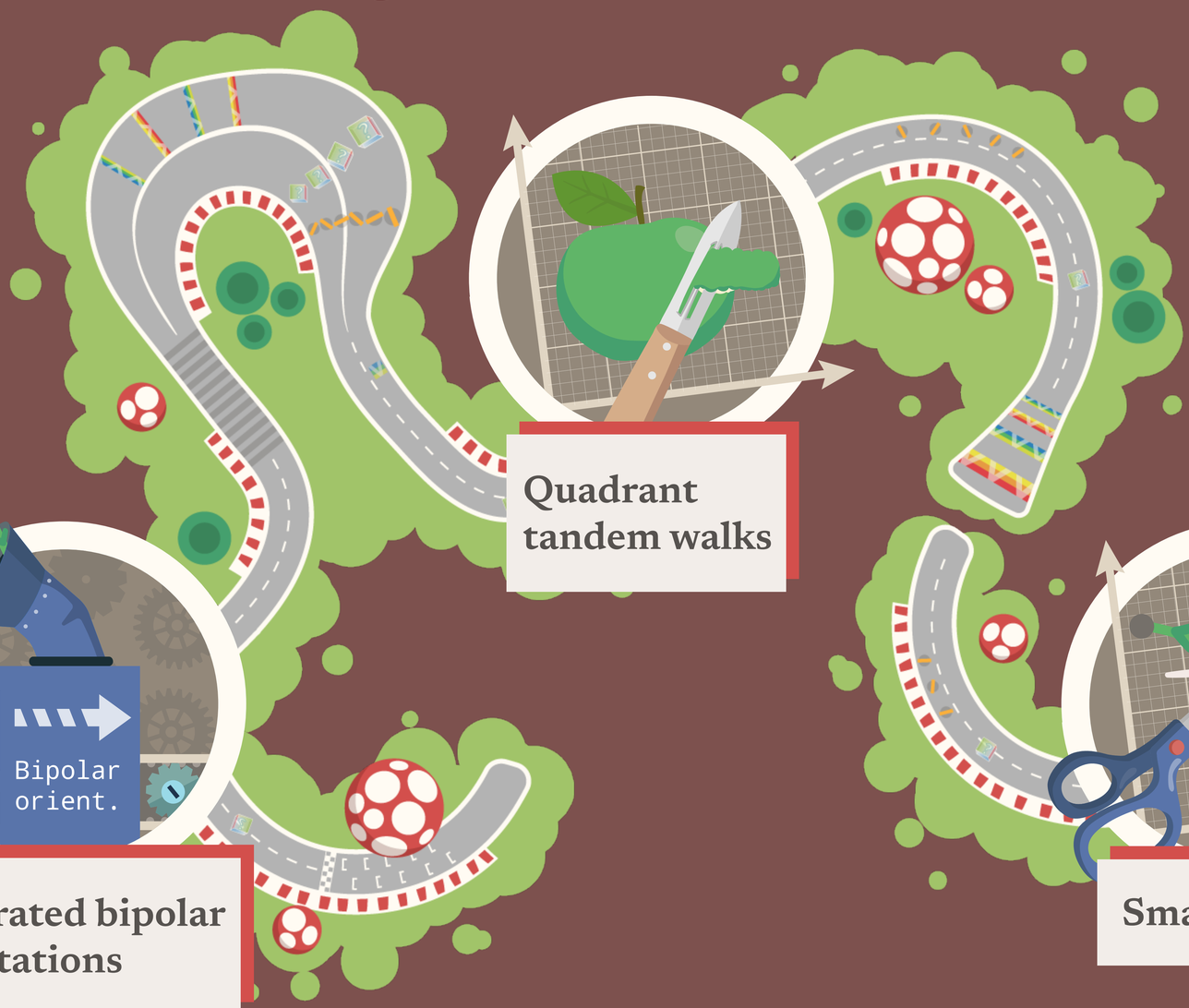
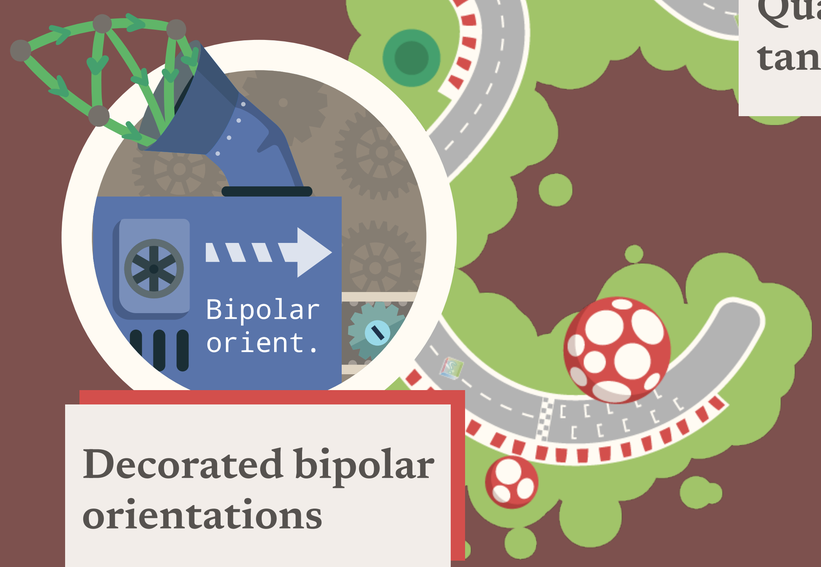
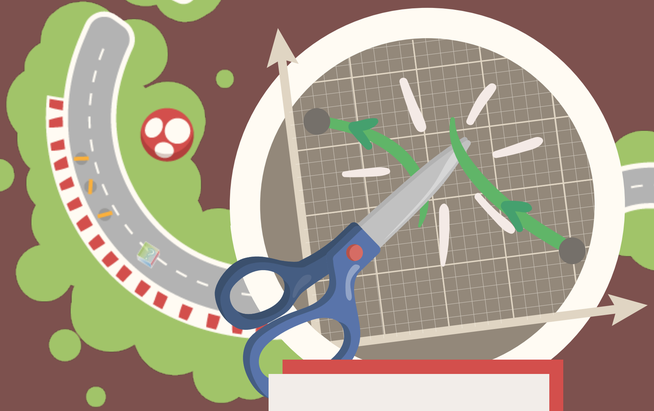


Quadrant tandem walks

Asymptotics



Decorated bipolar orientations



Small steps

bipolar posets by vertices

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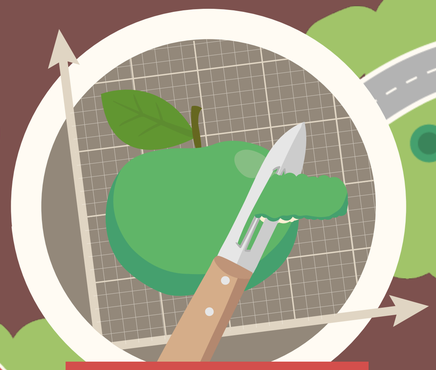
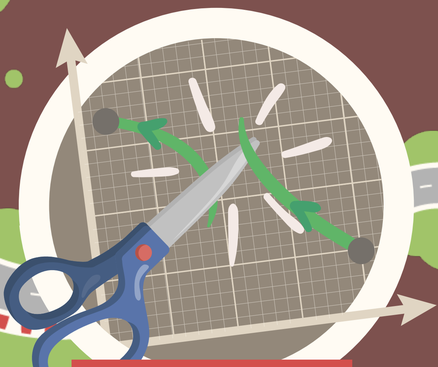


Quadrant tandem walks



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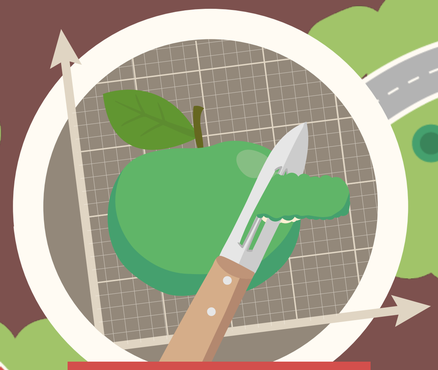

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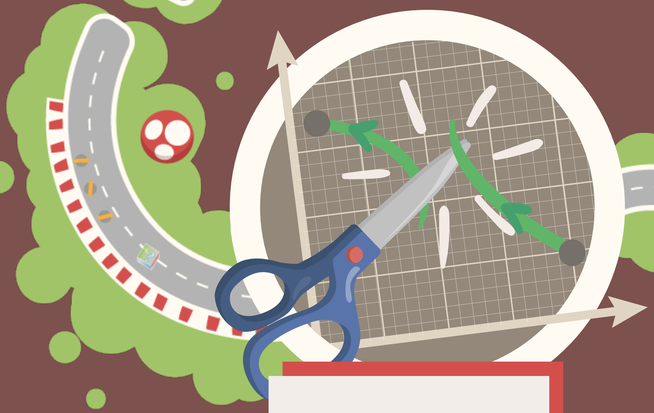
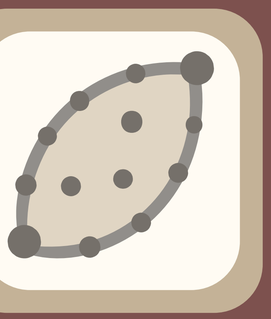


Small steps

Asymptotics



bipolar posets by vertices

bipolar posets by edges

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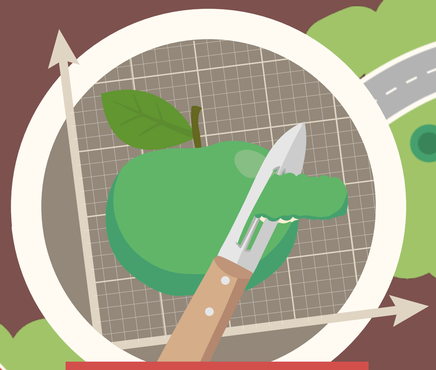
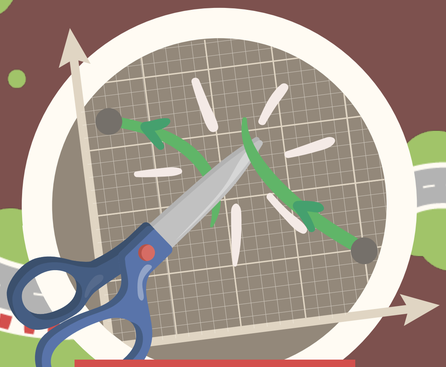
Decorated bipolar orientations



Quadrant tandem walks



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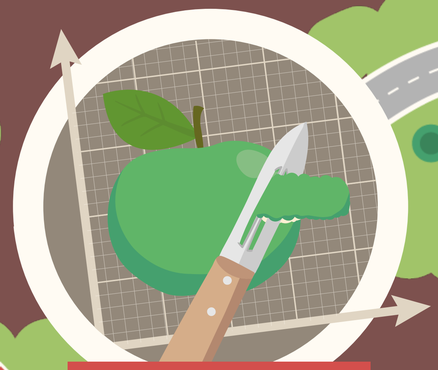
Decorated bipolar orientations



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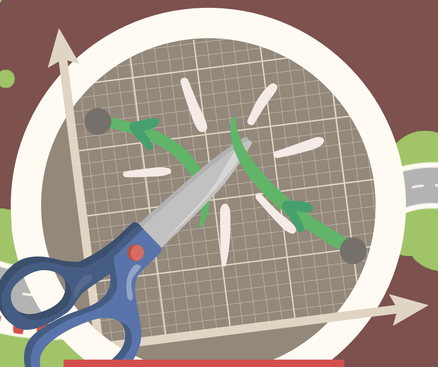


Small steps

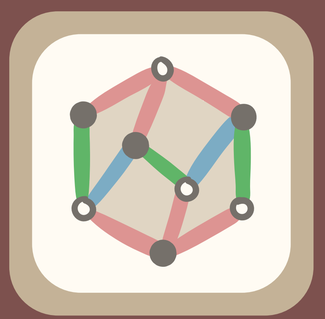
Asymptotics

bipolar posets by vertices

bipolar posets by edges

transversal structures

polyhedral orientations



Schnyder labelings

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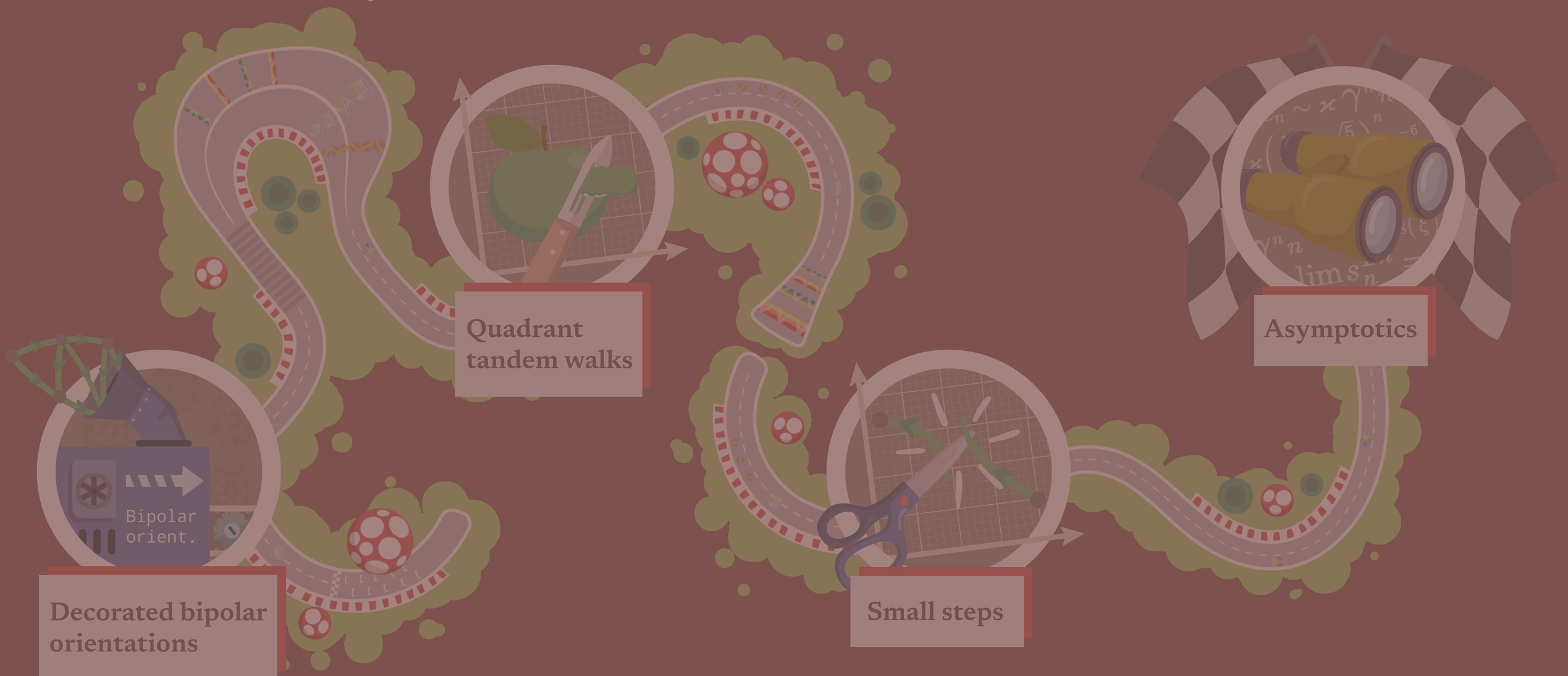

bipolar posets by vertices

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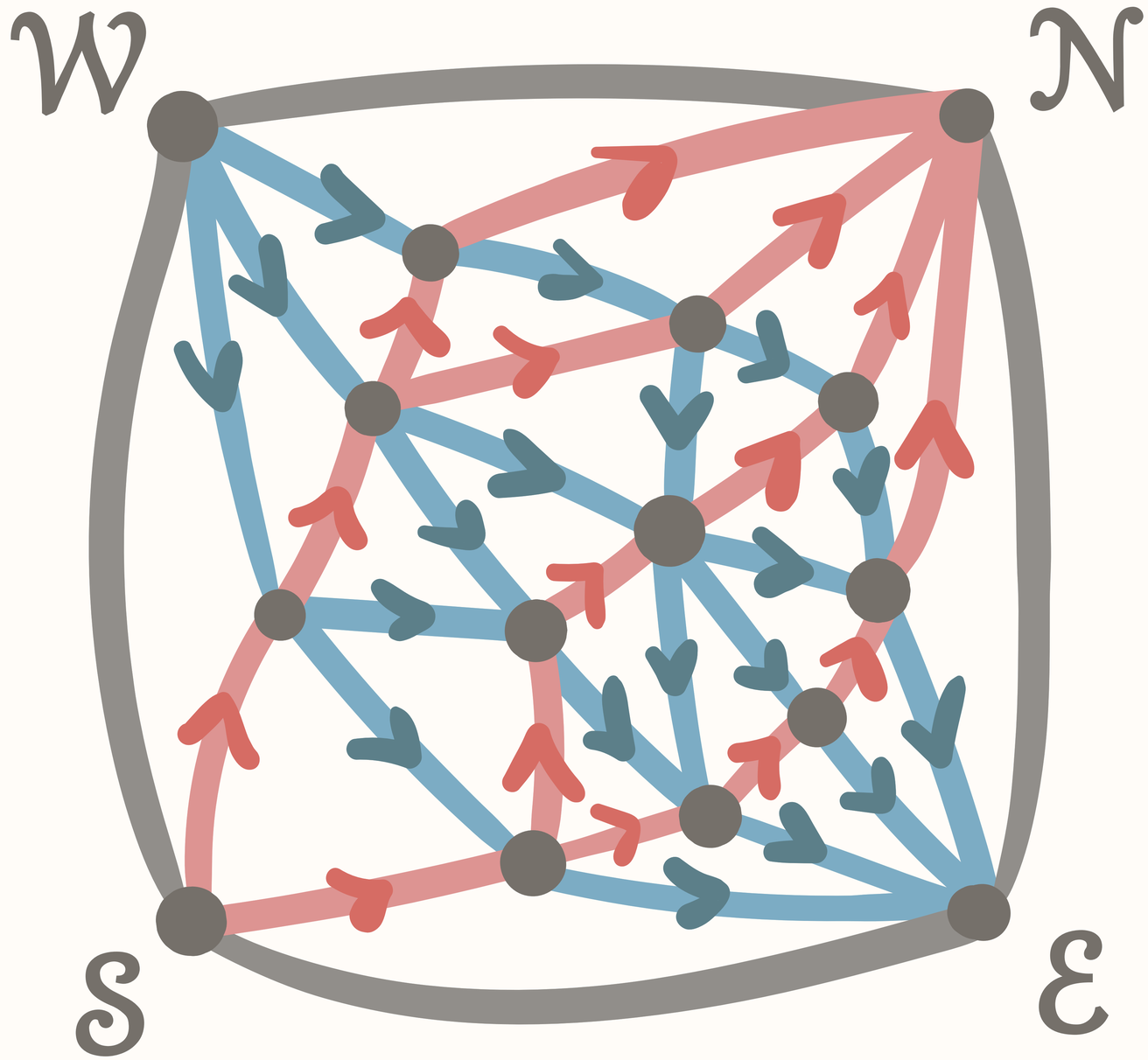
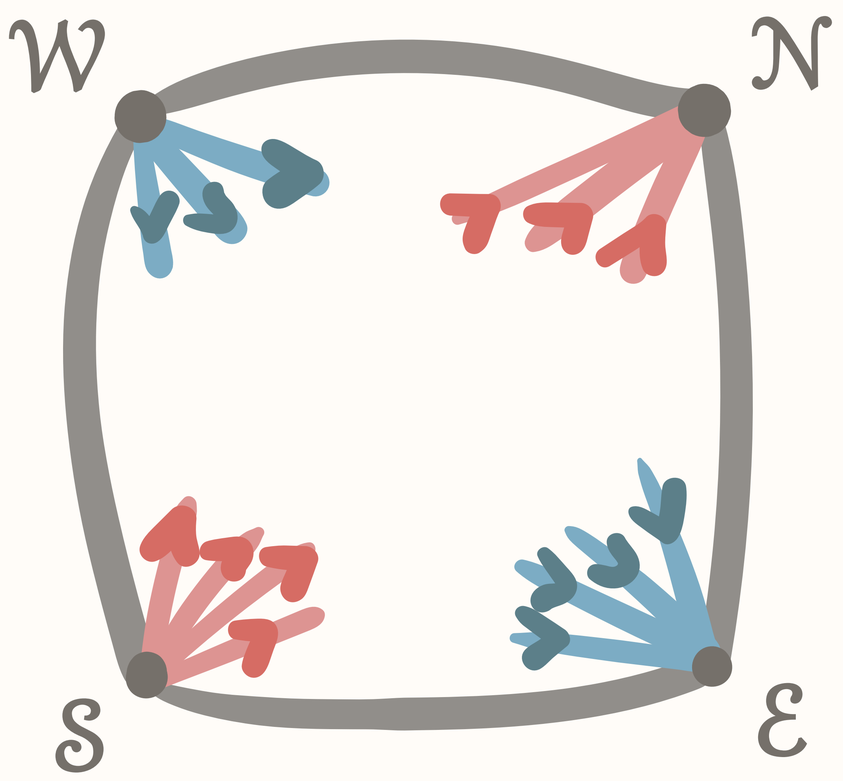
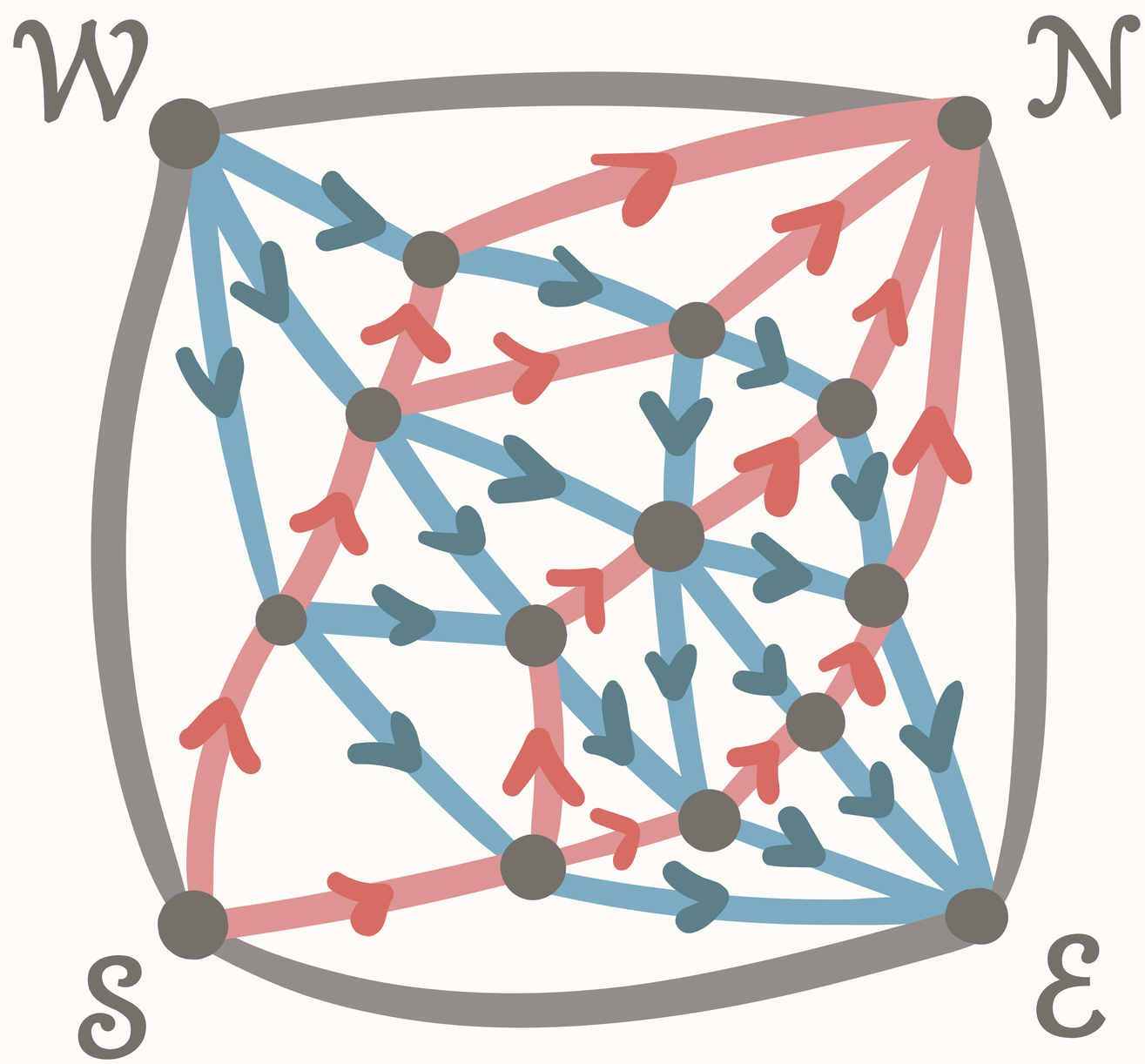


Transversal structures





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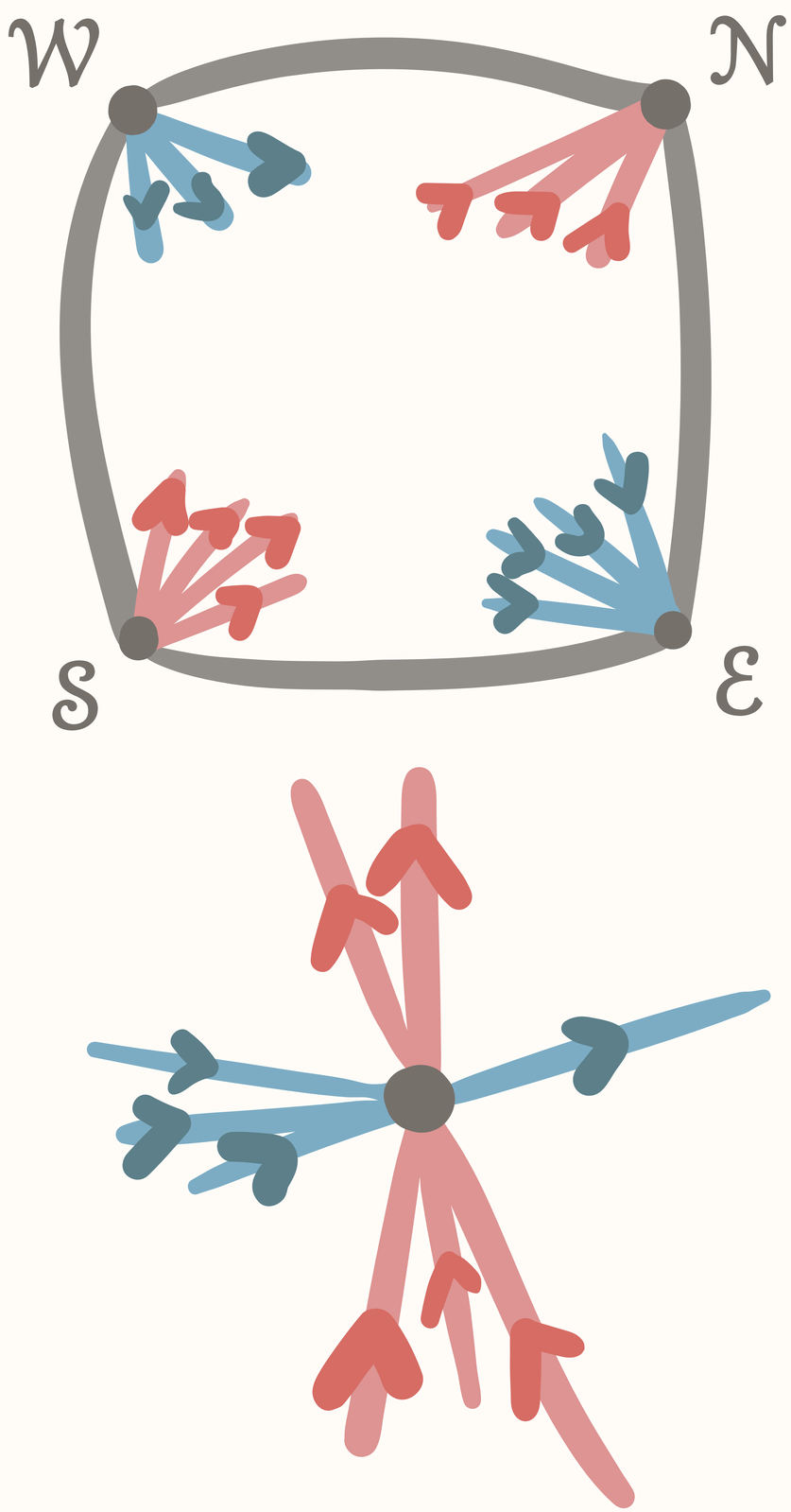
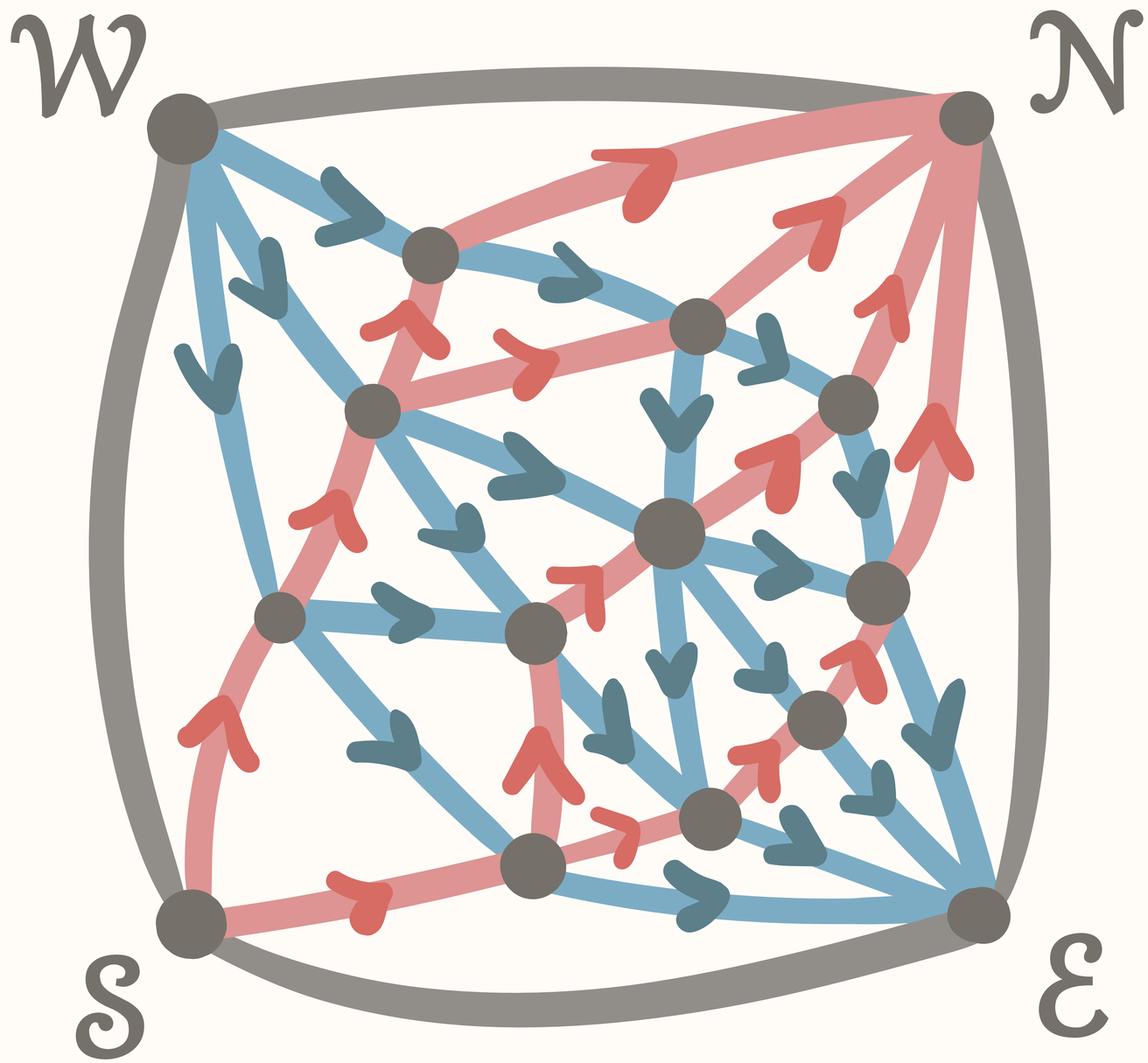
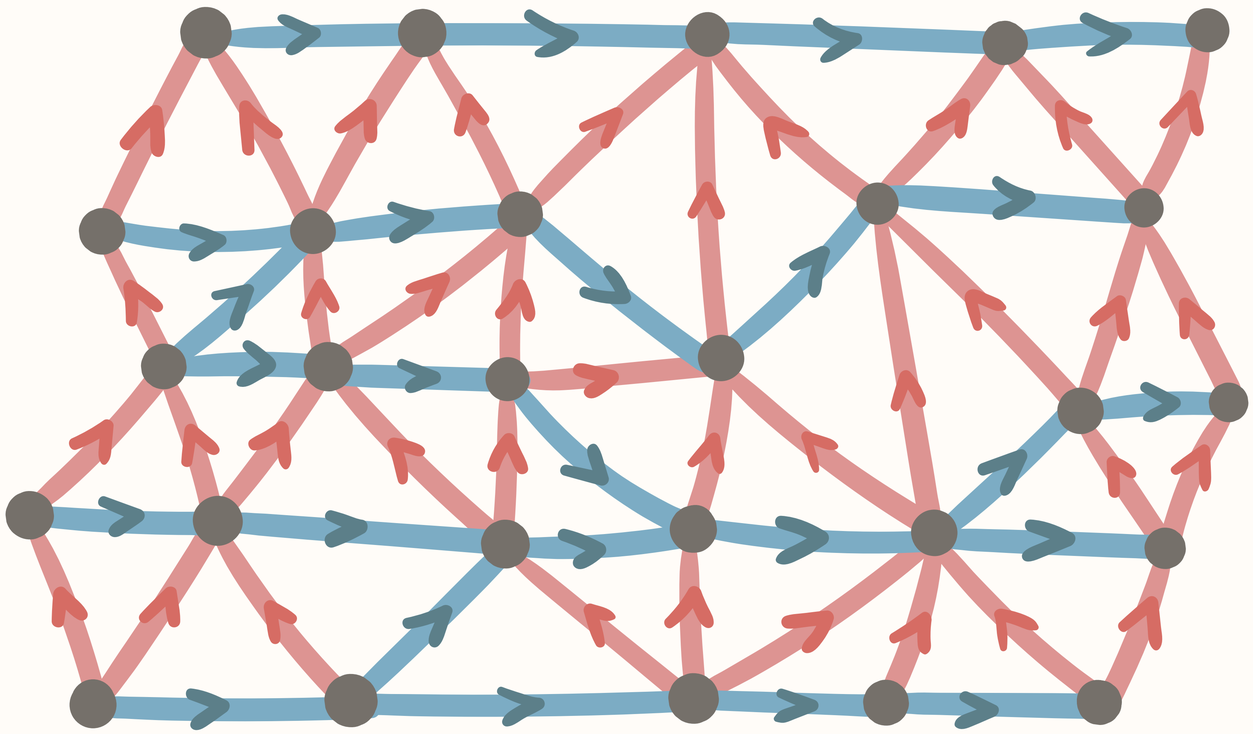


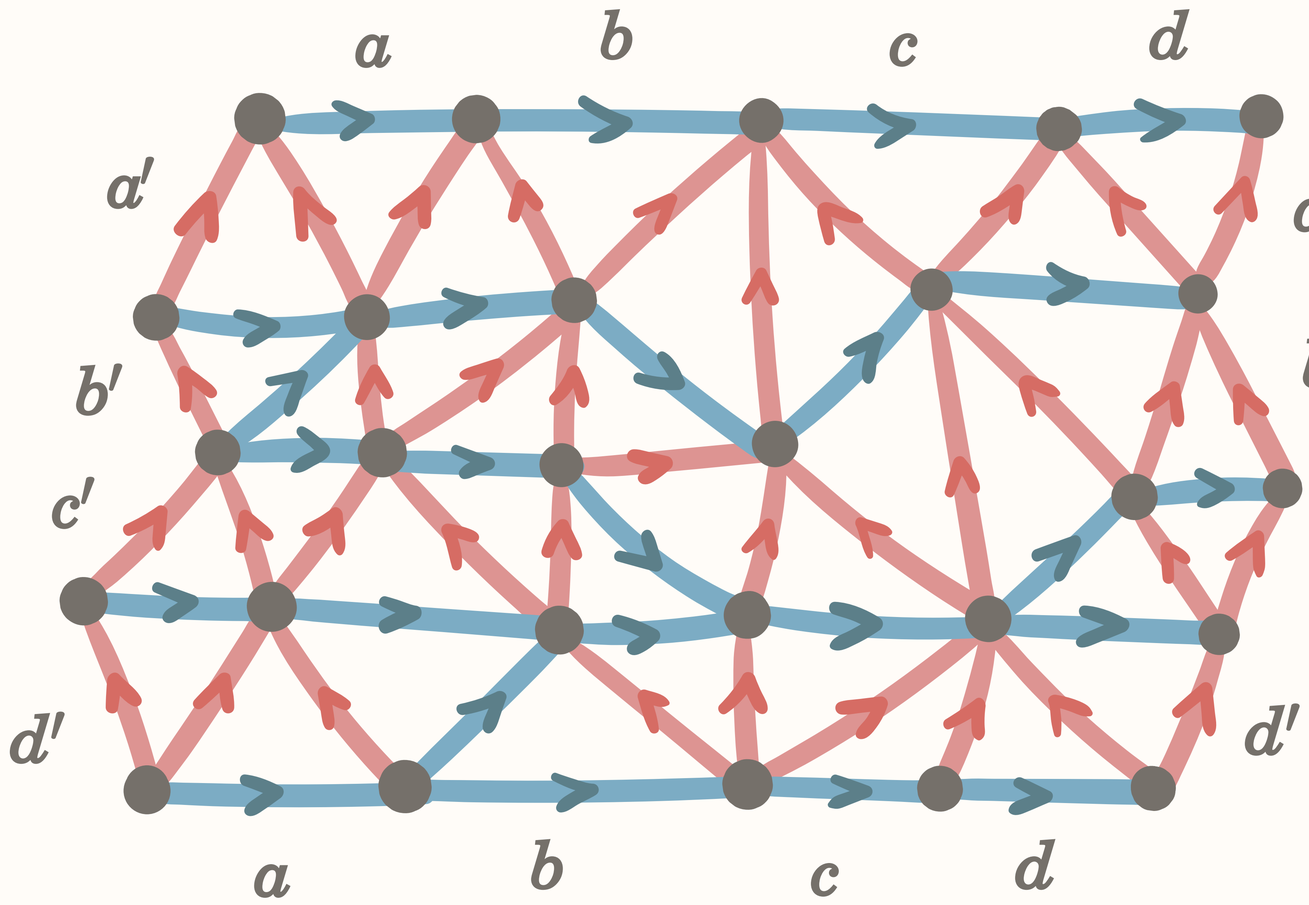


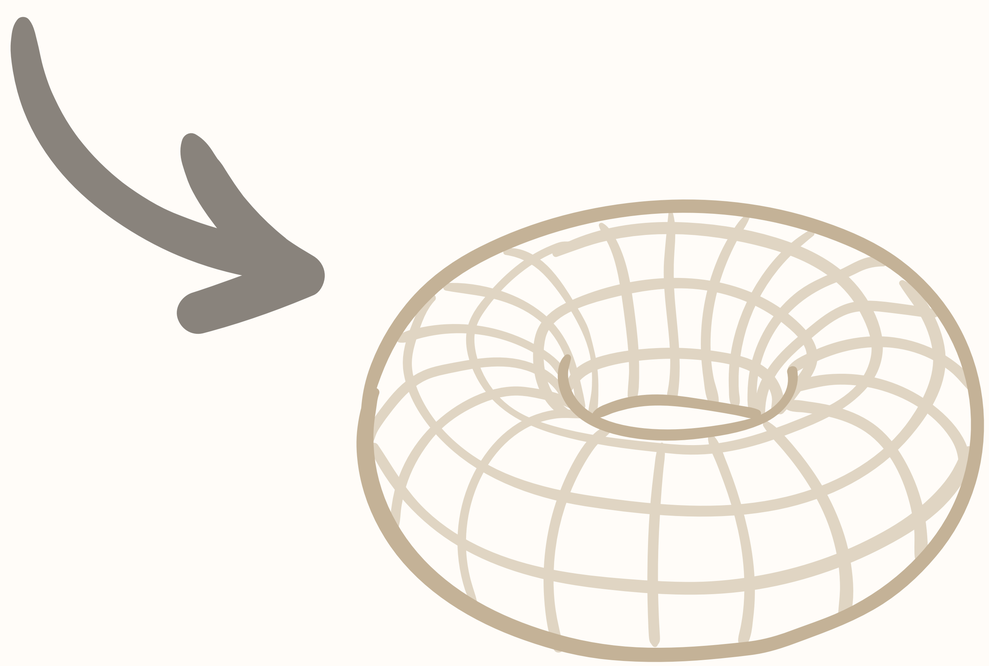
Locally causal dynamical triangulations



↠ *Locally causal dynamical triangulations in two dimensions,* R. Loll & B. Ruijl (2015)

Locally causal dynamical triangulations





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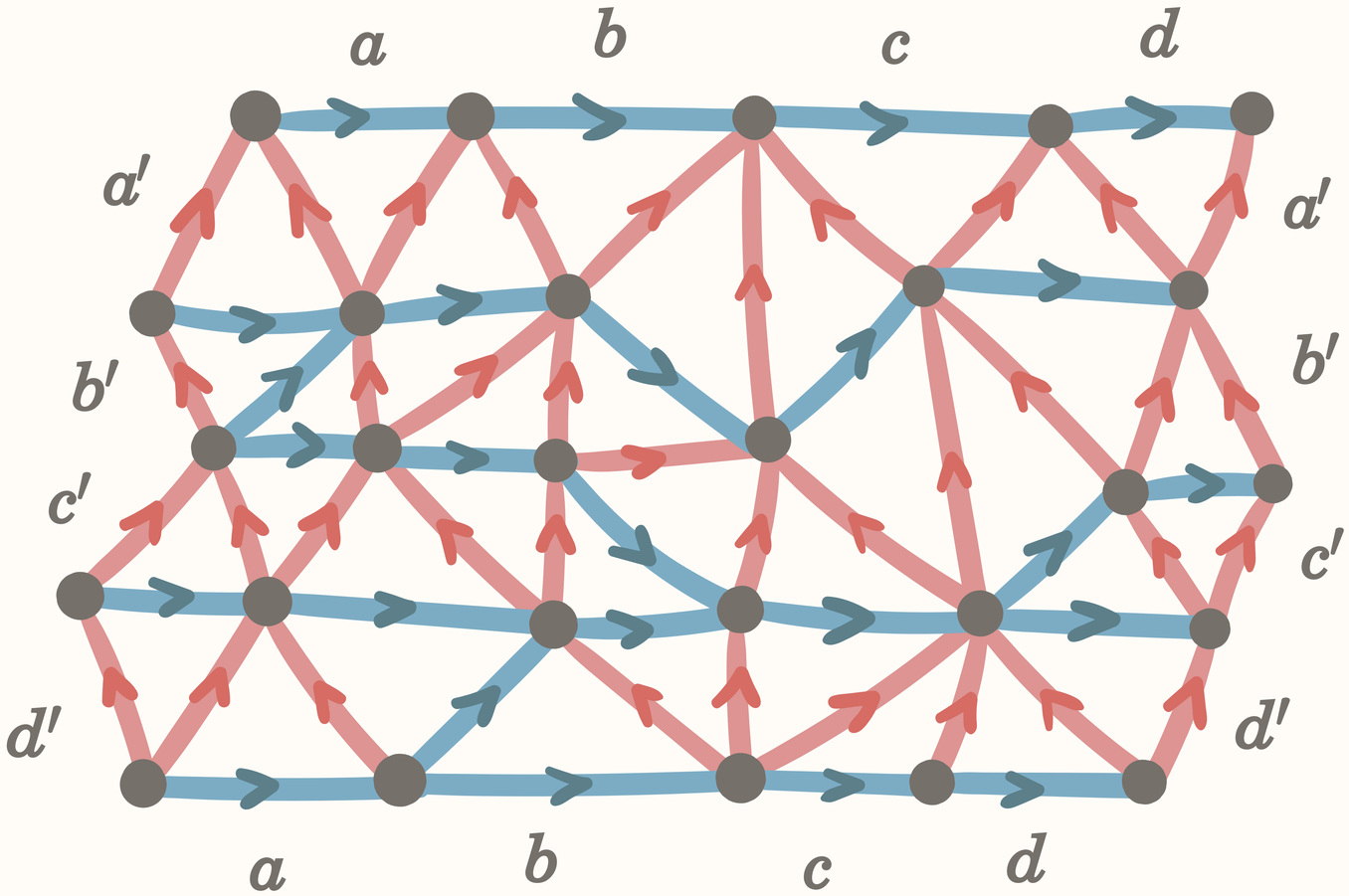
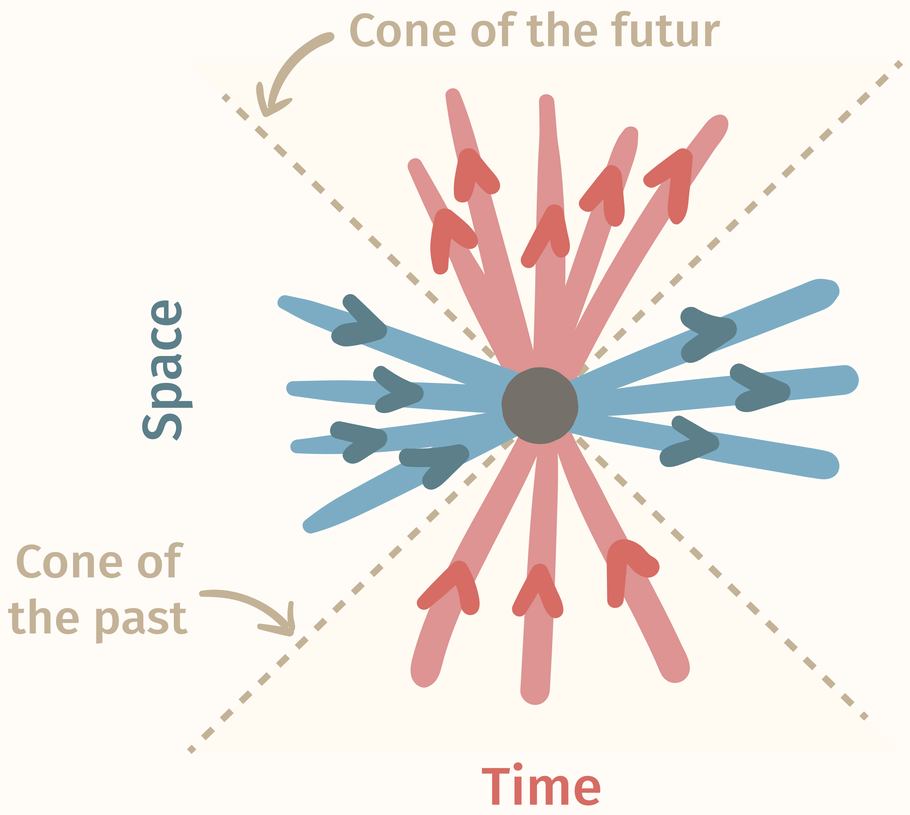
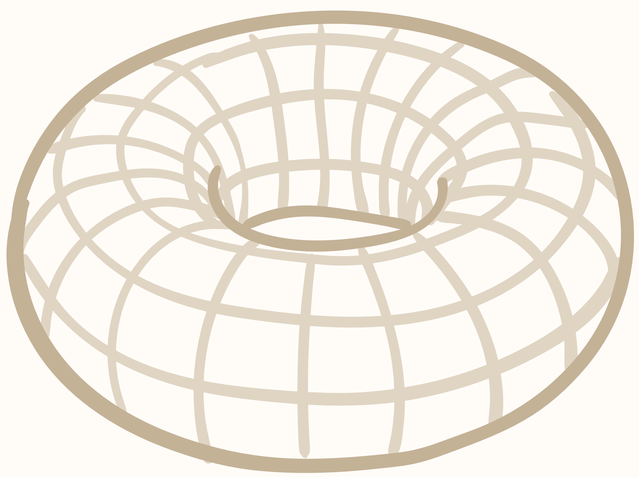
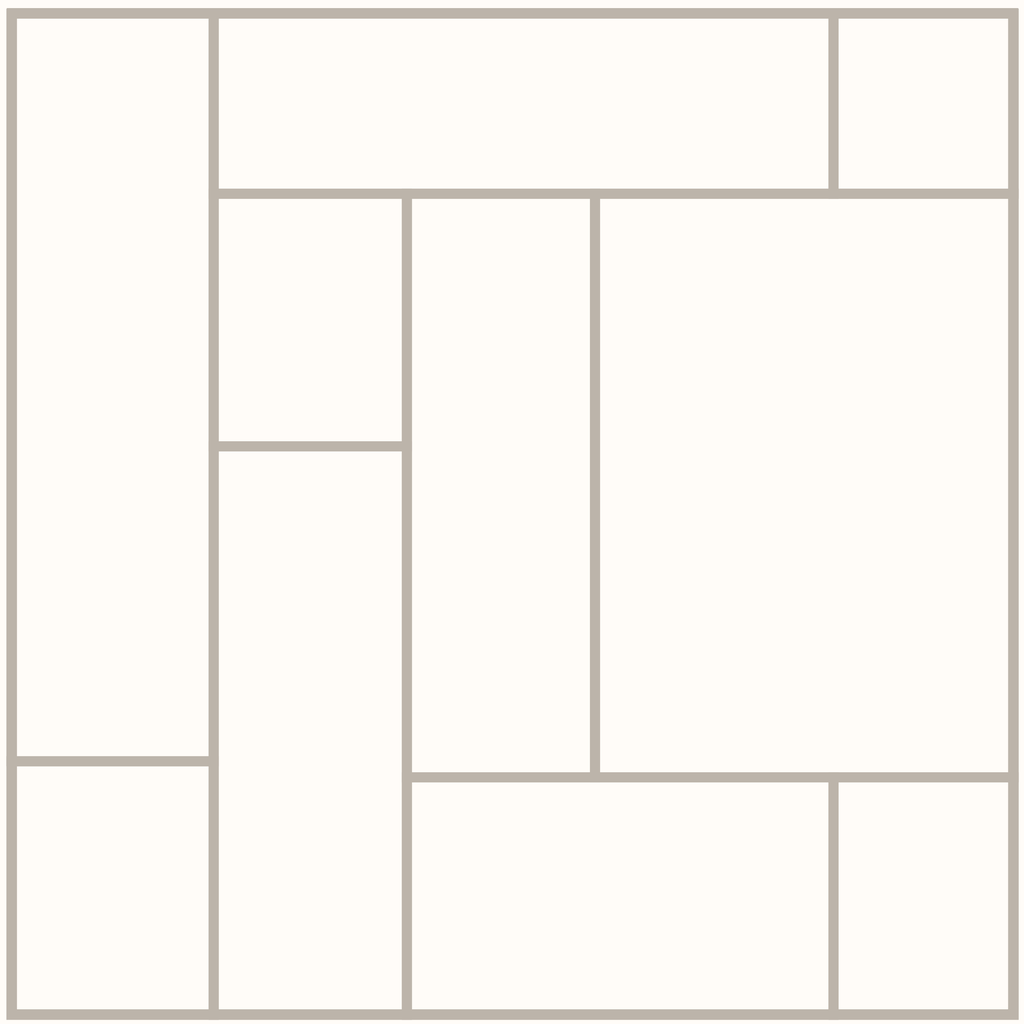






Locally causal dynamical triangulations in two dimensions, R. Loll & B. Ruijl (2015)

Link with rectangular tilings



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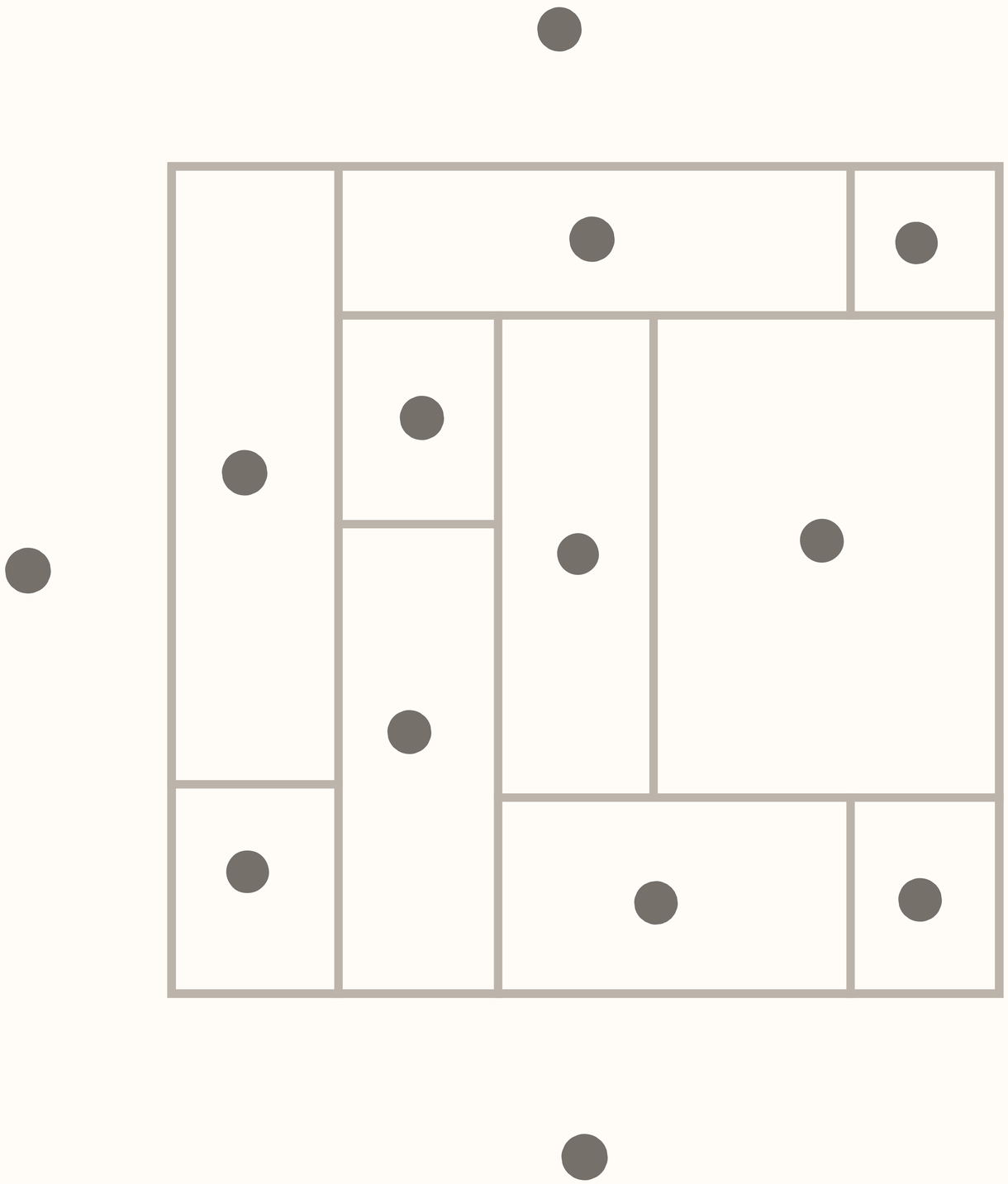

→ *Regular edge labellings of 4-connected plane graphes and its applications in graph drawing problems*, G. Kant & X. He (1997)

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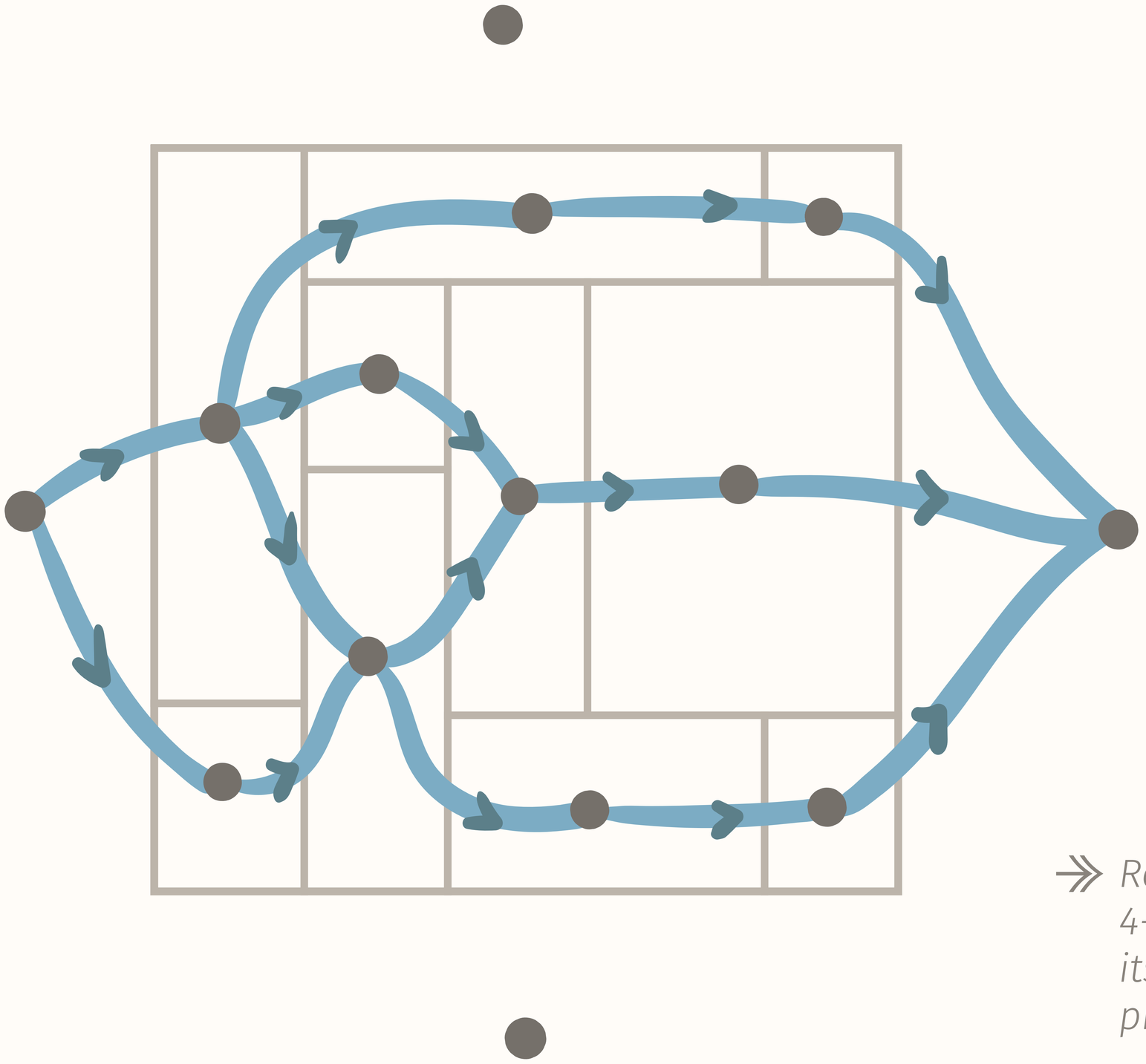

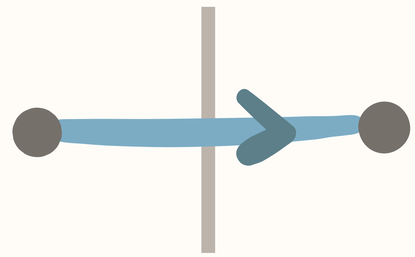

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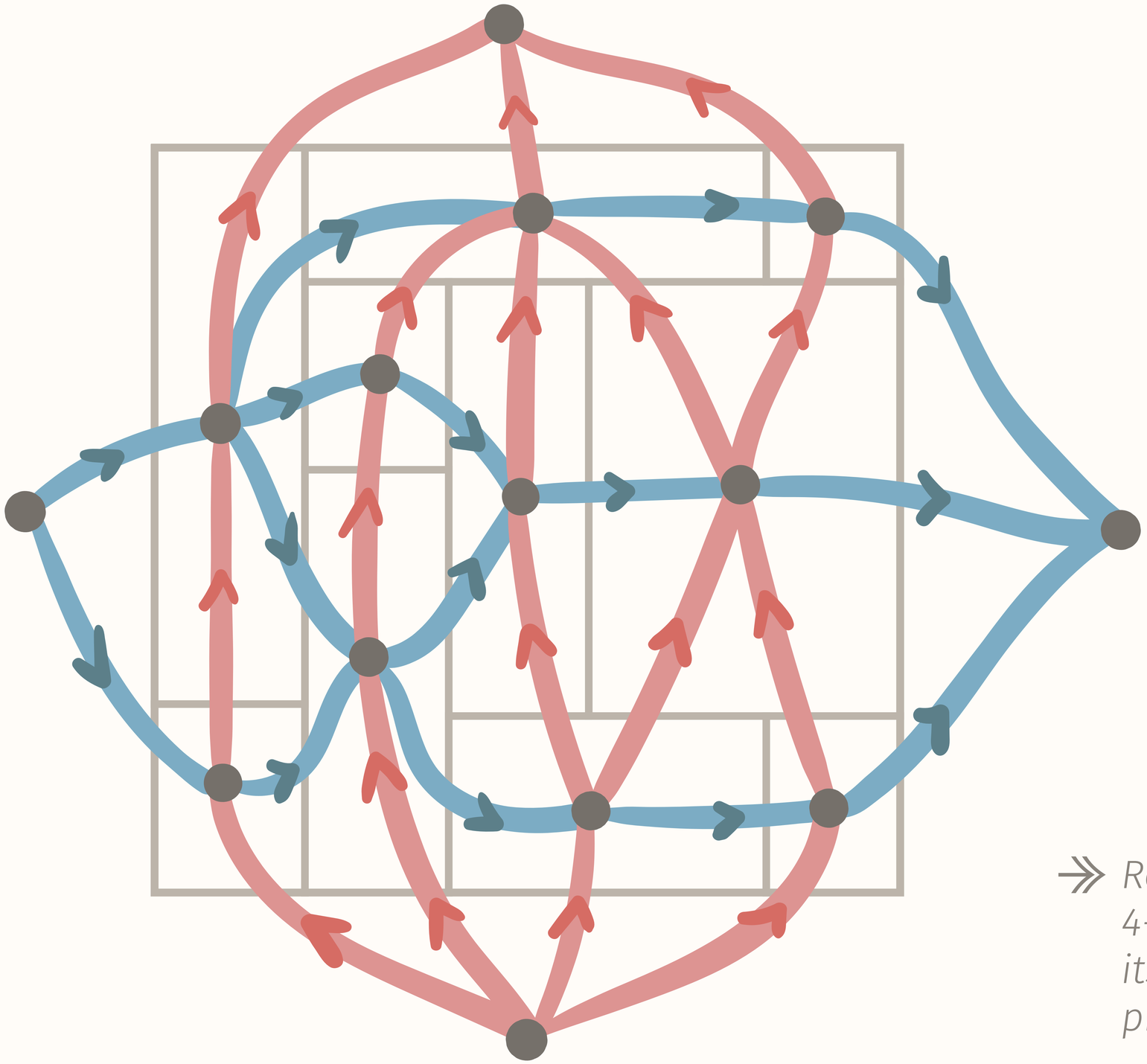

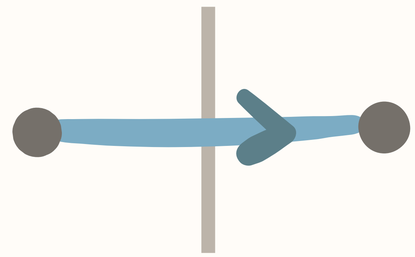

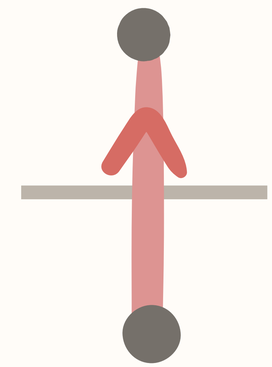

Regular edge labellings of 4-connected plane graphes and its applications in graph drawing problems, G. Kant & X. He (1997)

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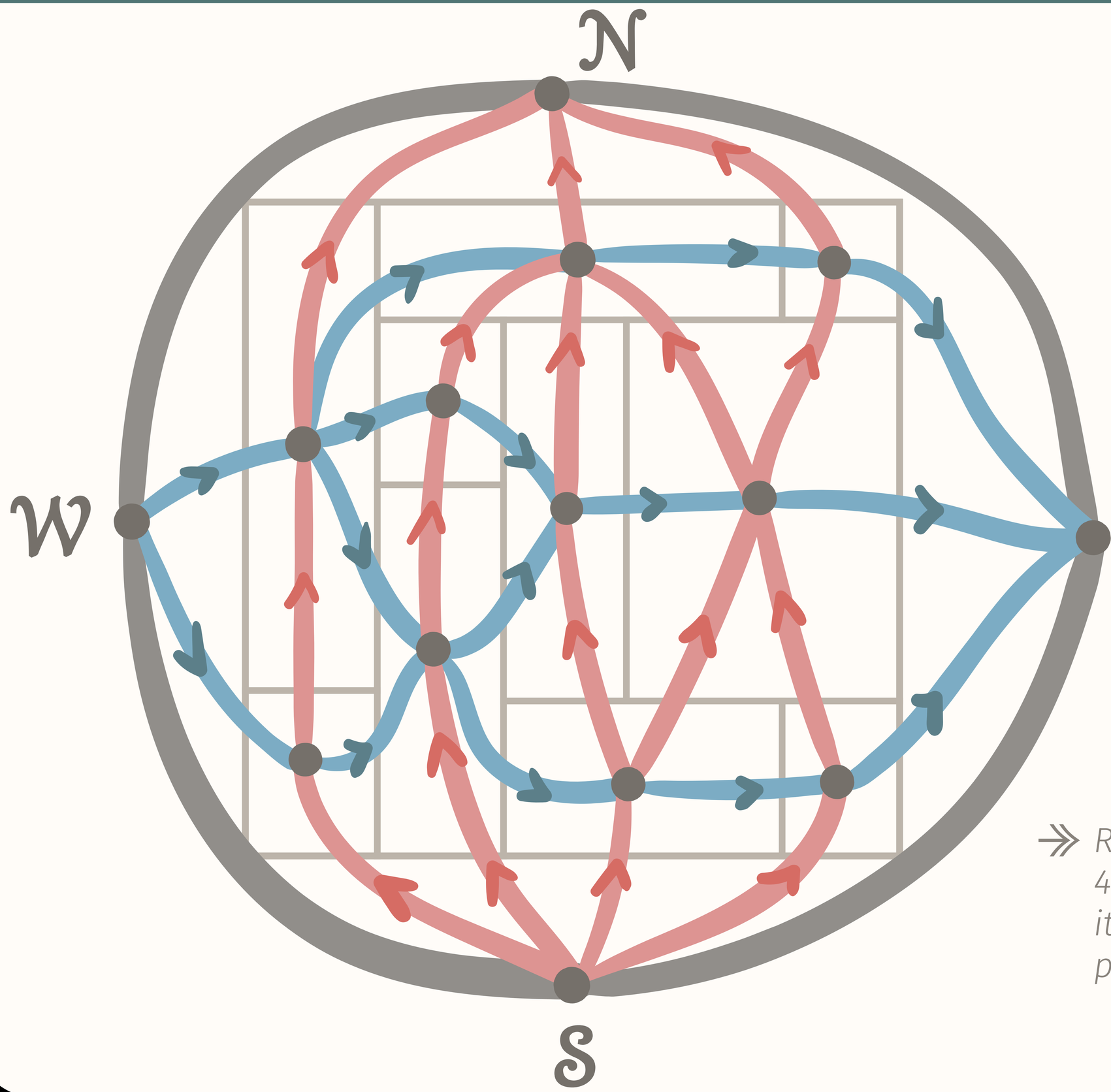

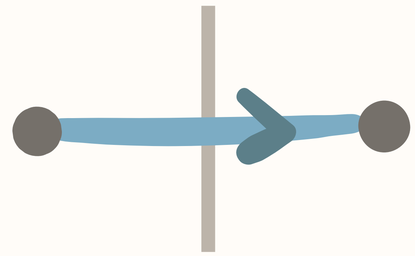

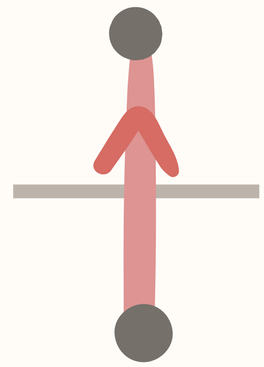

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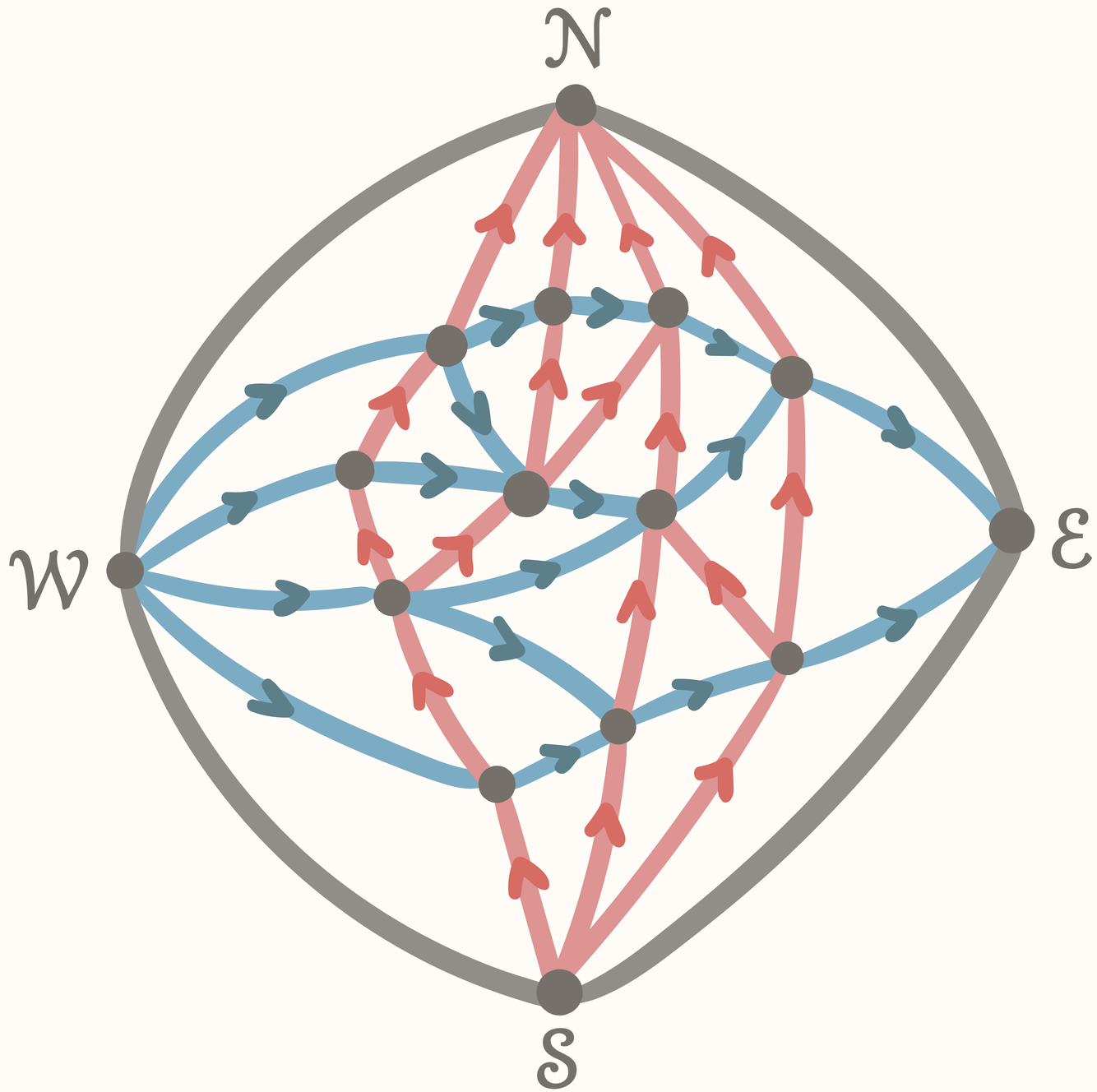
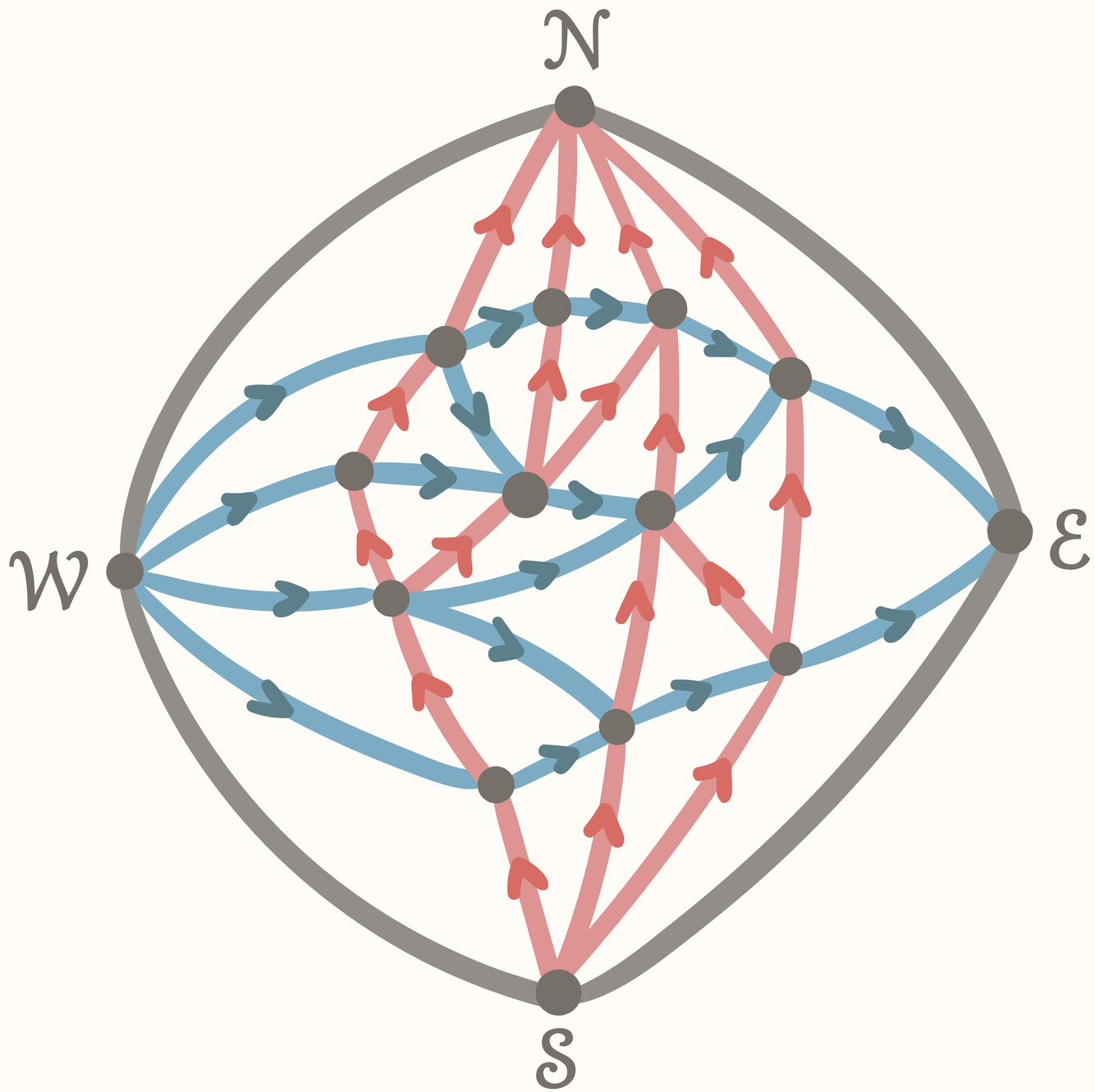
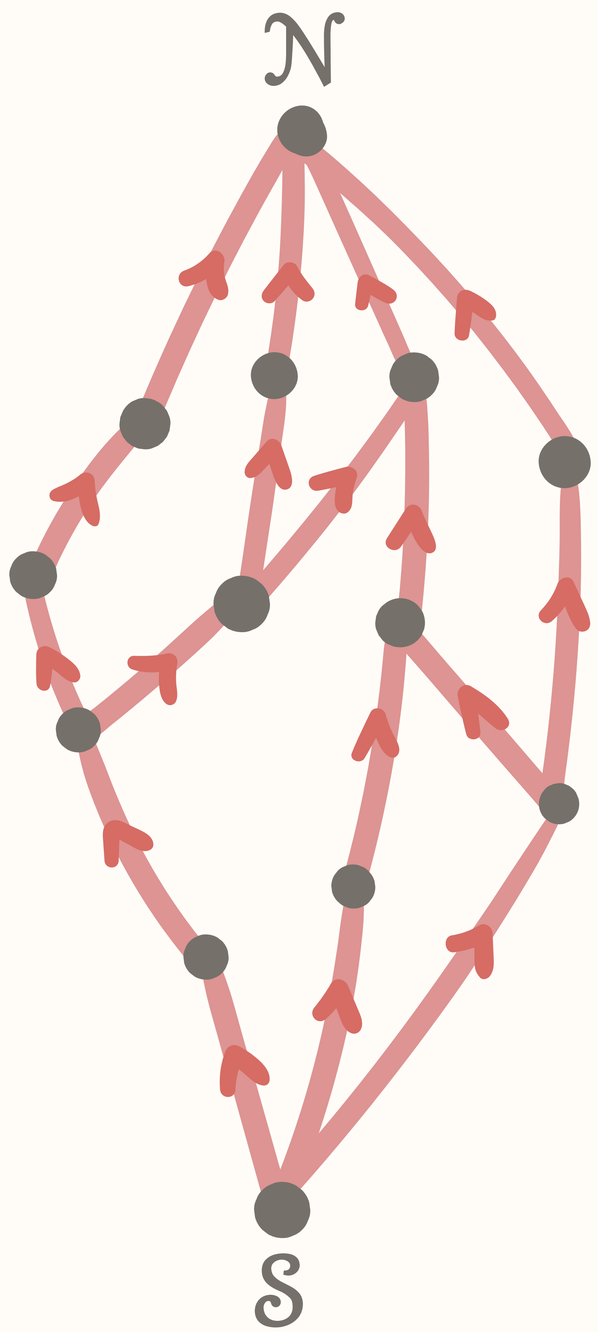
Schnyder labelings

Encoding with bipolar orientations



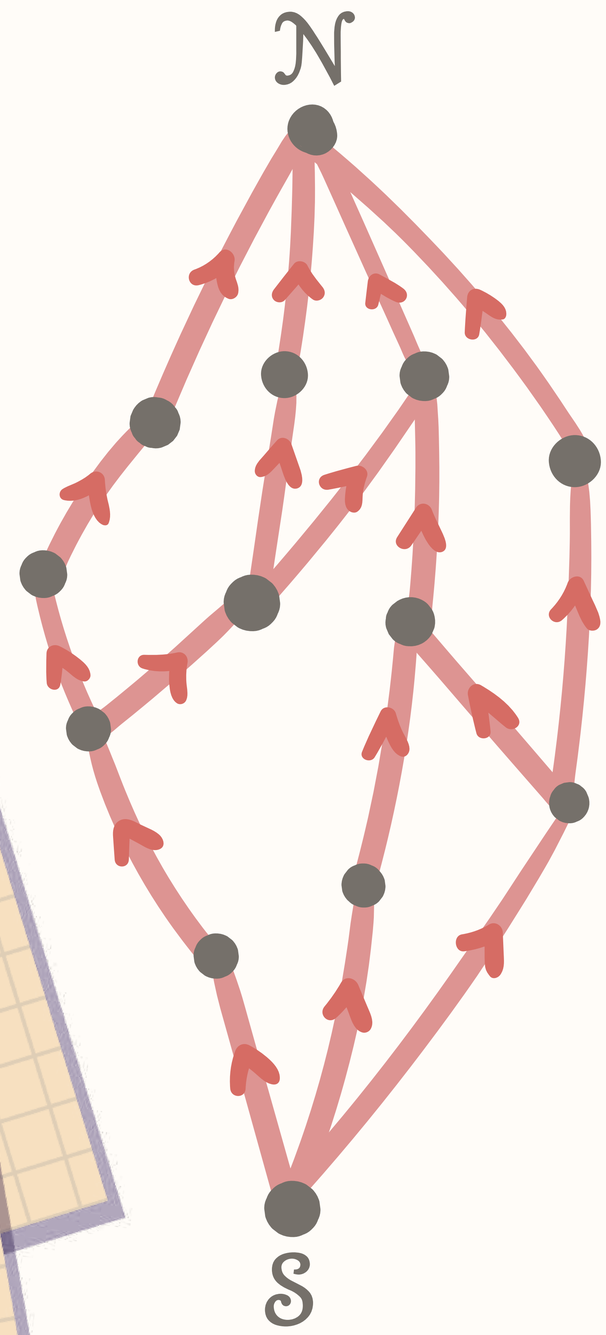
Encoding with bipolar orientations



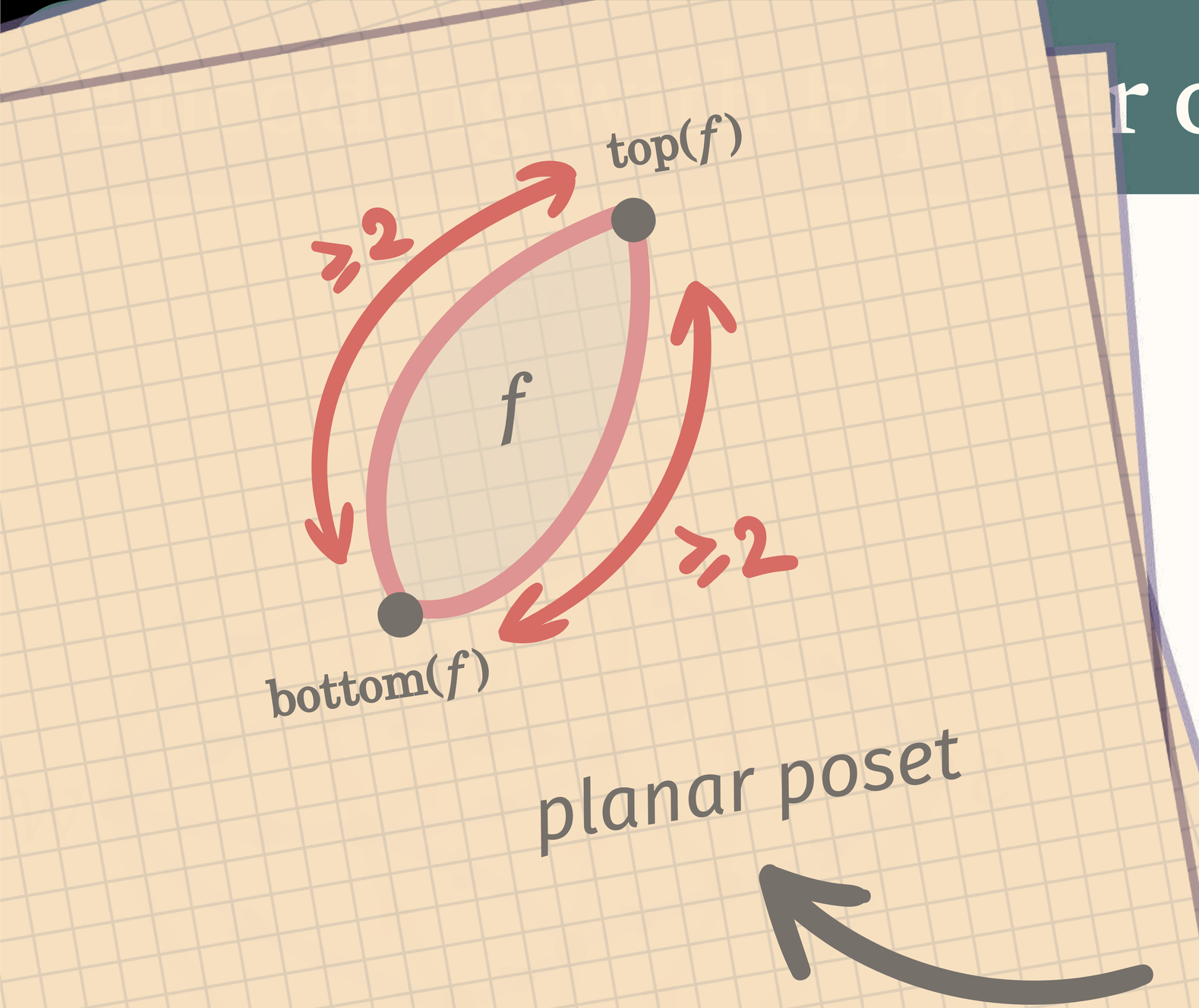


r orientations

planar poset



r orientations



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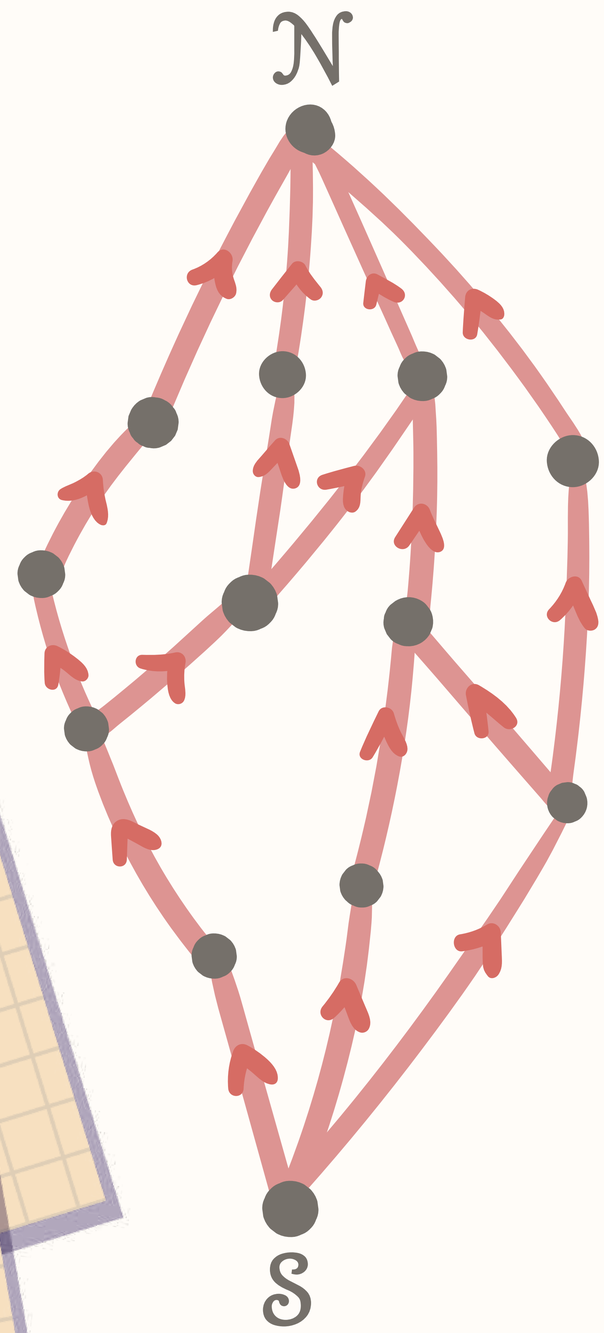

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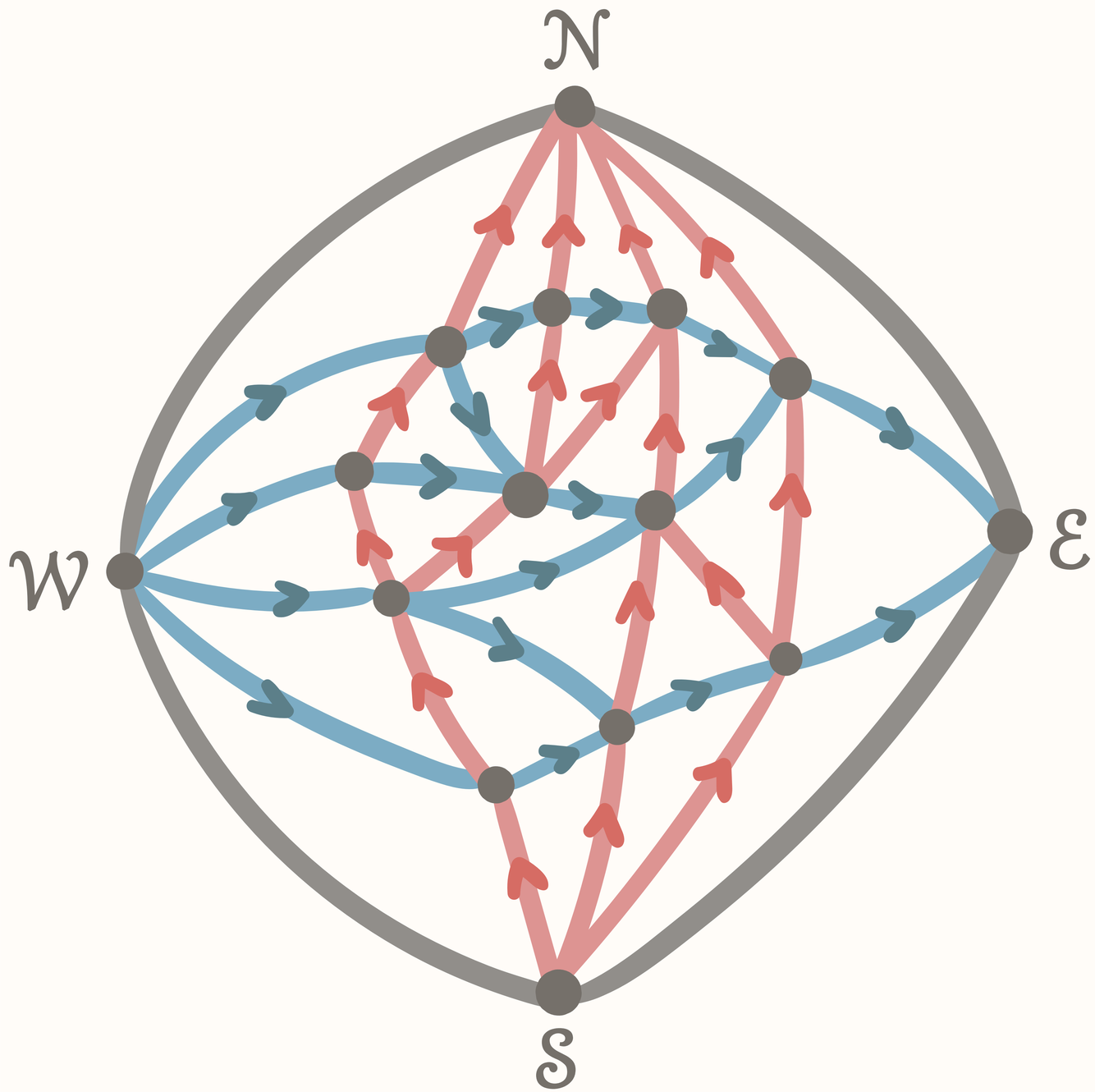

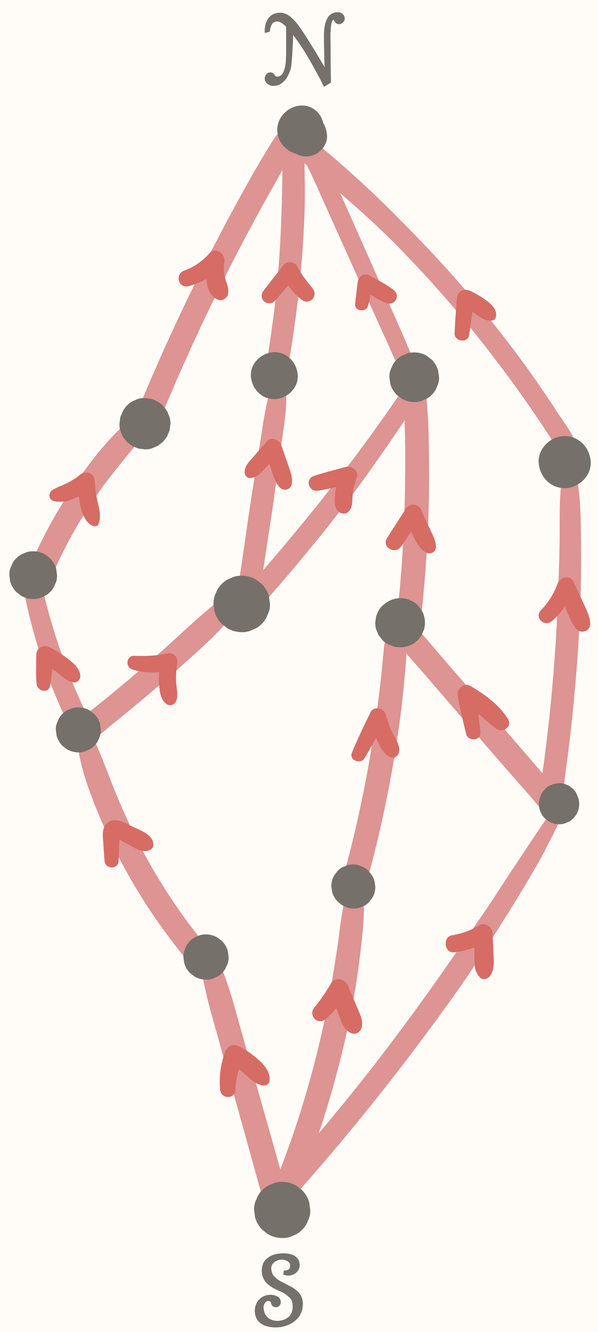

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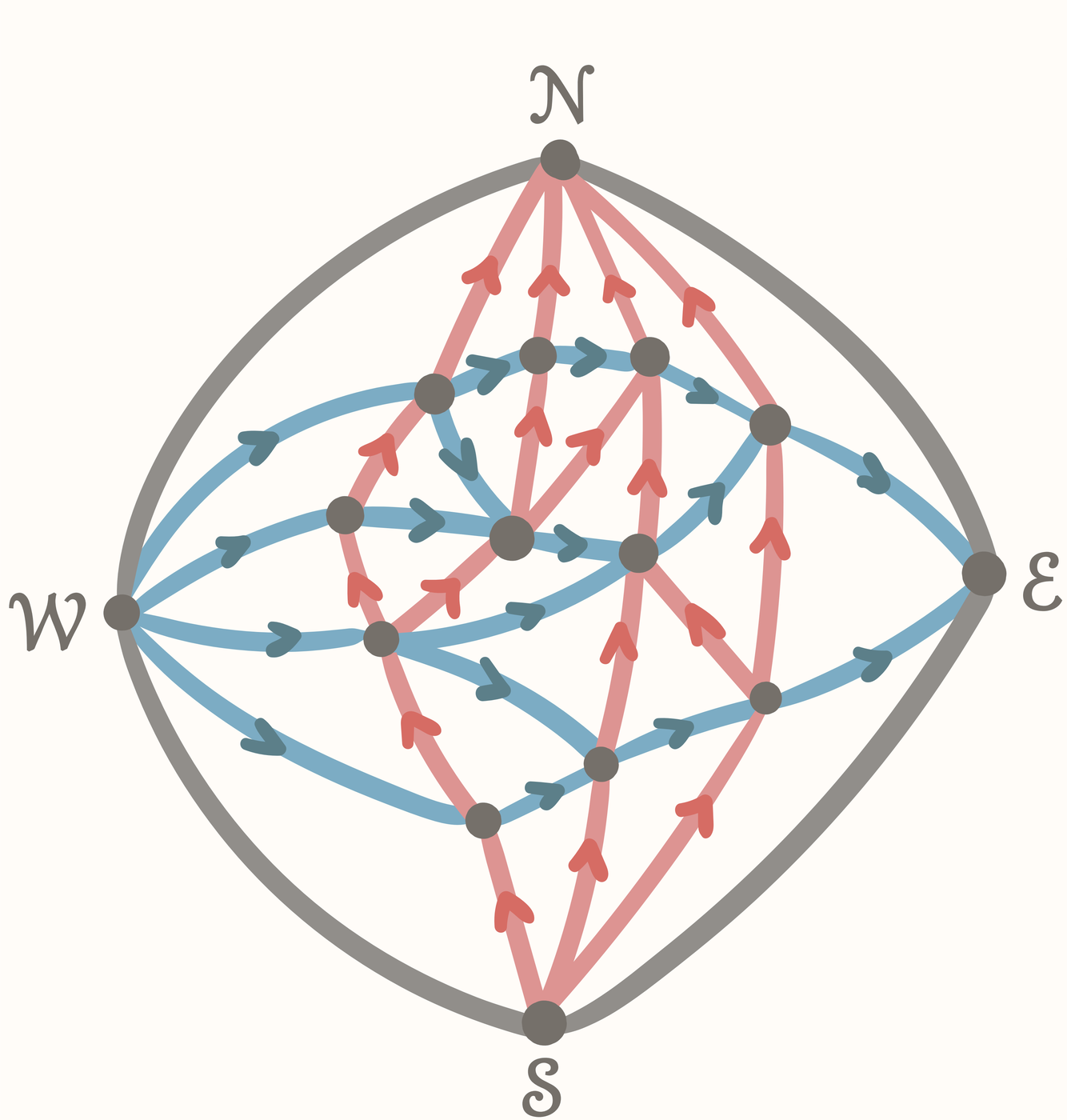

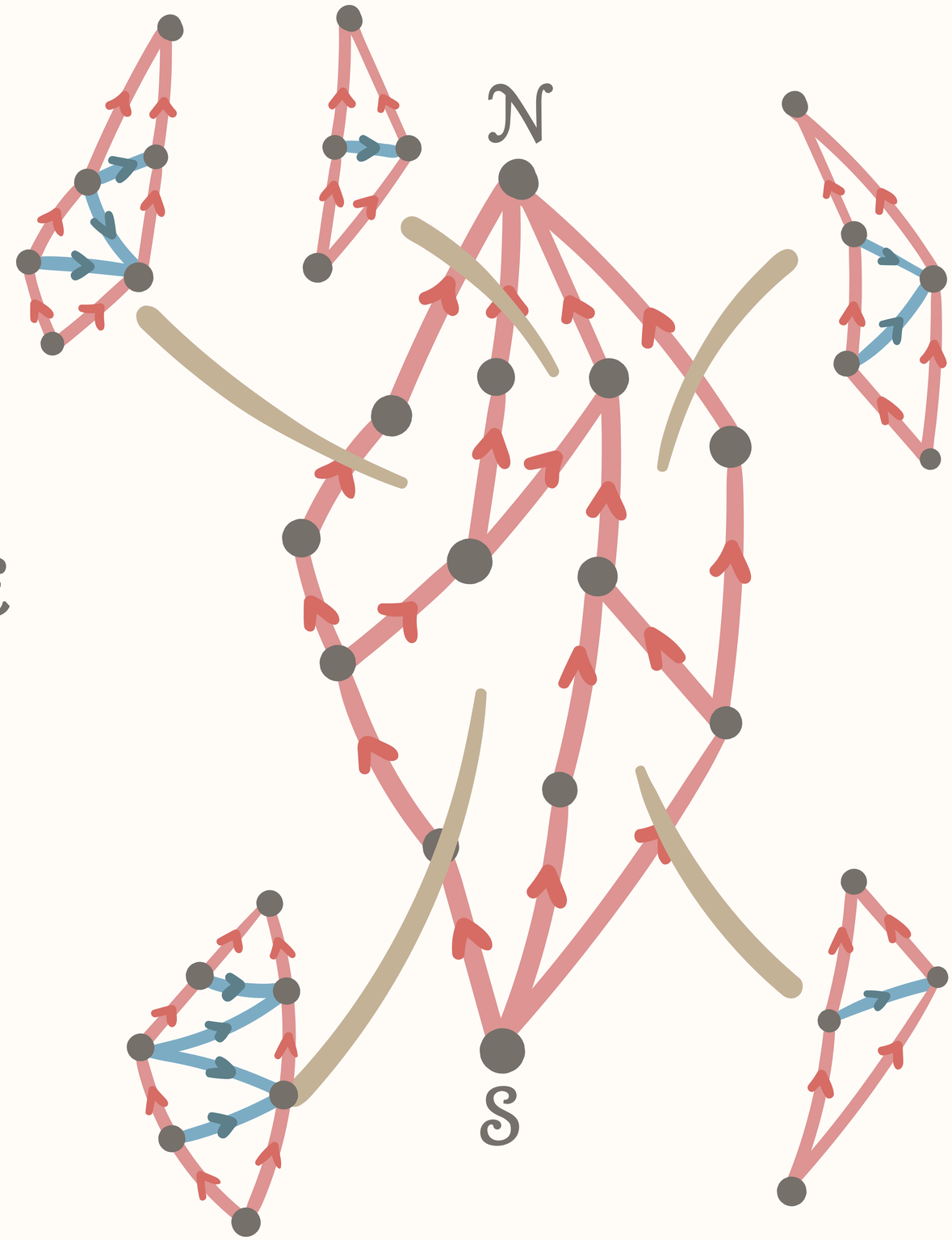

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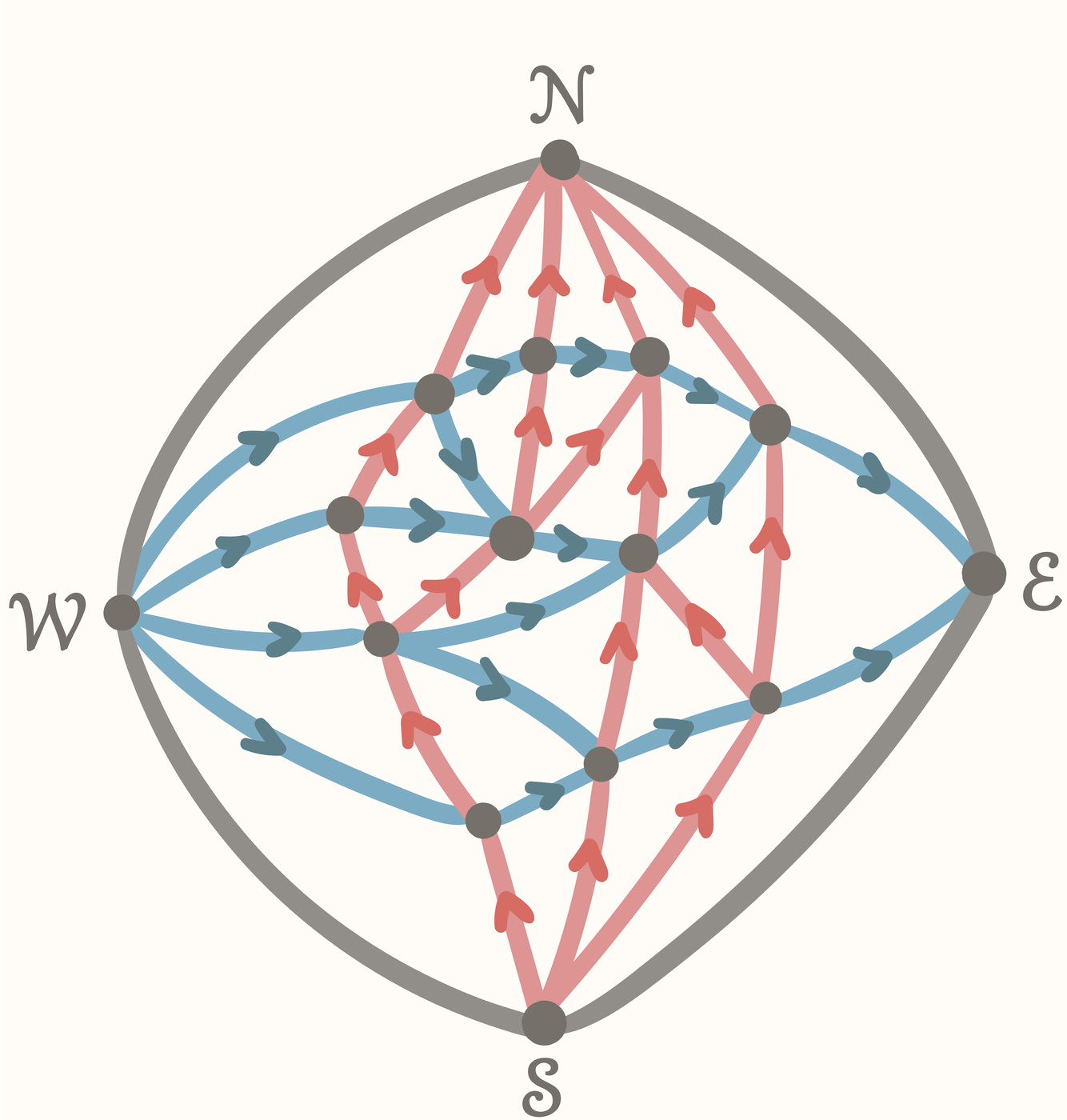

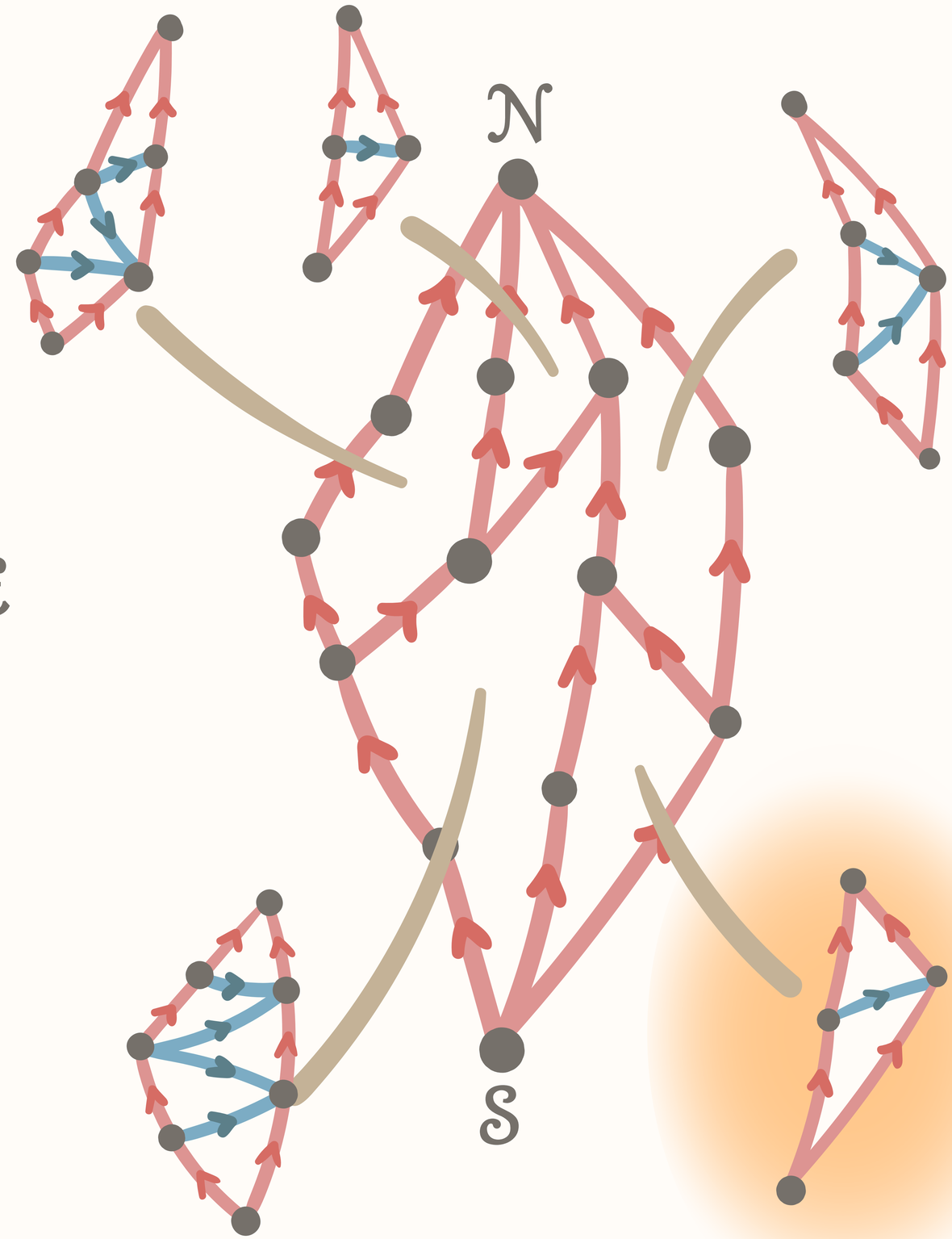

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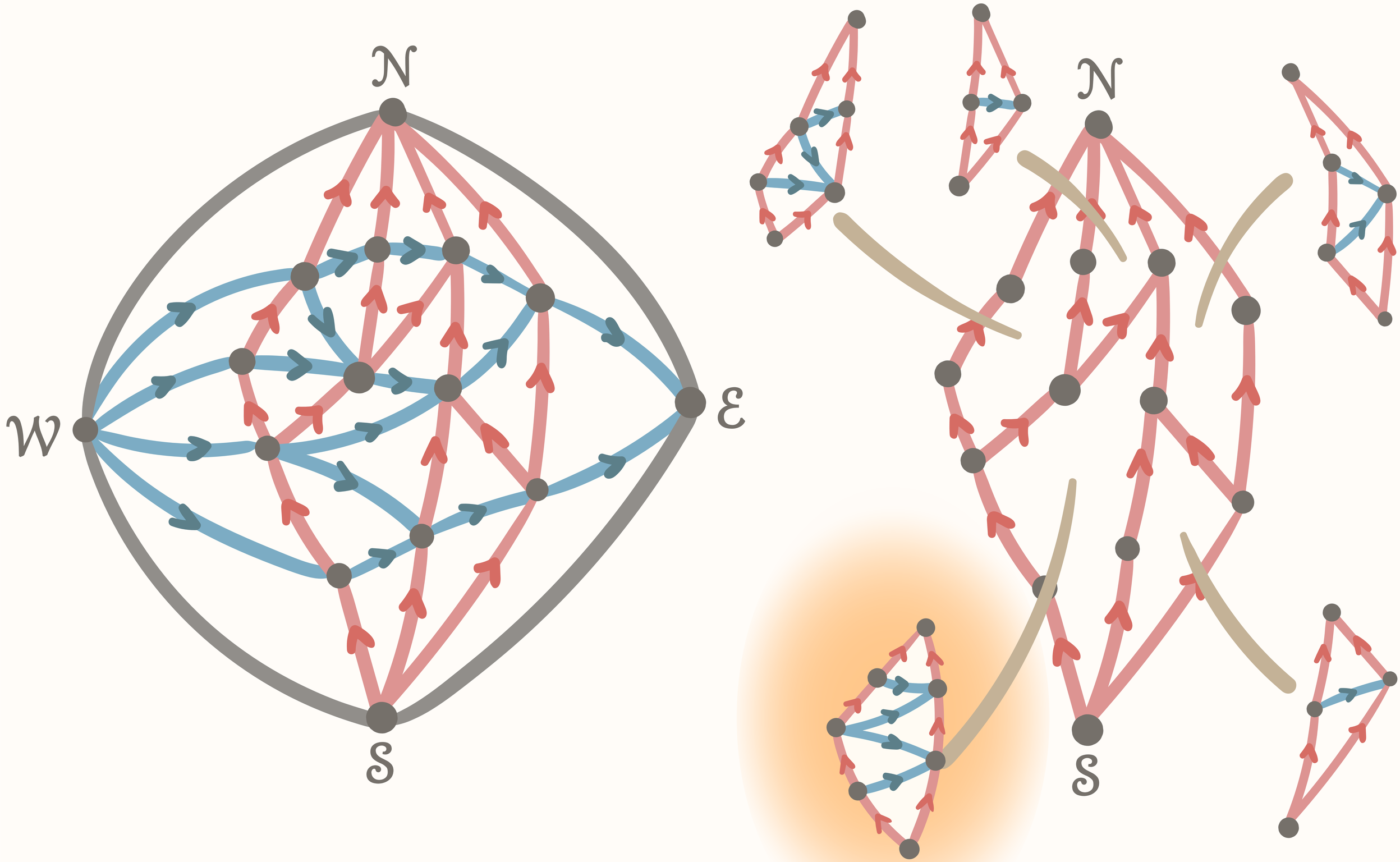

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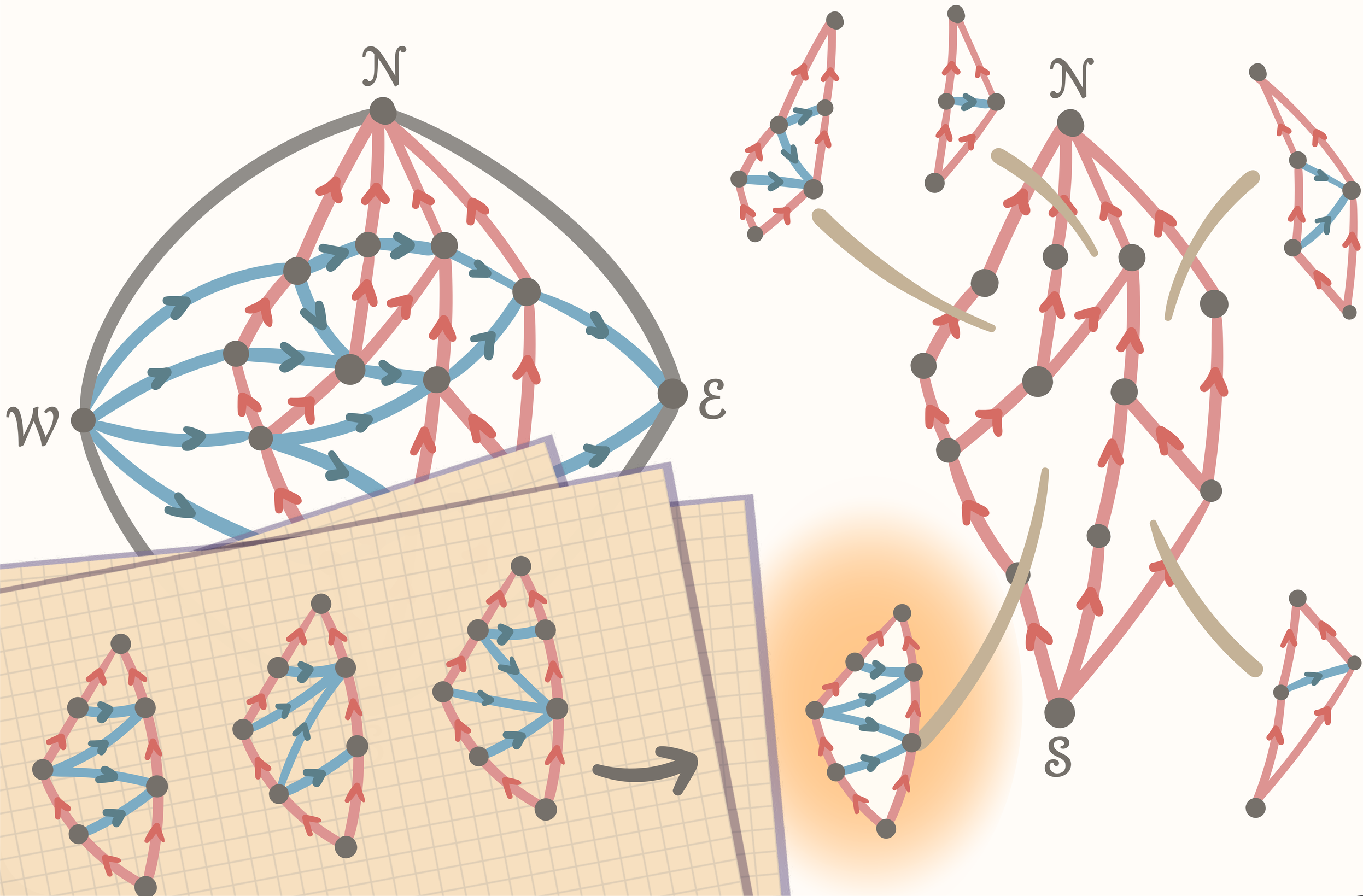

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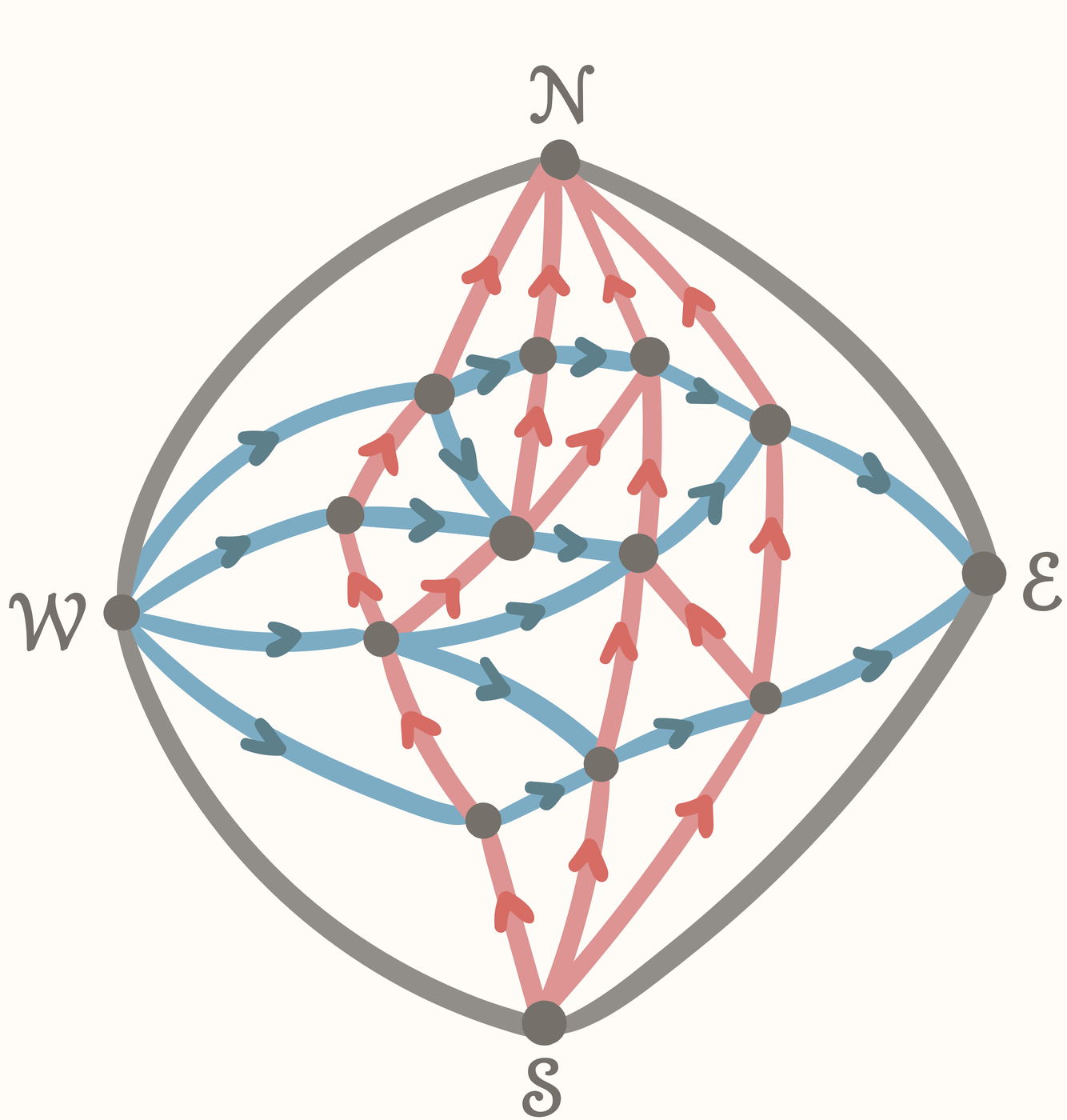

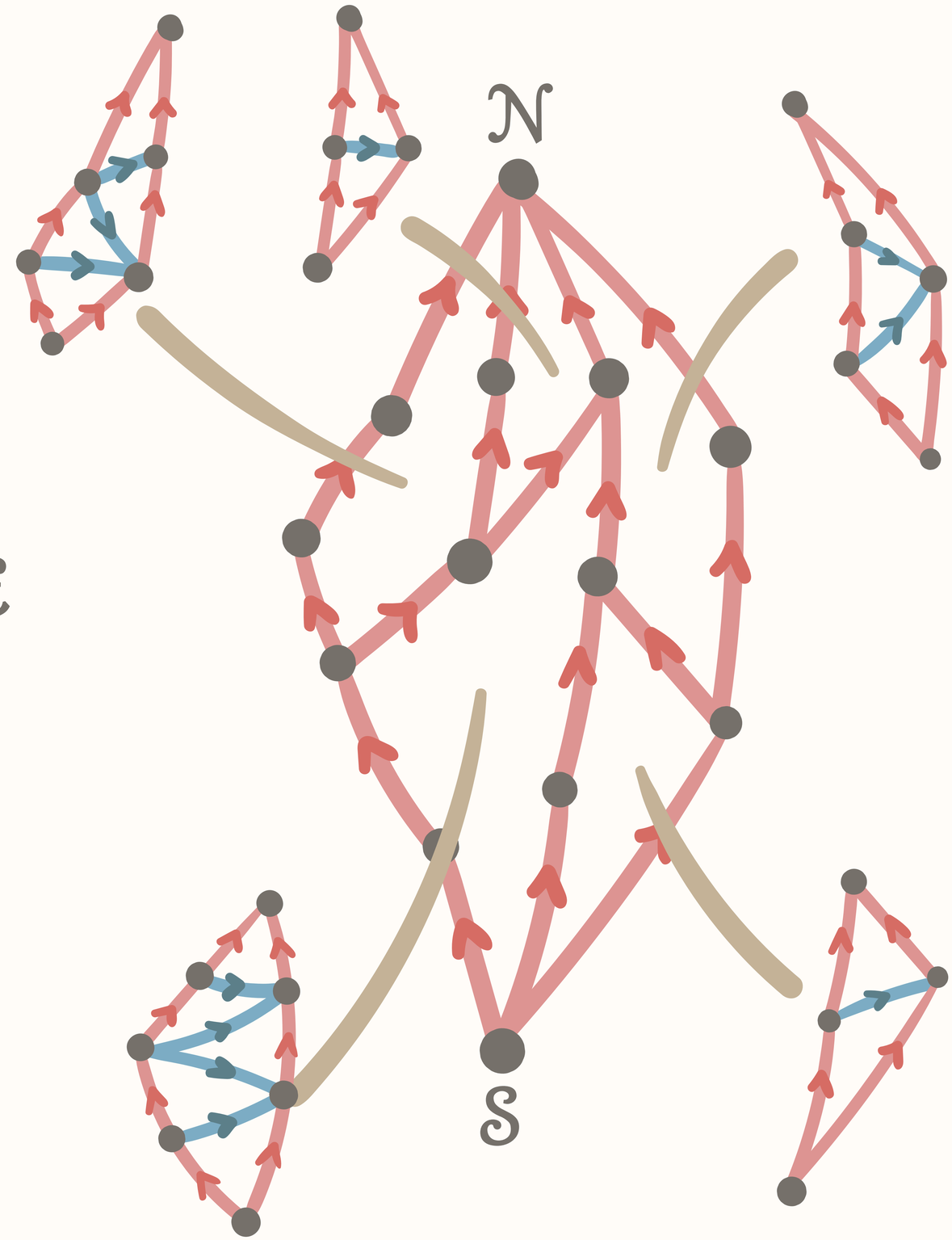

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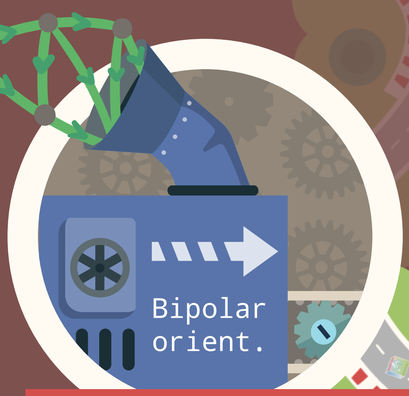
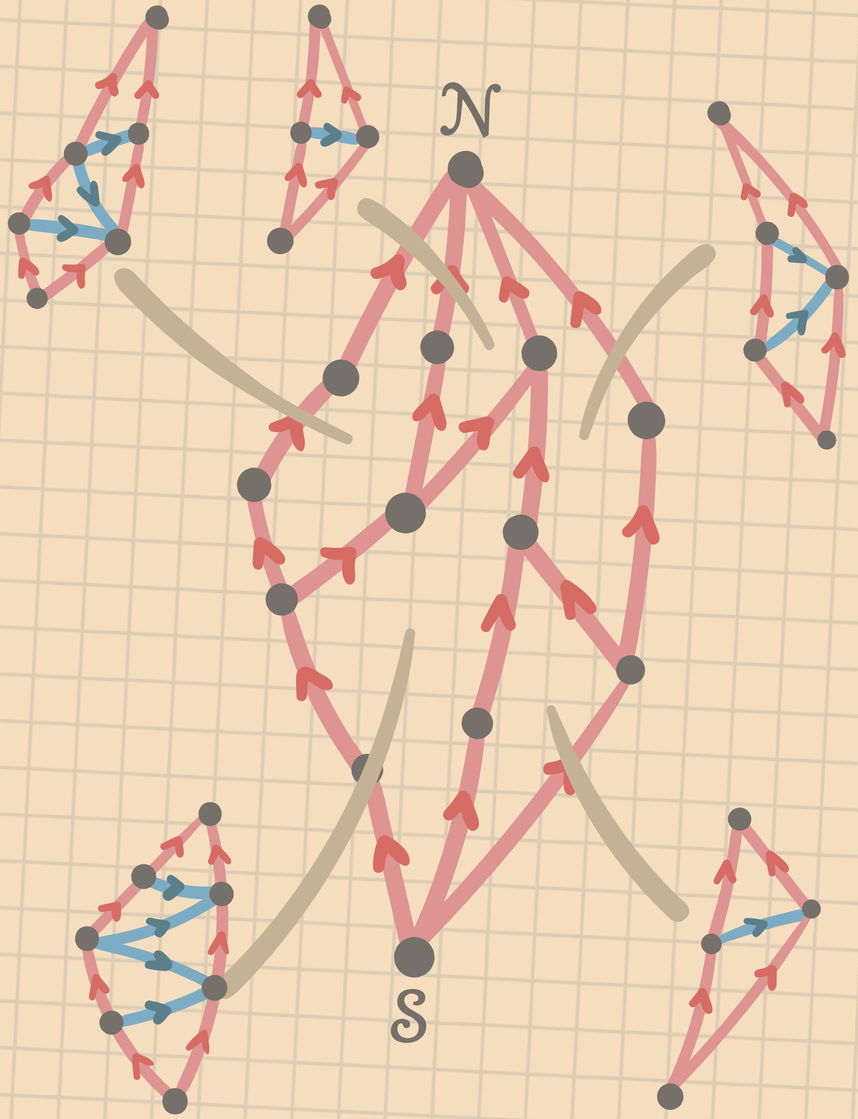


Decorated bipolar orientations



bipolar posets by vertices

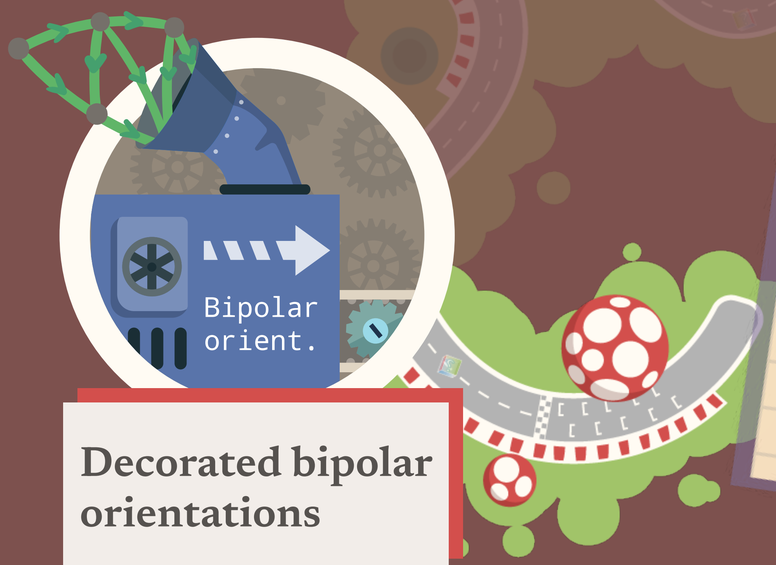
bipolar posets by edges

transversal structures

polyhedral orientations

Schnyder labelings

Summary



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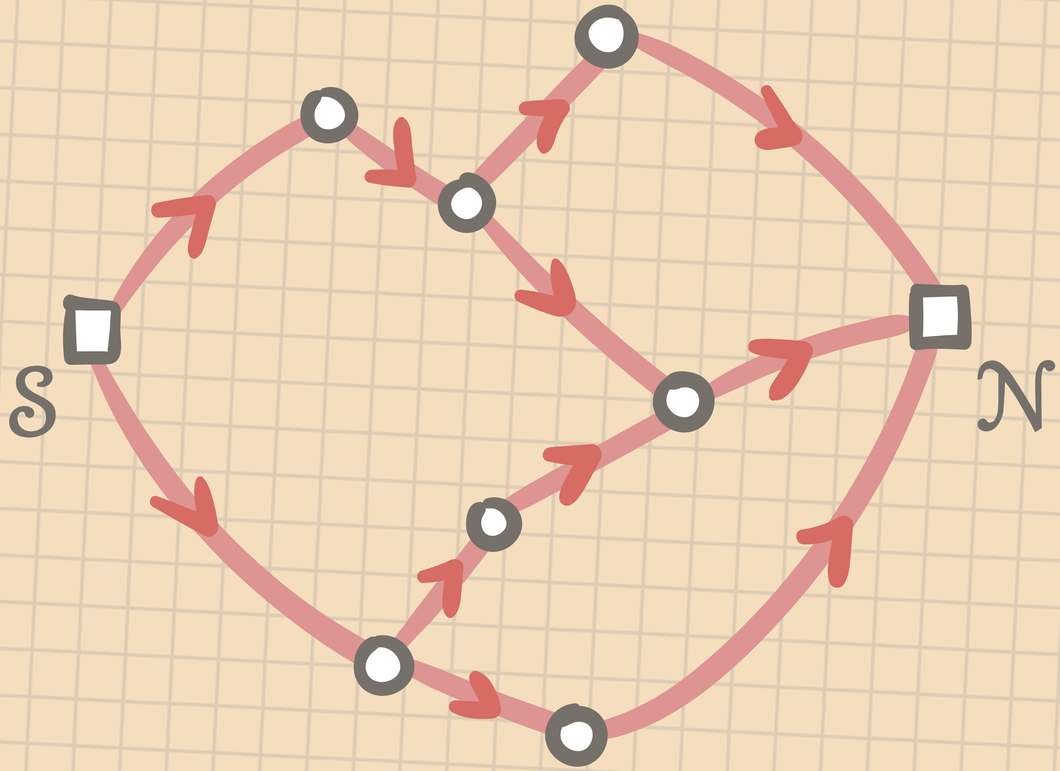

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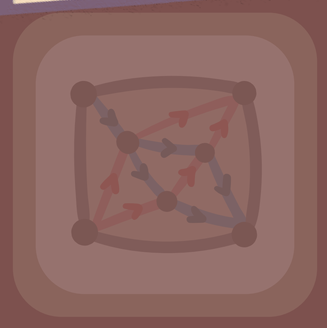

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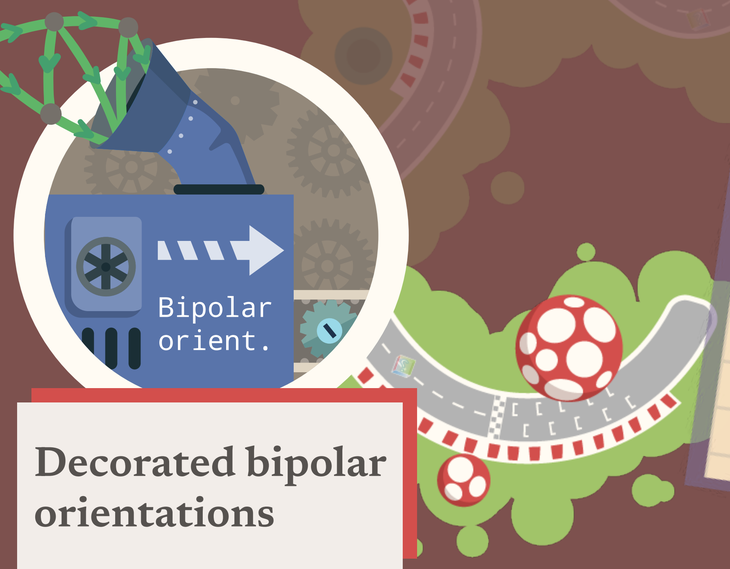

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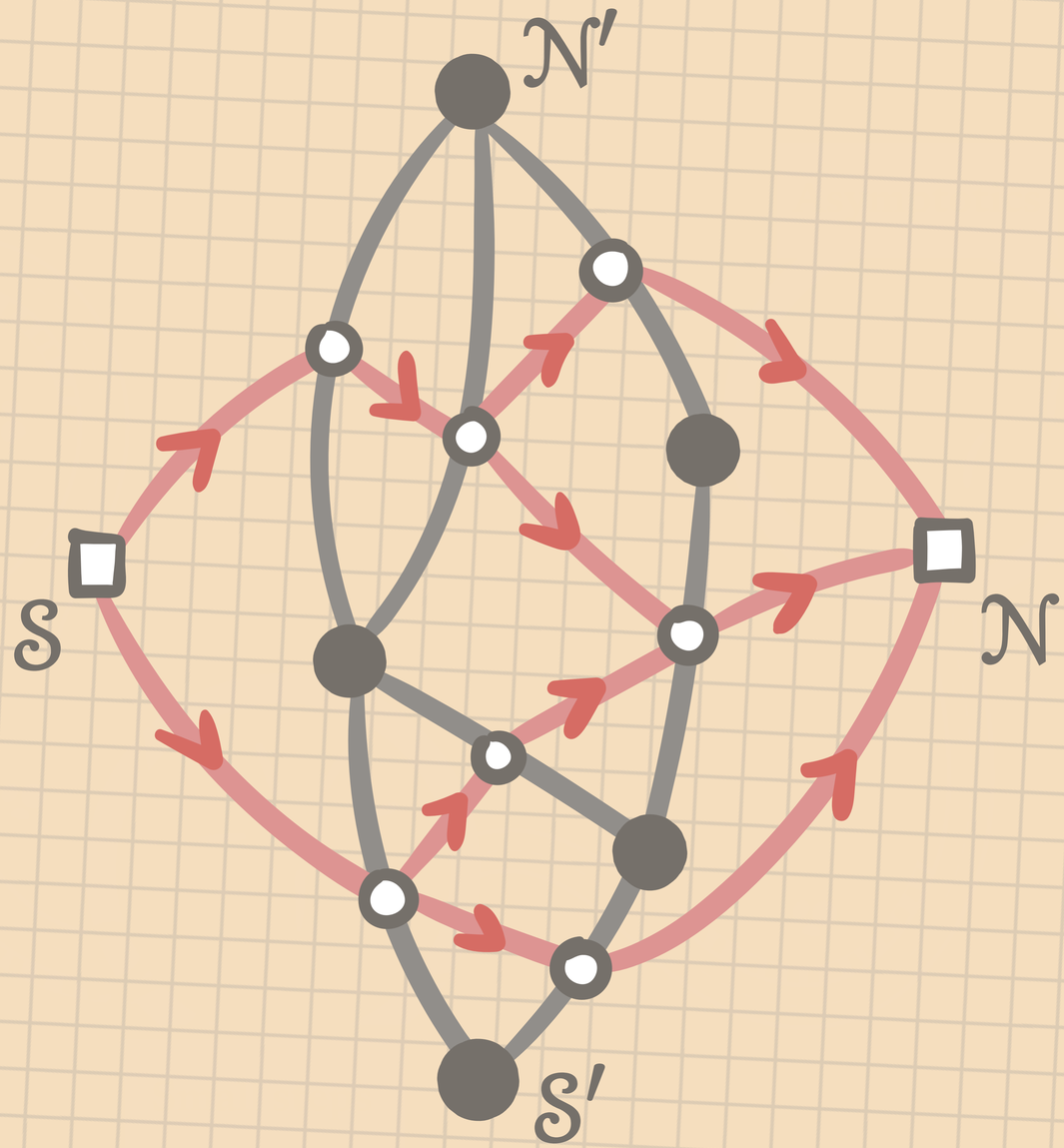

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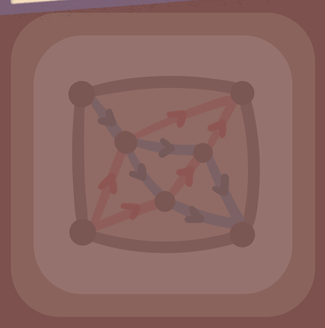

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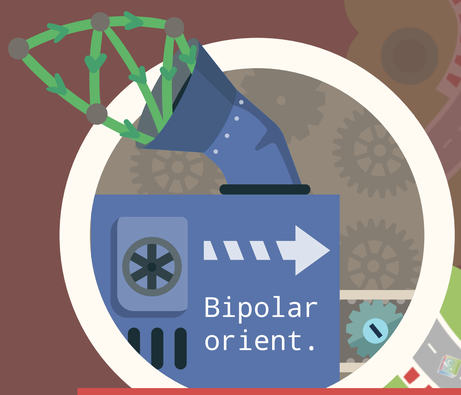

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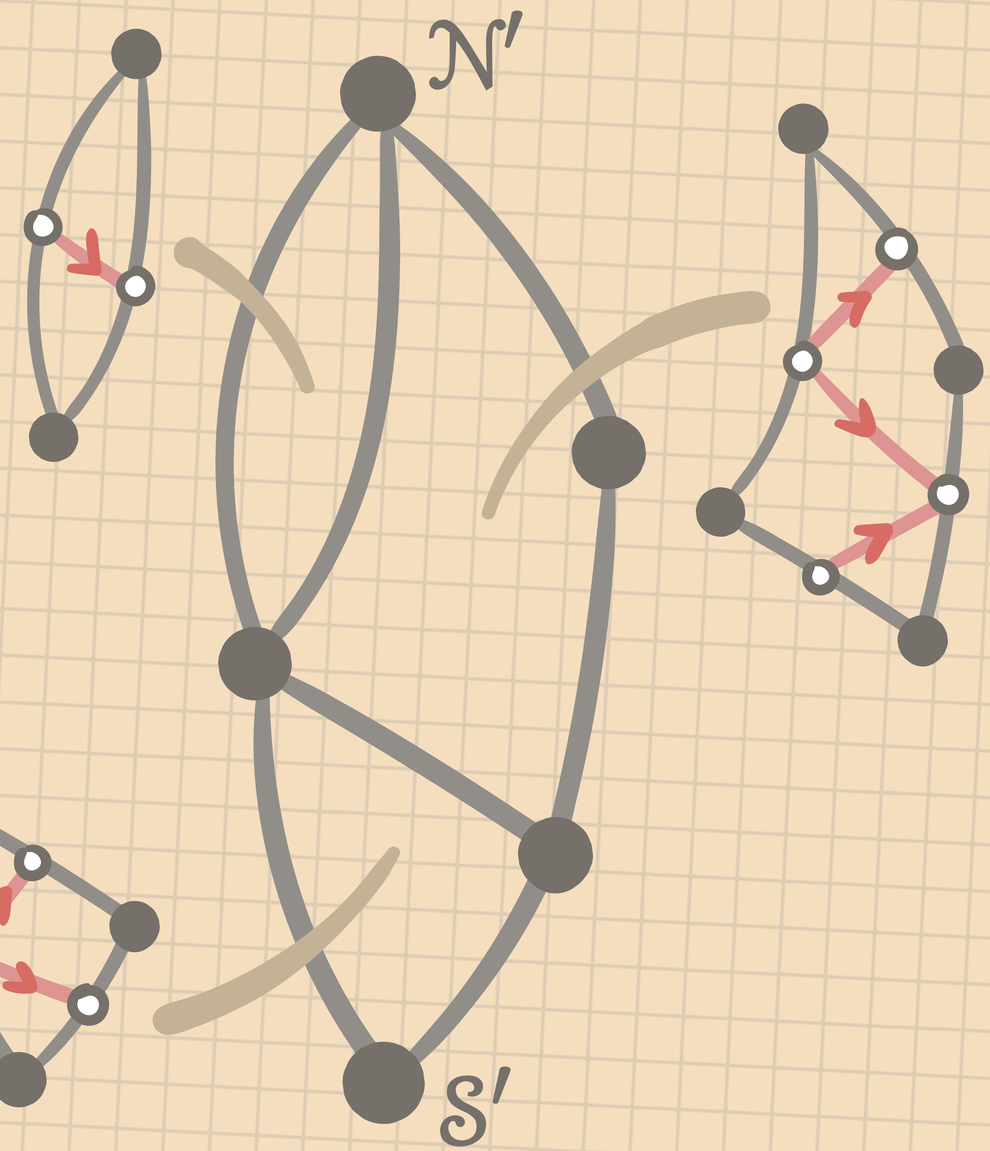

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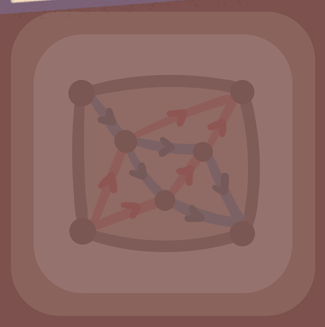

transversal structures

polyhedral orientations

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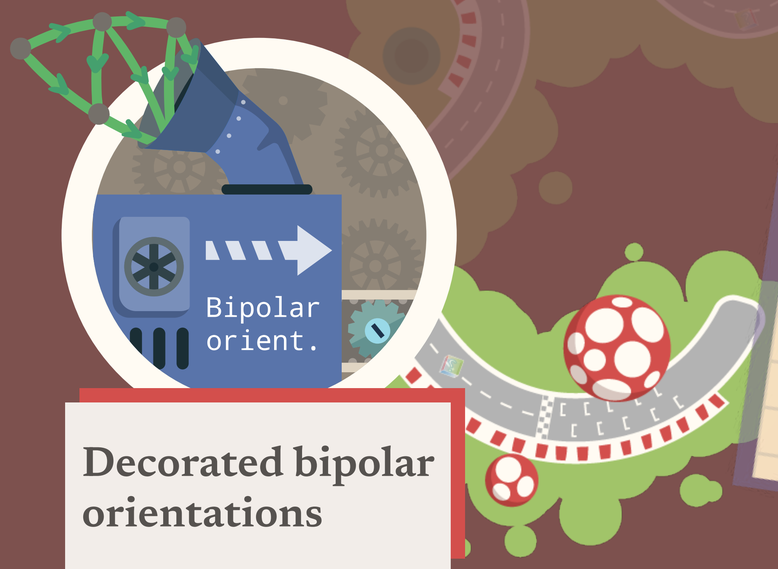
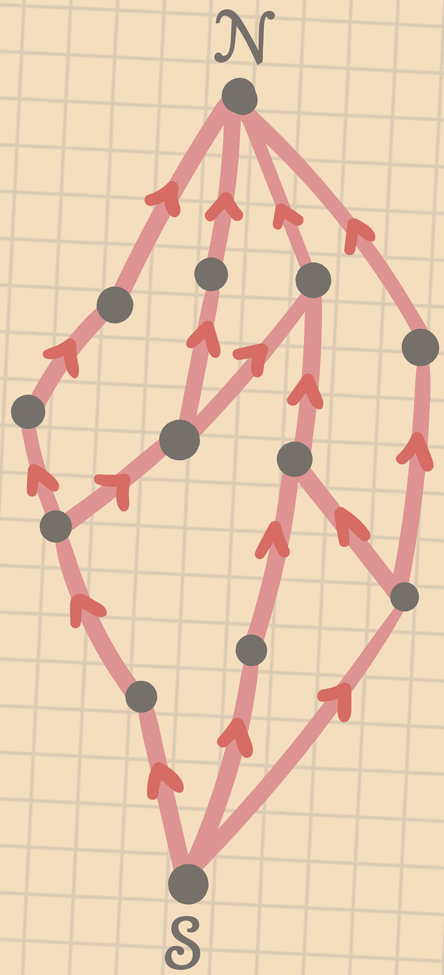


Decorated bipolar orientations



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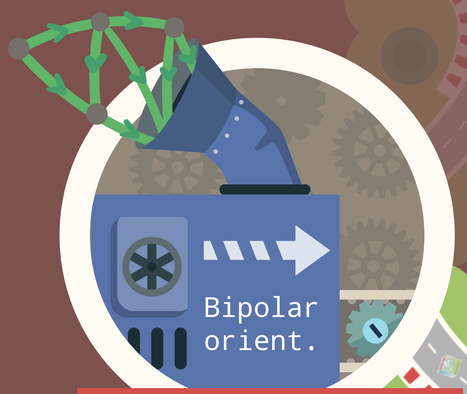

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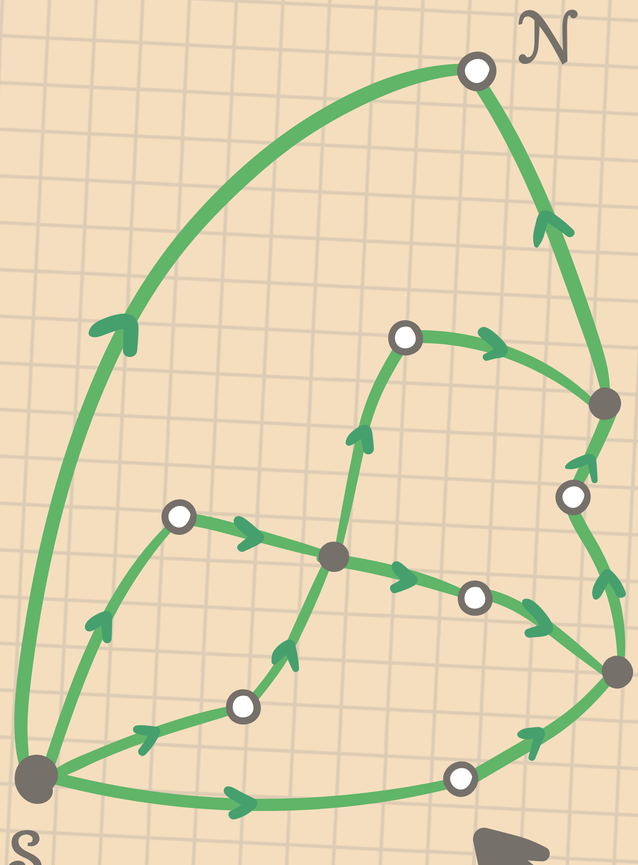

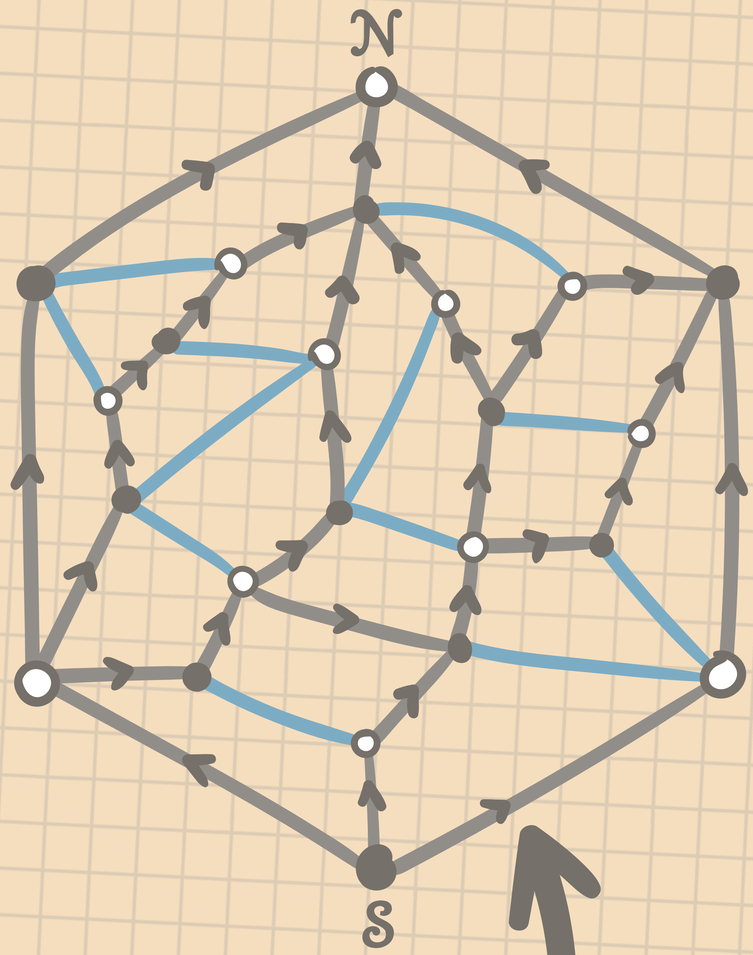

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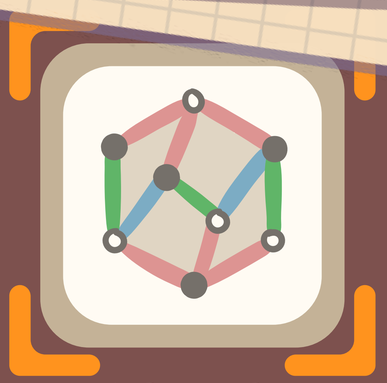
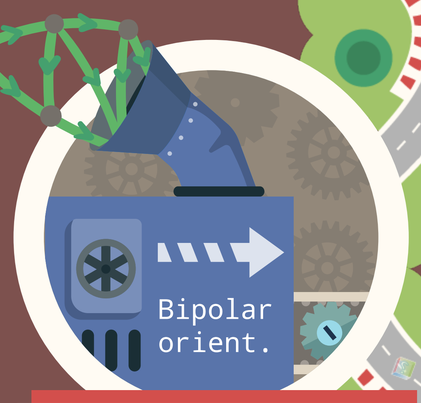
polyhedral orientations



Schnyder labelings

Summary

Quadrant tandem walks



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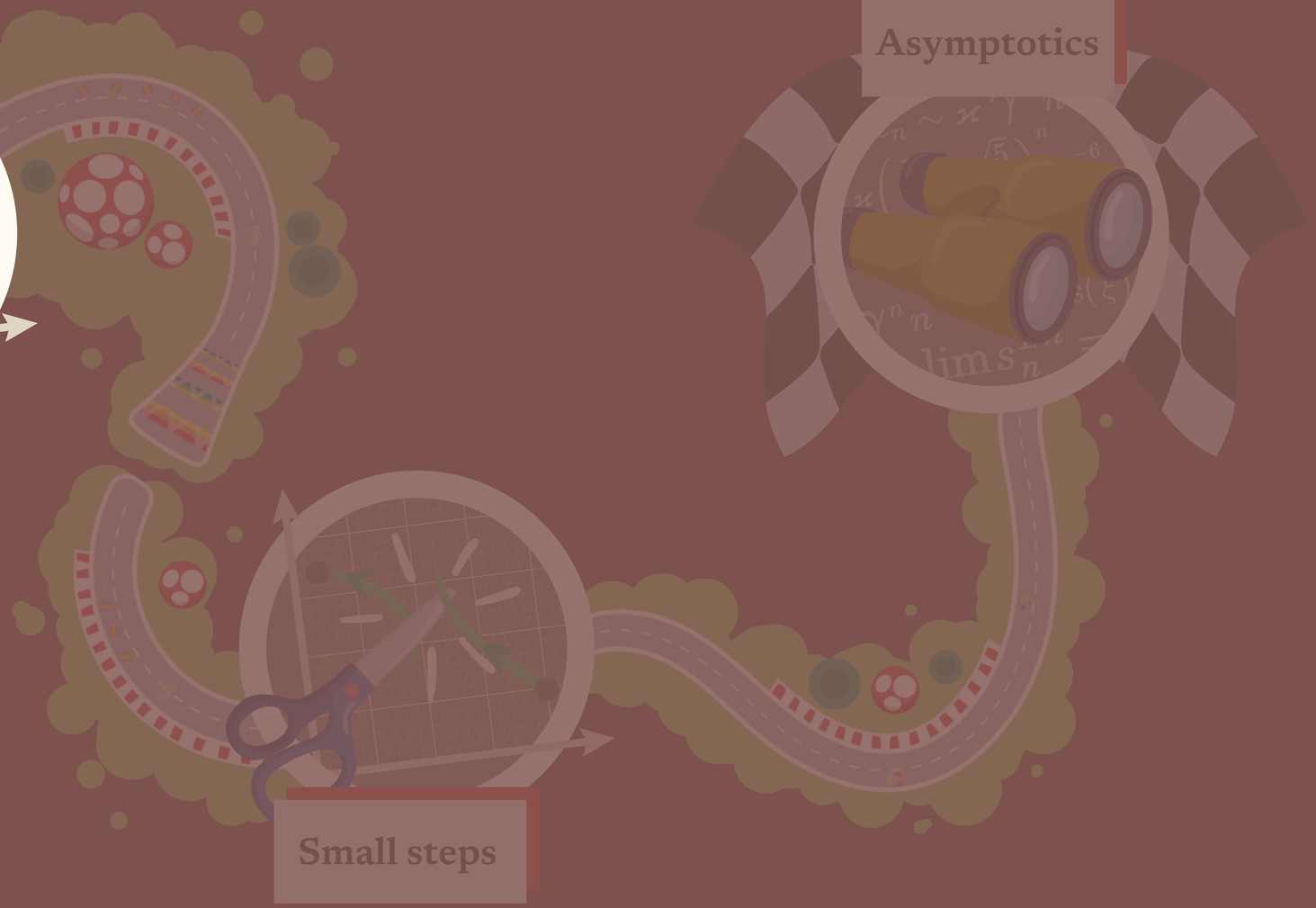


Small steps

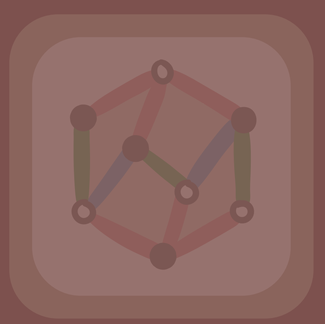
Asymptotics

bipolar posets by vertices

bipolar posets by edges

transversal structures

polyhedral orientations



Schnyder labelings

Summary

Decorated bipolar orientations

Quadrant tandem walks

Asymptotics

Small steps

a. The KMSW bijection

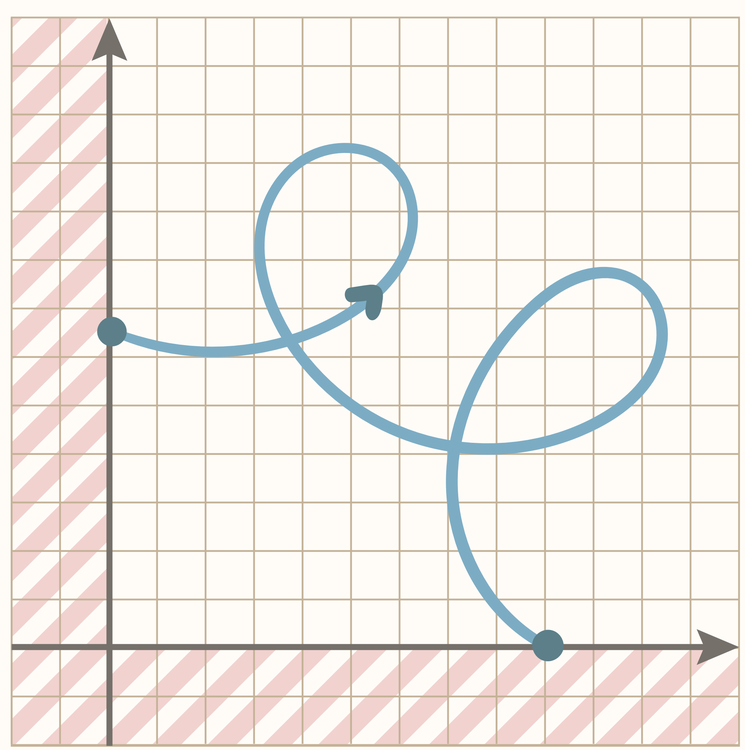
b. Application to transversal structures

The KMSW bijection

Plane bipolar orientations ⟷ *tandems walks in the quarter plane*

The KMSW bijection

Plane bipolar orientations ⟷ *tandems walks in the quarter plane*



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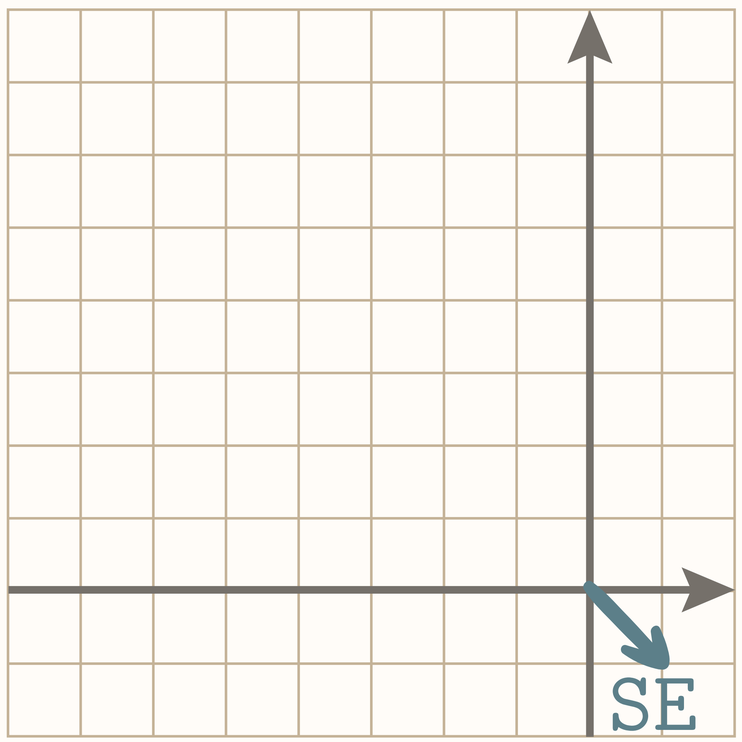

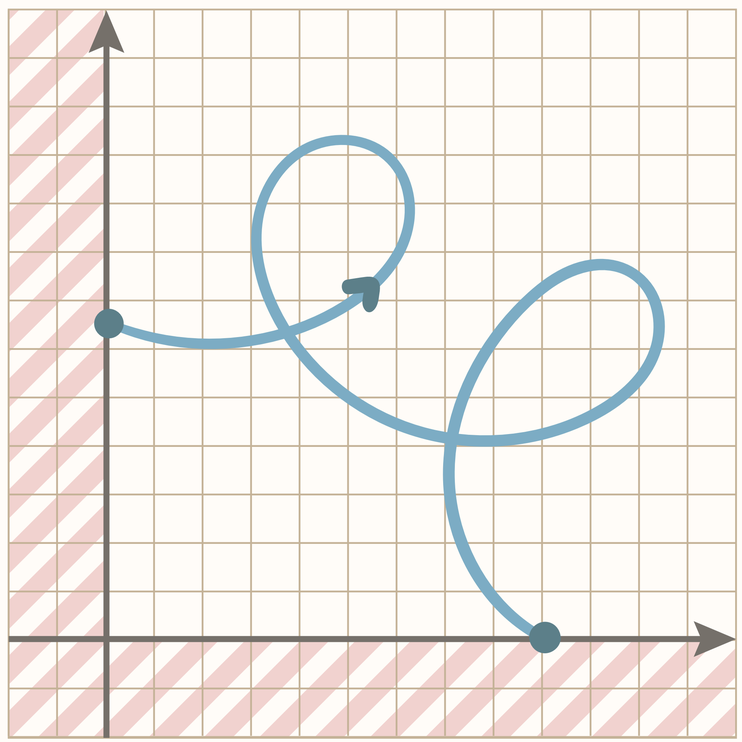

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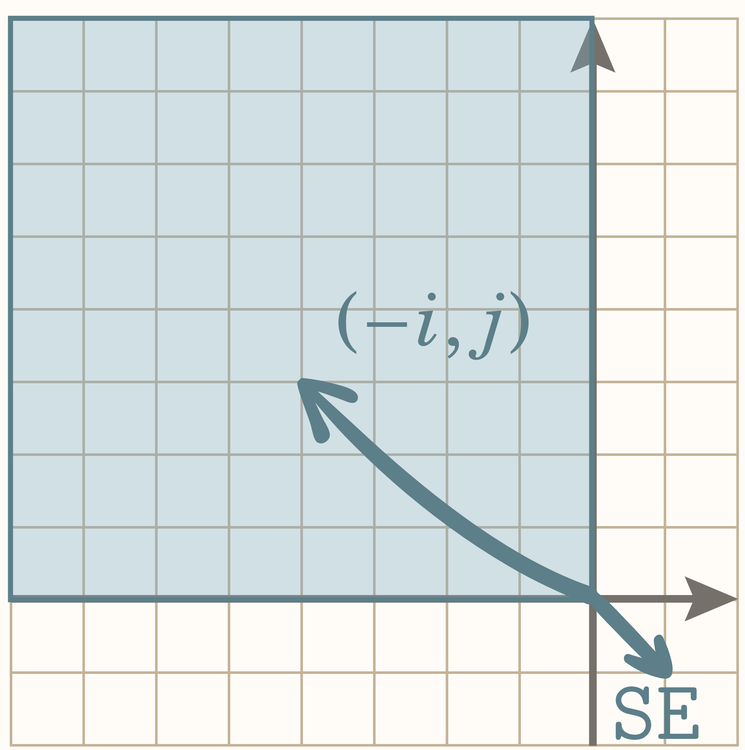

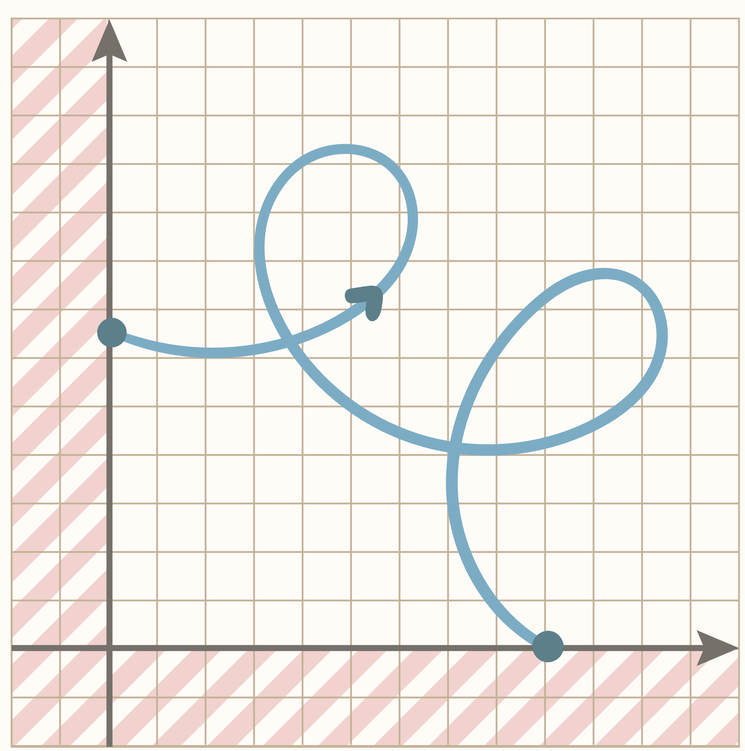

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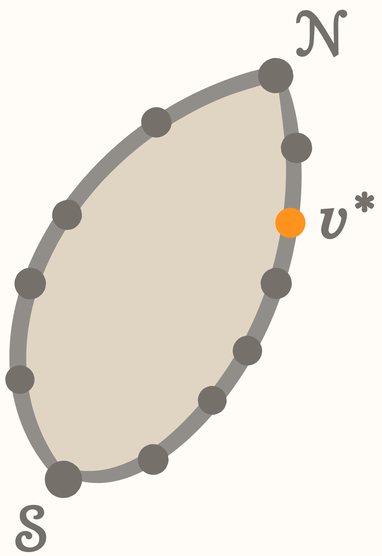

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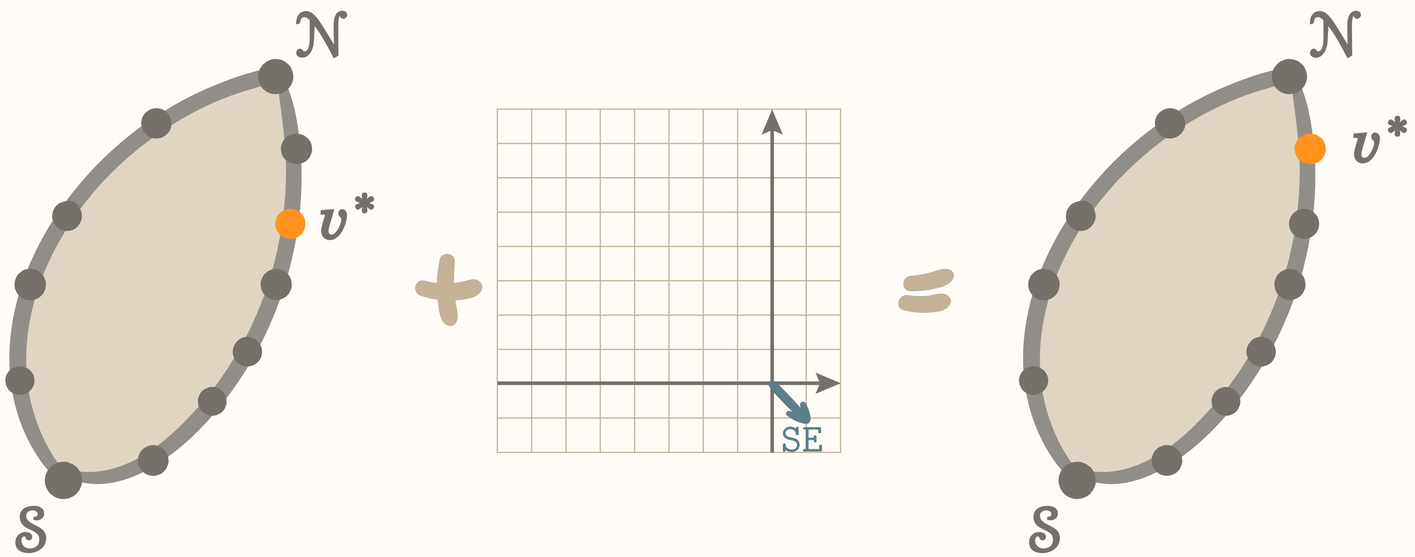

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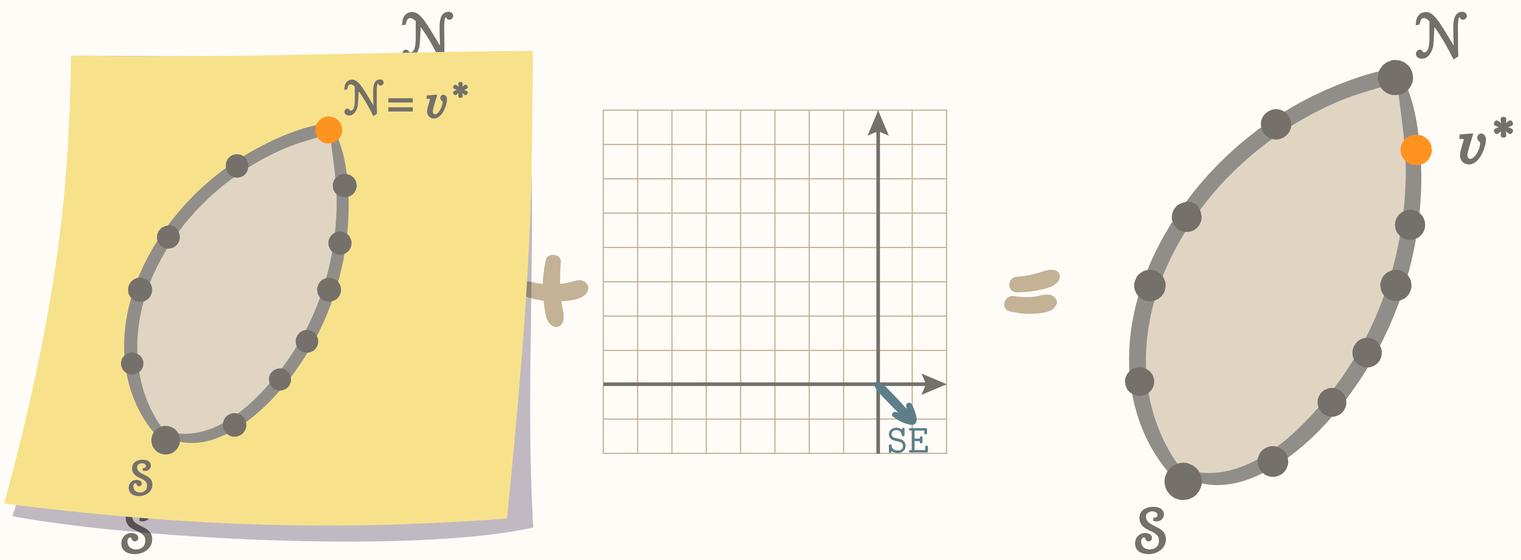

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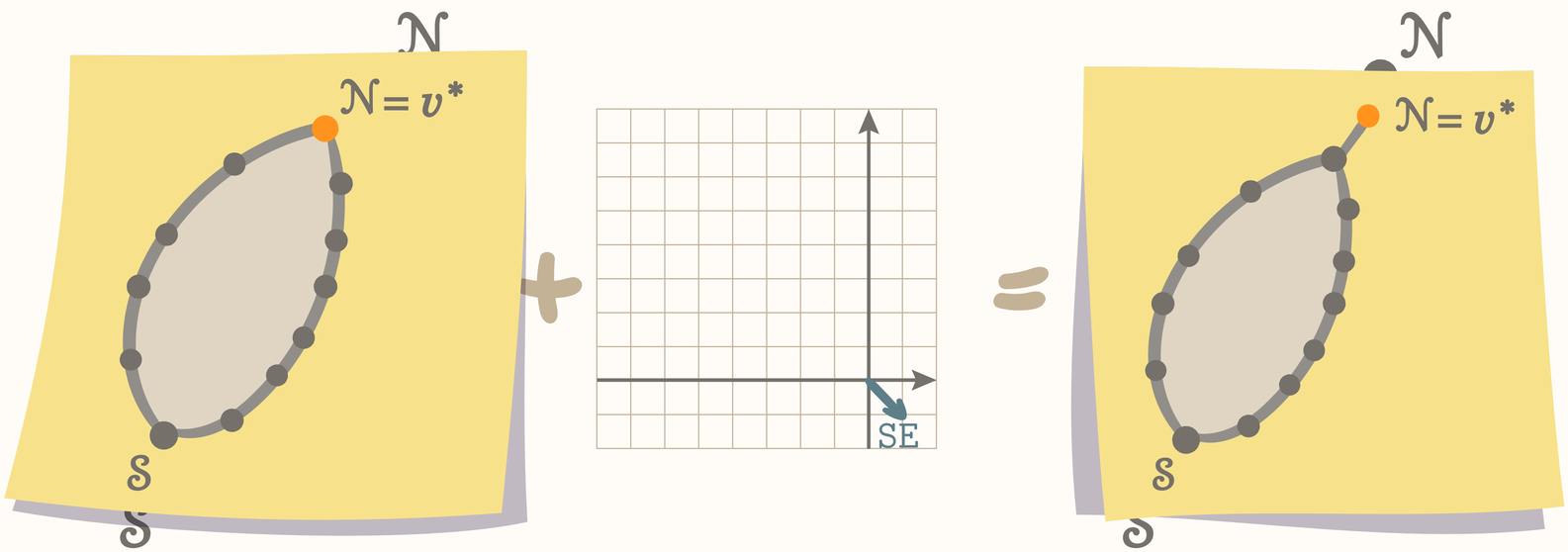

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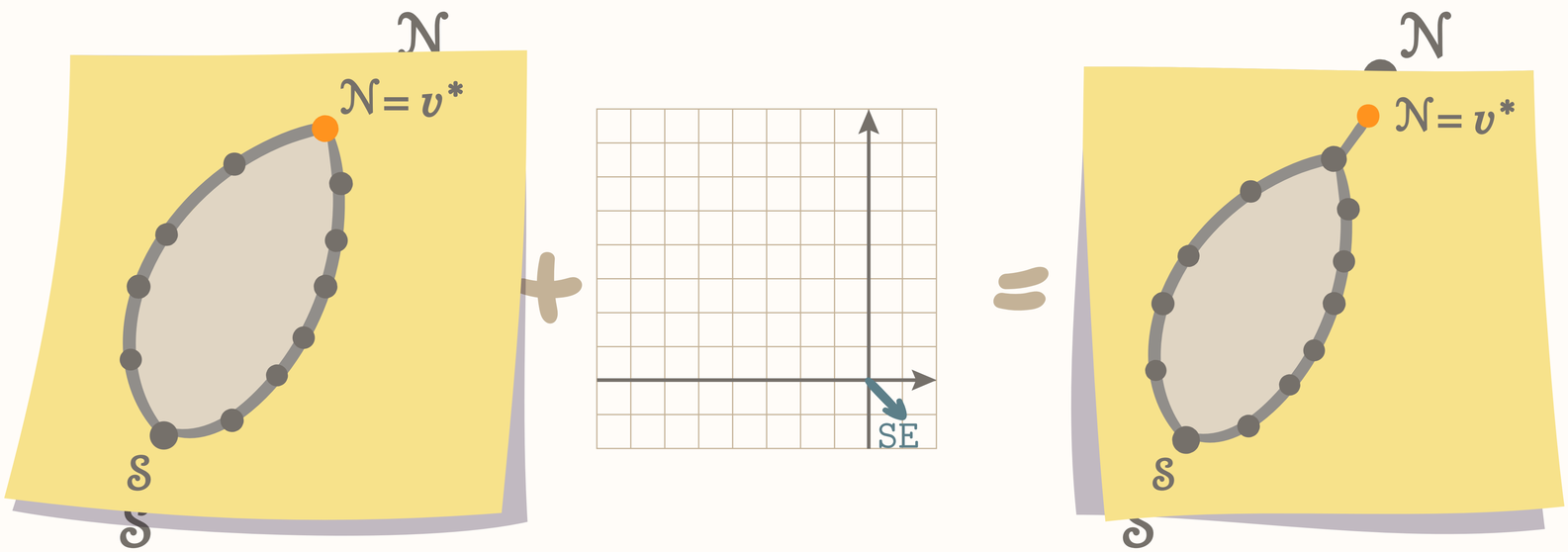

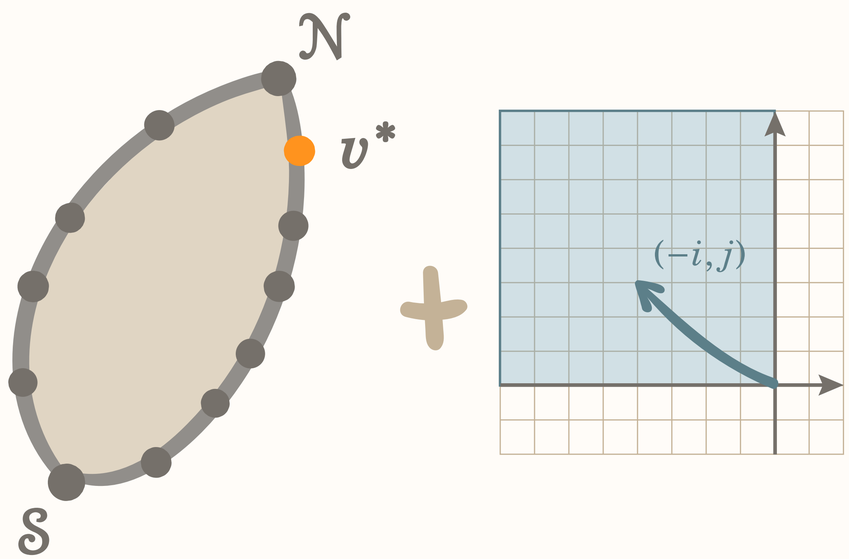

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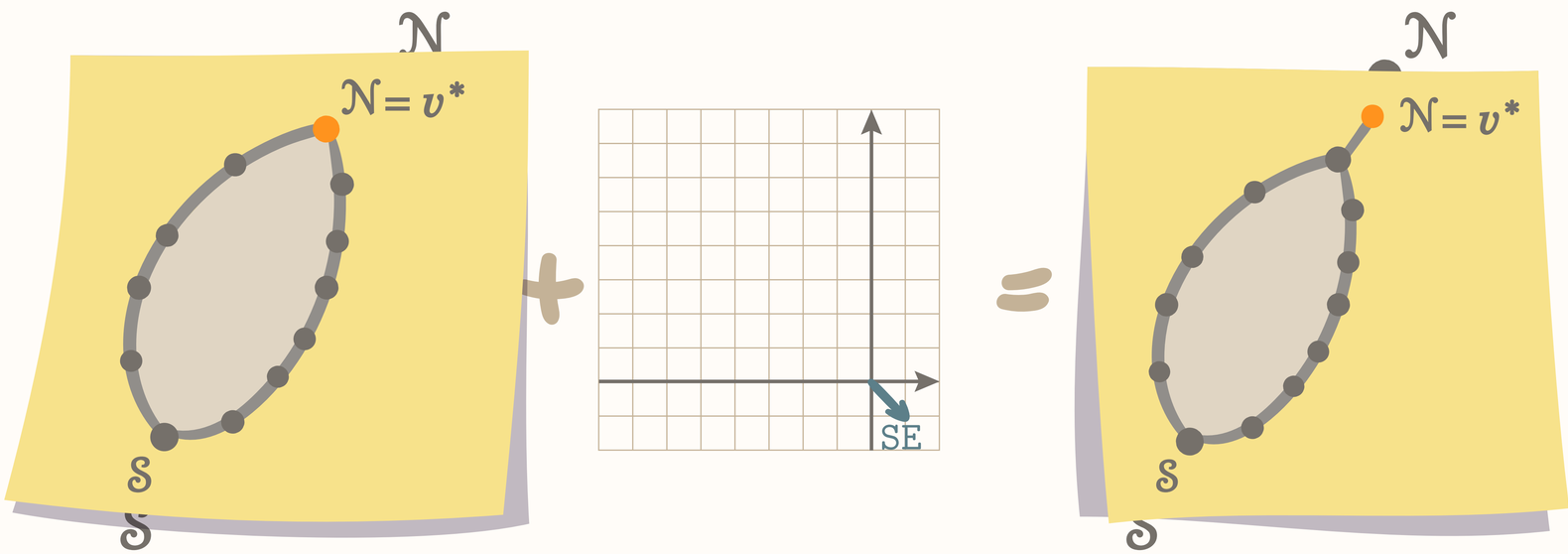

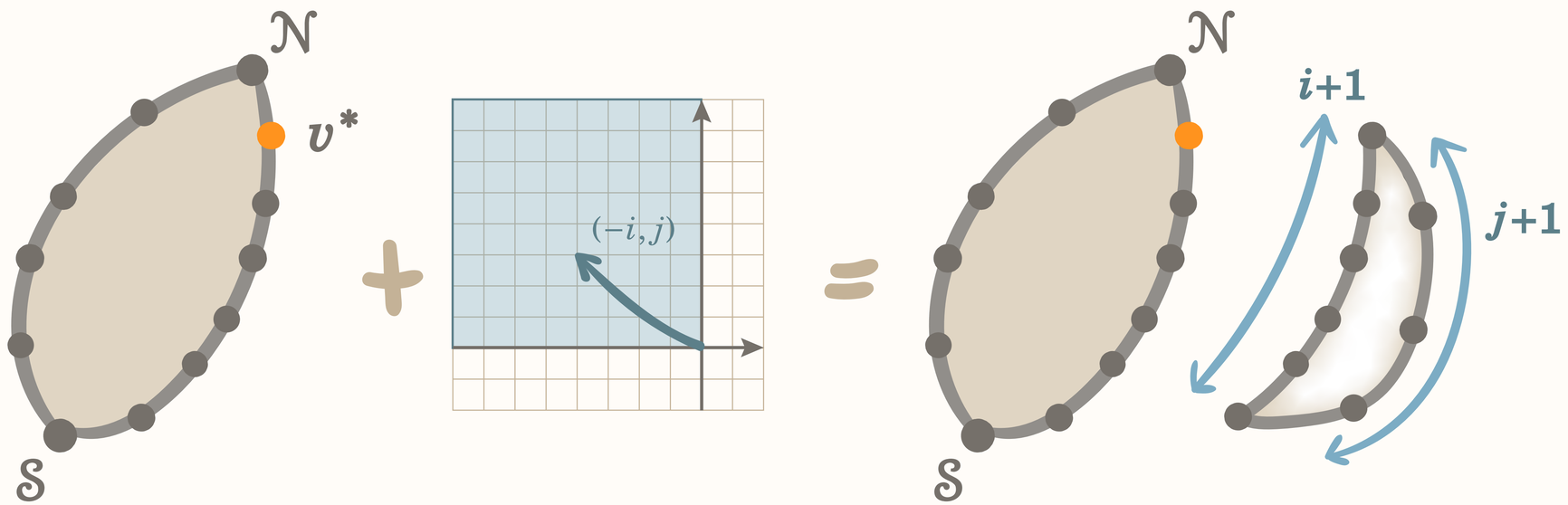

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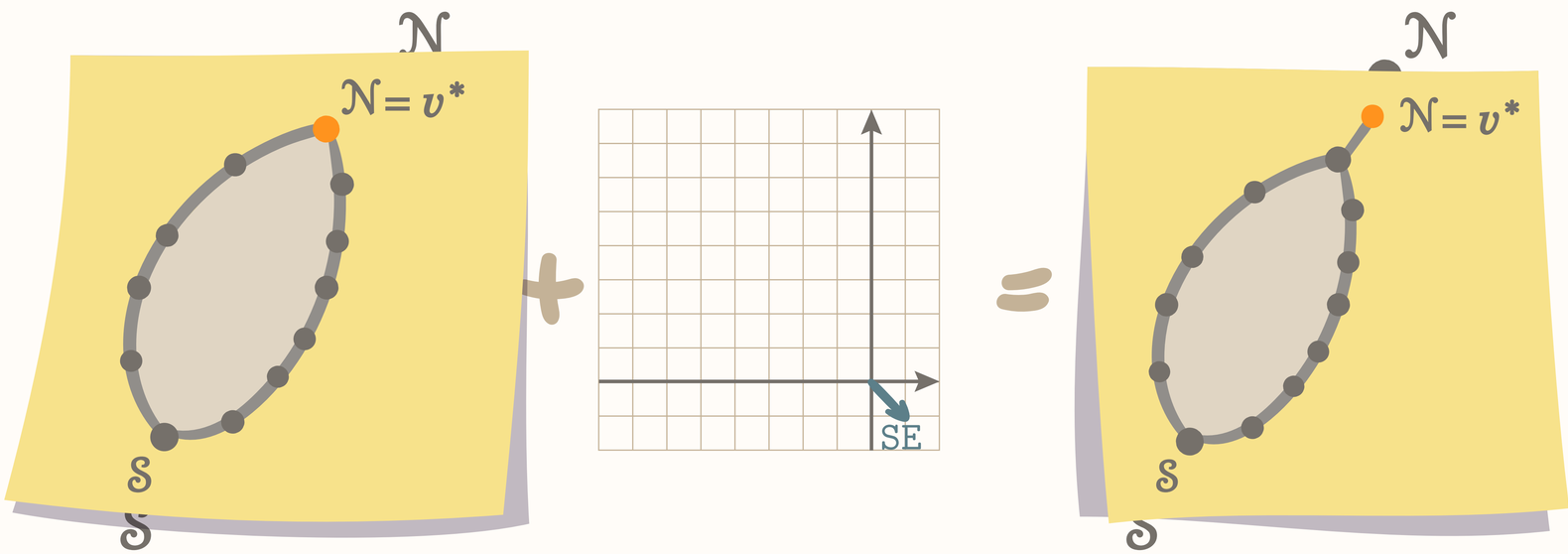

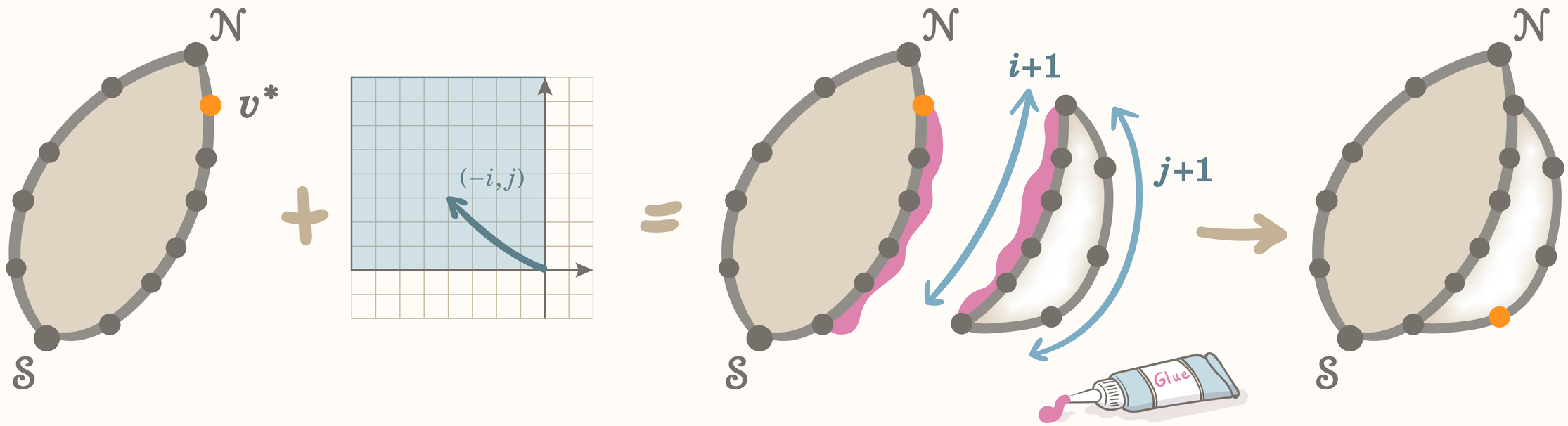

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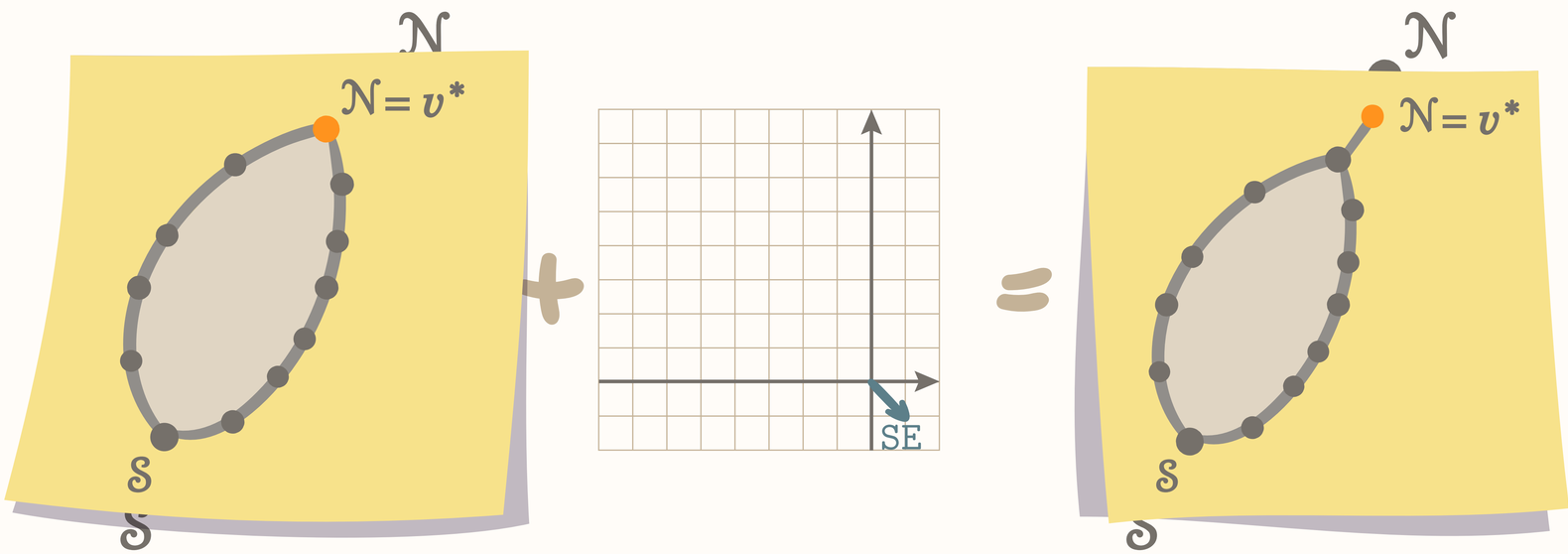
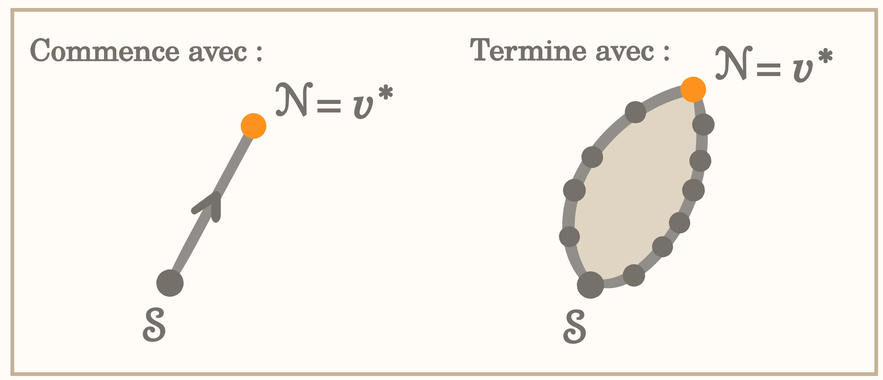
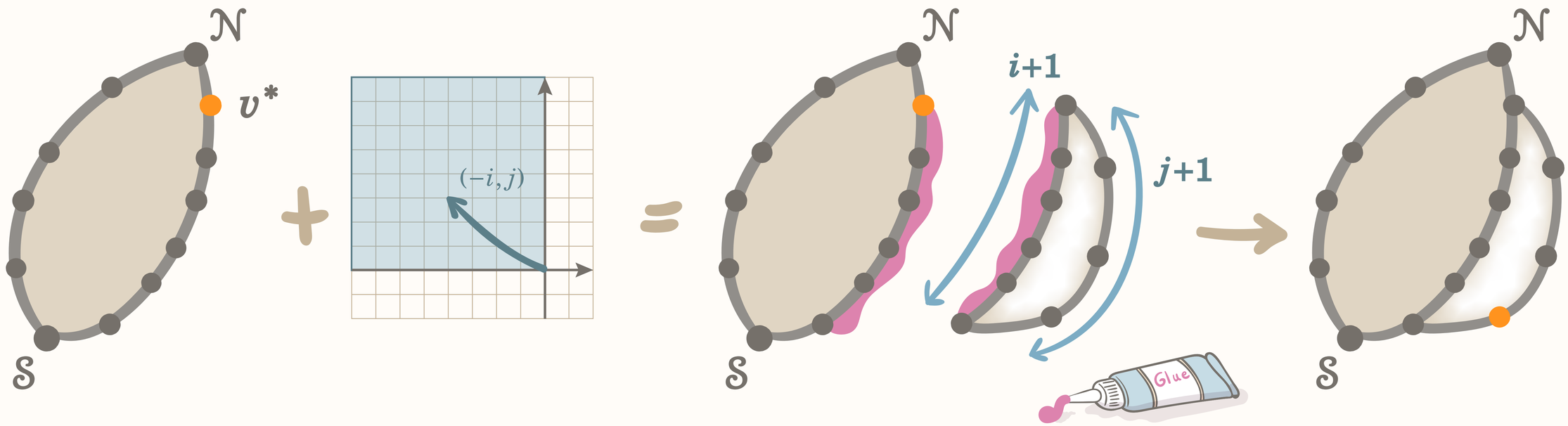
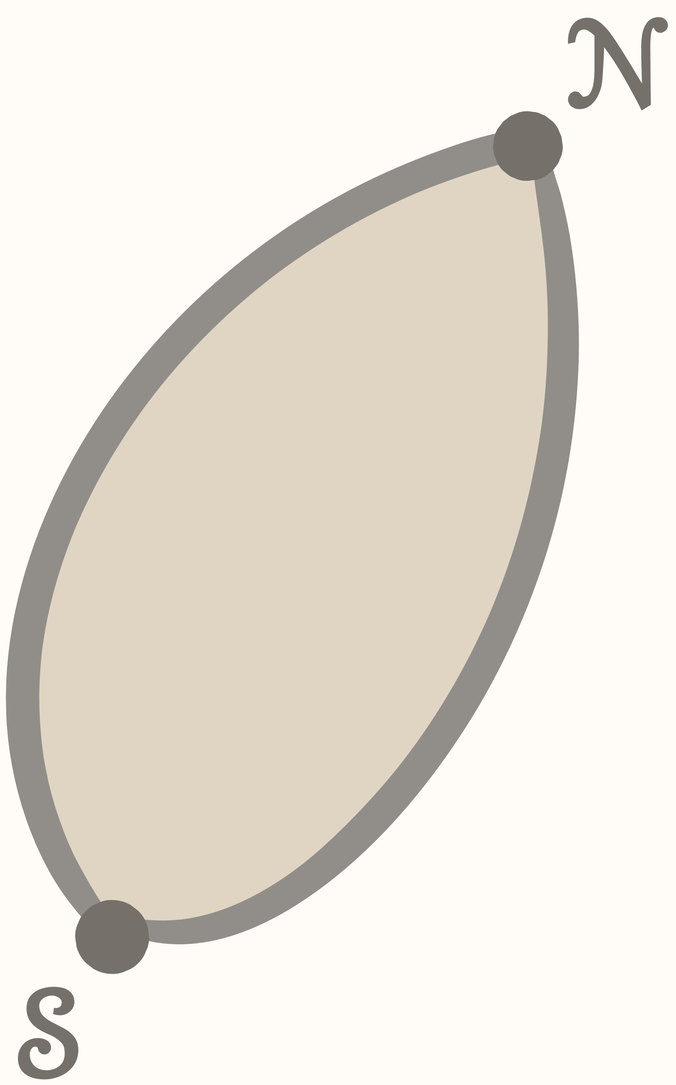


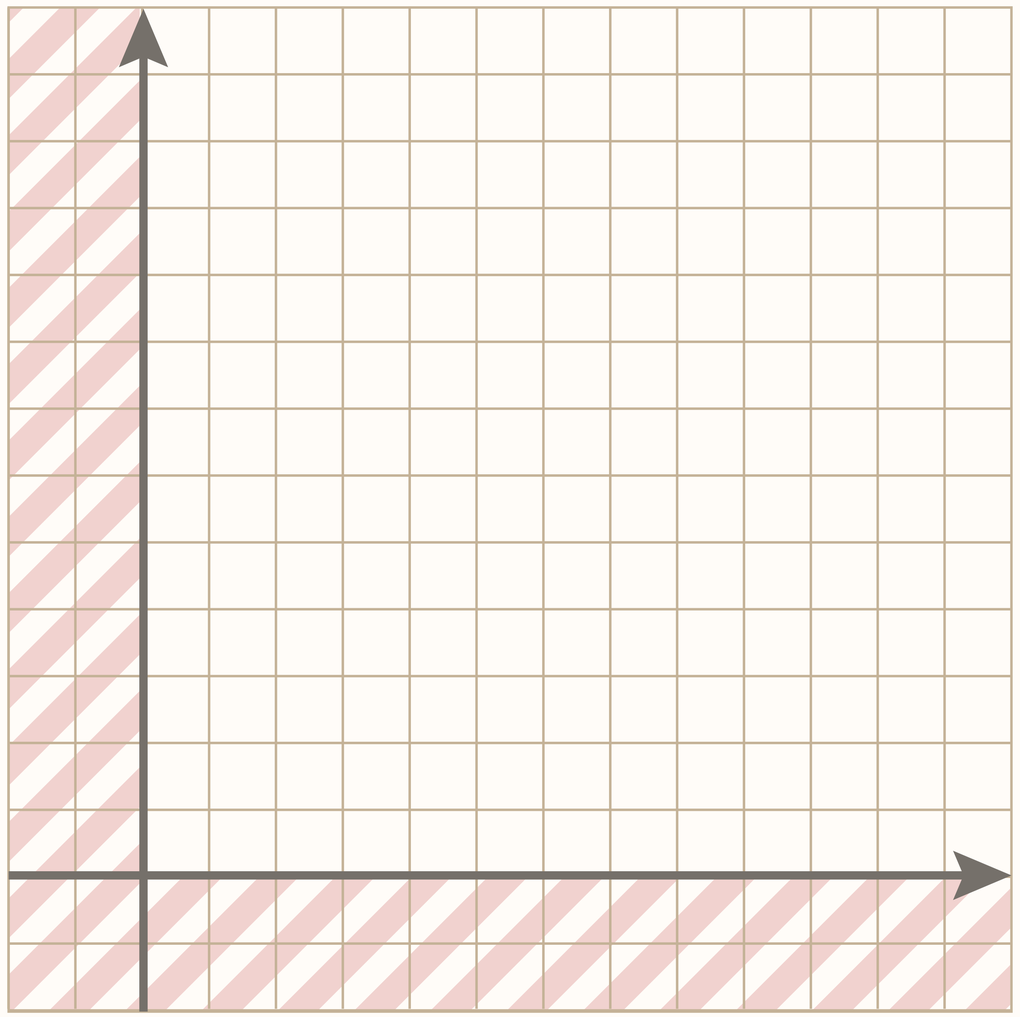




The KMSW bijection

Plane bipolar orientations ⟷ *tandems walks in the quarter plane*





The KMSW bijection

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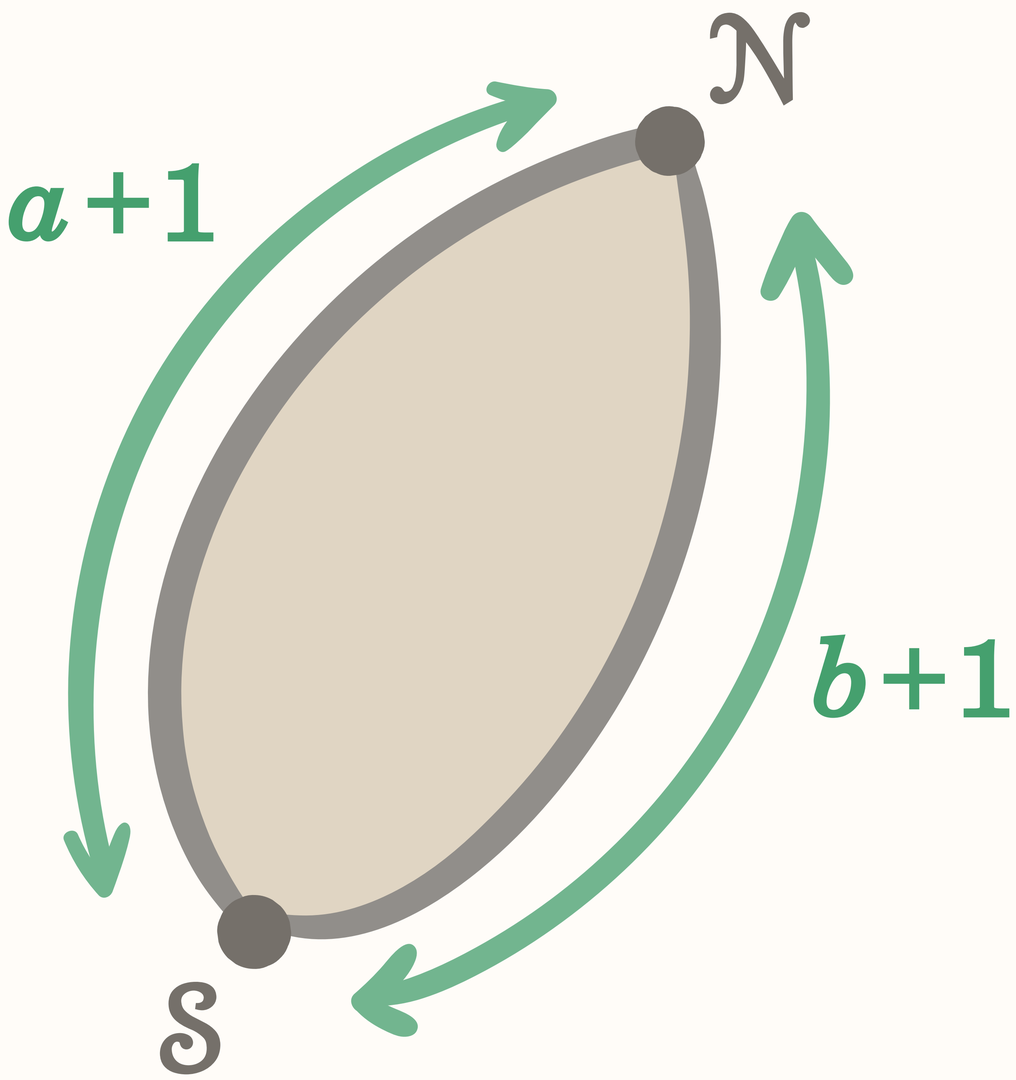

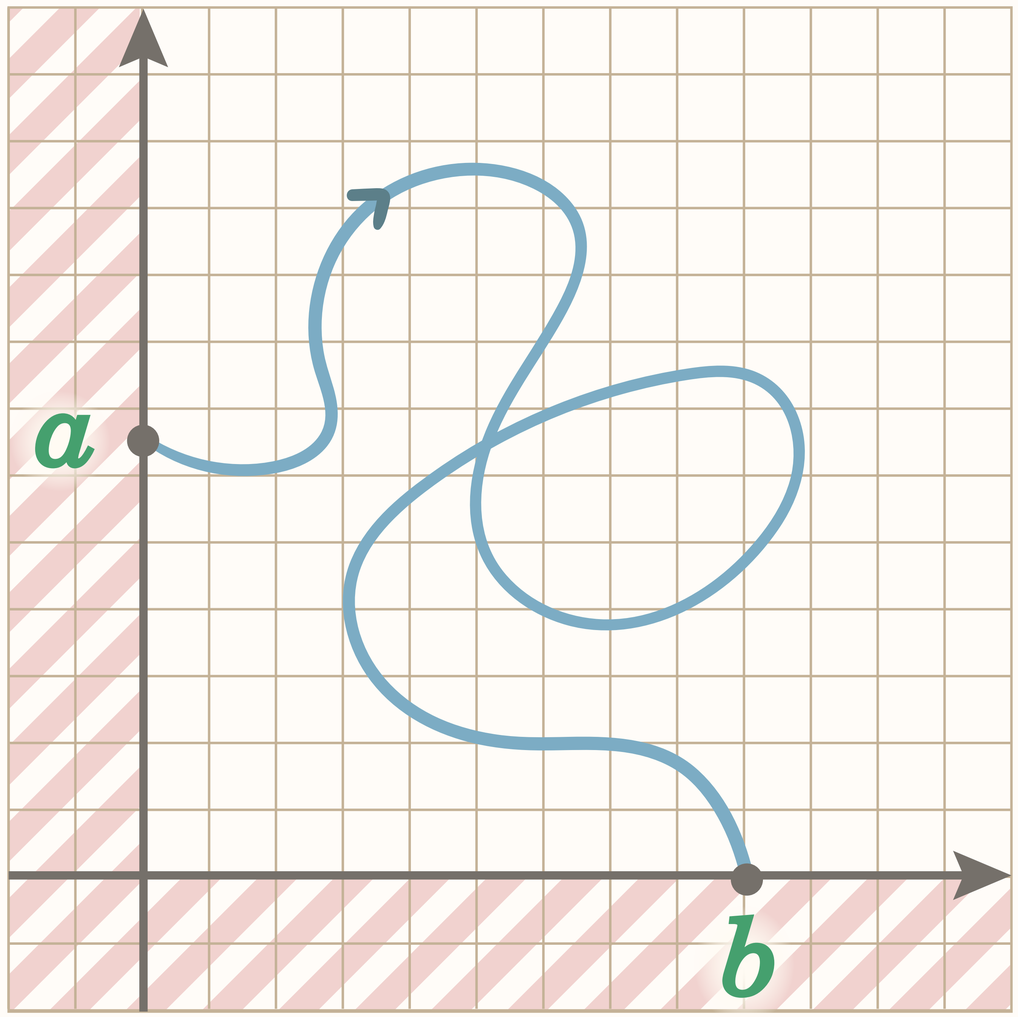

The KMSW bijection

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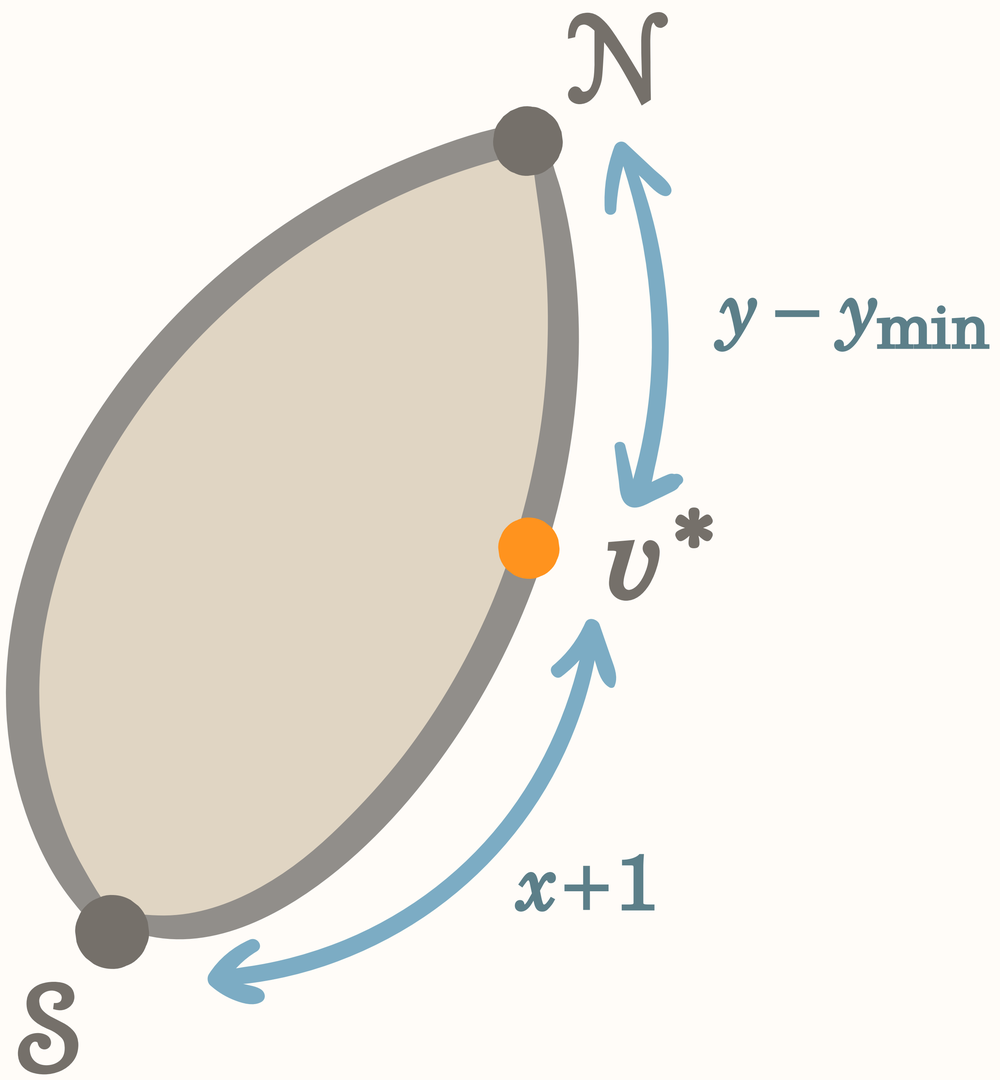
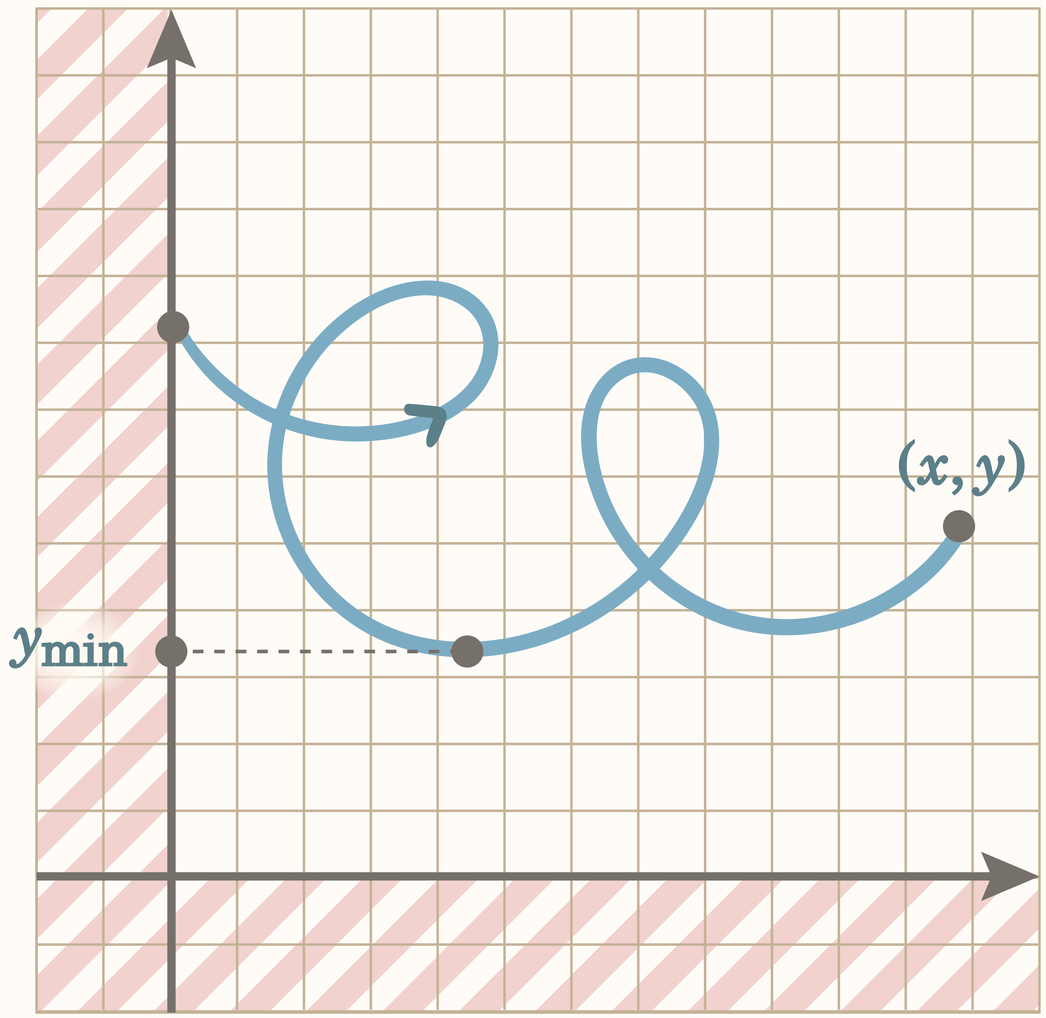
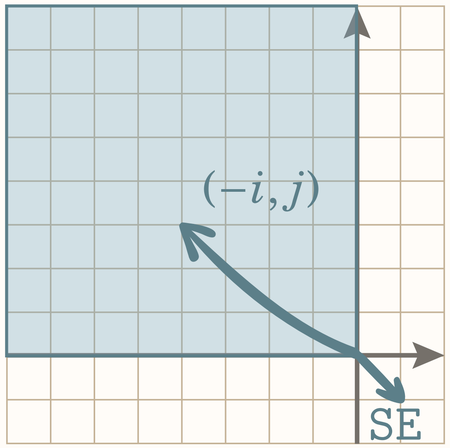
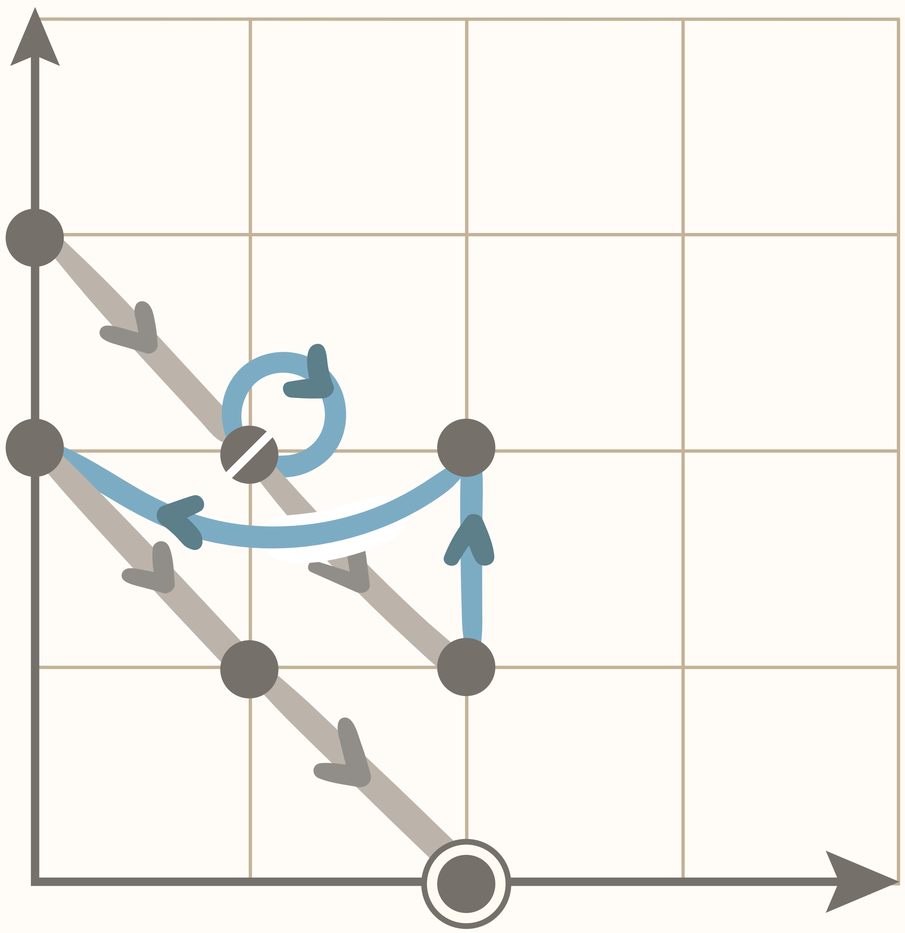
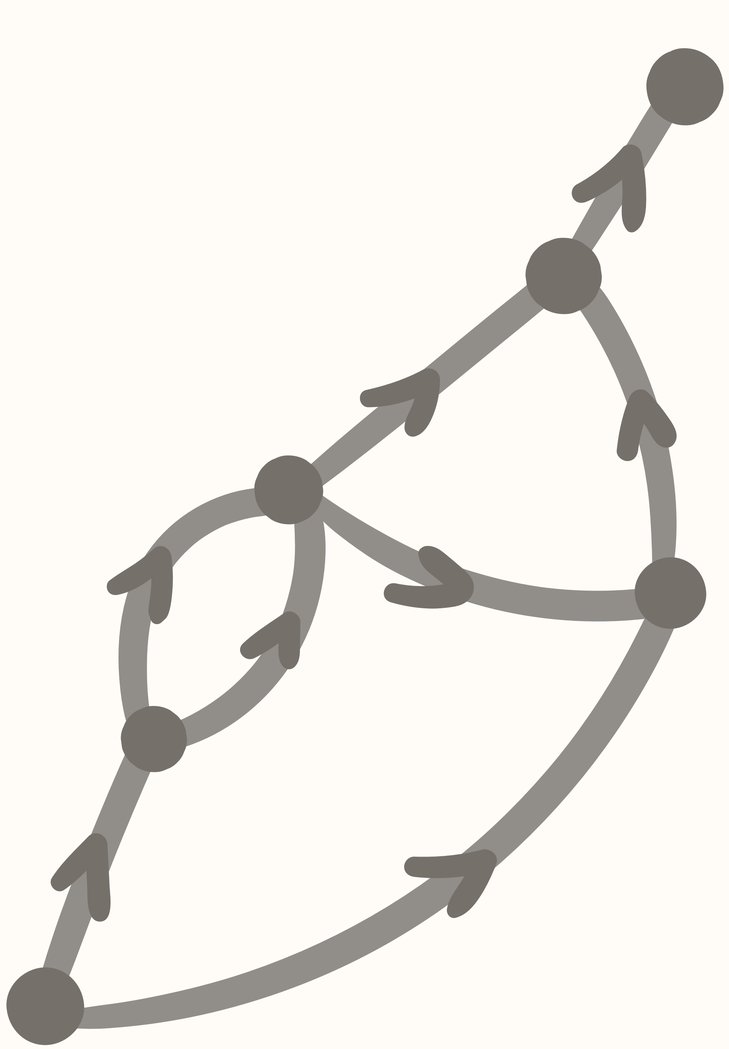




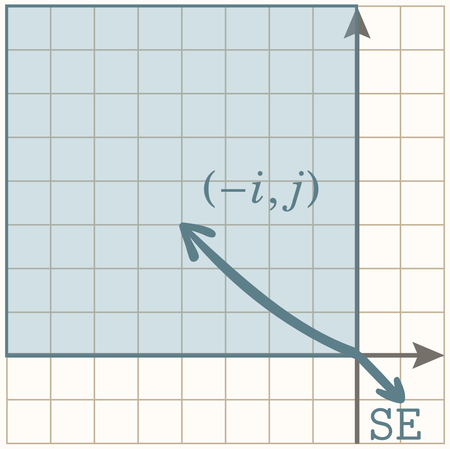
KMSW bijection example

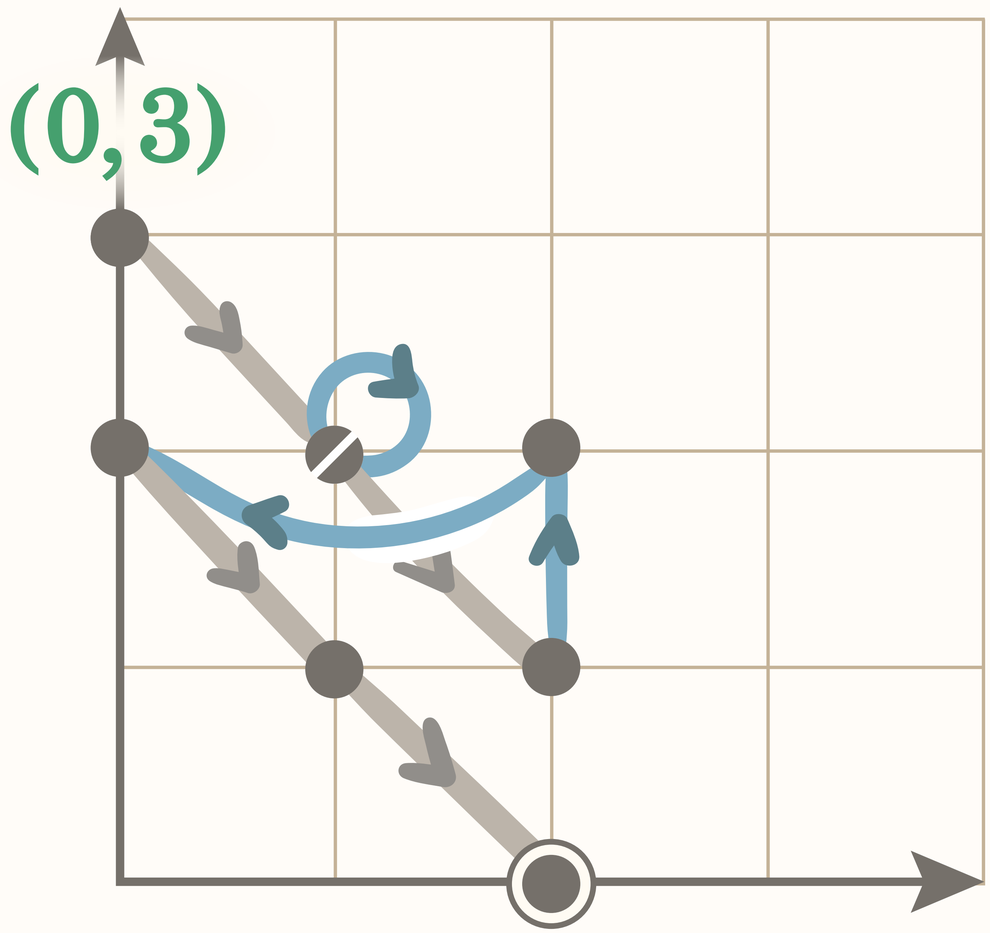


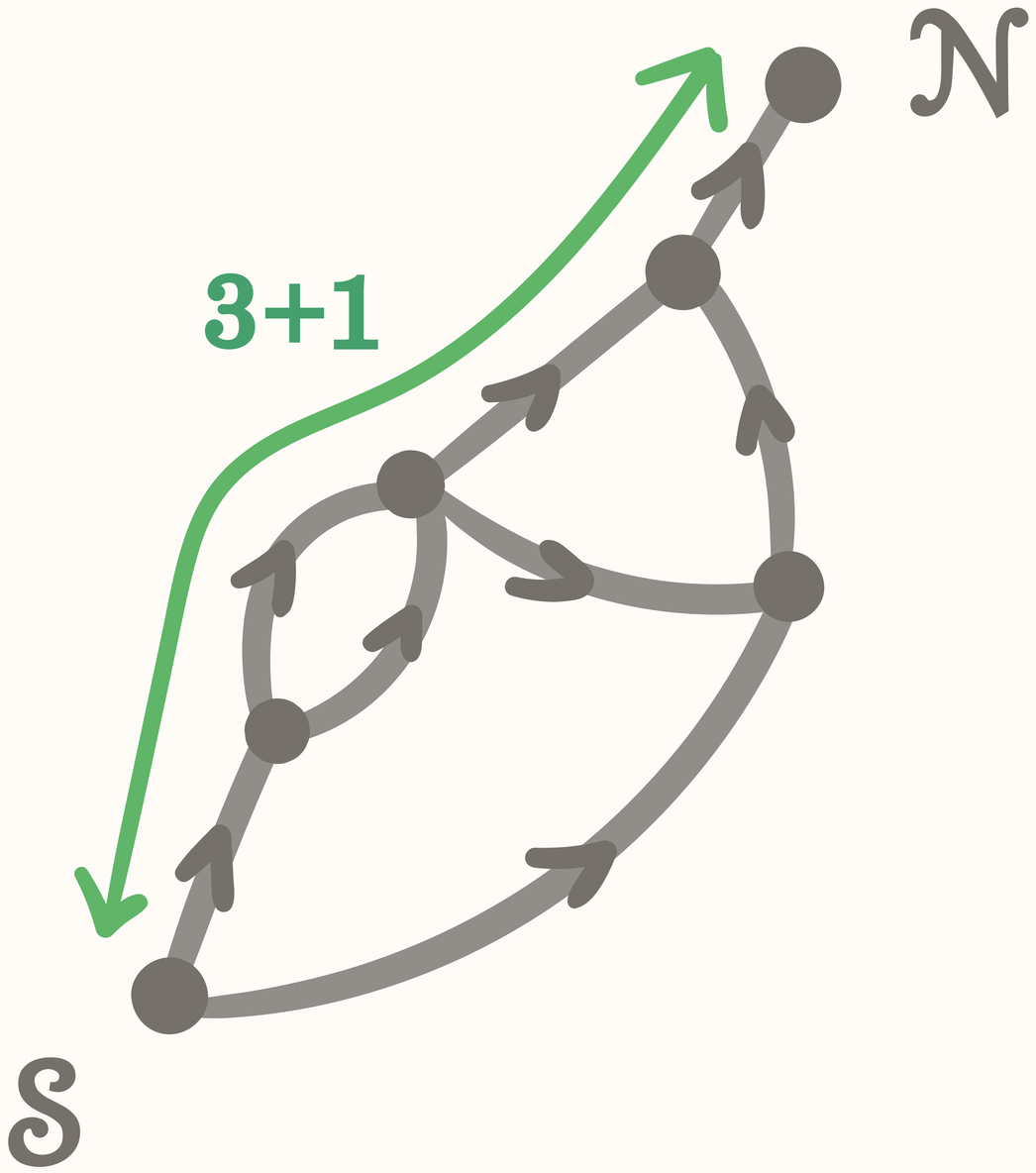




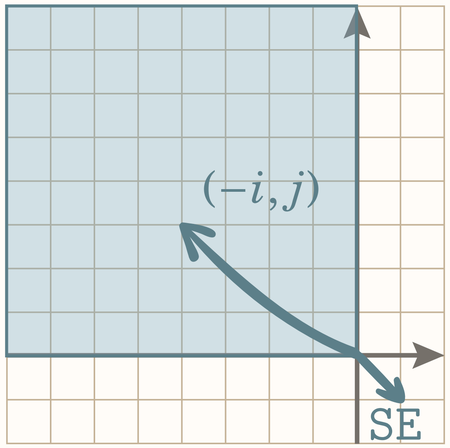
KMSW bijection example

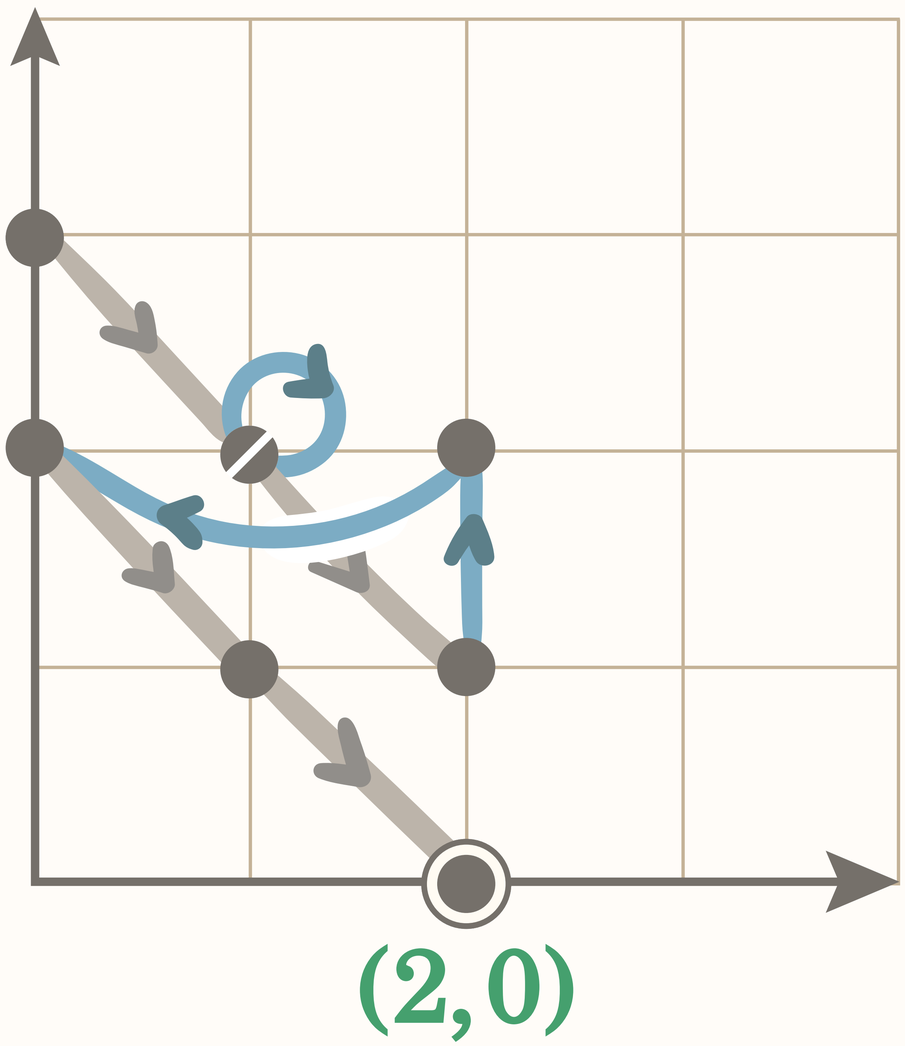


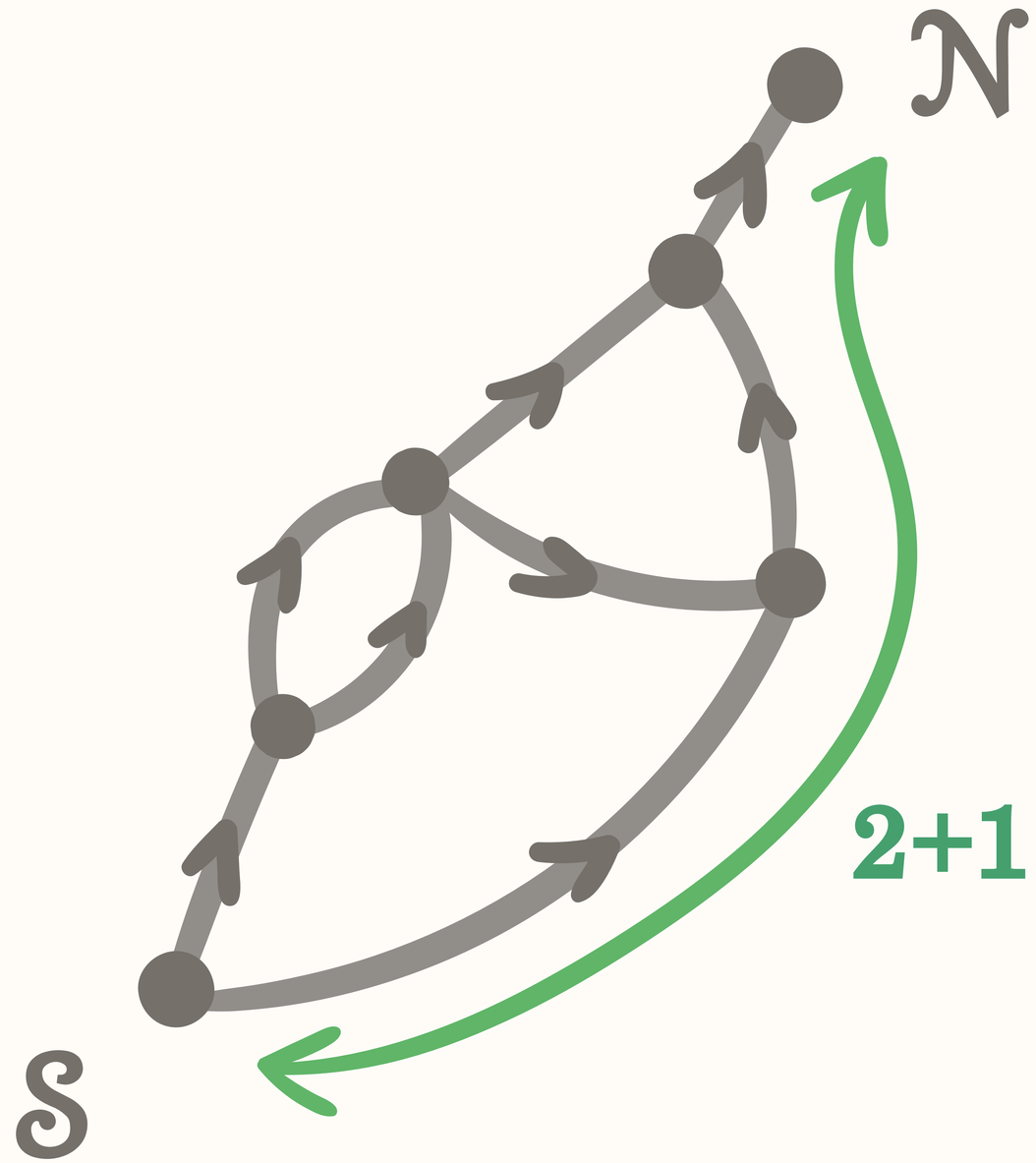




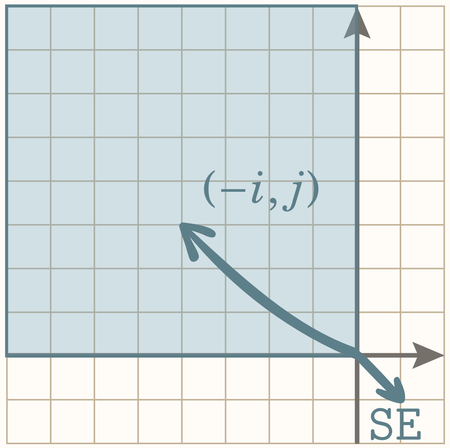
KMSW bijection example

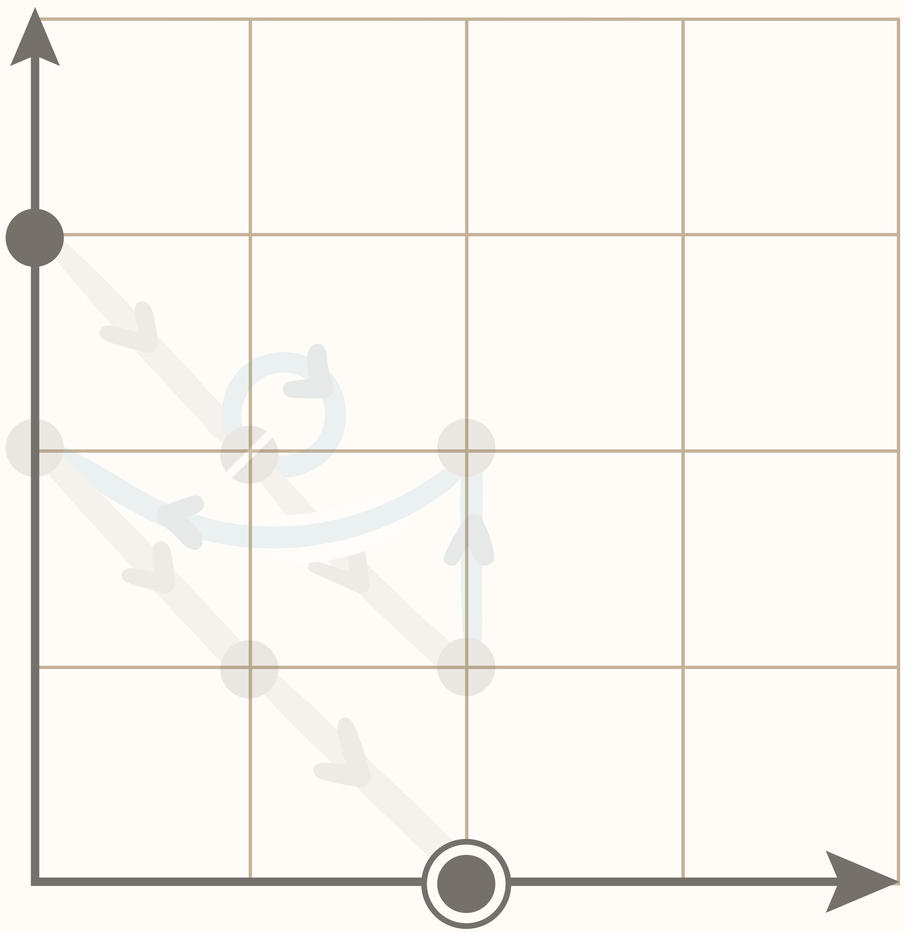


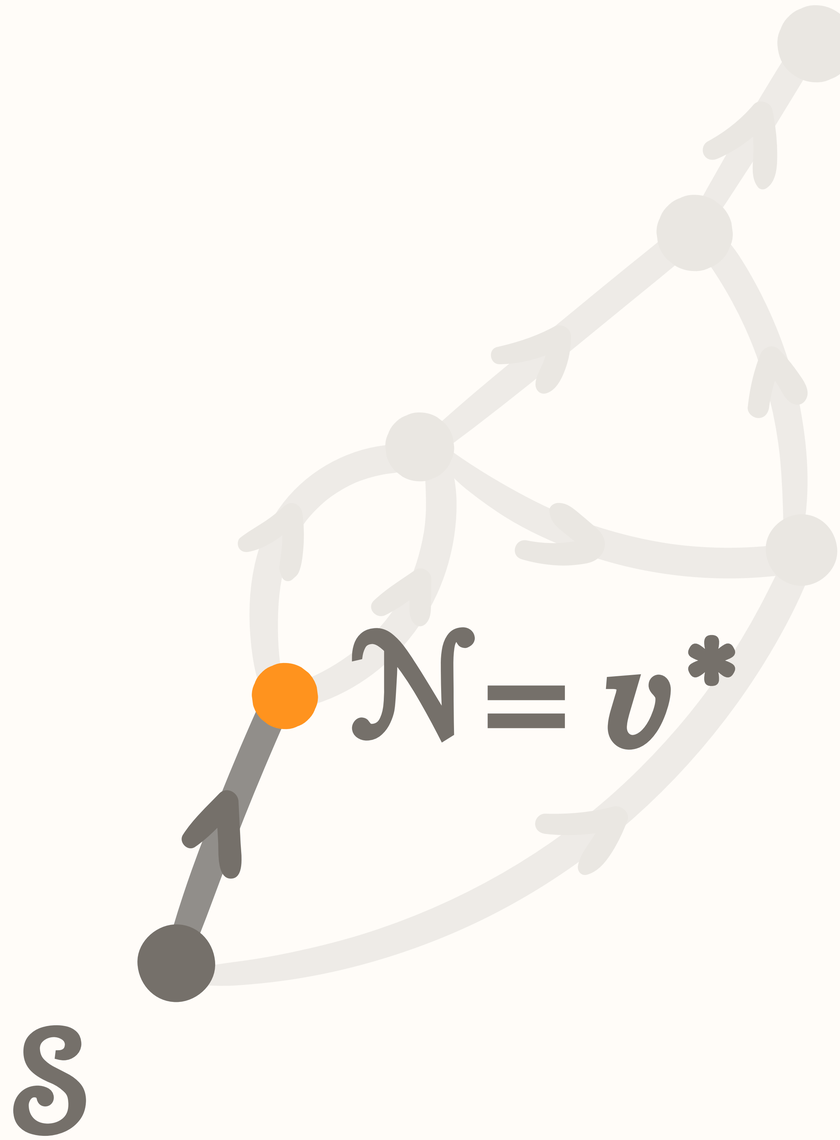




KMSW bijection example







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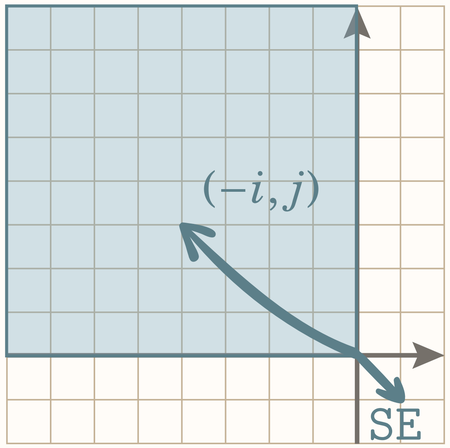

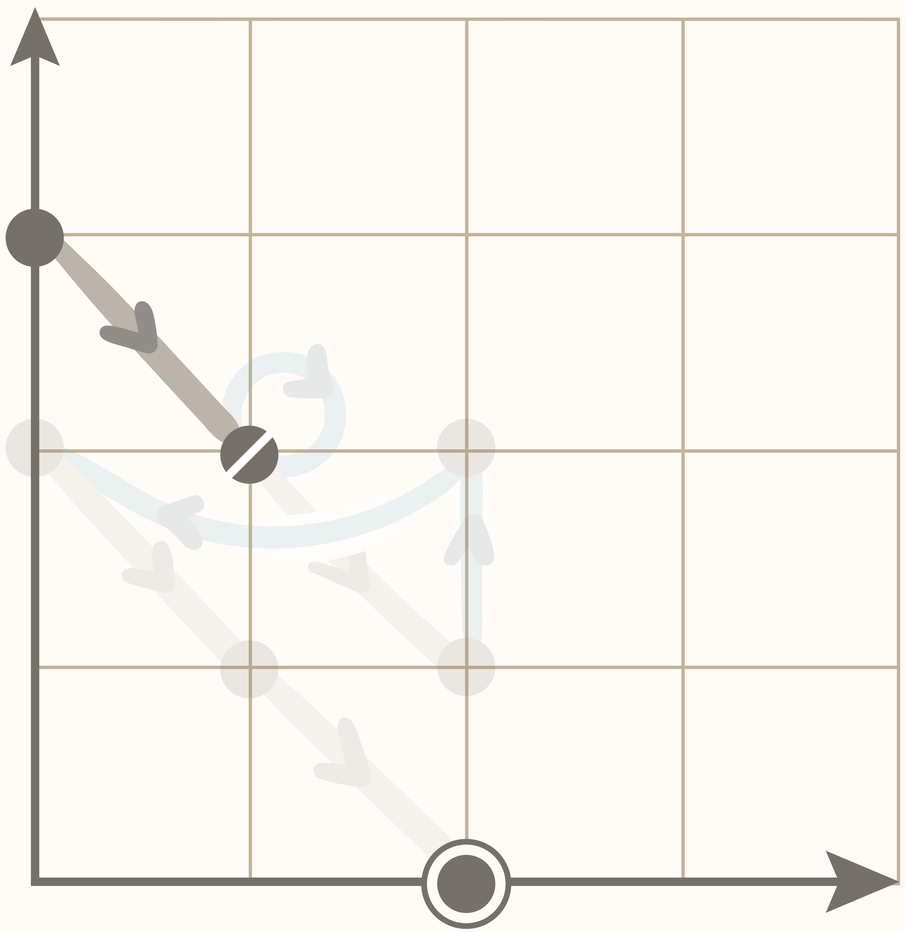

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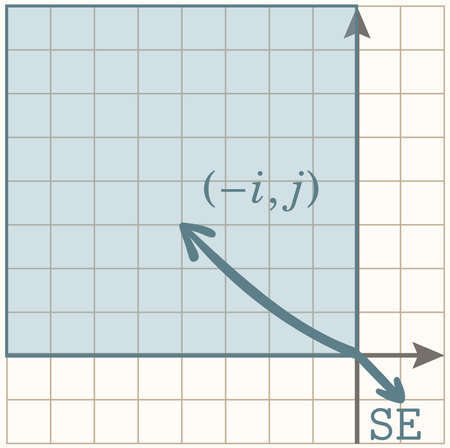
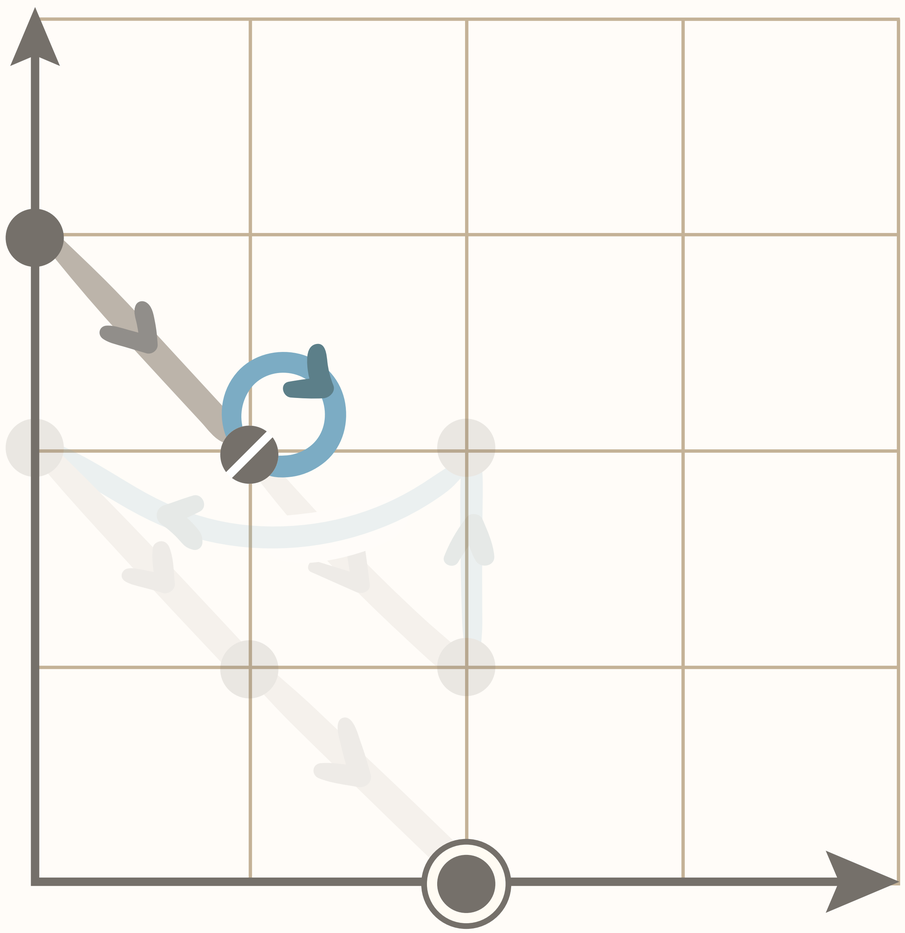
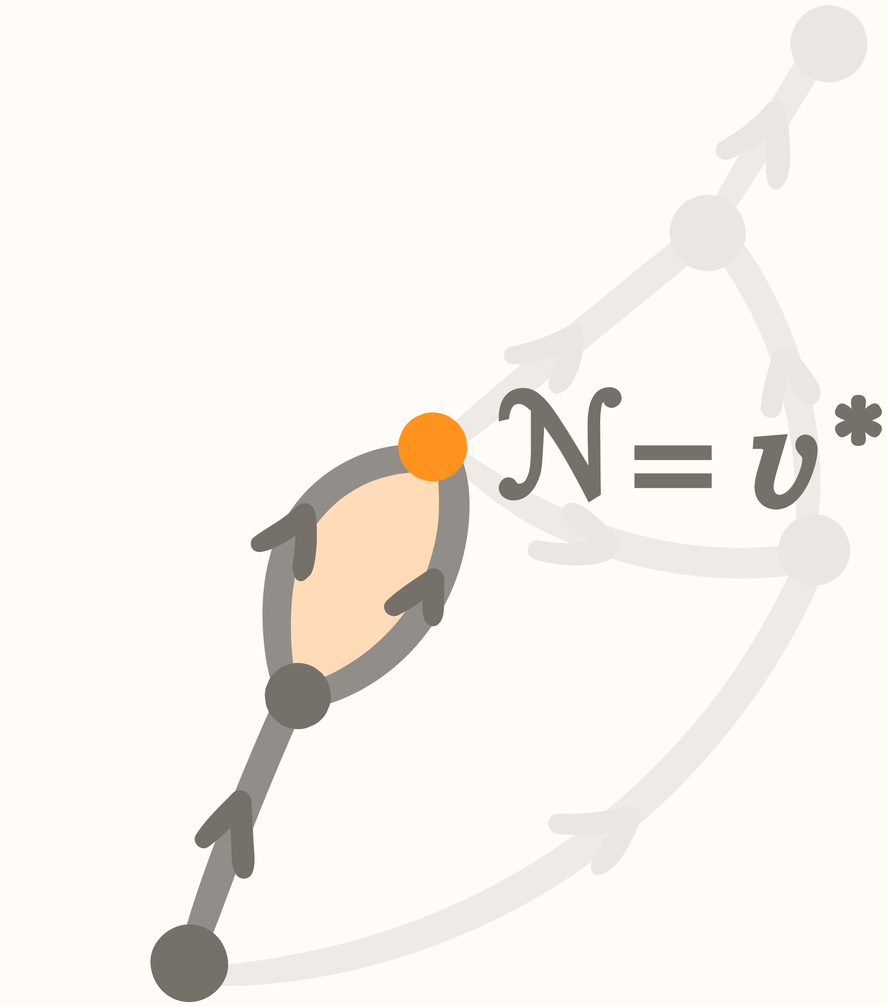
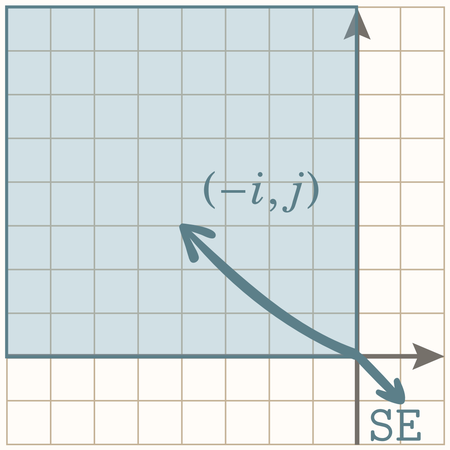
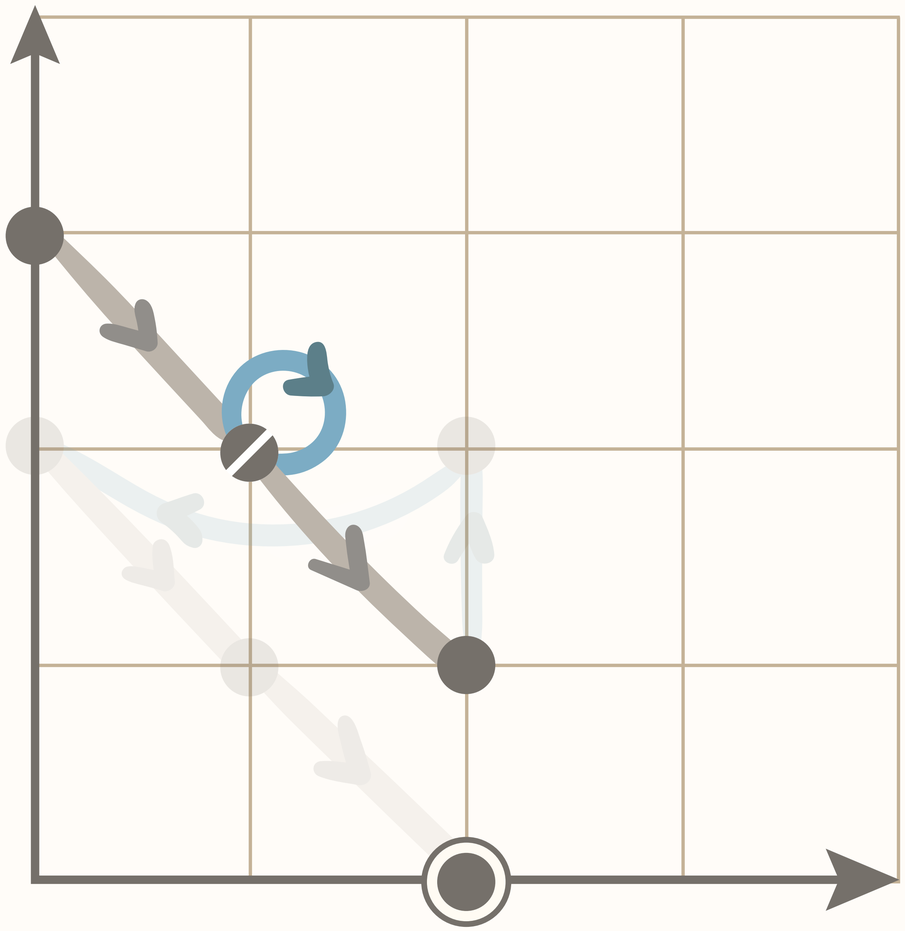


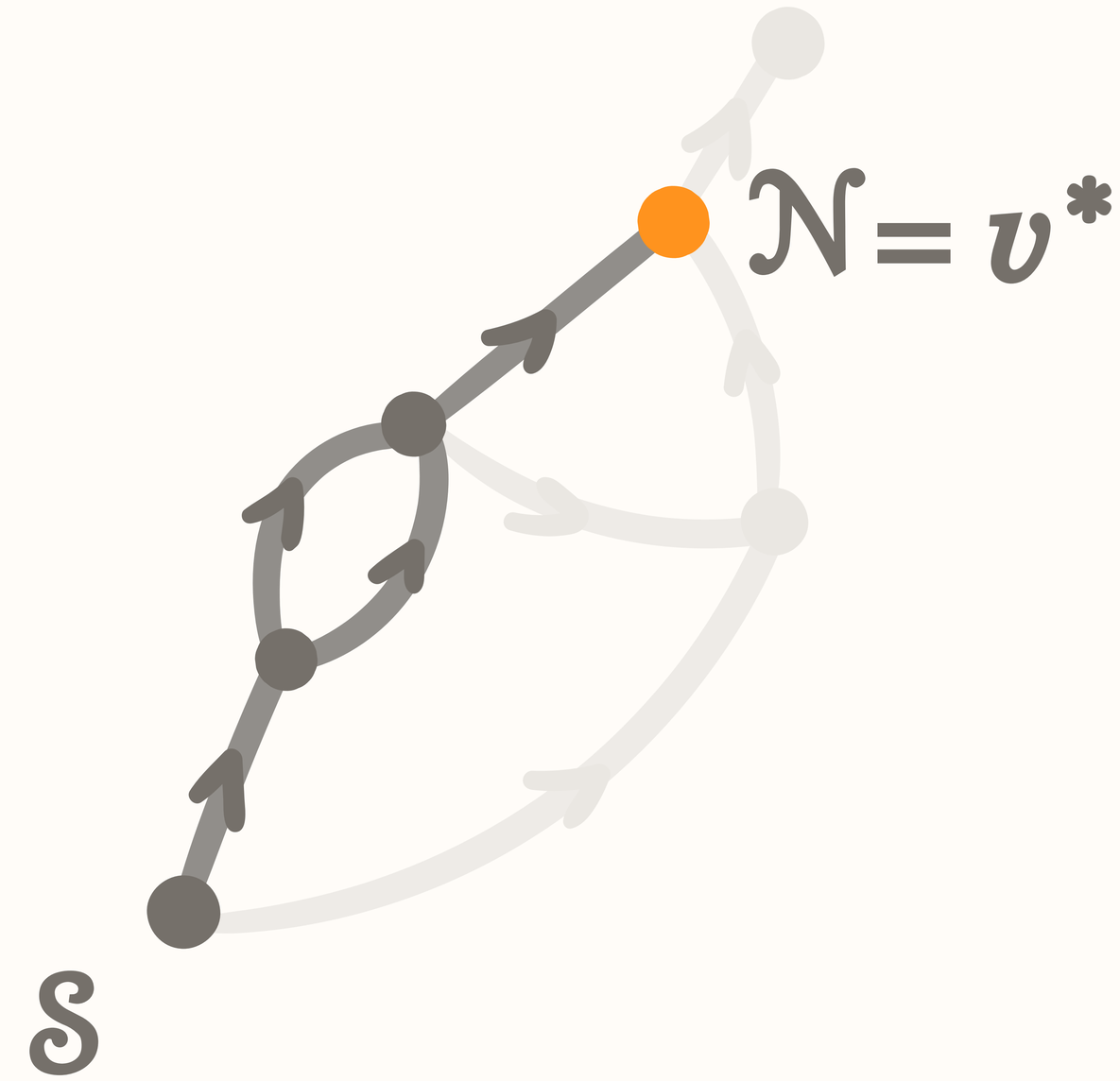




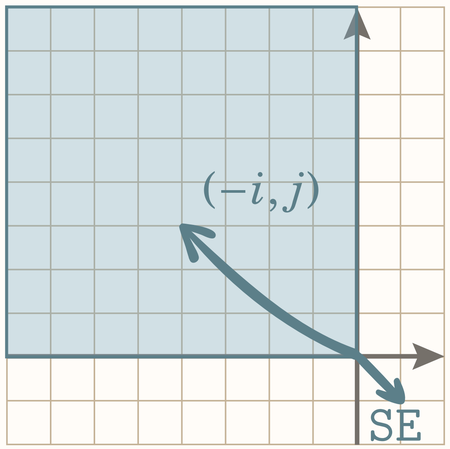
KMSW bijection example

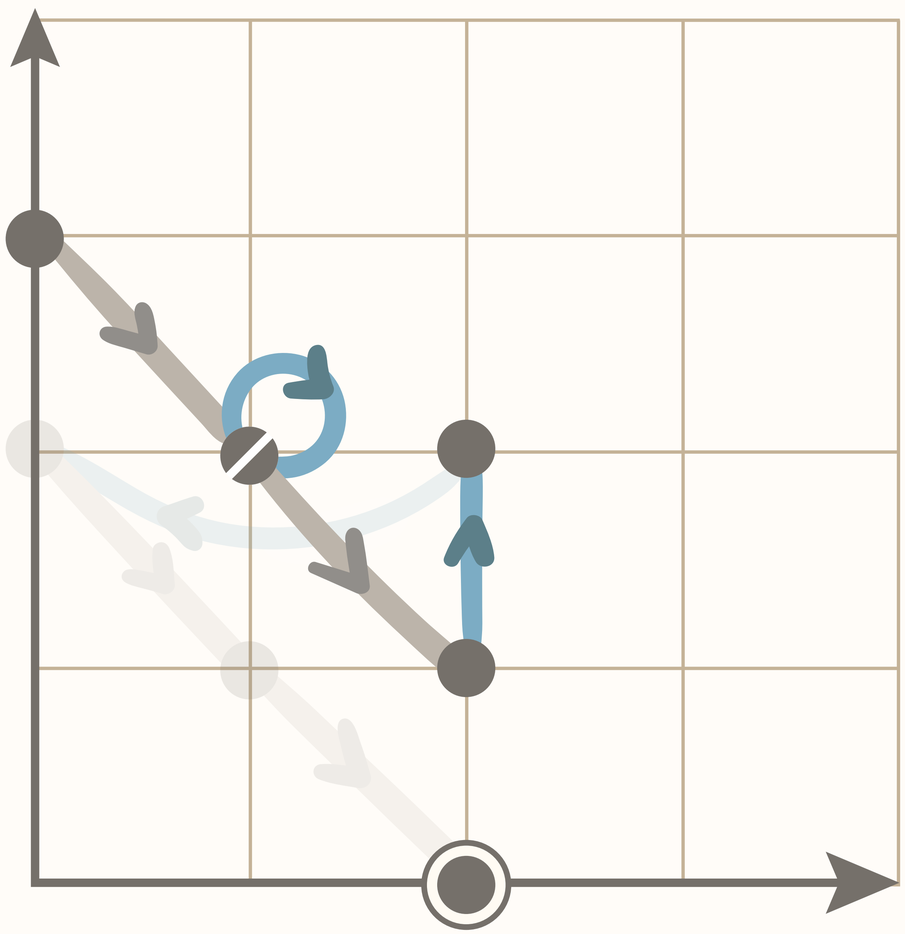


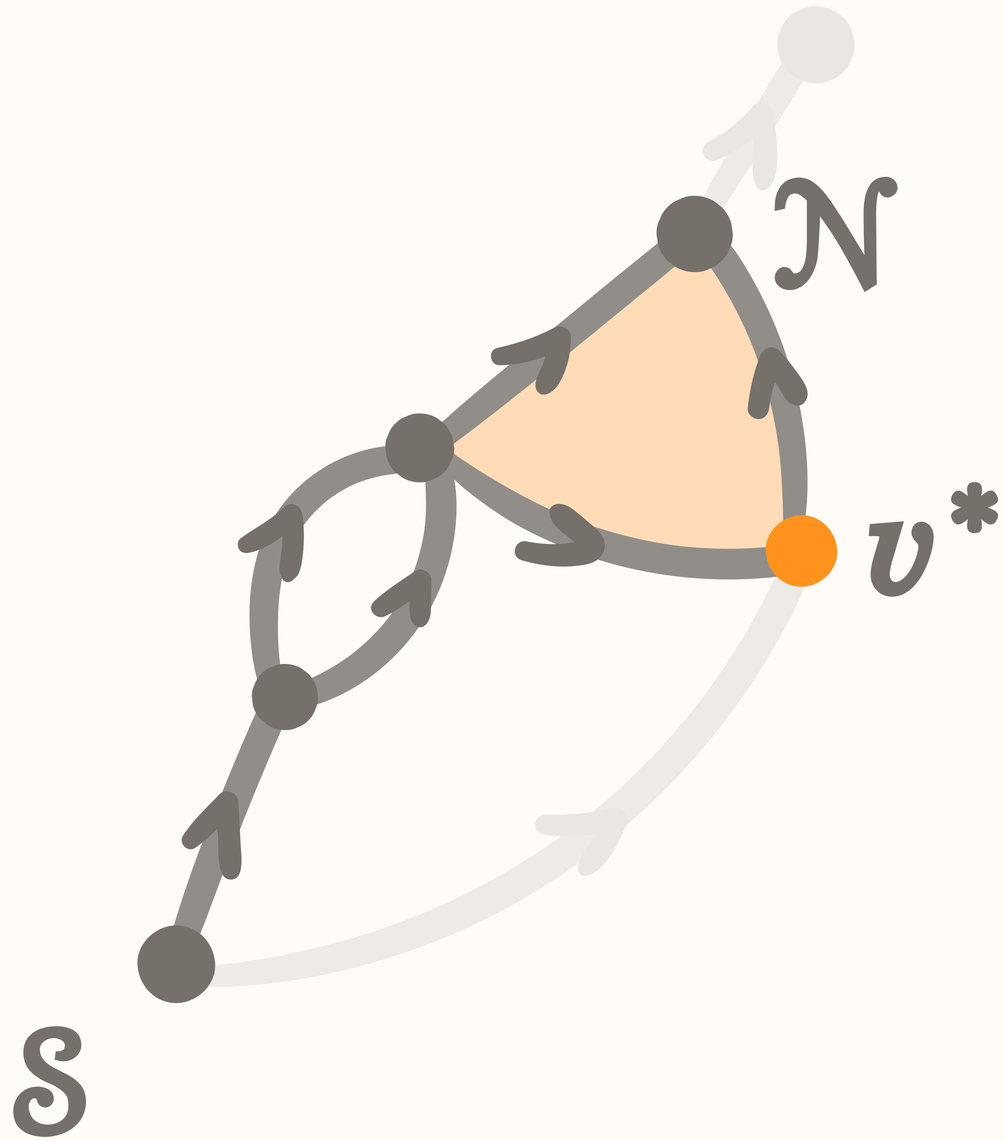




KMSW bijection example







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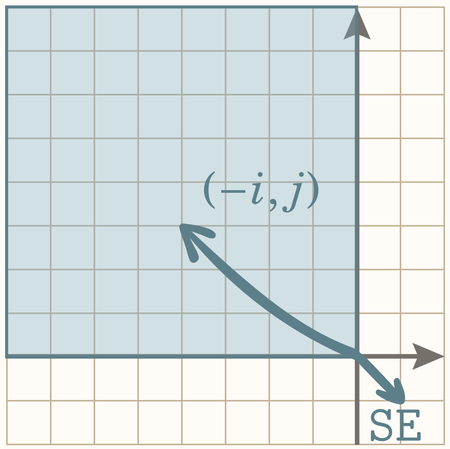
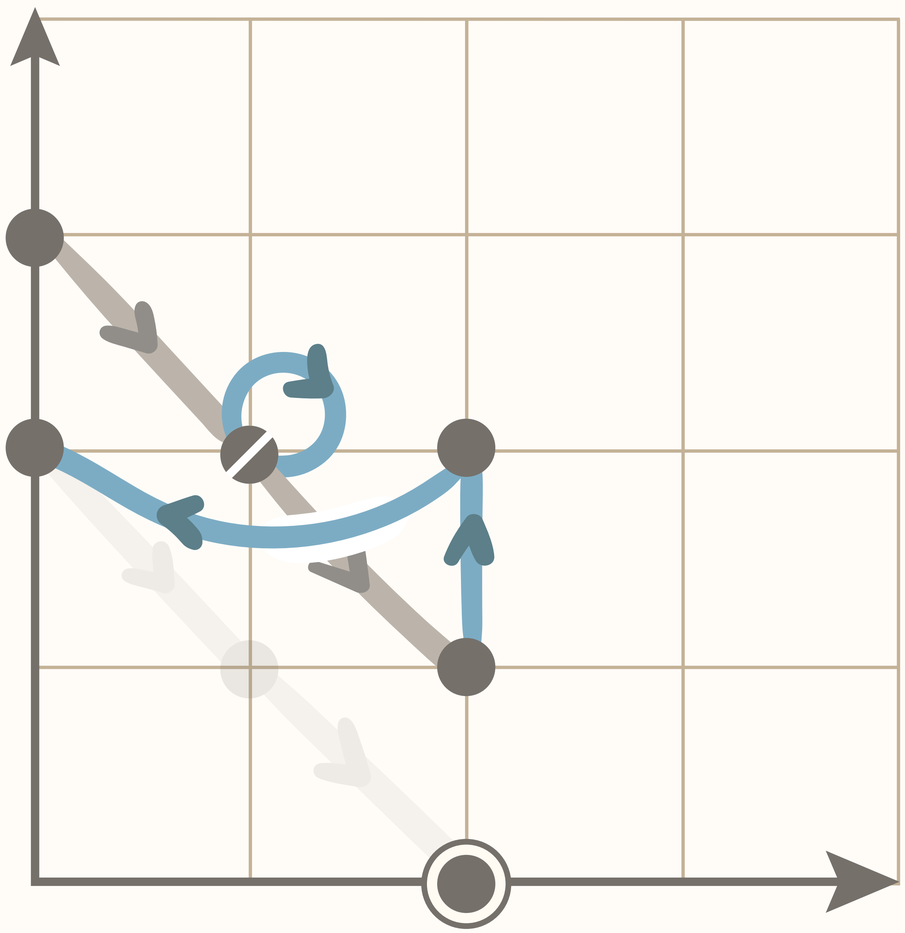
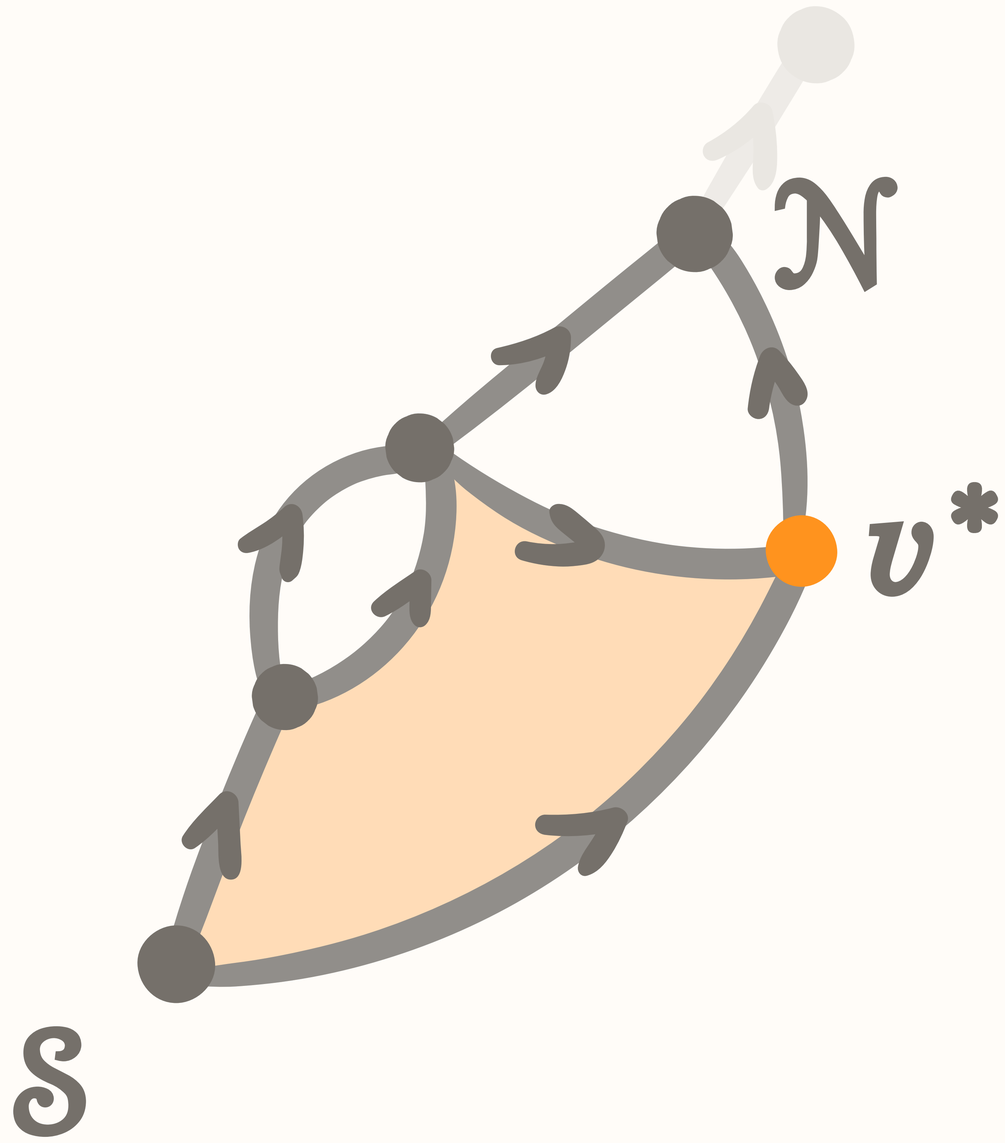
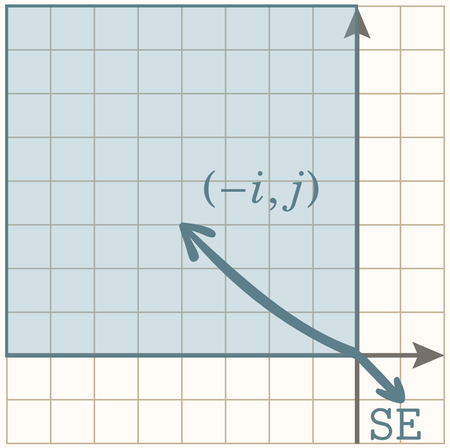
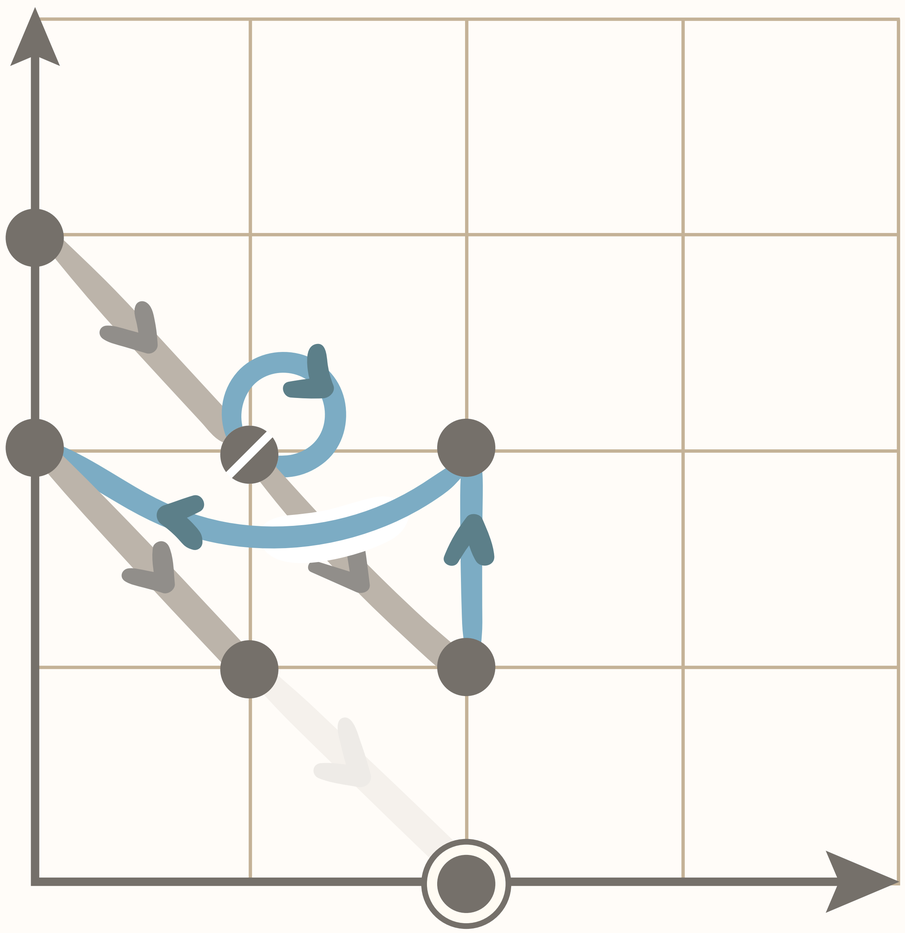


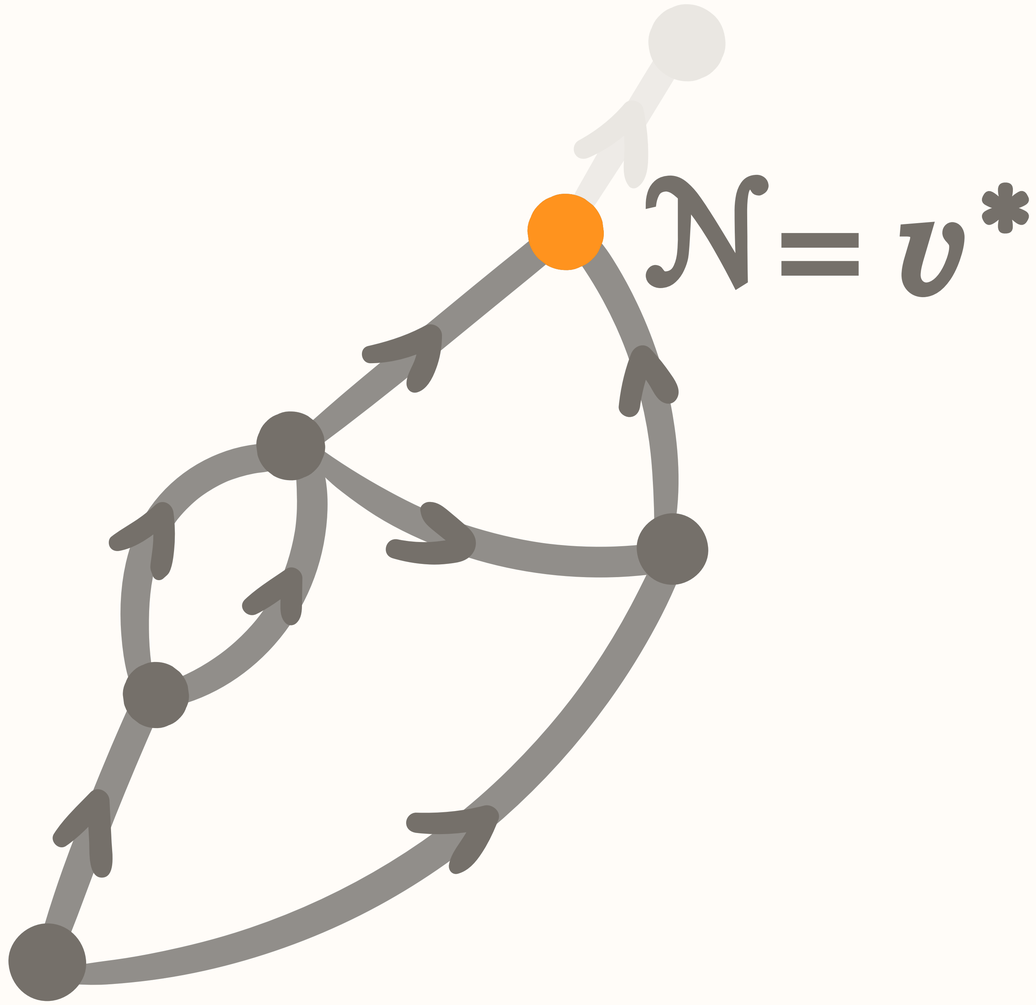




KMSW bijection example







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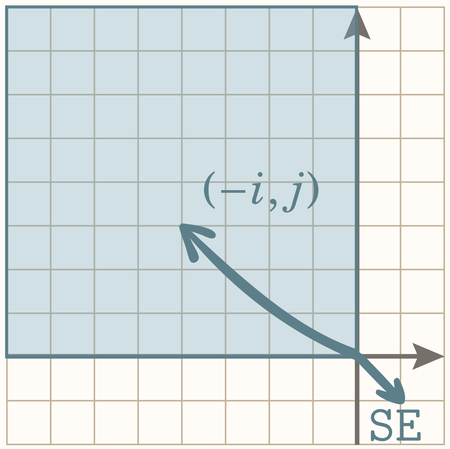
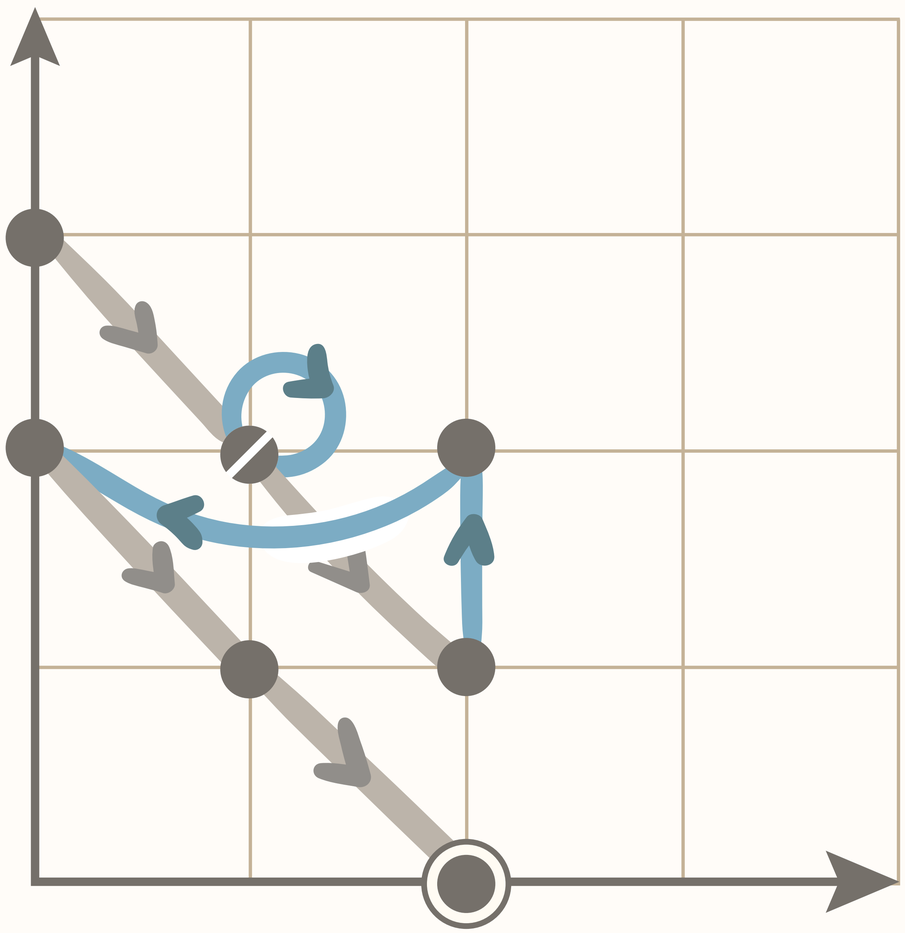
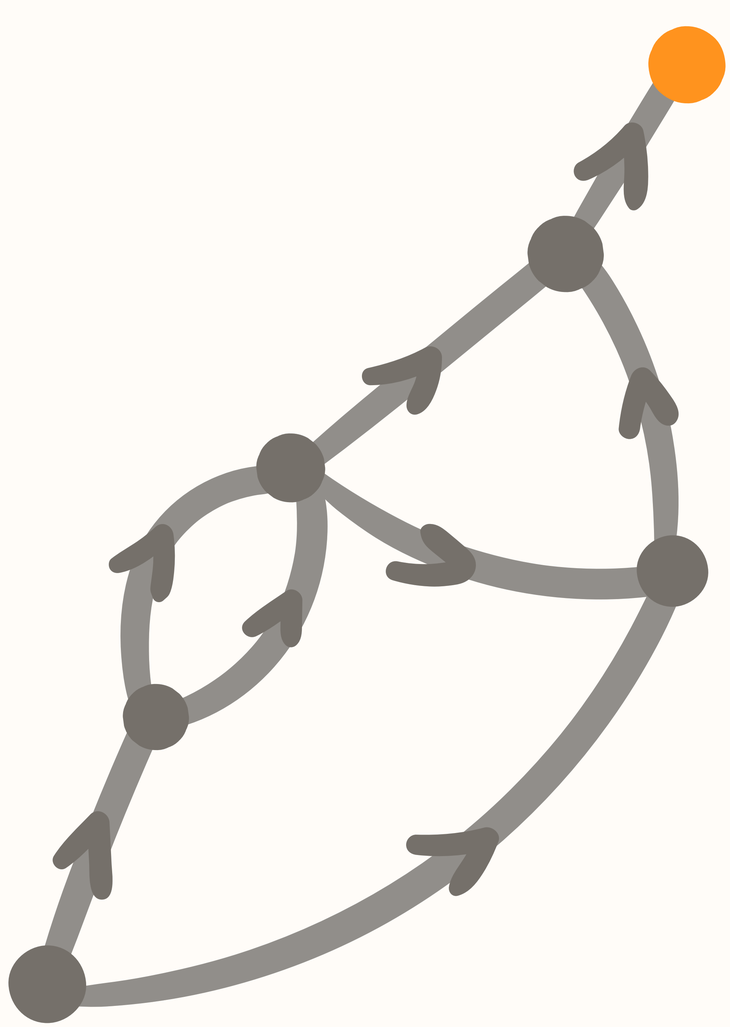
KMSW bijection example



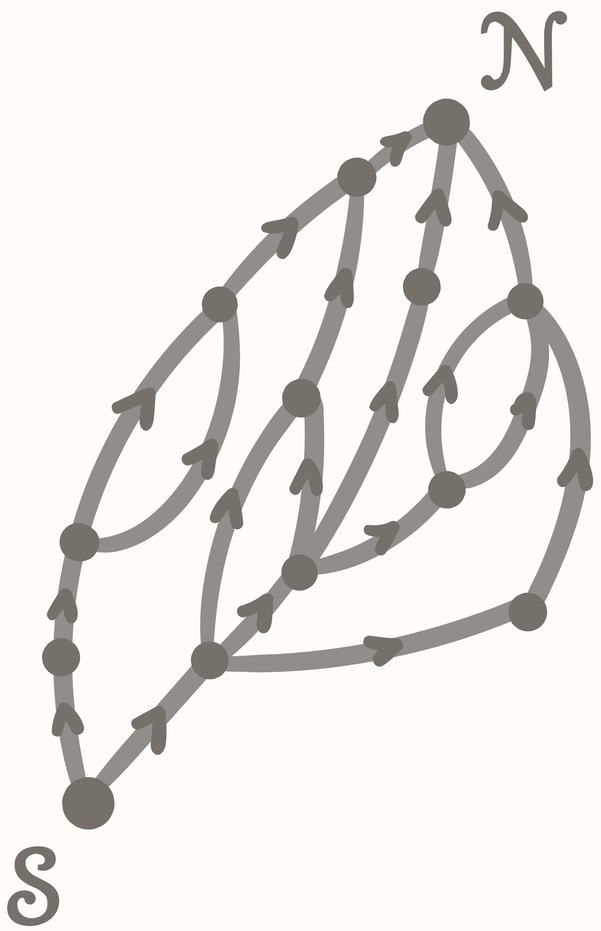




$\mathcal{S}$

The KMSW bijection

bipolar orientations ⟷ *tandem walks in the quarter plane*



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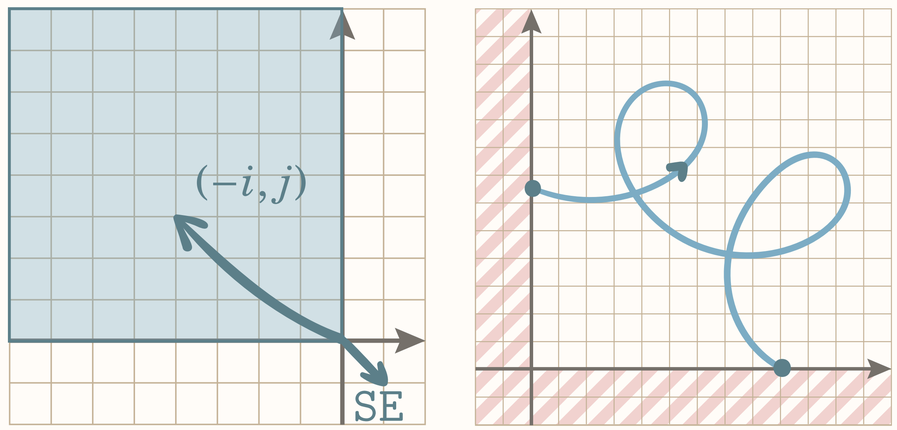

↠ *Bipolar orientations on planar maps and SLE_{12},* R. Kenyon, J. Miller, S. Sheffield and D. Wilson (2015)

The KMSW bijection

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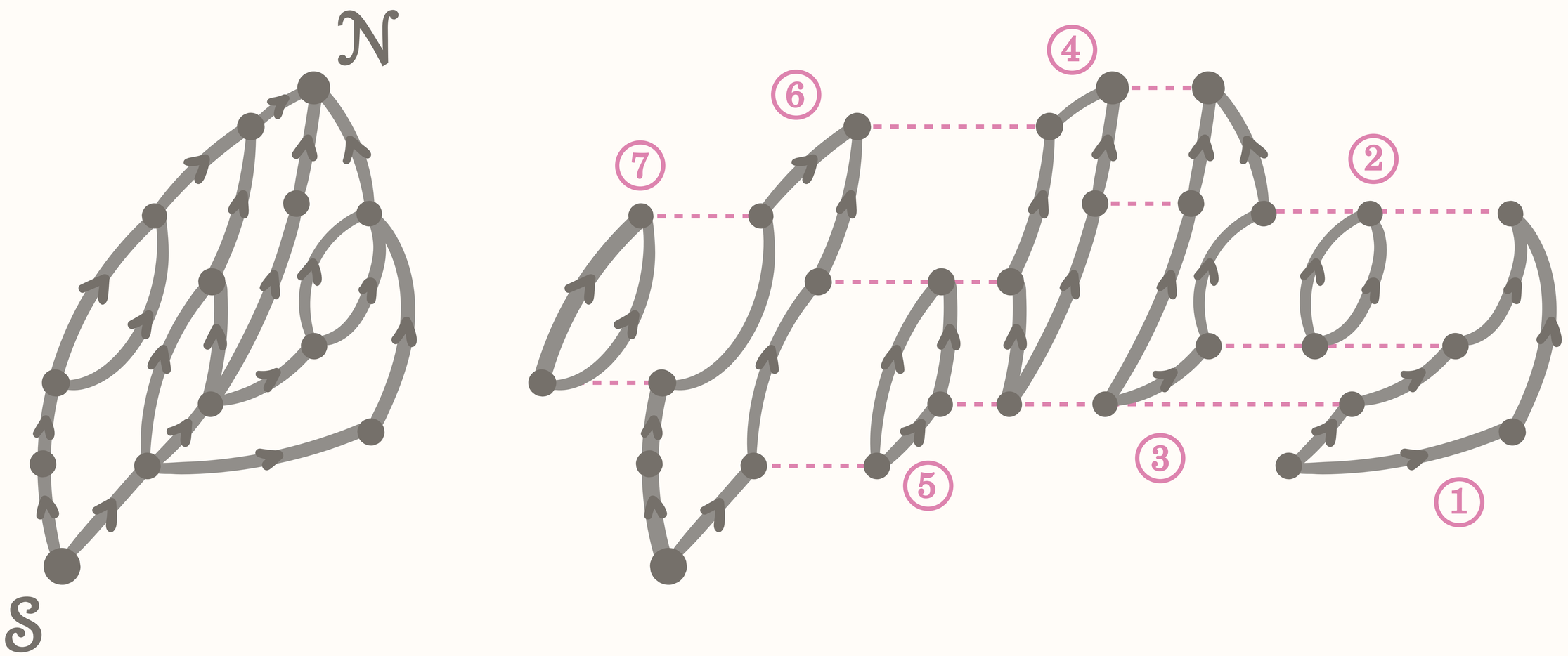
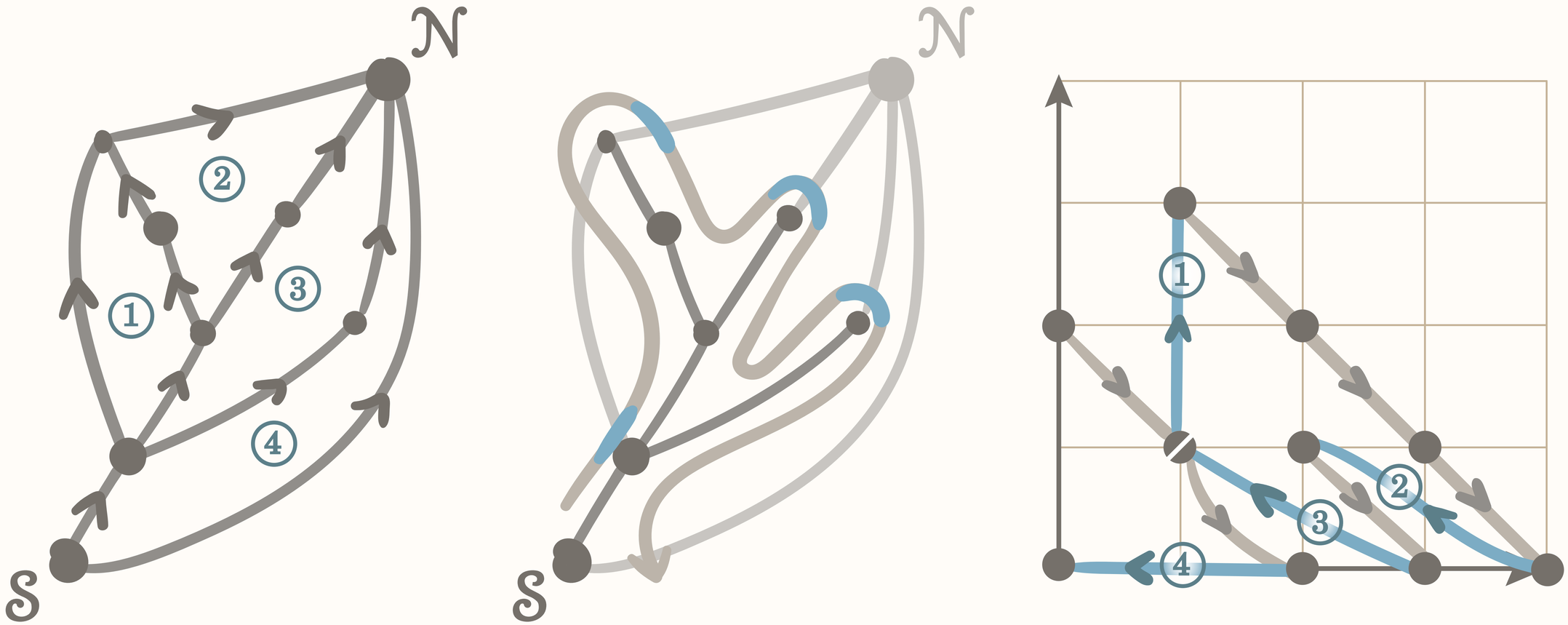


→≫ *Bipolar orientations on planar maps and SLE_{12},* **R. Kenyon, J. Miller, S. Sheffield and D. Wilson (2015)**

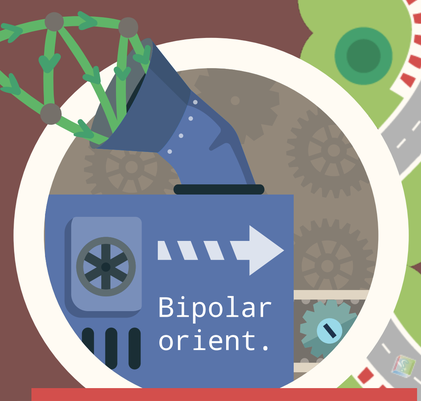
The KMSW bijection

bipolar orientations ⟷ *tandem walks in the quarter plane*

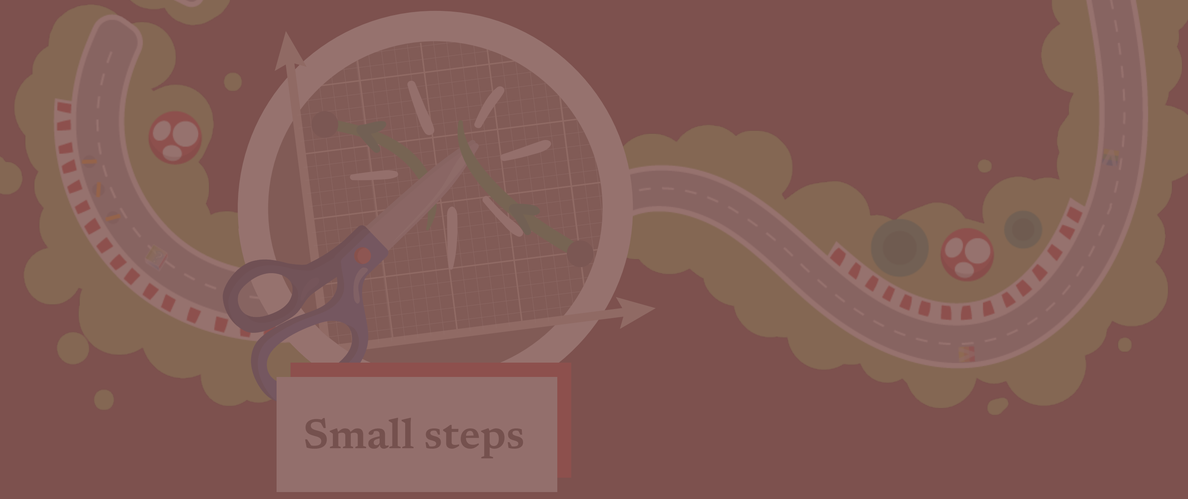


→ *Bipolar orientations on planar maps and SLE_{12},* R. Kenyon, J. Miller, S. Sheffield and D. Wilson (2015)

Summary



Decorated bipolar orientations

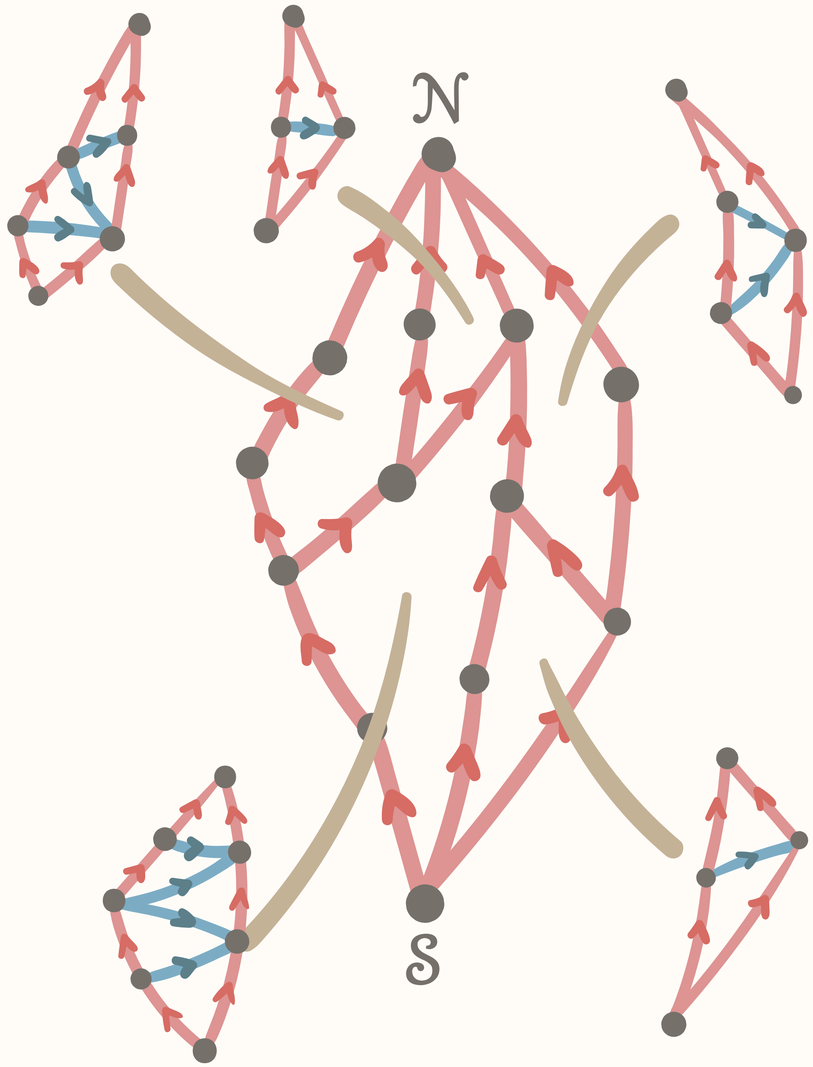


Small steps

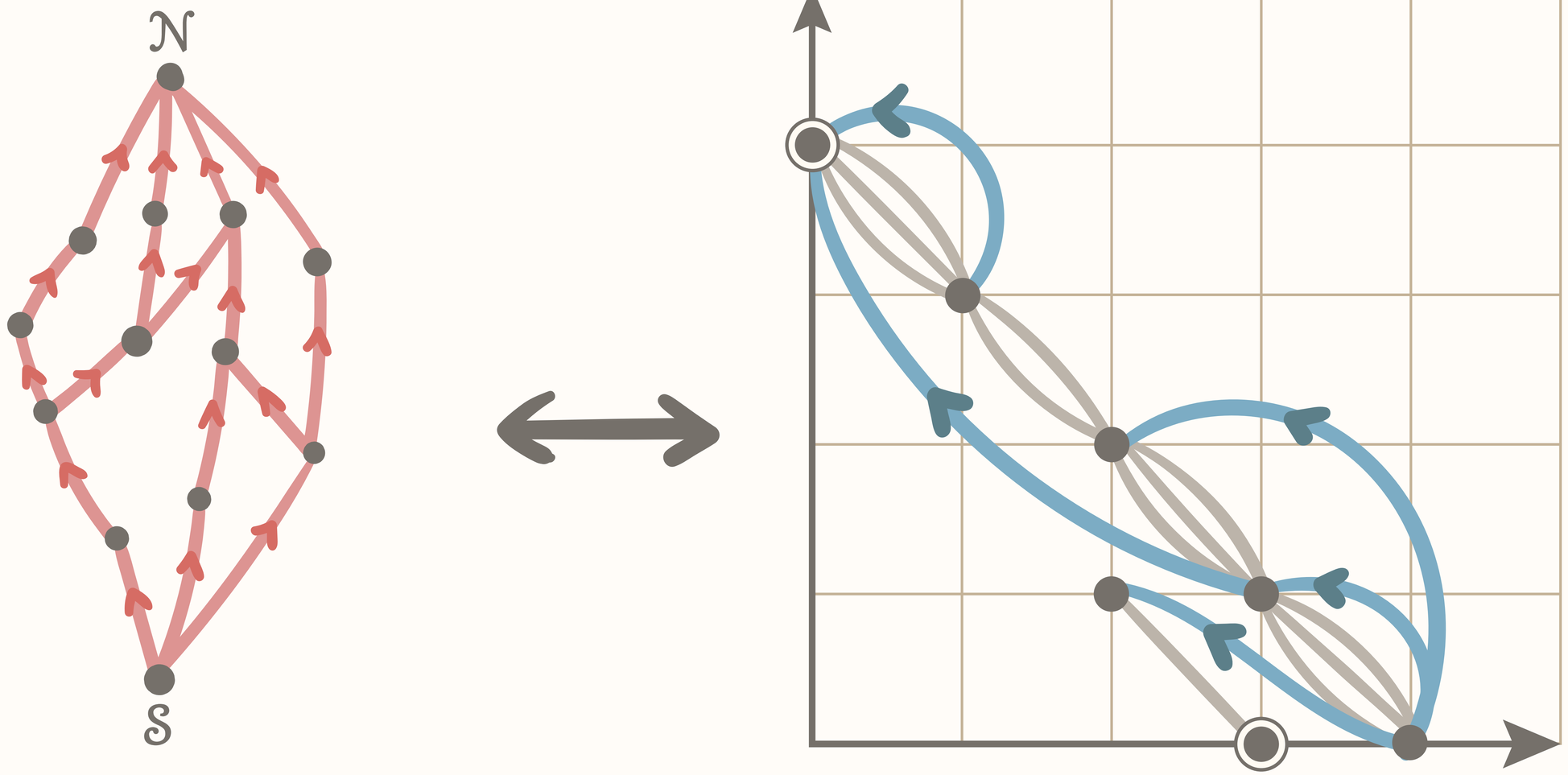
a. The KMSW bijection

b. Application to transversal structures

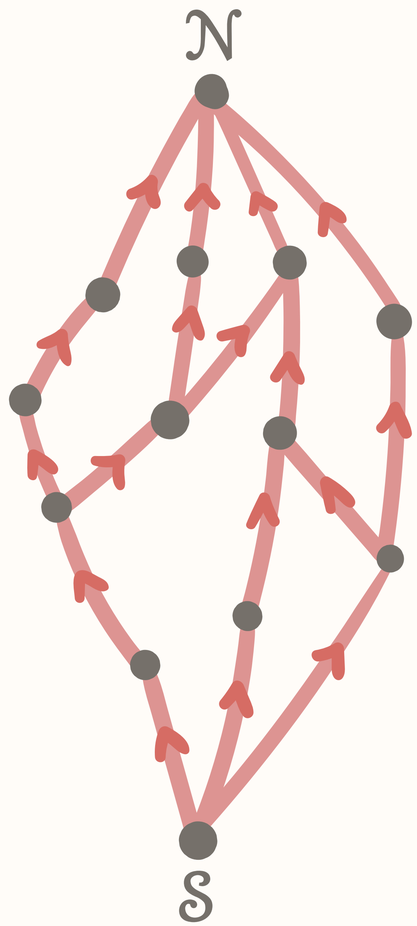
Application to transversal structures

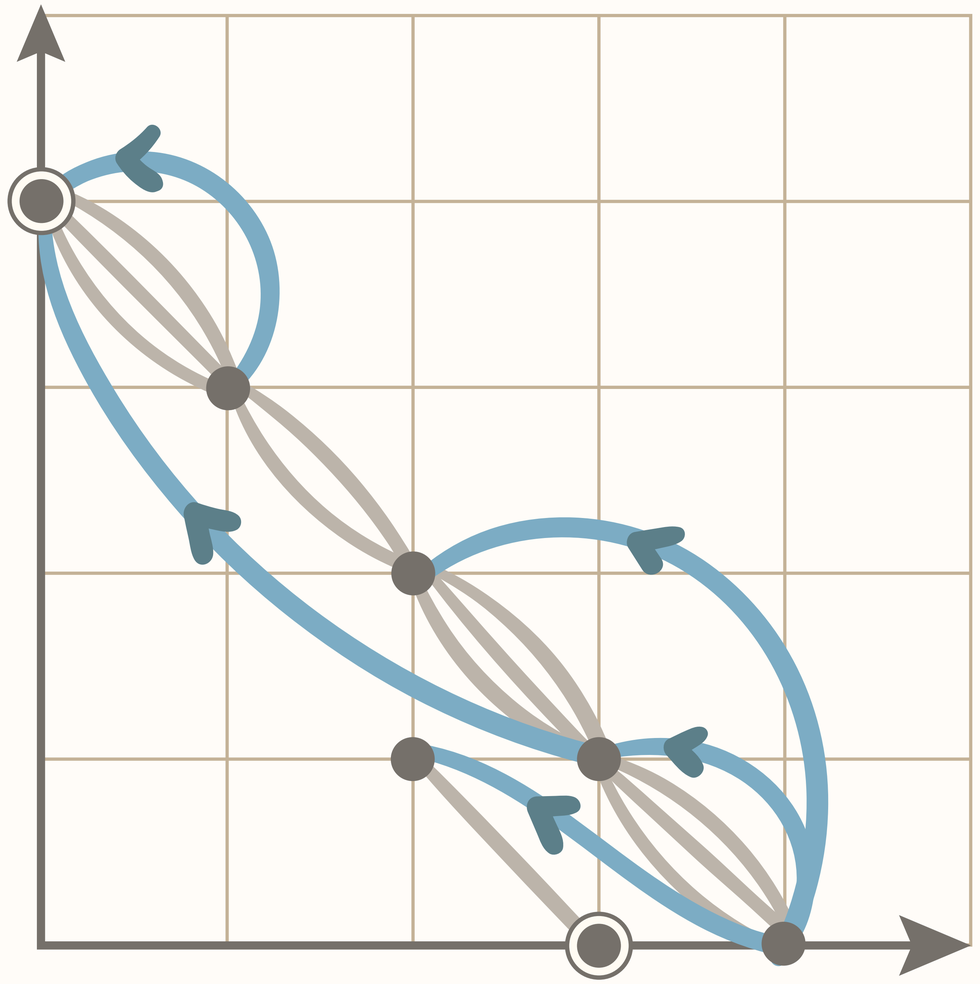


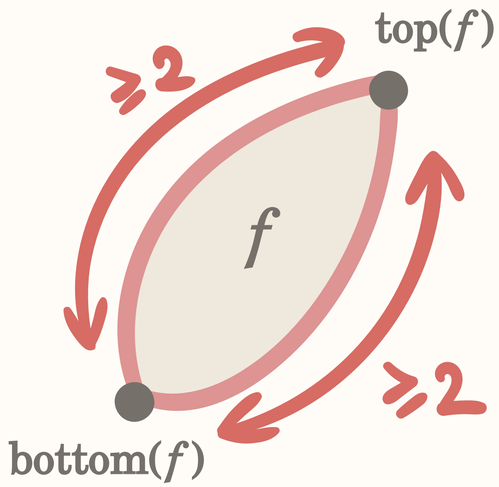
Application to transversal structures

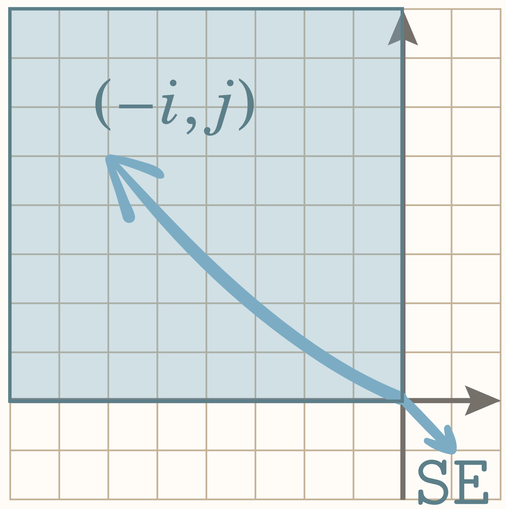


Application to transversal structures

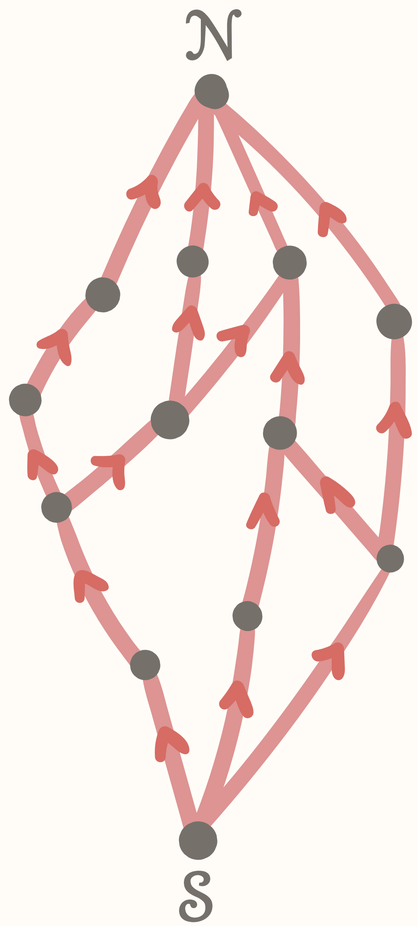


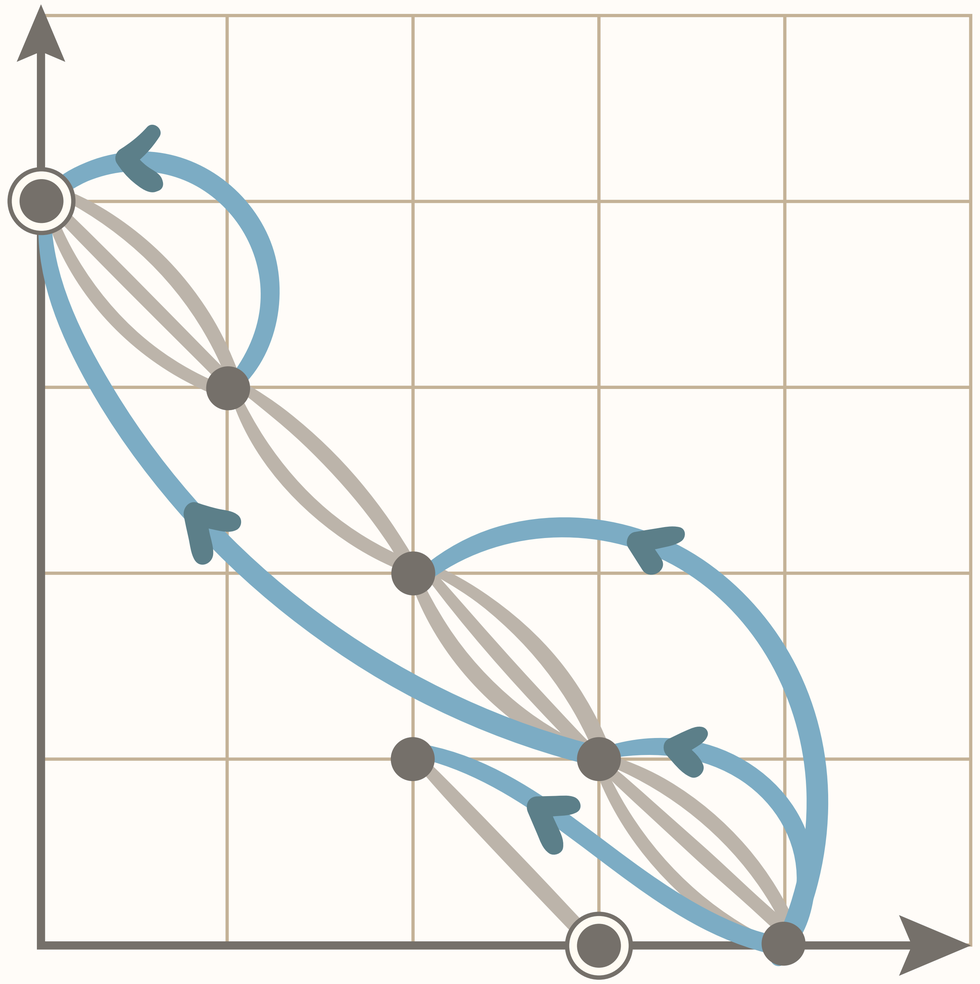


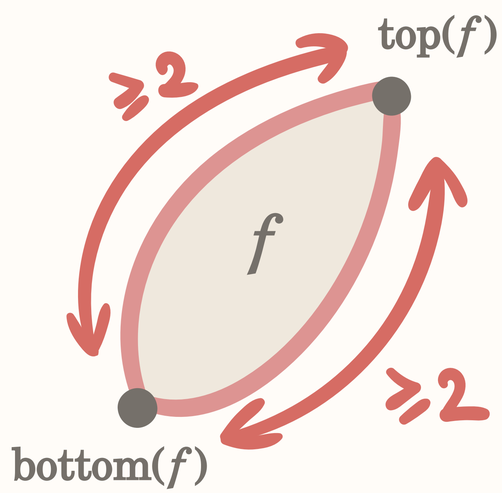




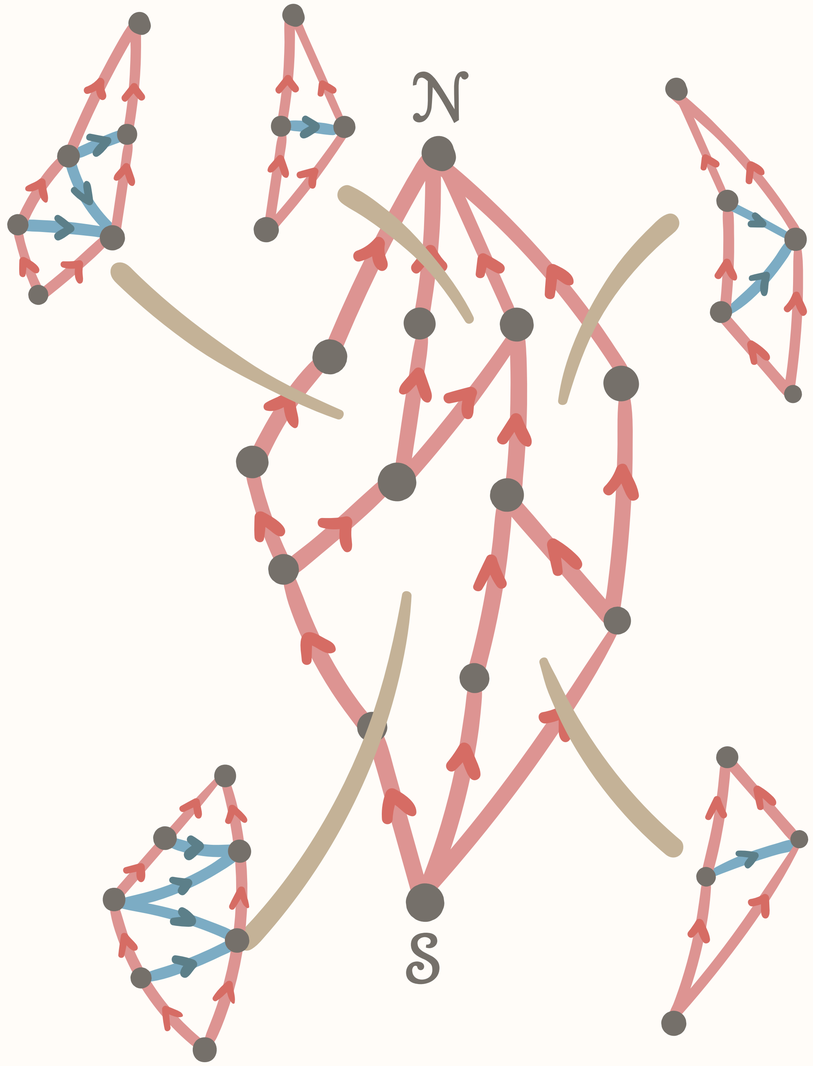
Application to transversal structures

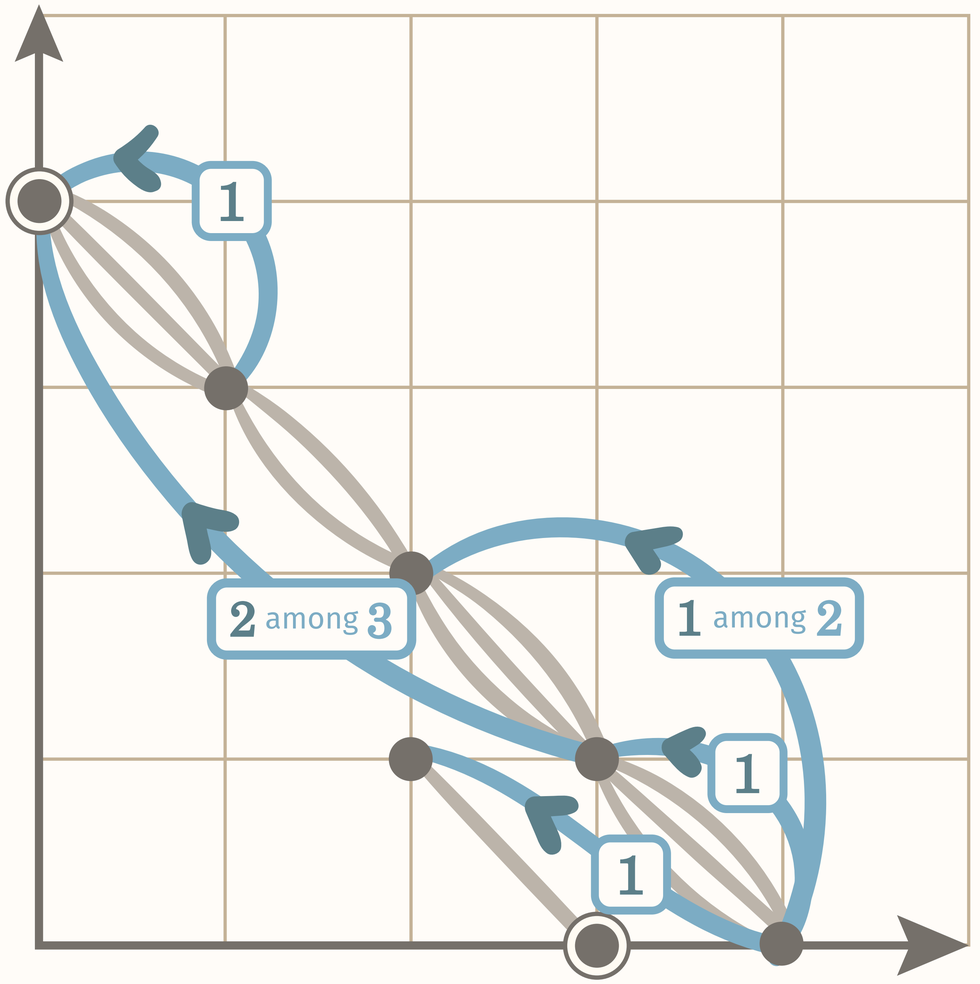


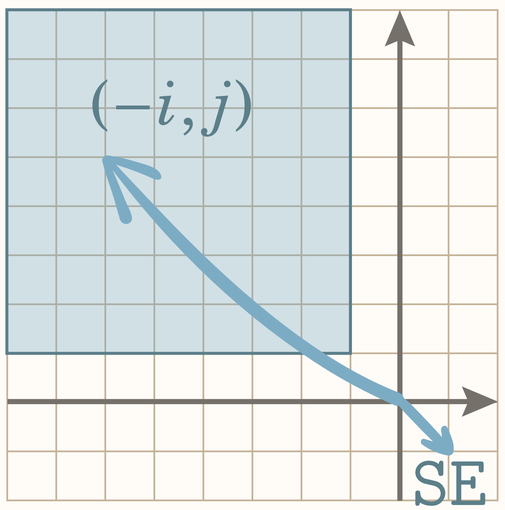




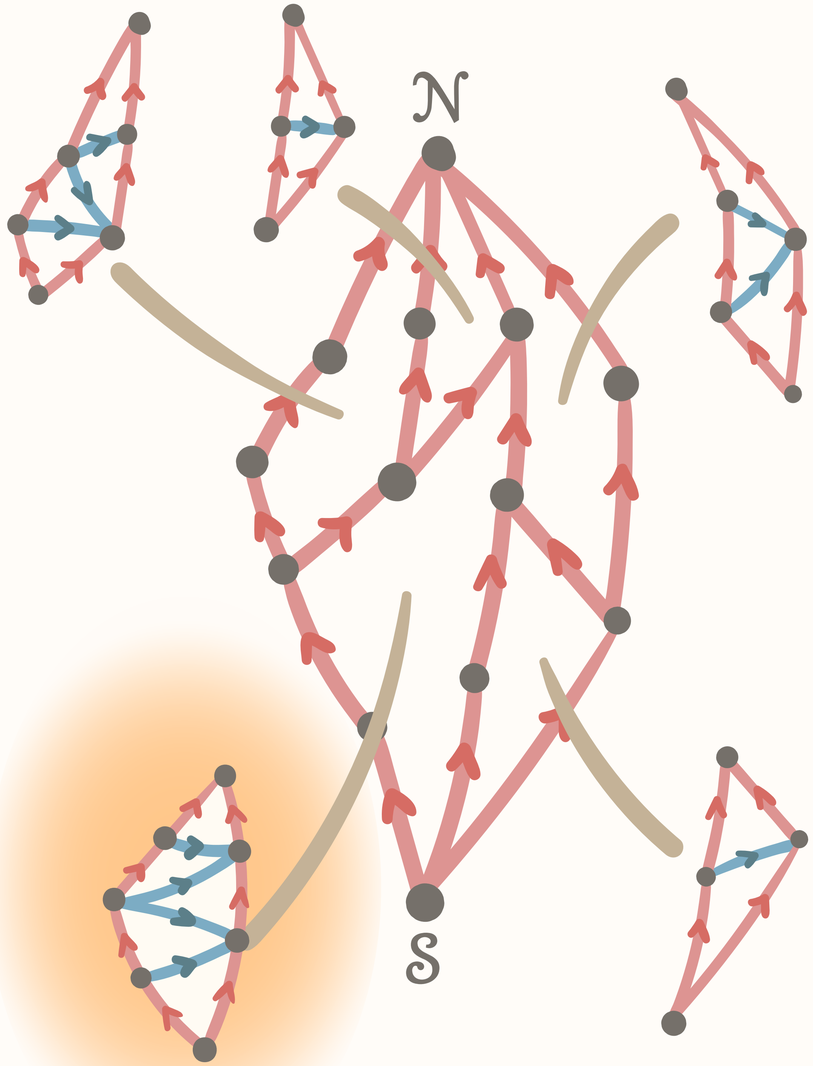
Application to transversal structures

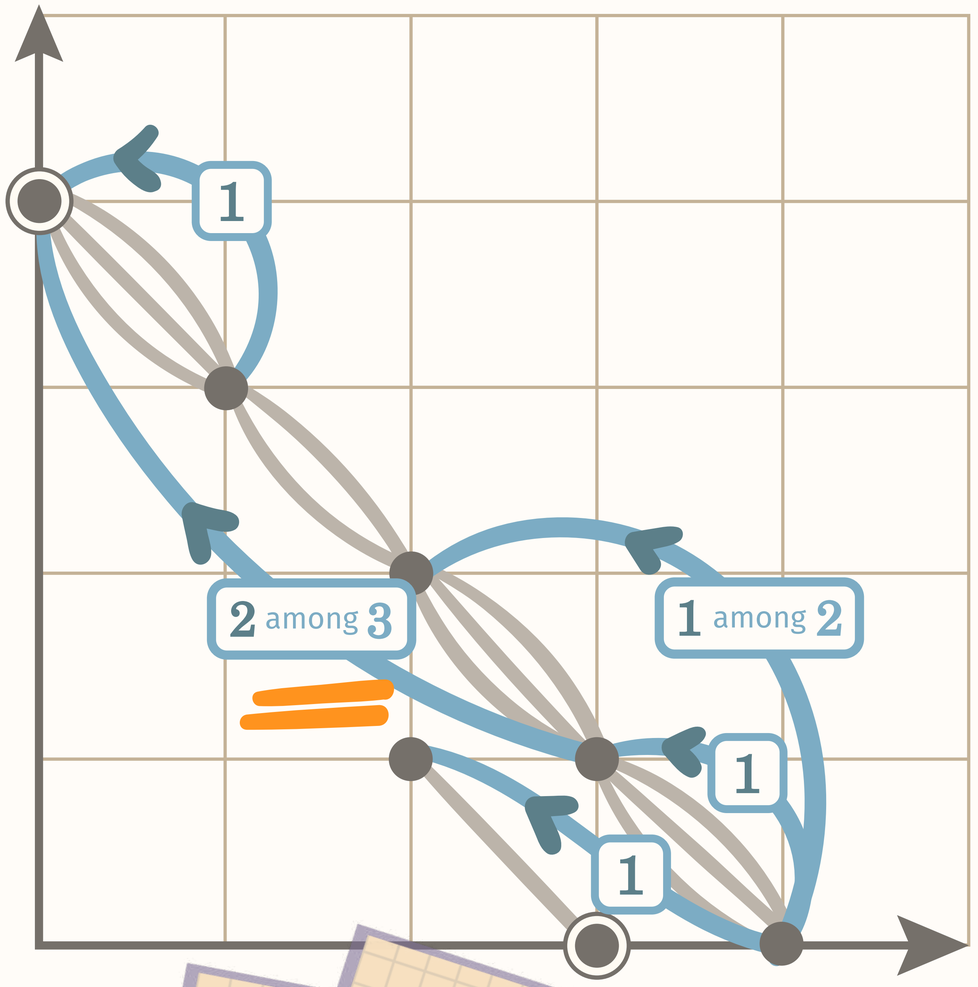


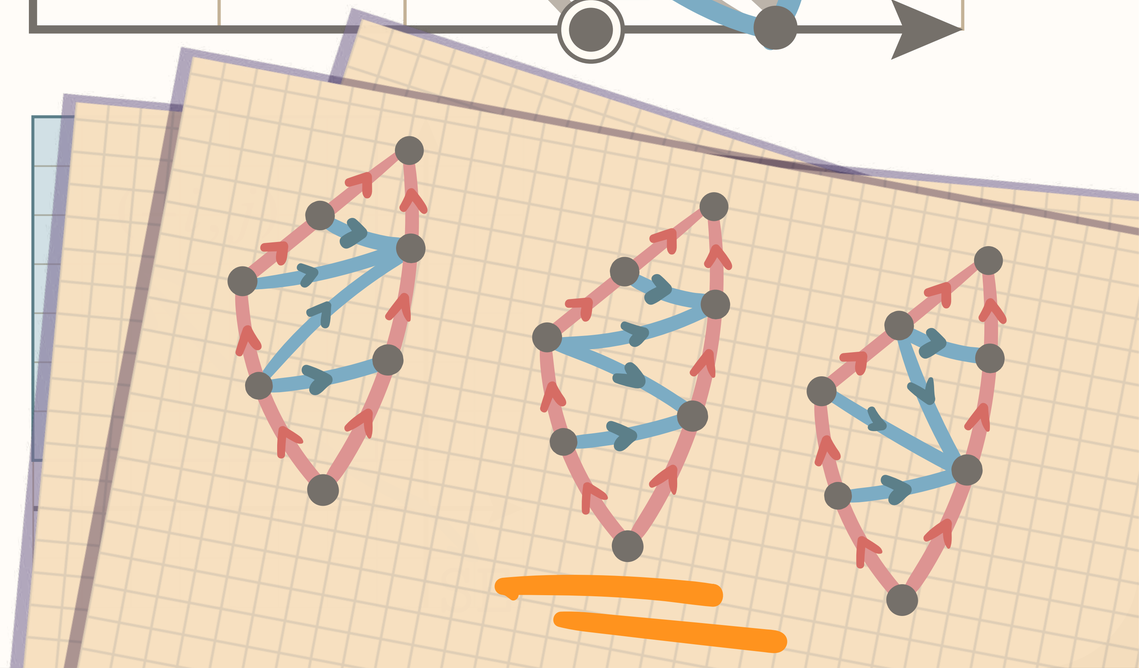




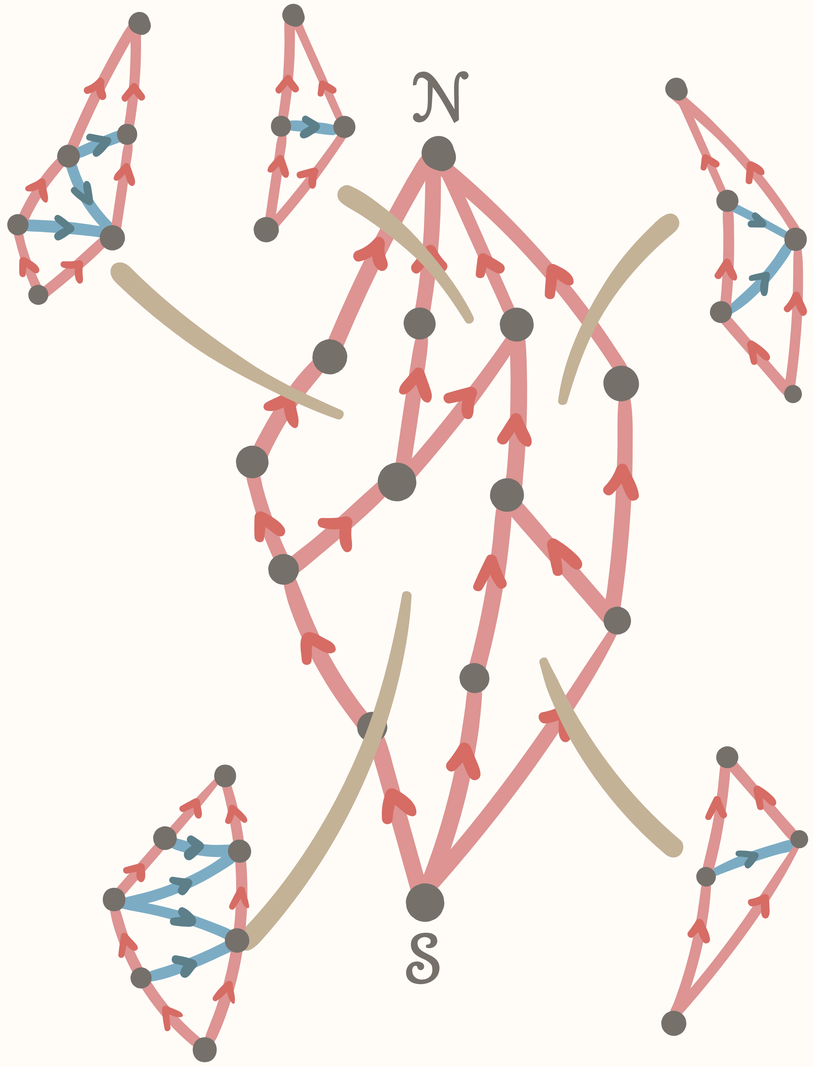
Application to transversal structures

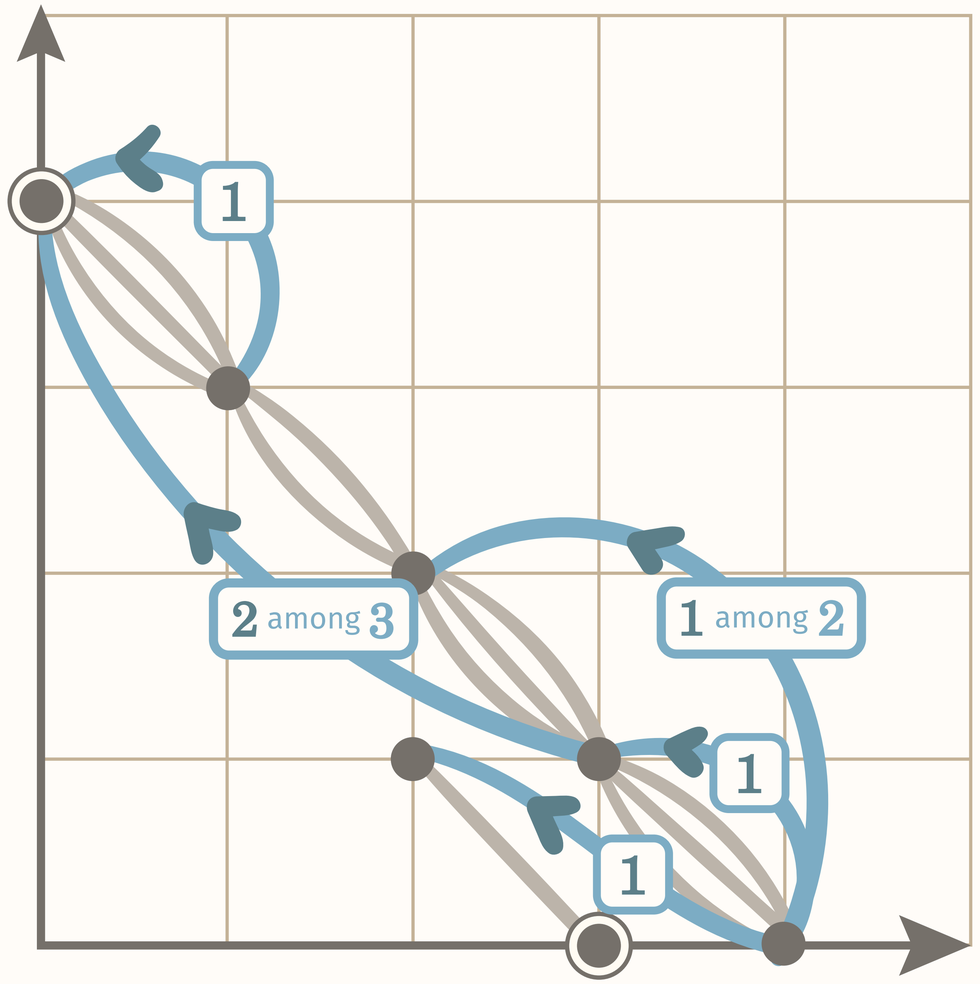


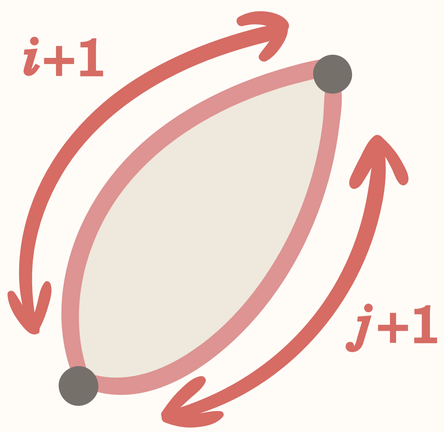


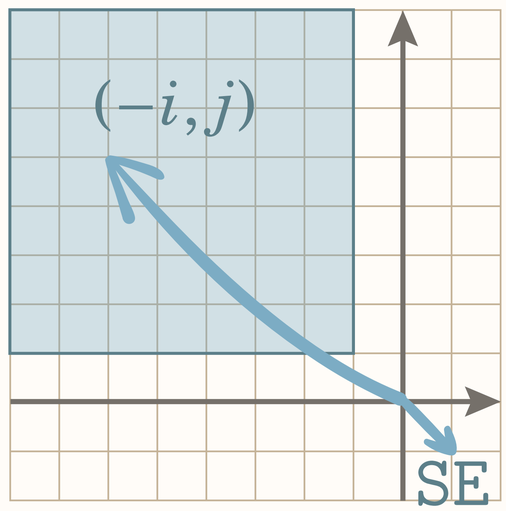


Application to transversal structures

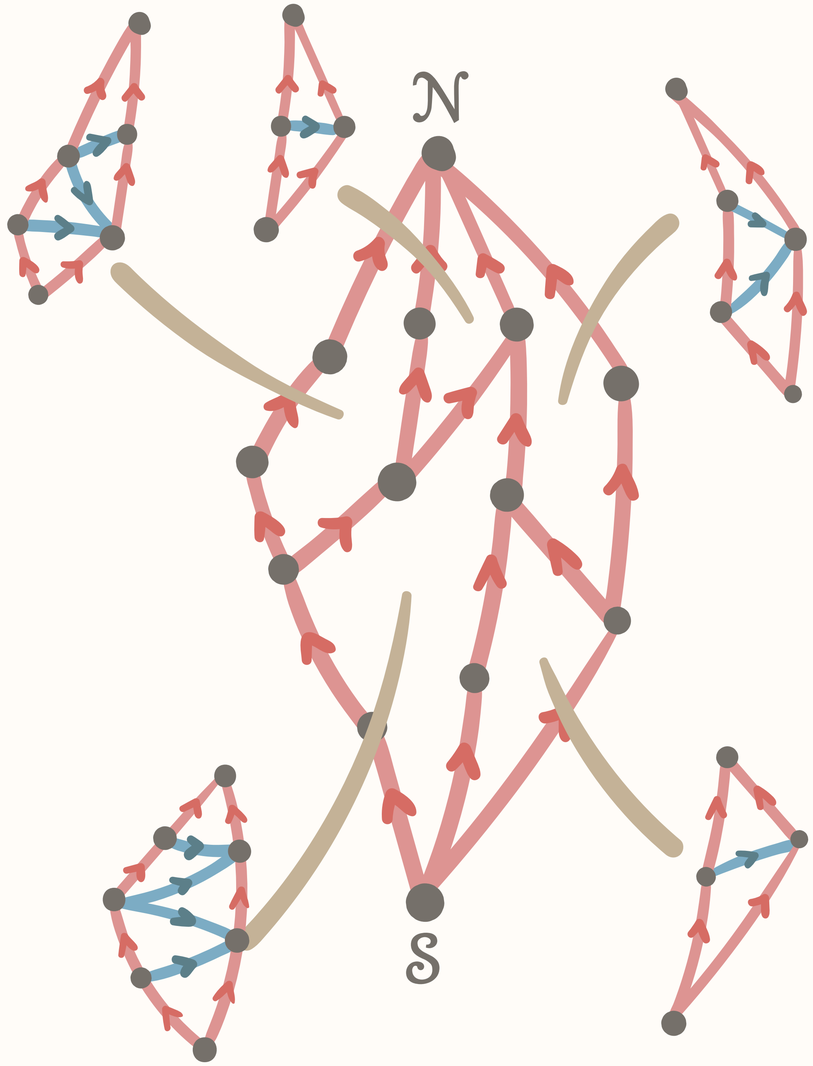


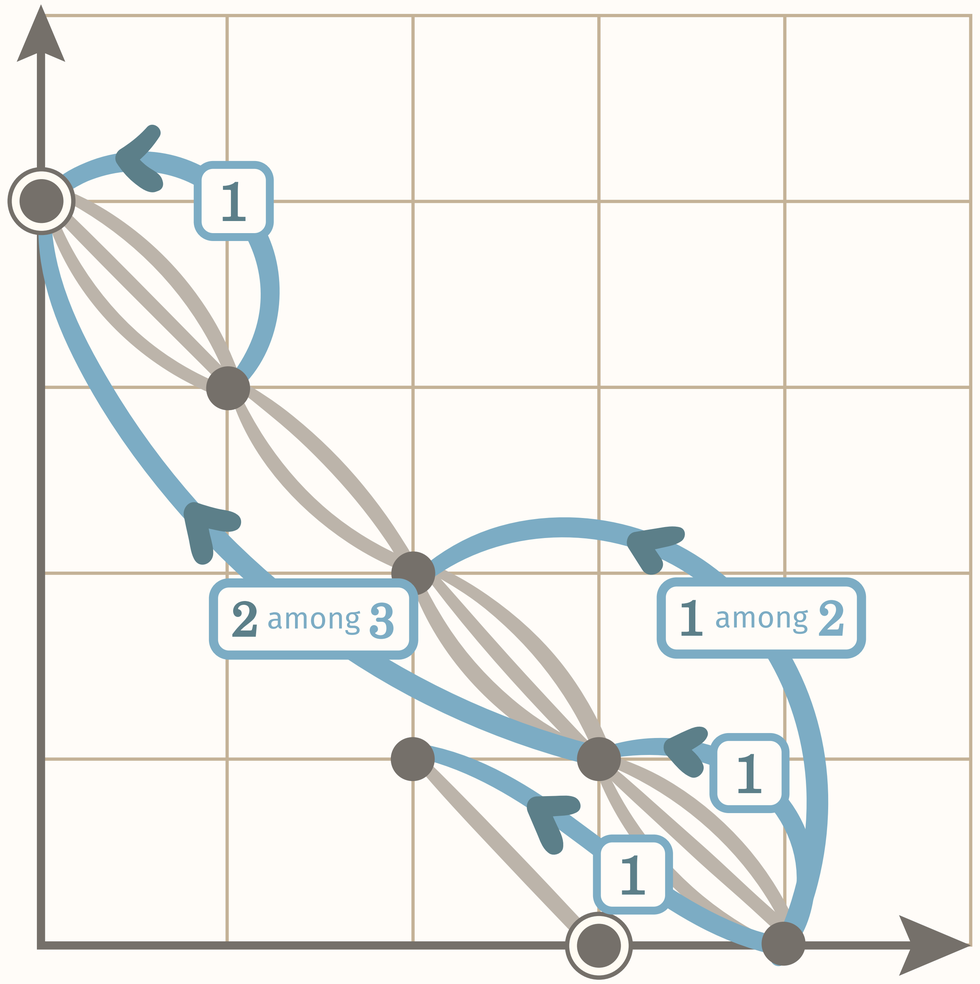


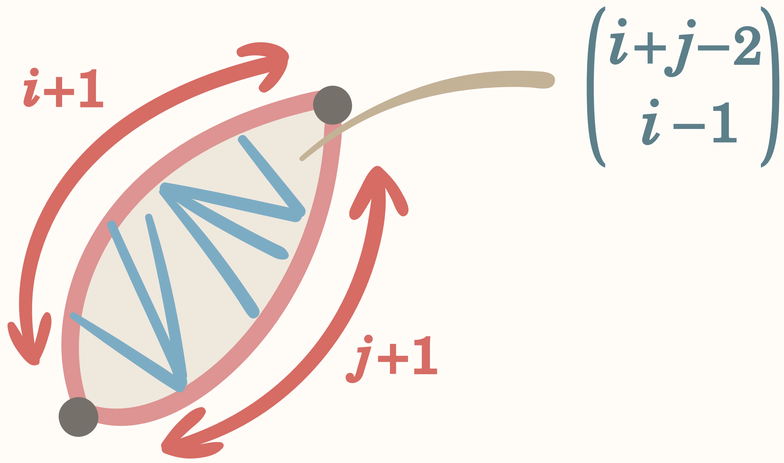


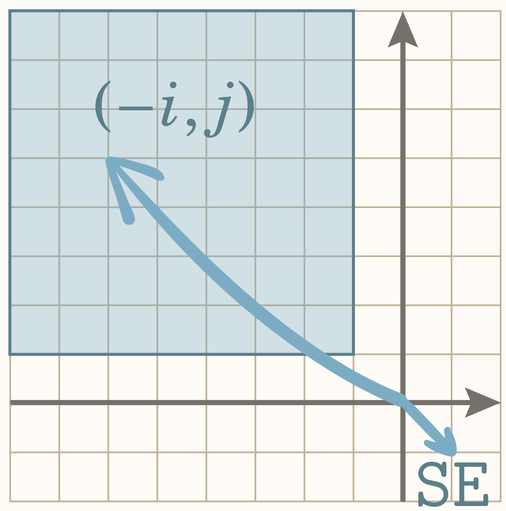


Application to transversal structures

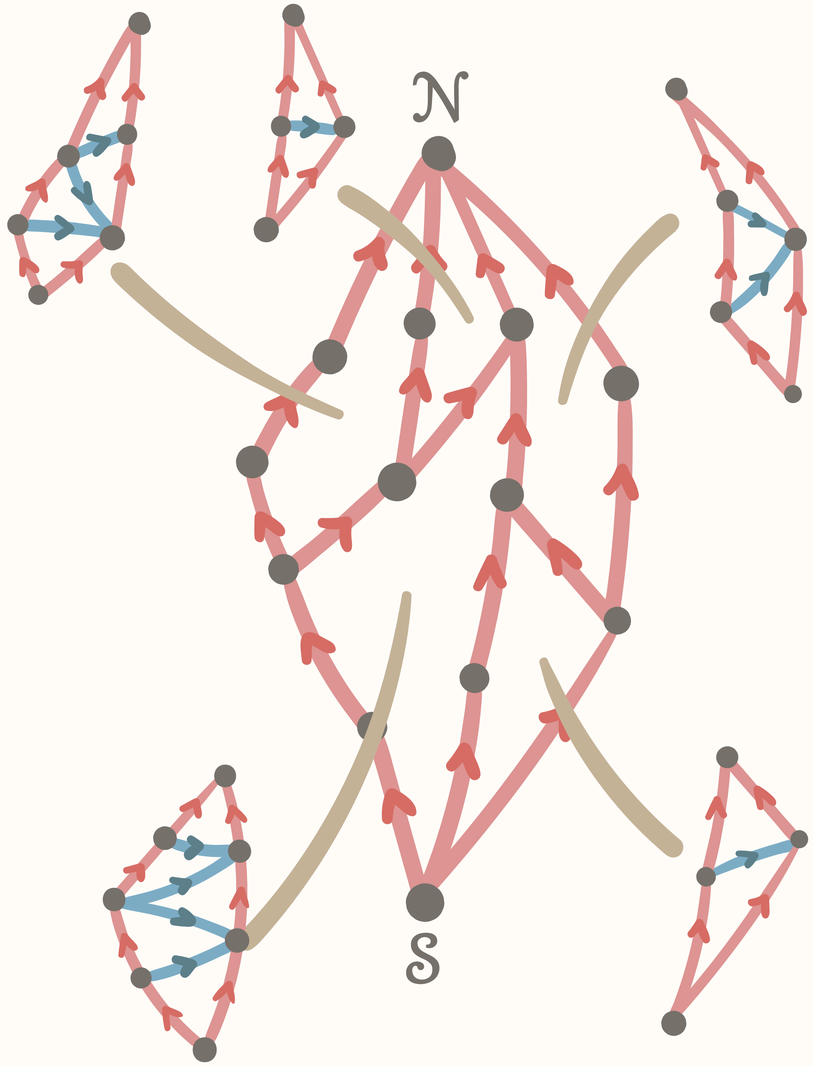


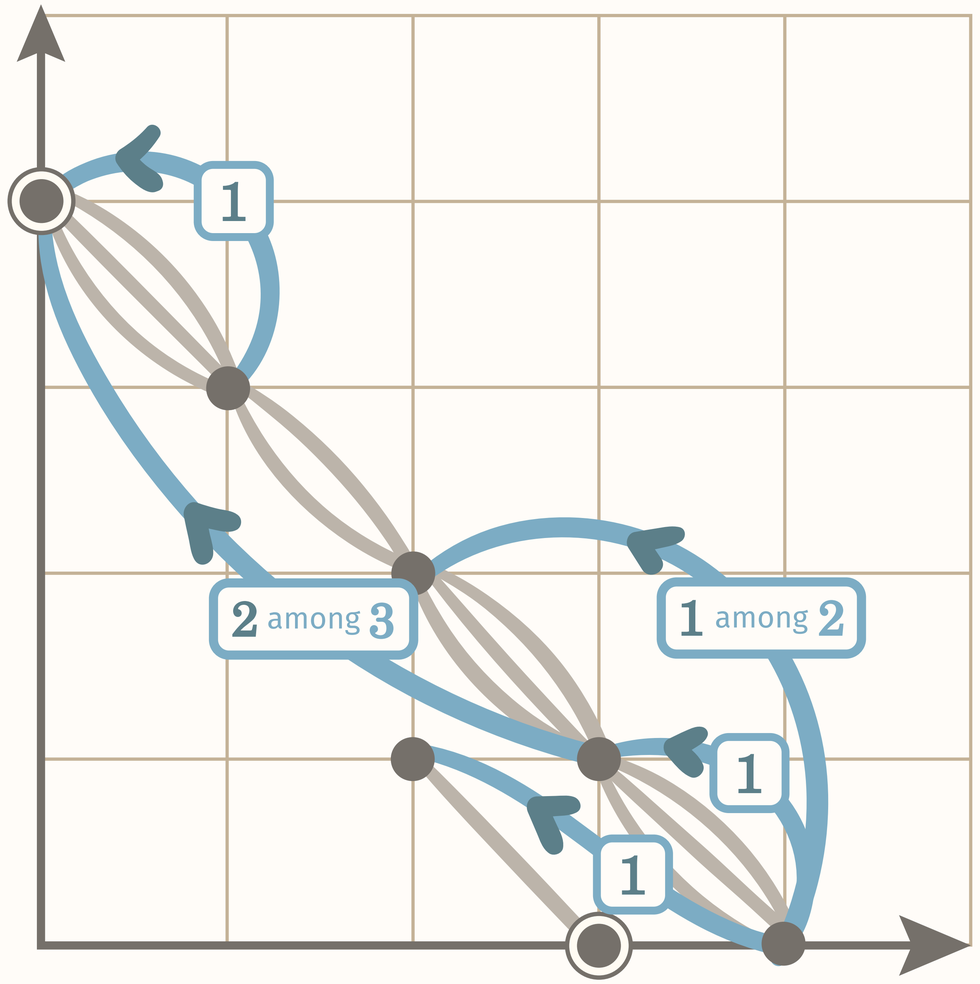


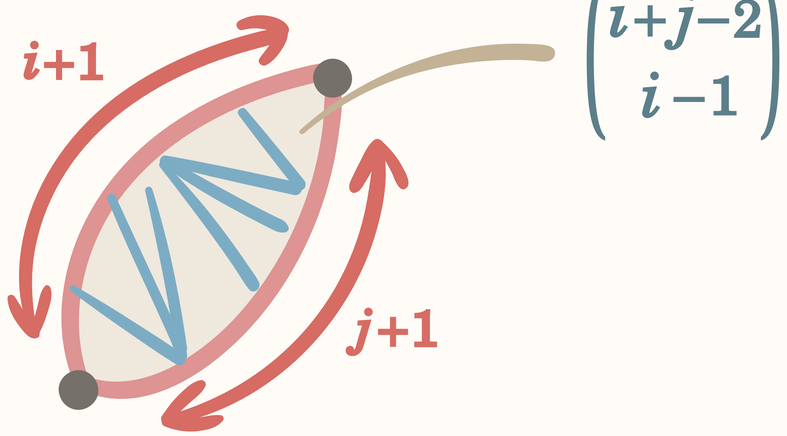


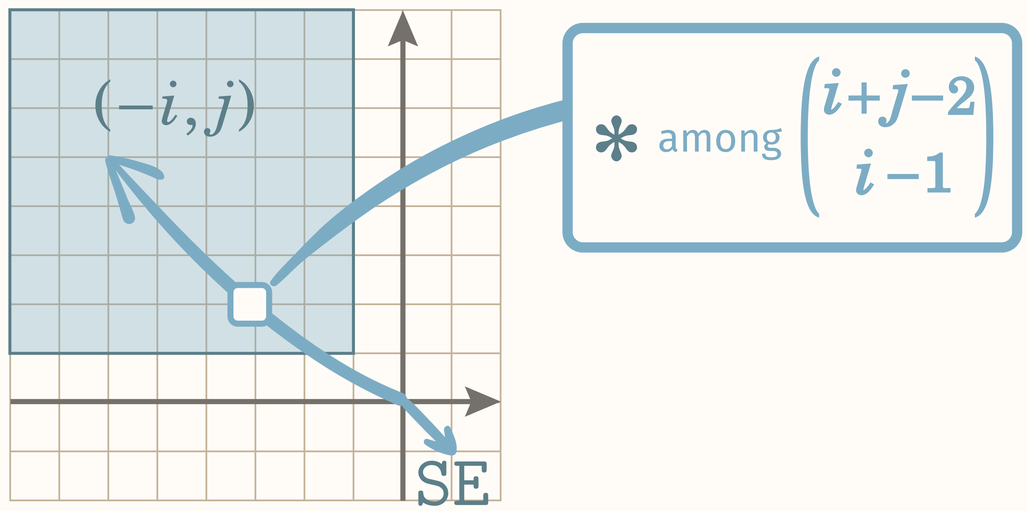


Application to transversal structures





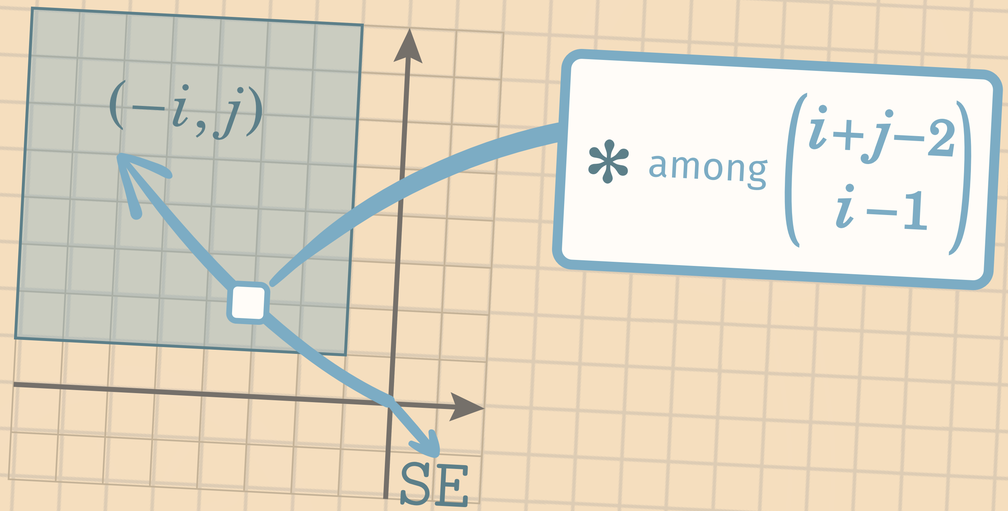




Summary

Quadrant tandem walks

Decorated bipolar orientations



bipolar posets by vertices

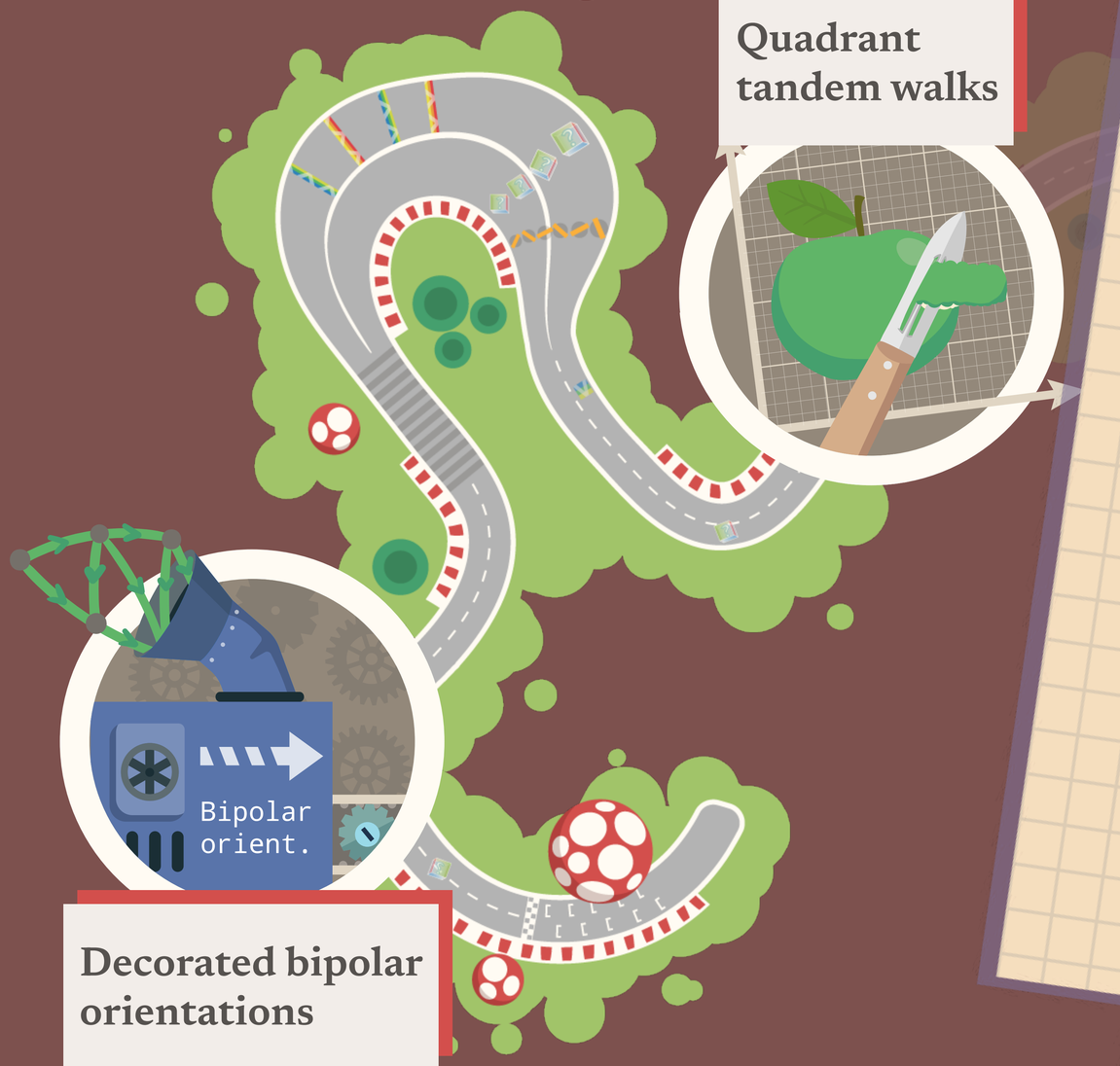
bipolar posets by edges

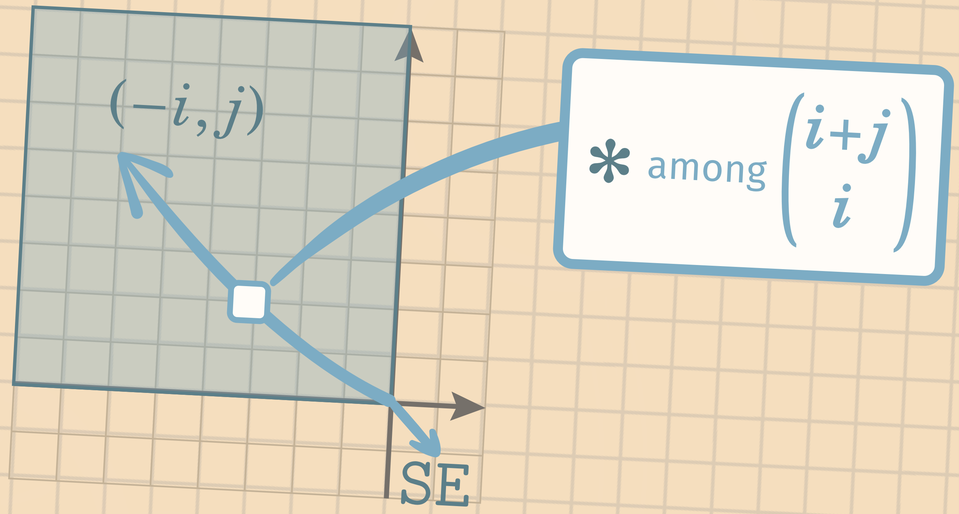
transversal structures

polyhedral orientations

Schnyder labelings

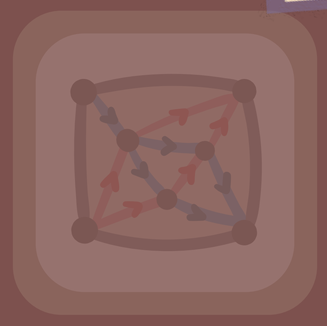
Summary





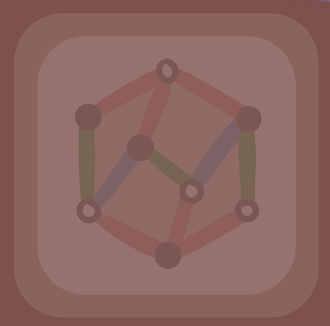
bipolar posets by vertices

bipolar posets by edges



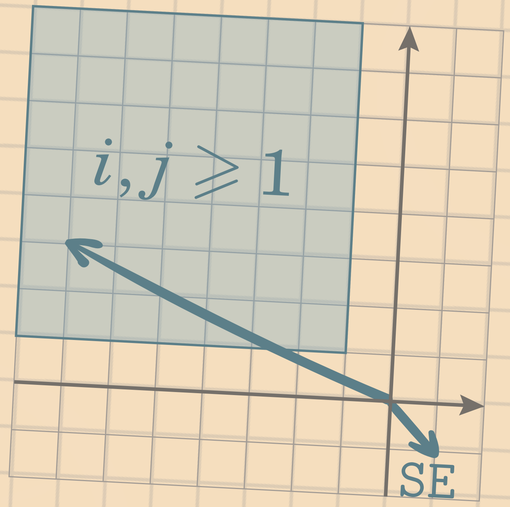
transversal structures

polyhedral orientations



Schnyder labelings

Summary



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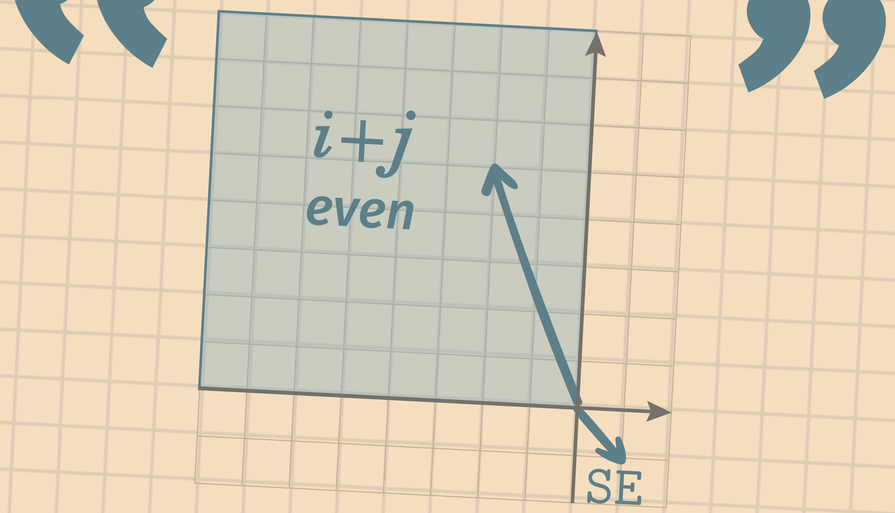
polyhedral orientations

Schnyder labelings

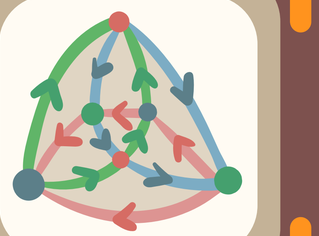
Summary



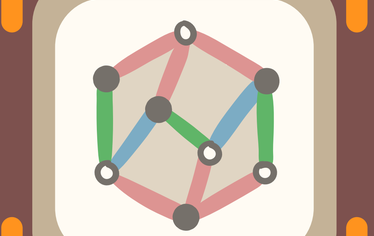
bipolar posets by vertices

bipolar posets by edges

transversal structures

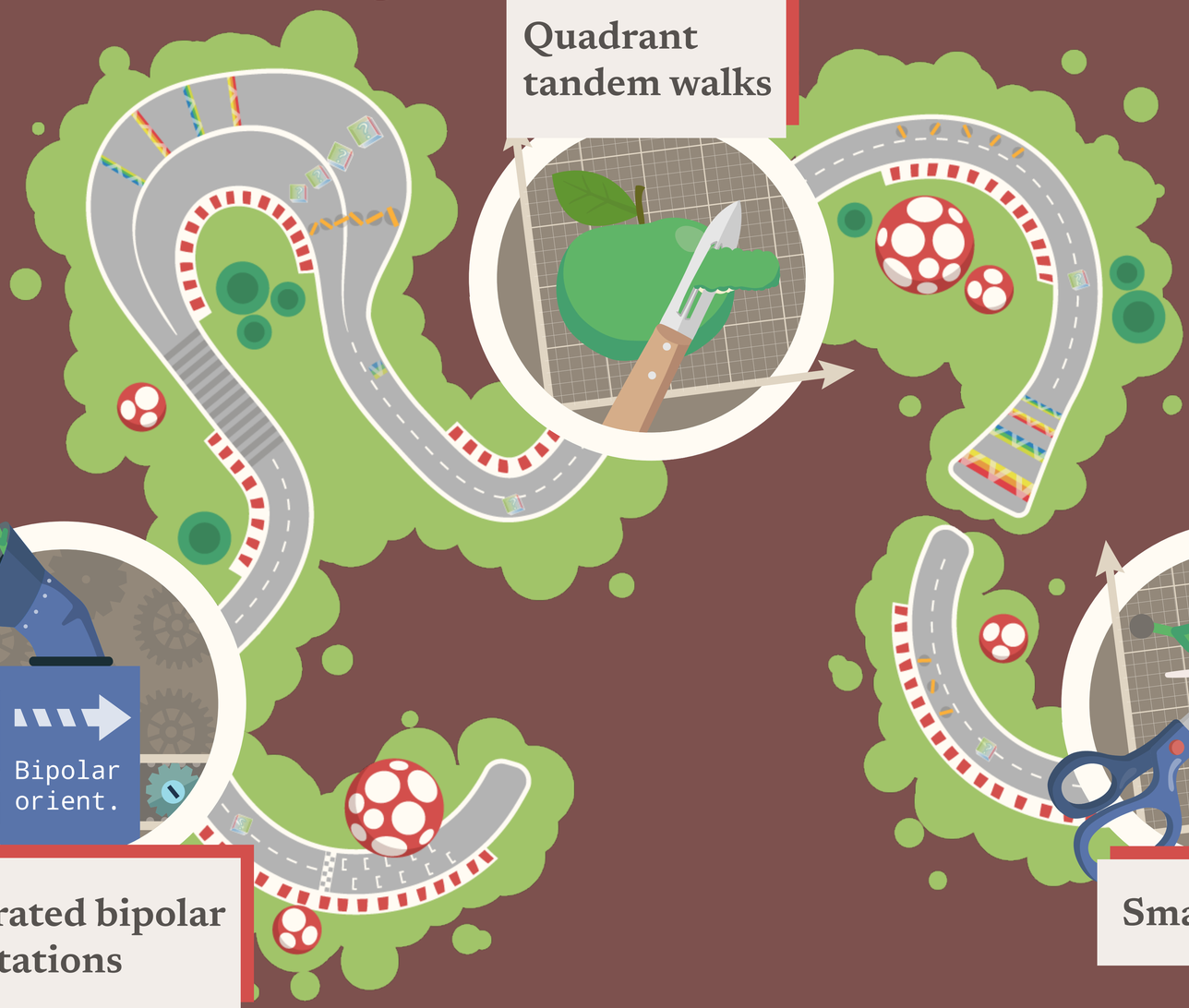


polyhedral orientations

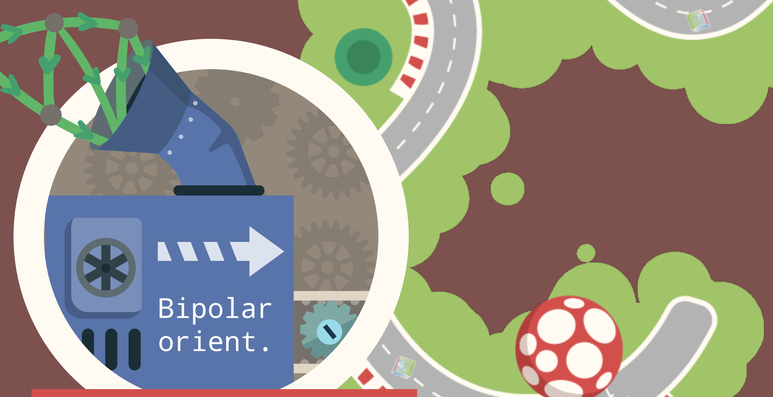


Schnyder labelings

Summary



Quadrant tandem walks



Decorated bipolar orientations

Small steps

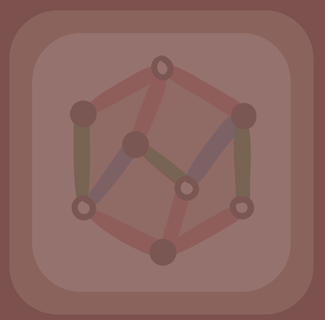
Asymptotics

bipolar posets by vertices

bipolar posets by edges

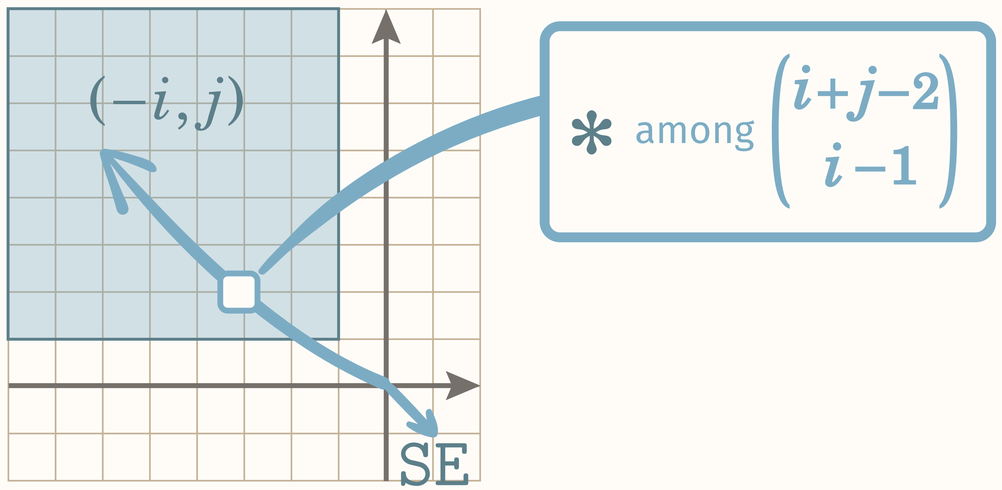
transversal structures

polyhedral orientations



Schnyder labelings

Breaking into small steps



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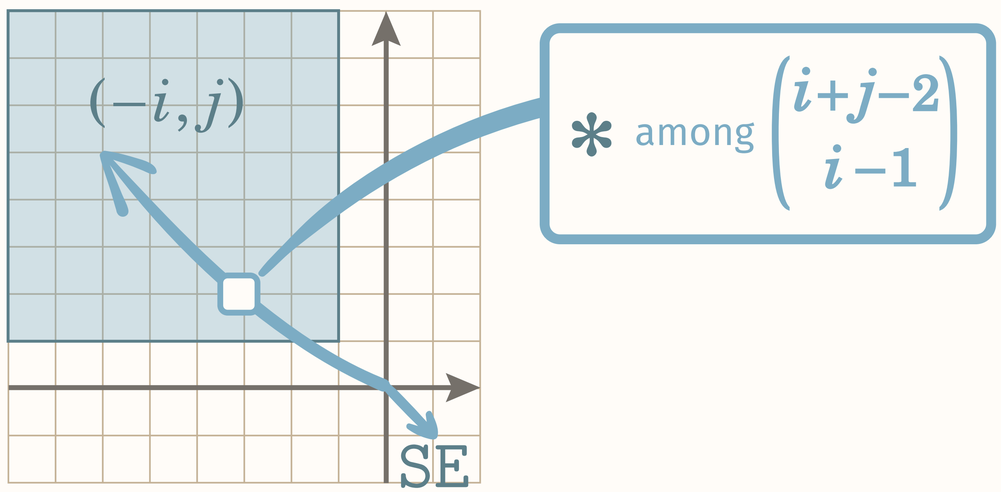

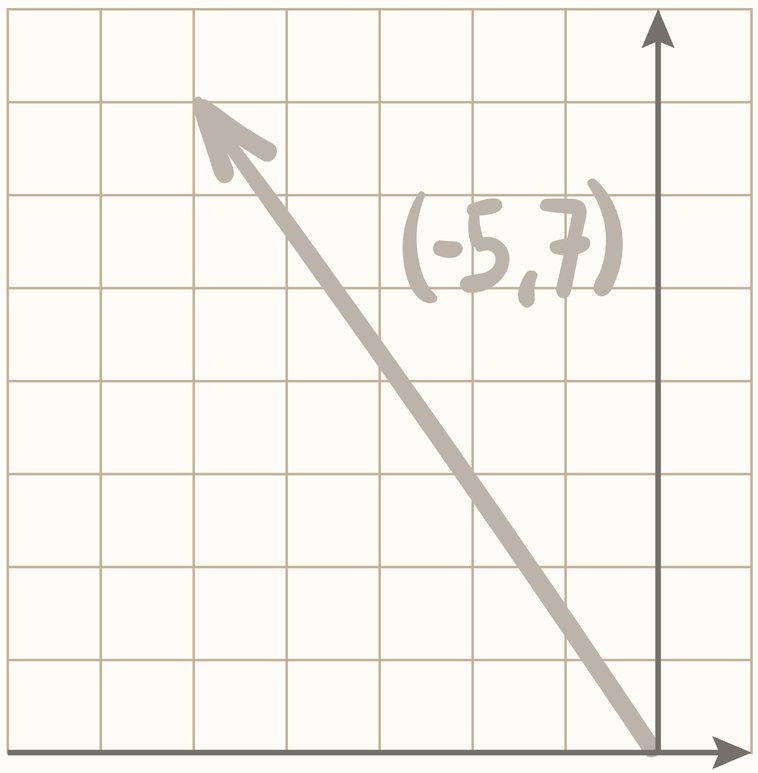

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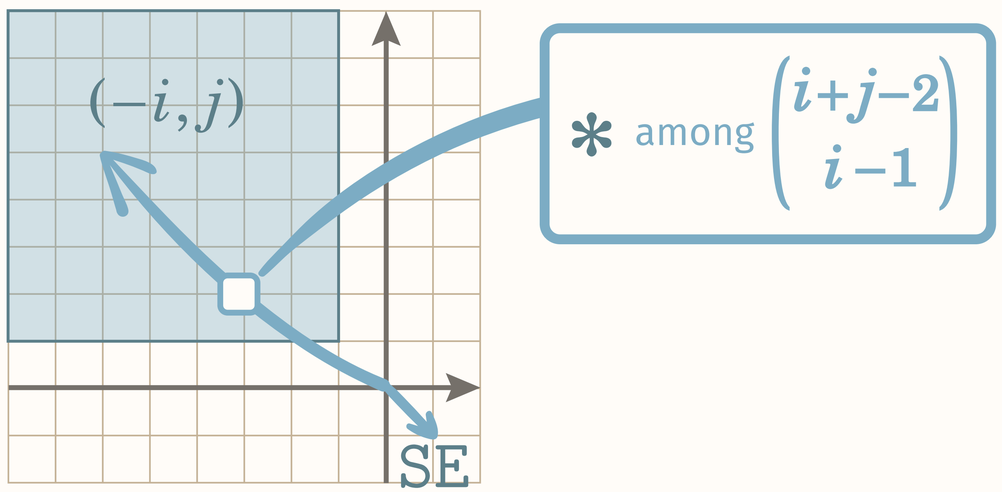

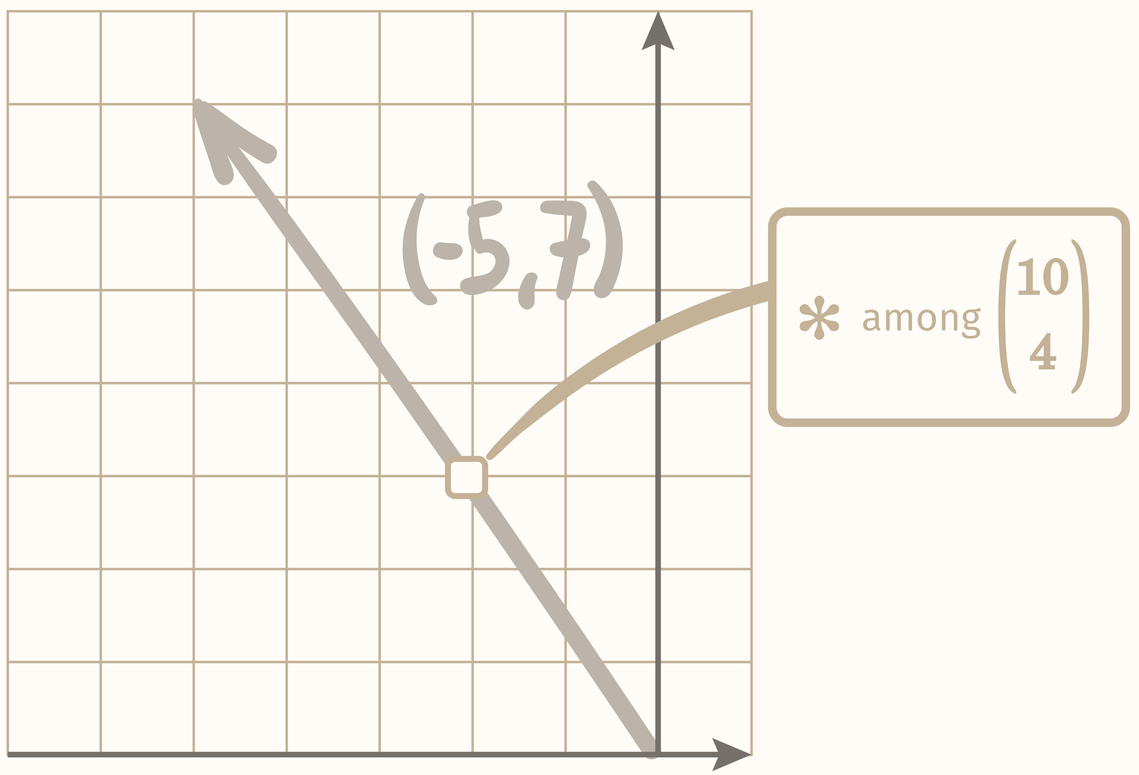

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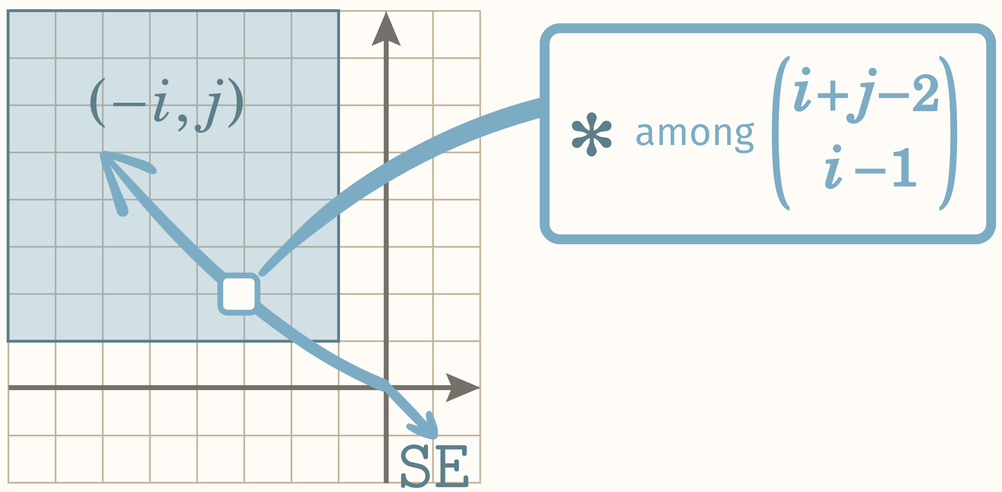

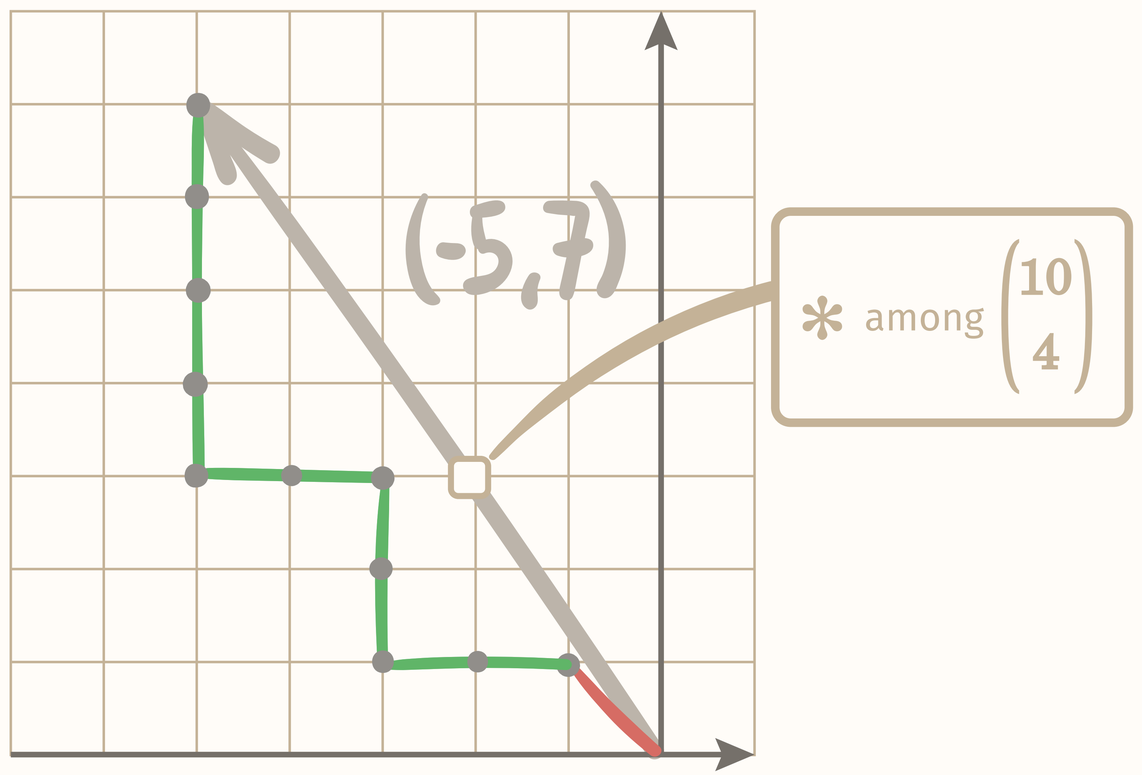

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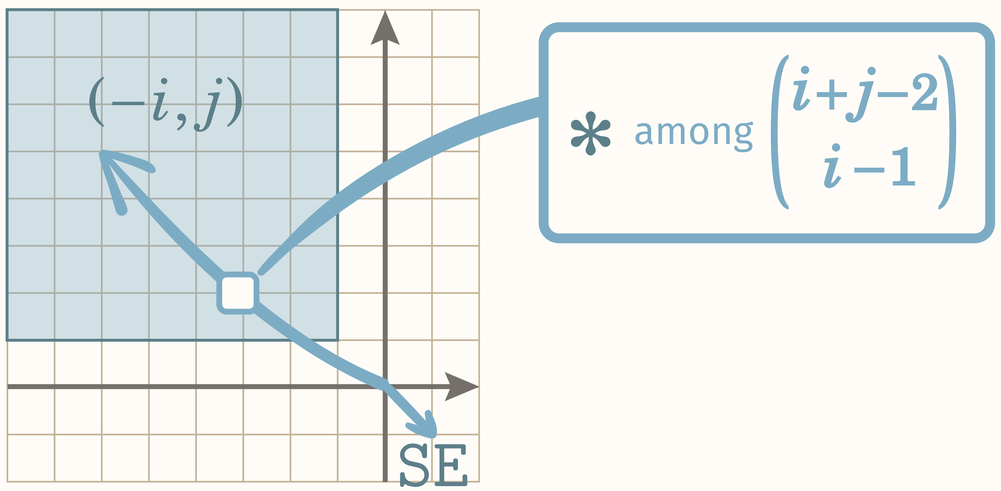

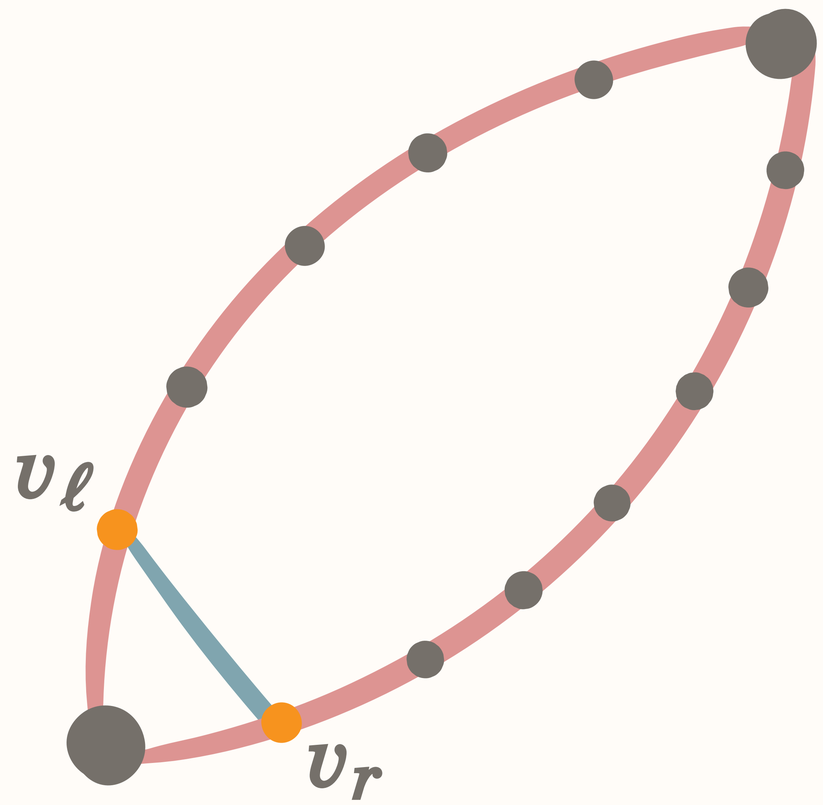

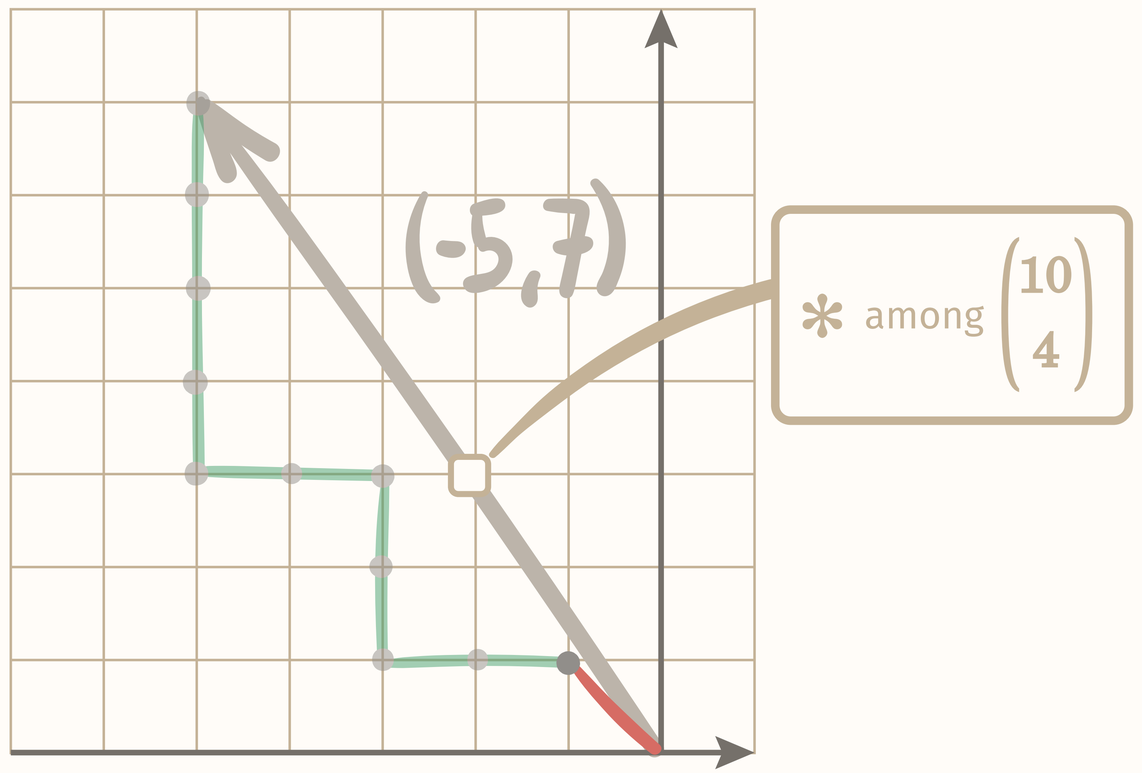

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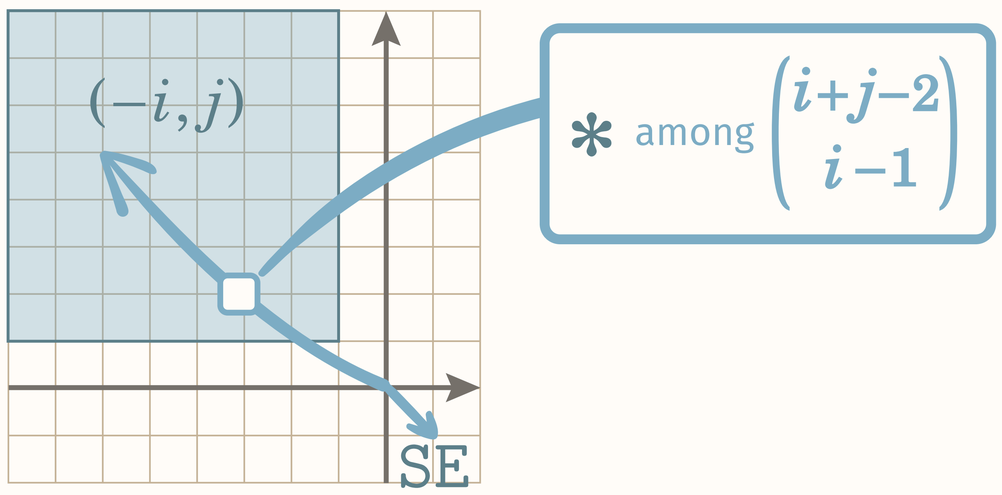

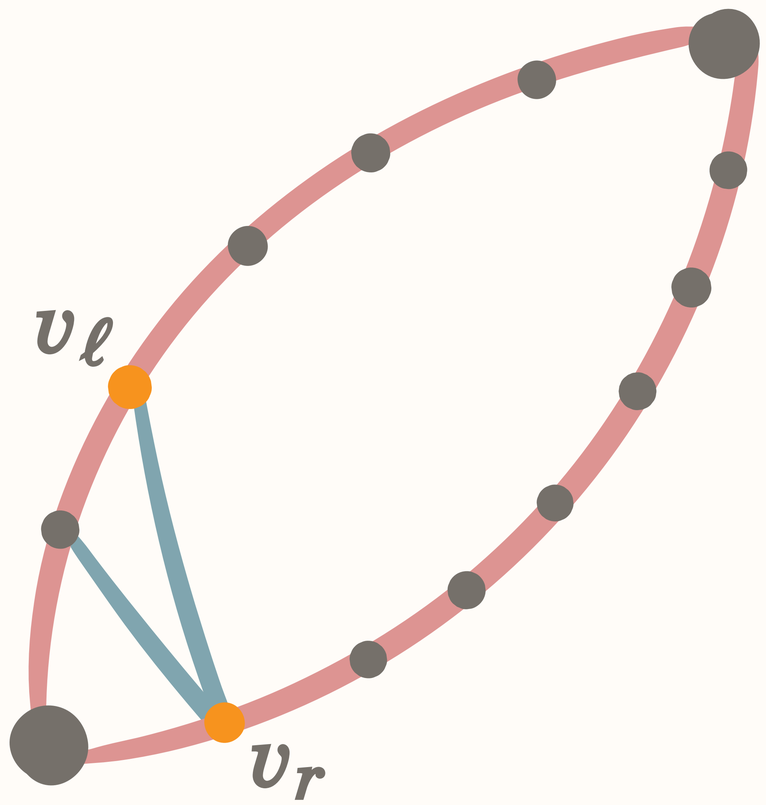

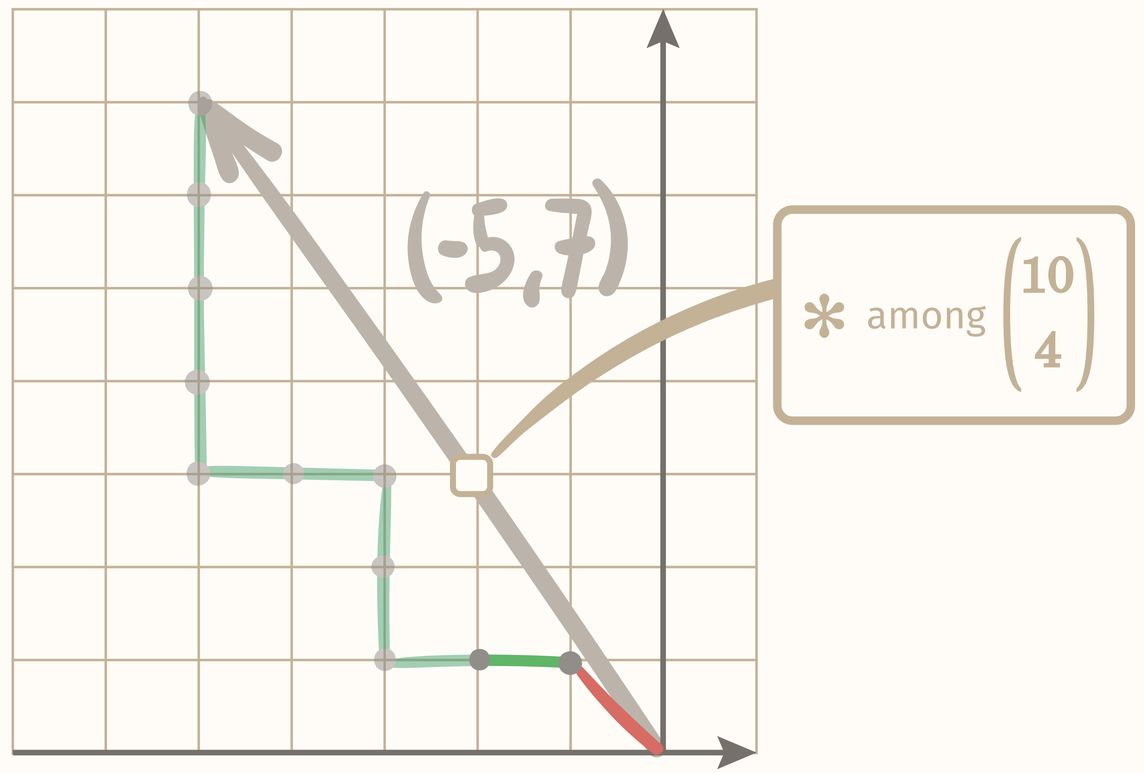

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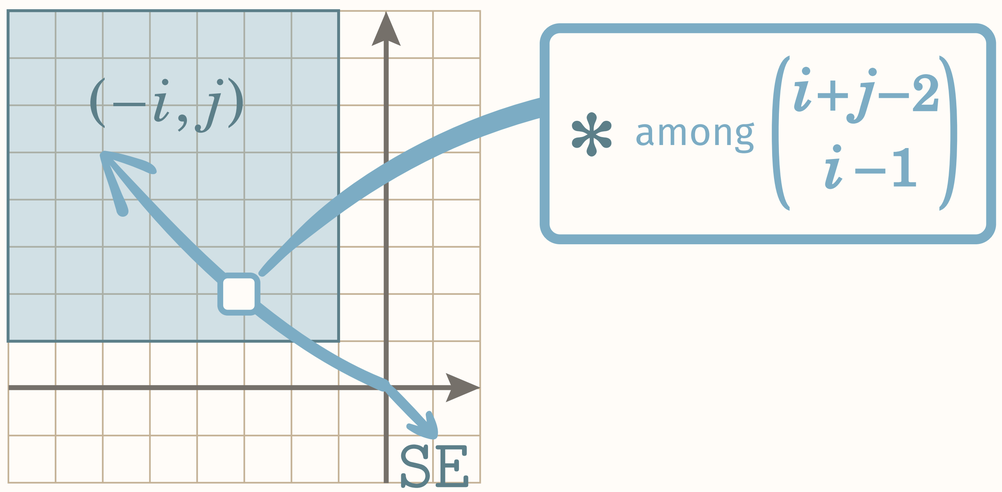

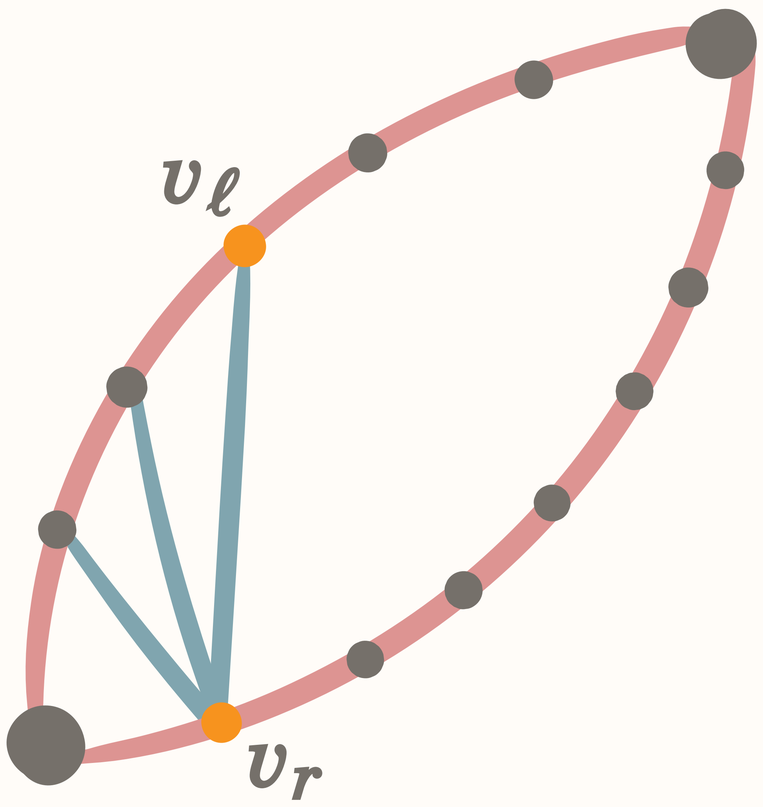

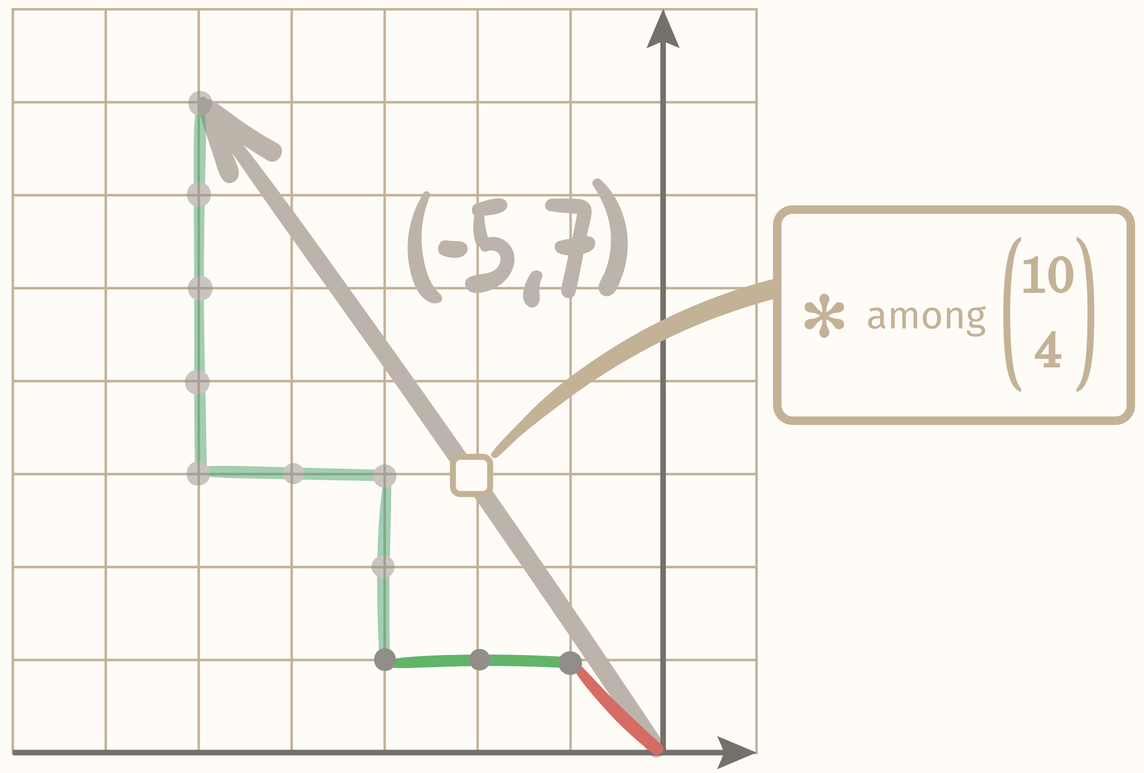

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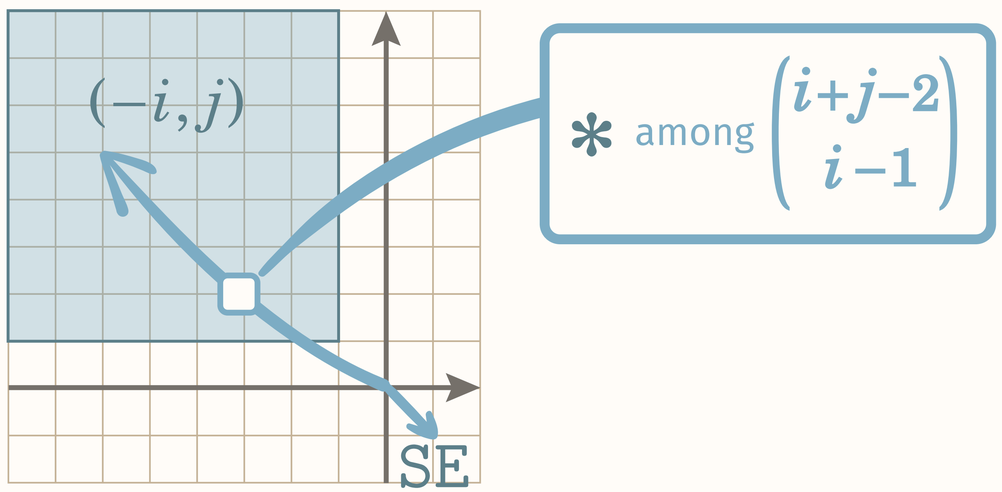

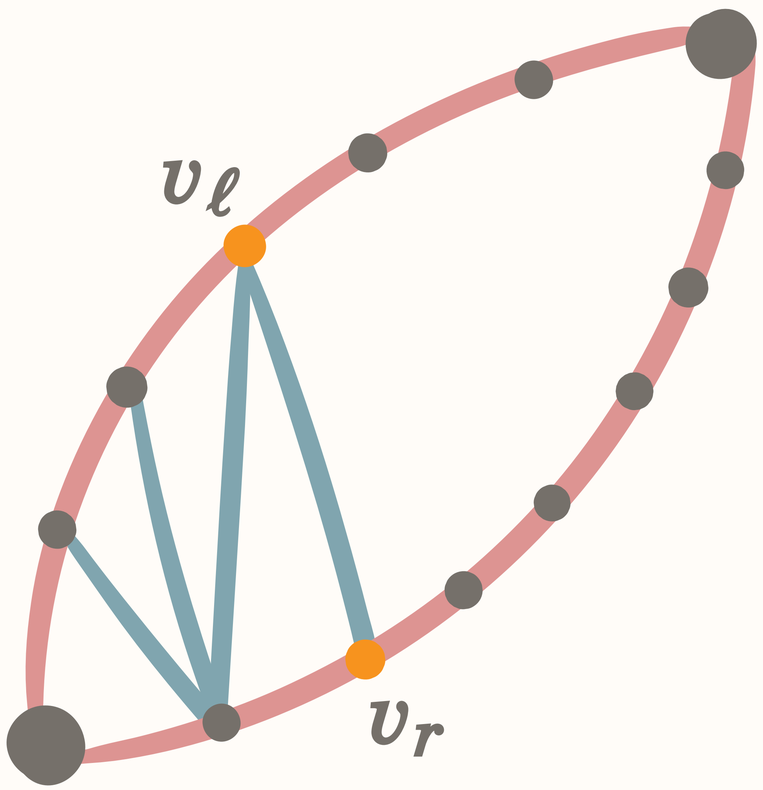

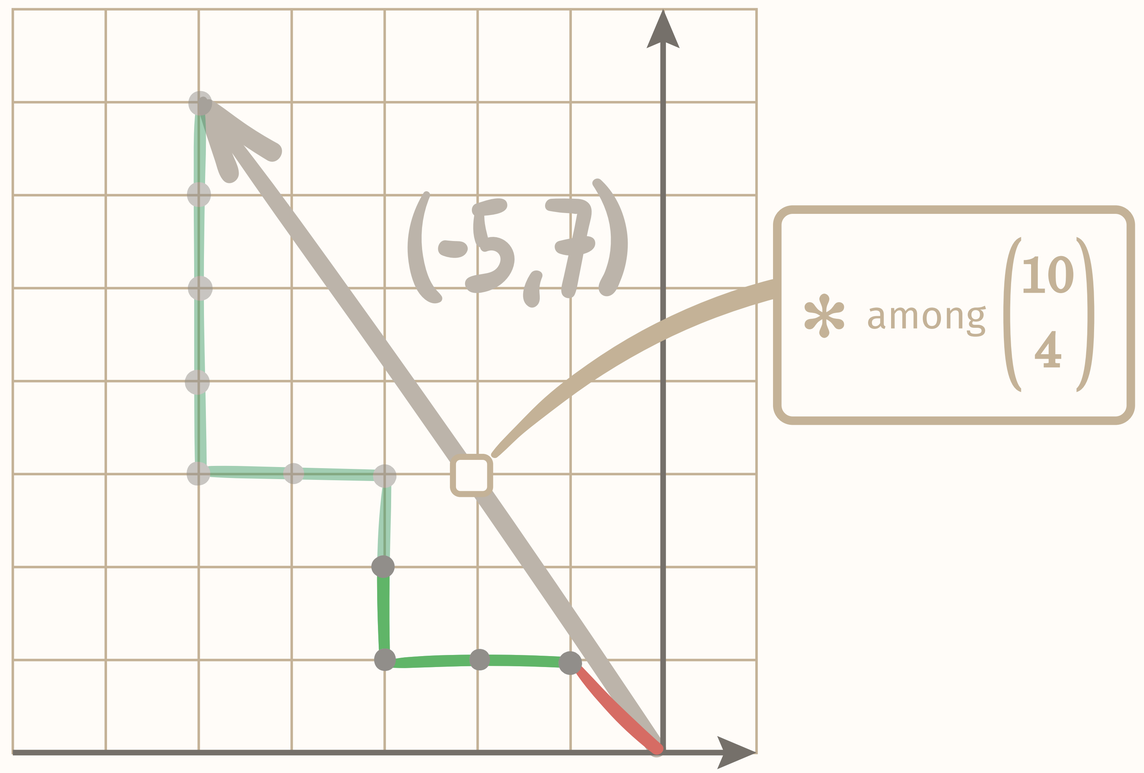

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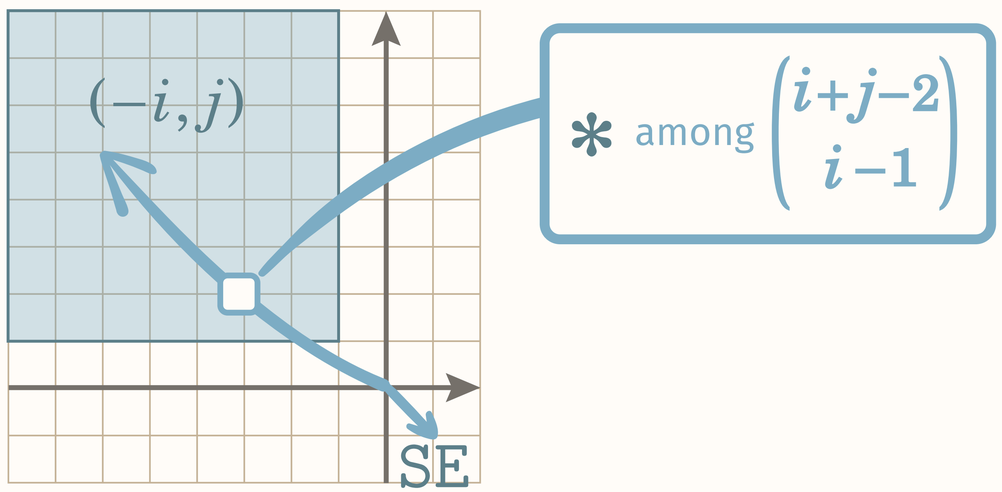

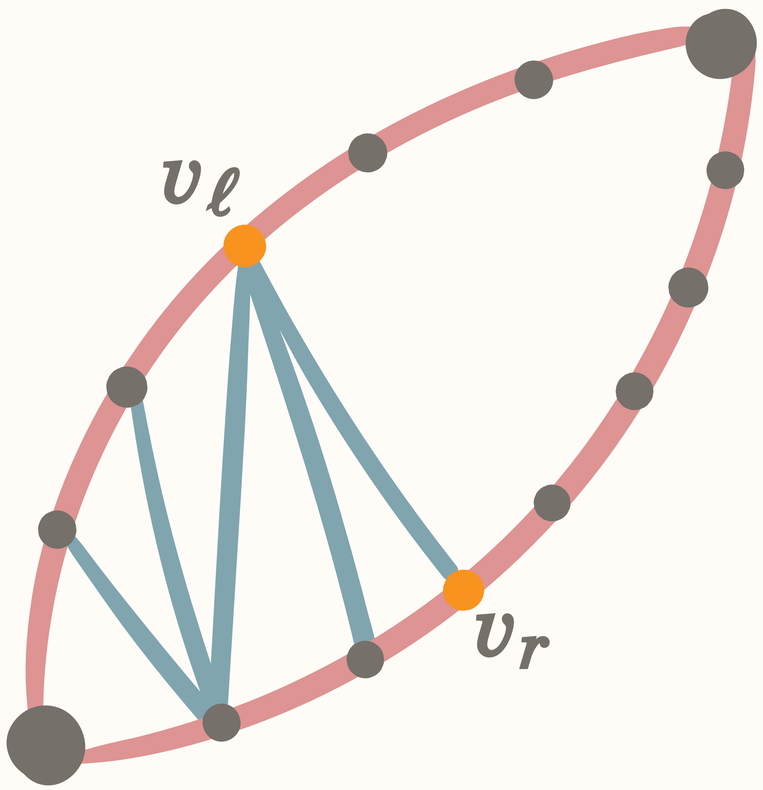

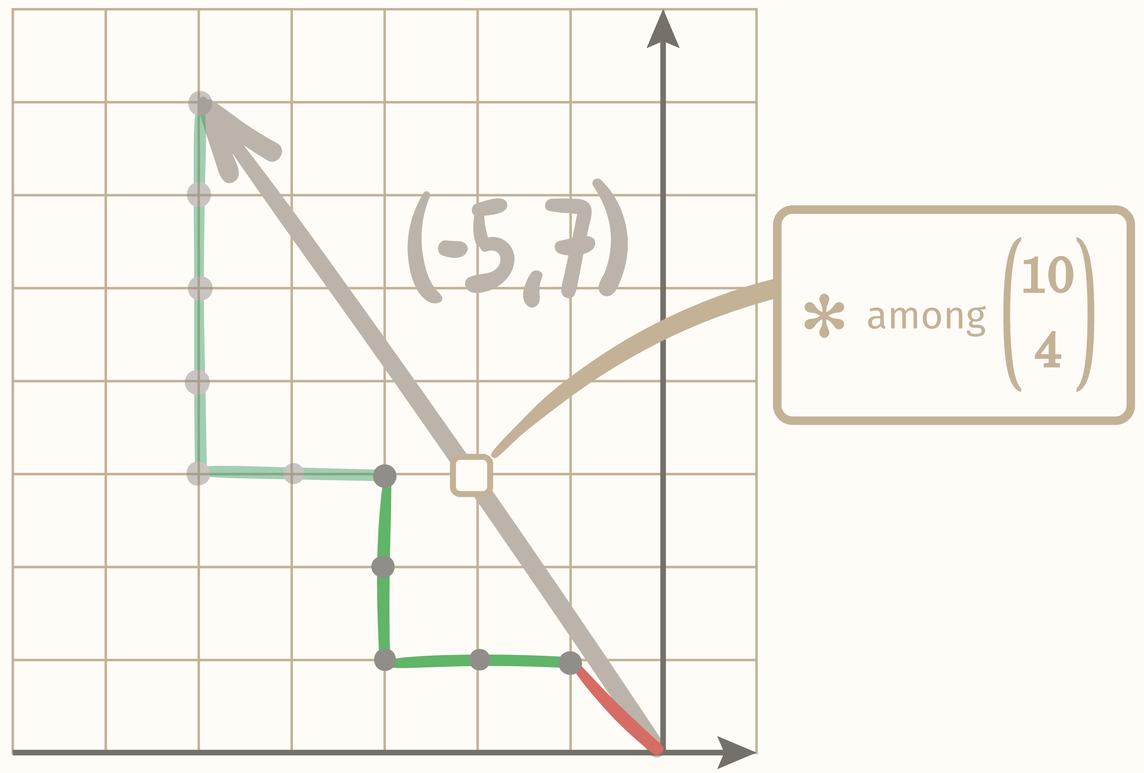

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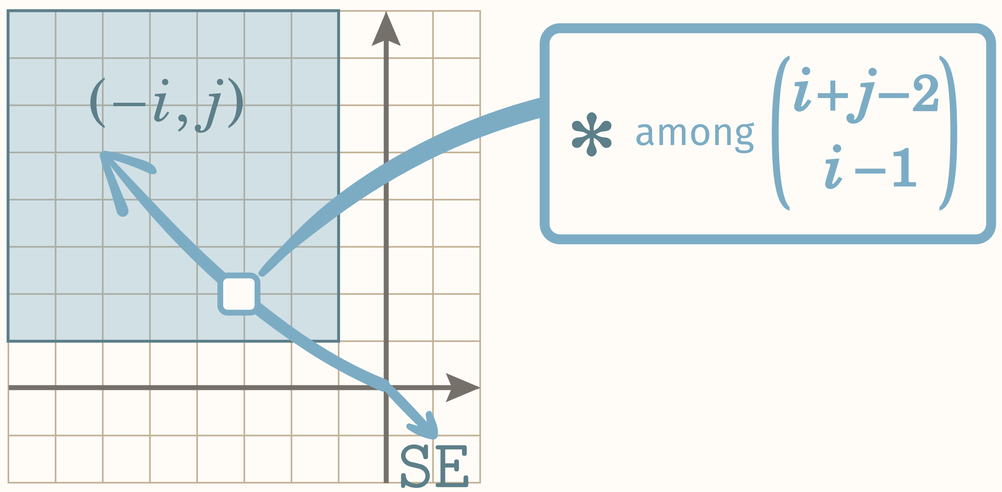
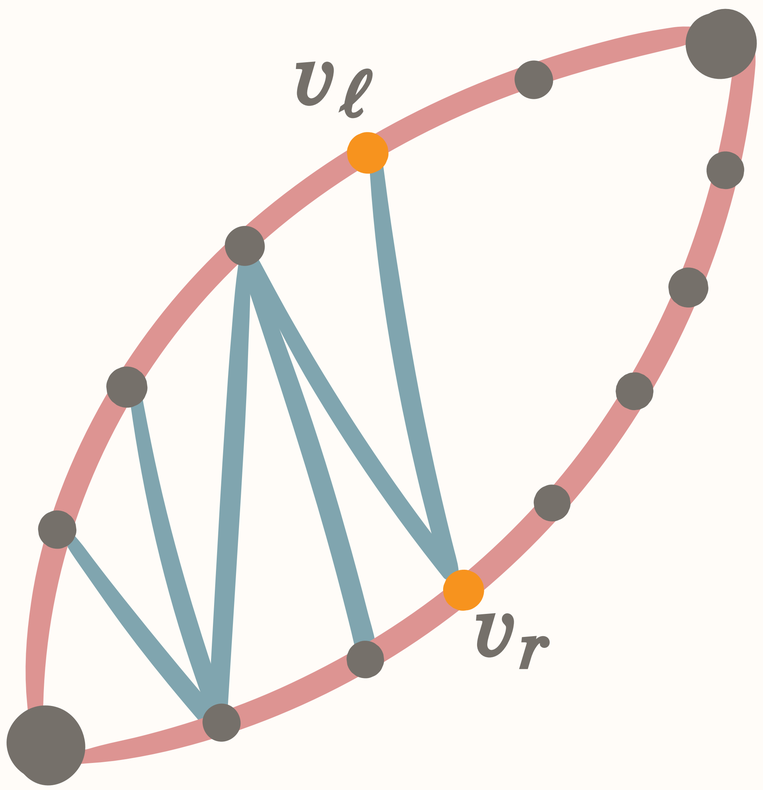
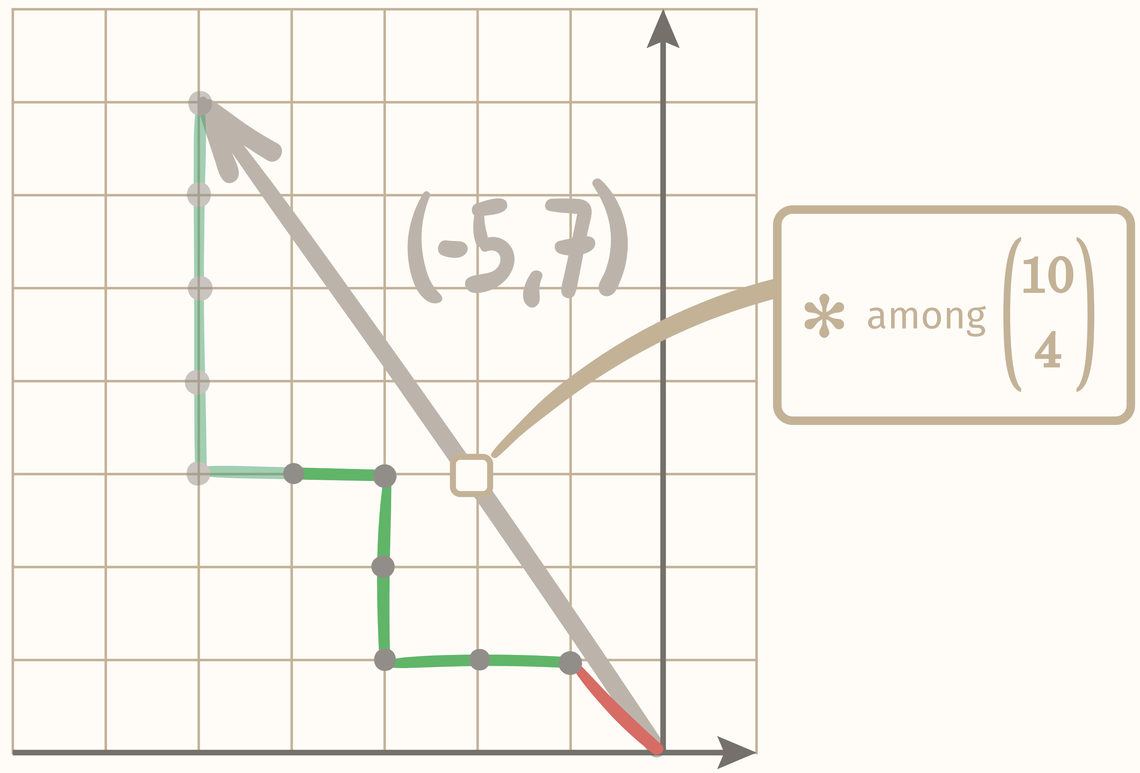
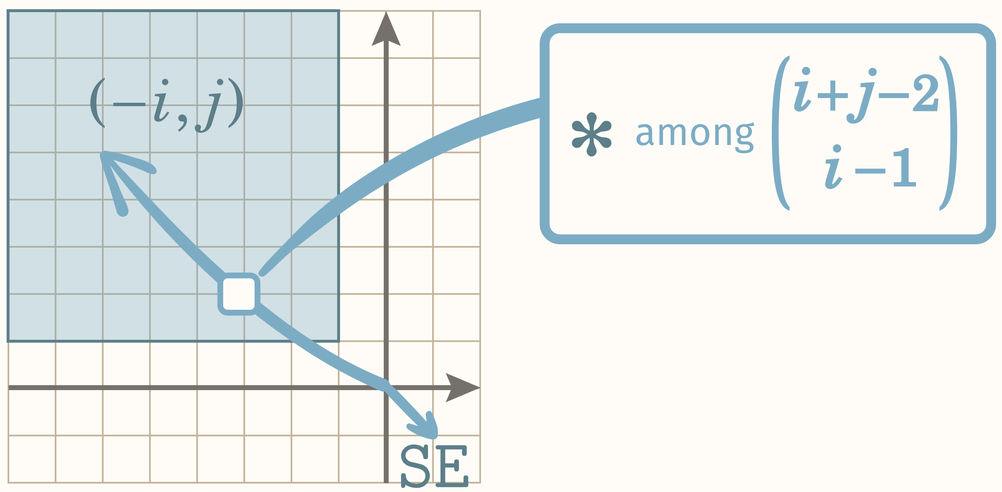
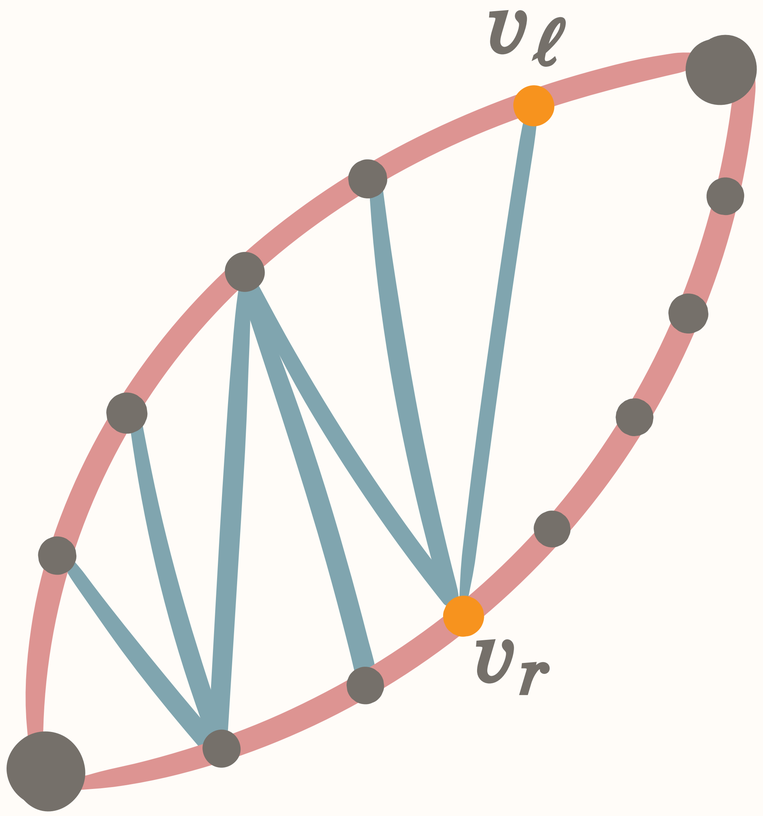


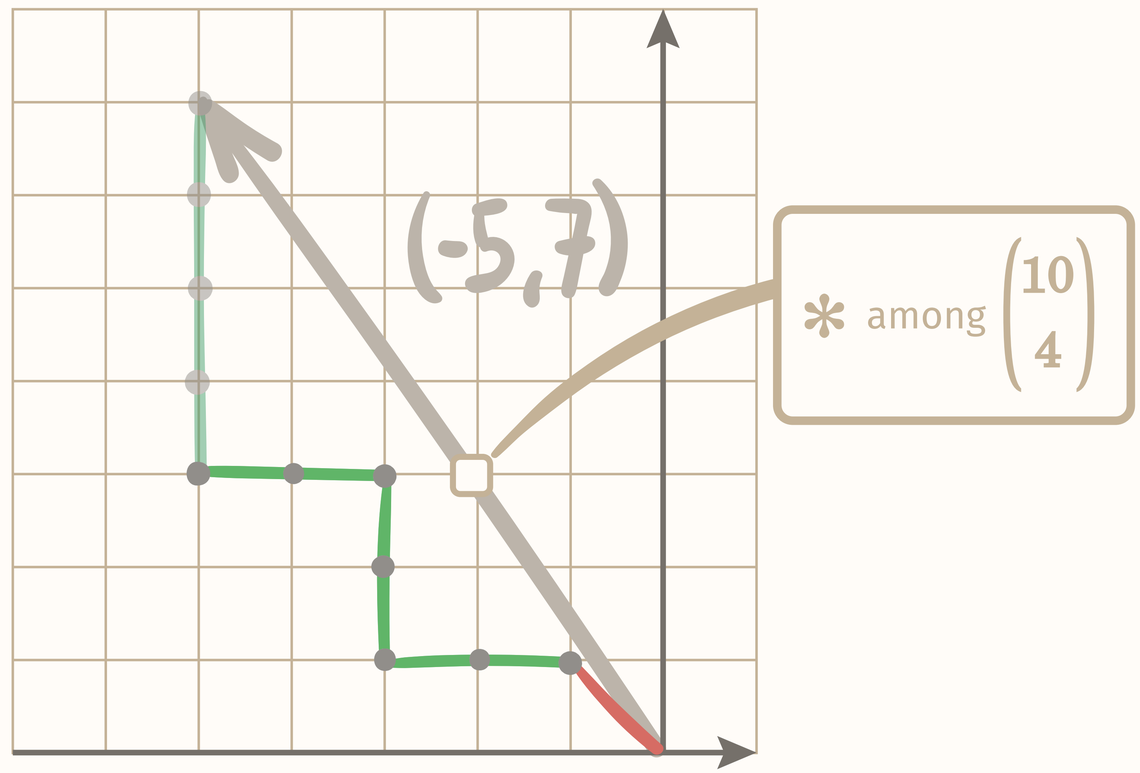




Breaking into small steps







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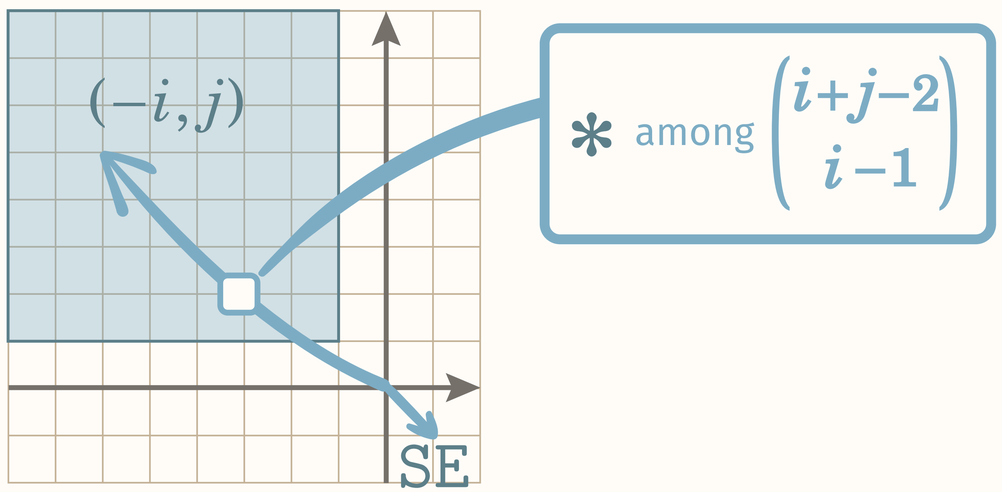

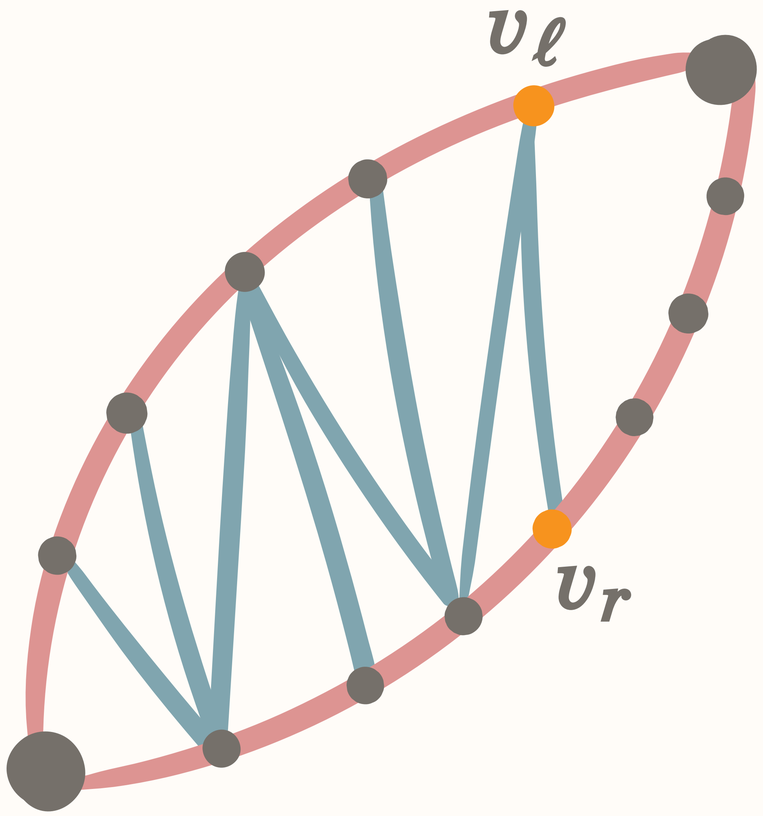

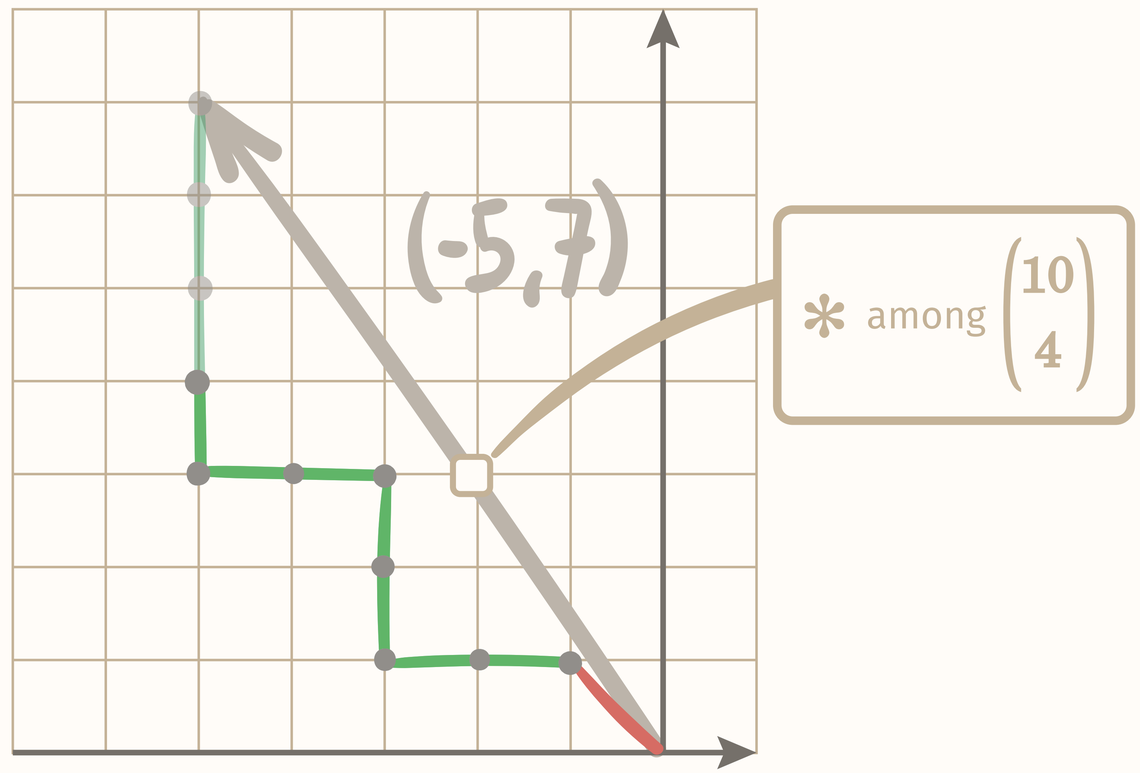

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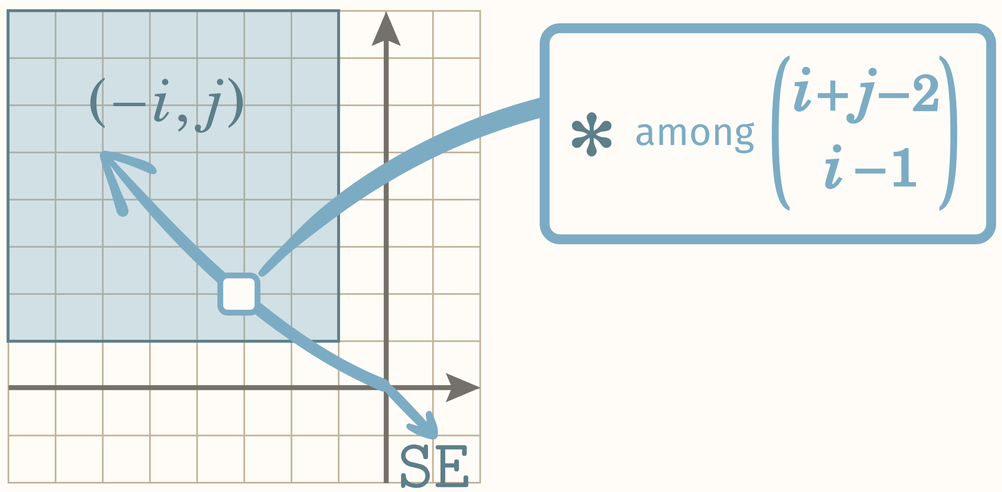

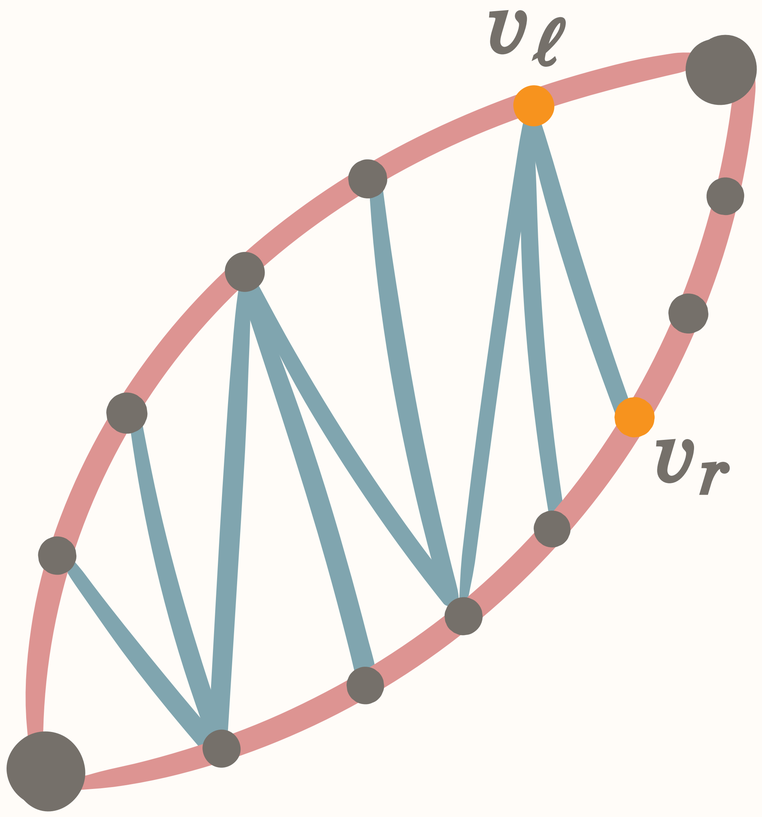

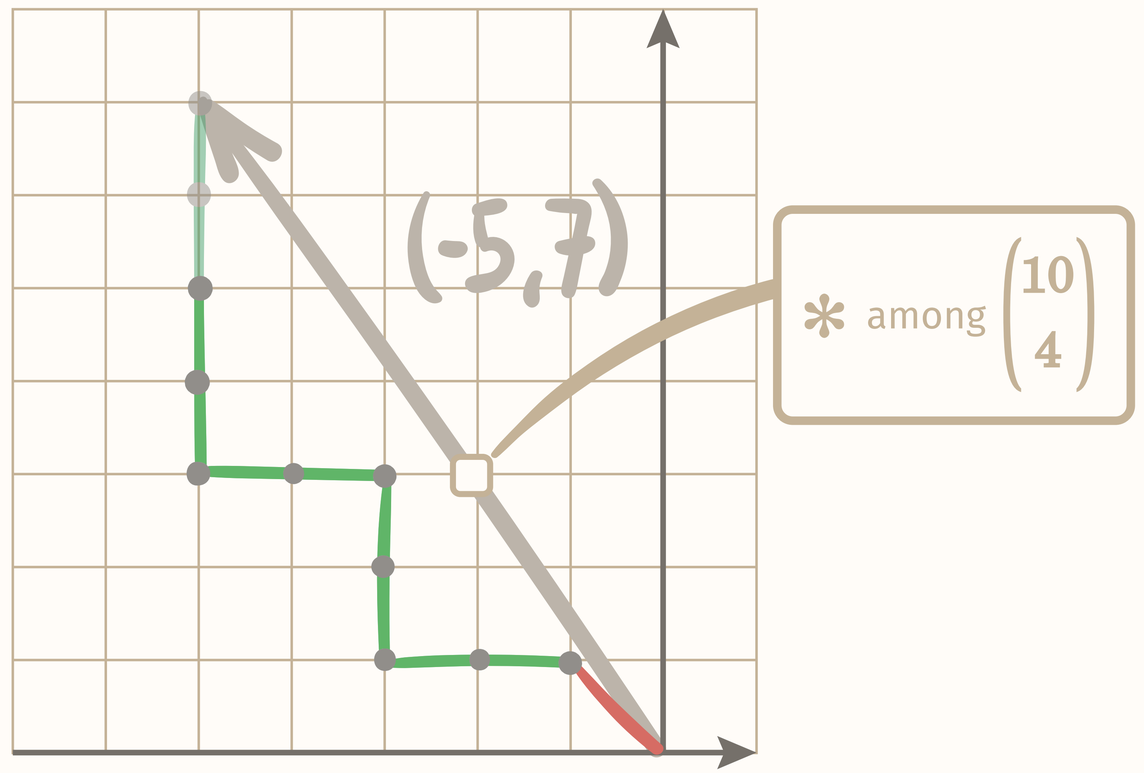

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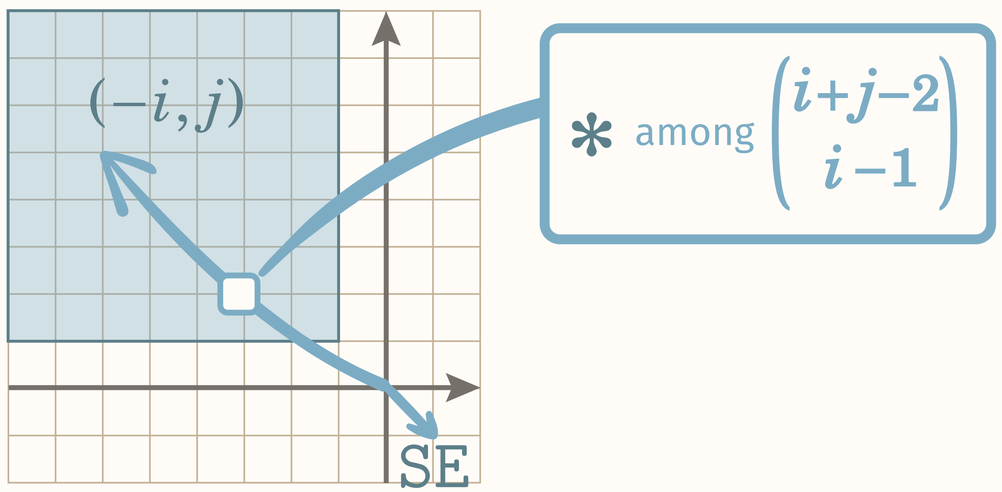

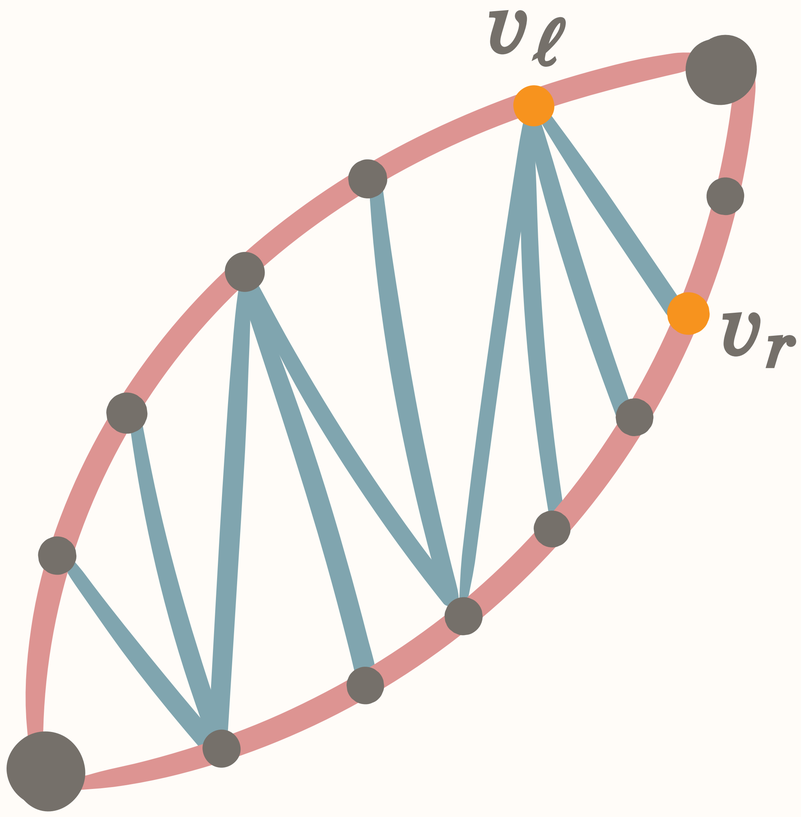

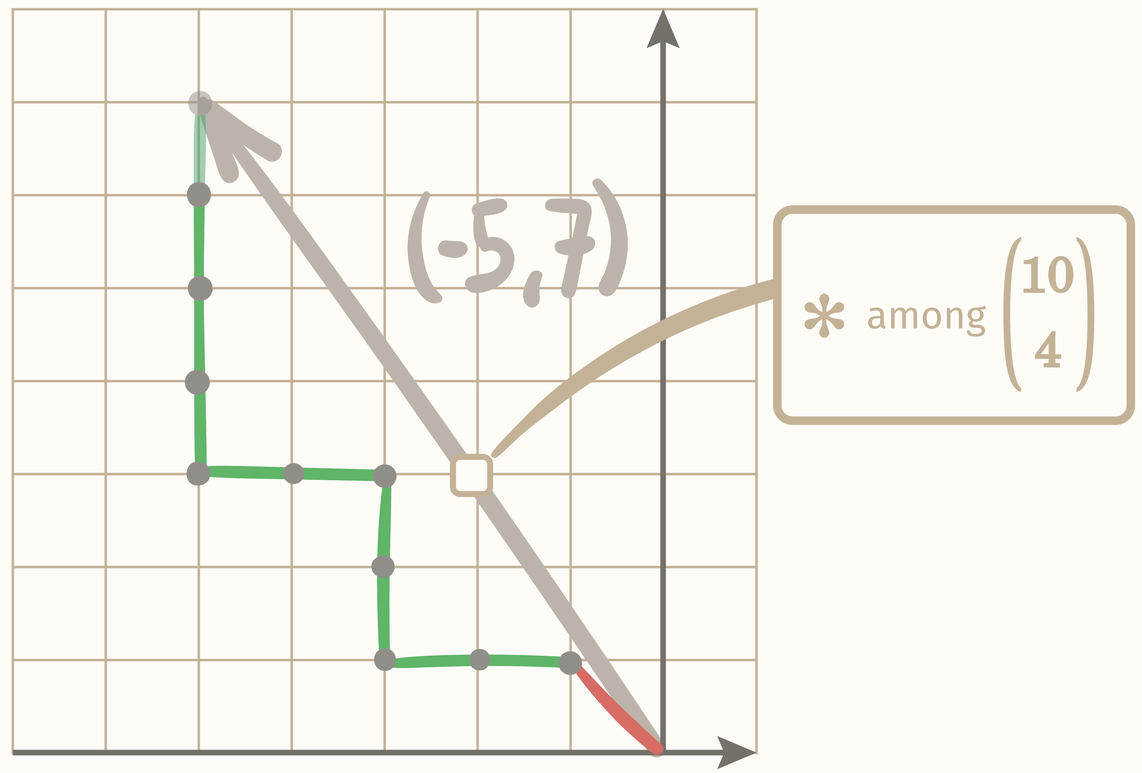

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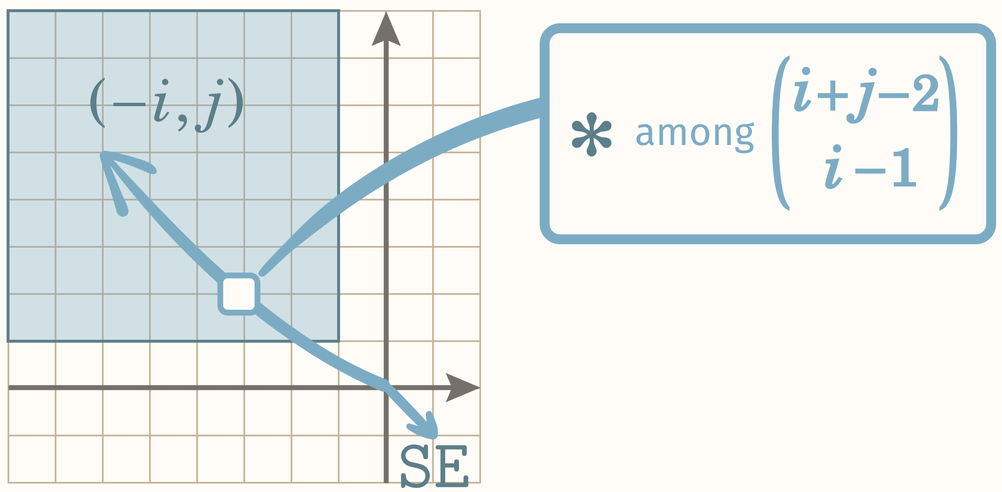

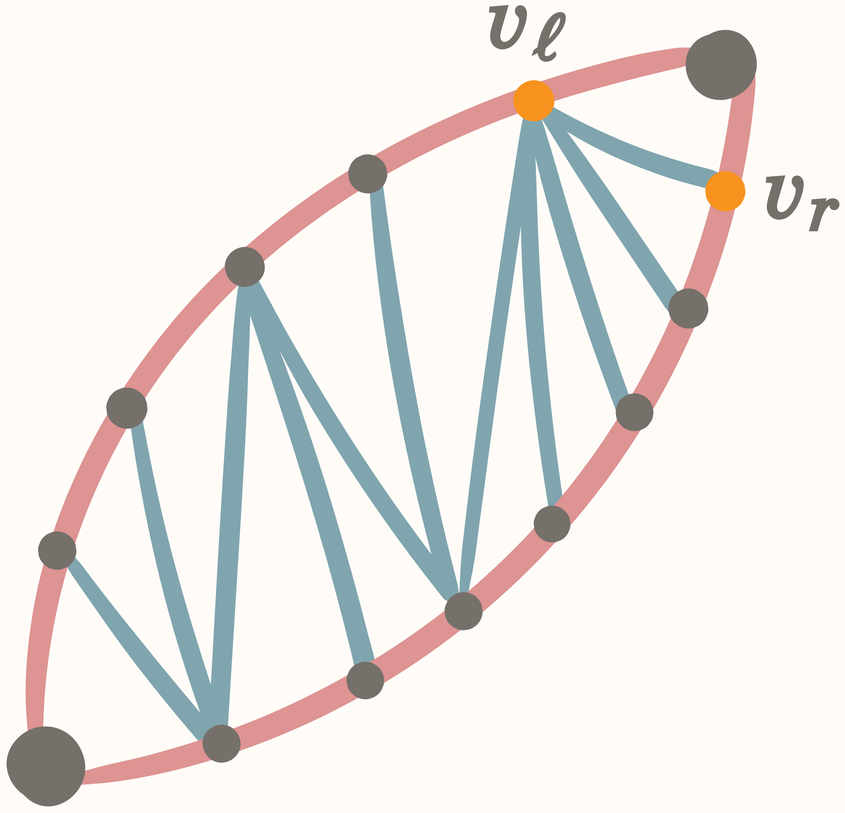

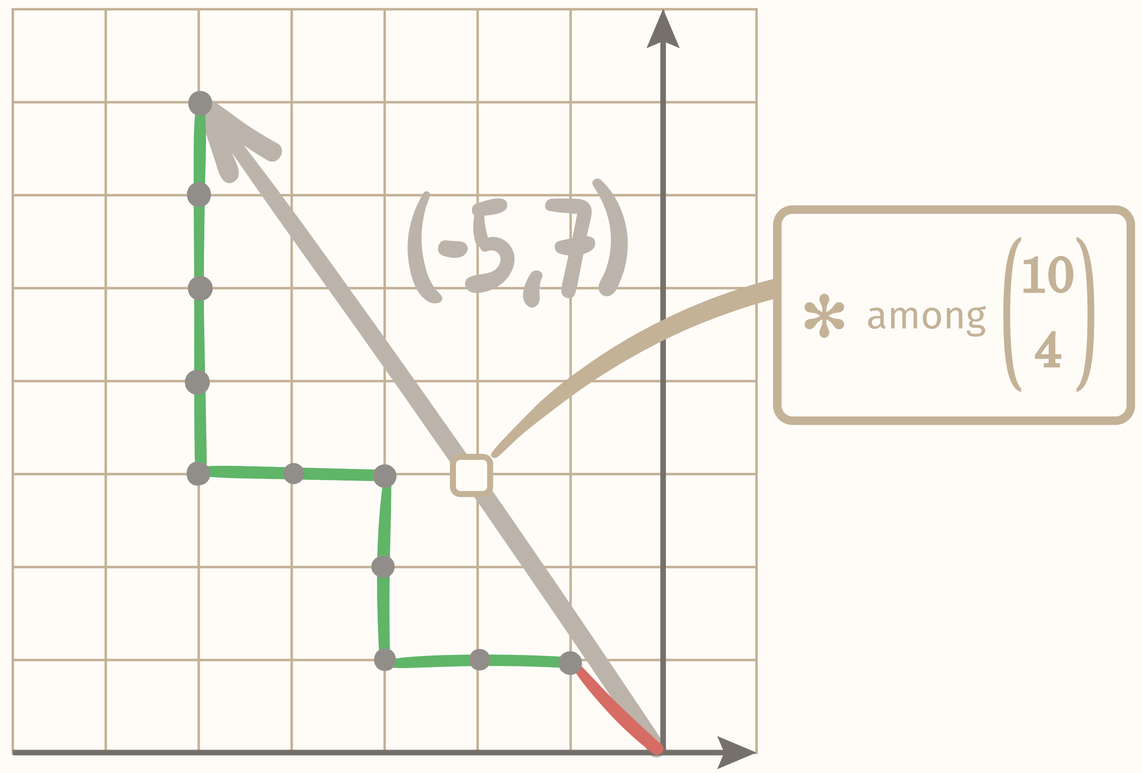

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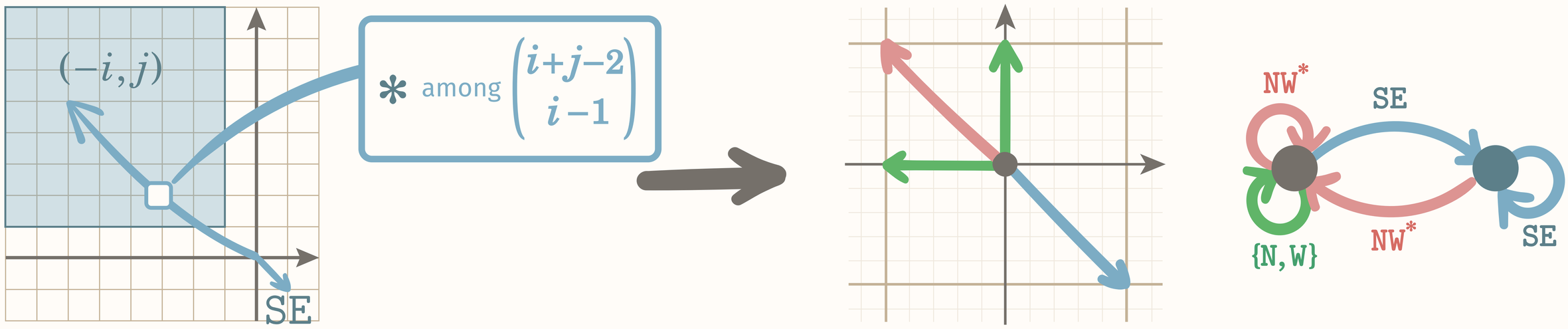

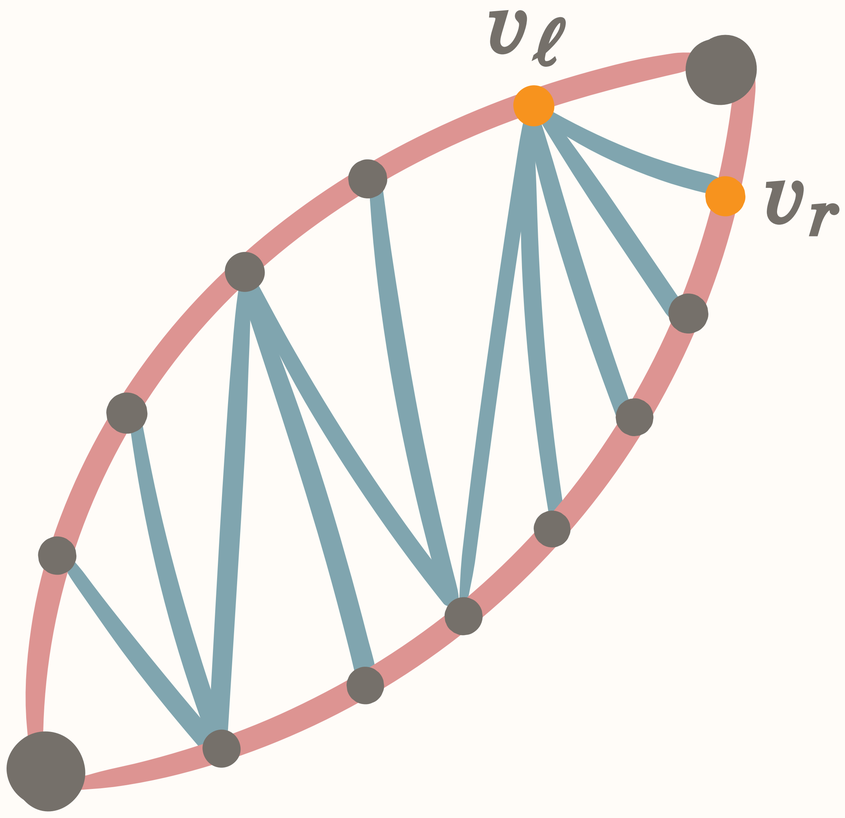

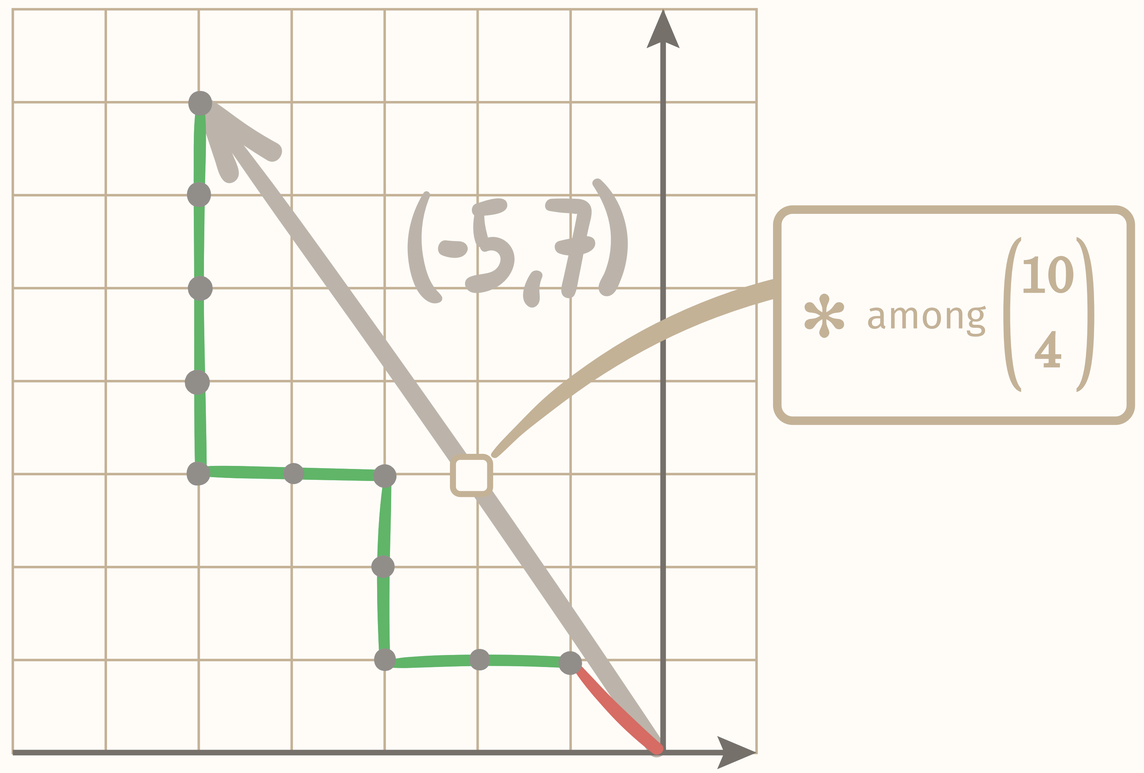

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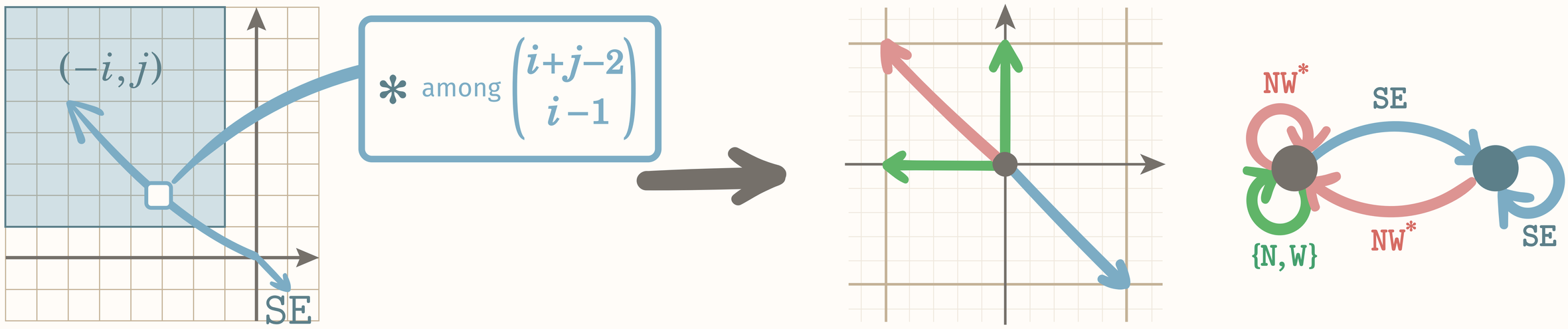


$$1, 1, 2, 6, 24, 116, 642, 3938, 26194, 186042, \ldots$$

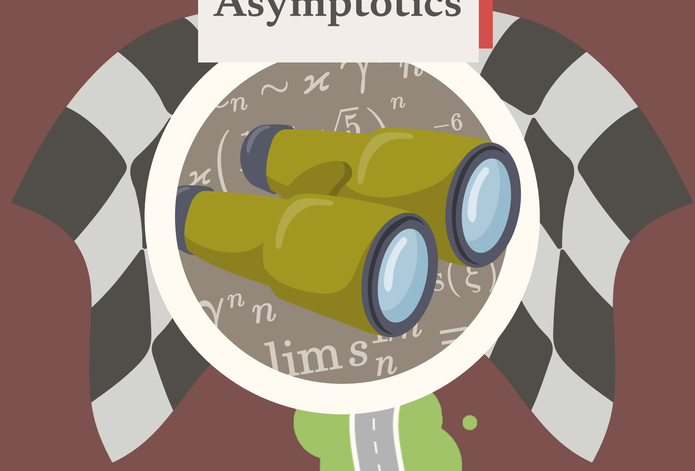
↠ *Counting rectangular drawings* T. Takahashi, R. Fujimaki; Y. Inoue (2009)

Summary

Decorated bipolar orientations

Quadrant tandem walks

Small steps



Asymptotics

bipolar posets by vertices

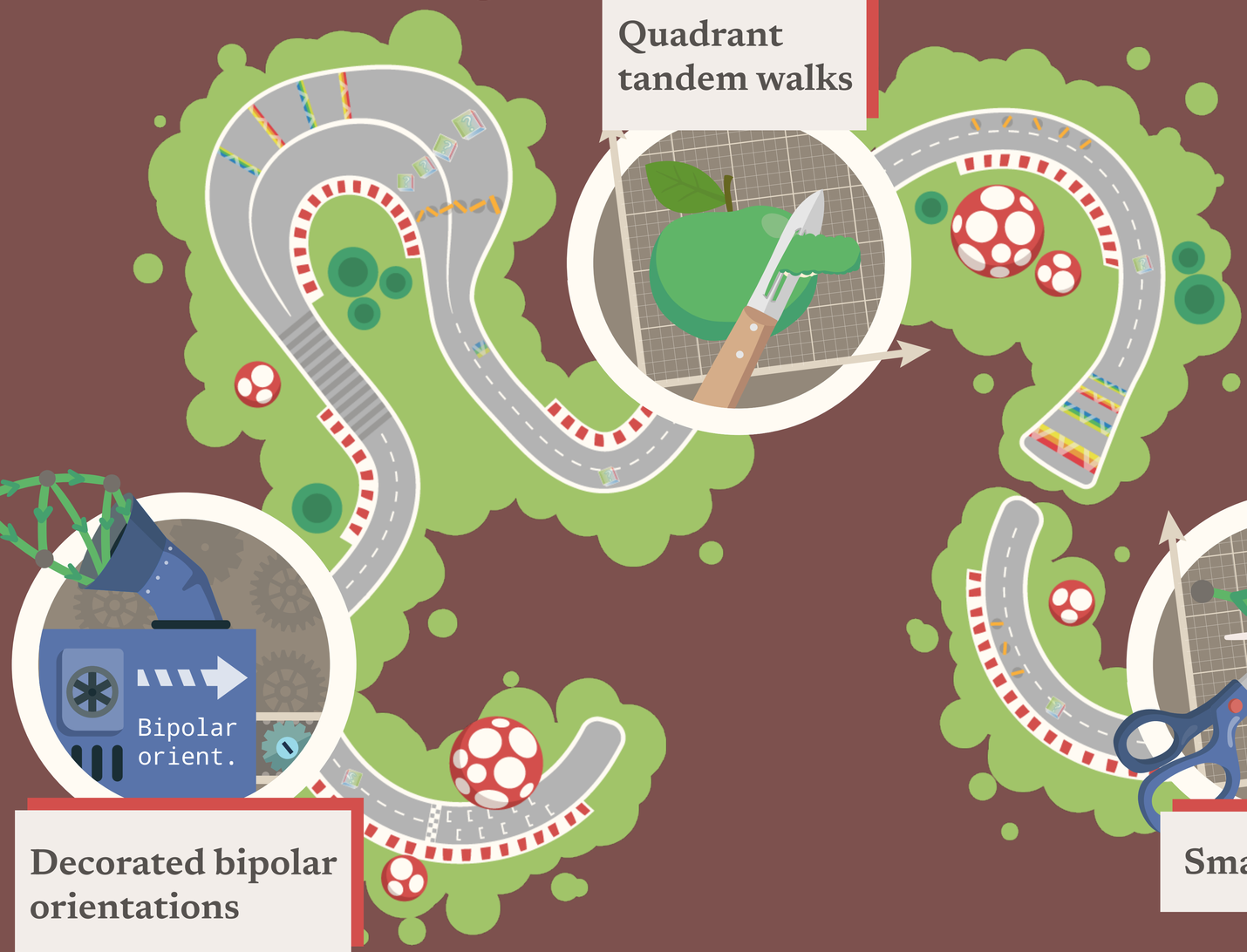
bipolar posets by edges

transversal structures

polyhedral orientations

Schnyder labelings

Summary



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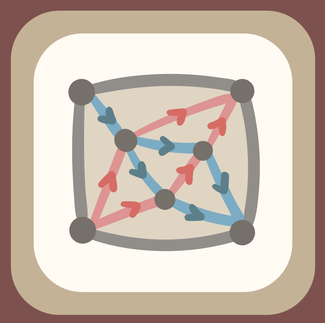

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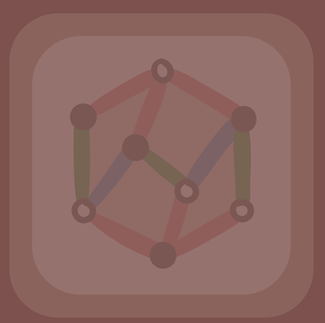

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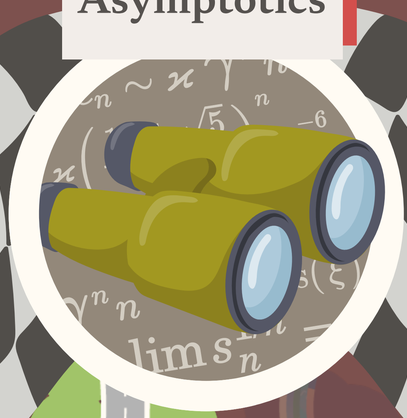

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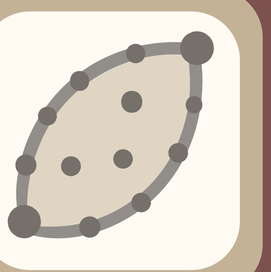

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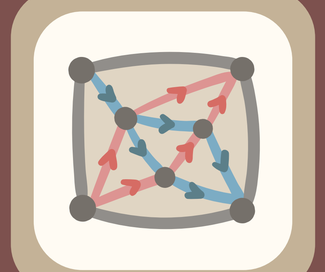

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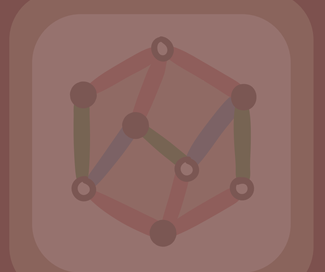
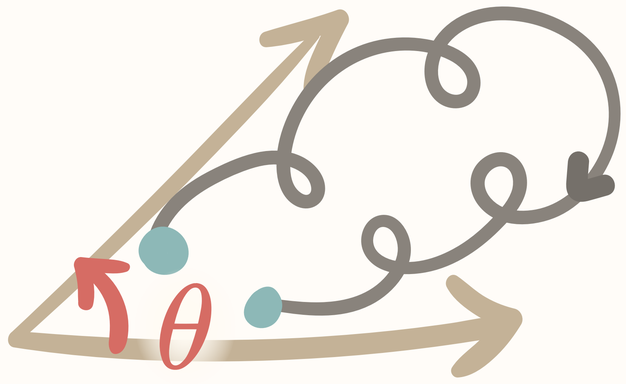


Schnyder labelings

Asymptotic counting results



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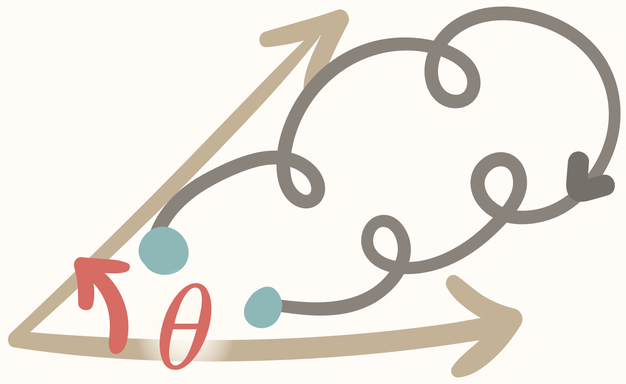

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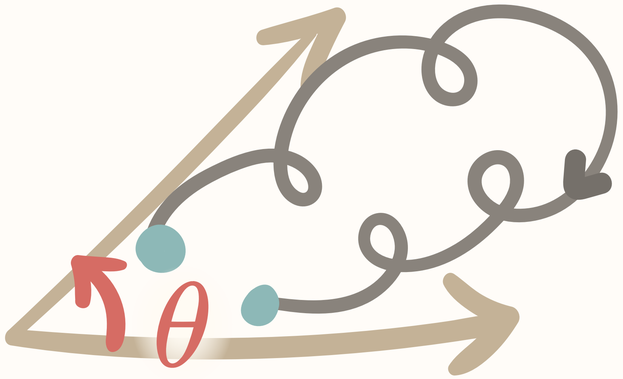

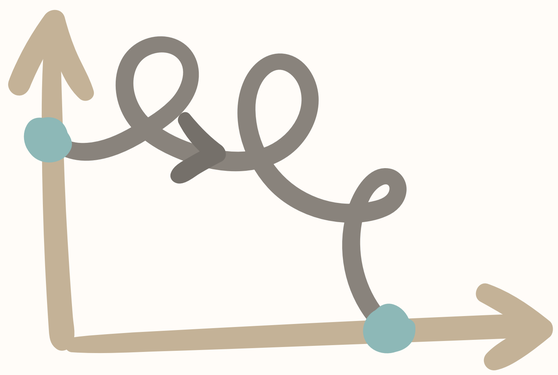

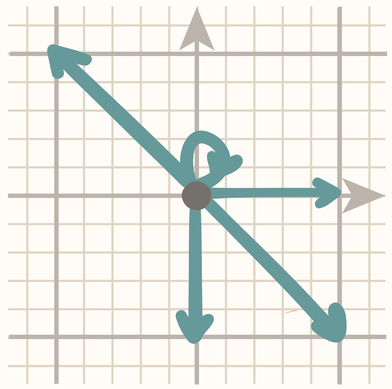

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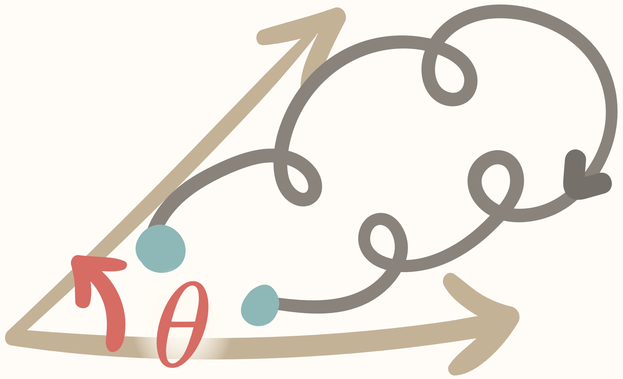

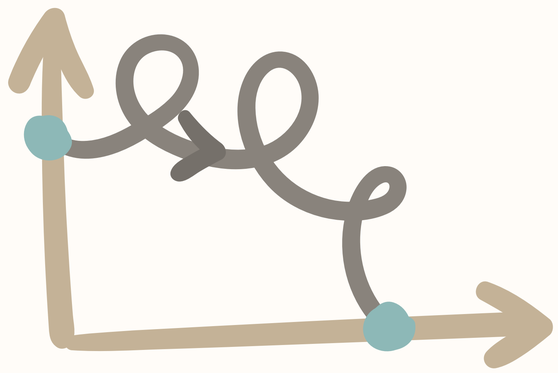

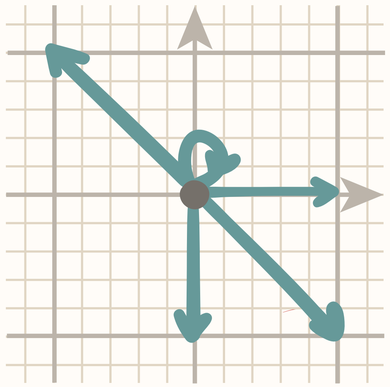

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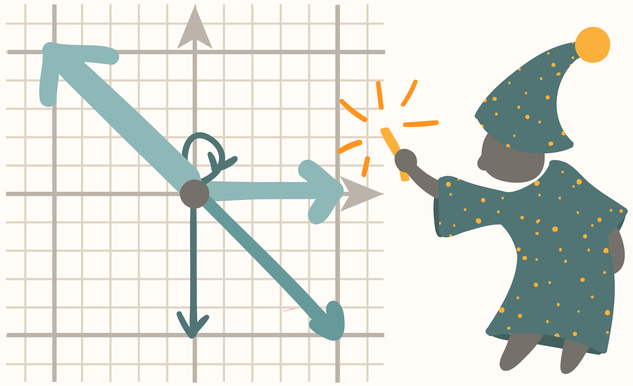
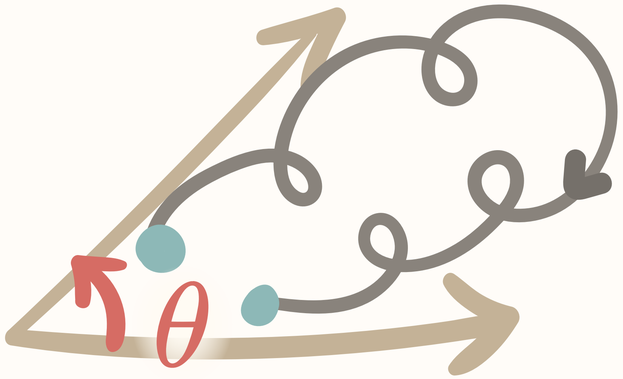
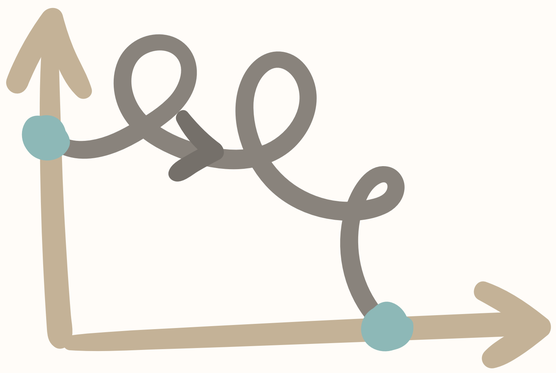
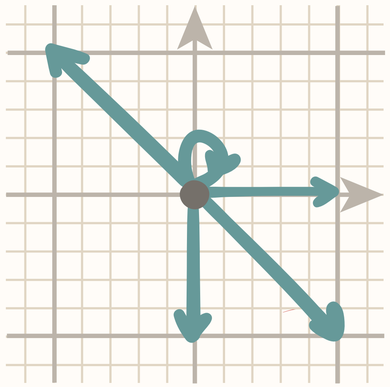


Weighted steps

Asymptotic counting results







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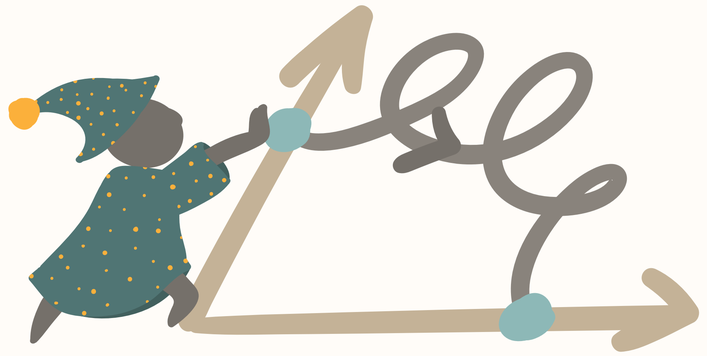
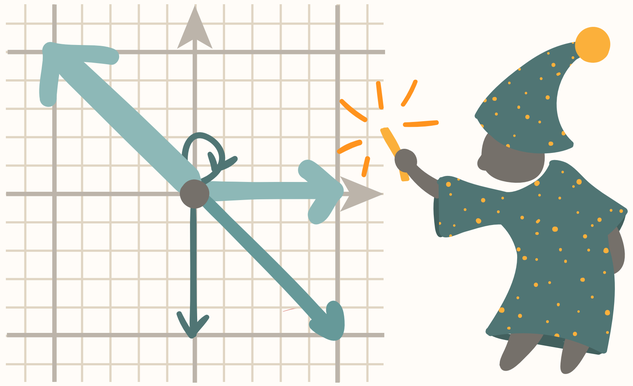


Shear transformation



Weighted steps

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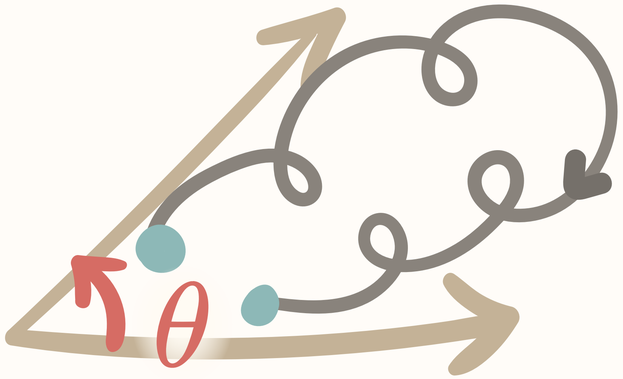
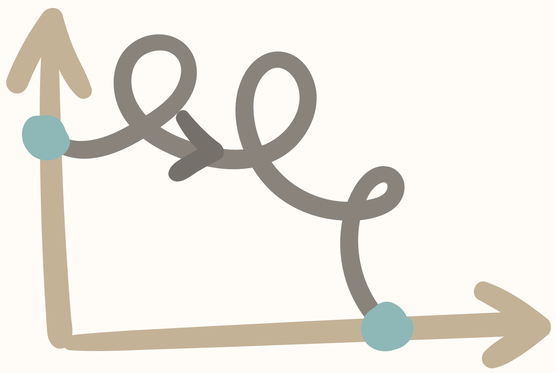
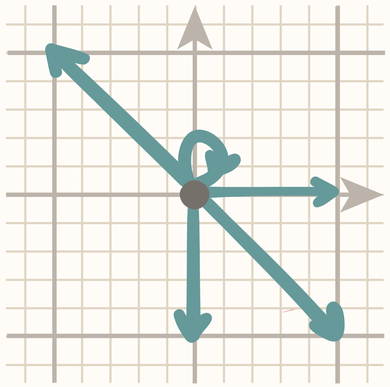
Asymptotic counting results







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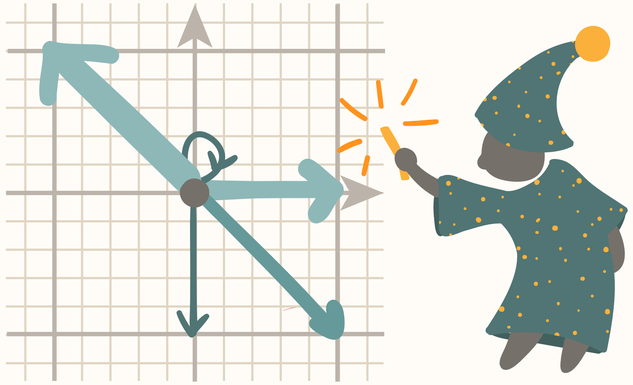
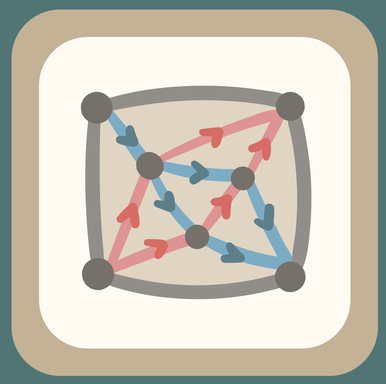


Weighted steps

Asymptotics

Asymptotic counting results

Model	*Asymptotics*
Transversal structures *n vertices* 	$t_n \sim \varkappa \left(\frac{27}{2}\right)^n n^{-1-\frac{\pi}{\arccos(7/8)}}$ ⋙ *A (4n-4)-Bit reprentation of a rectangular drawing or foorplan,* Y. Inoue, T. Takahashi & R. Fujimaki (2009)

Asymptotic counting results

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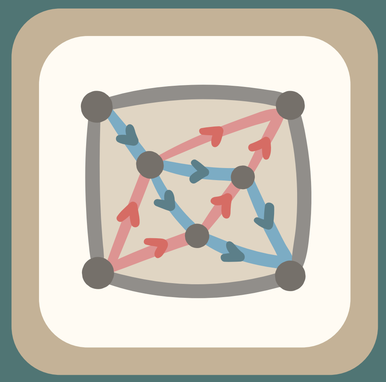
Transversal structures *n vertices* 	$t_n \sim \varkappa \left(\frac{27}{2}\right)^n n^{-1-\frac{\pi}{\arccos(7/8)}}$ ⋙ *A (4n-4)-Bit reprentation of a rectangular drawing or foorplan,* Y. Inoue, T. Takahashi & R. Fujimaki (2009)
Posets *n+2 edges*	$e_n \sim \varkappa \gamma^n n^{-\alpha}$ $\qquad \alpha = 1+\frac{\pi}{\arccos(\xi)}$ γ *and* ξ *are algebraic* $\gamma \approx 4.80\ldots$ $\alpha \approx 5.14\ldots$
Posets *n vertices*	$b_n \sim \varkappa \left(\frac{11+\sqrt{5}}{2}\right)^n n^{-6}$

PLANE PERMUTATIONS

$$p_n \sim \varkappa \left(\frac{11+\sqrt{5}}{2}\right)^n \cdot n^{-6}$$

» *Semi-Baxter and strong-Baxter : two relatives of Baxter Sequences,* **M. Bouvel, V. Guerrini, A. Rechnitzer & S. Rinaldi (2018)**

Posets
n+2 edg

Posets
n vertices

$$b_n \sim \varkappa \left(\frac{11+\sqrt{5}}{2}\right)^n n^{-6}$$

Posets
n+2 edg

Posets
n vertices

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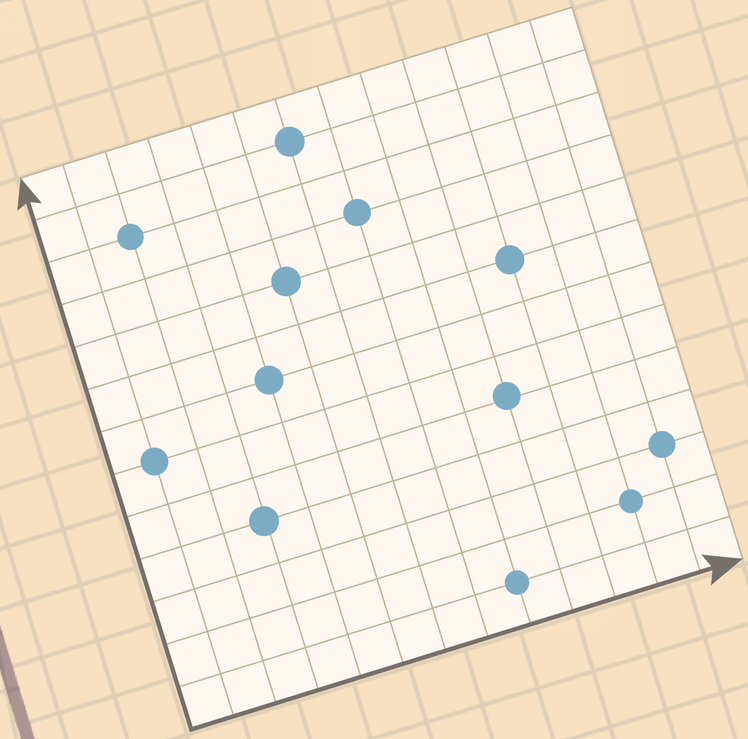

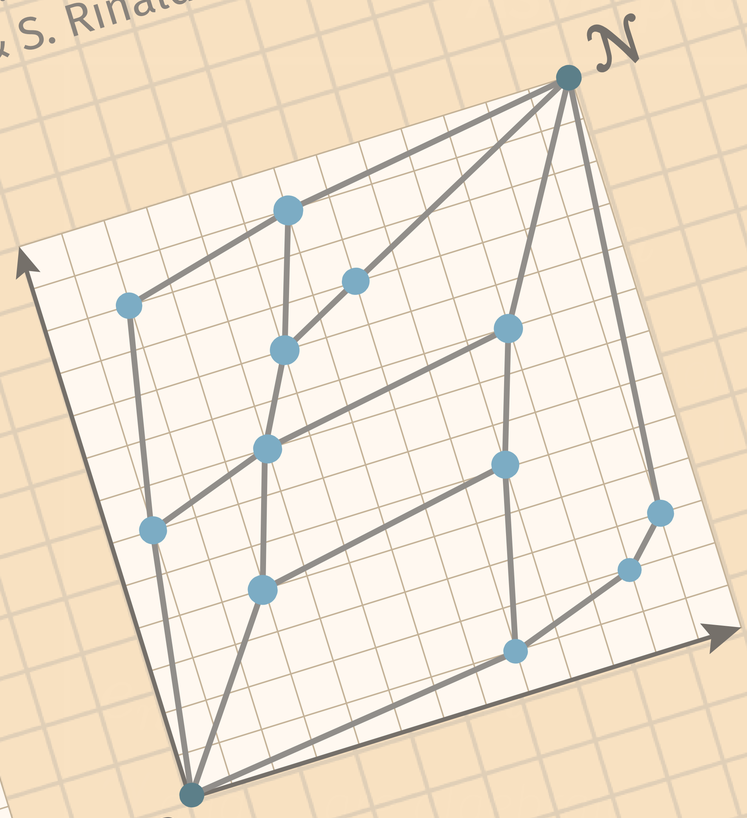

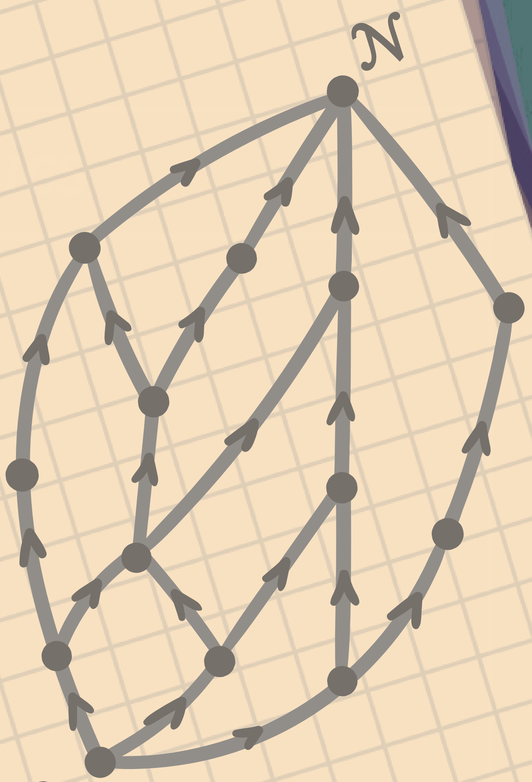

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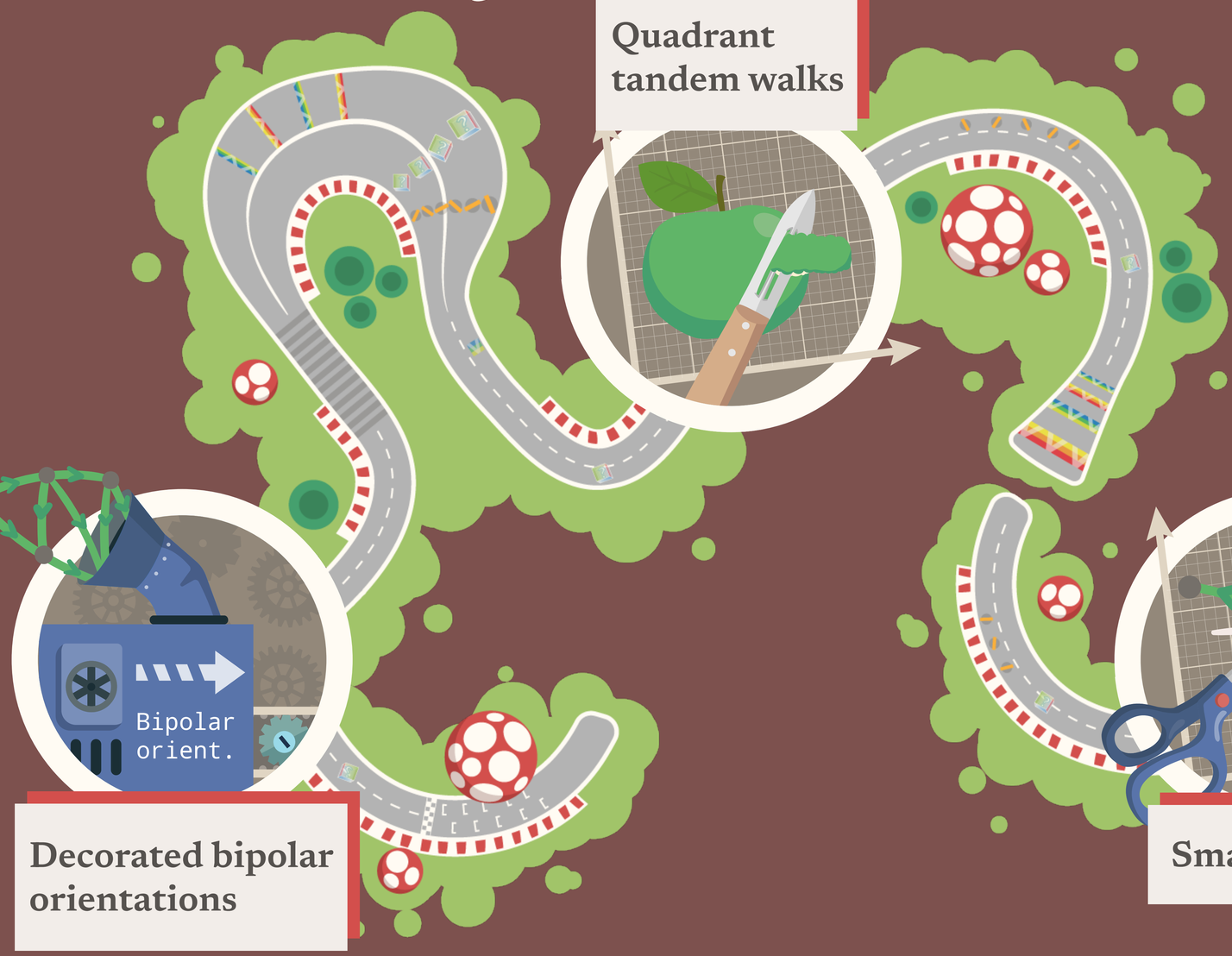
Summary



bipolar posets by vertices

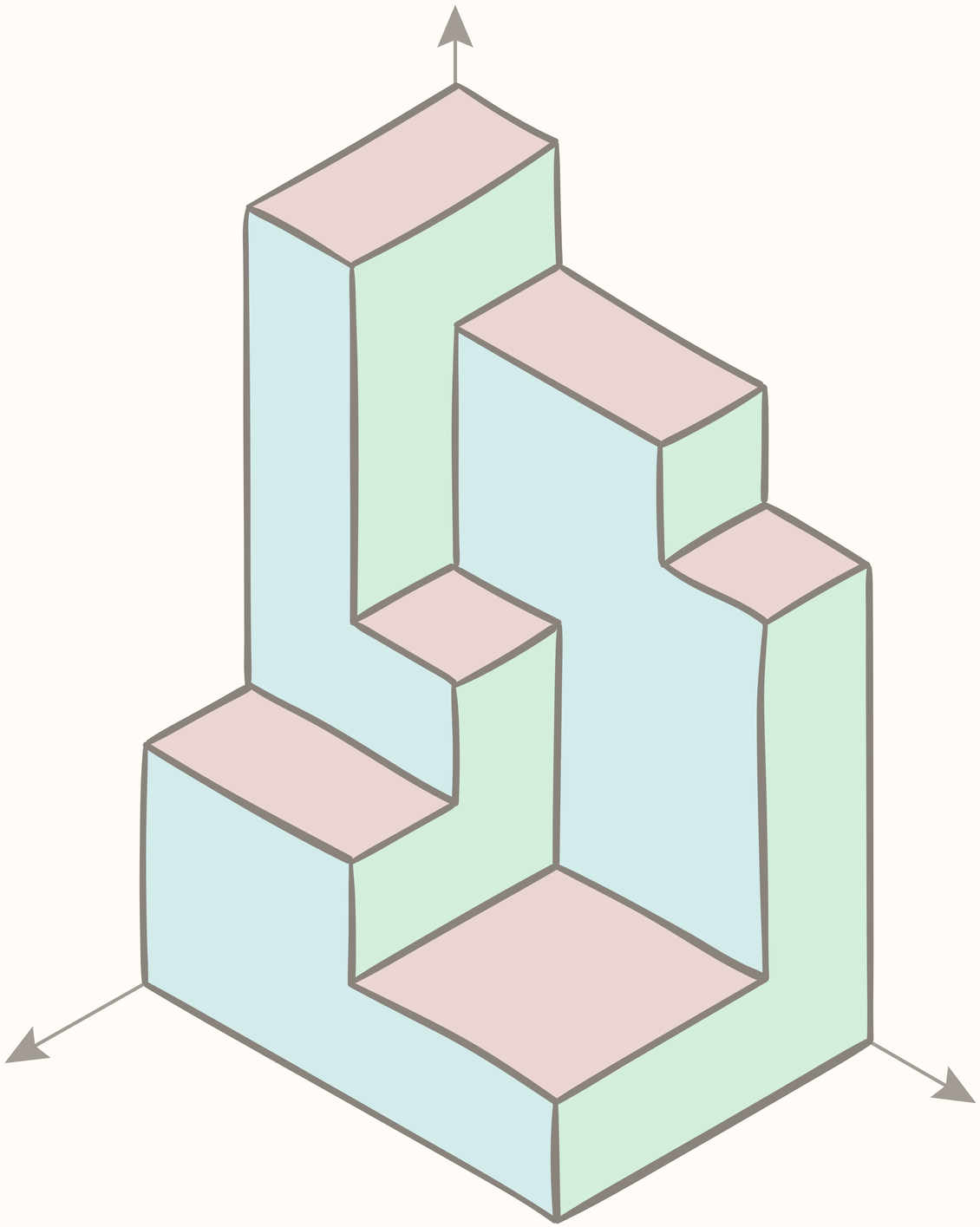
bipolar posets by edges

transversal structures

polyhedral orientations

Schnyder labelings

Corner polyhedra



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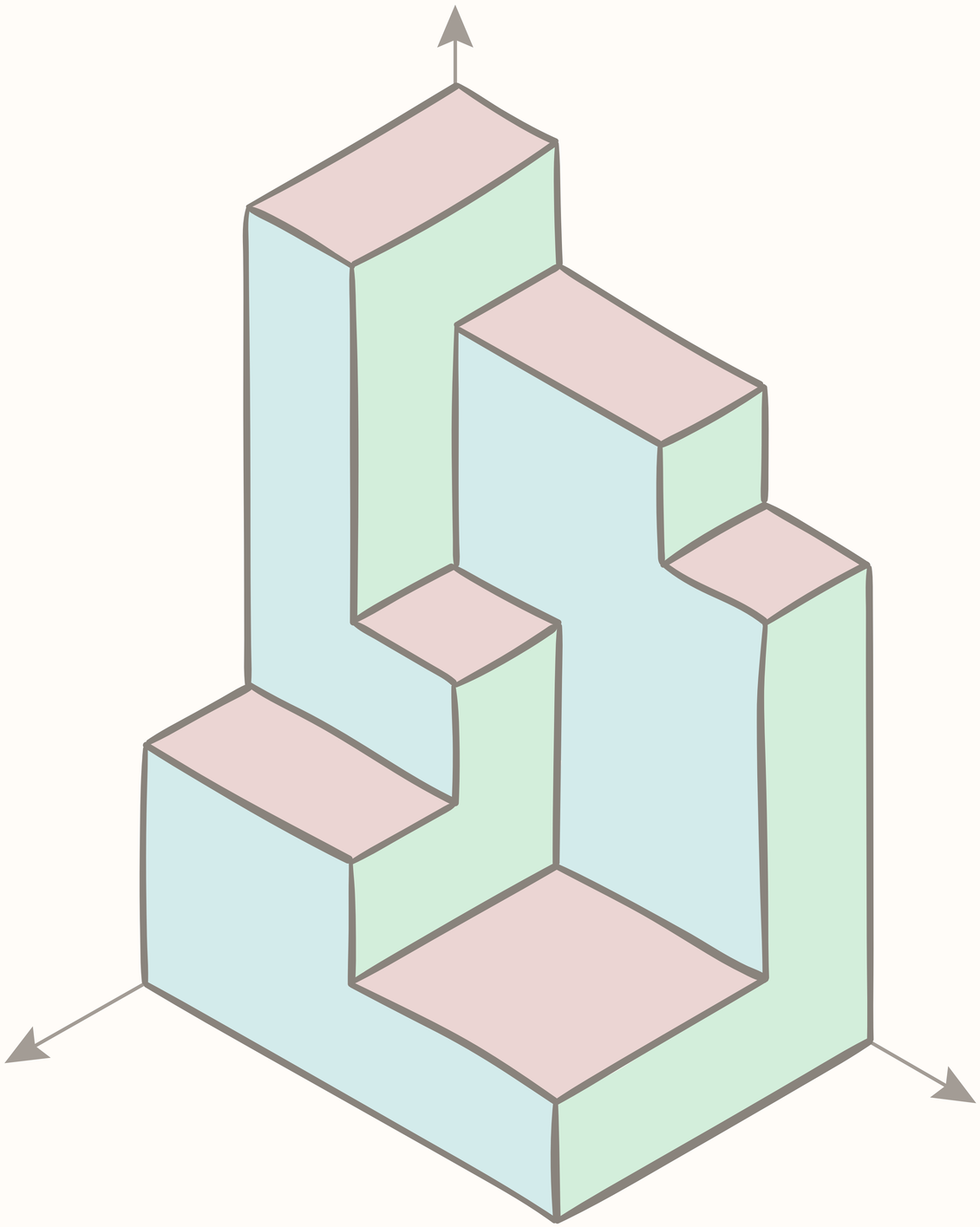

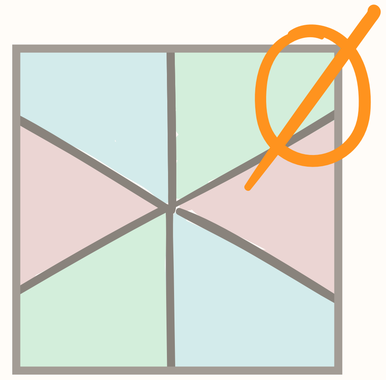

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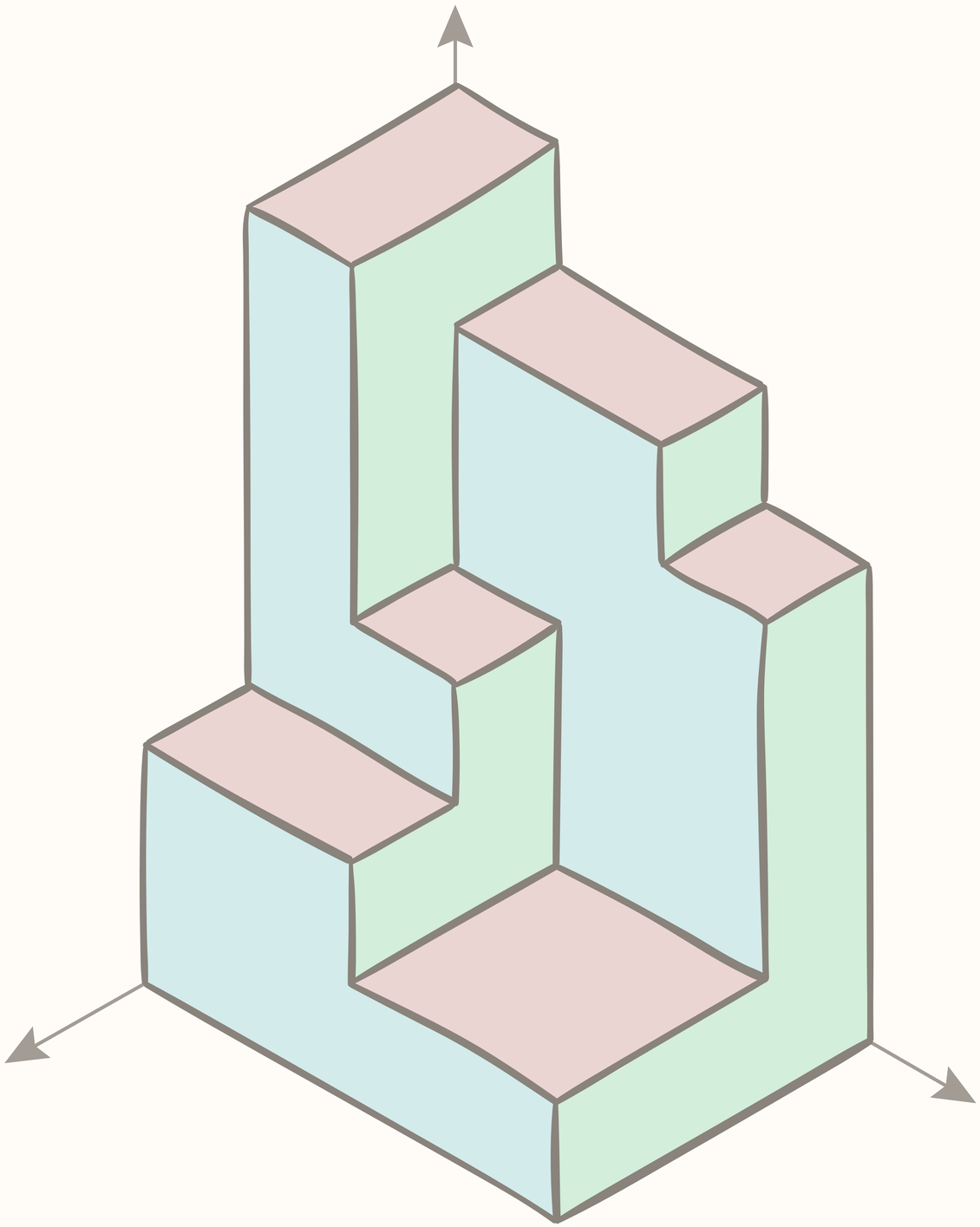
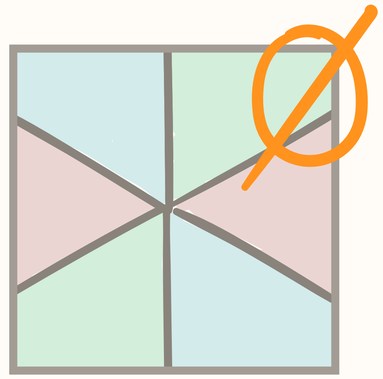
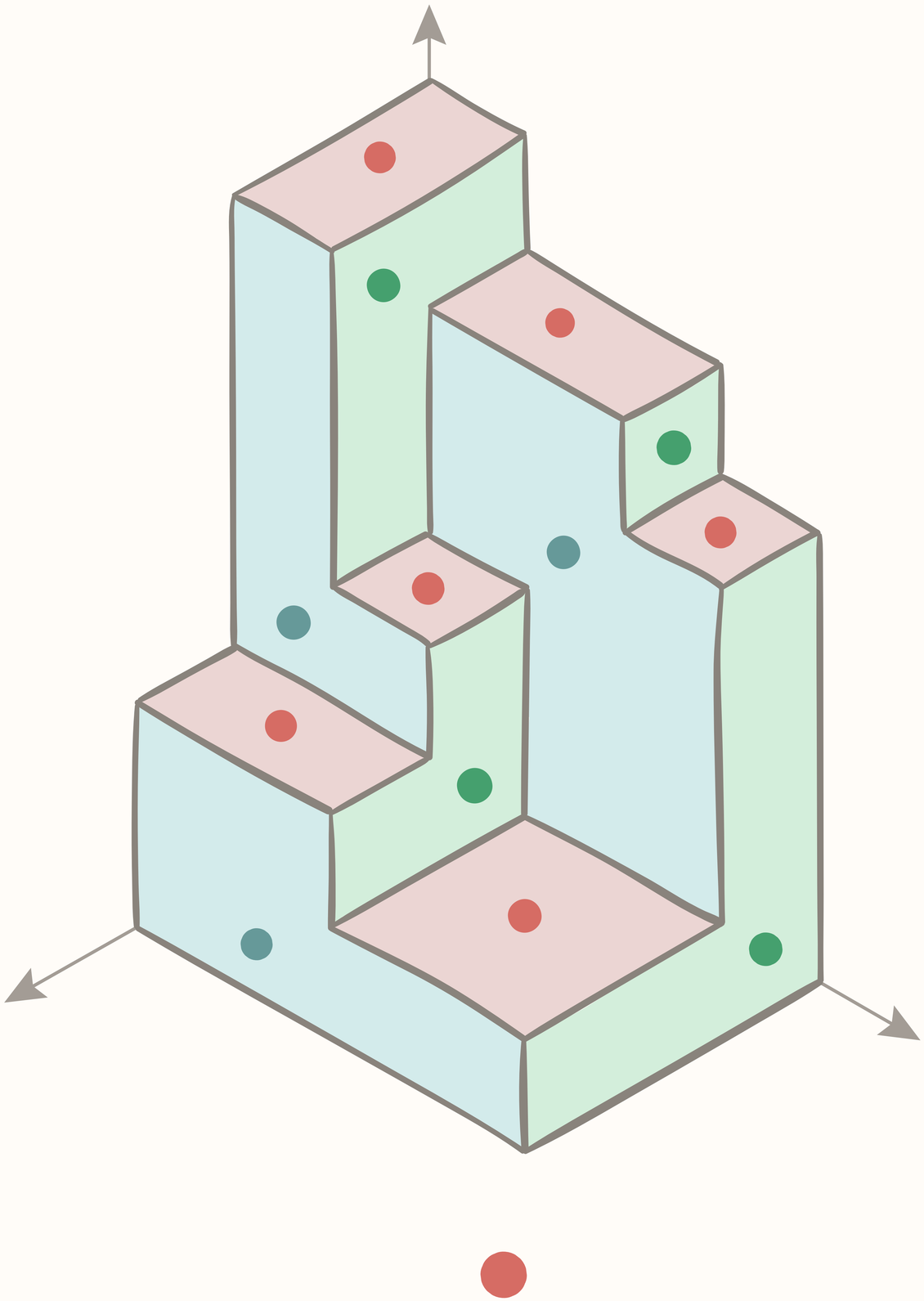


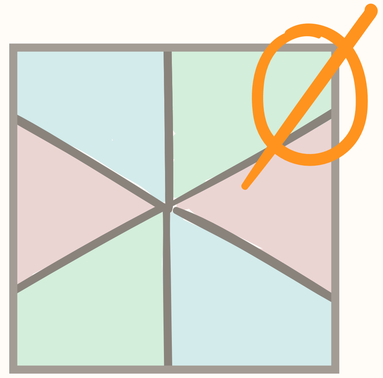


corner polyhedra ↔ ***polyhedral orientations***

↠ *Steinitz theorems for orthogonal polyhera,*
D. Eppstein & E. Mumford (2010)

Corner polyhedra





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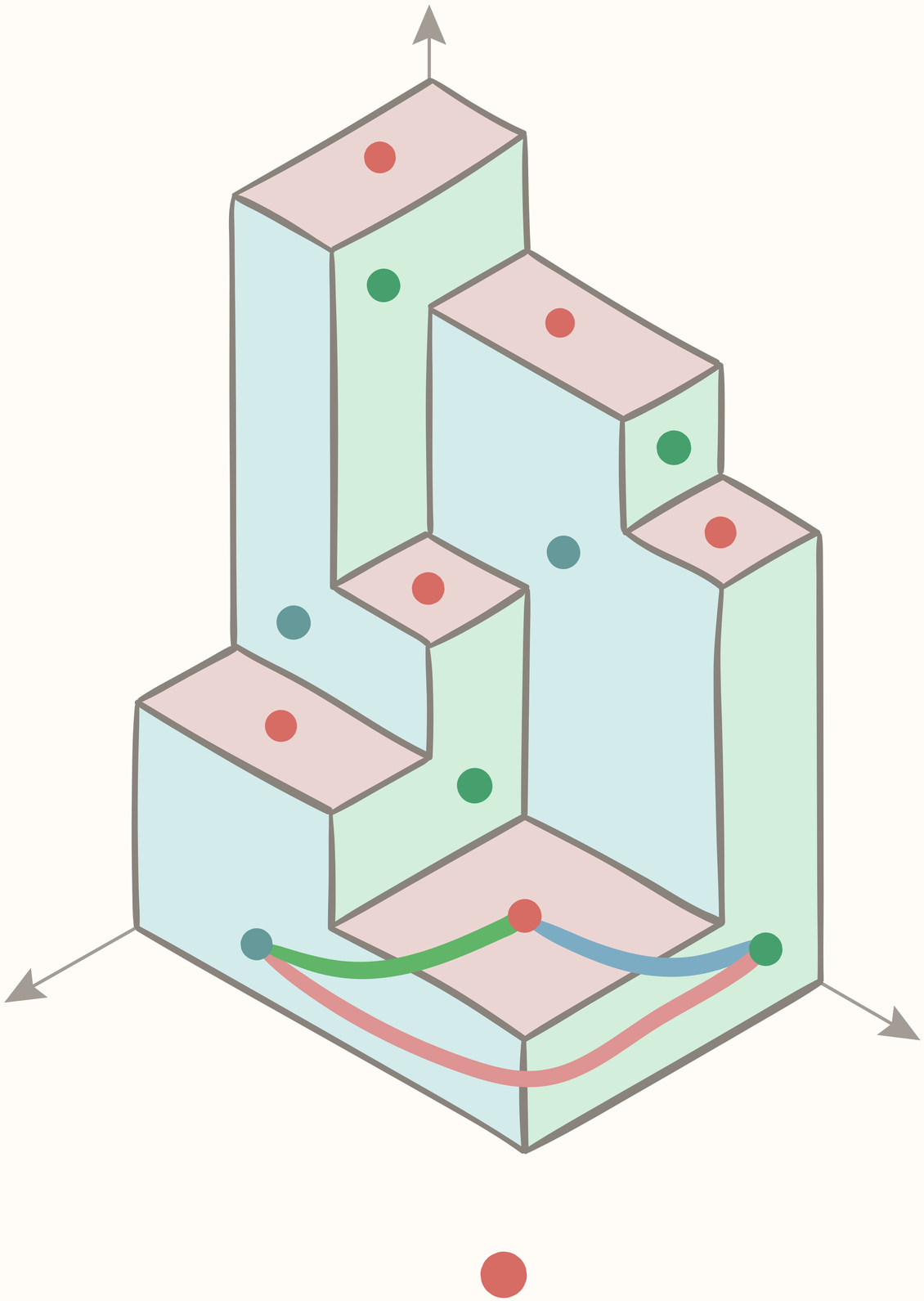

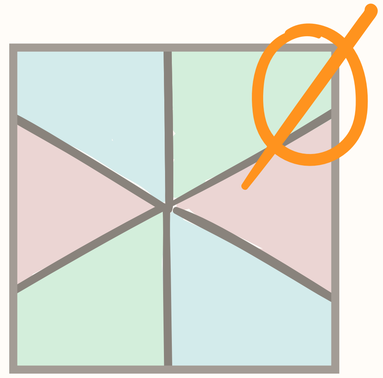

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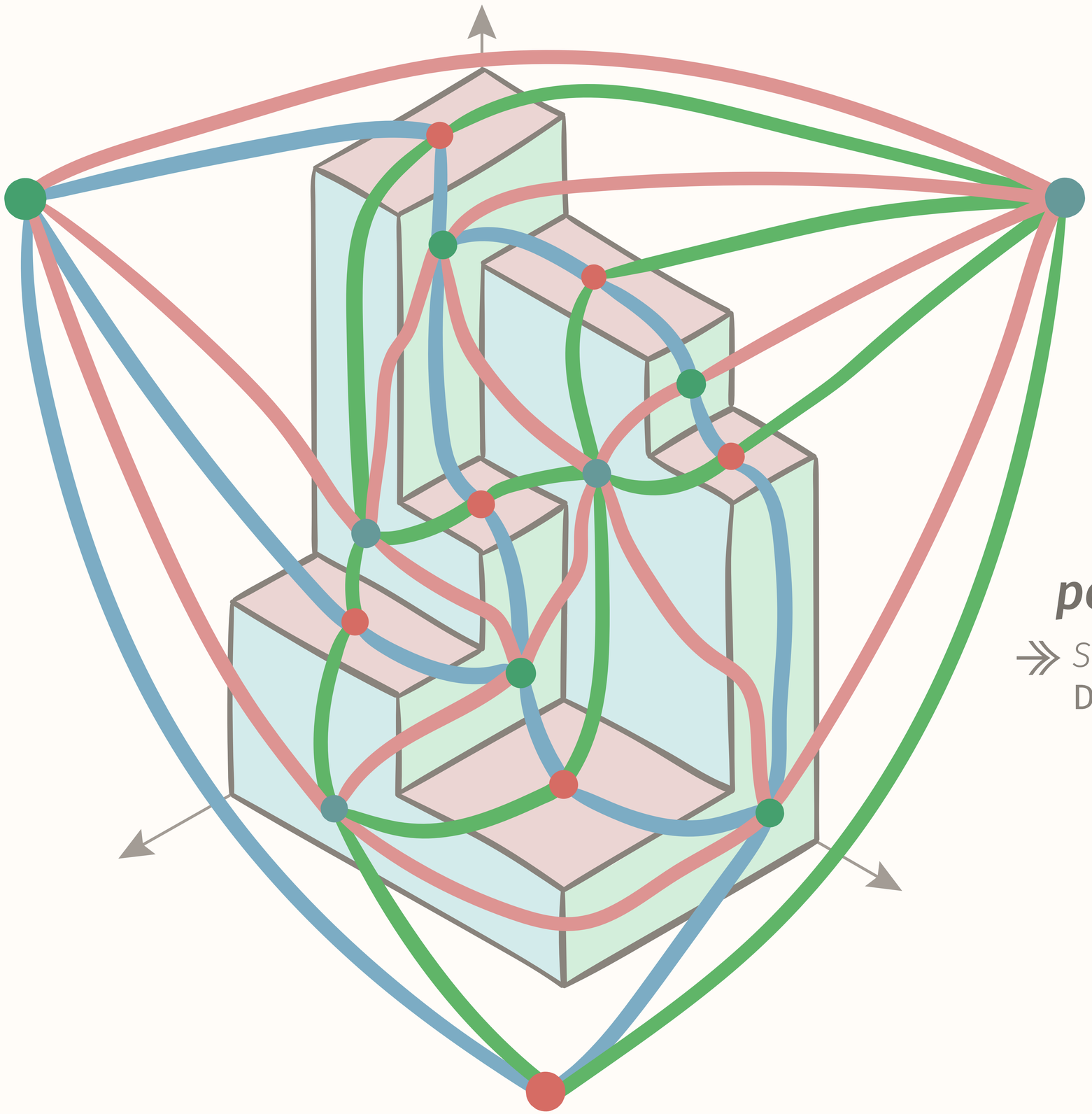

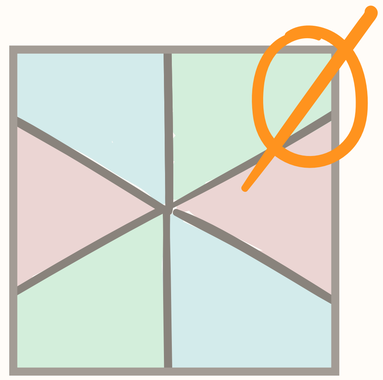

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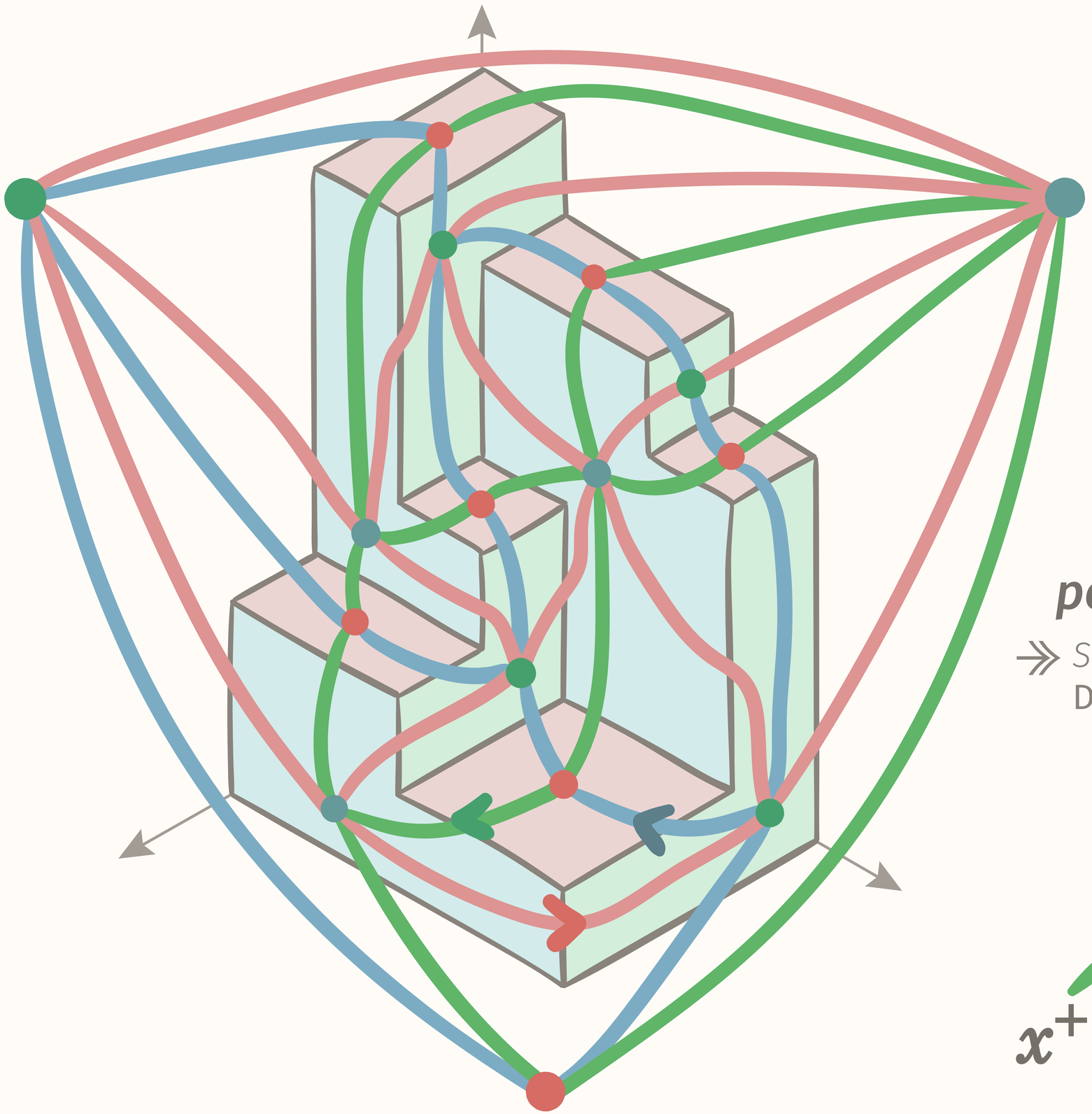

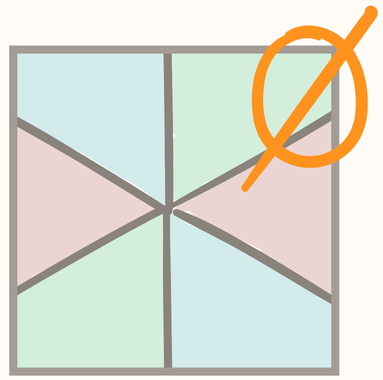

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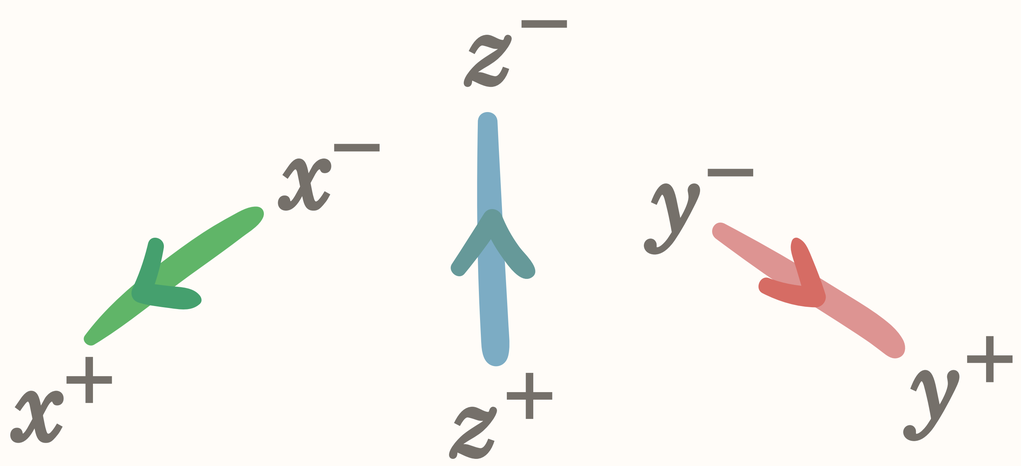

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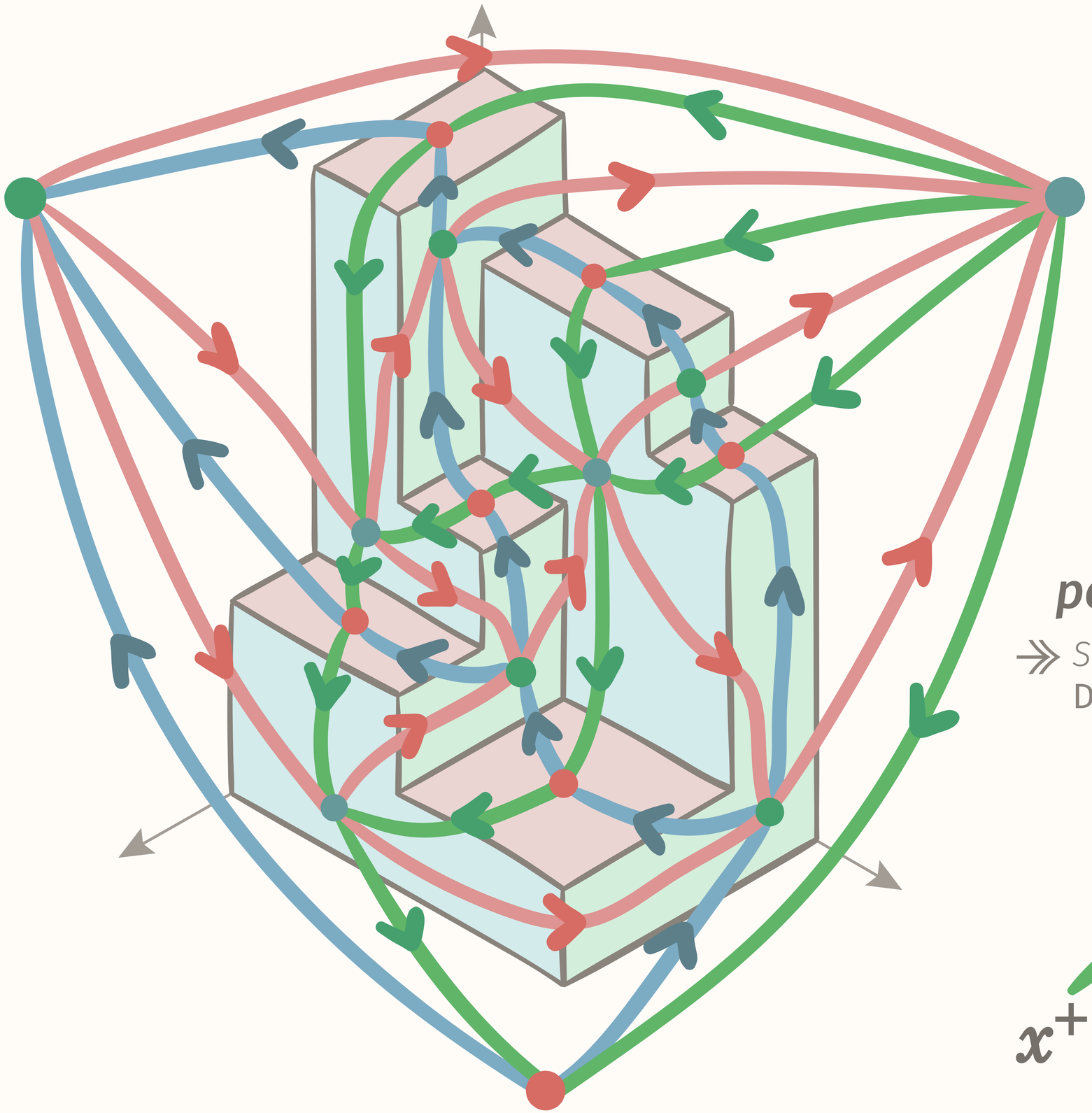

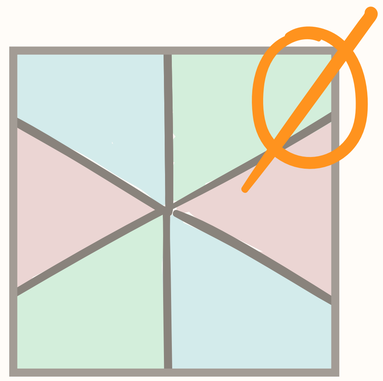

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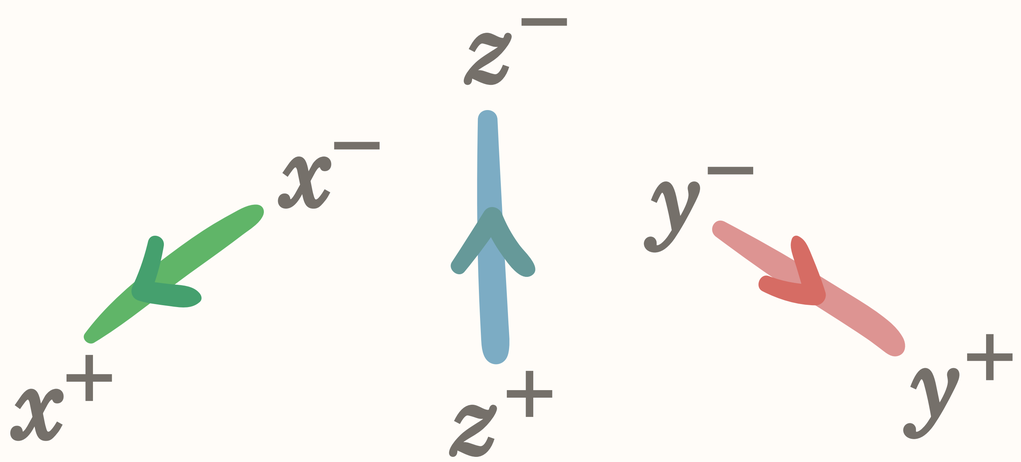

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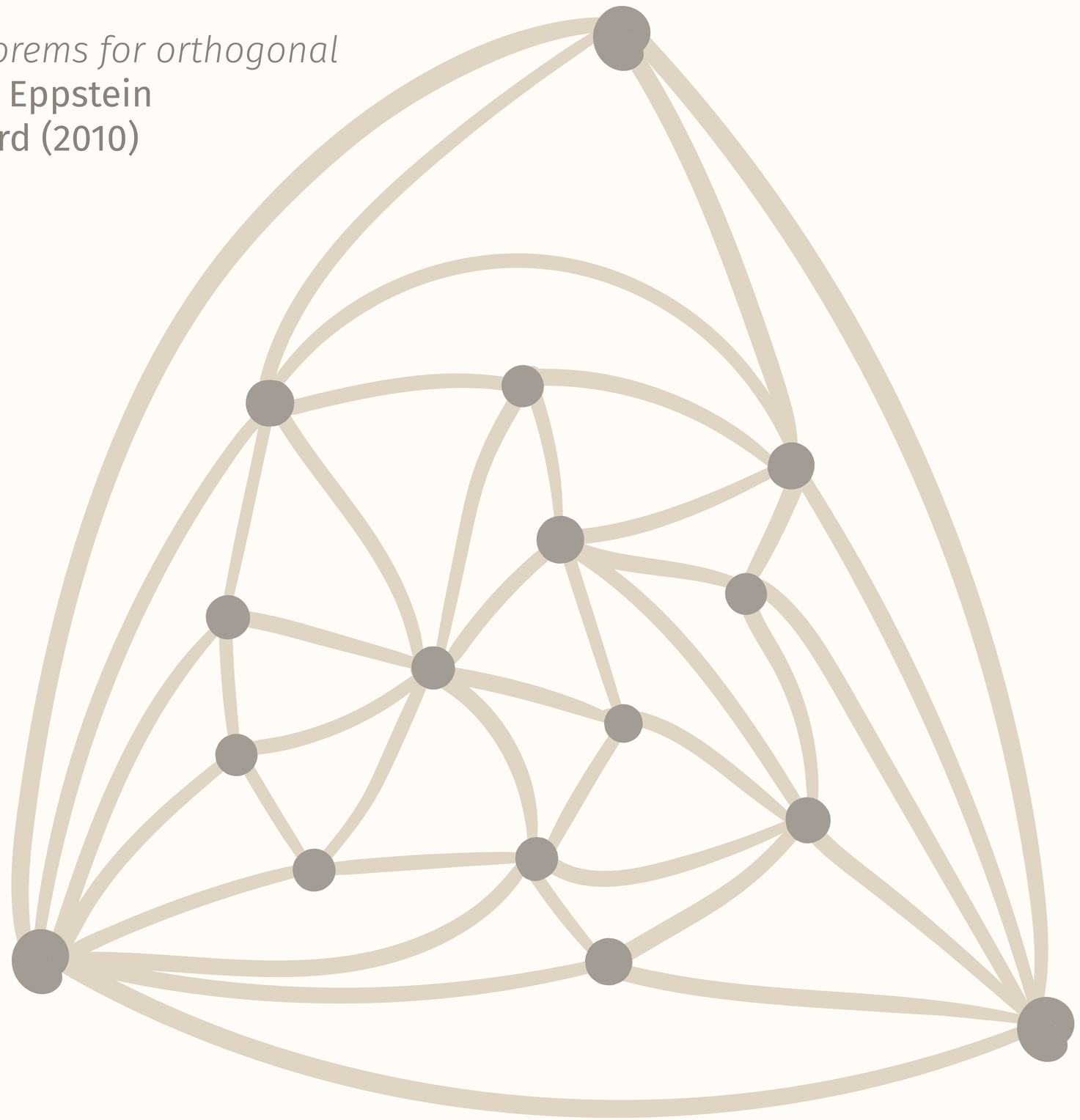

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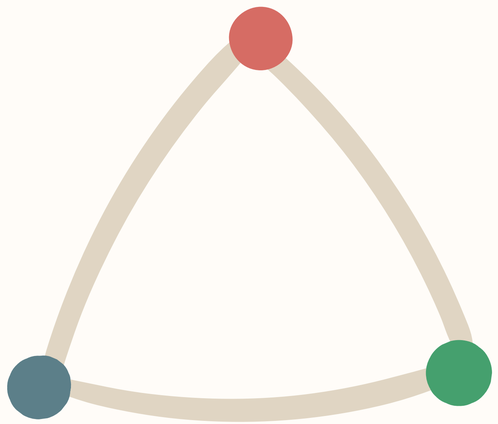

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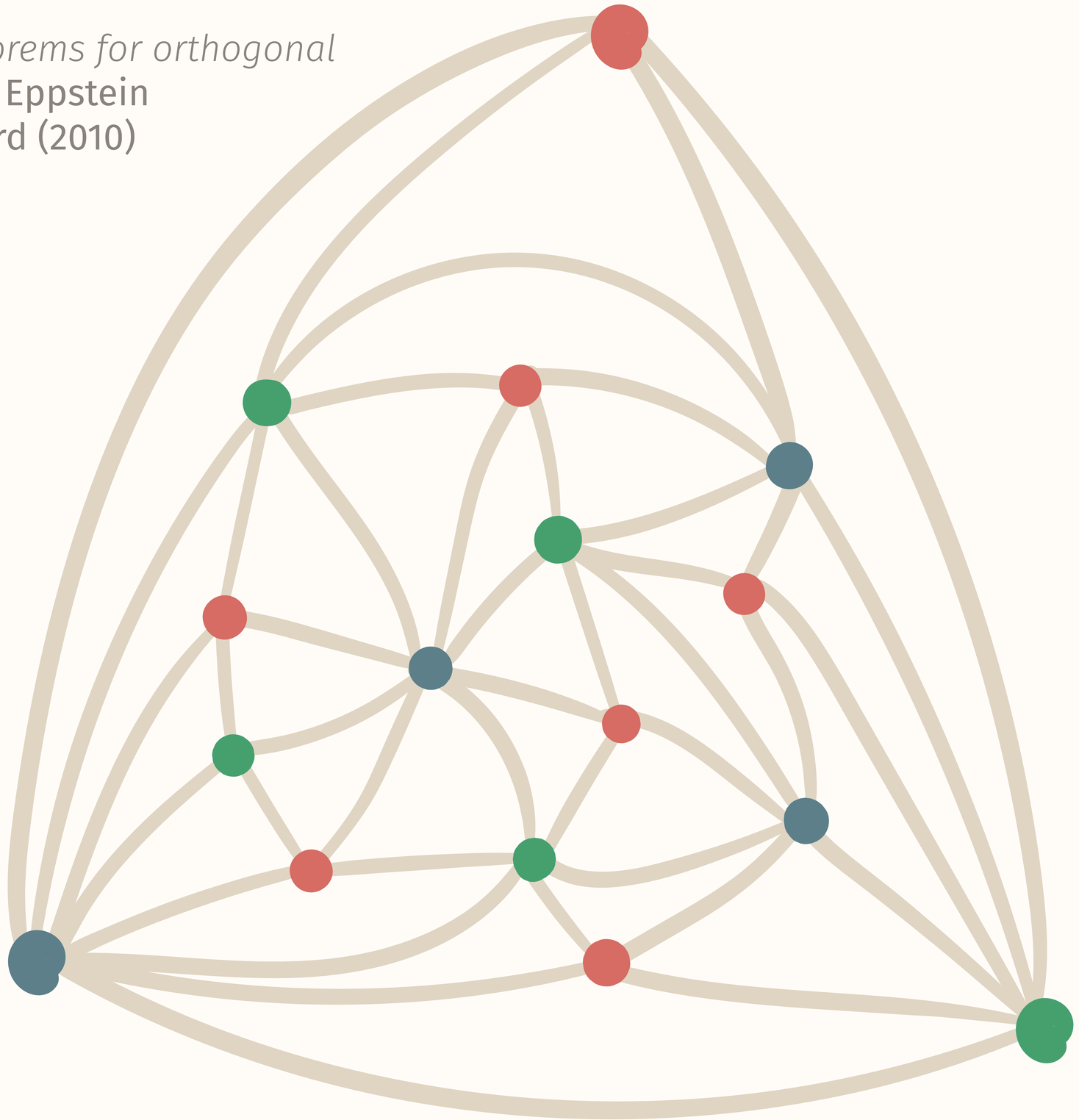

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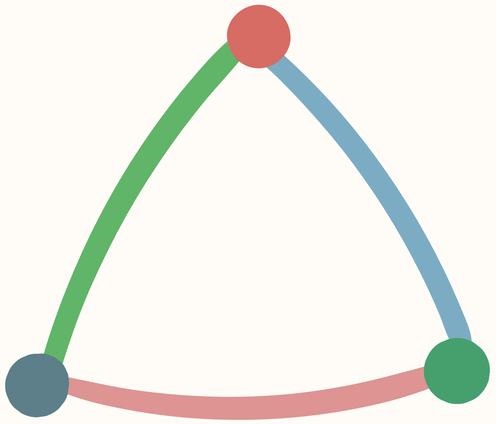

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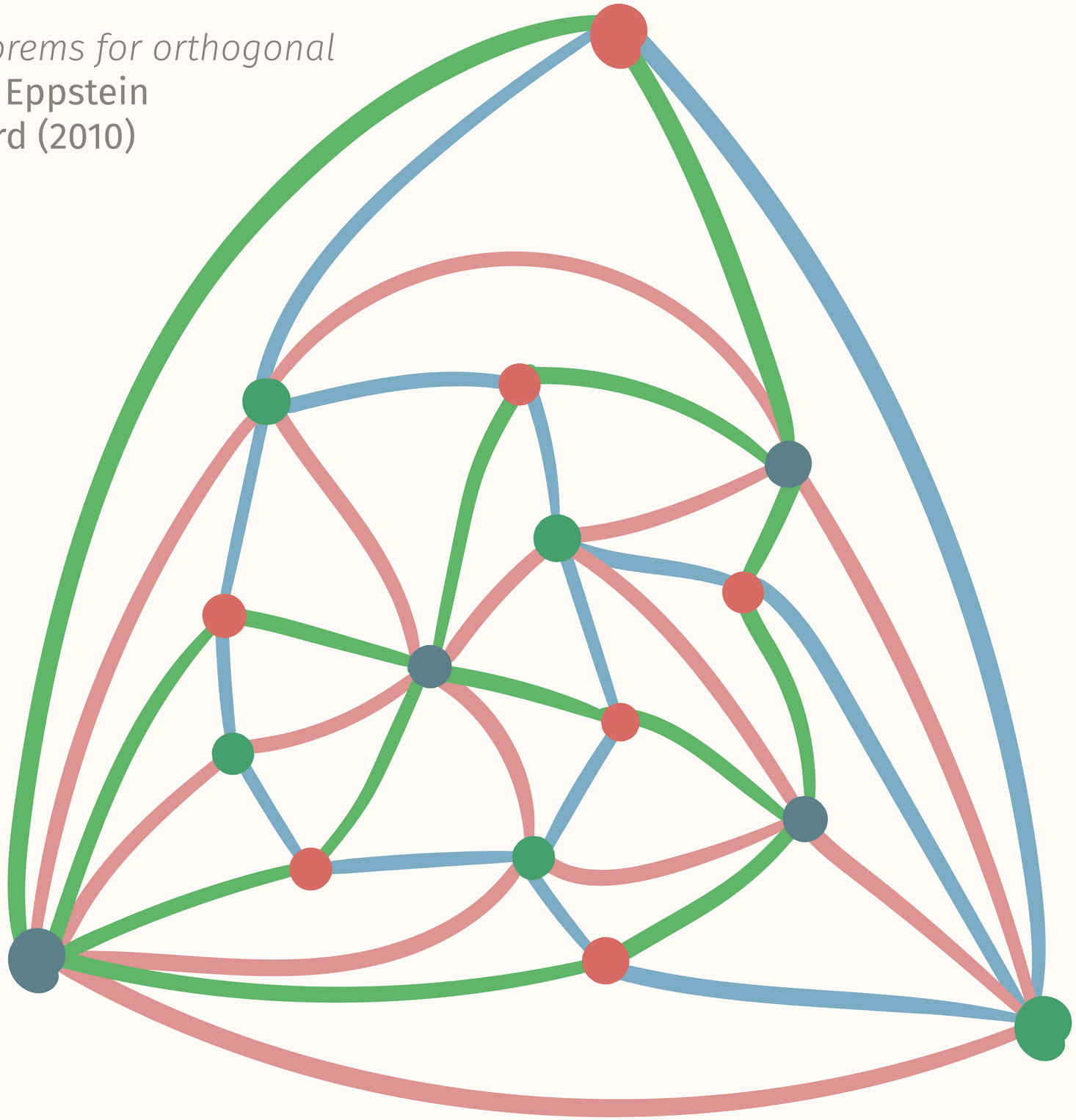

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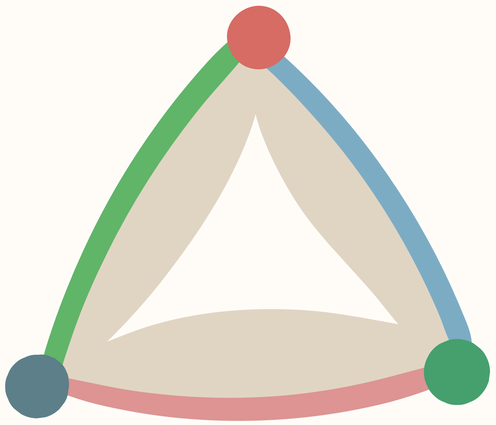

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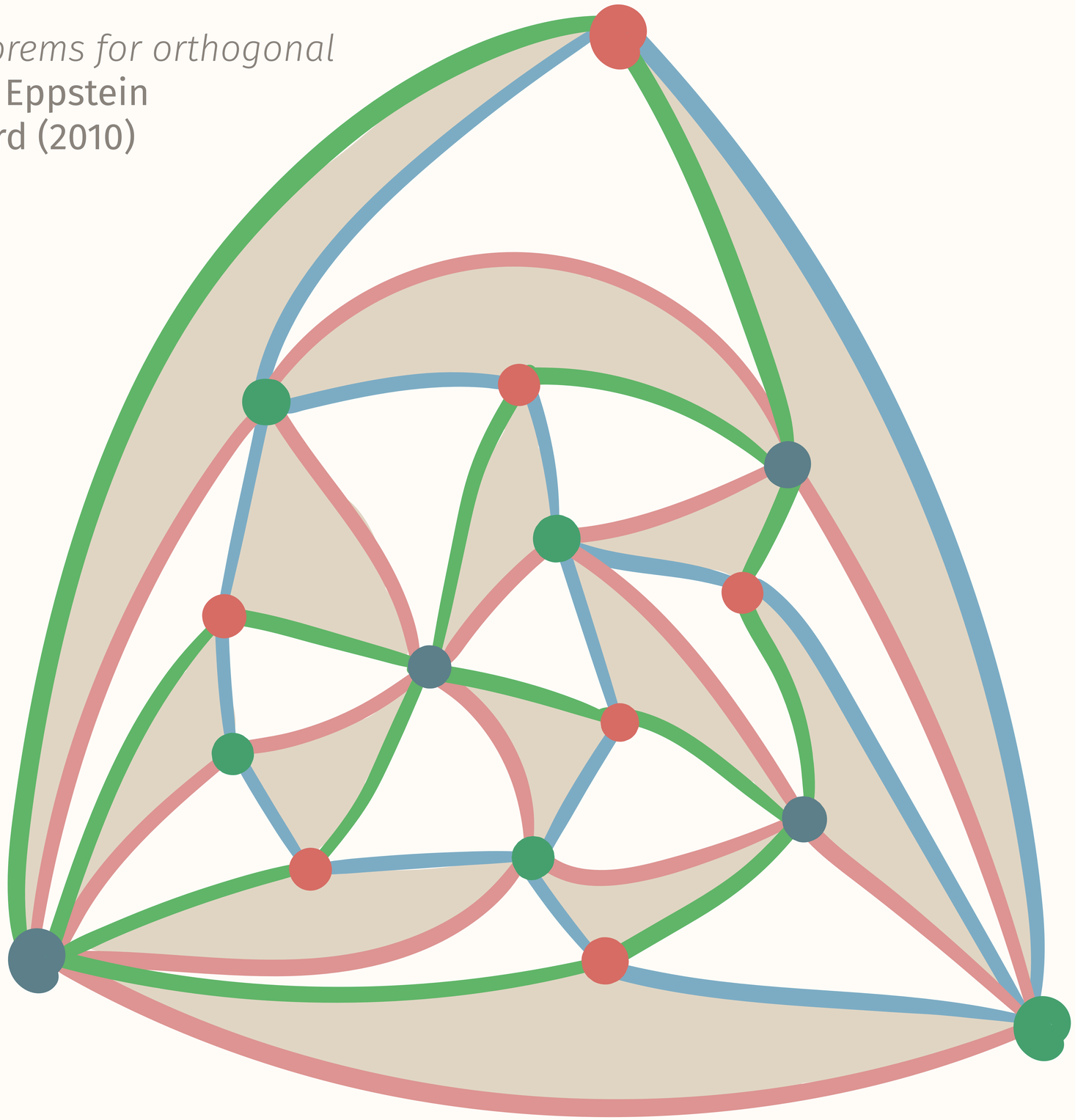

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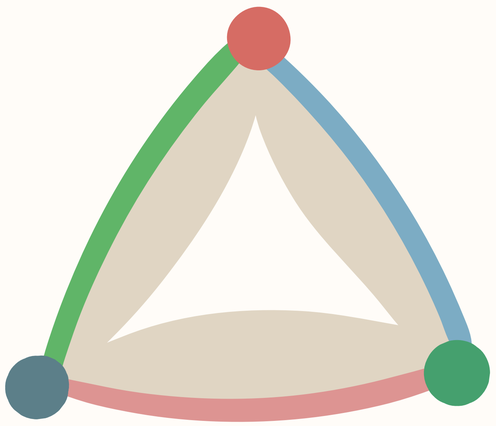

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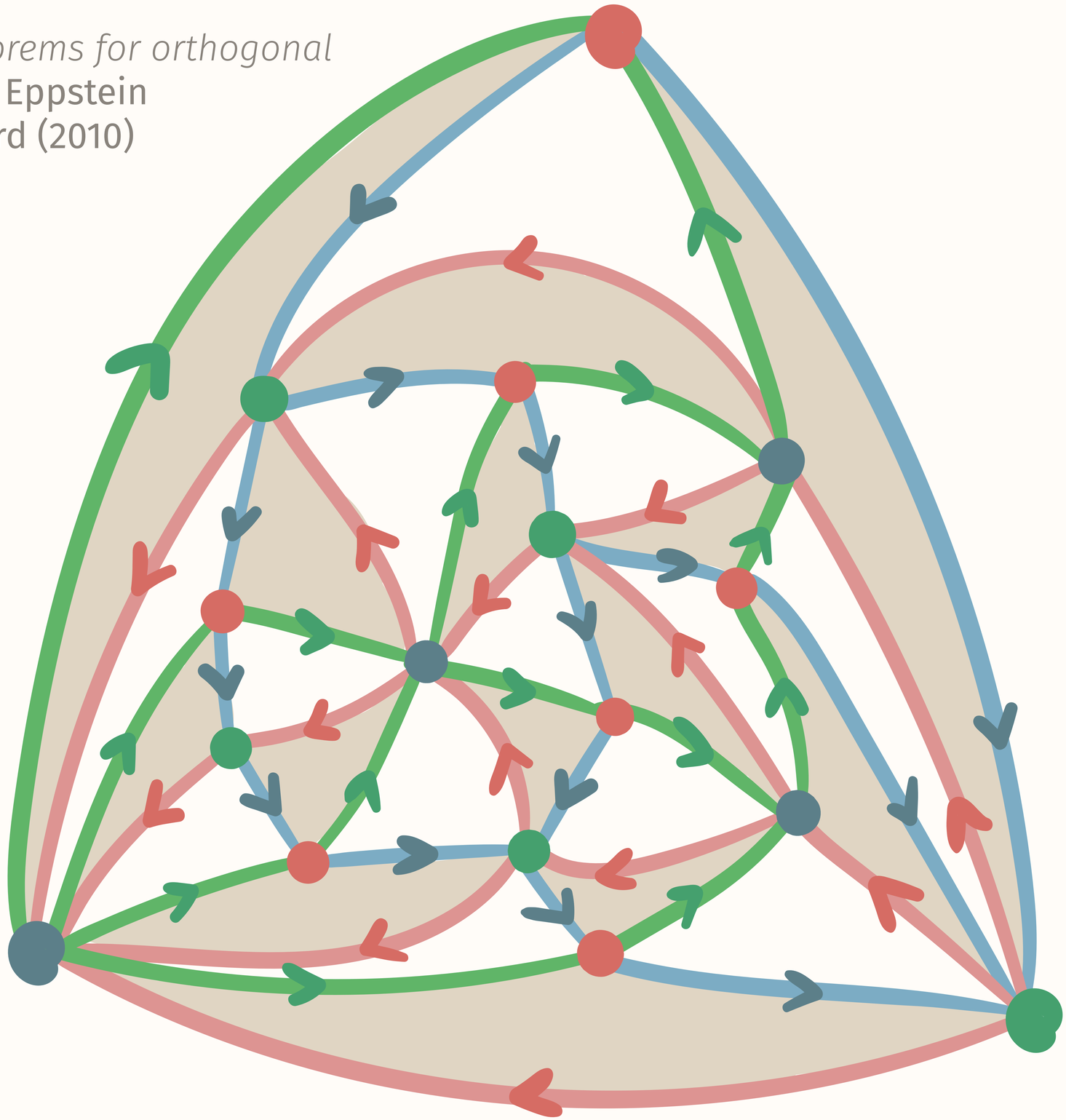

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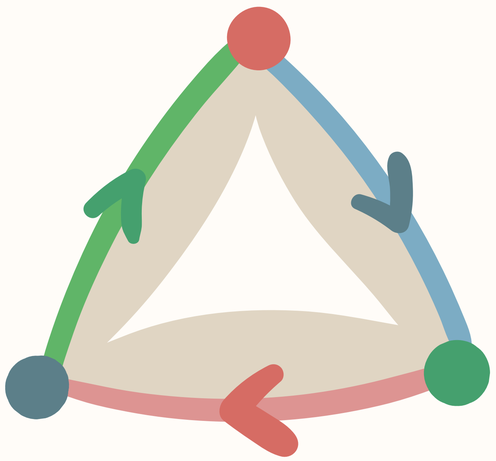

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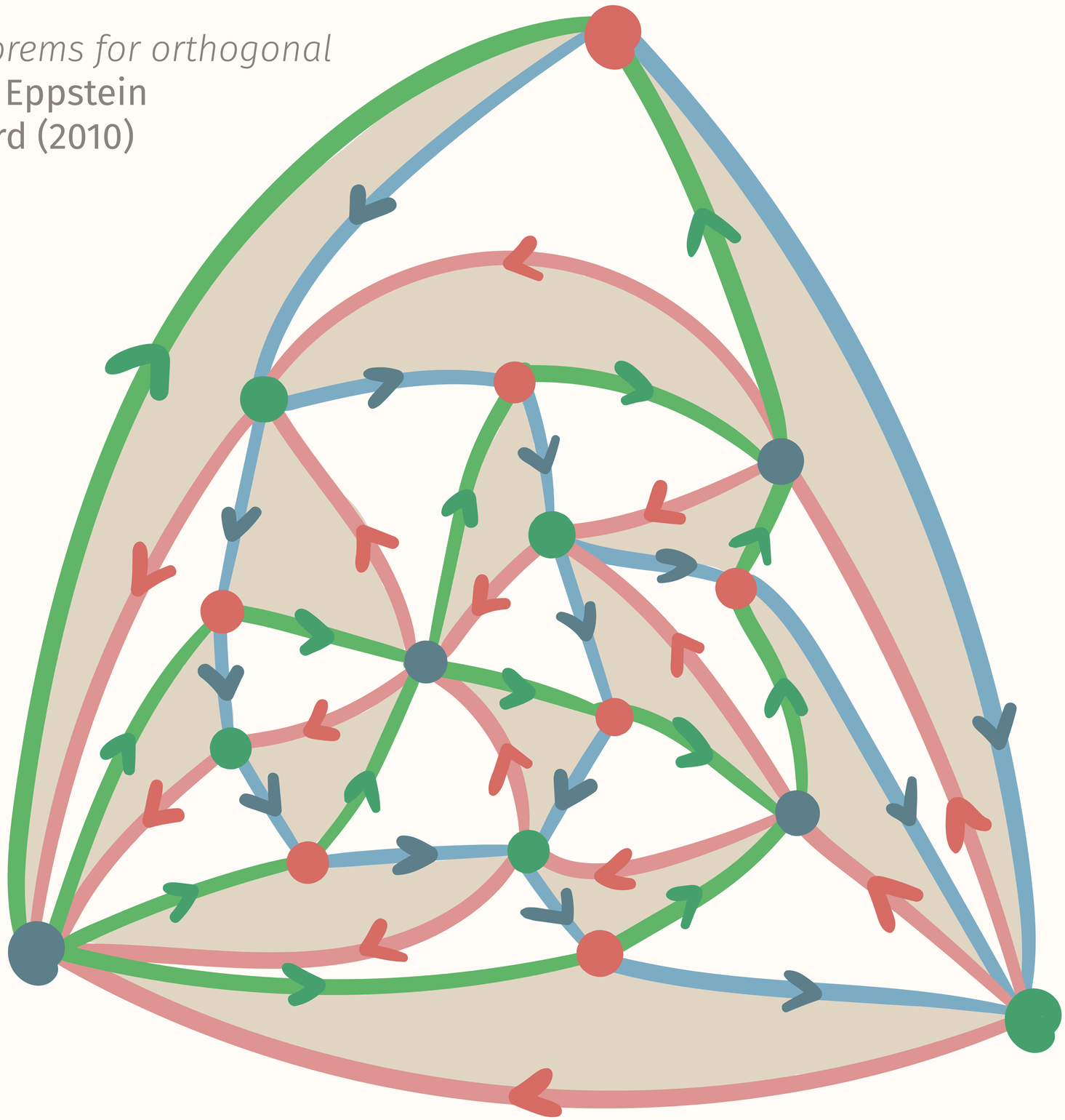

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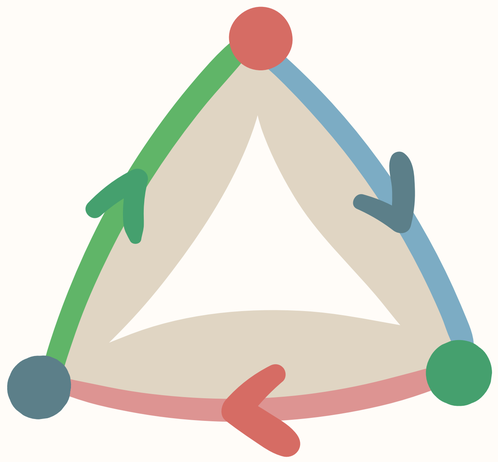

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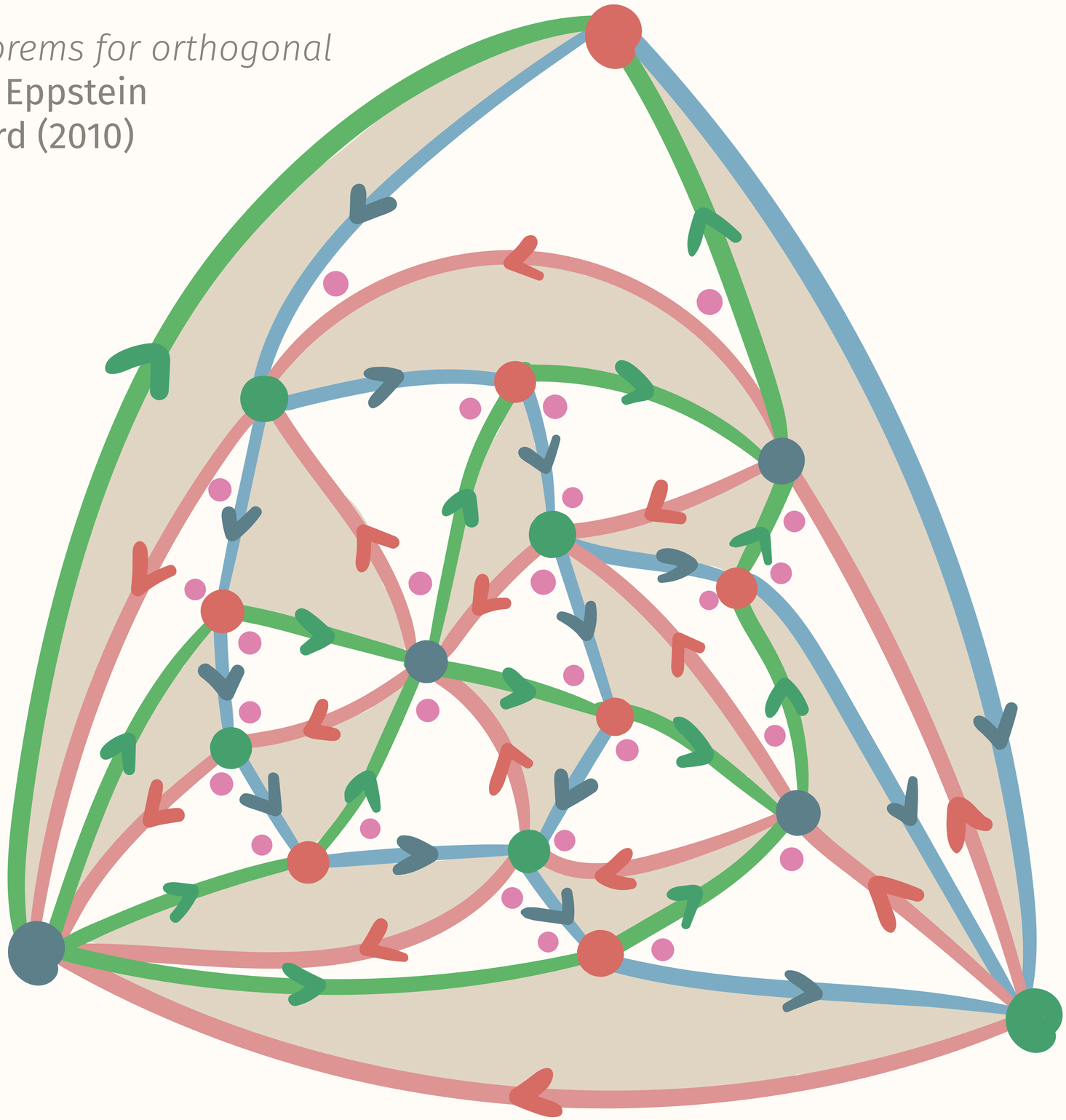

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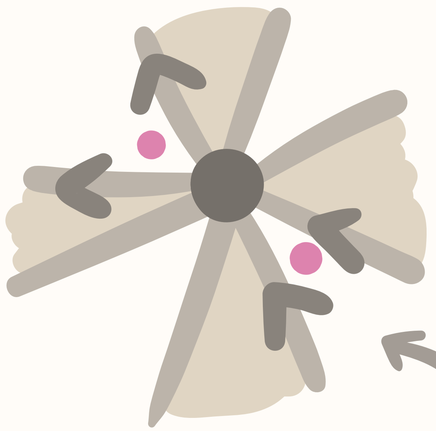
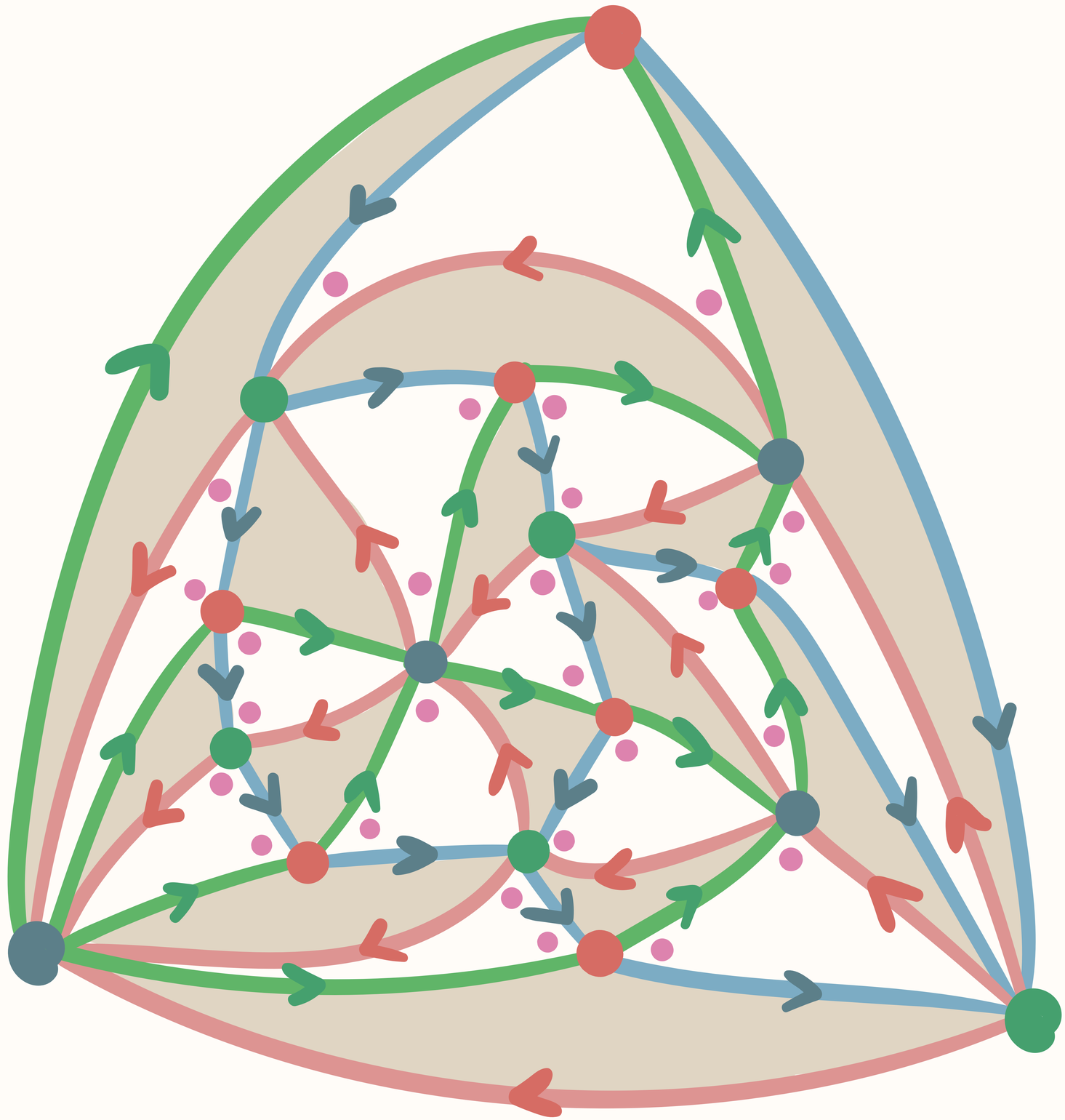
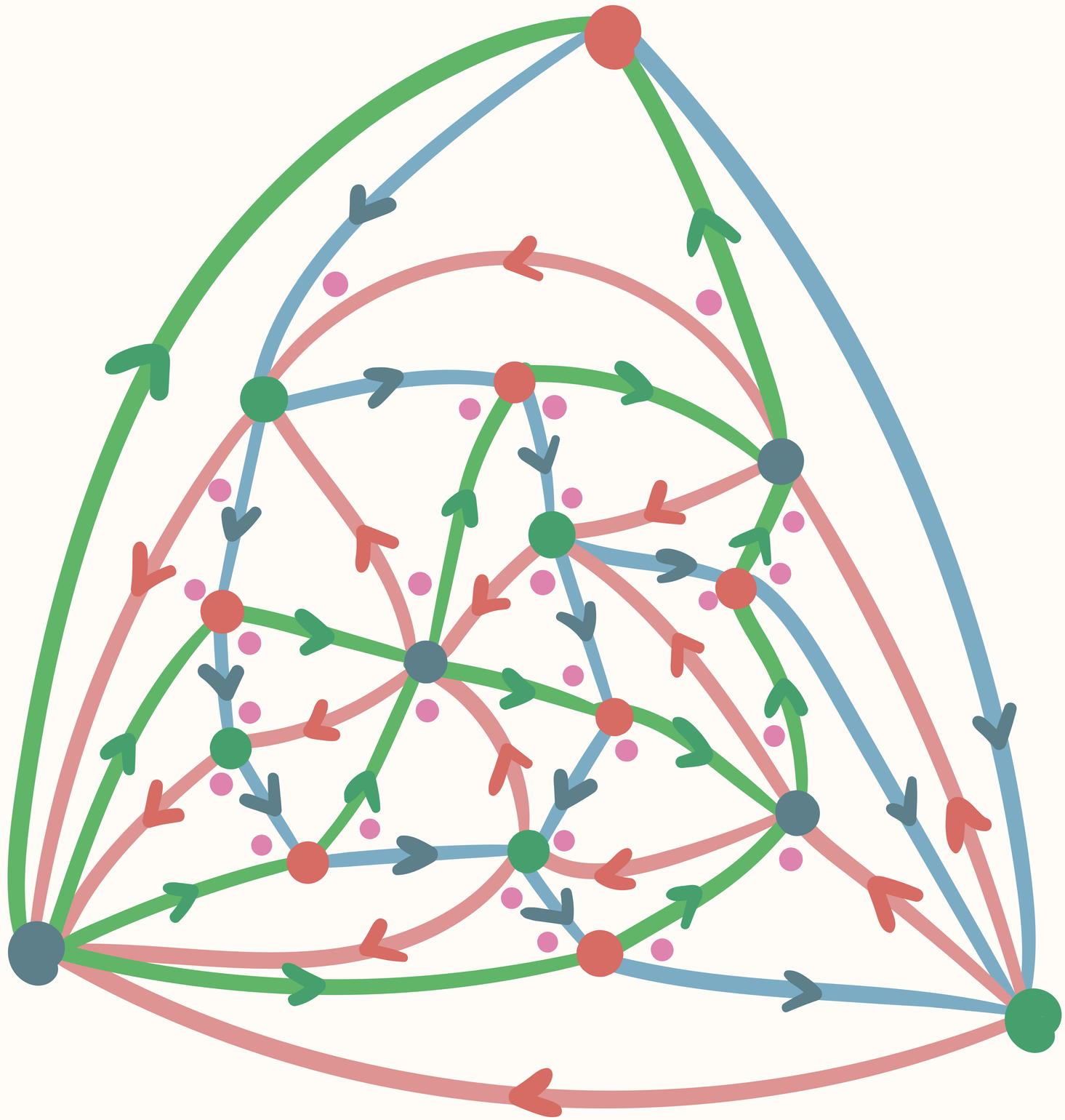


extremal x 2

Polyhedral orientations



Polyhedral orientations



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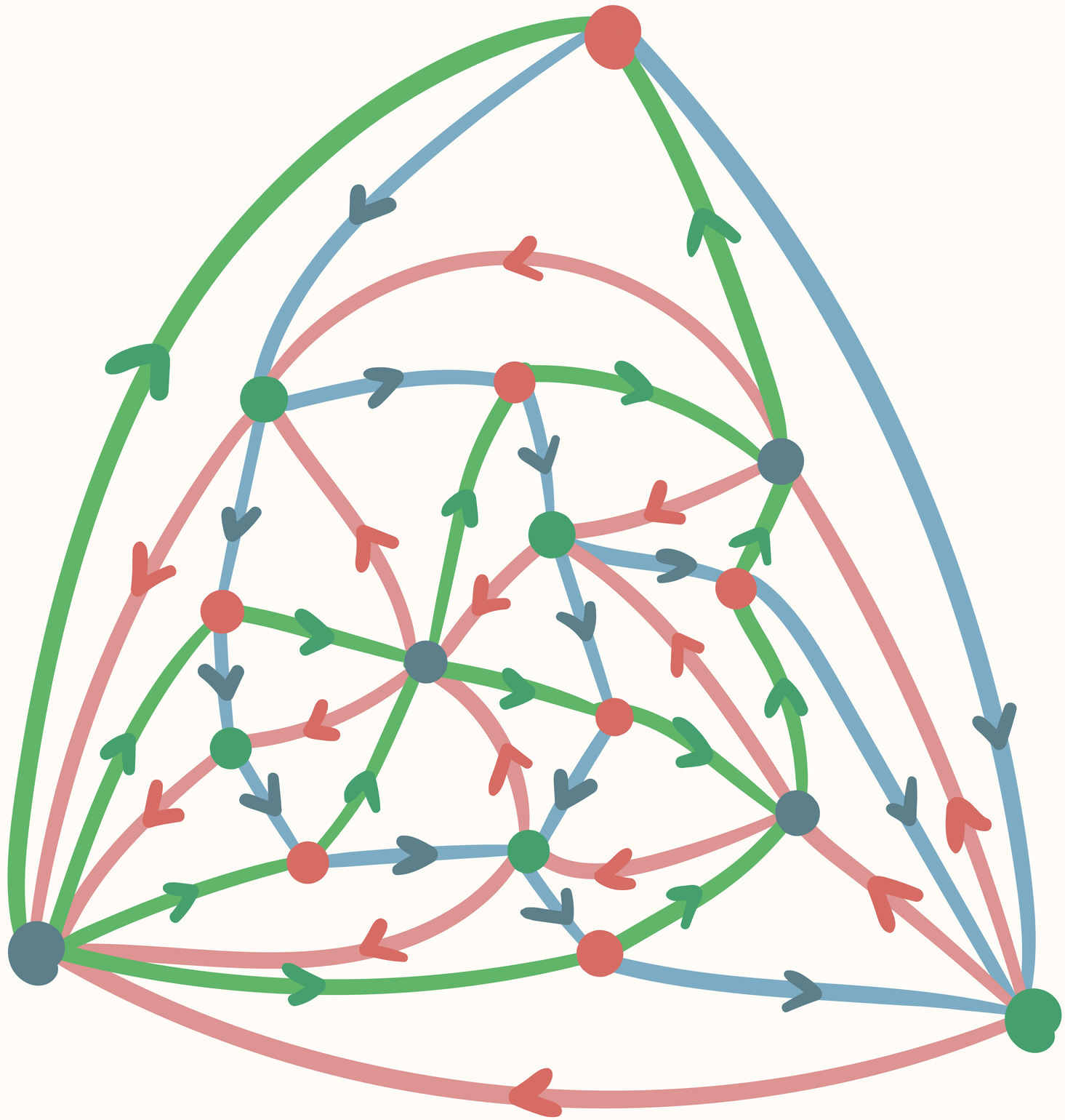

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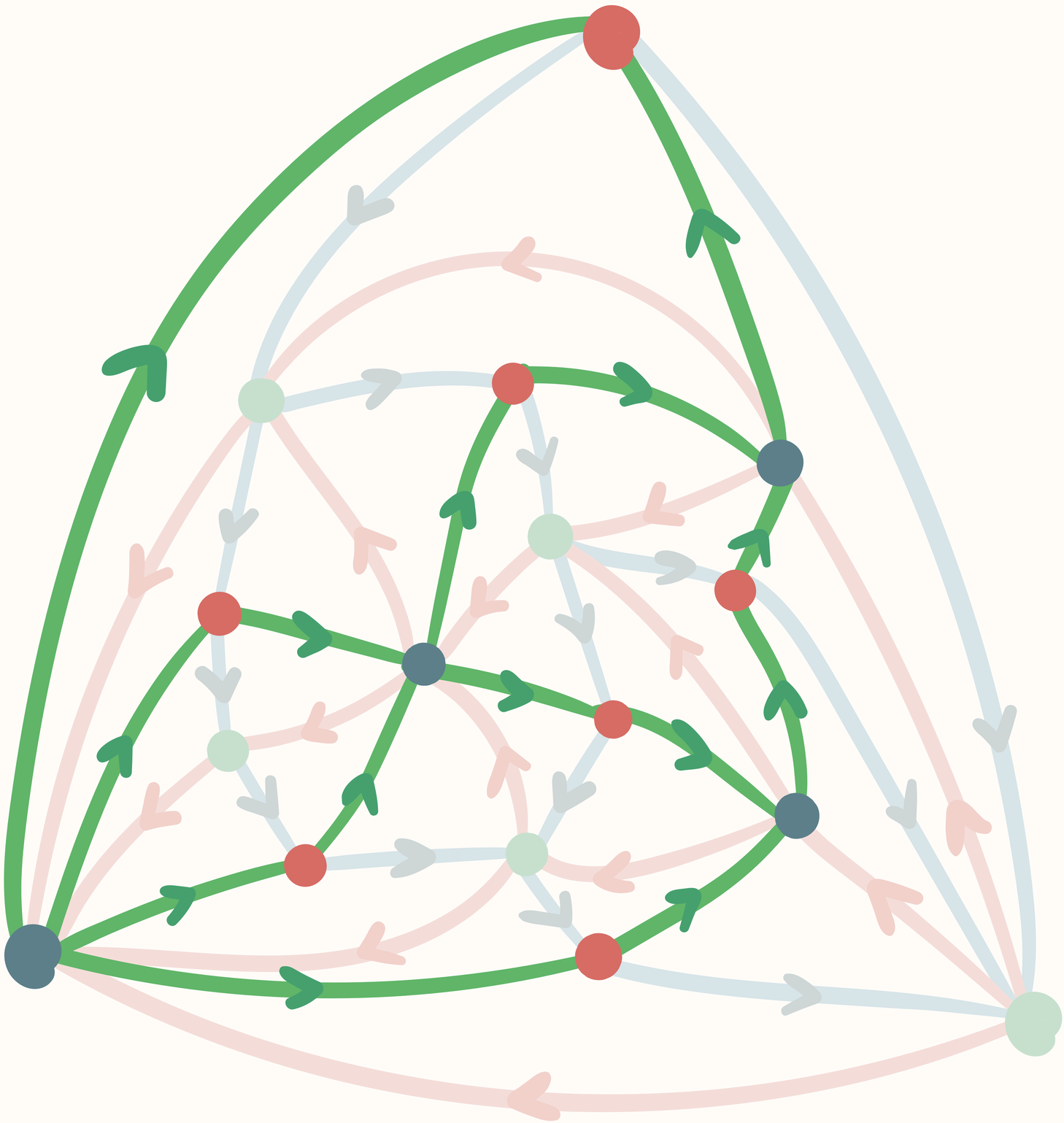

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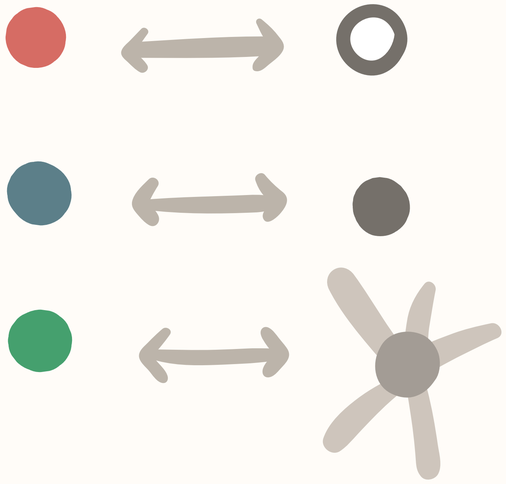

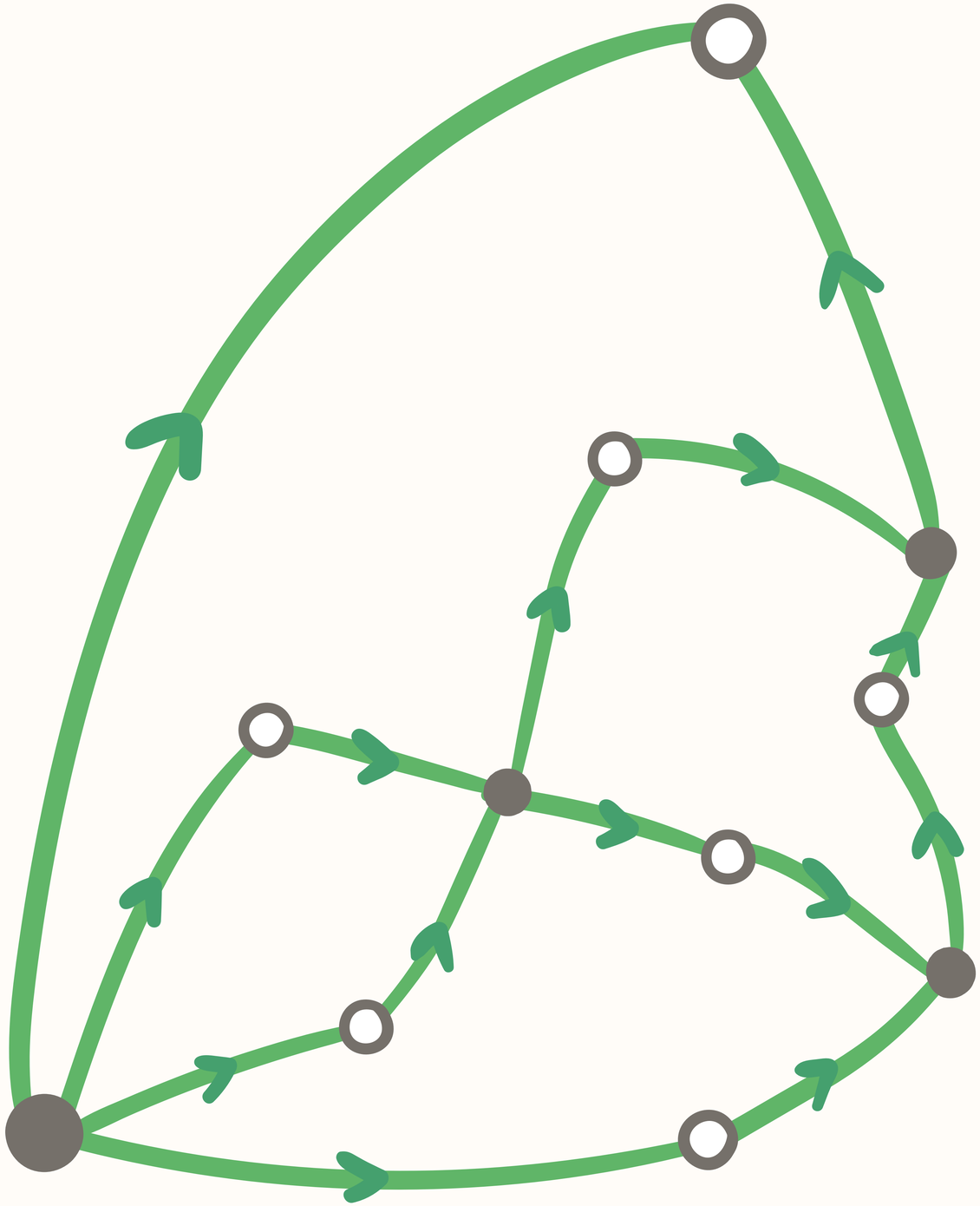

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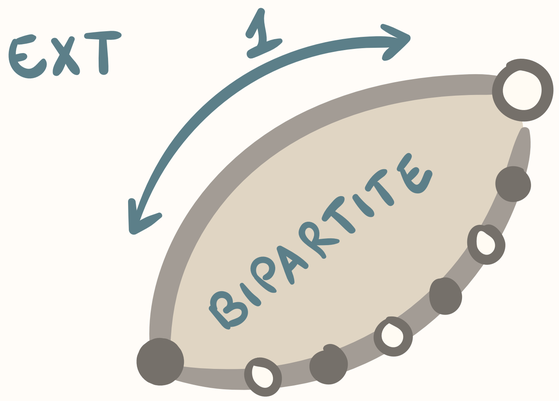
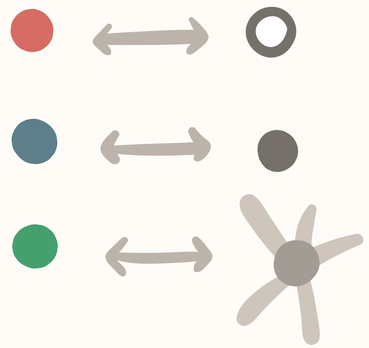
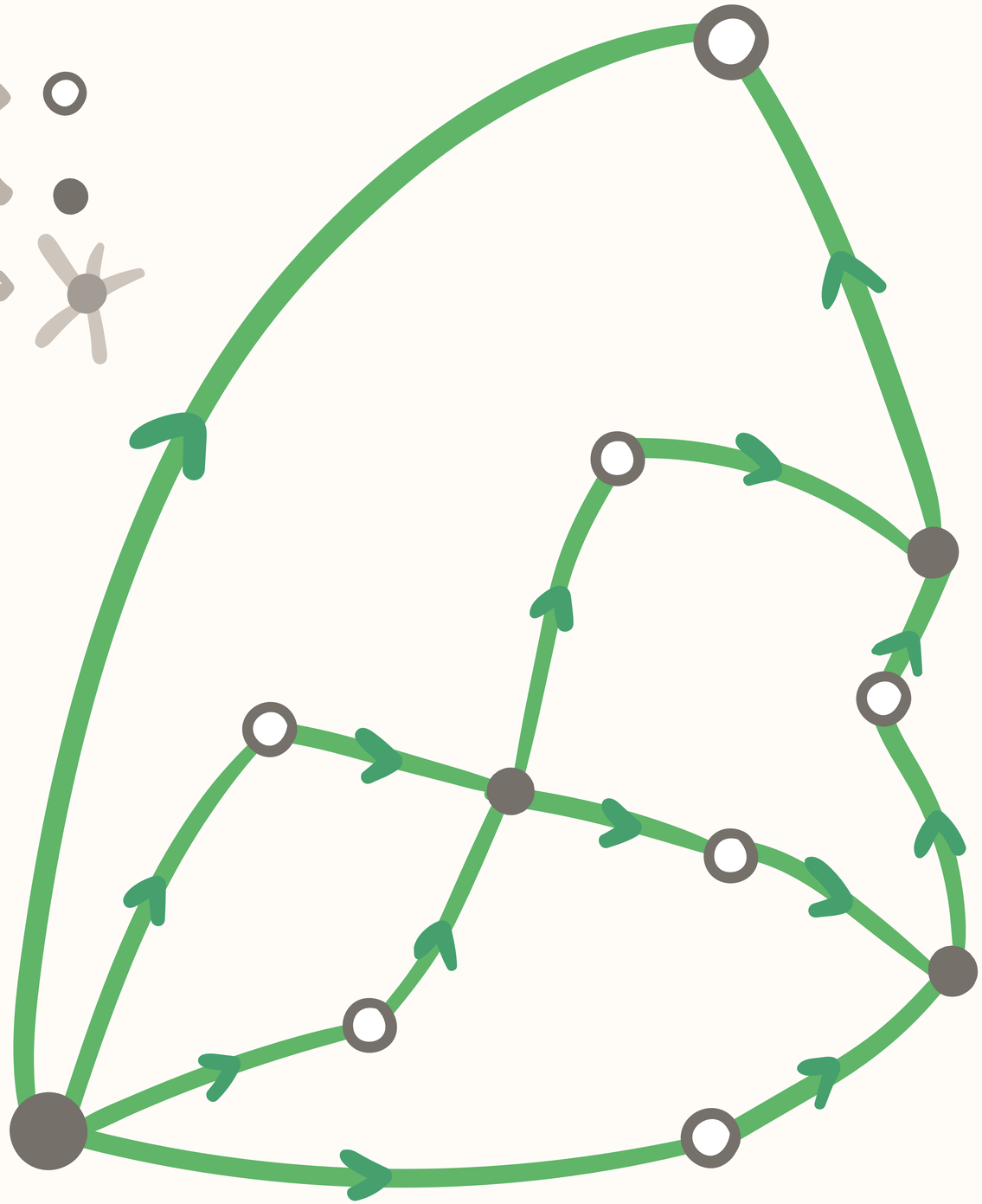
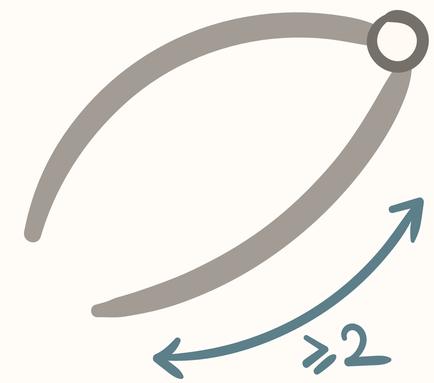
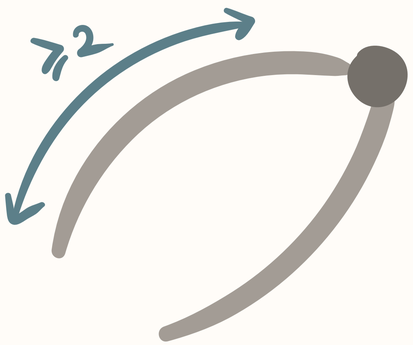


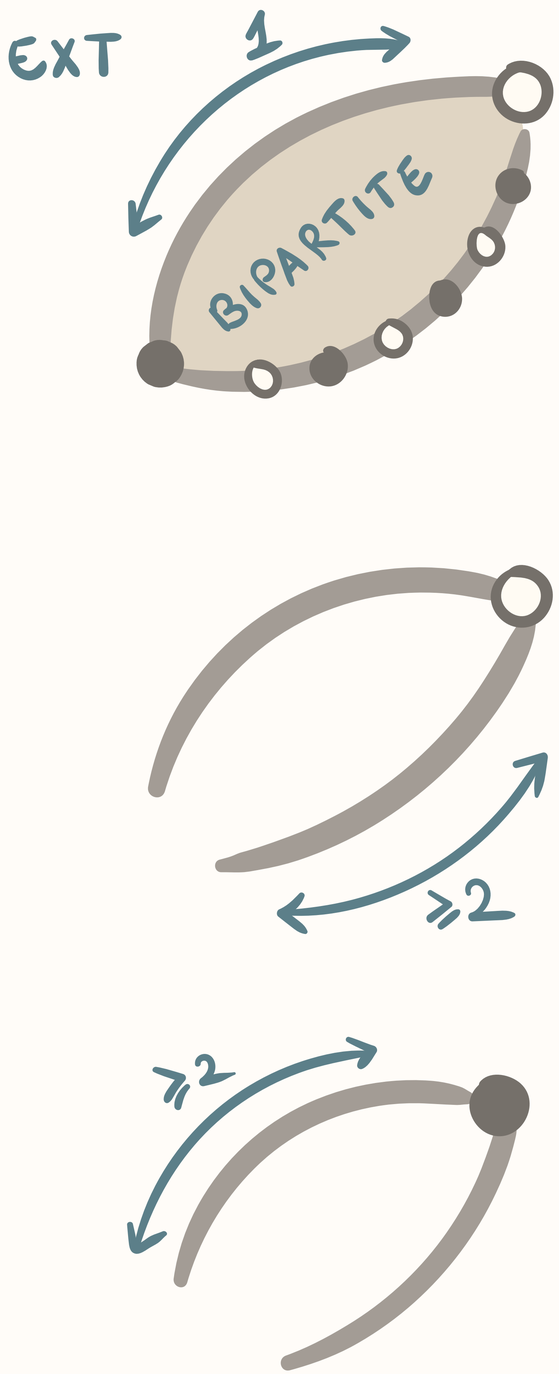








Polyhedral orientations via KMSW



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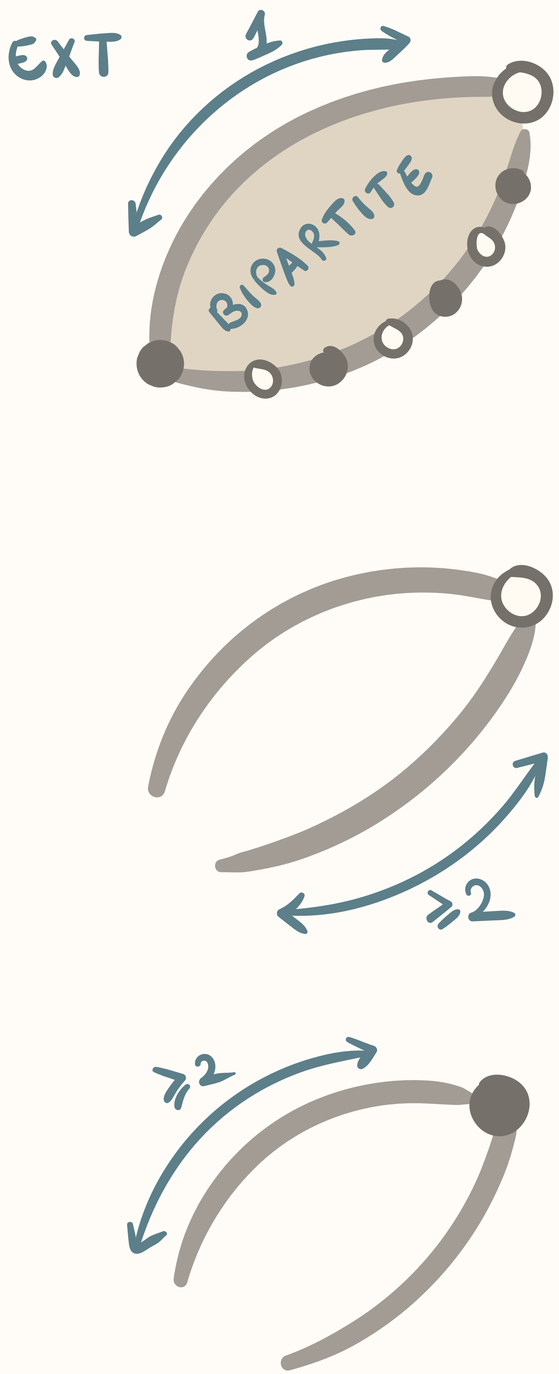

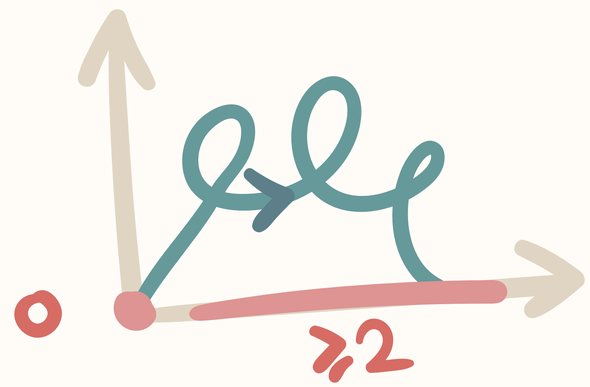

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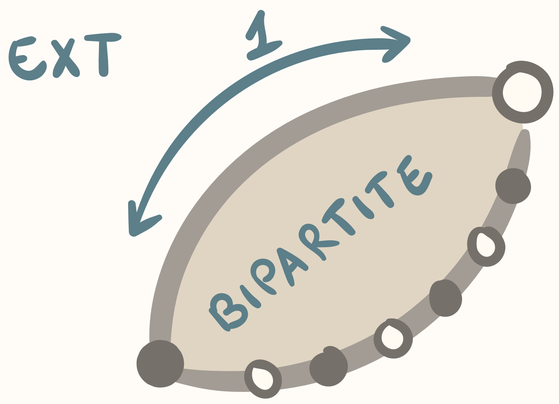
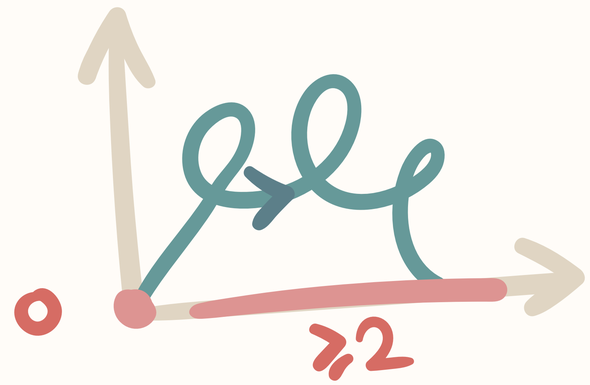
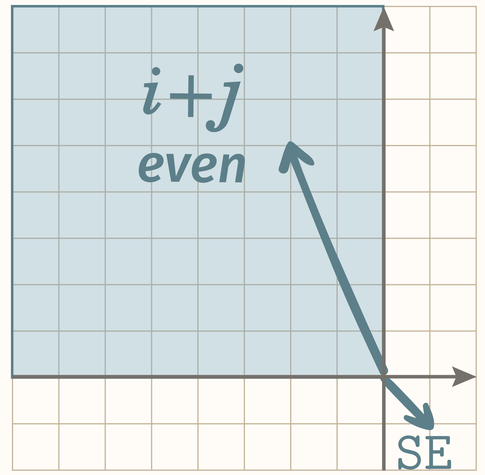
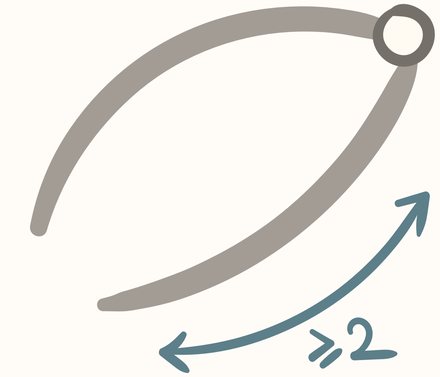
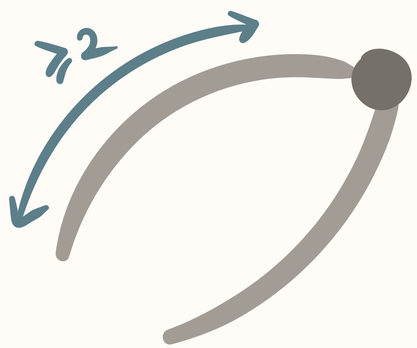


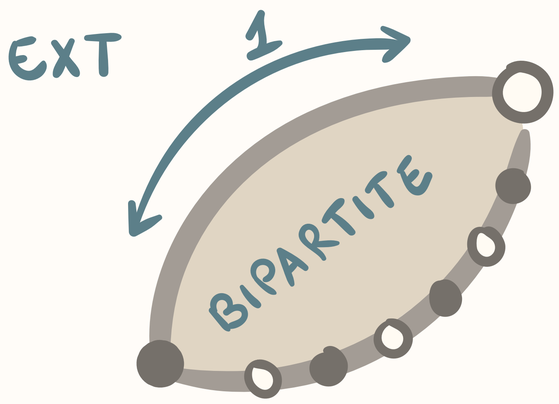


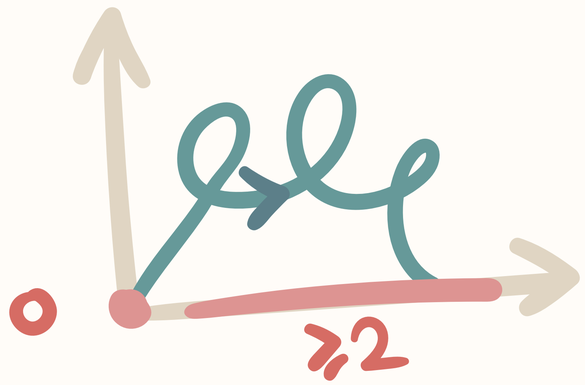


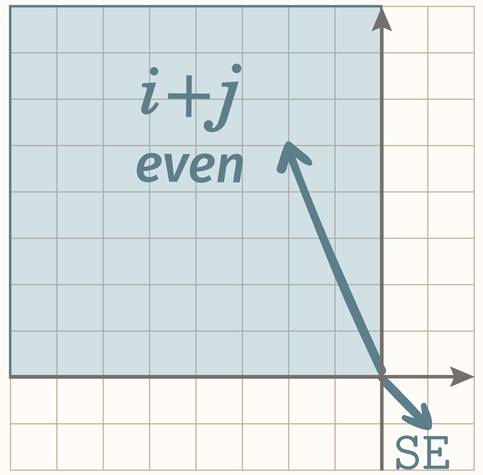


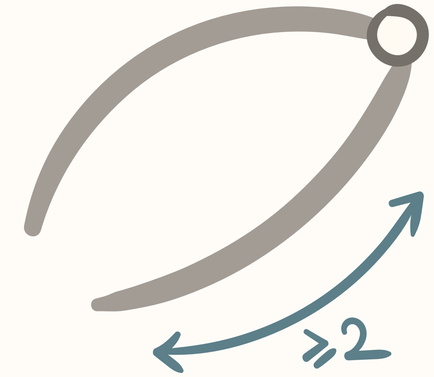


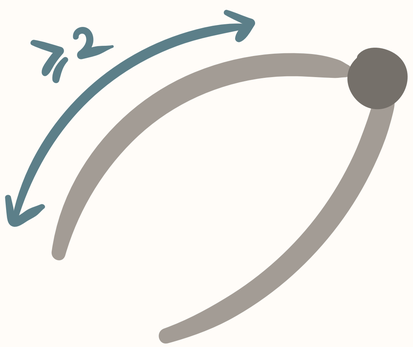
Polyhedral orientations via KMSW

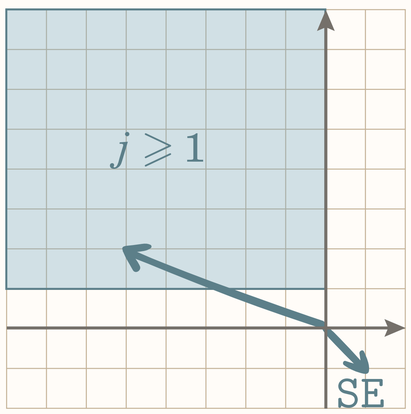






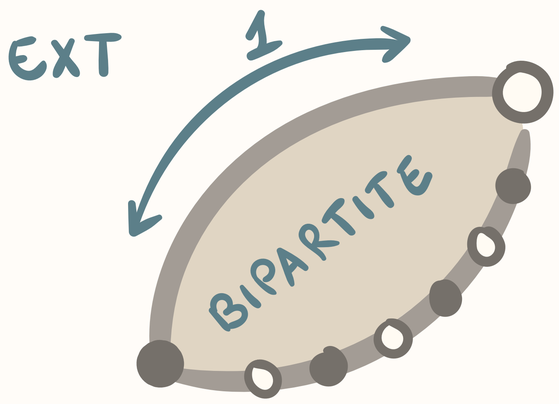


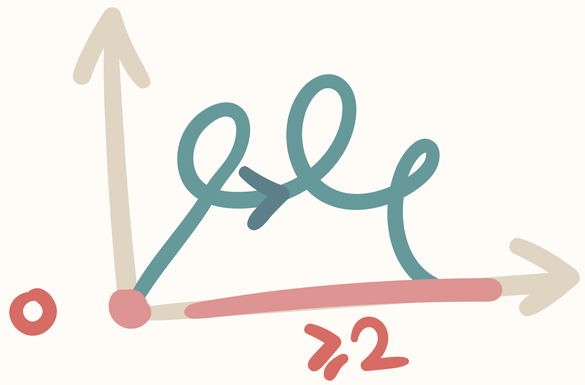


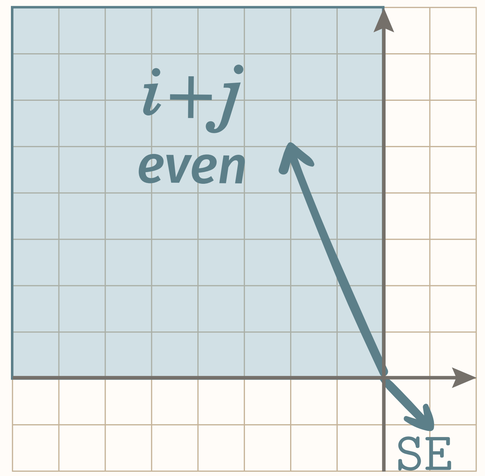


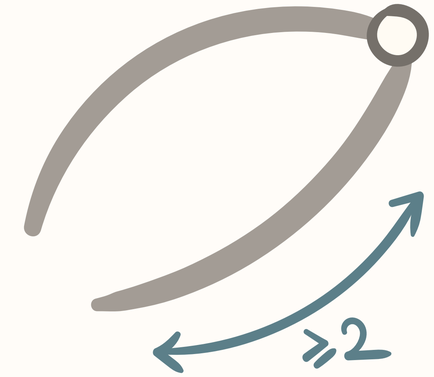
when x, y are even

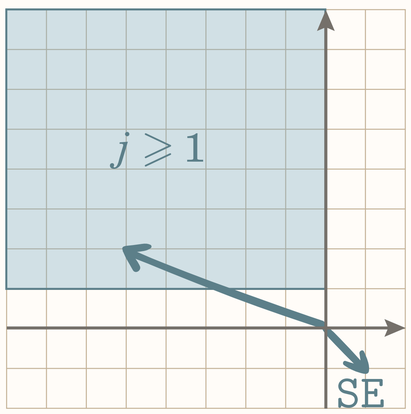
Polyhedral orientations via KMSW



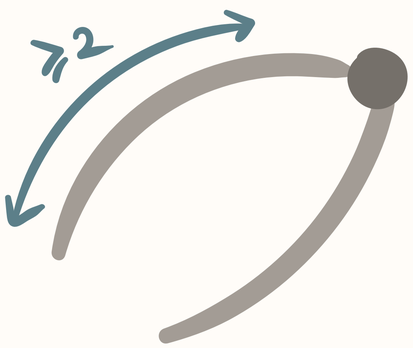


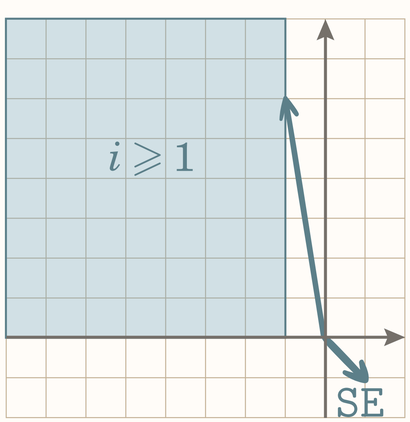






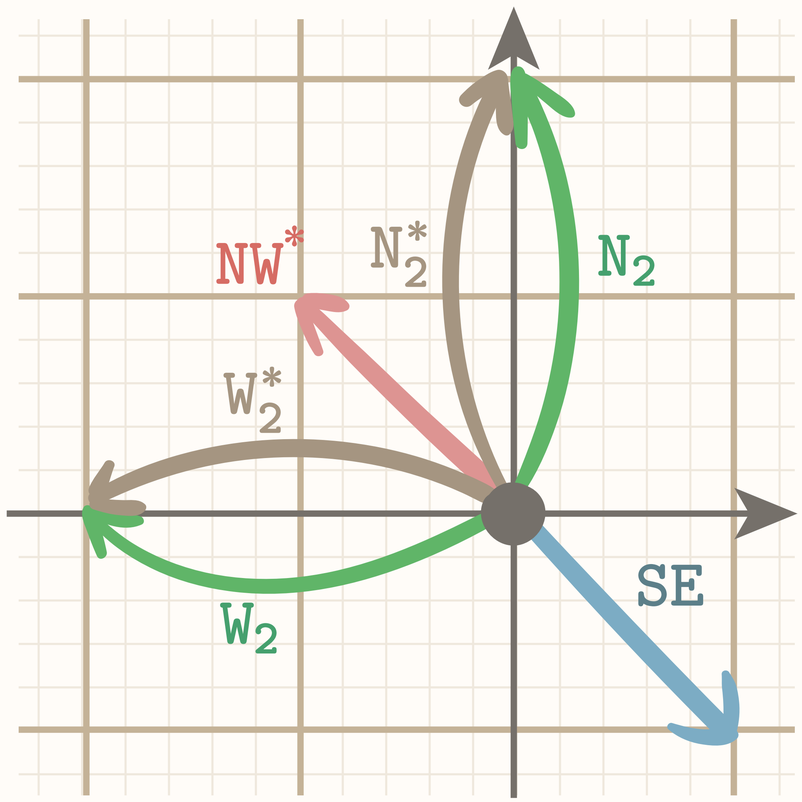
when x, y are even



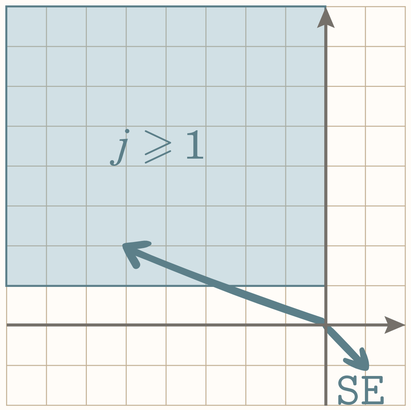


when x, y are odd

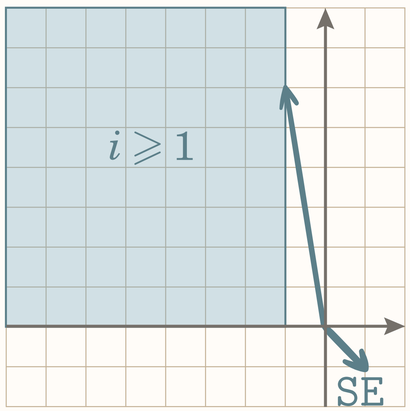
Polyhedral orientations via KMSW



$\Longleftrightarrow$

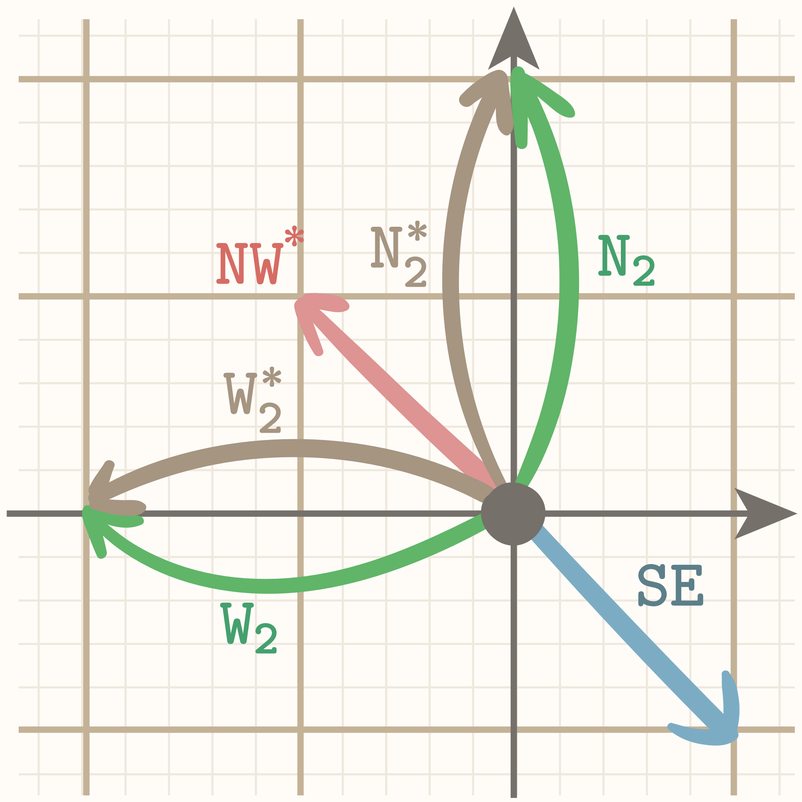


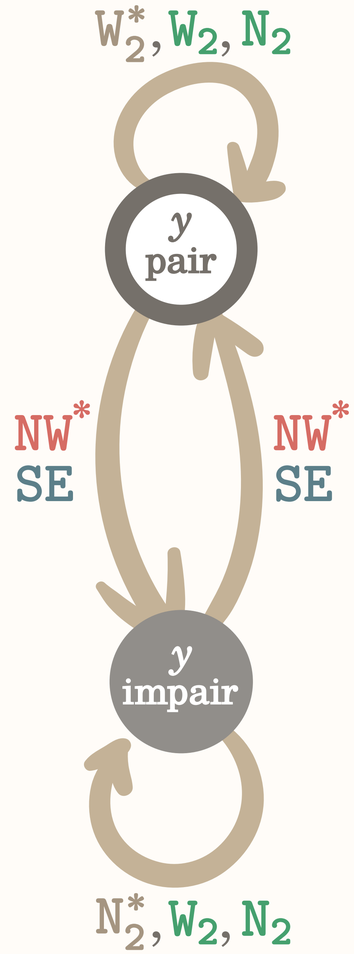
when x, y are even

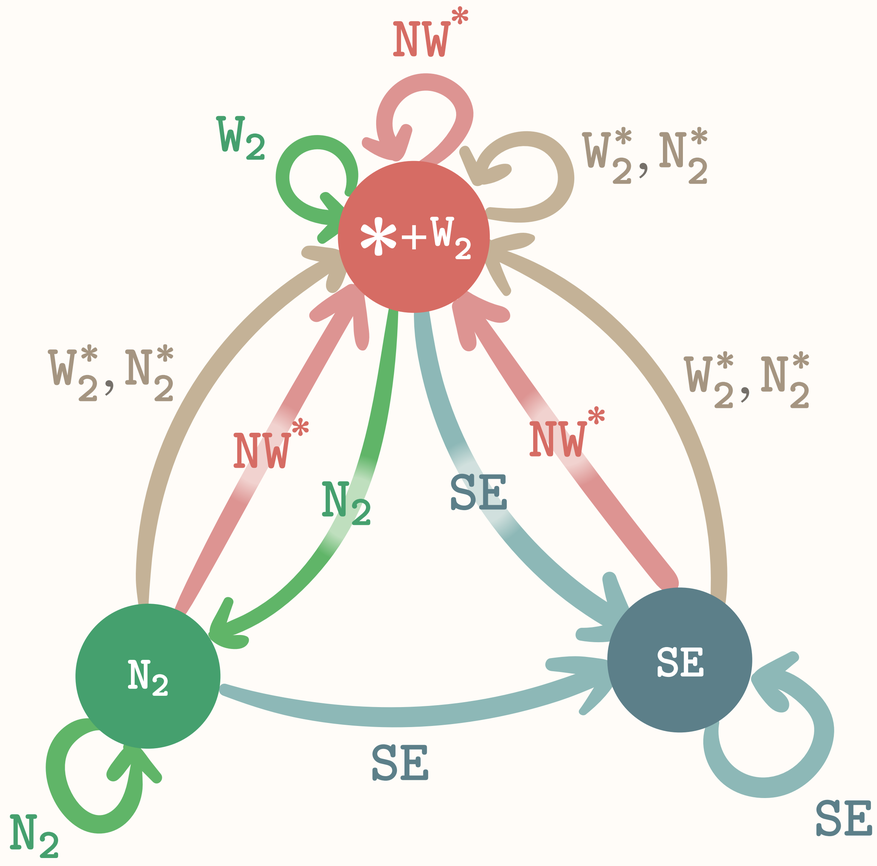


when x, y are odd

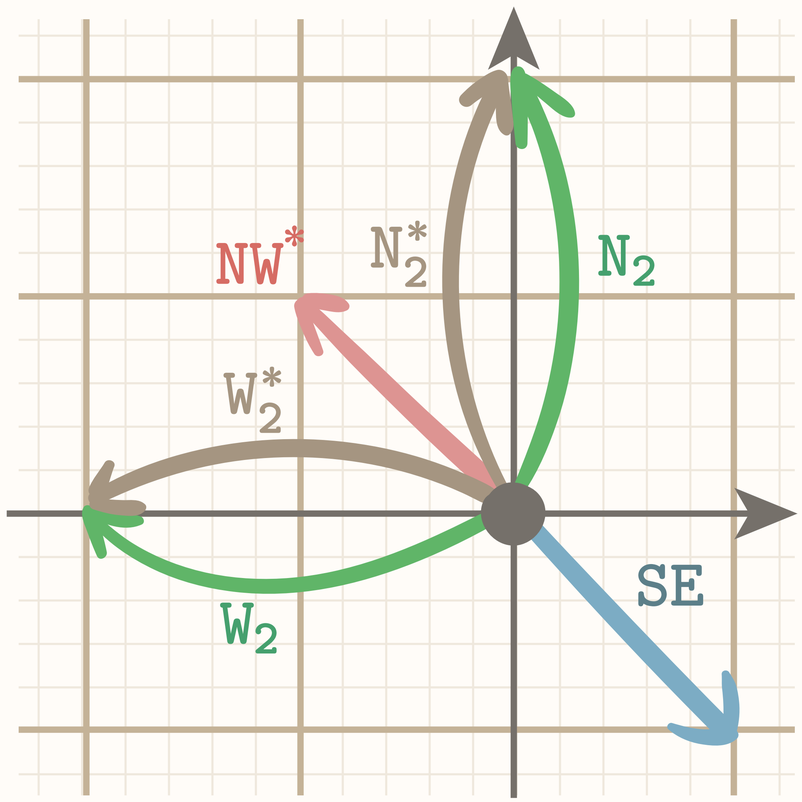
Polyhedral orientations via KMSW

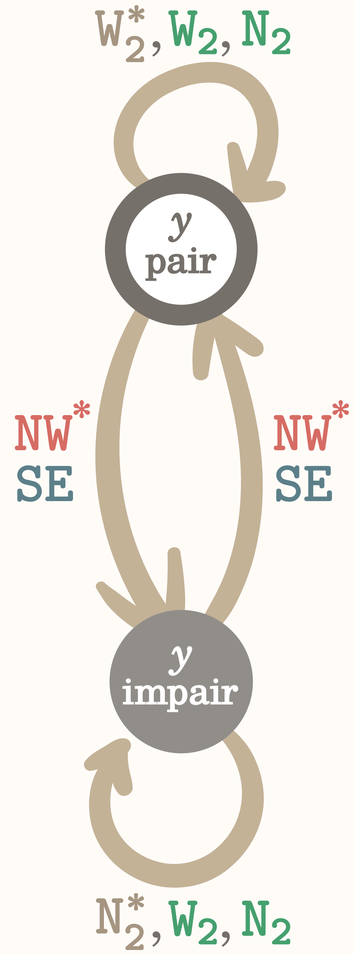


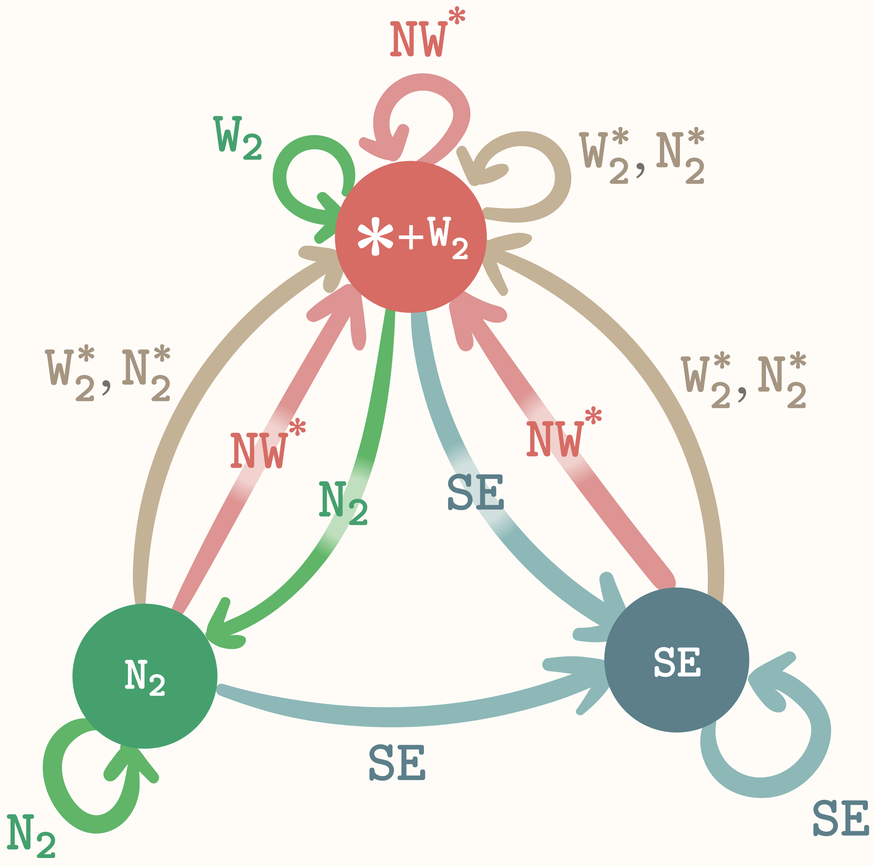




Polyhedral orientations via KMSW

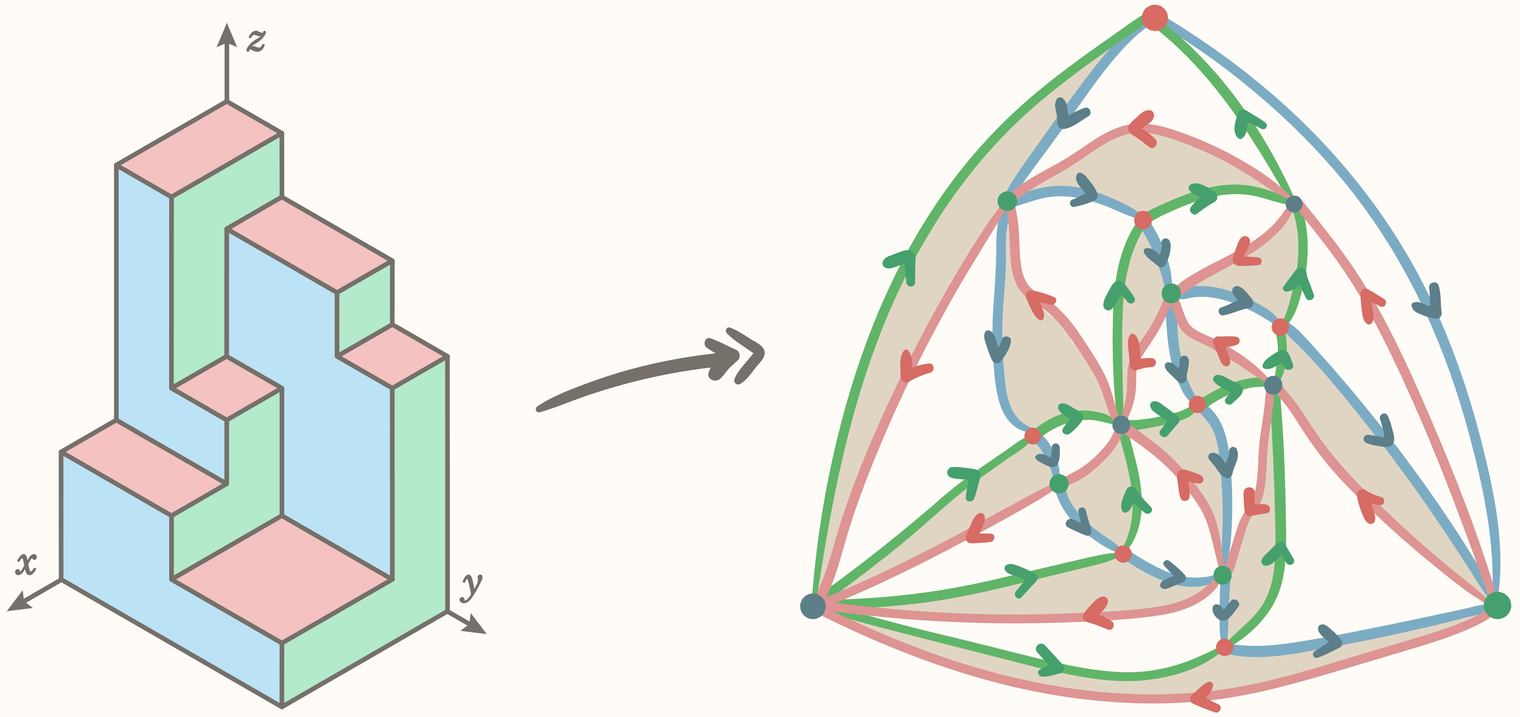




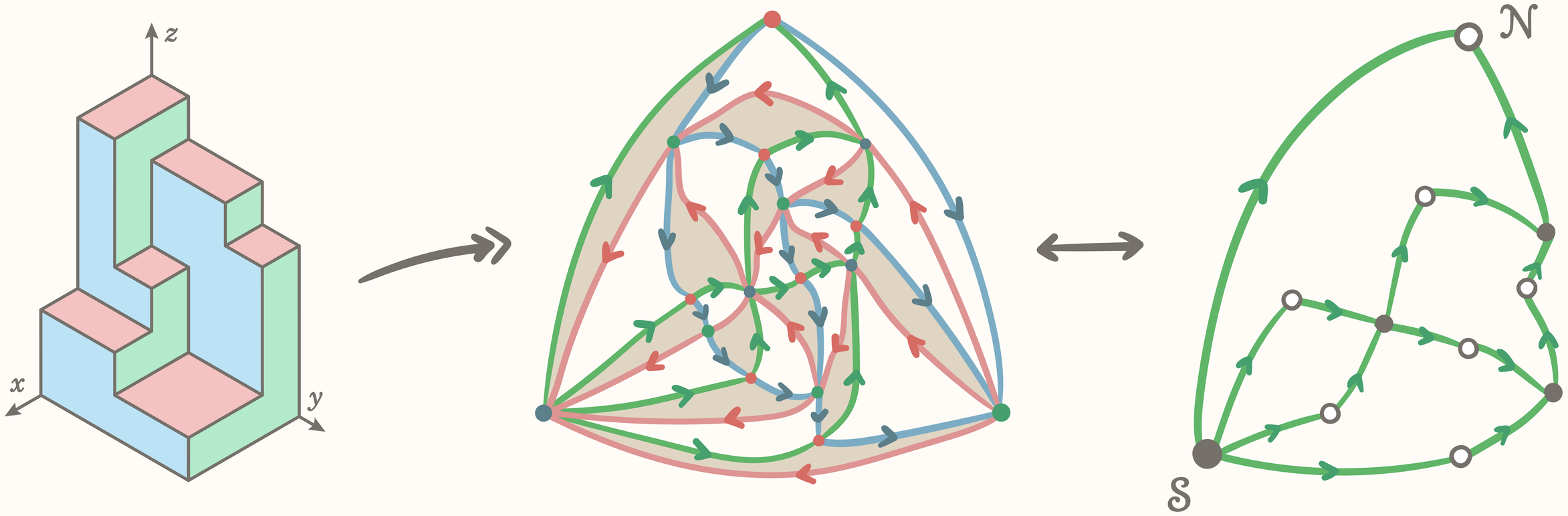


$$t^3+3t^5+4t^6+15t^7+39t^8+122t^9+375t^{10}+1212t^{11}+3980t^{12}+O(t^{13})$$

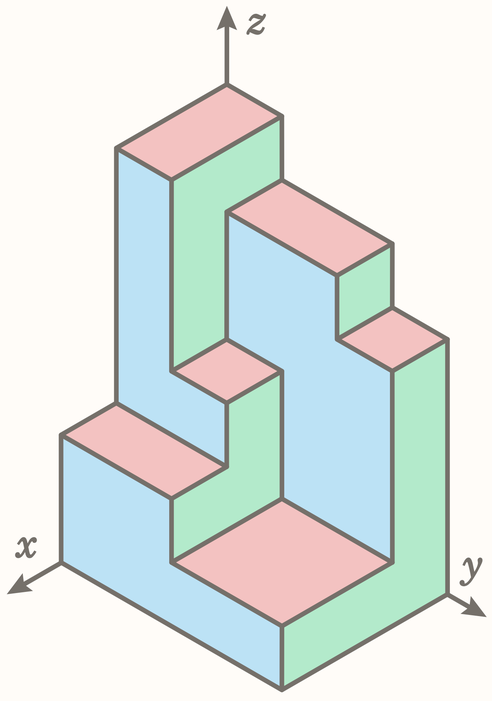
Recap polyhedral orientations

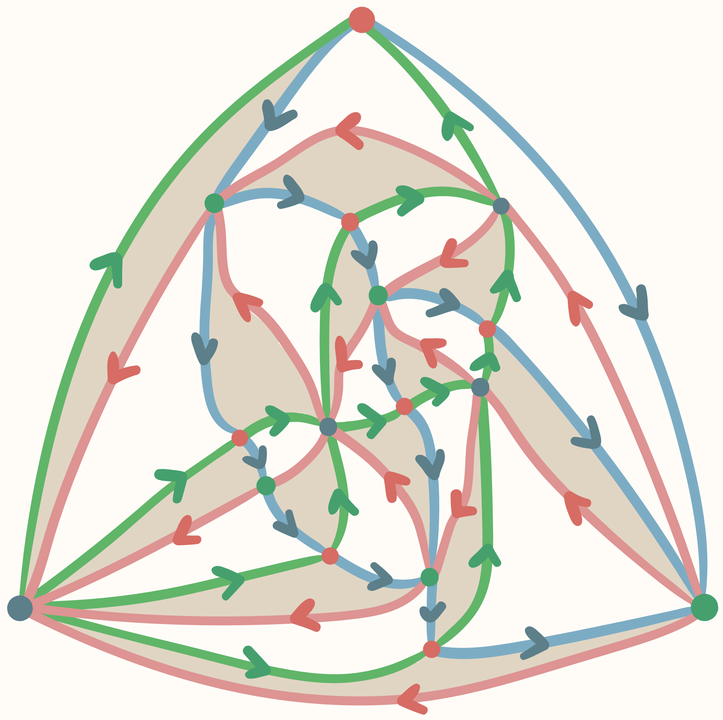


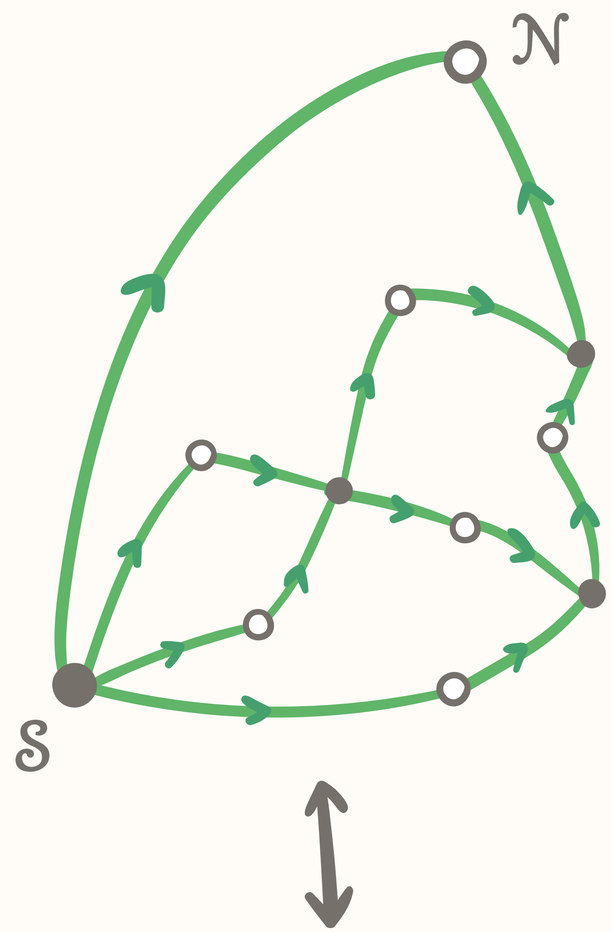
Recap polyhedral orientations

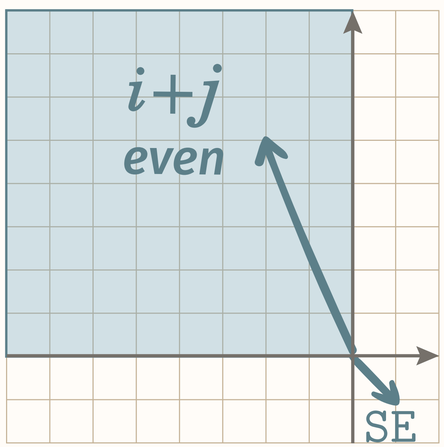


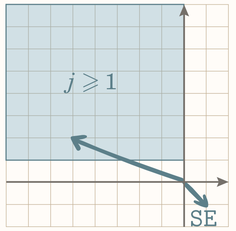
Recap polyhedral orientations



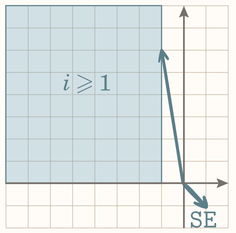






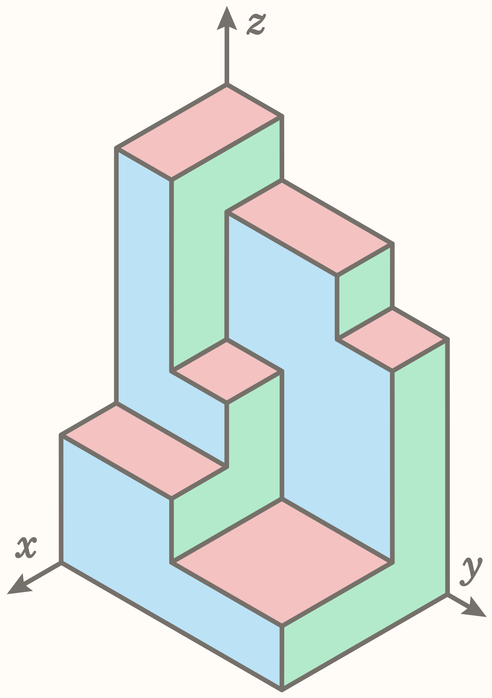


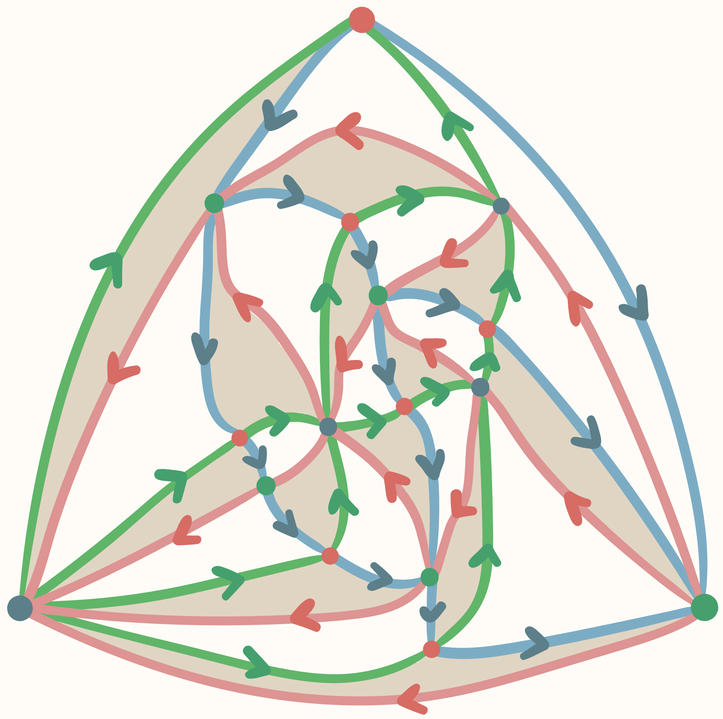
when x, y are even

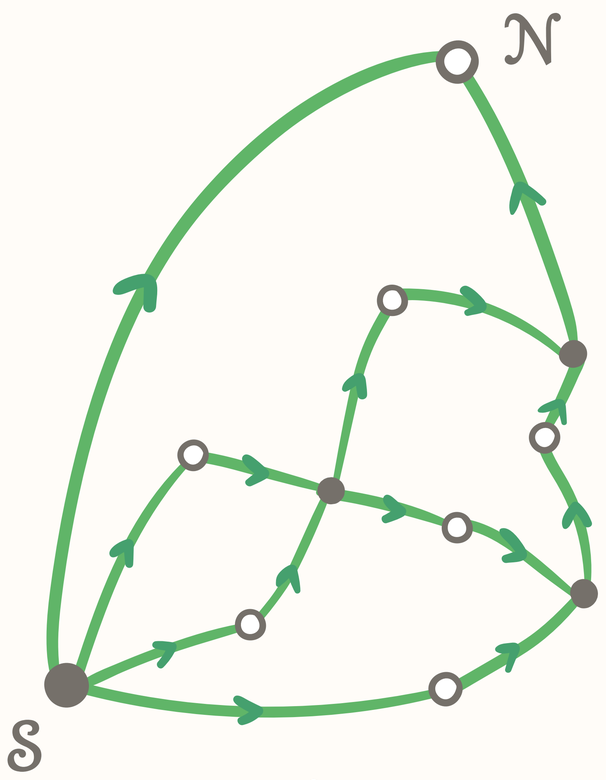


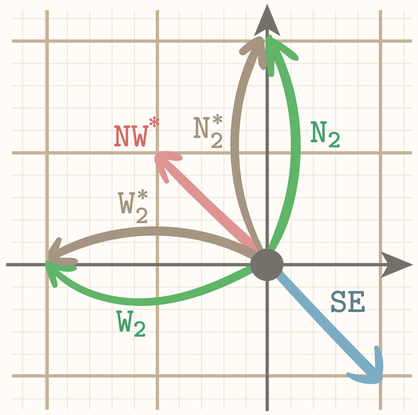
when x, y are odd

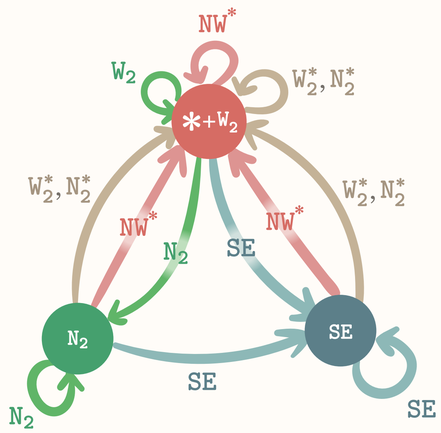
Recap polyhedral orientations

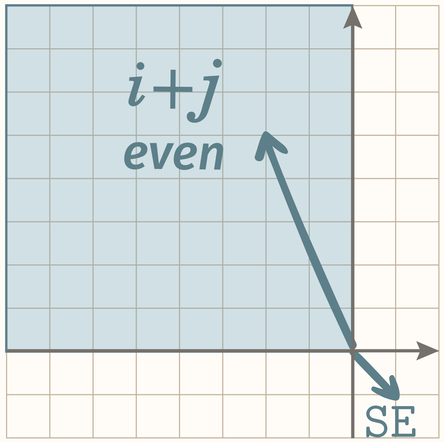


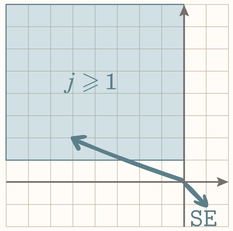




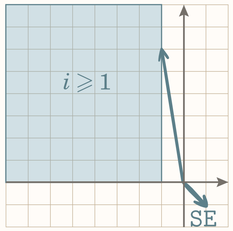








when x, y are even



when x, y are odd

Summary

Quadrant tandem walks

Asymptotics

Bipolar orient.

Decorated bipolar orientations

Small steps

bipolar posets by vertices

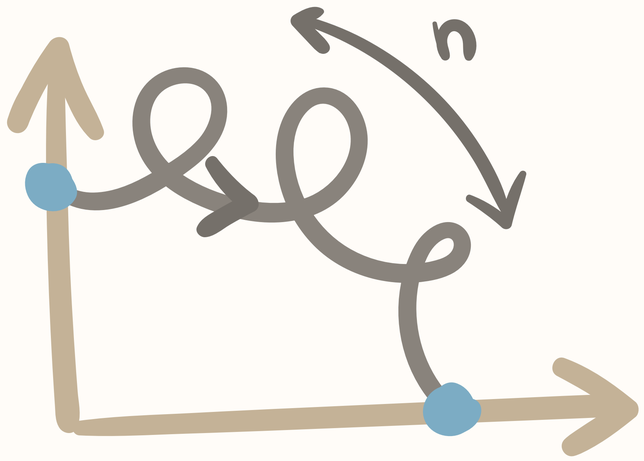
bipolar posets by edges

transversal structures

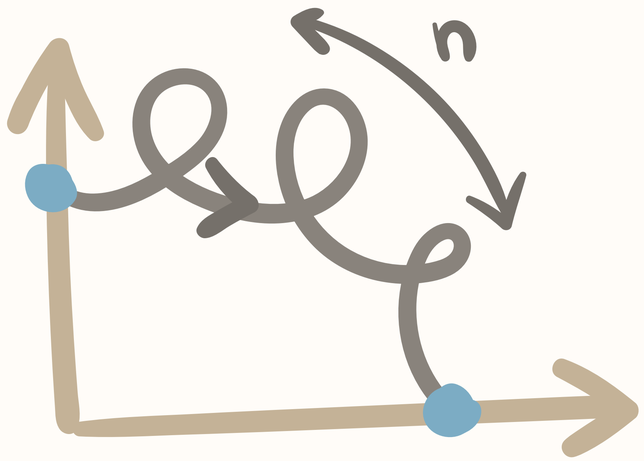
polyhedral orientations

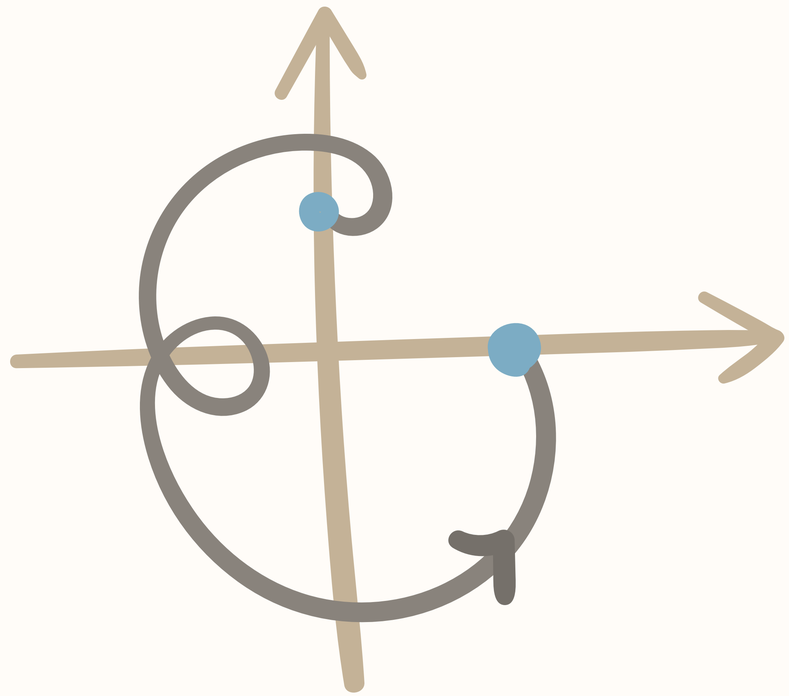
Schnyder labelings

Asymptotic counting results



Asymptotic counting results





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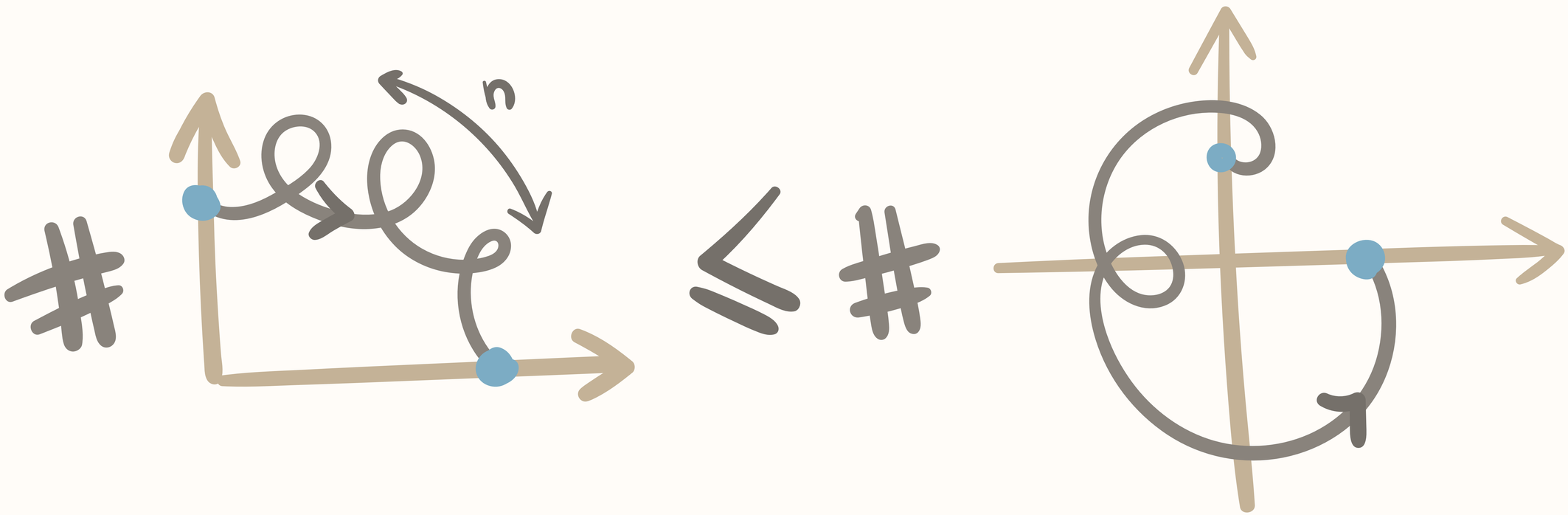

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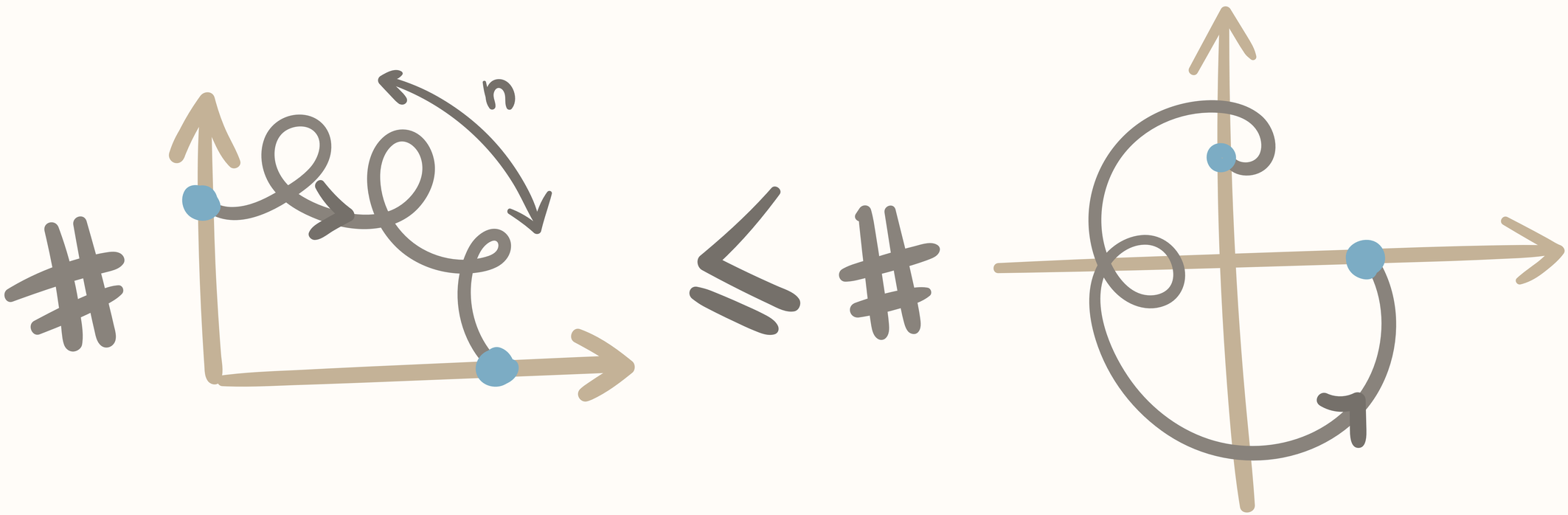
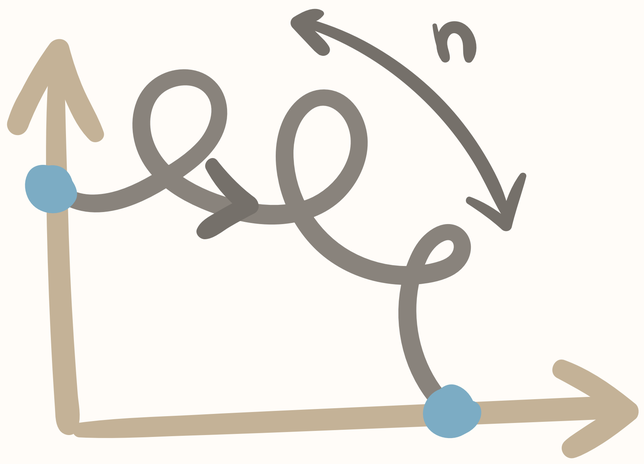


$$\#\left\{\text{tetrahedron}\right\}_n \leqslant \left(\frac{9}{2}\right)^n$$

$$\#\left\{\text{cube}\right\}_n \leqslant 2\cdot\left(\frac{16}{3}\right)^n$$

Asymptotic counting results



$$\#\,\Big|_n \leqslant \left(\frac{9}{2}\right)^n$$

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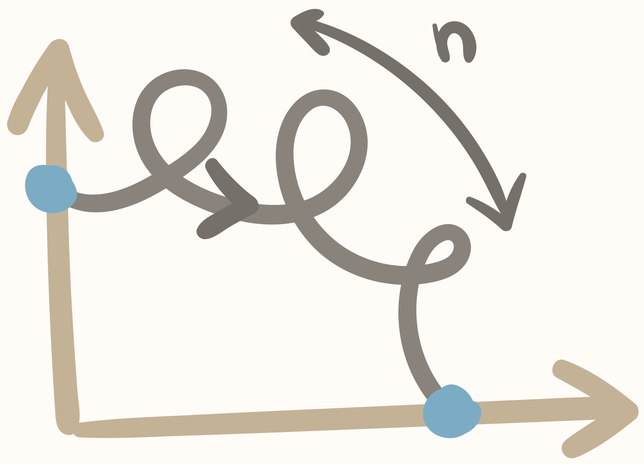
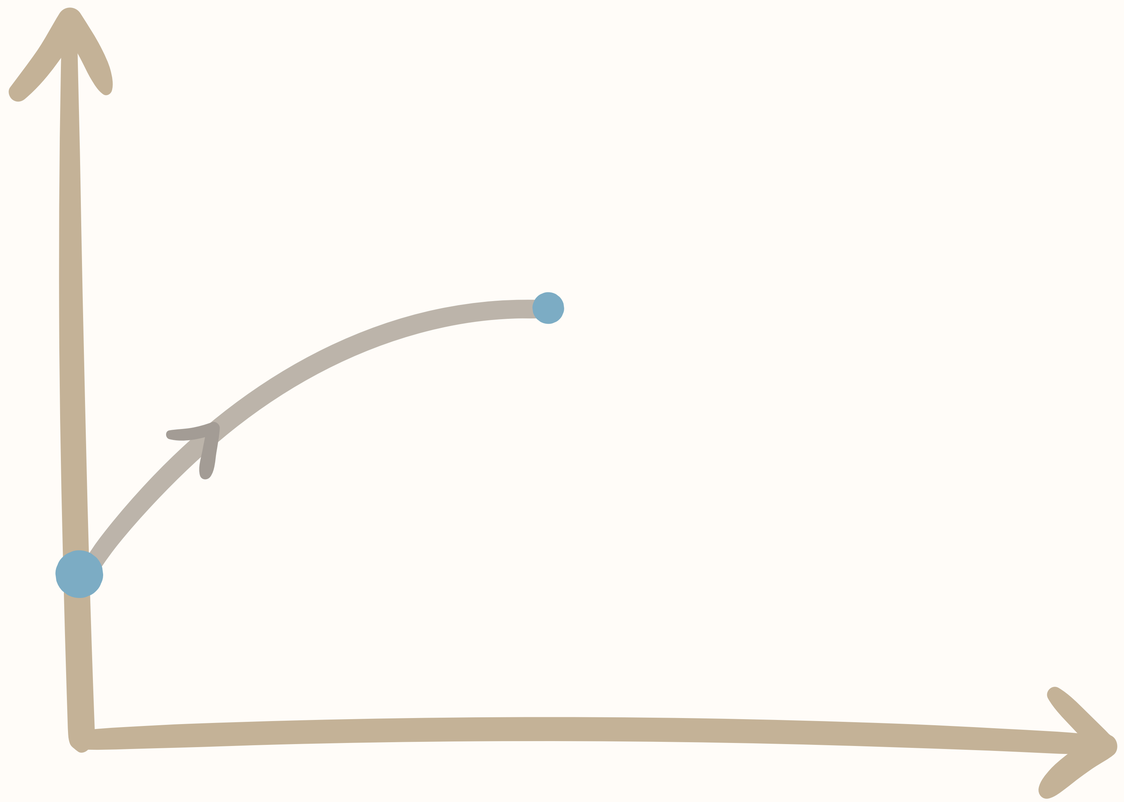
Asymptotic counting results





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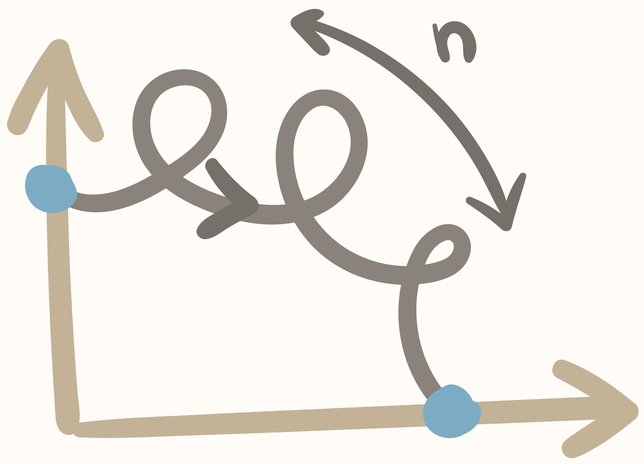
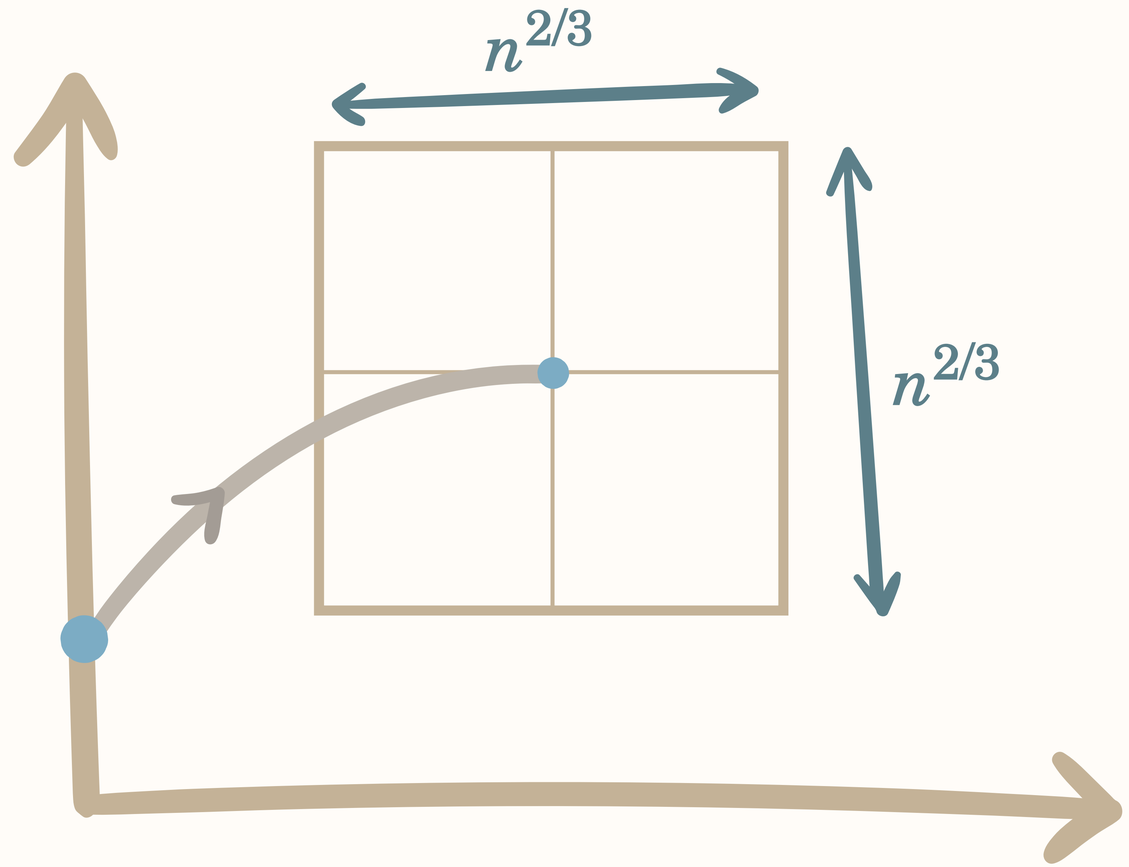
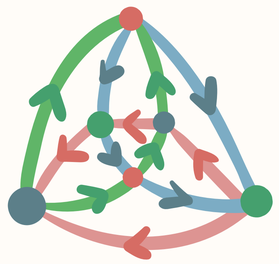
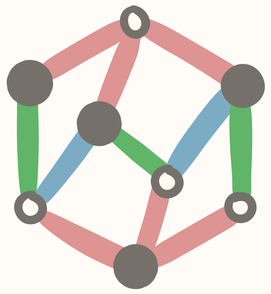
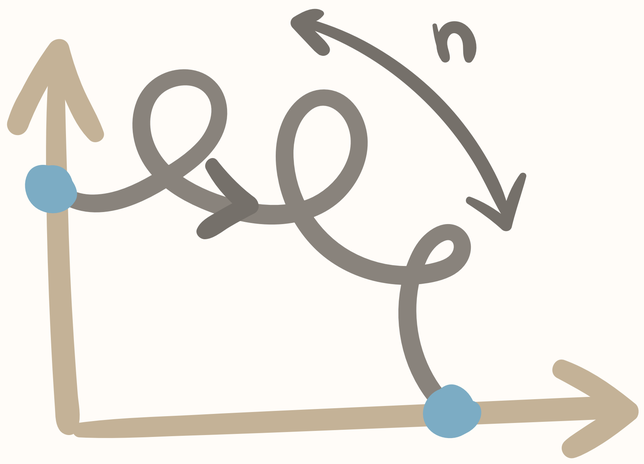


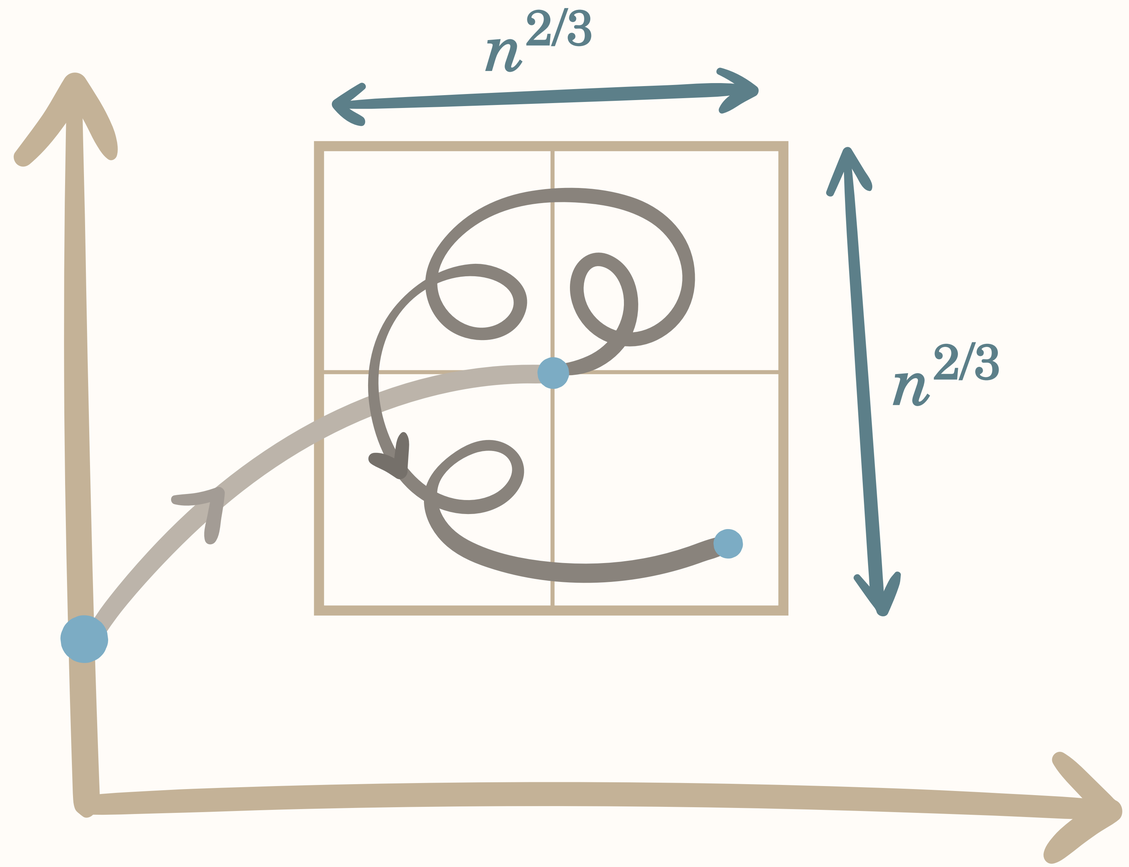


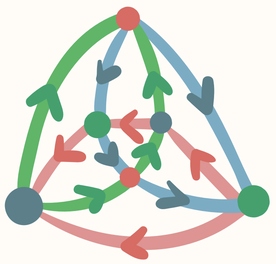
$$\#\left\{\text{  }\right\}_n \leq \left(\frac{9}{2}\right)^n$$

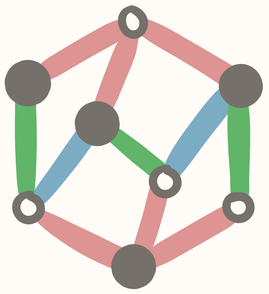
$$\#\left\{\text{  }\right\}_n \leq 2\cdot\left(\frac{16}{3}\right)^n$$

Asymptotic counting results

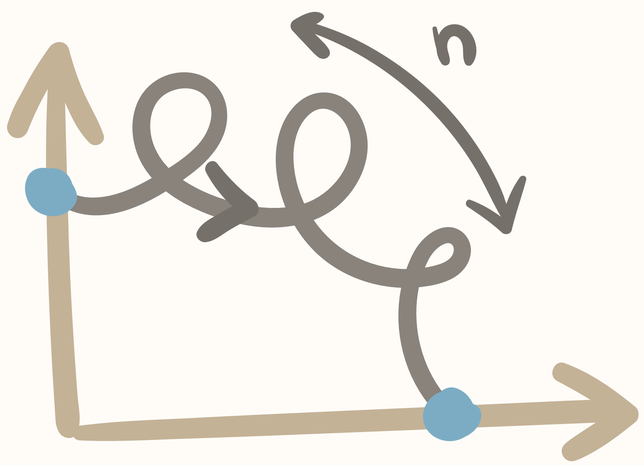


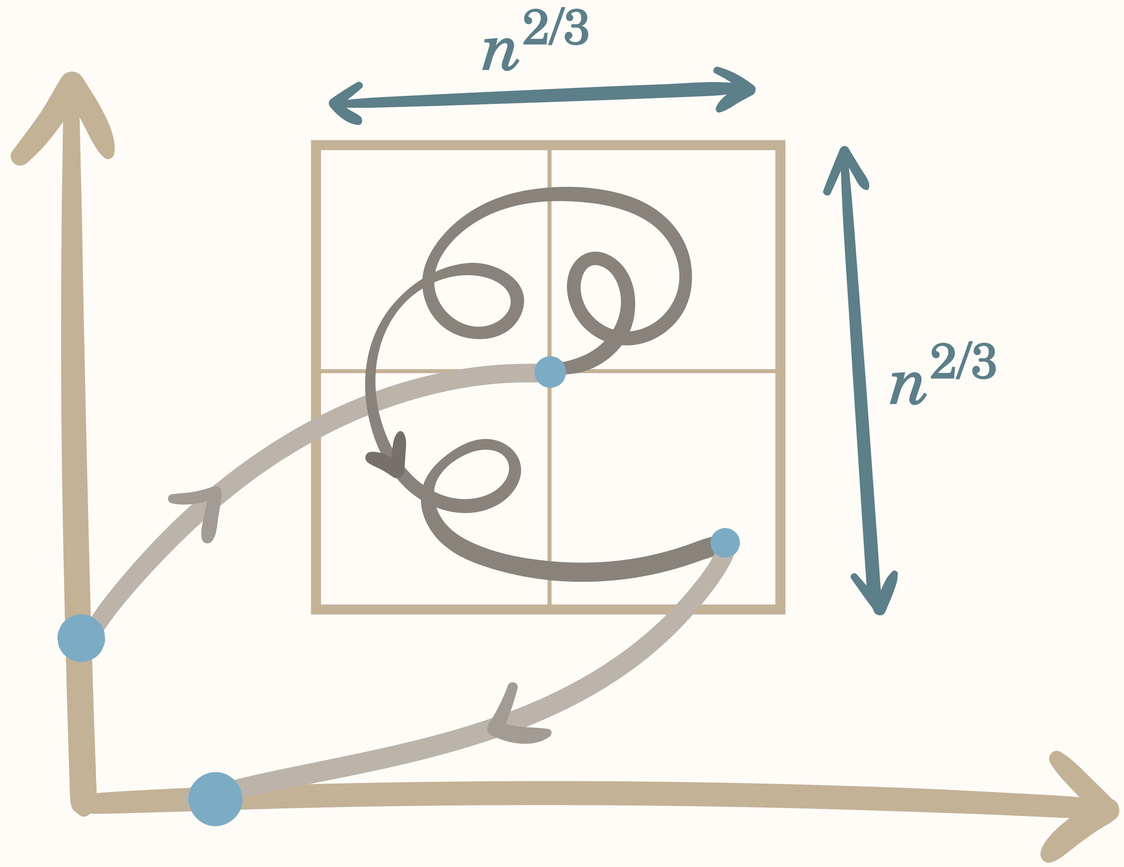


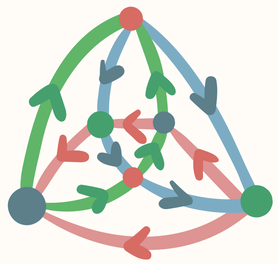
$$\#\left\{\text{  }\right\}_n \leq \left(\frac{9}{2}\right)^n$$

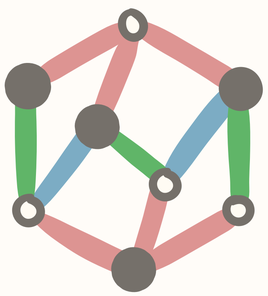
$$\#\left\{\text{  }\right\}_n \leq 2\cdot\left(\frac{16}{3}\right)^n$$

Asymptotic counting results

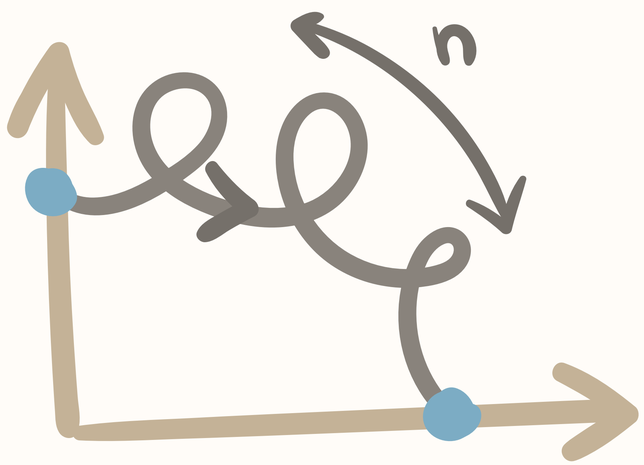


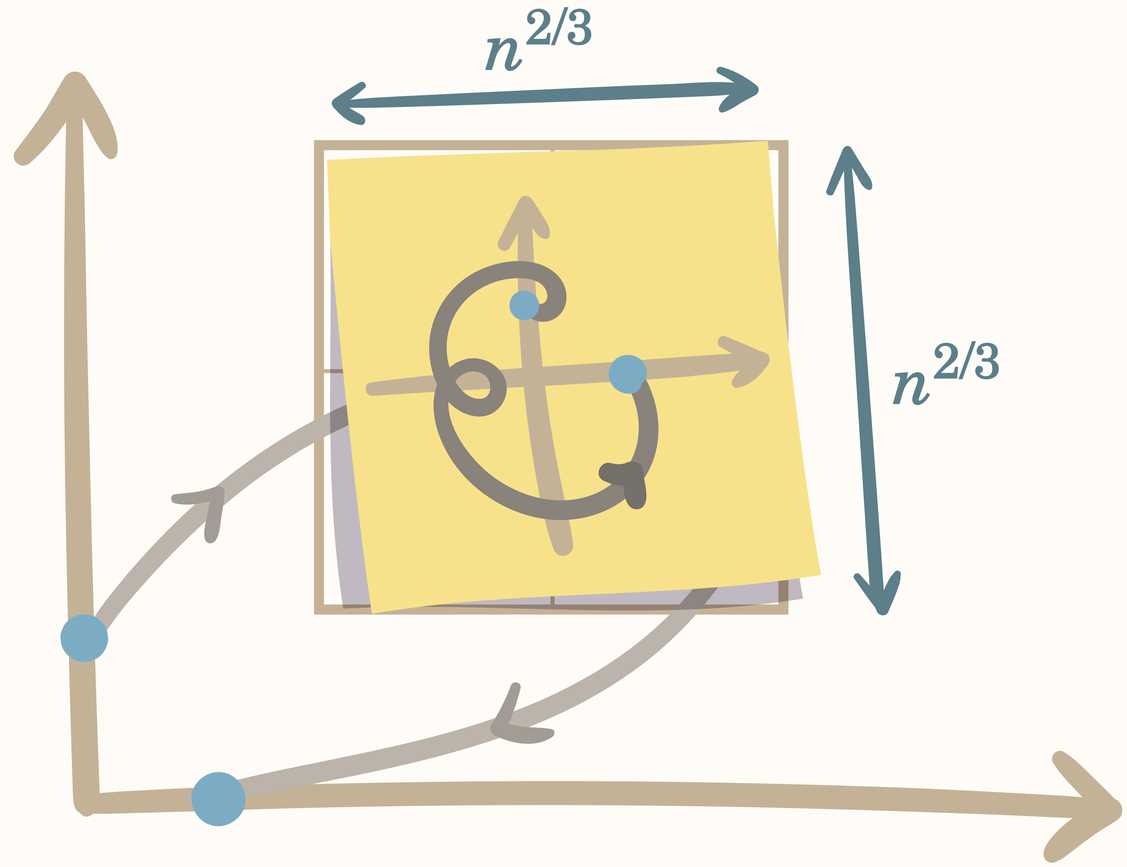


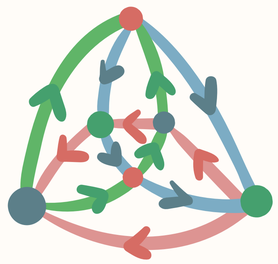
$$\#\left\{\text{  }\right\}_n \leqslant \left(\frac{9}{2}\right)^n$$

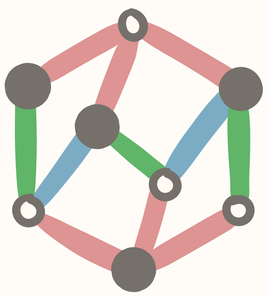
$$\#\left\{\text{  }\right\}_n \leqslant 2\cdot\left(\frac{16}{3}\right)^n$$

Asymptotic counting results

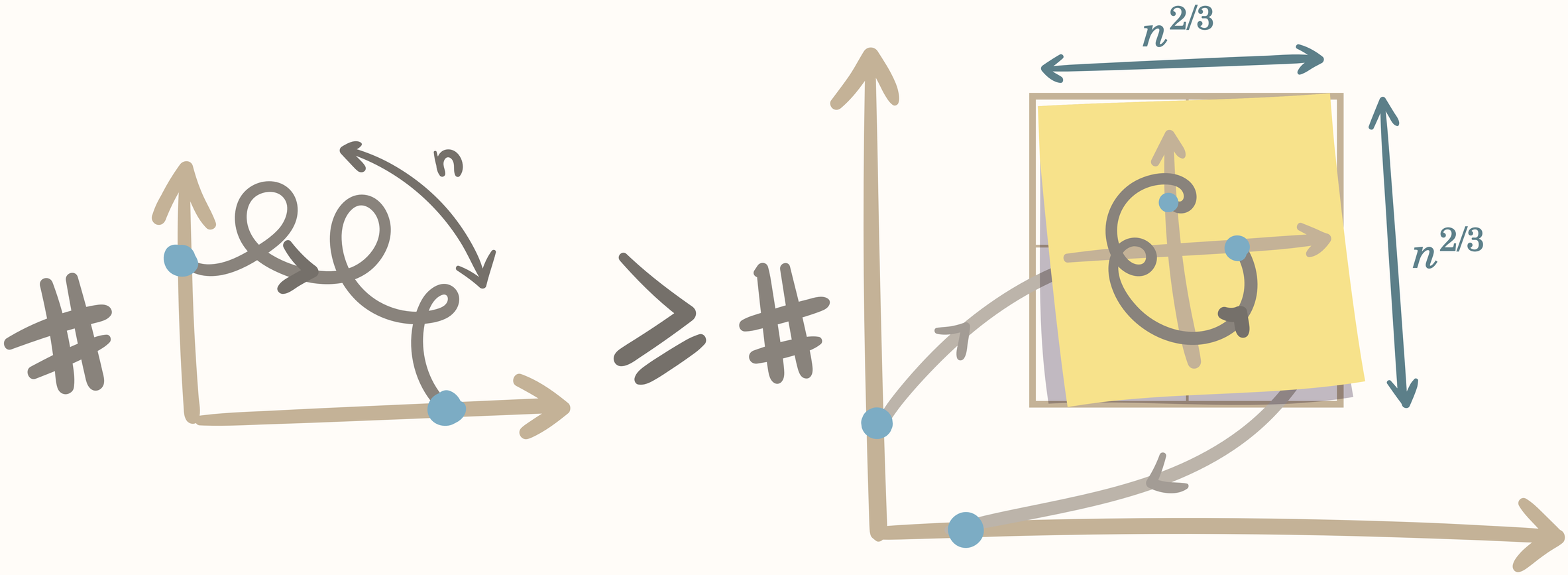




$$\#\left\{\text{  }\right\}_n \leqslant \left(\frac{9}{2}\right)^n$$

$$\#\left\{\text{  }\right\}_n \leqslant 2\cdot\left(\frac{16}{3}\right)^n$$

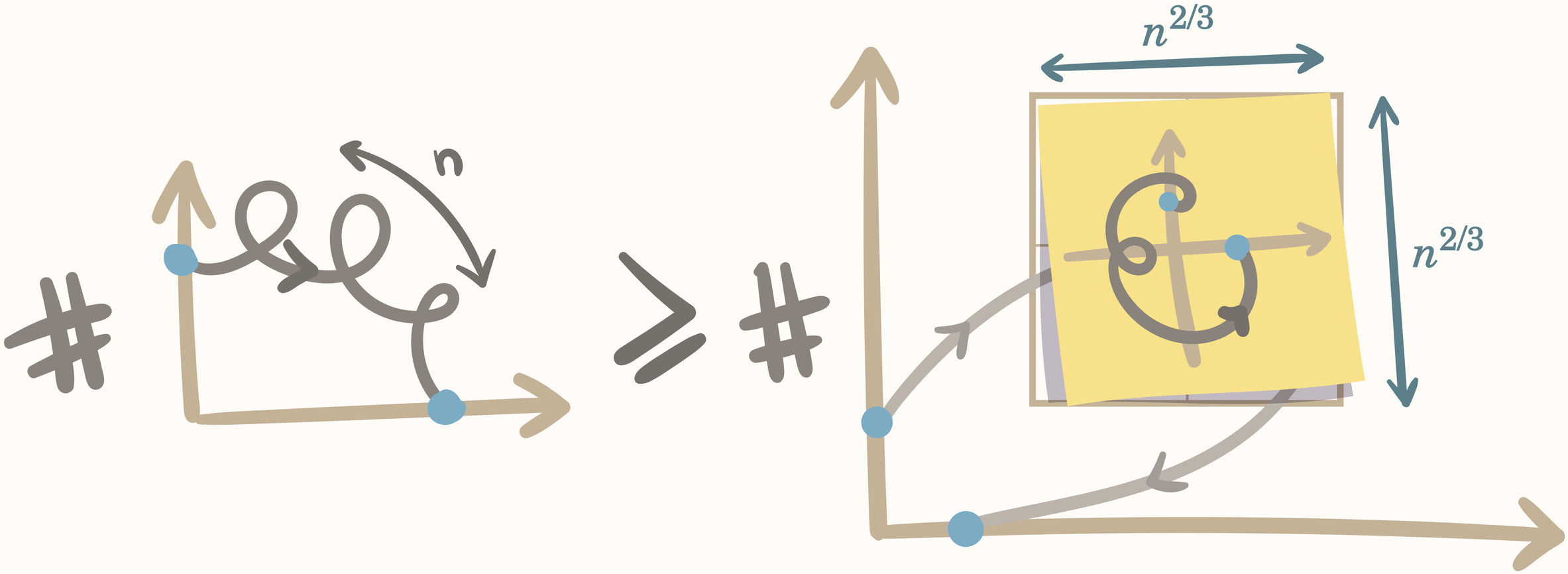
Asymptotic counting results



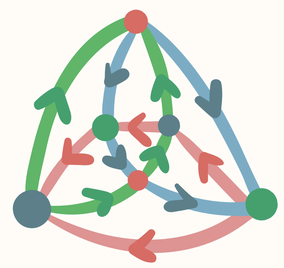
$$\# \left| _n \leqslant \left(\frac{9}{2}\right)^n \right.$$

$$\# \left| _n \leqslant 2 \cdot \left(\frac{16}{3}\right)^n \right.$$

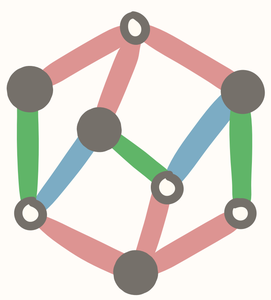
Asymptotic counting results



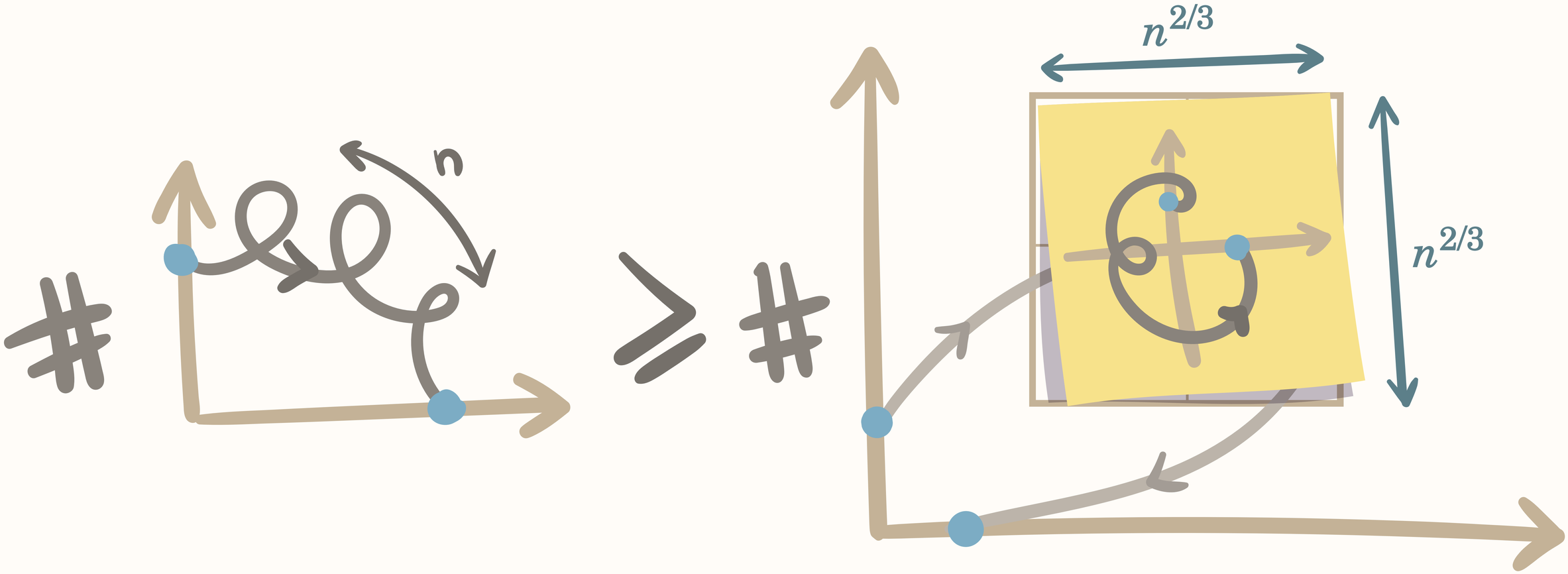
$$\left(\frac{9}{2}\right)^{n+o(n)} \leqslant \#\ \Big|_n \leqslant \left(\frac{9}{2}\right)^{n}$$



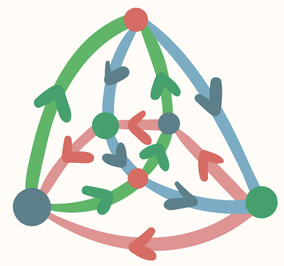
$$\left(\frac{16}{3}\right)^{n+o(n)} \leqslant \#\ \Big|_n \leqslant 2\cdot\left(\frac{16}{3}\right)^{n}$$



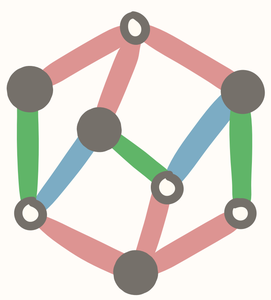
Asymptotic counting results



$$\left(\frac{9}{2}\right)^{n+o(n)} \leqslant \# \left|_n \leqslant \left(\frac{9}{2}\right)^{n}\right.$$



$$\left(\frac{16}{3}\right)^{n+o(n)} \leqslant \# \left|_n \leqslant 2\cdot\left(\frac{16}{3}\right)^{n}\right.$$

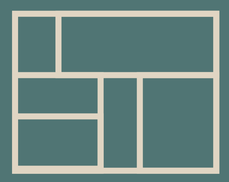
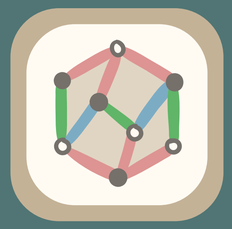


$$\lim p_n^{1/n} = 9/2$$

$$\lim s_n^{1/n} = 16/3$$

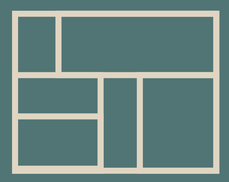
Summary

$a_n \sim \kappa \cdot \gamma^n \, n^\alpha$ | $\alpha = 1 + \pi / \arccos(\xi)$

Model		Asymtotics	
Transversal structures *n blue edges*		$\gamma = 27/2$	non D-Finite
		$\alpha \approx 7.21$	$\xi = 7/8$
Polyhedral orientations *n inner faces*			
Schnyder colorings *n inner faces* 			
Posets *n vertices*		$\gamma = (11 + 5\sqrt{5})/2$	D-Finite
		$\alpha = 6$	$\xi = (1 + \sqrt{5})/4$
Posets *n+2 edges*		$\gamma \approx 4.80$	non D-Finite
		$\alpha \approx 5.14$	$\xi \approx 0.73$

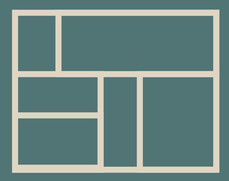
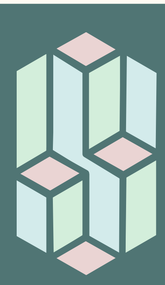
Summary

$\lim a_n^{1/n} = \gamma$ | $a_n \sim \varkappa \cdot \gamma^n \, n^\alpha$ | $\alpha = 1 + \pi / \arccos(\xi)$

Model		Asymtotics	
Transversal structures *n blue edges*		$\gamma = 27/2$	non D-Finite
		$\alpha \approx 7.21$	$\xi = 7/8$
Polyhedral orientations *n inner faces*		$\gamma = 9/2$	
Schnyder colorings *n inner faces*		$\gamma = 16/3$	
Posets *n vertices*		$\gamma = (11 + 5\sqrt{5})/2$	D-Finite
		$\alpha = 6$	$\xi = (1 + \sqrt{5})/4$
Posets *n+2 edges*		$\gamma \approx 4.80$	non D-Finite
		$\alpha \approx 5.14$	$\xi \approx 0.73$

Summary

$\lim a_n^{1/n} = \gamma$	$a_n \sim \varkappa \cdot \gamma^n \, n^\alpha$	$\alpha = 1 + \pi / \arccos(\xi)$

Model		*Asymtotics*	
Transversal structures *n blue edges*		$\gamma = 27/2$	non D-Finite
		$\alpha \approx 7.21$	$\xi = 7/8$
Polyhedral orientations *n inner faces*		$\gamma = 9/2$	(non D-Finite)
		$(\alpha \approx 4.23)$	$(\xi = 9/16)$
Schnyder colorings *n inner faces*		$\gamma = 16/3$	(non D-Finite)
		$(\alpha \approx 6.08)$	$(\xi = 22/27)$
Posets *n vertices*		$\gamma = (11 + 5\sqrt{5})/2$	D-Finite
		$\alpha = 6$	$\xi = (1 + \sqrt{5})/4$
Posets *n+2 edges*		$\gamma \approx 4.80$	non D-Finite
		$\alpha \approx 5.14$	$\xi \approx 0.73$

Merci

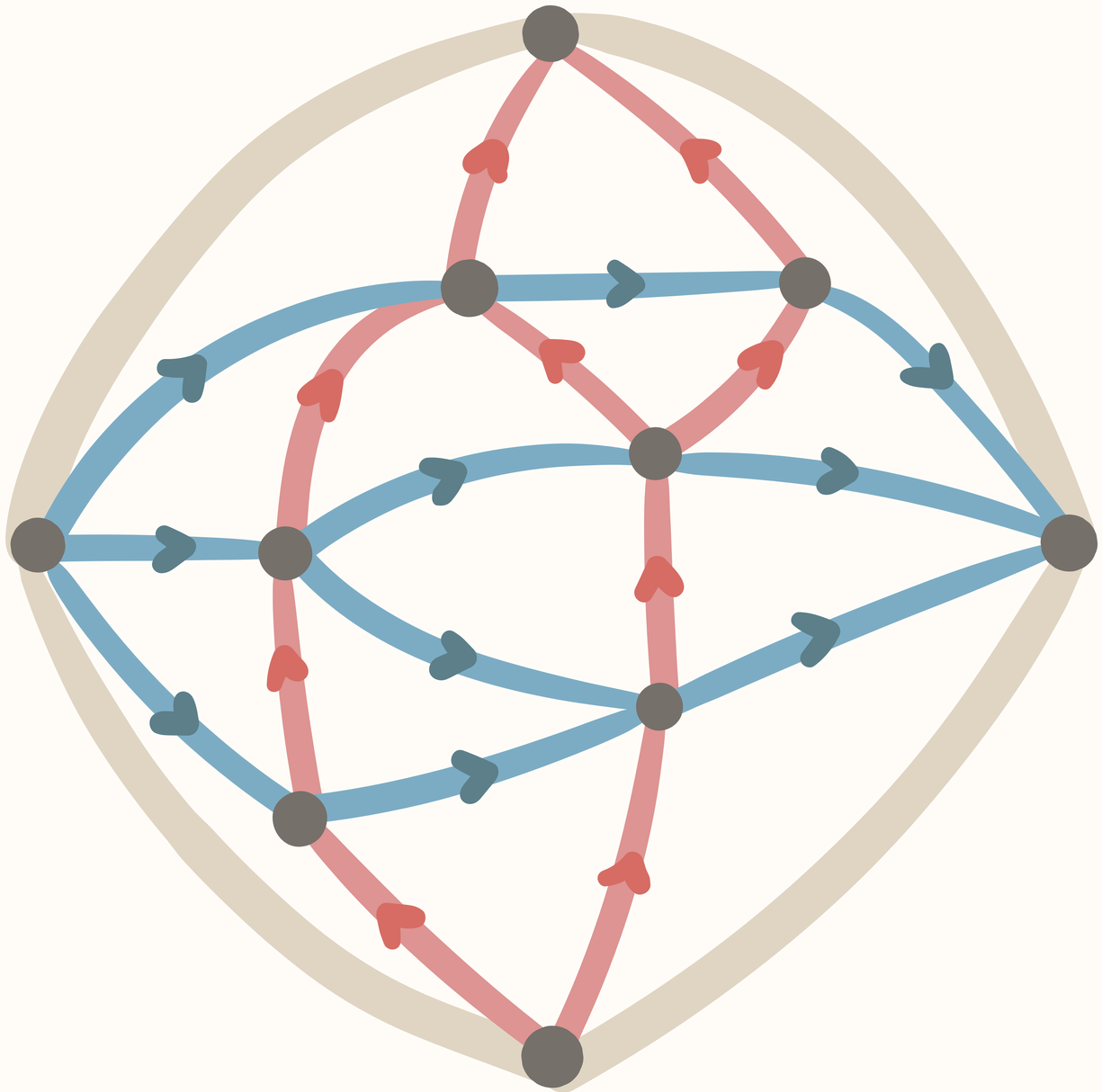
ANNEXES

A. Generalization for transversal structures

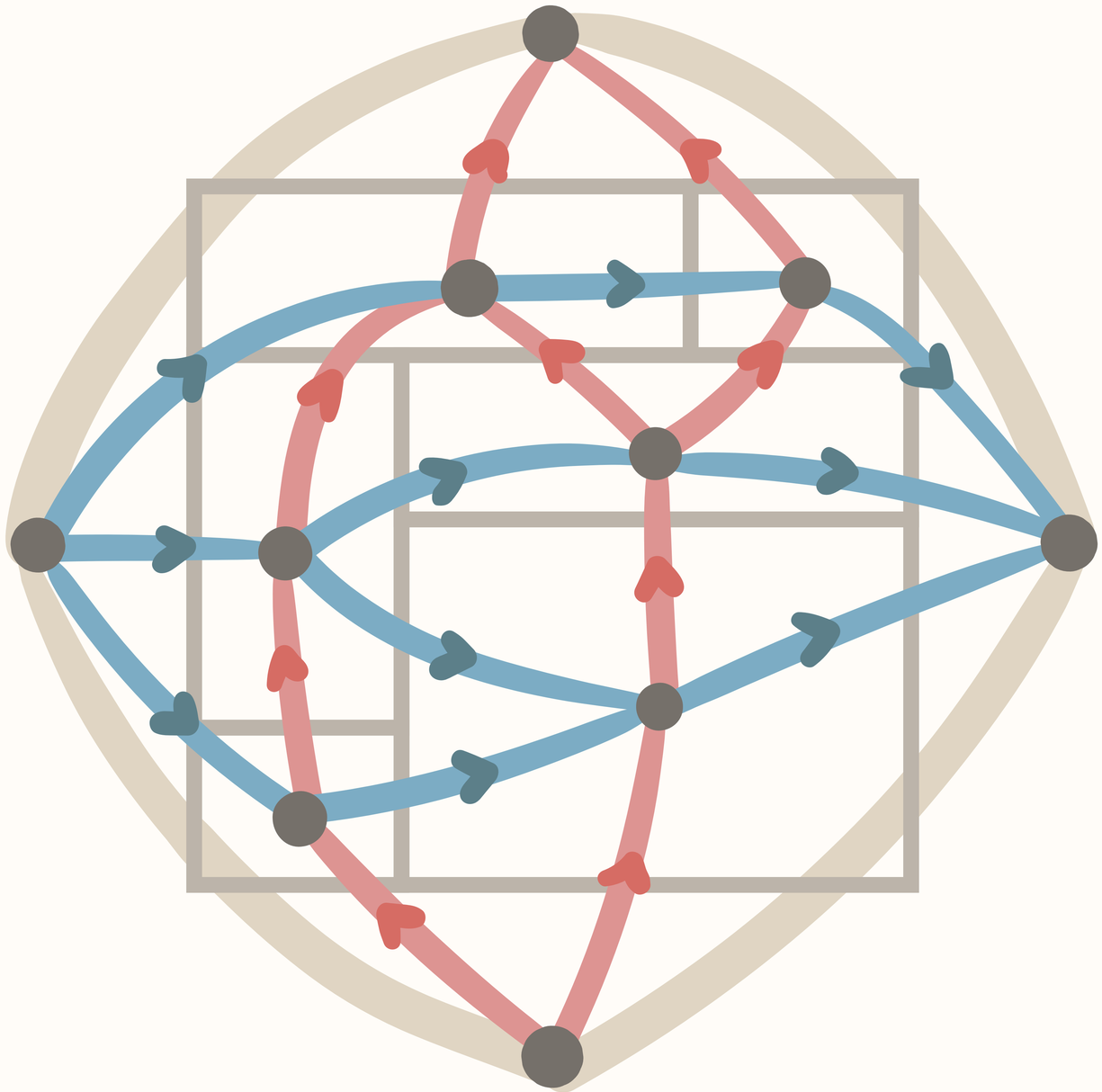
B. Schnyder labelings

C. Bijection for plane permutations

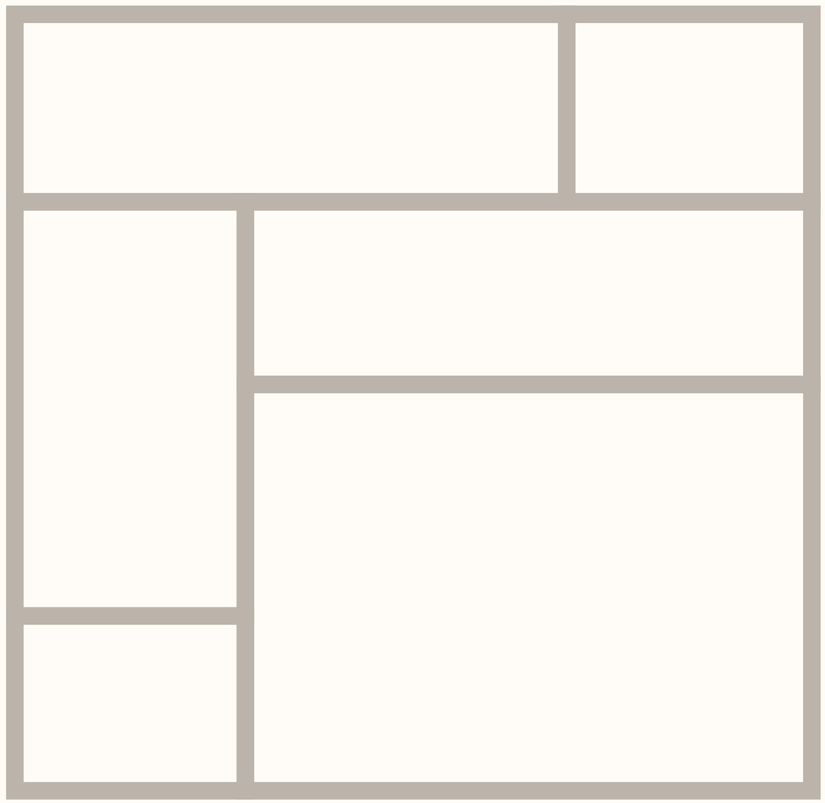
Generic transversal structures



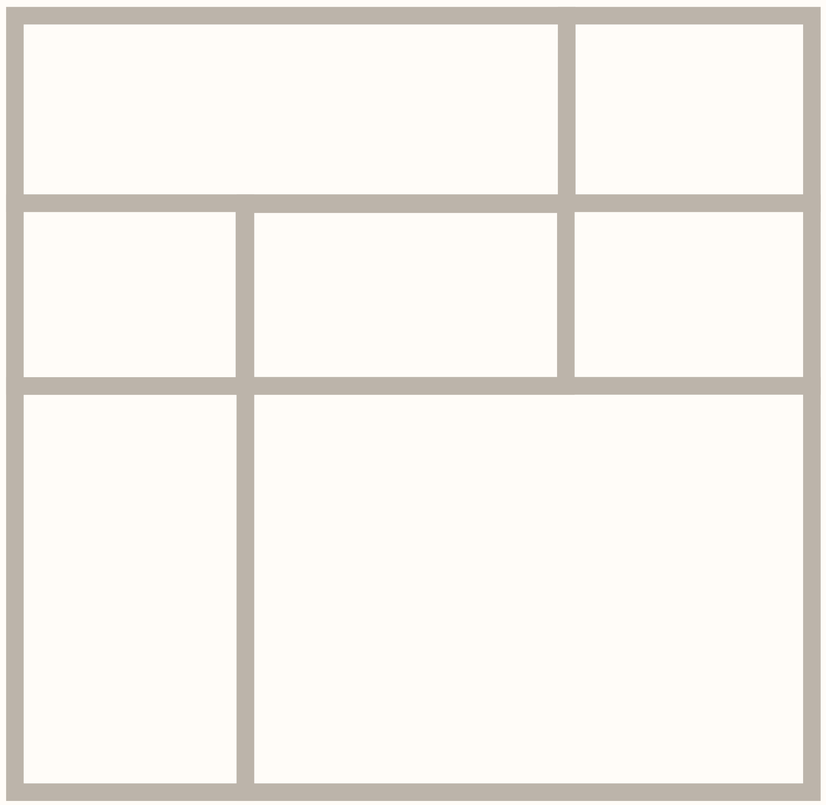
Generic transversal structures



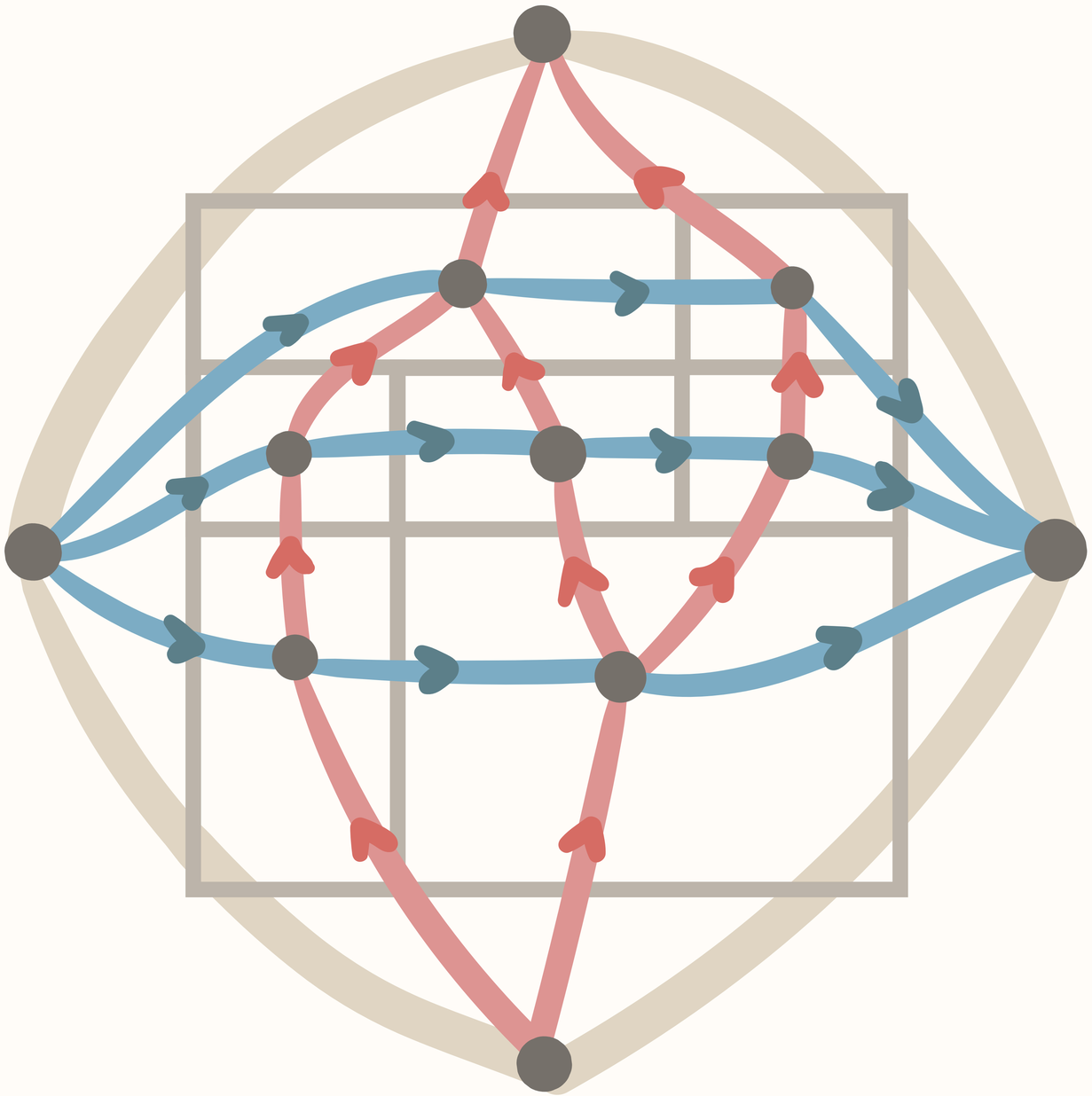
Generic transversal structures



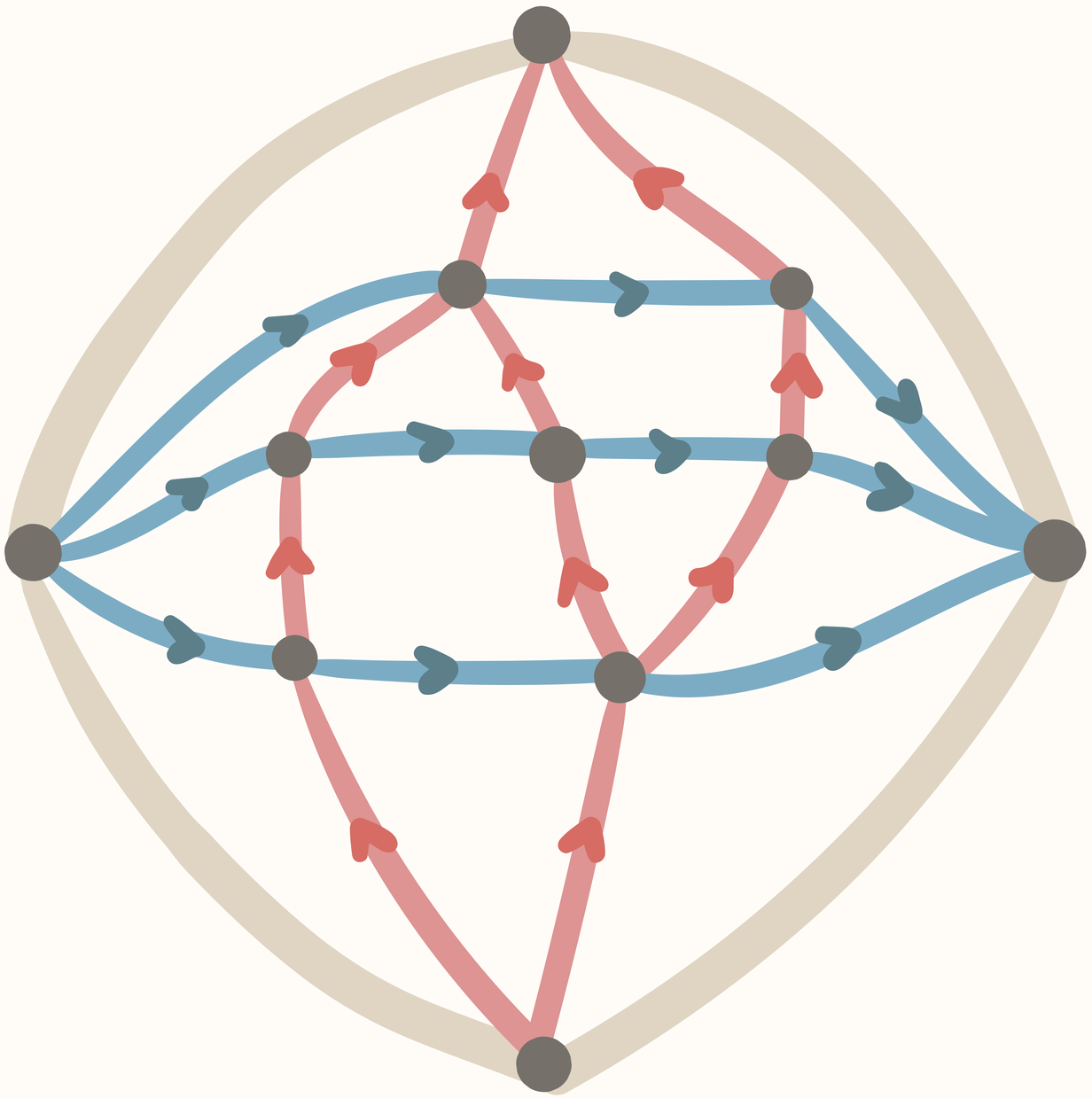
Generic transversal structures

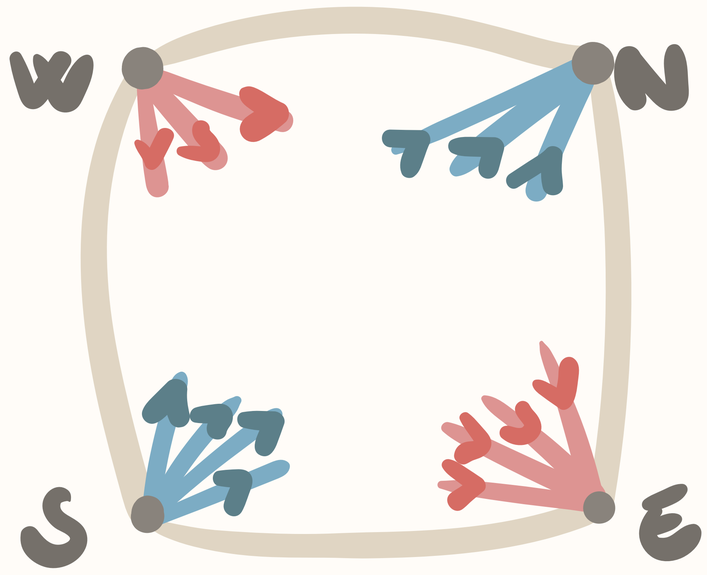


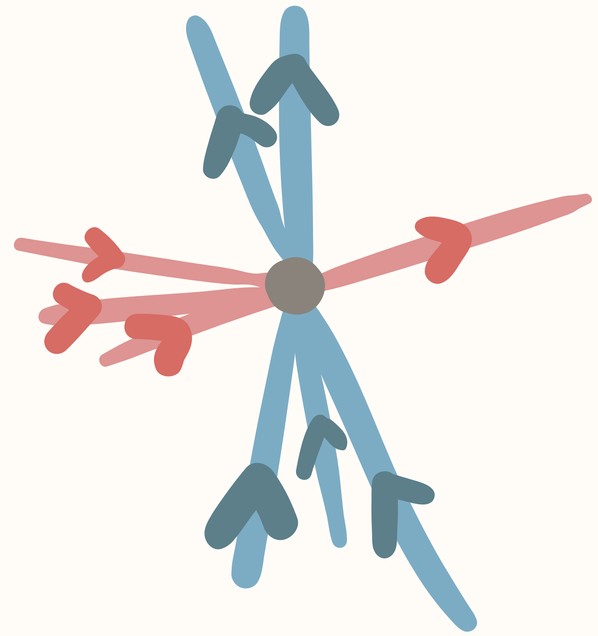
Generic transversal structures



Generic transversal structures

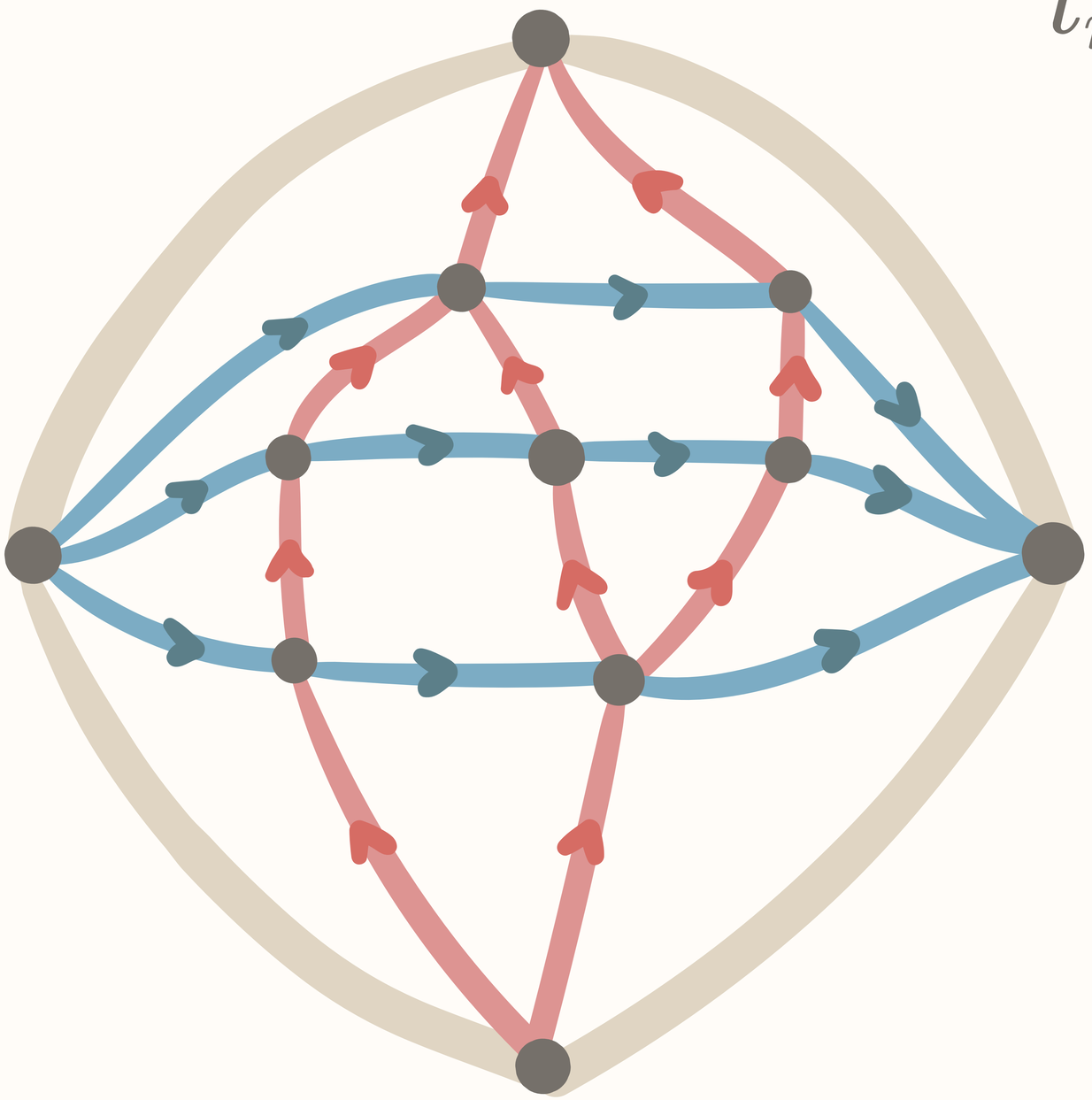




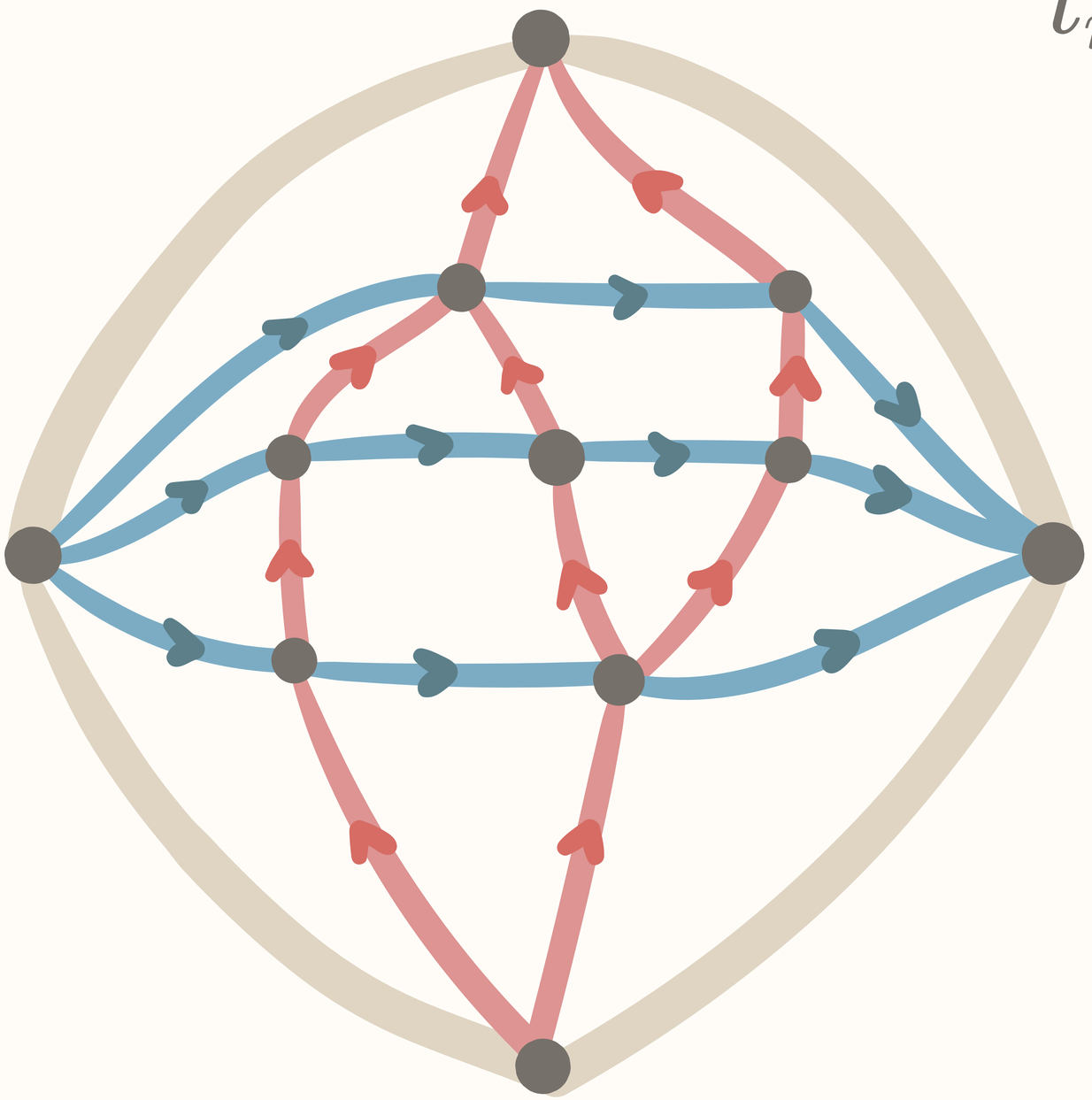


Generic transversal structures

$$t_n \sim \varkappa \ \gamma(v)^n \ n^{-1-\frac{\pi}{\arccos(\xi(v))}}$$



Generic transversal structures



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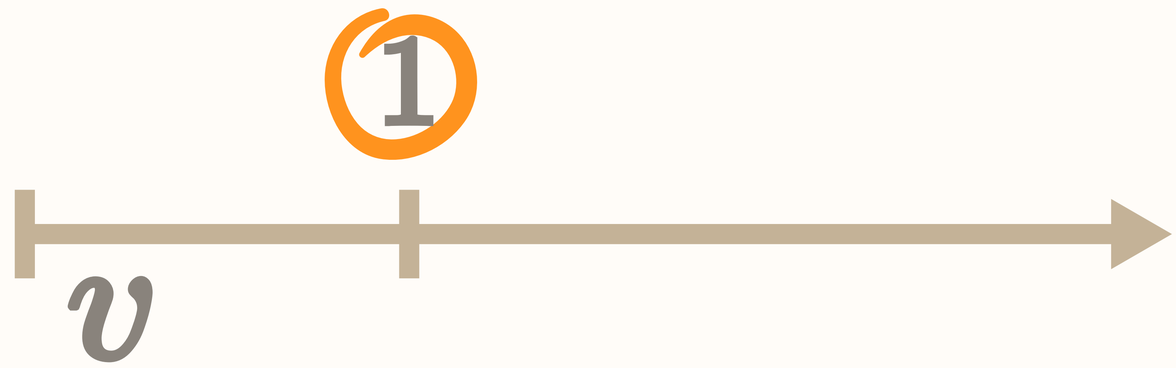
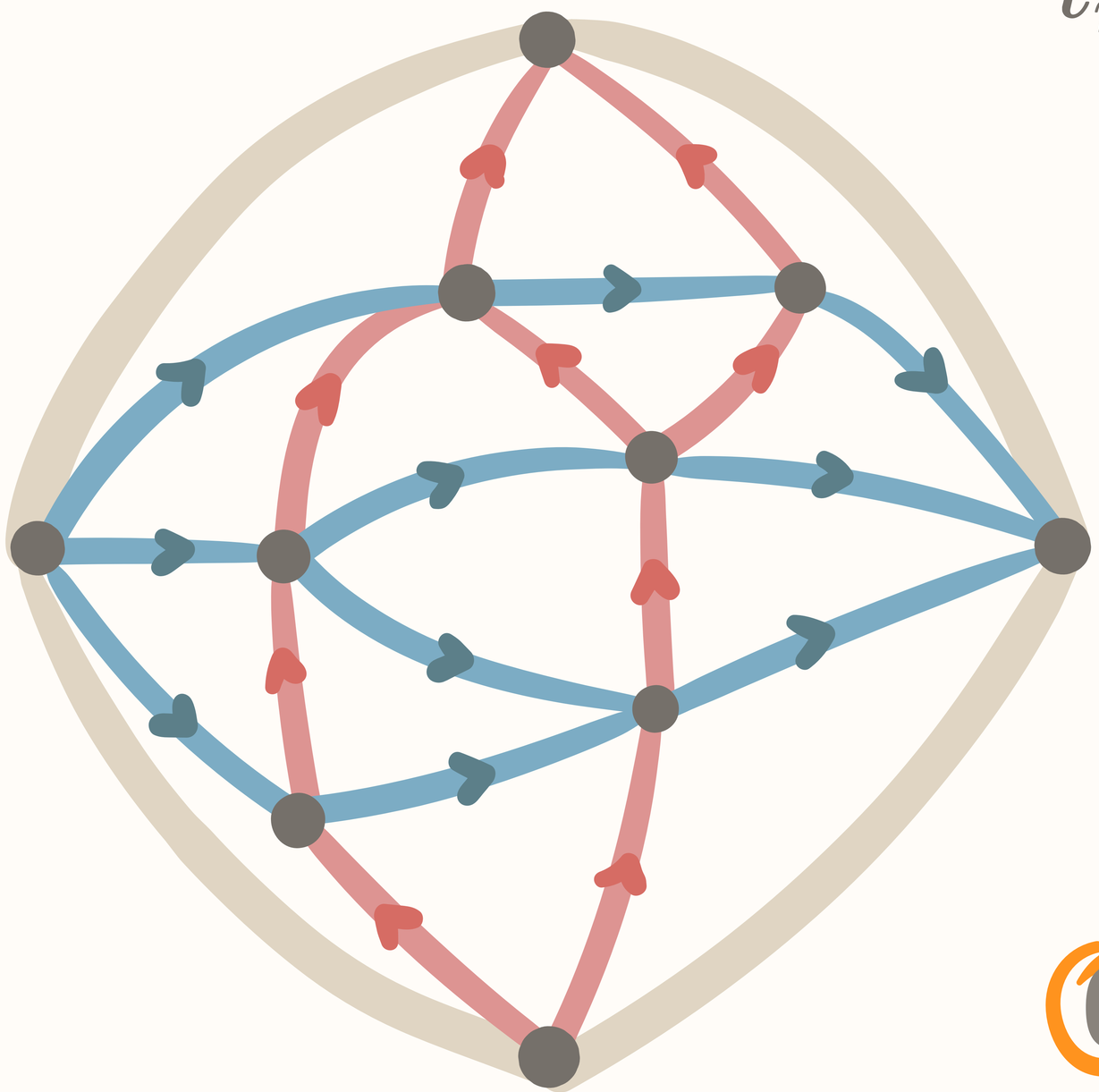


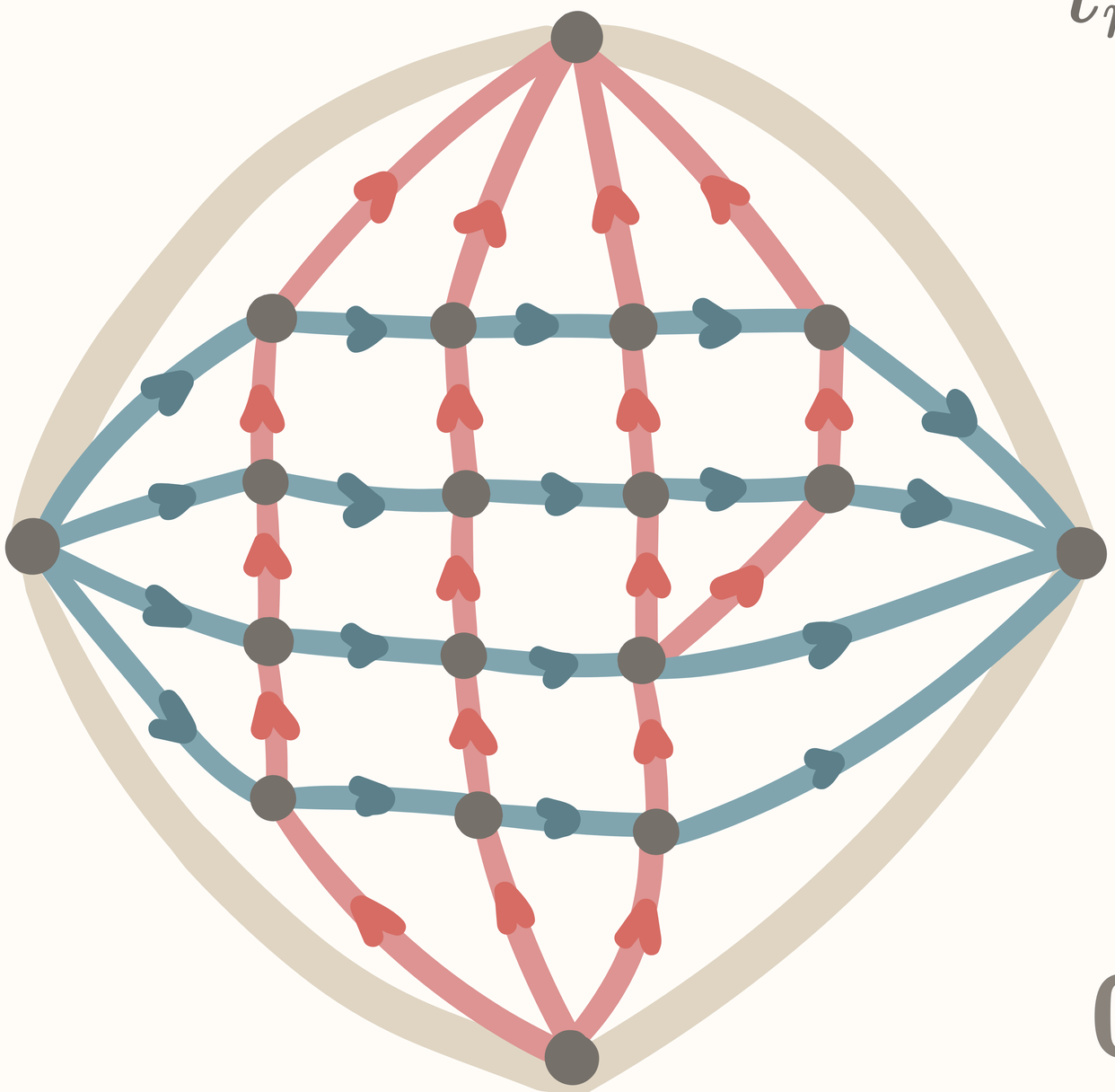
Generic transversal structures

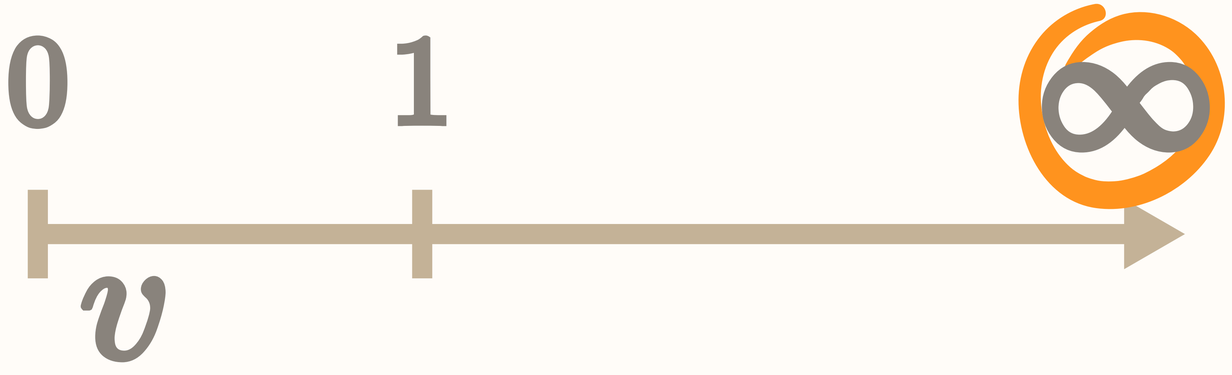
$$t_n \sim \varkappa \left(\frac{27}{2}\right)^n n^{-1-\frac{\pi}{\arccos(7/8)}}$$



Generic transversal structures

$$t_n \sim \varkappa \ \gamma(v)^n \ n^{-1-\frac{\pi}{\arccos(\xi(v))}}$$





Generic transversal structures

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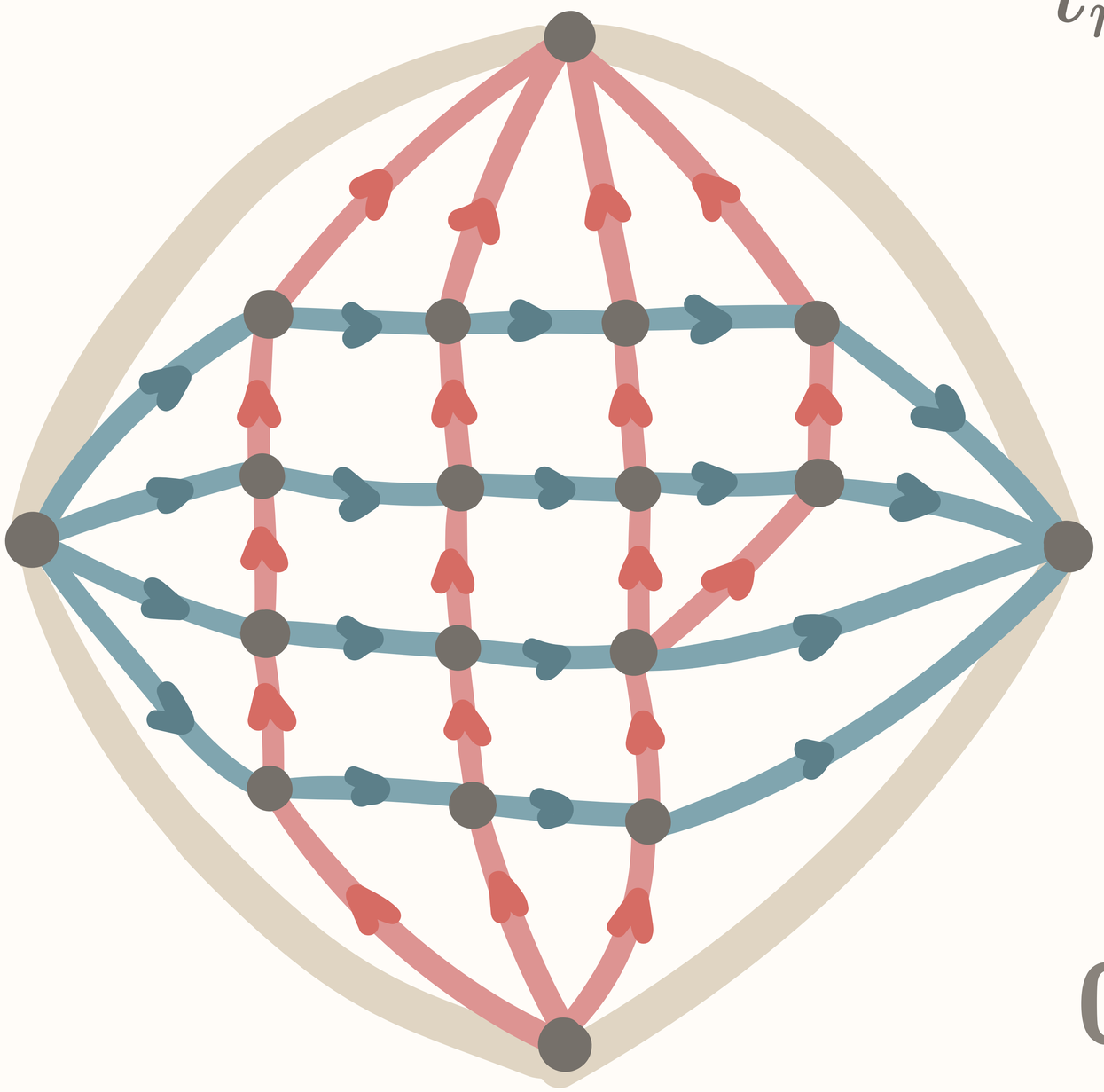

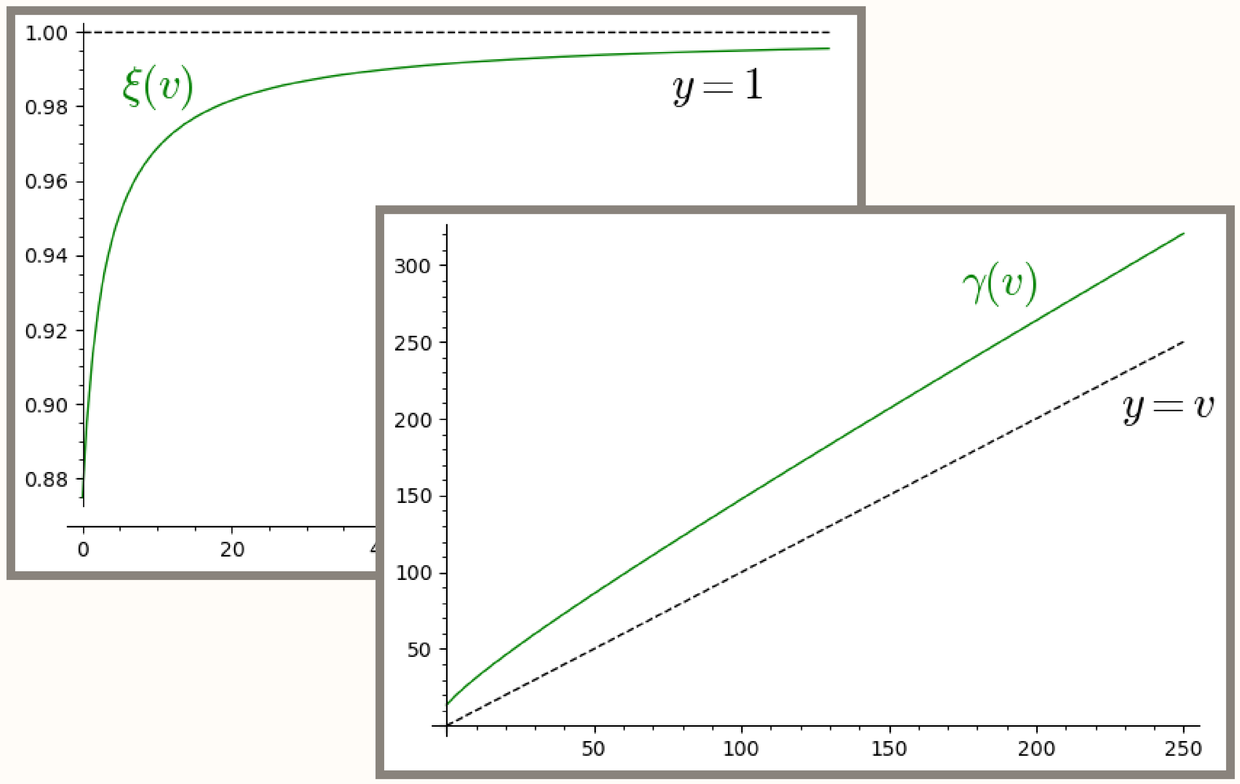

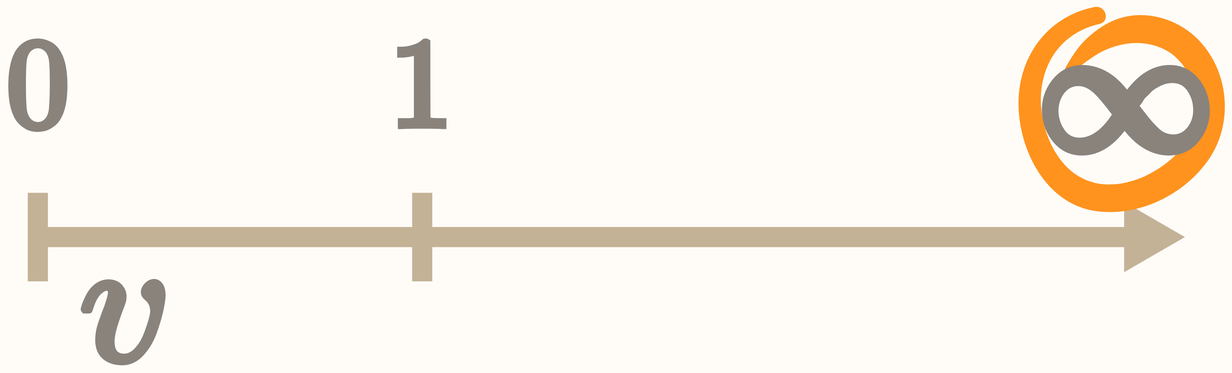

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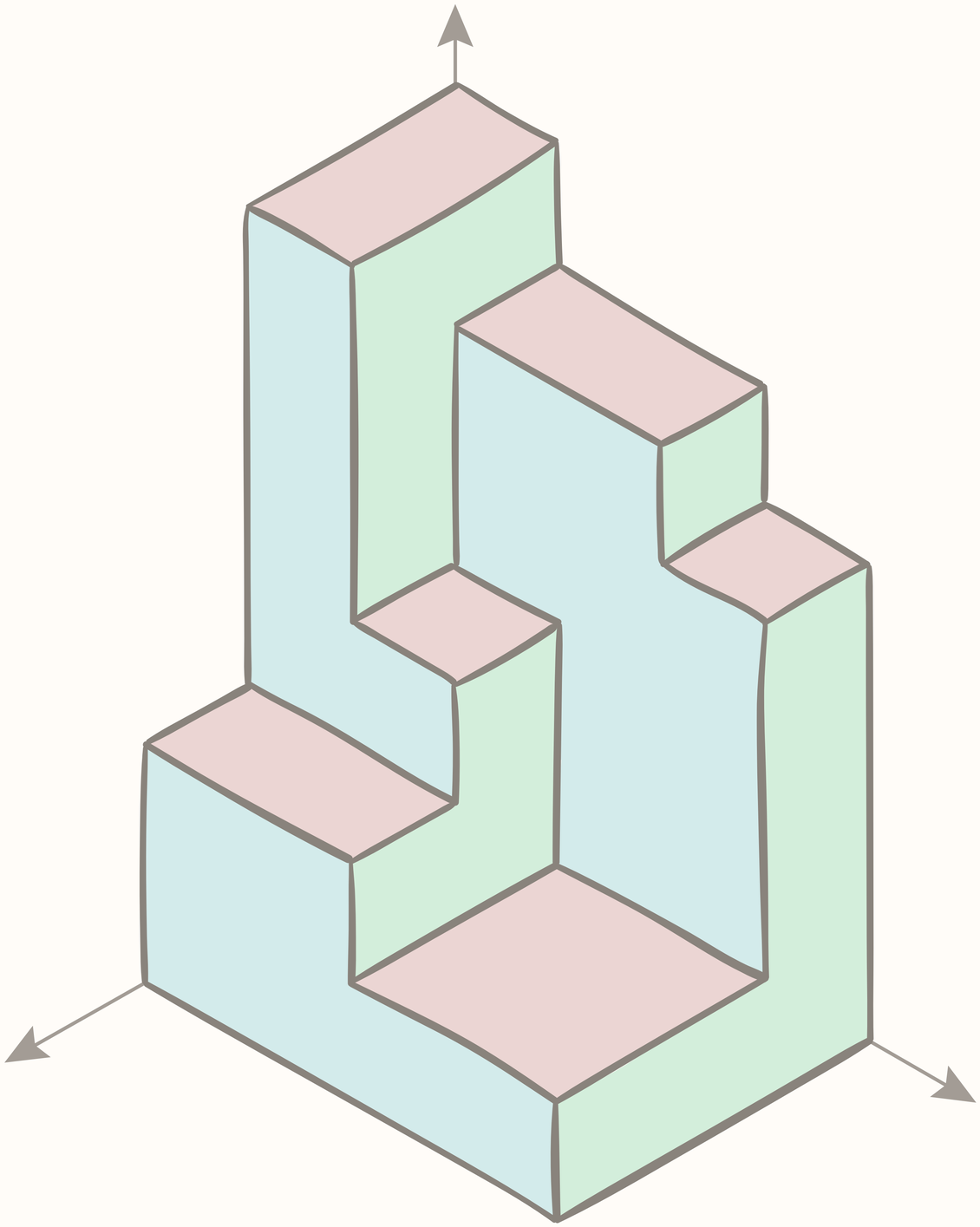
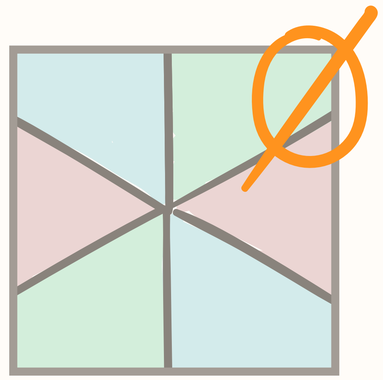
B. Schnyder labelings

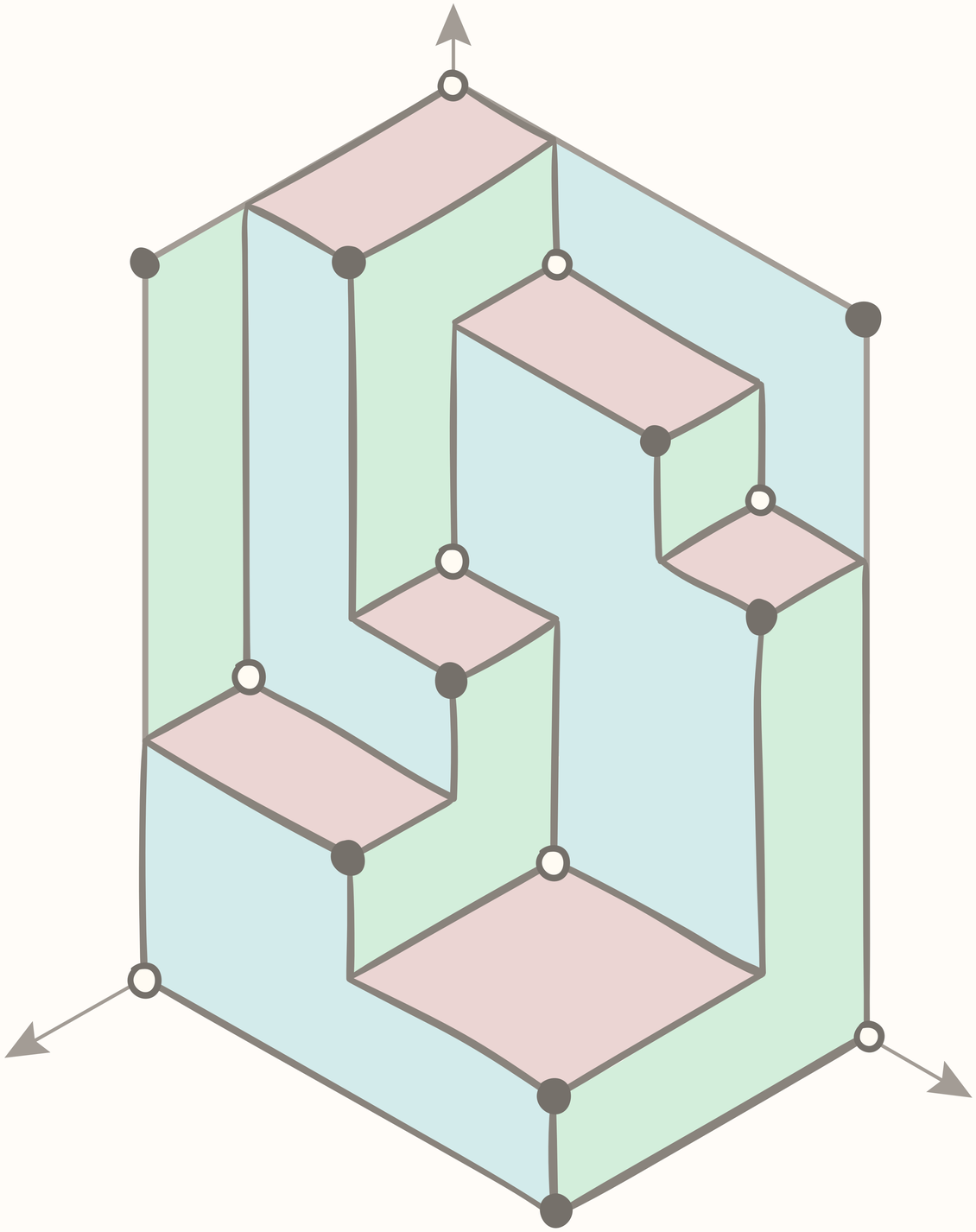
C. Bijection for plane permutations

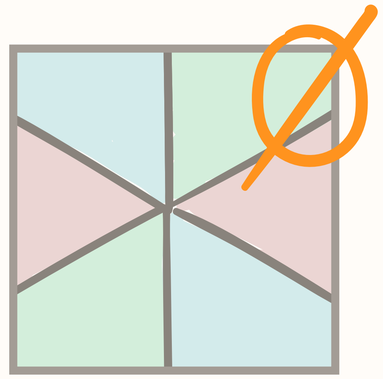
Rigid corner polyhedra



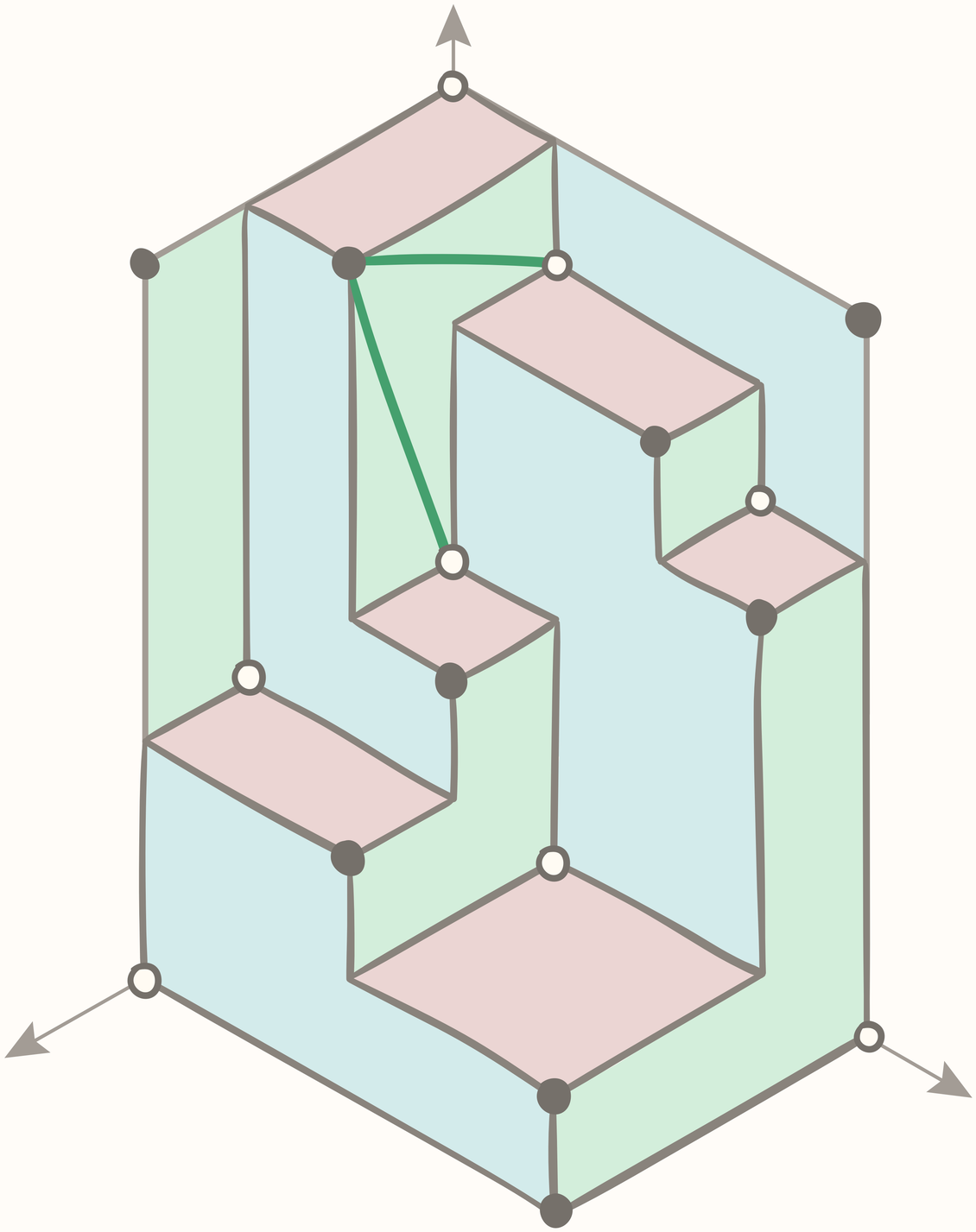


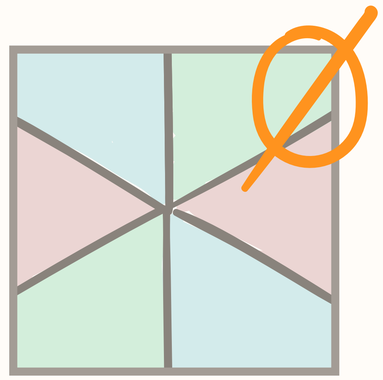
Rigid corner polyhedra





Rigid corner polyhedra





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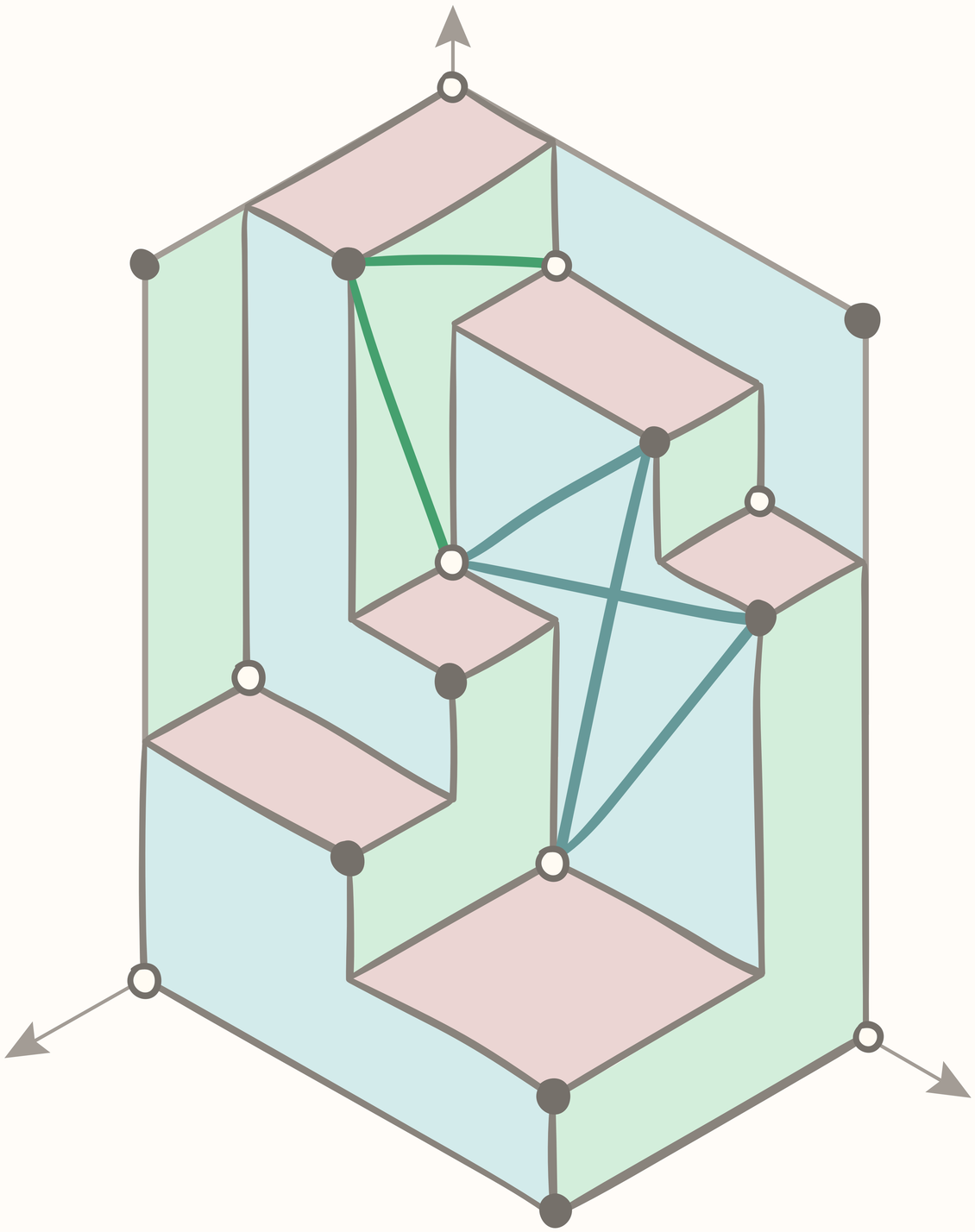

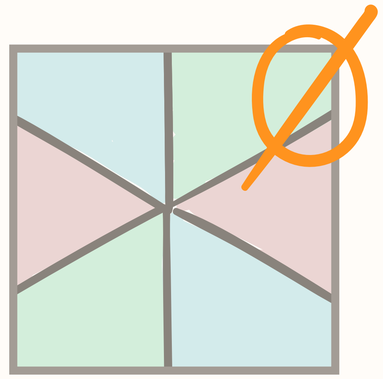

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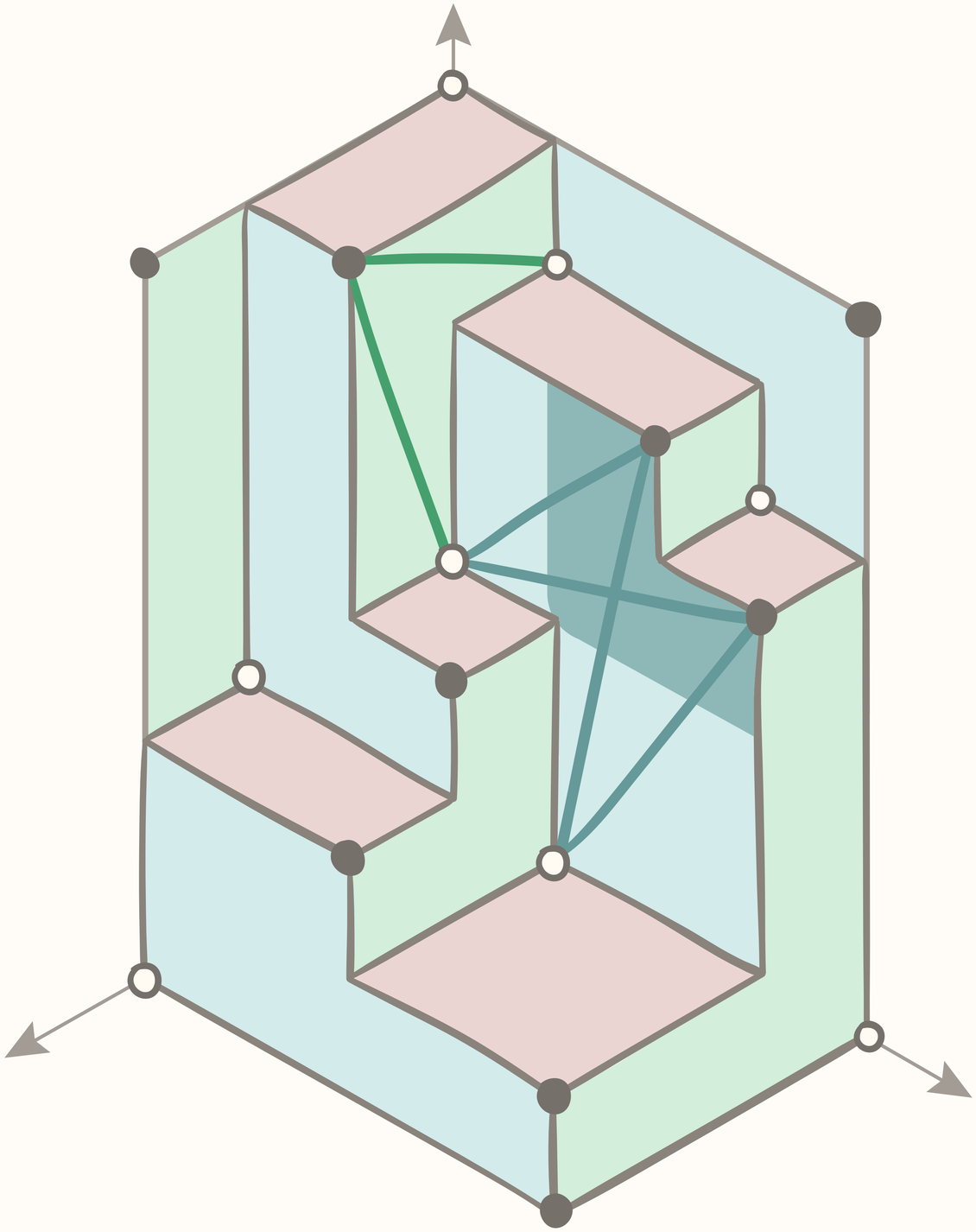

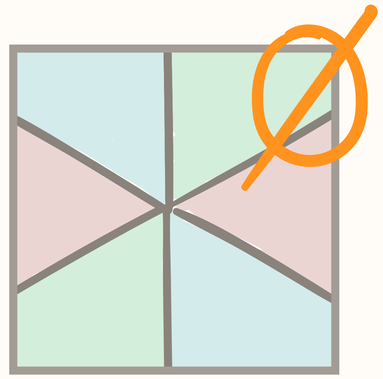

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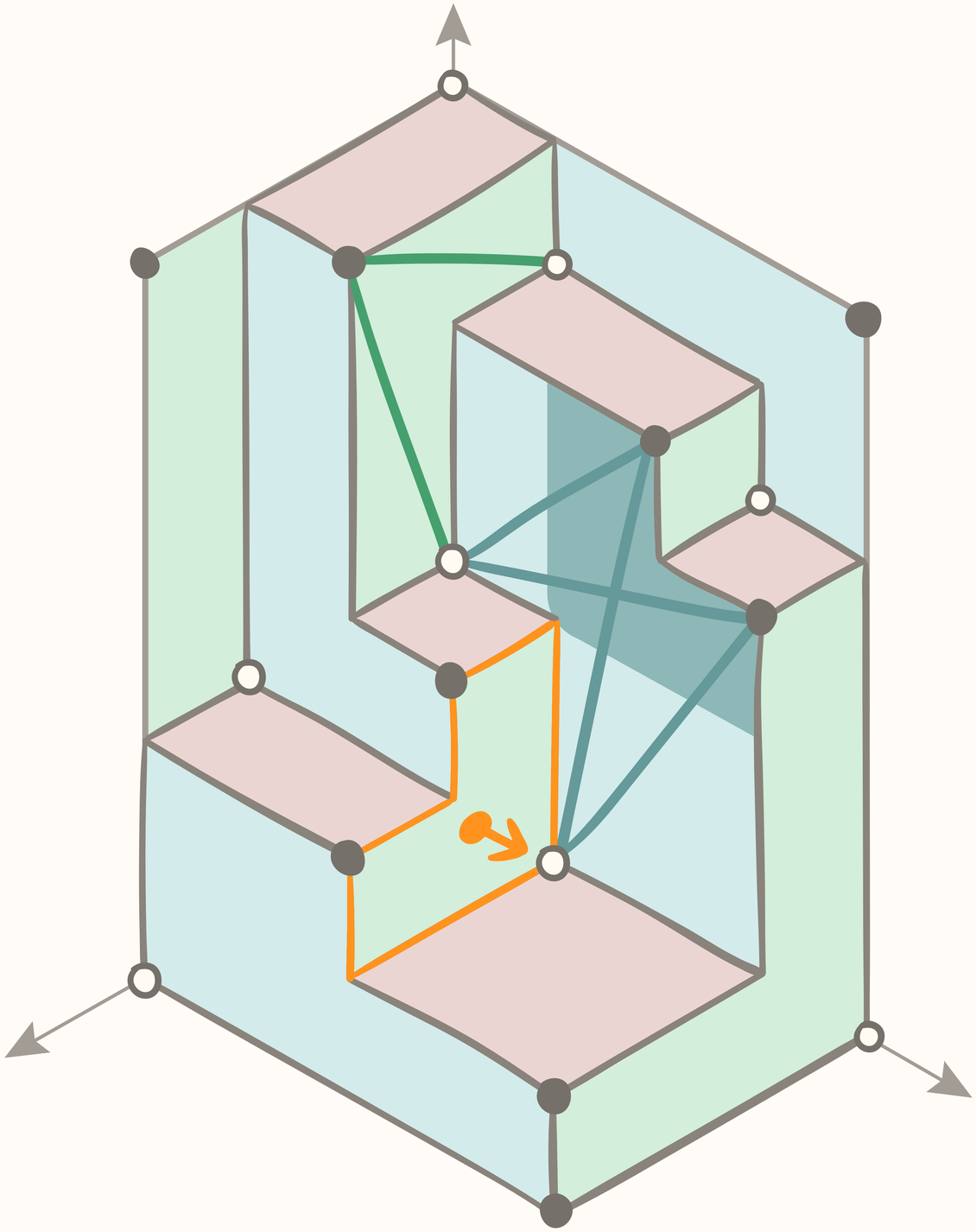

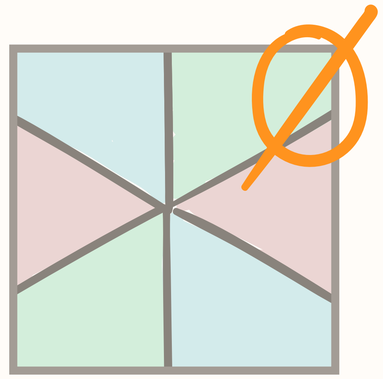

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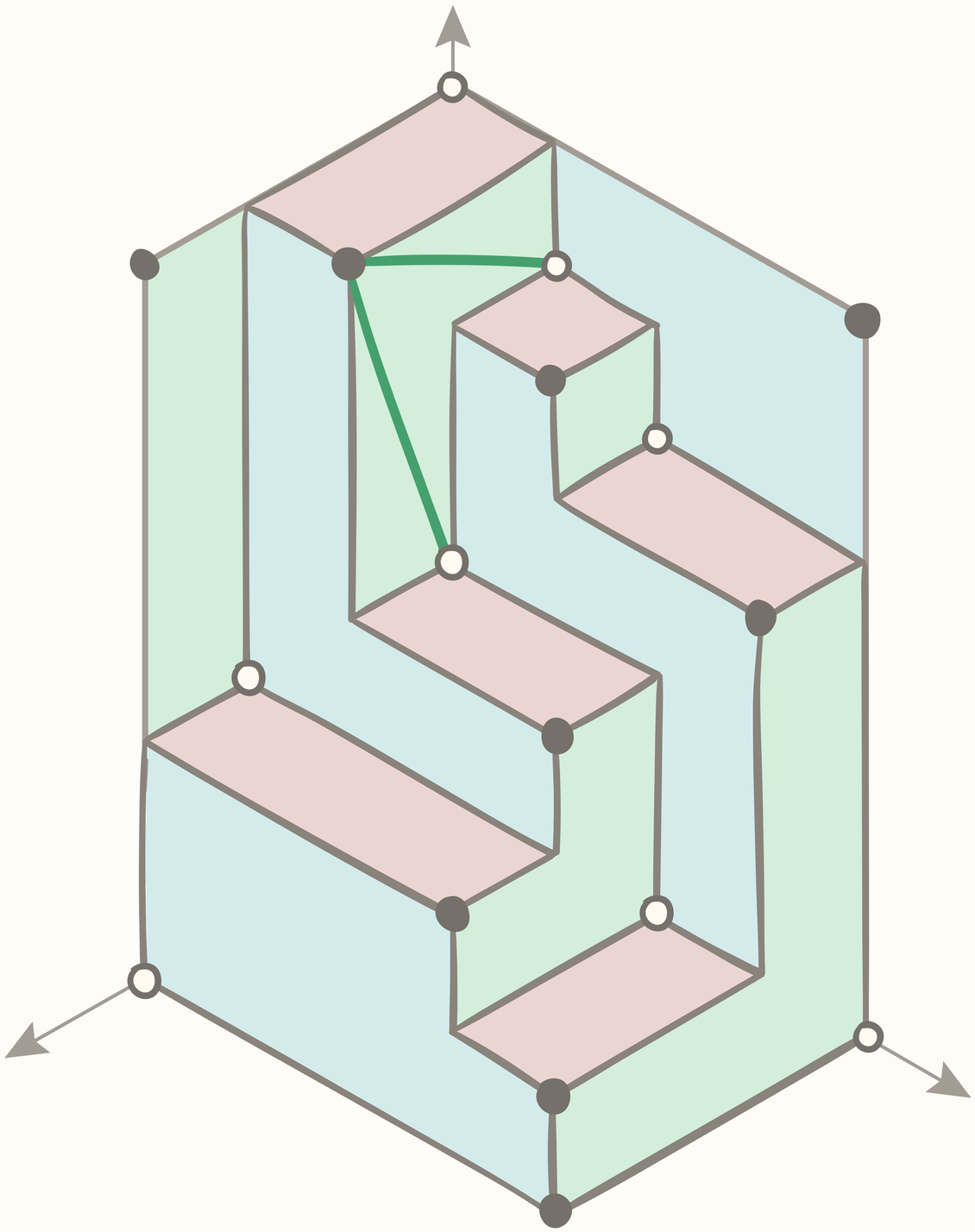

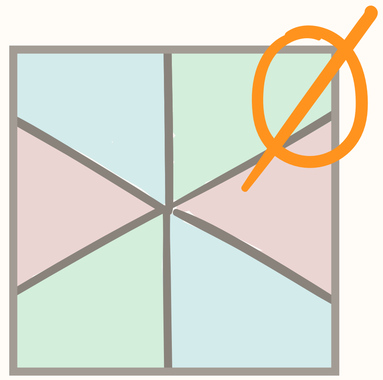

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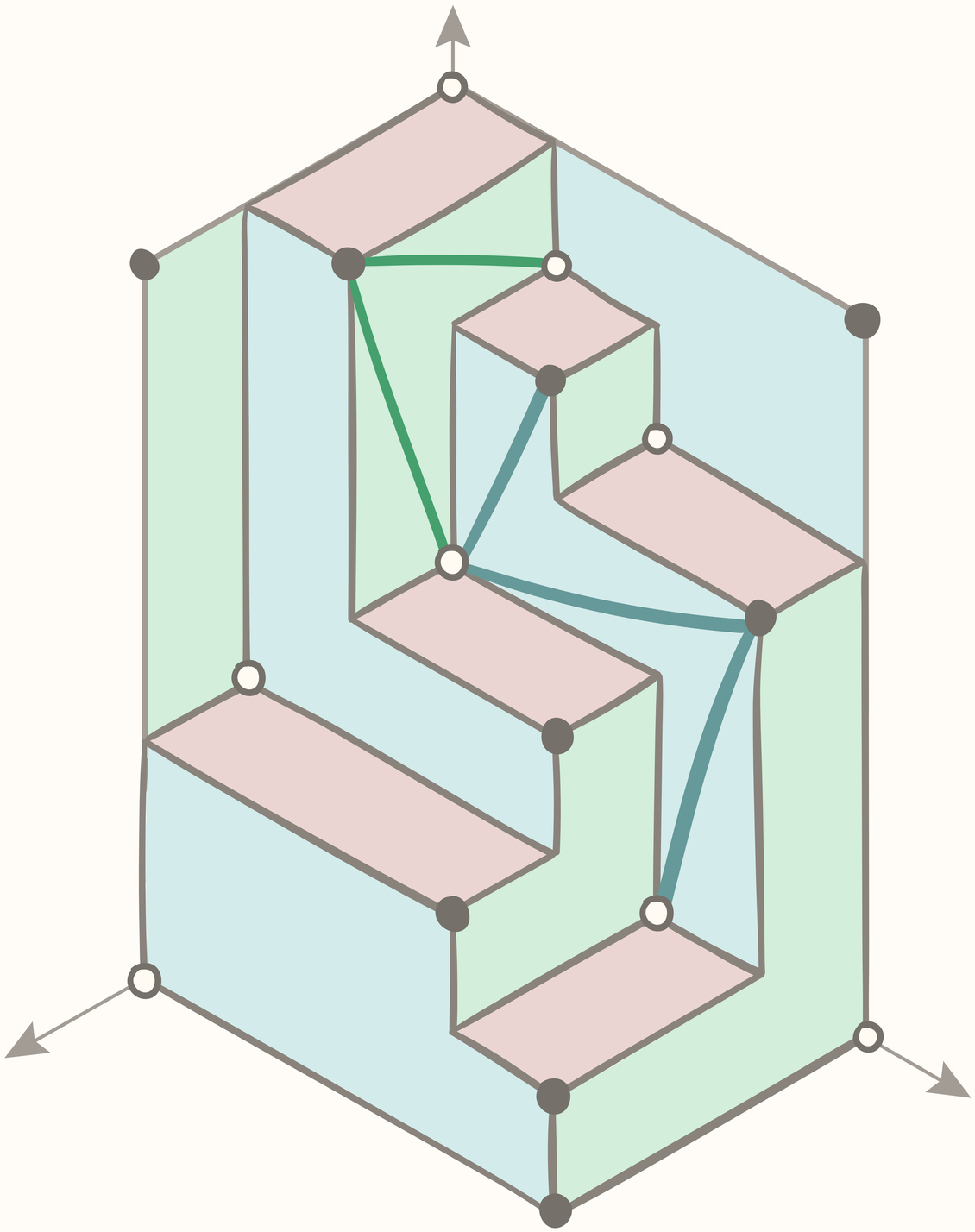

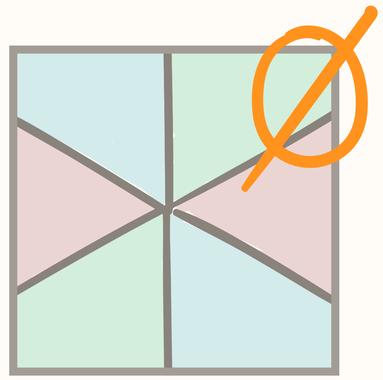

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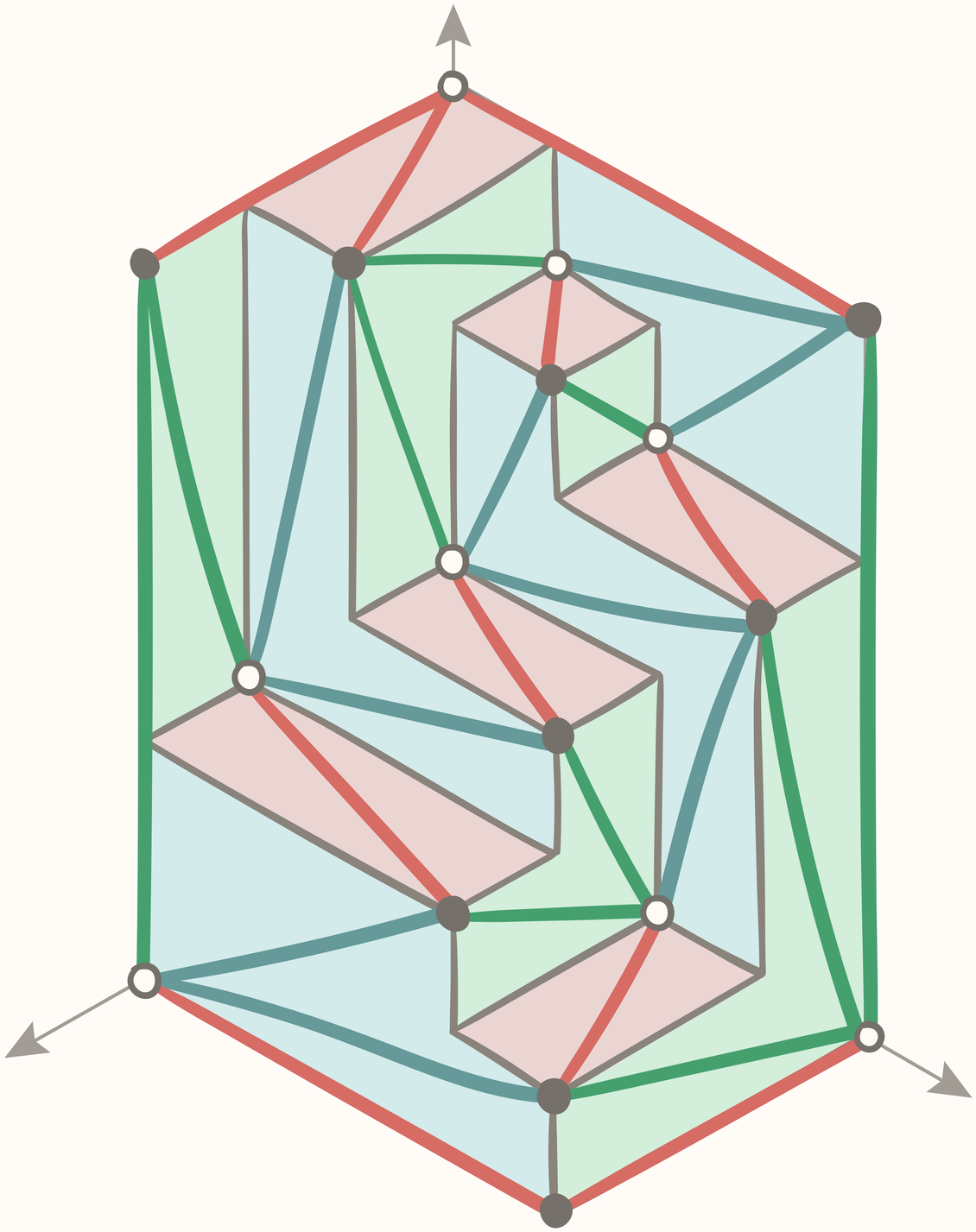
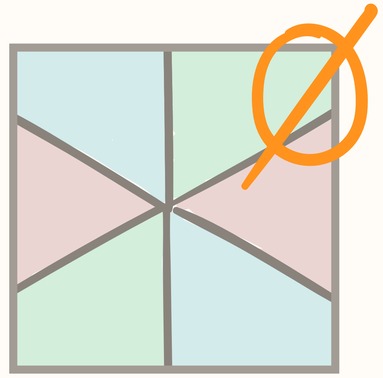
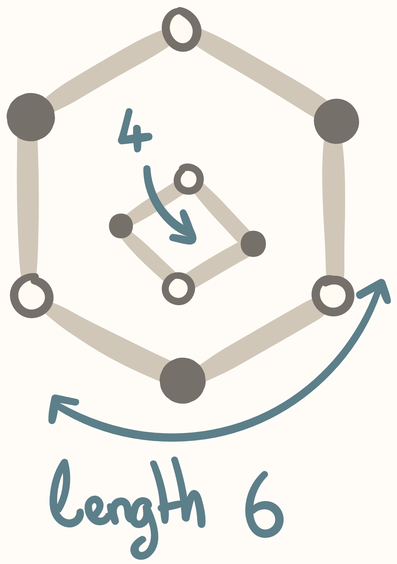


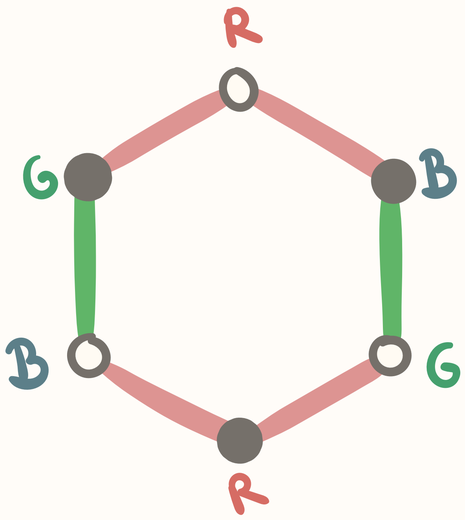


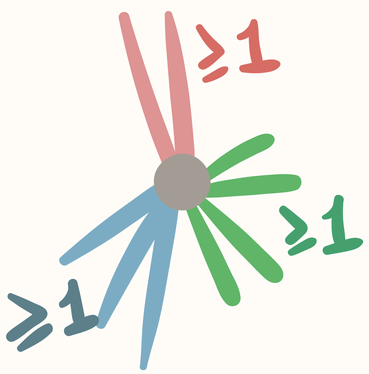
rigid corner polyhedra ↔ ***Schnyder labellings***

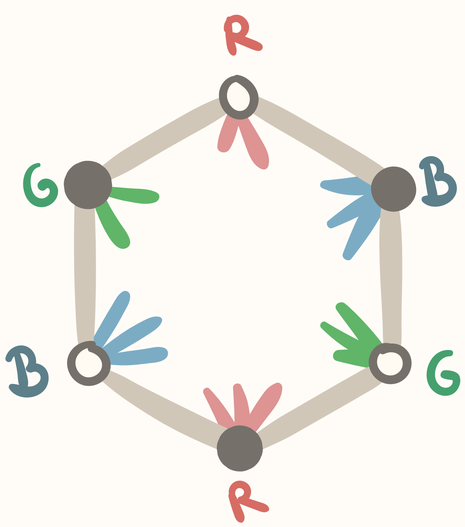
↠ *Geodesic embeddings and planar graphs,*
S. Felsner (2003)

Schnyder labellings









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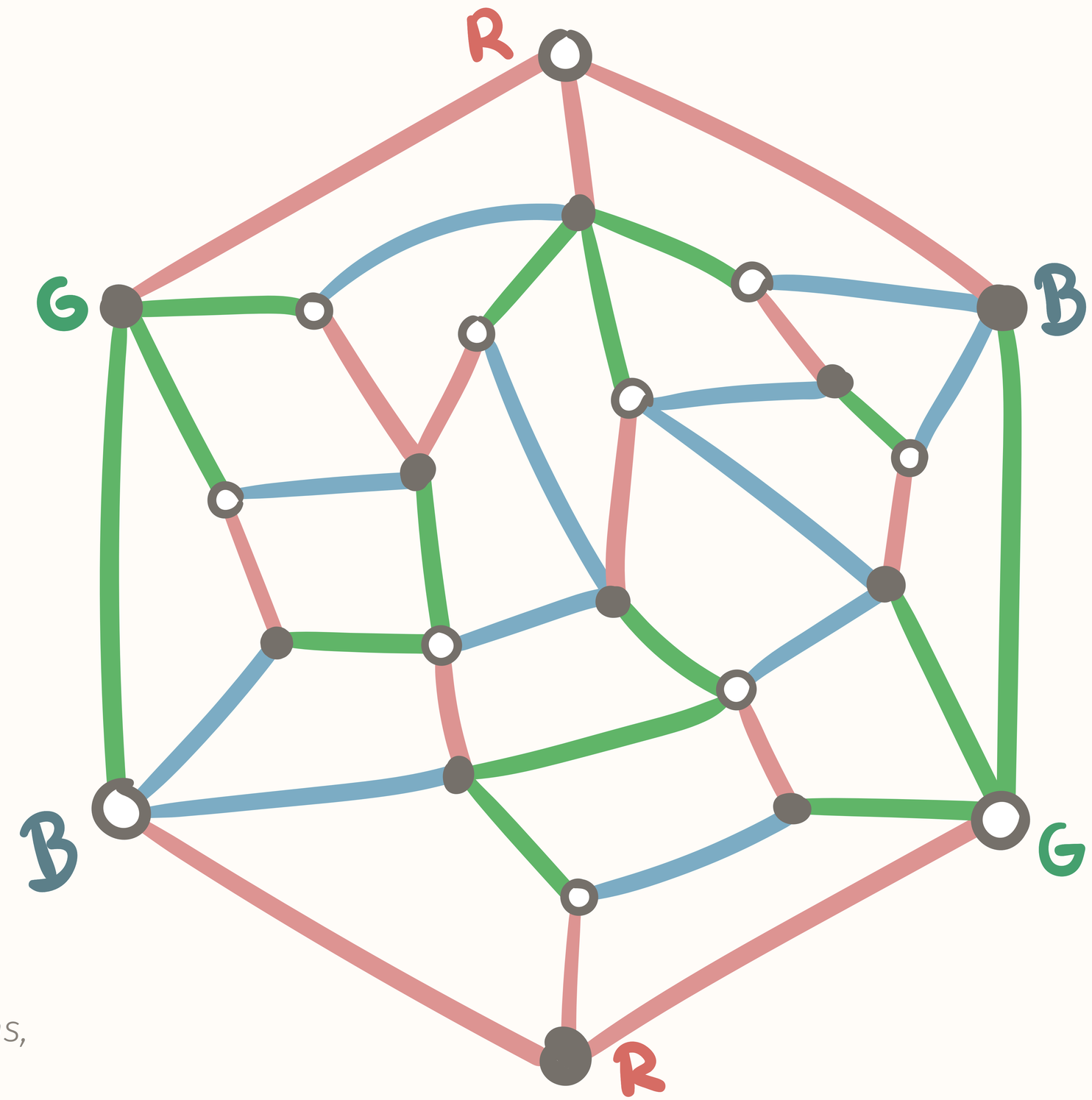
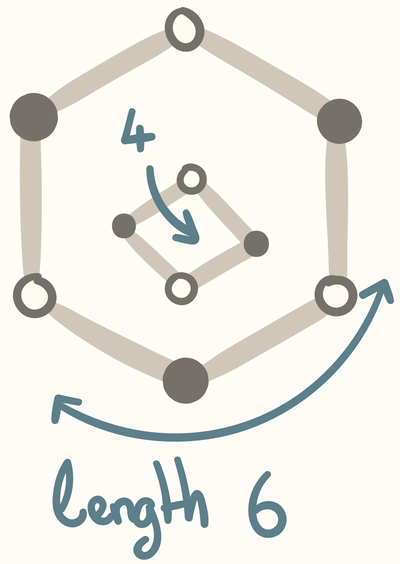
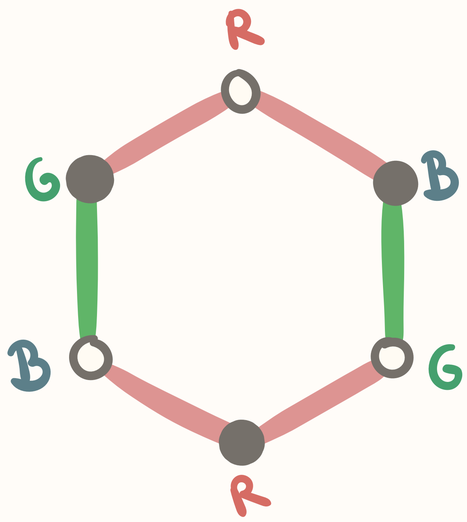
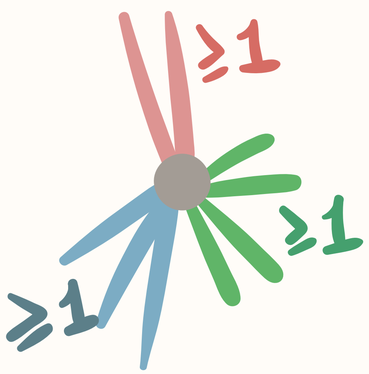
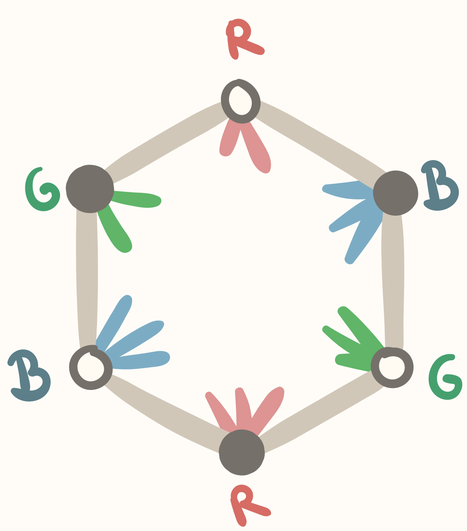


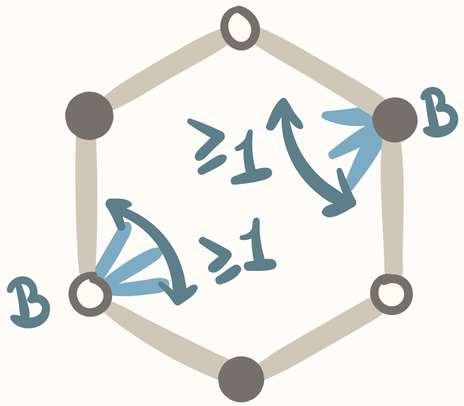
Schnyder labellings

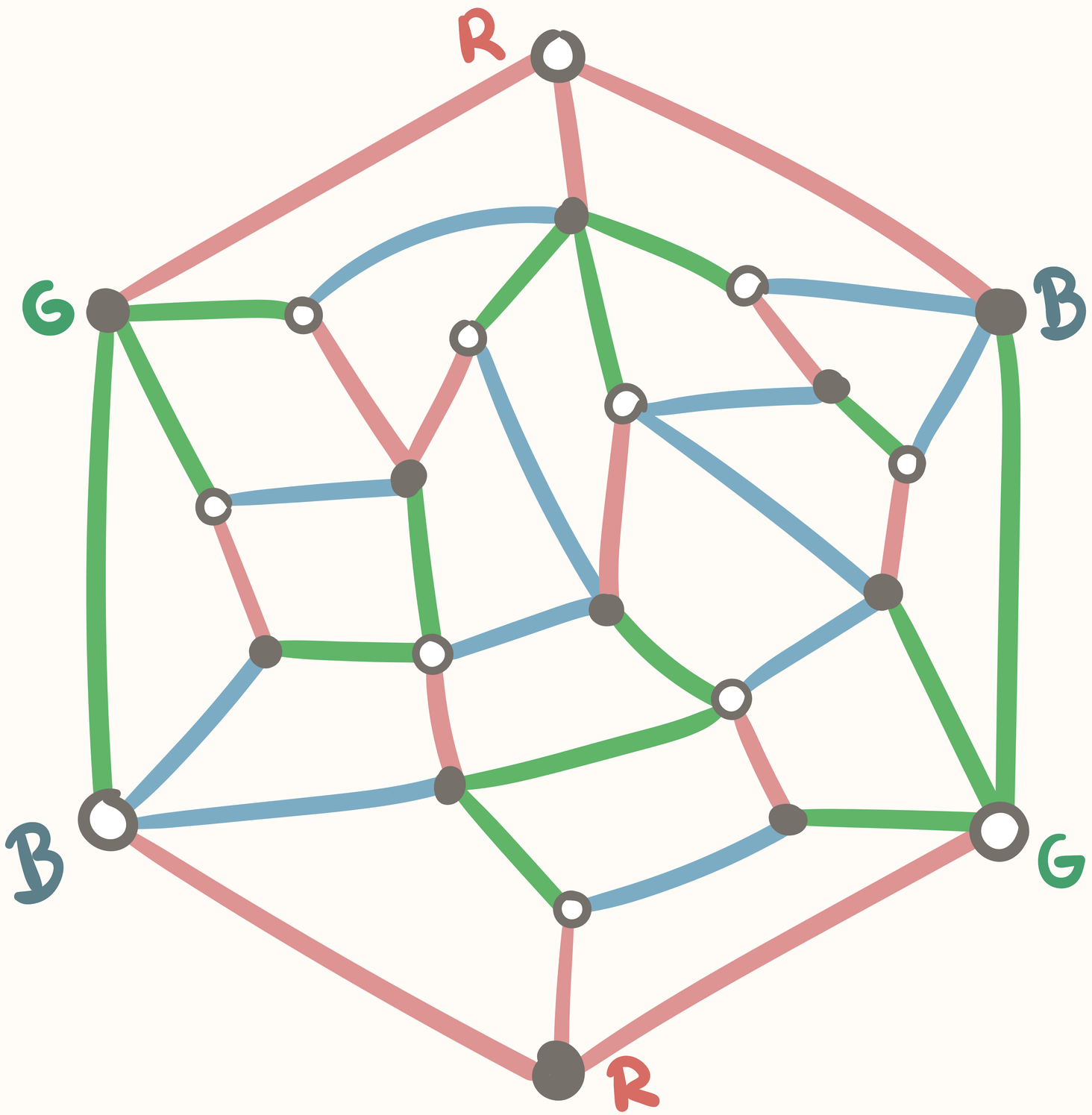












Schnyder labelli

ALL SCHNYDER LABELLINGS

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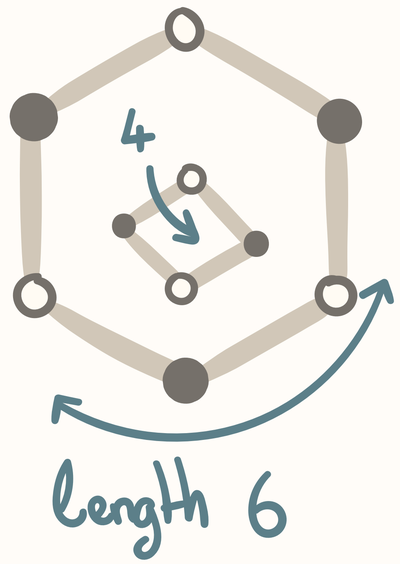
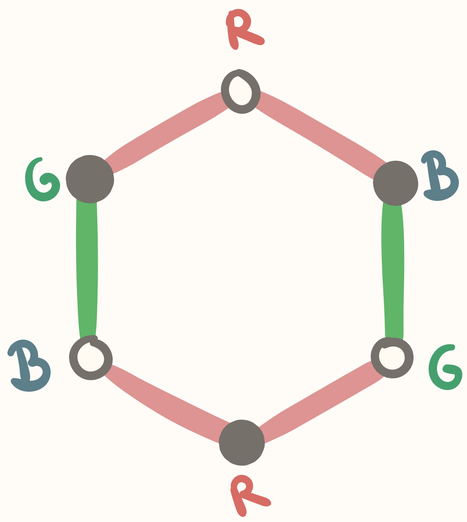
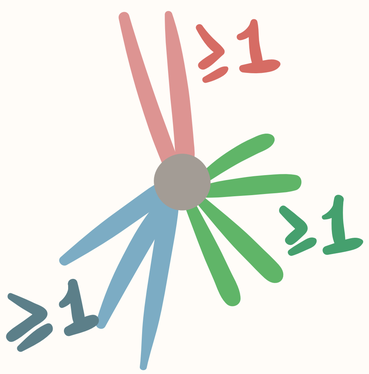
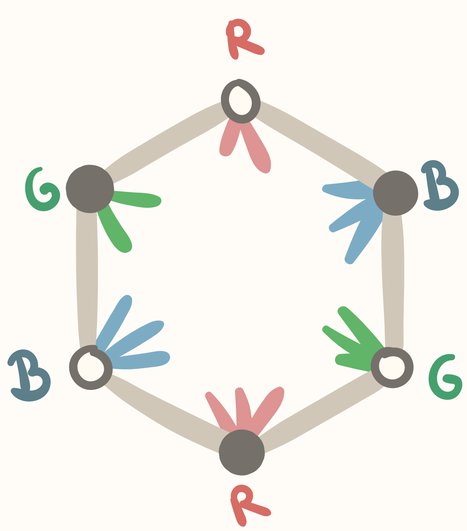
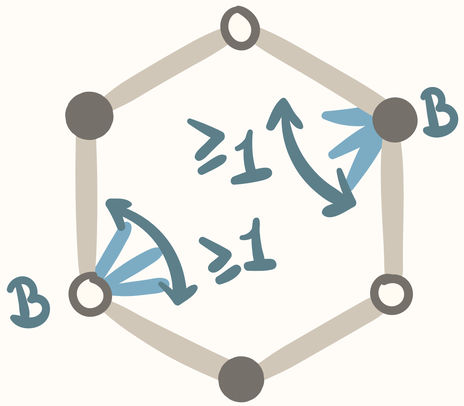
Schnyder labellings

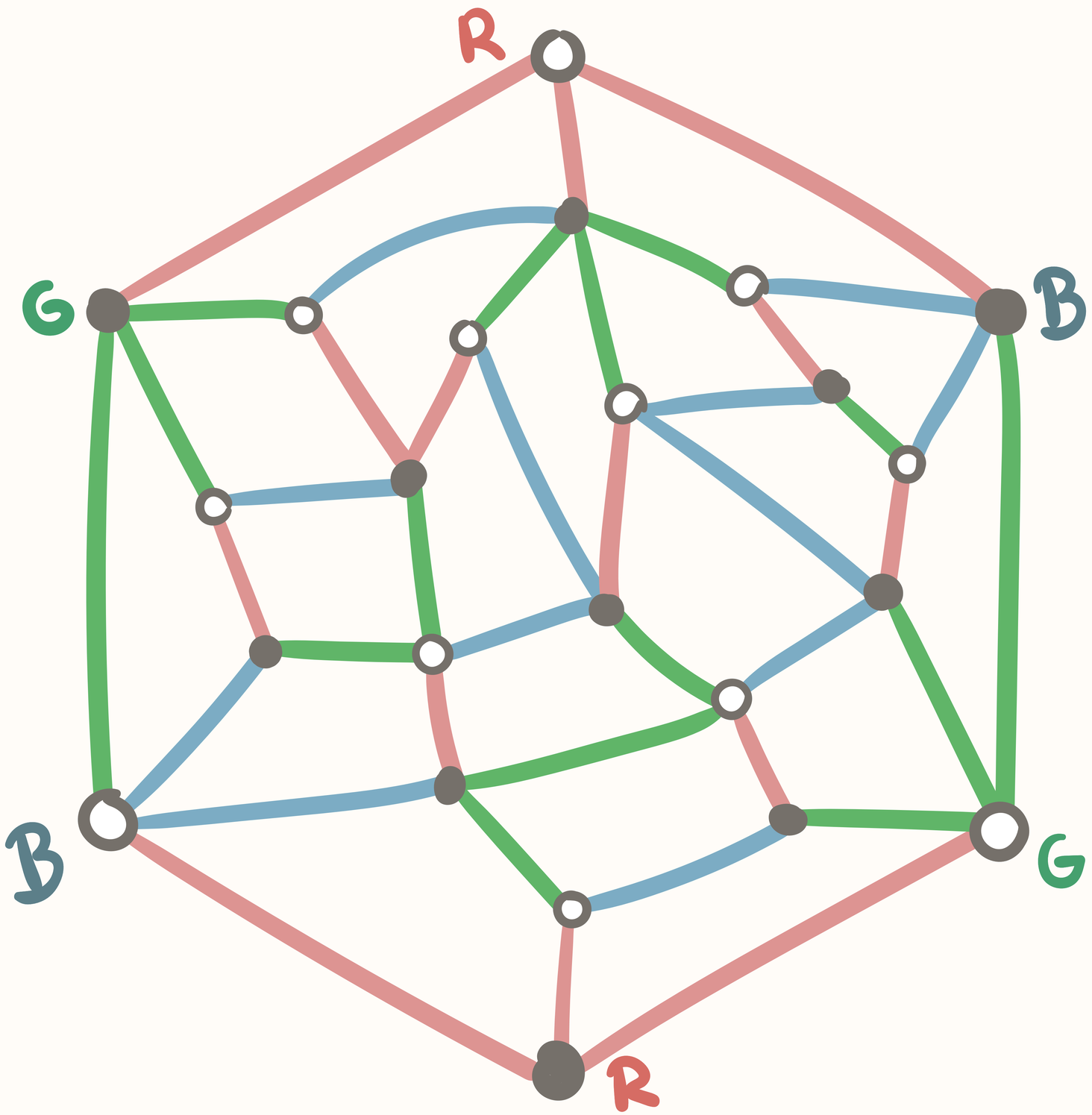




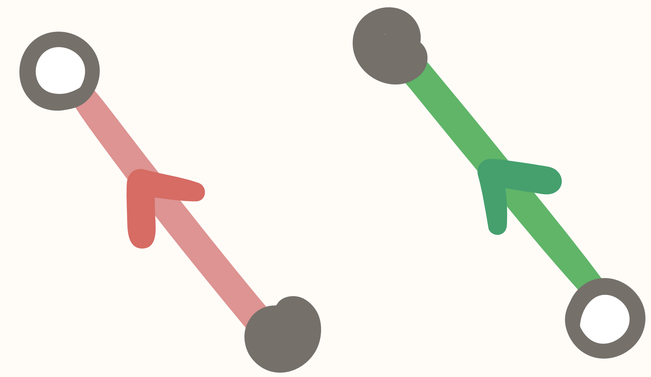


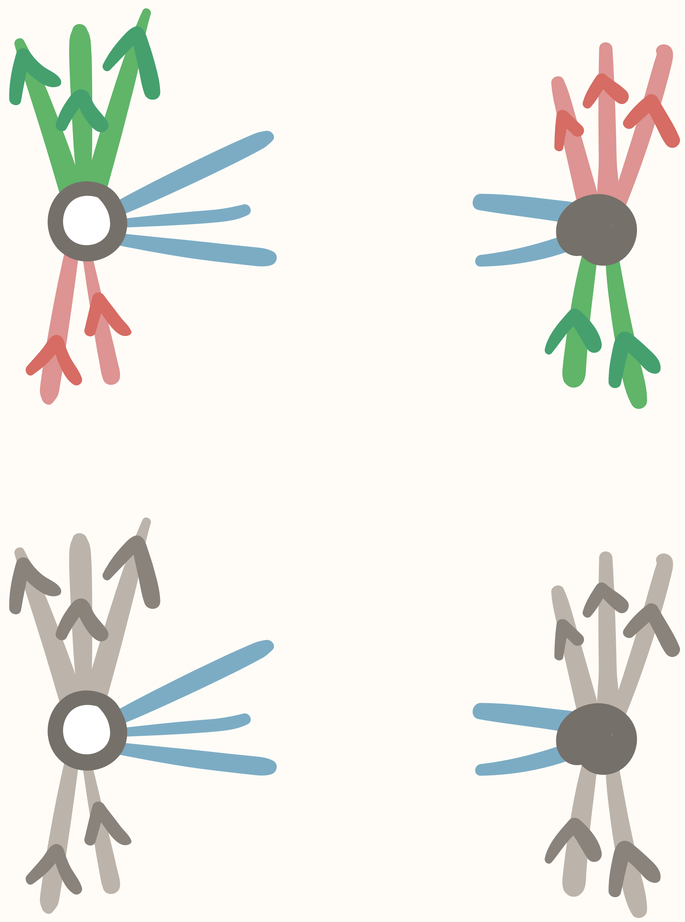


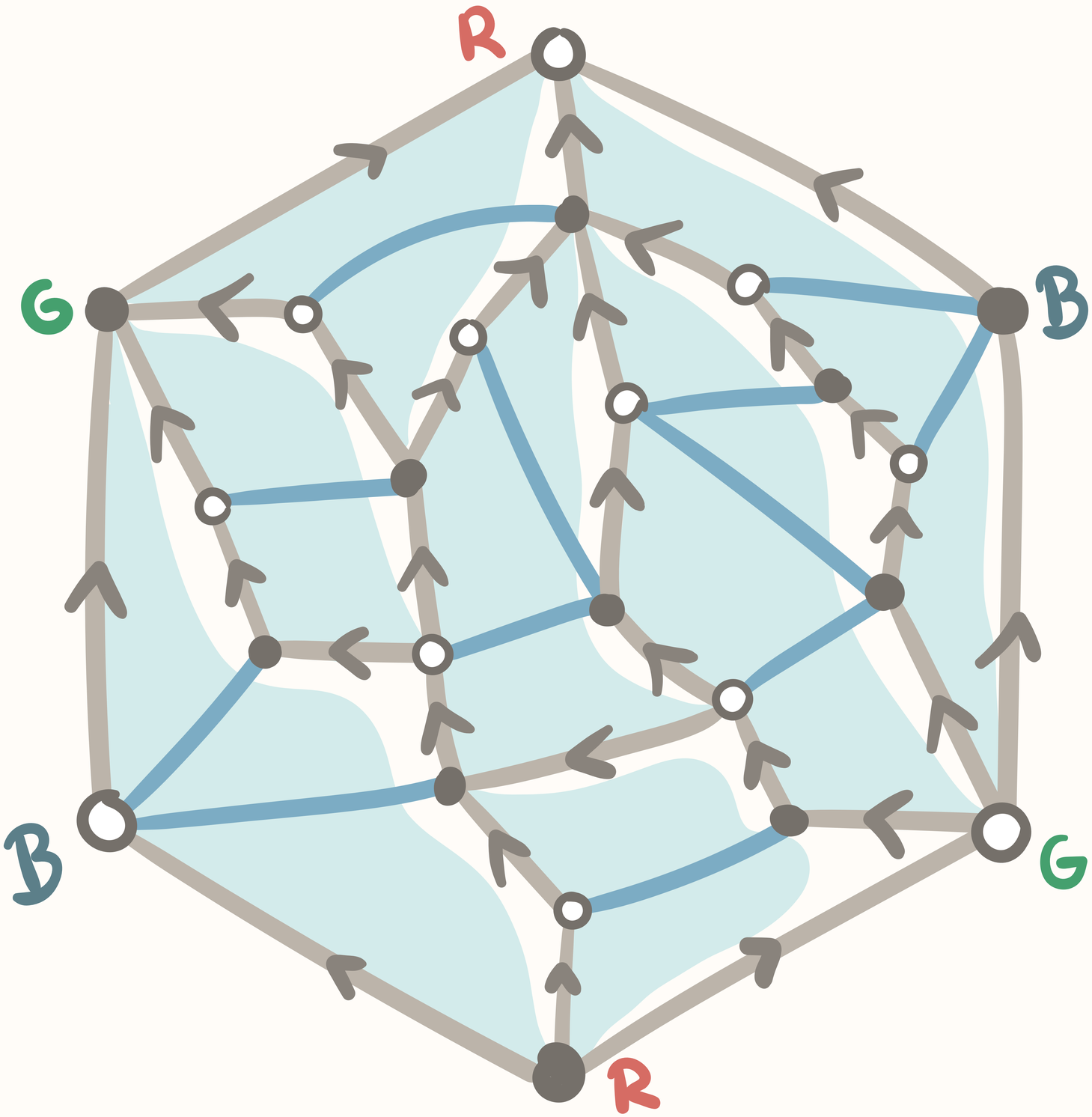




Schnyder labellings





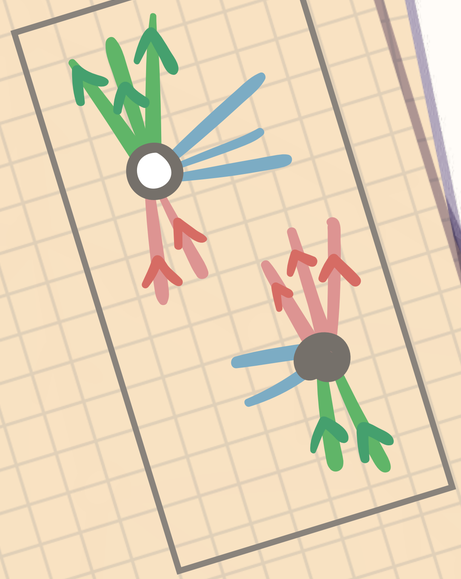


Schnyder labelli

IT IS A BIPOLAR ORIENTATION :

1) ACYCLIC

REMINDER



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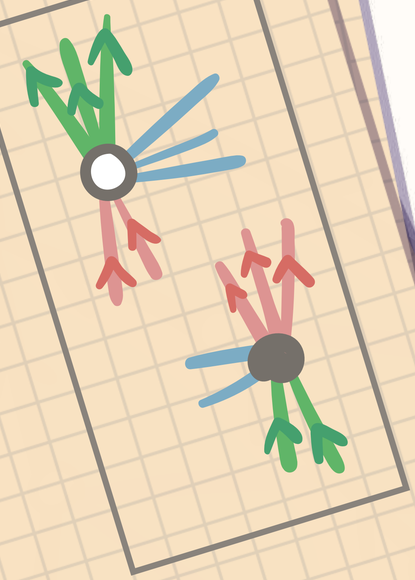
Schnyder labelli

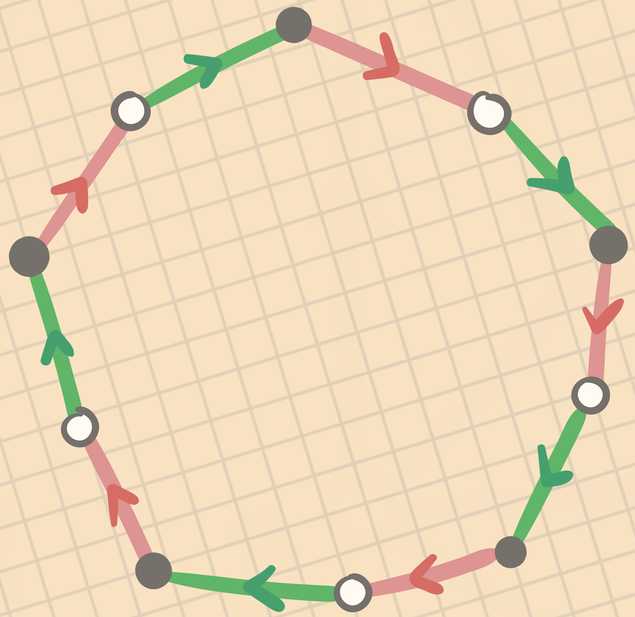
IT IS A BIPOLAR ORIENTATION :

1) ACYCLIC

ASSUME ∃ CYCLE, TAKEN MINIMAL

REMINDER





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Schnyder labelli

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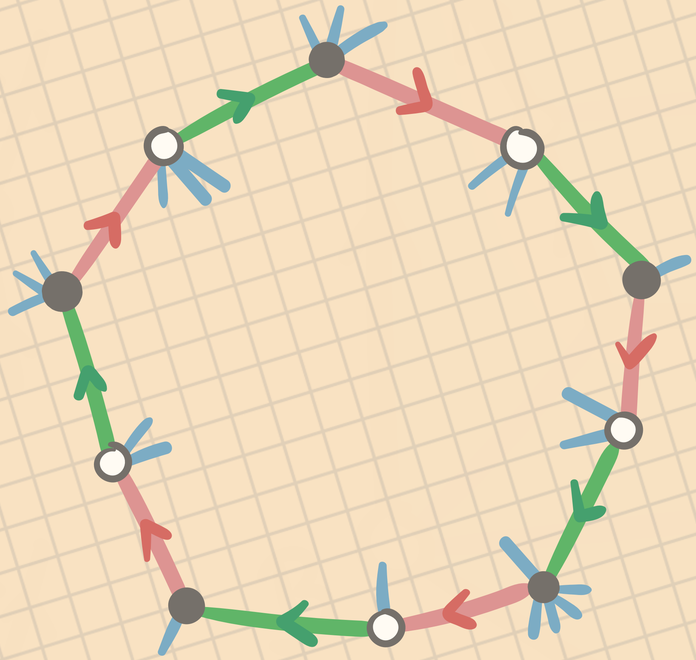

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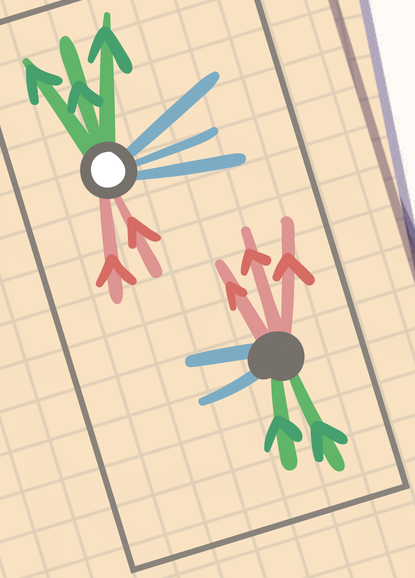

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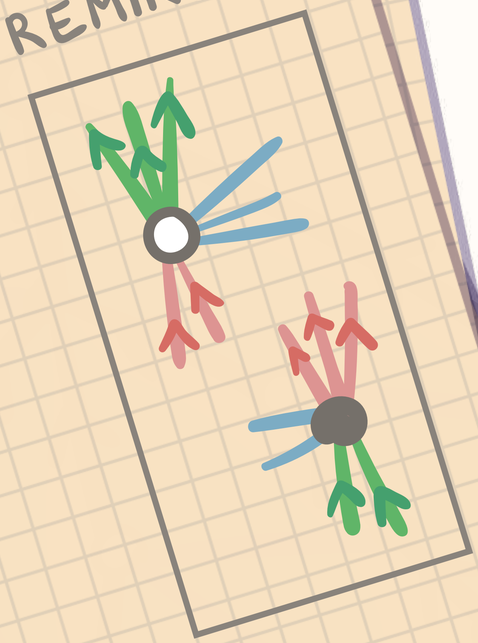

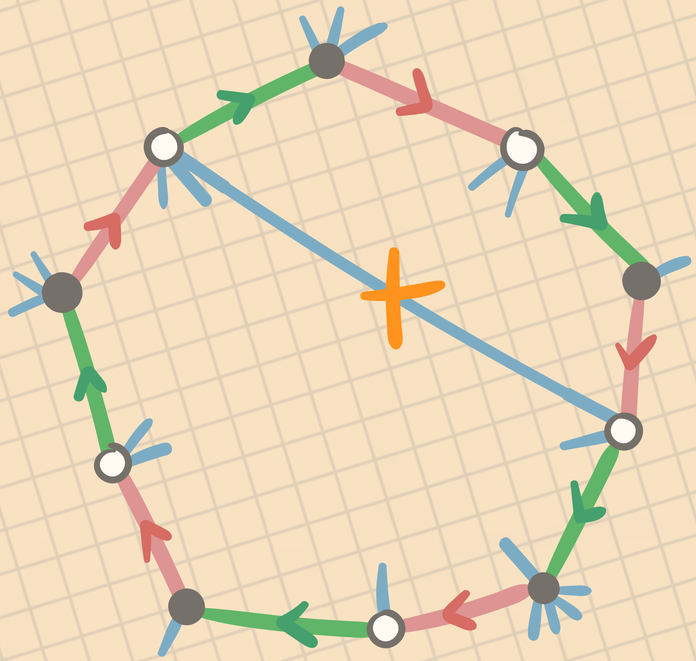

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ASSUME ∃ CYCLE, TAKEN MINIMAL

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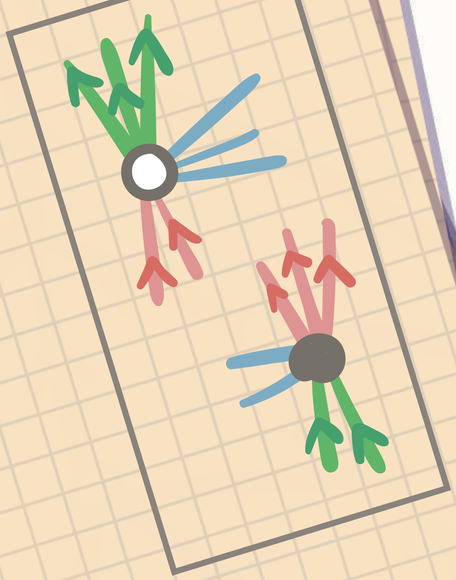

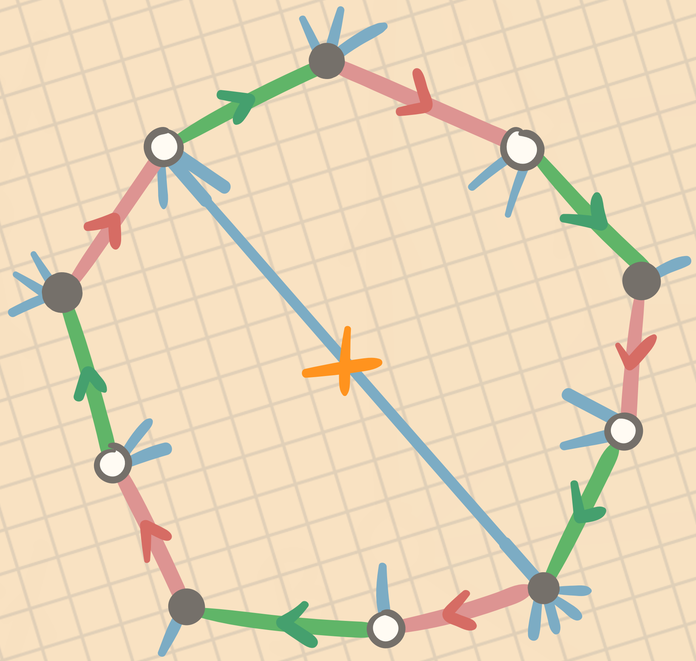

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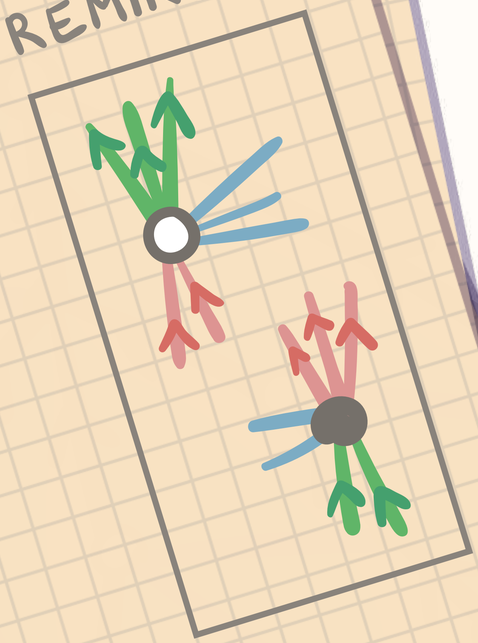

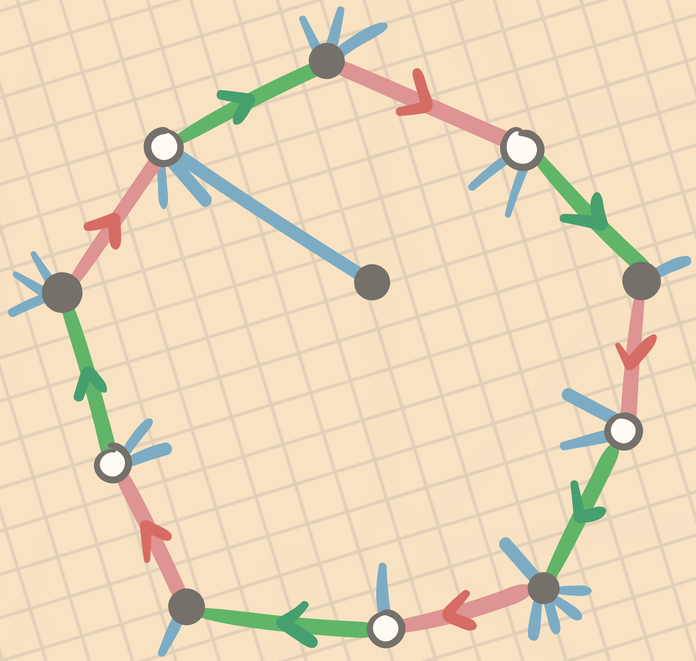

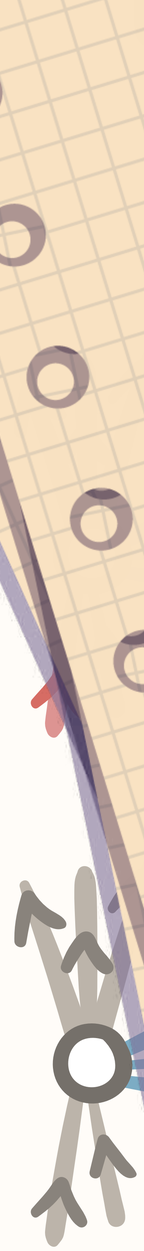

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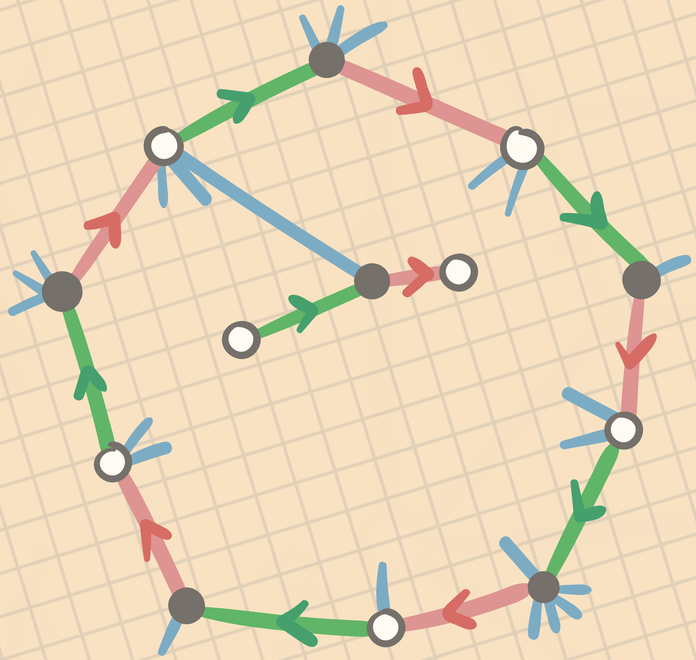

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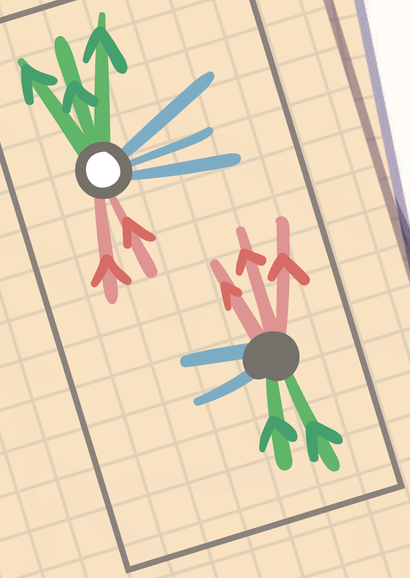

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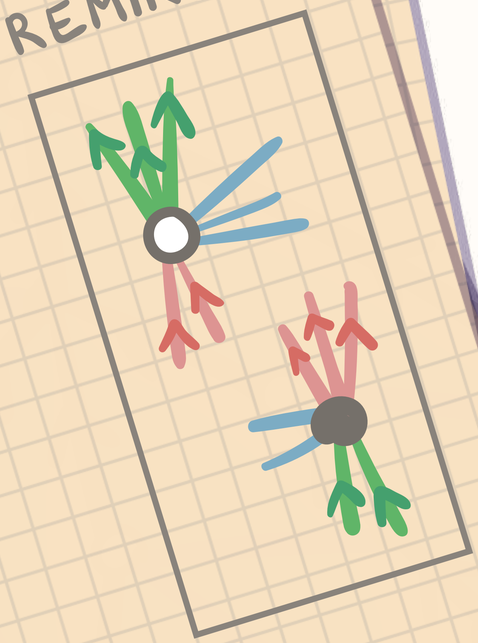

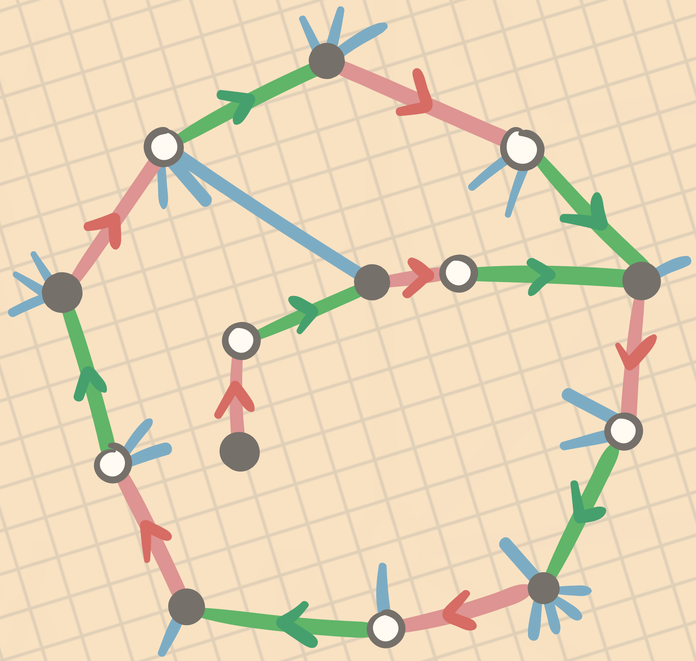

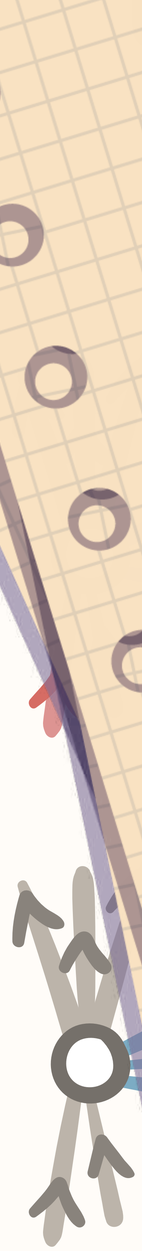

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Schnyder labelli

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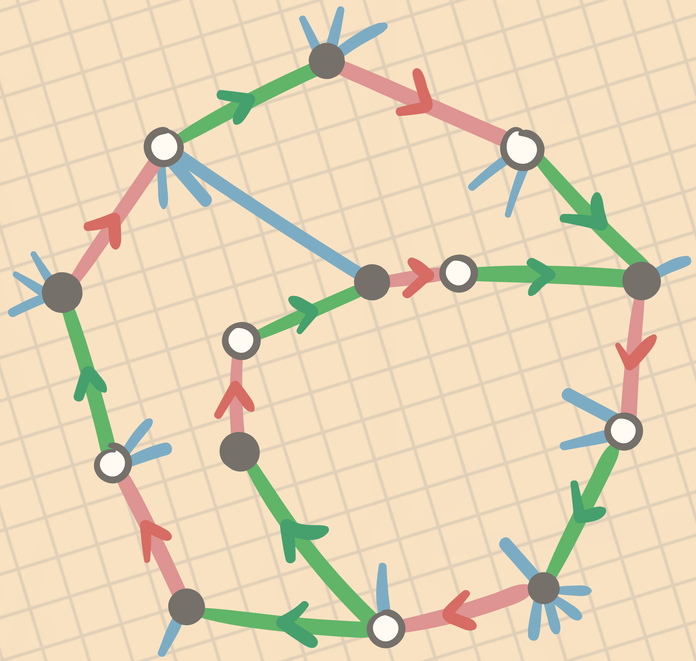
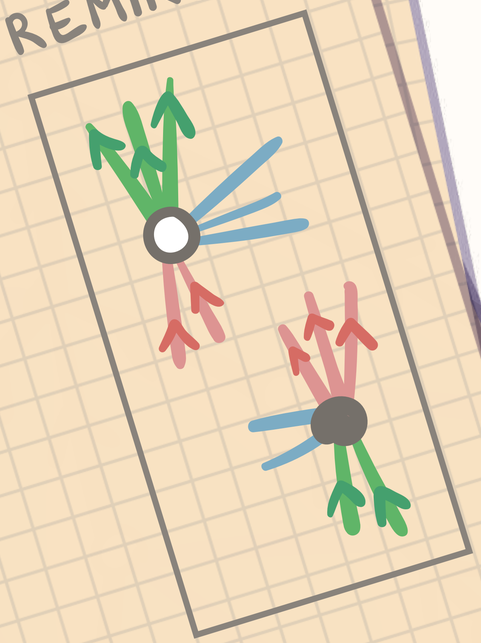
ASSUME ∃ CYCLE, TAKEN MINIMAL



NOT MINIMAL

REMINDER



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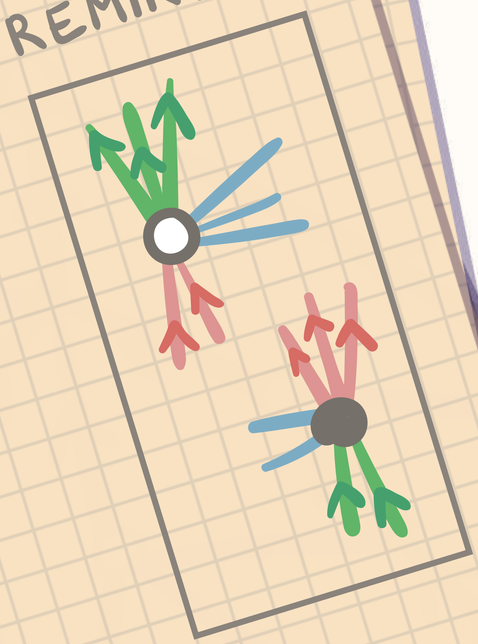
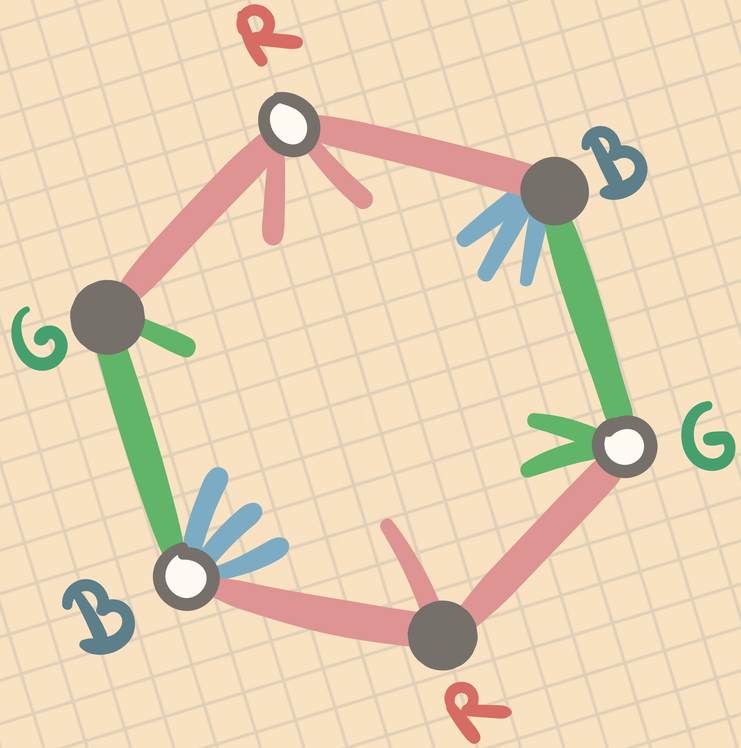
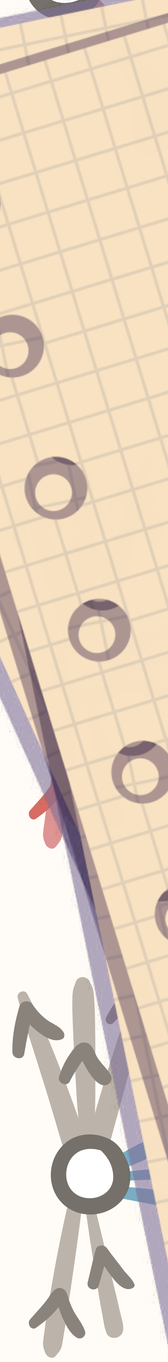
Schnyder labelli

IT IS A BIPOLAR ORIENTATION :

1) ACYCLIC
2) SINGLE SINK / SOURCE







Schnyder labelli

IT IS A BIPOLAR ORIENTATION :

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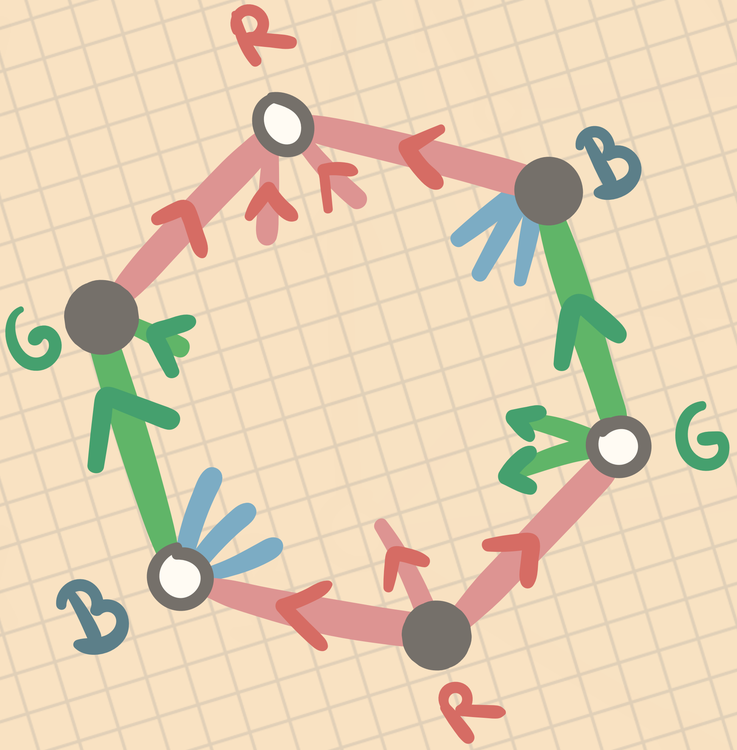
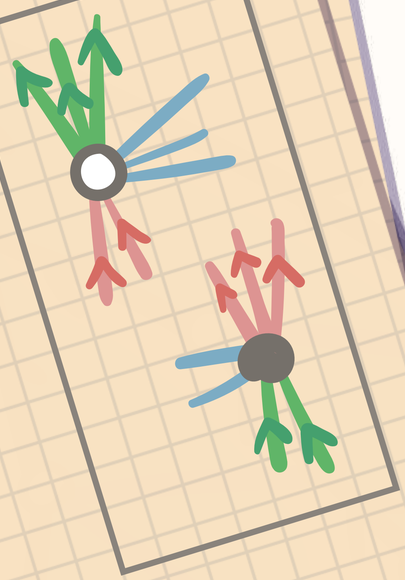
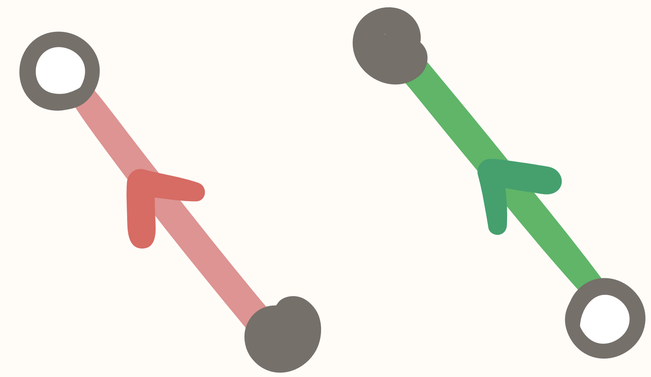
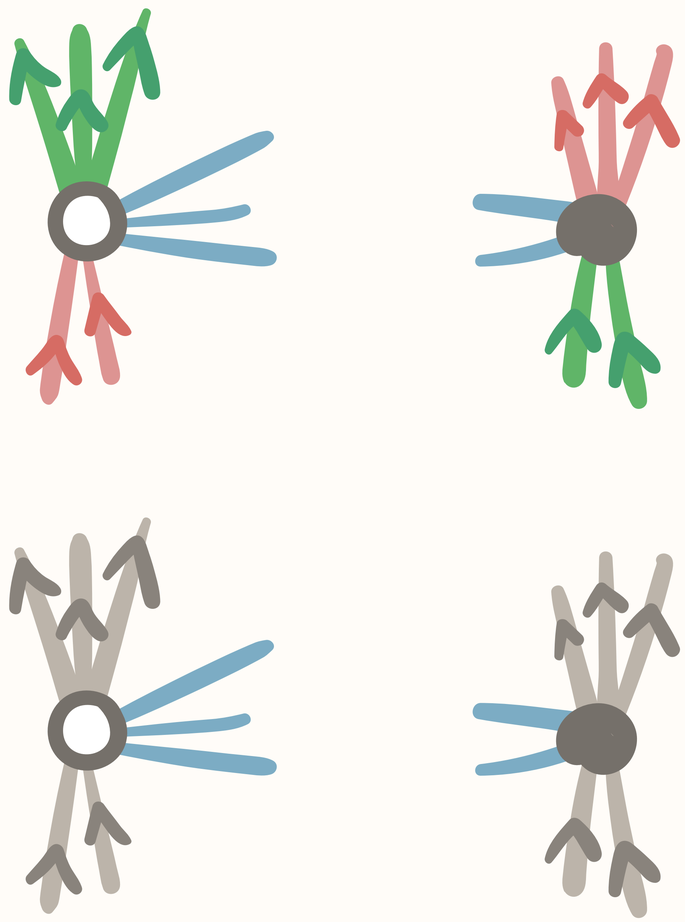


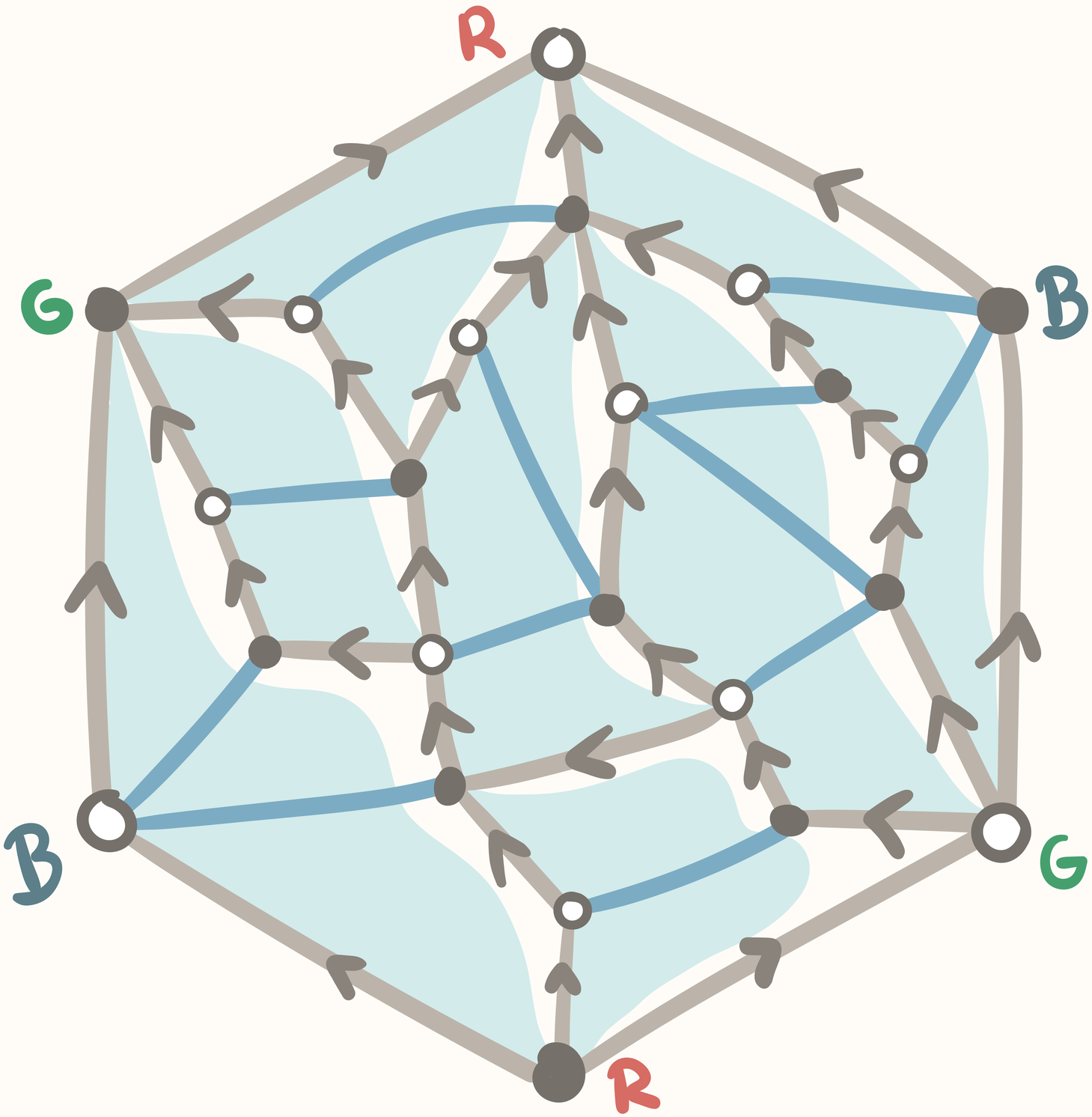
REMINDER



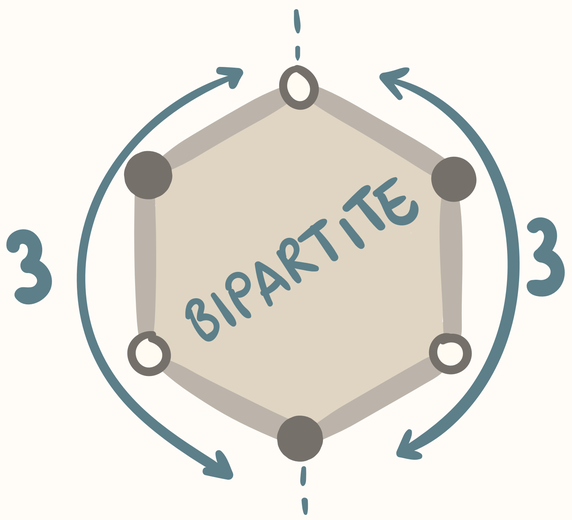
Schnyder labellings

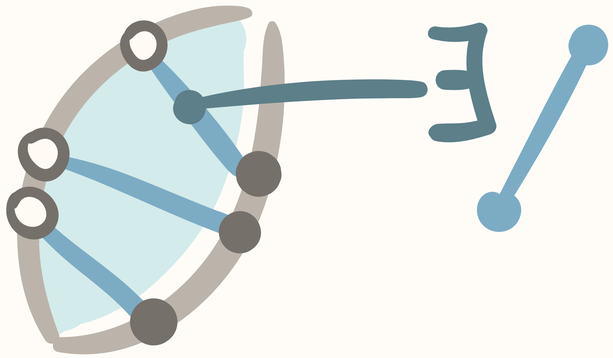


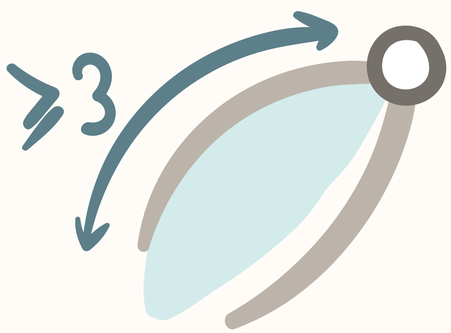


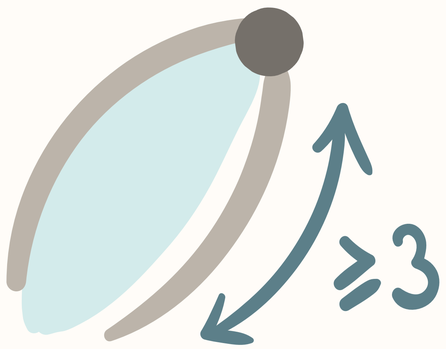


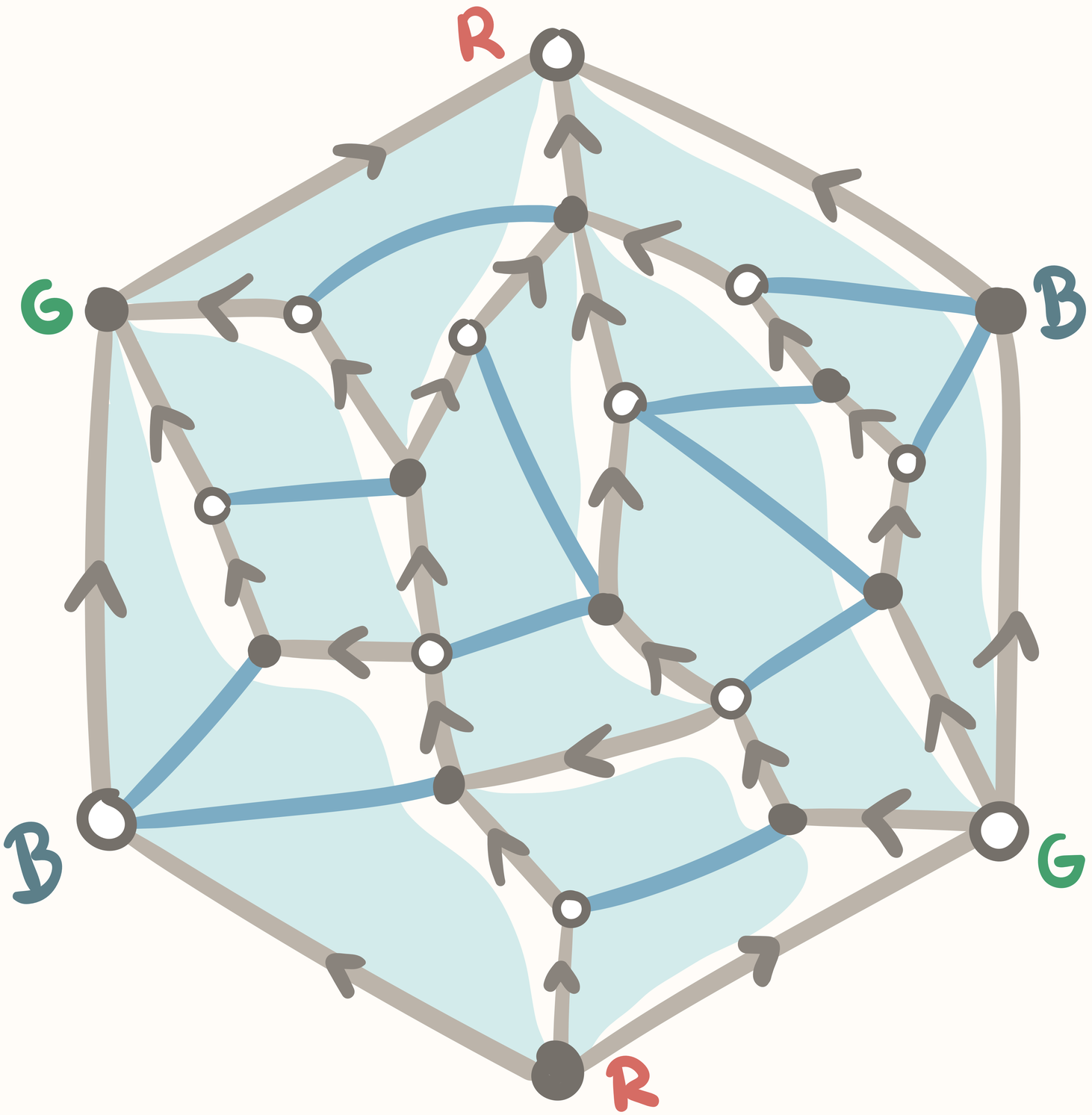
Schnyder labellings



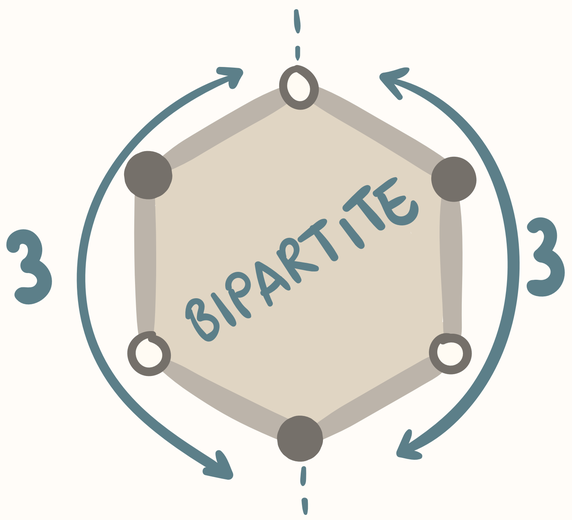


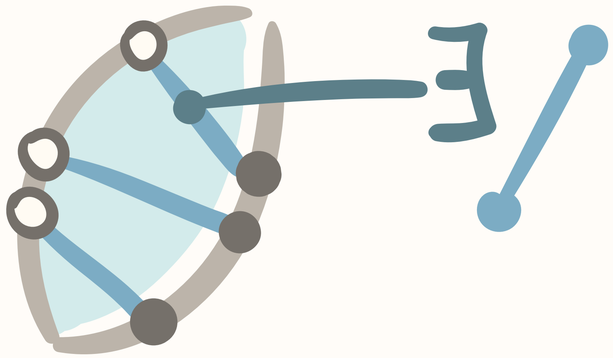


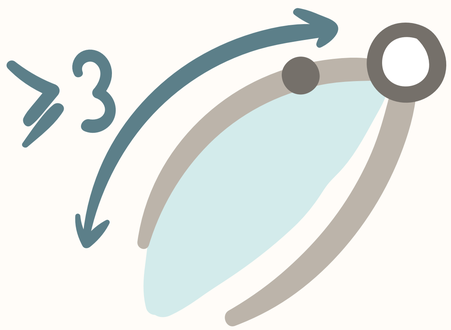


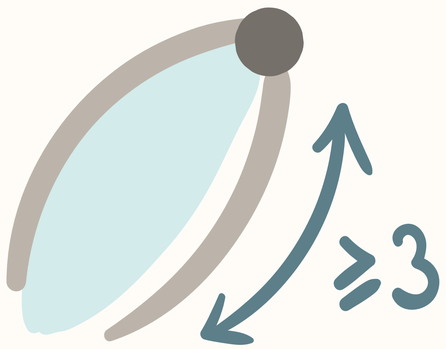


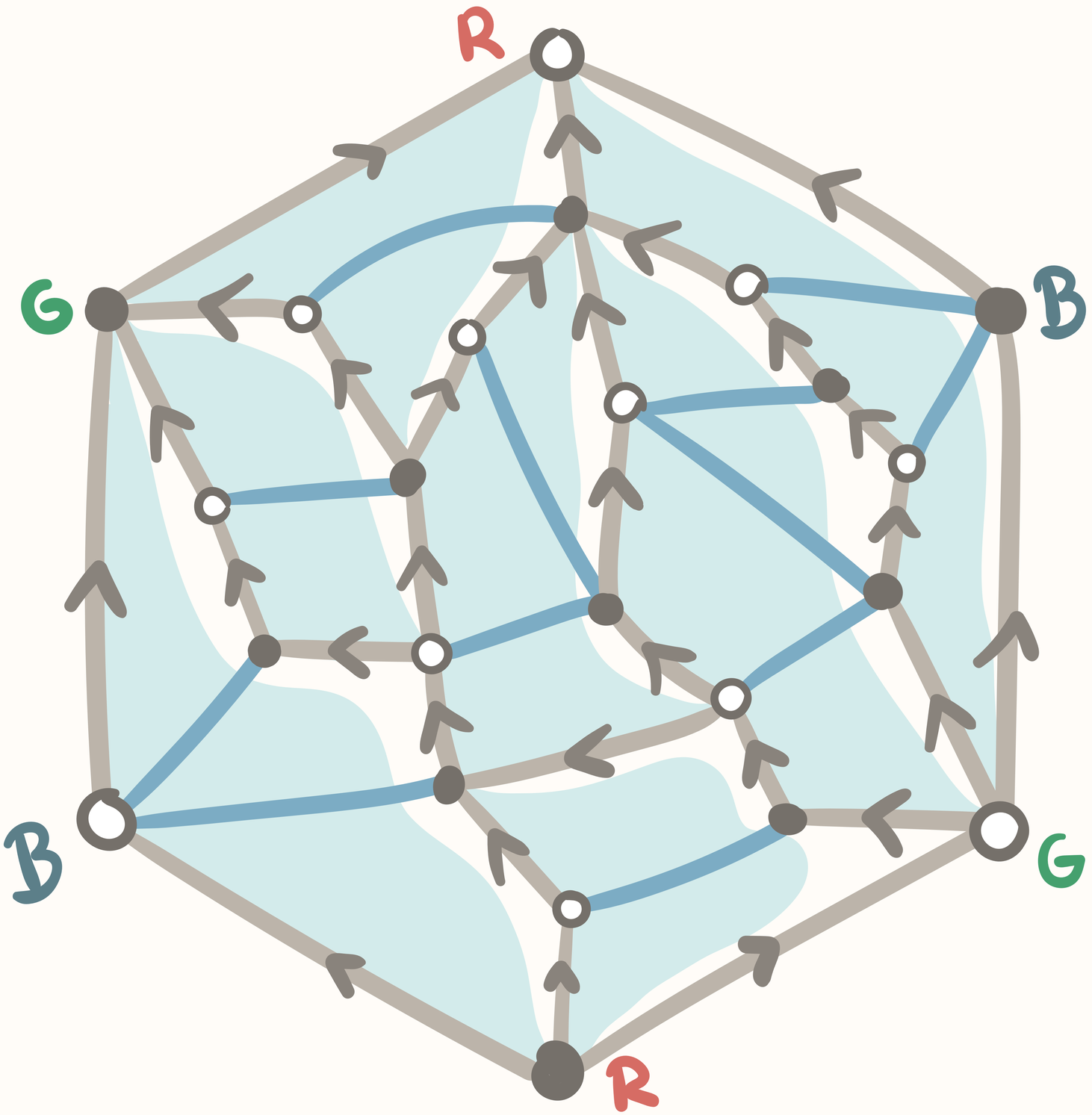
Schnyder labellings



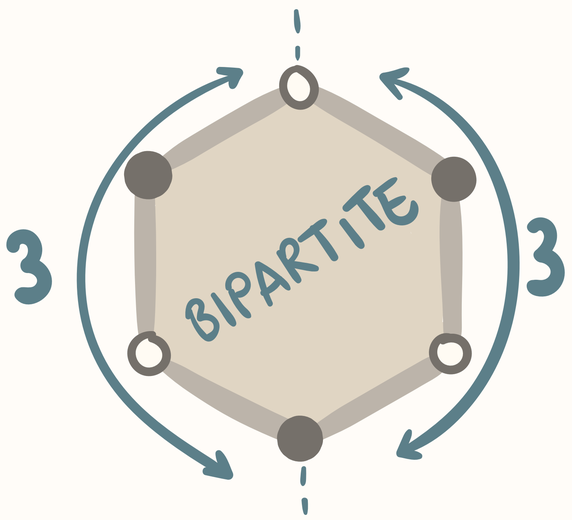


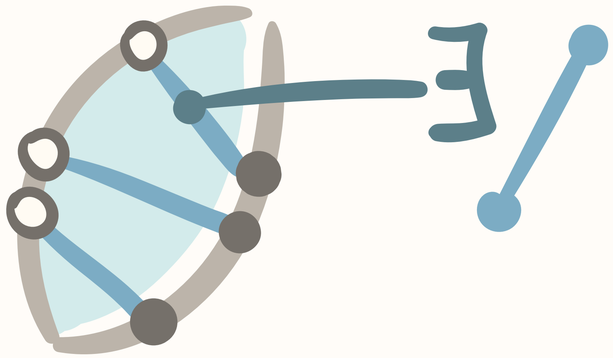


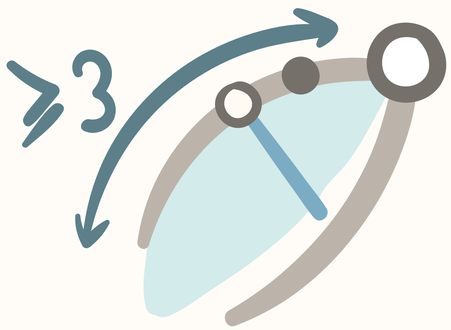


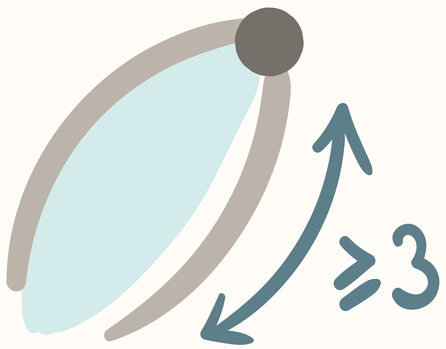


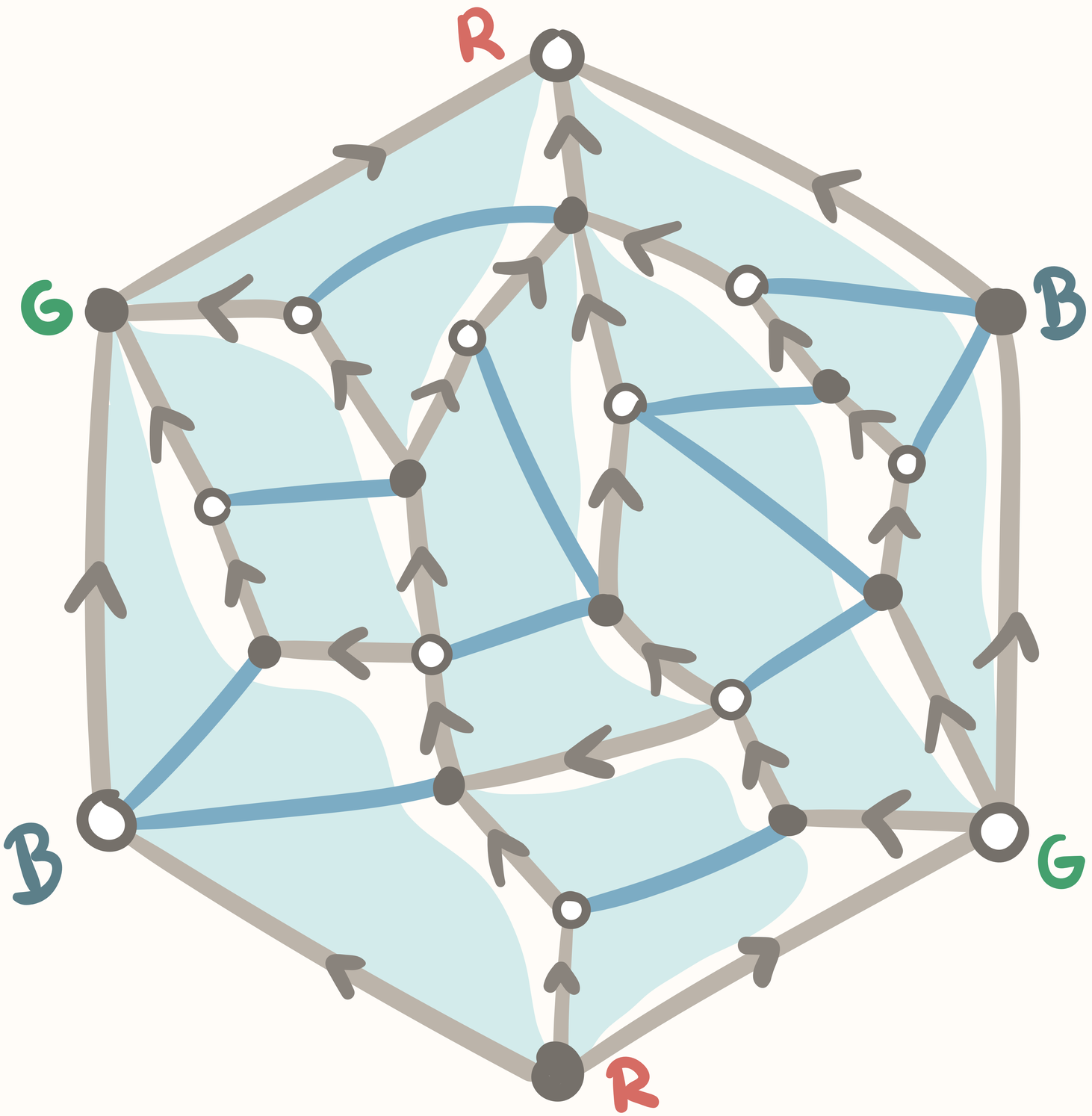
Schnyder labellings



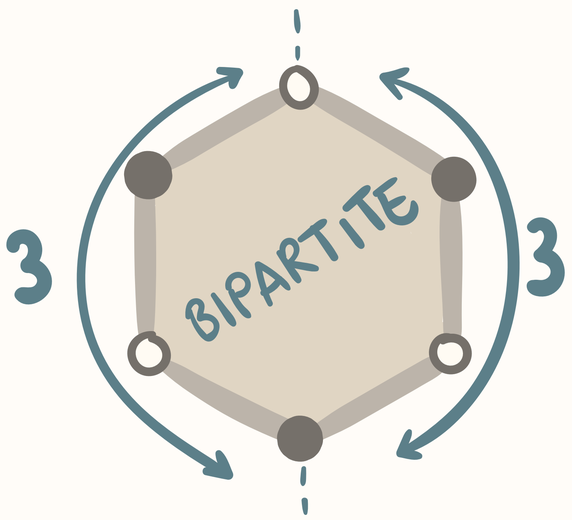


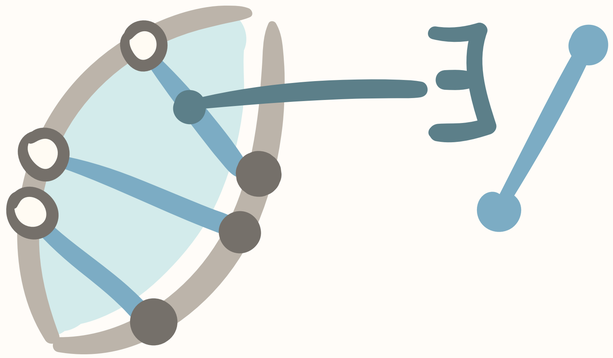


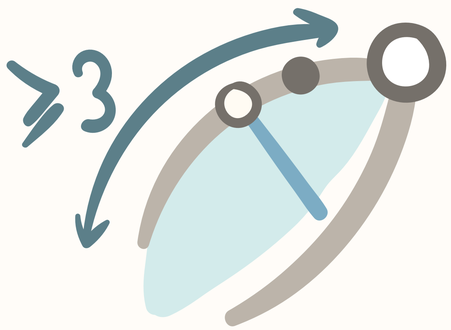


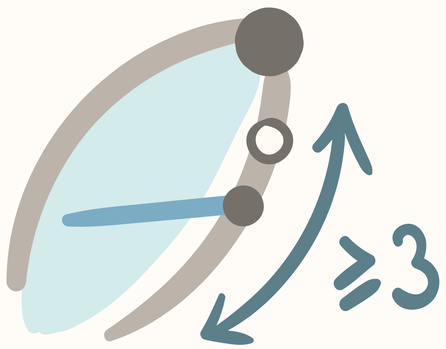


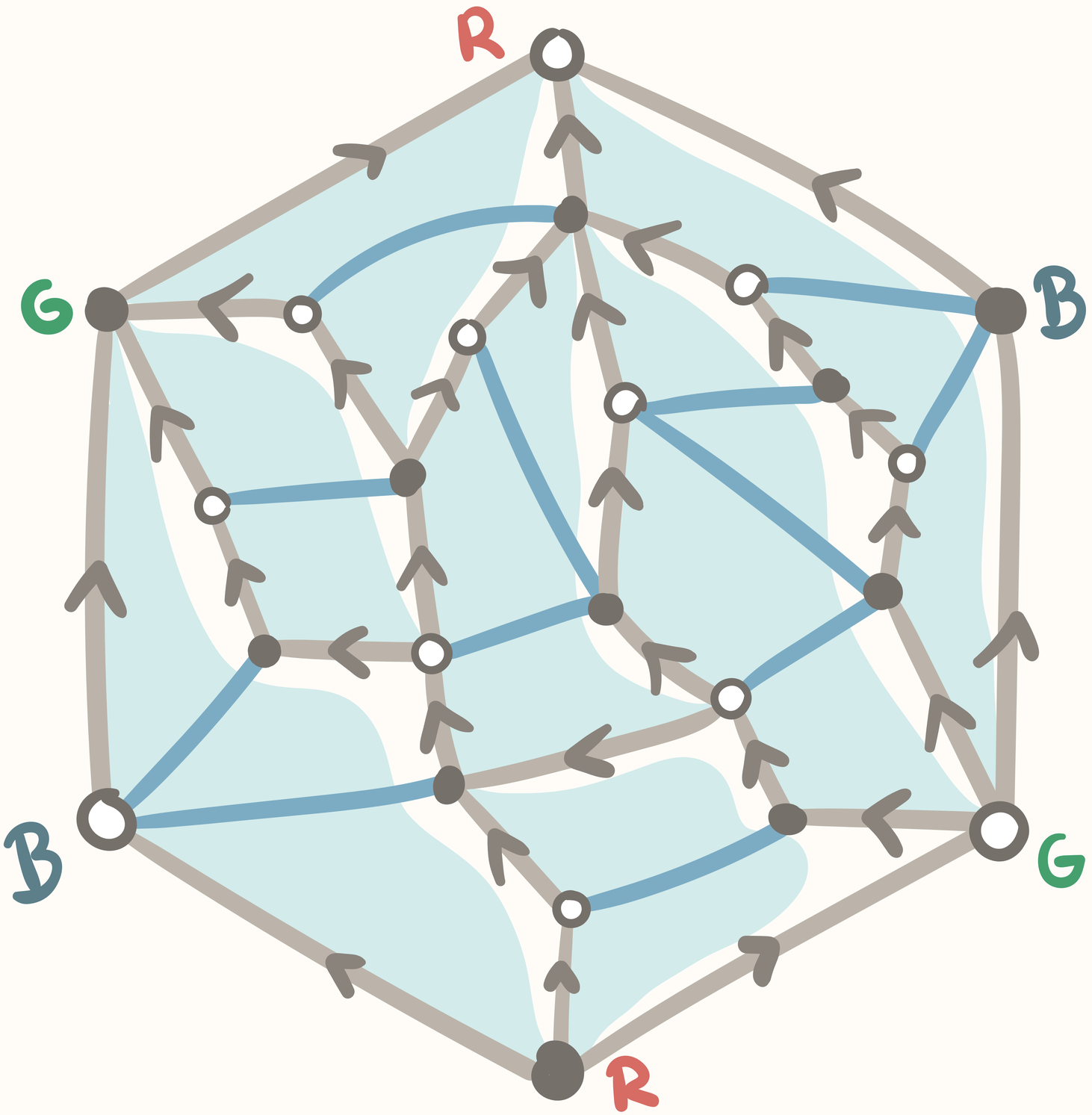
Schnyder labellings



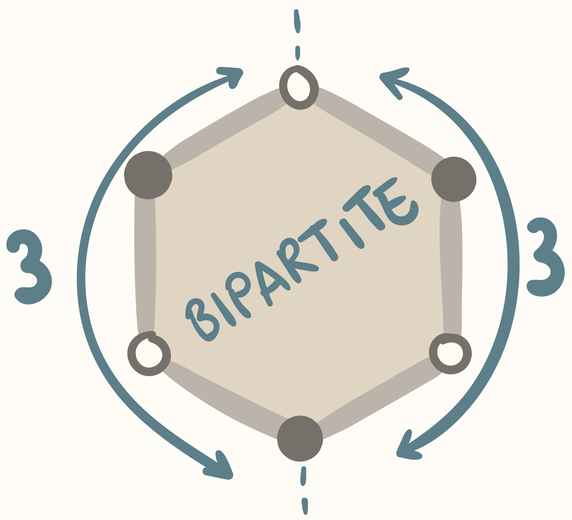


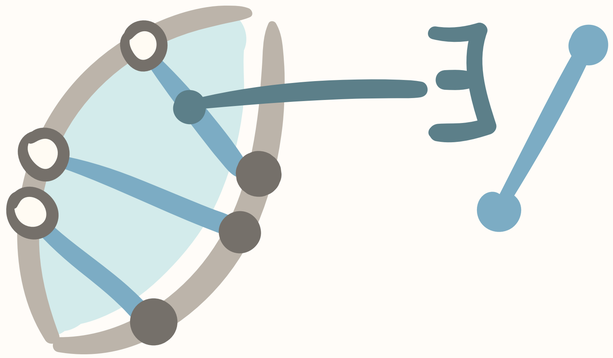


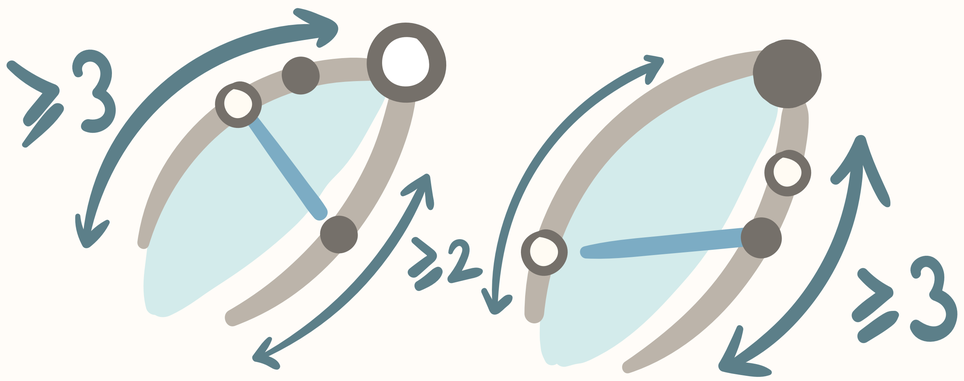


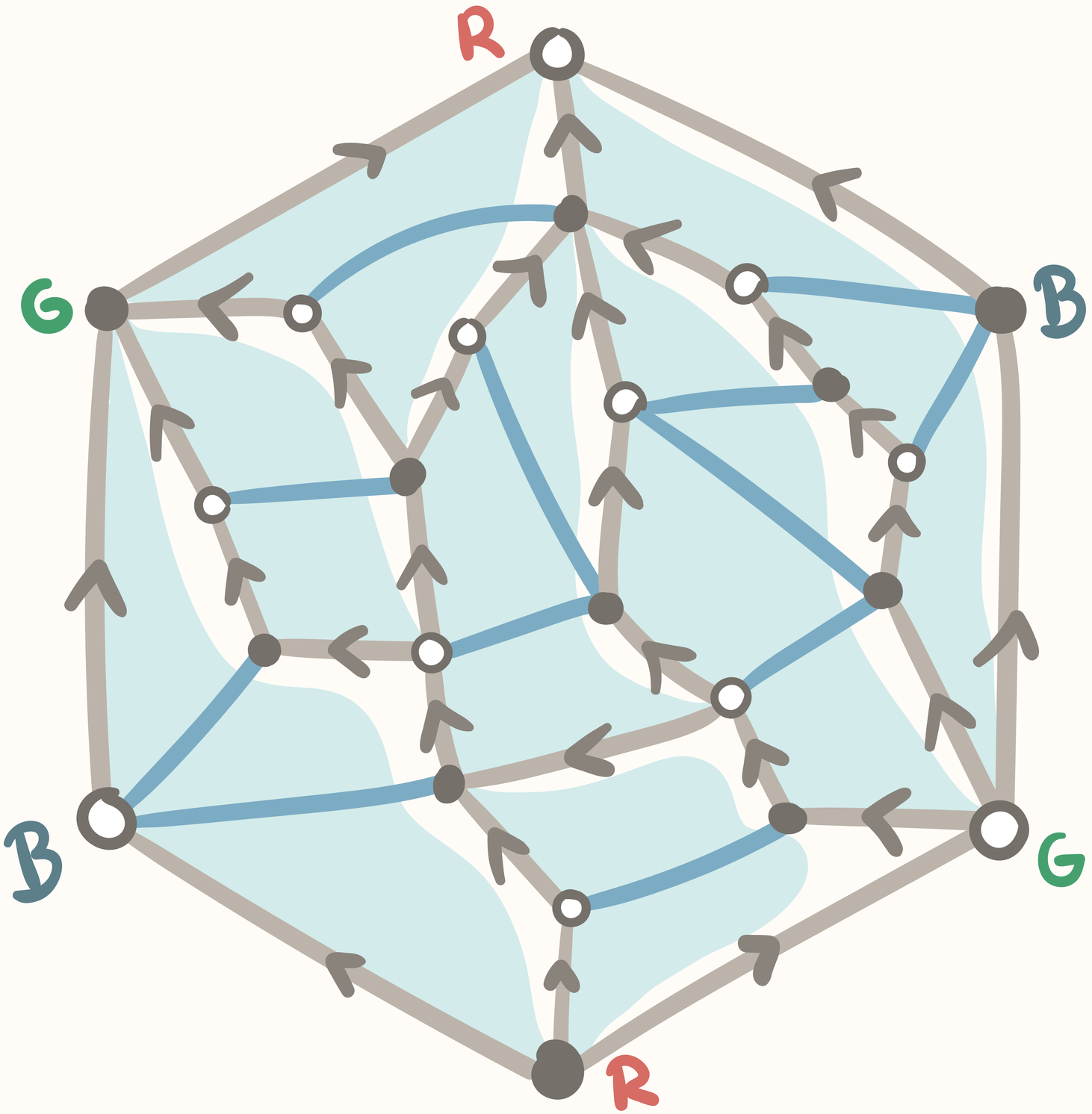


Schnyder labellings

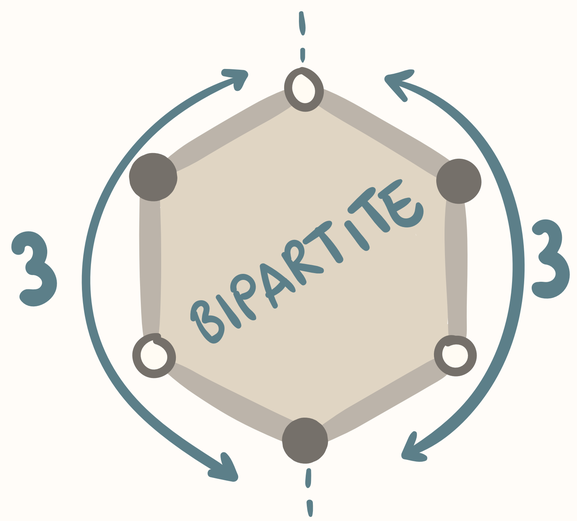


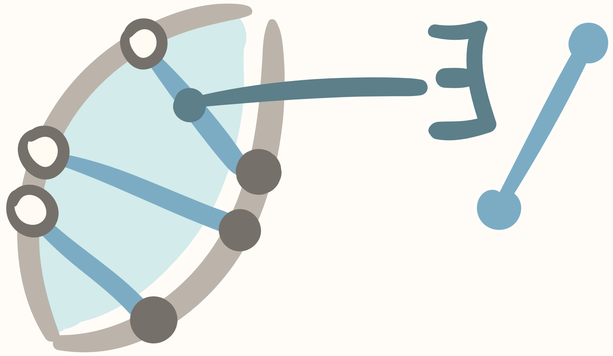


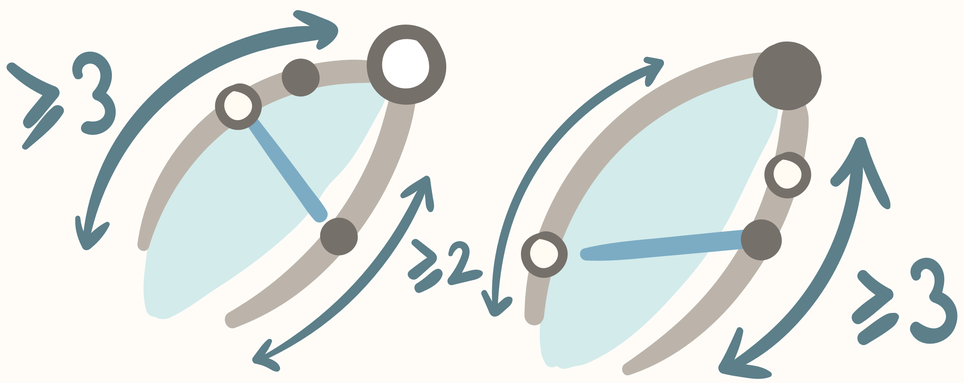




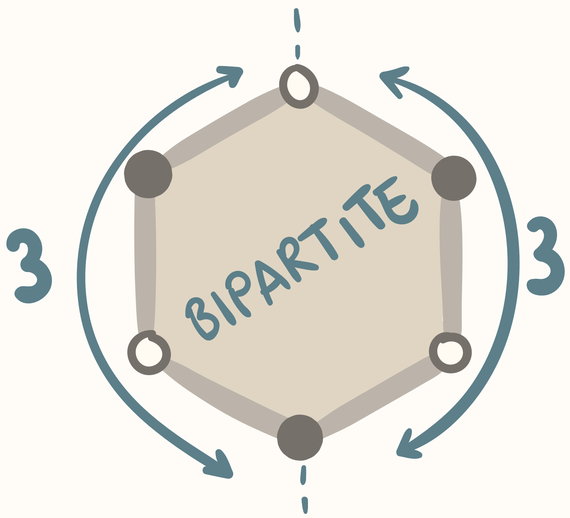
Schnyder labellings KMSW

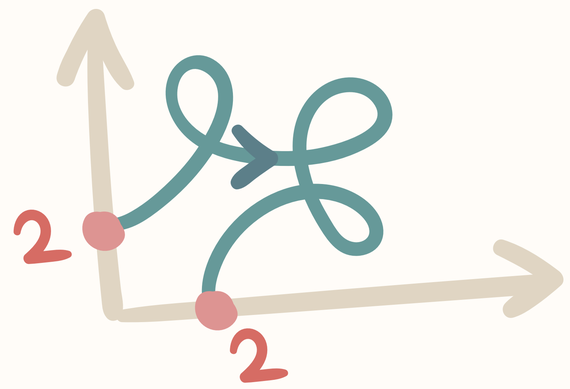


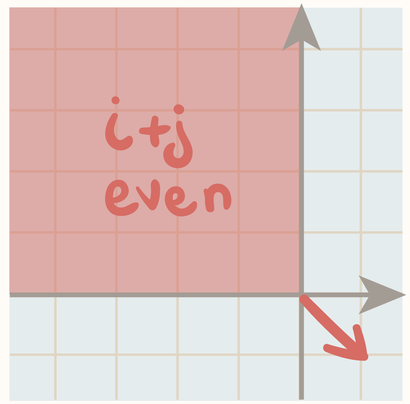


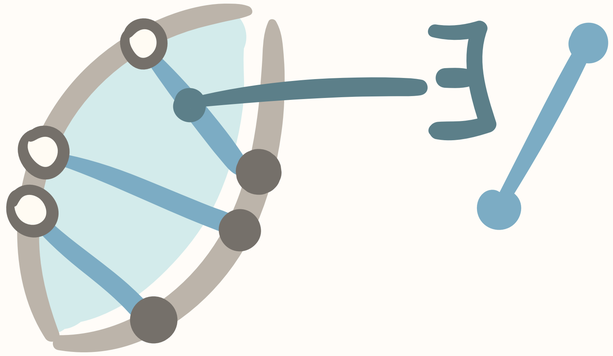


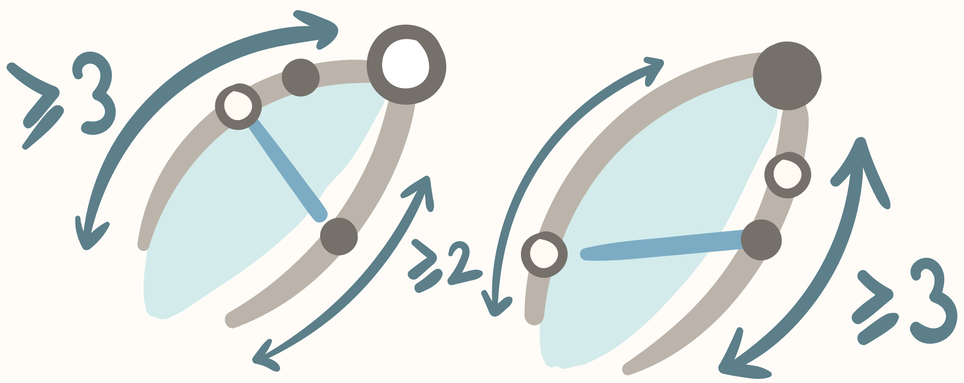
Schnyder labellings KMSW



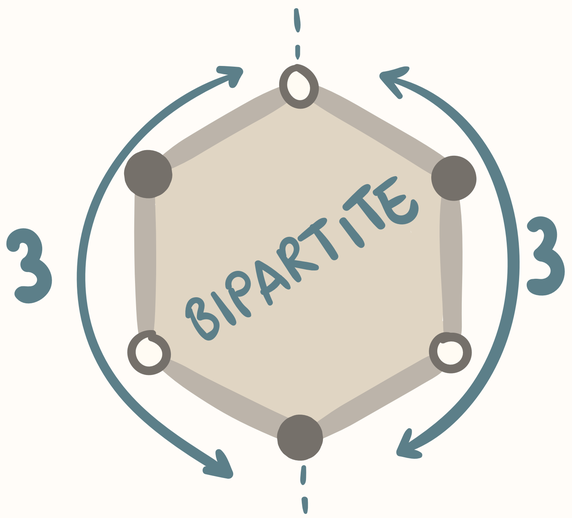


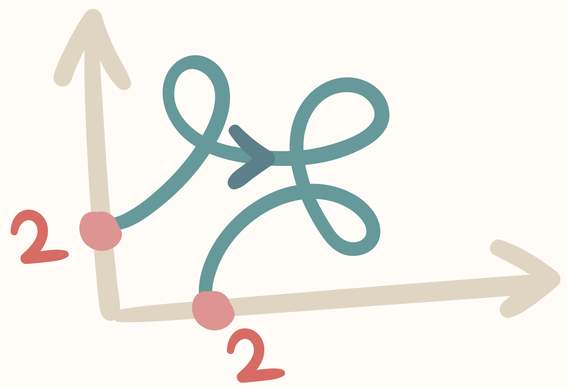


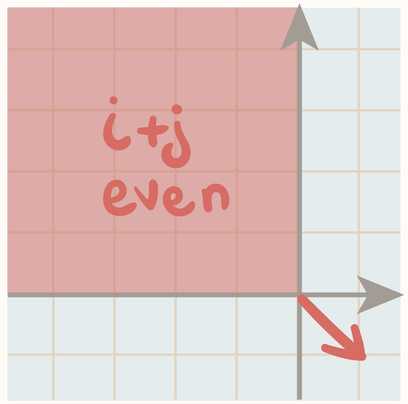


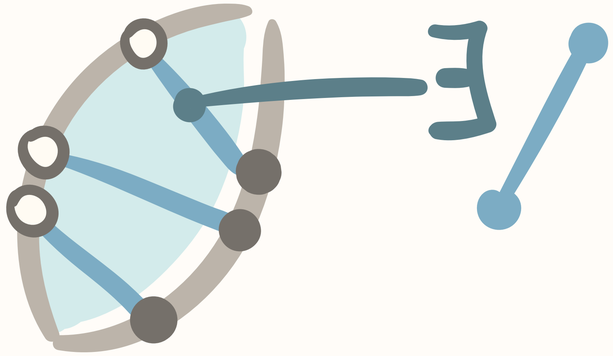


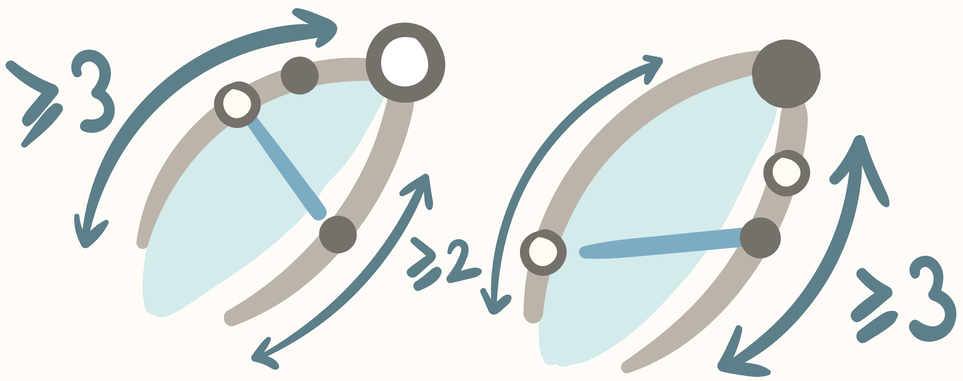
Schnyder labellings KMSW

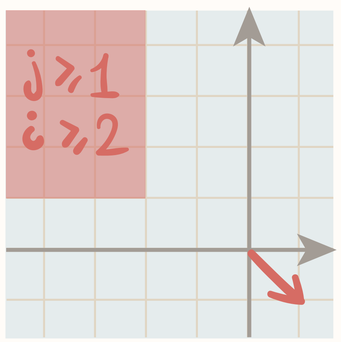




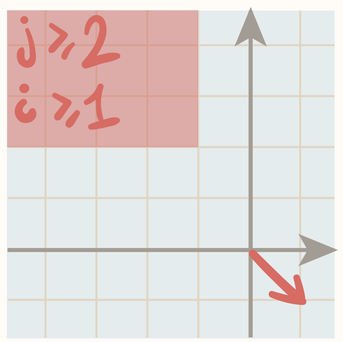






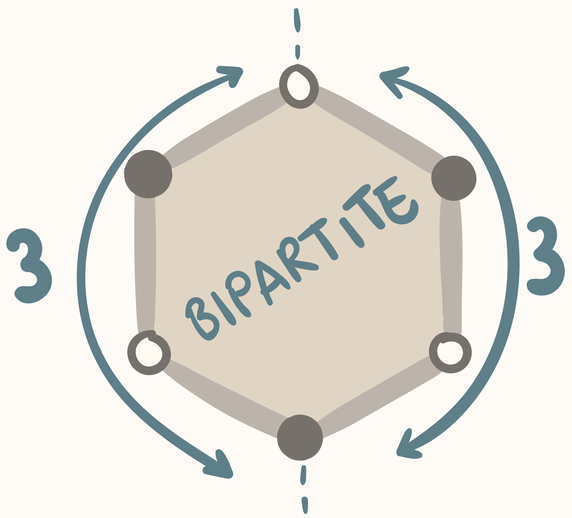


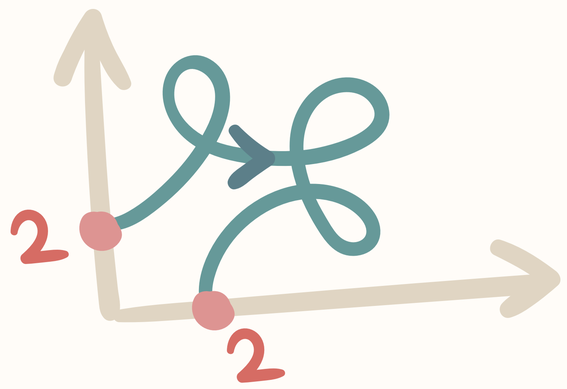
when x,y are even

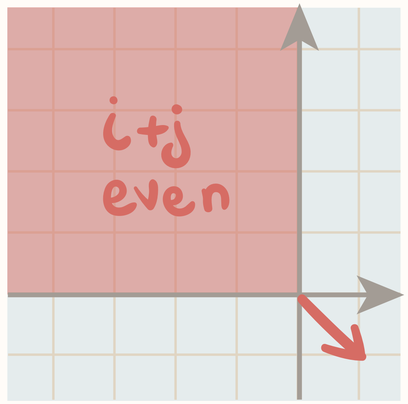


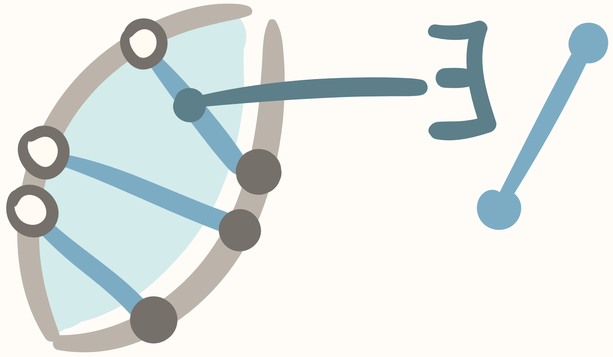
when x,y are odd

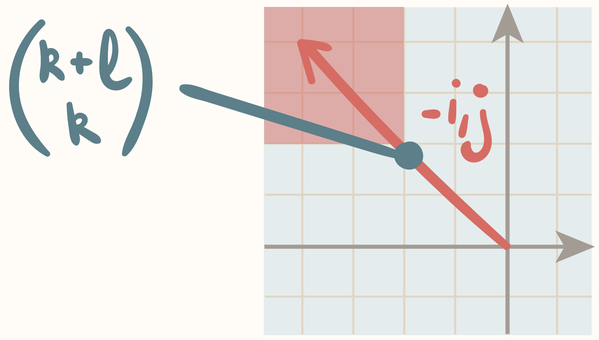
Schnyder labellings KMSW





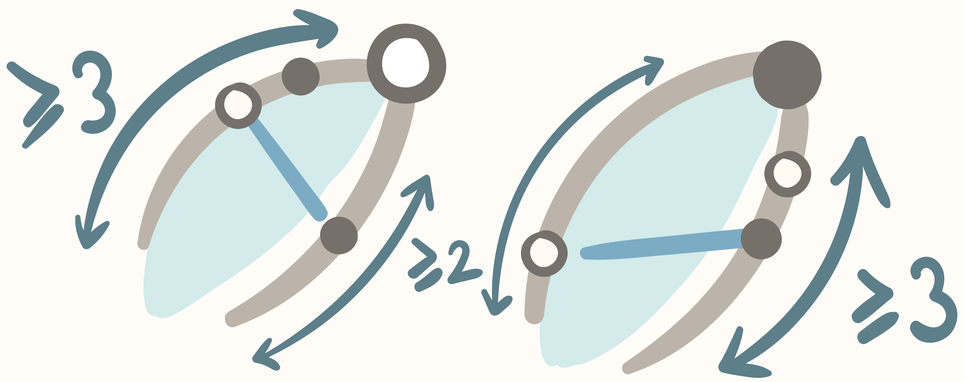


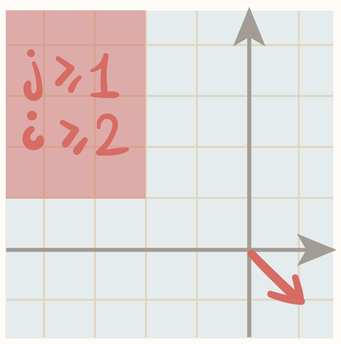




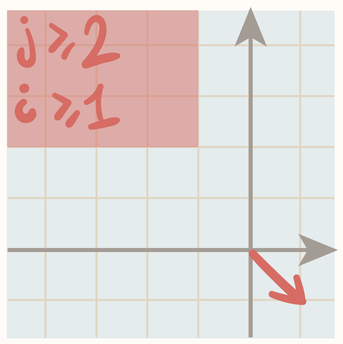
regarding x,y parity

with
i = 2k+2
j = 2l+2



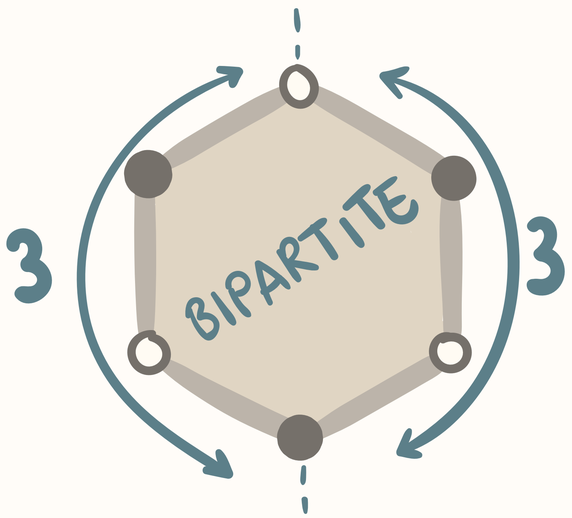


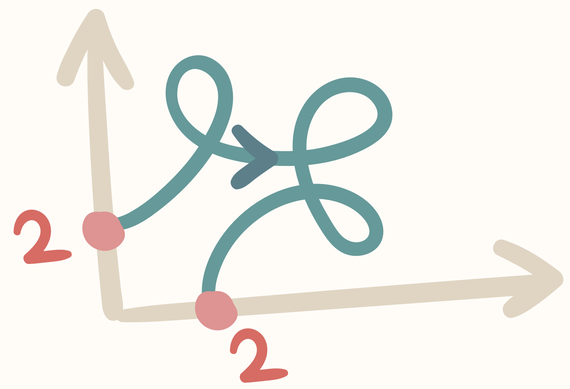
when x,y are even

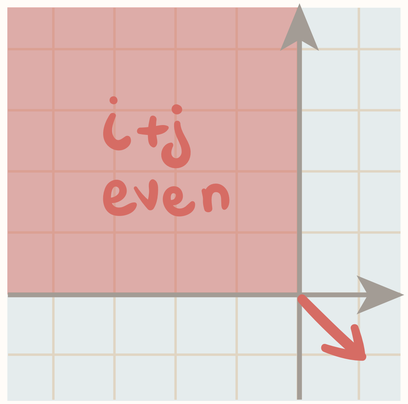


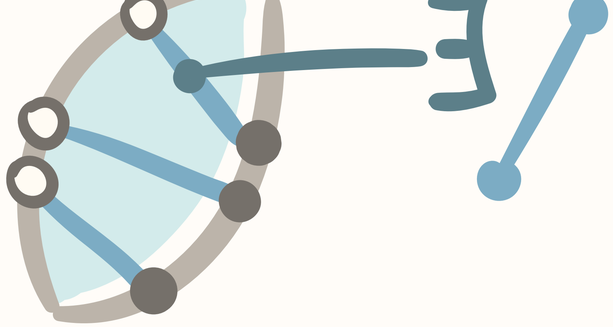
when x,y are odd

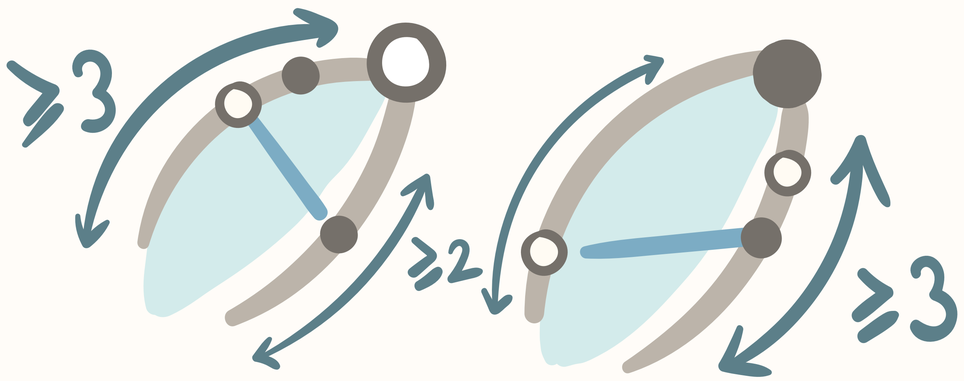
Schnyder labellings KMSW

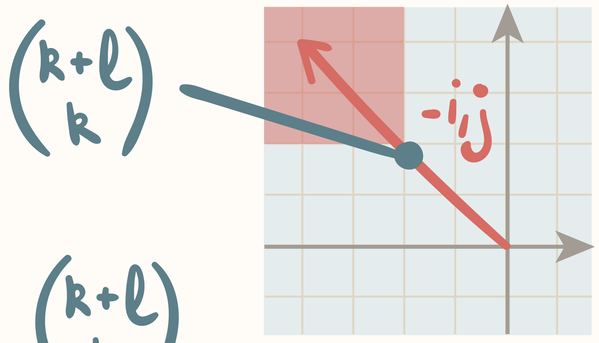






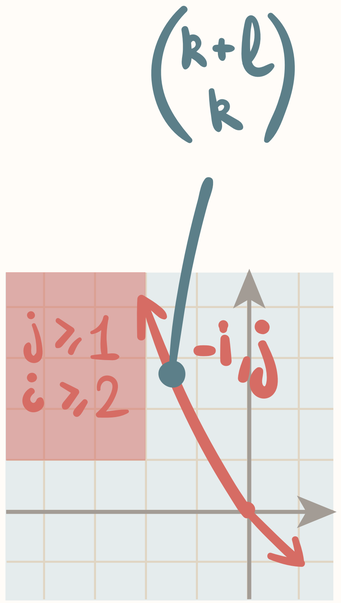






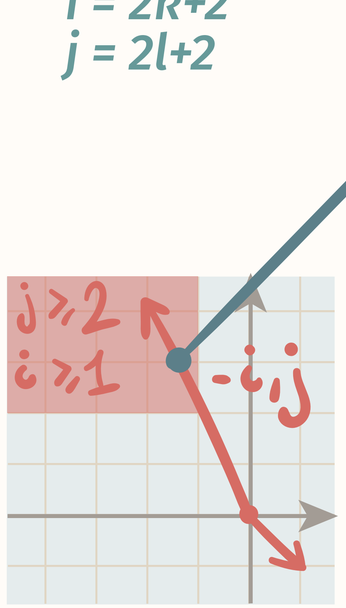
regarding x,y parity

with
i = 2k+2
j = 2l+2



when x,y are even

with
i = 2k+3
j = 2l+1



when x,y are odd

with
i = 2k+1
j = 2l+3

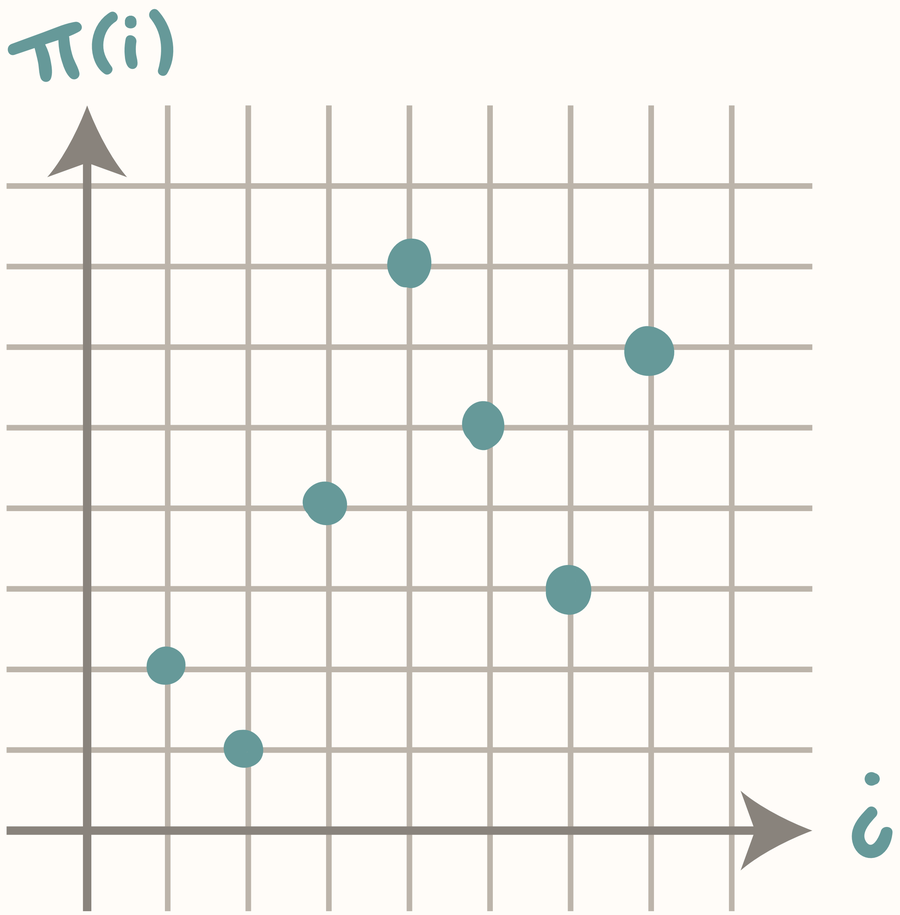
ANNEXES

A. Generalization for transversal structures

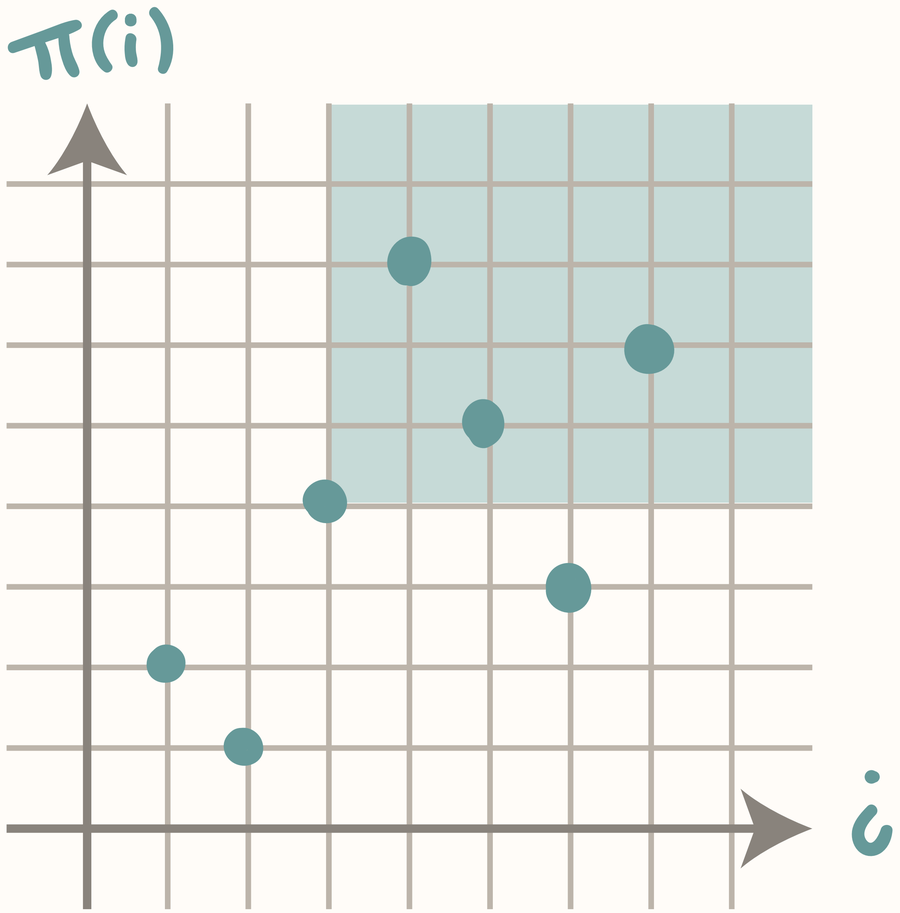
B. Schnyder labelings

C. Bijection for plane permutations

Plane permutations

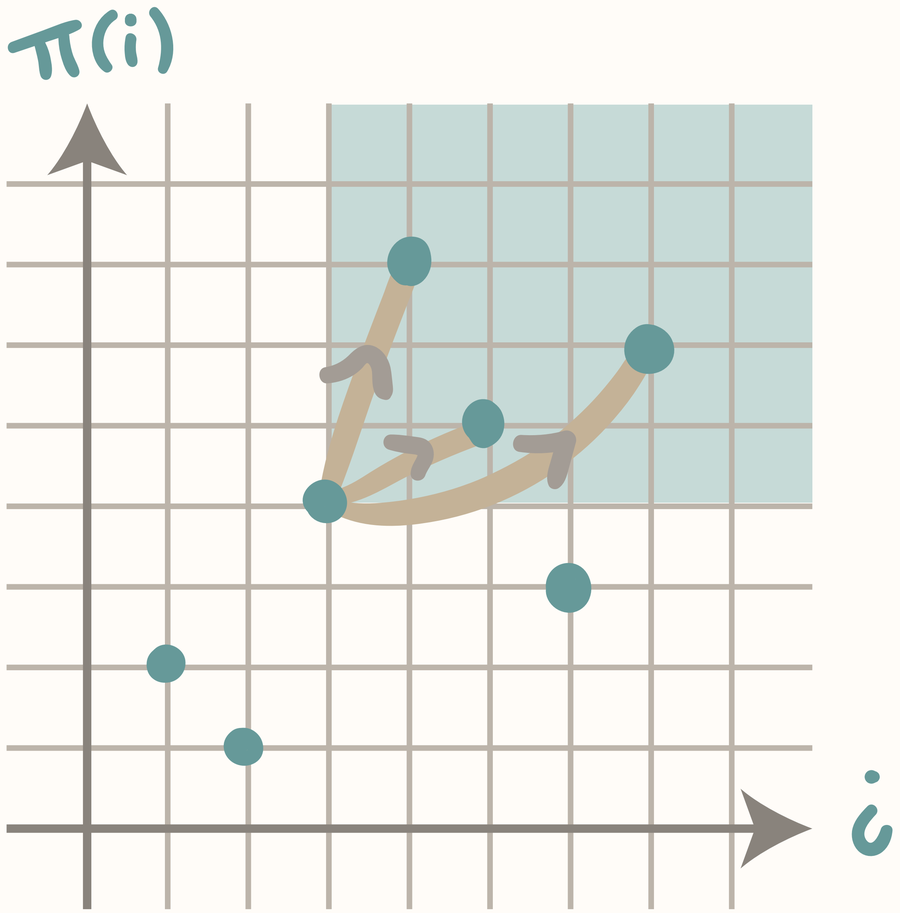


Plane permutations



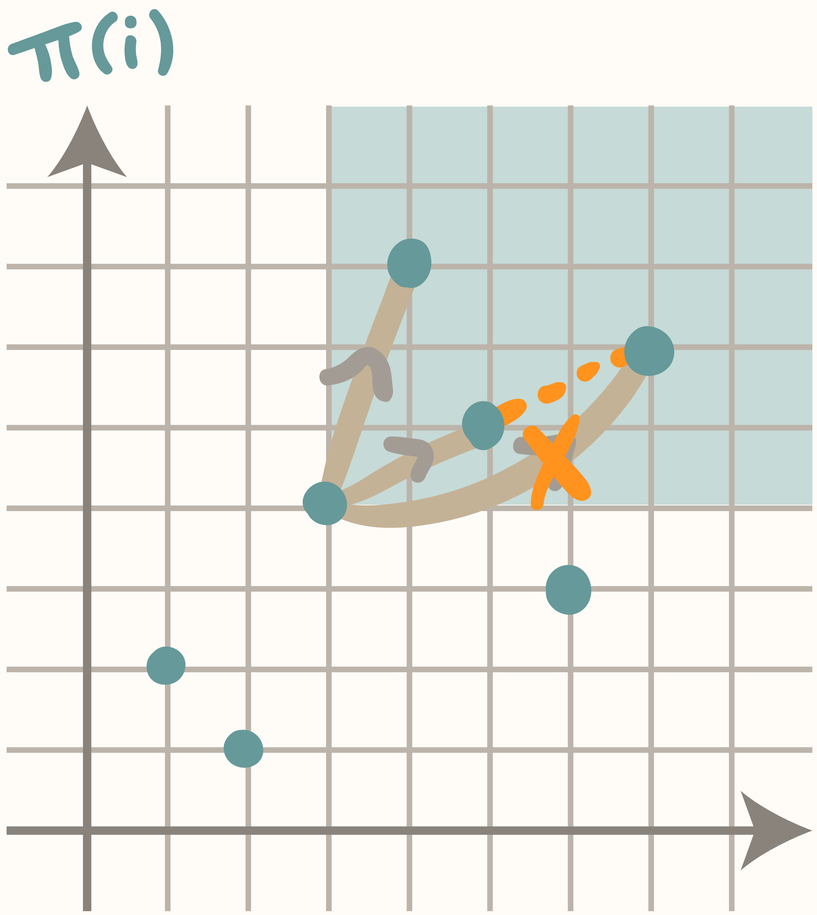
Dominance relation

Plane permutations



Dominance relation

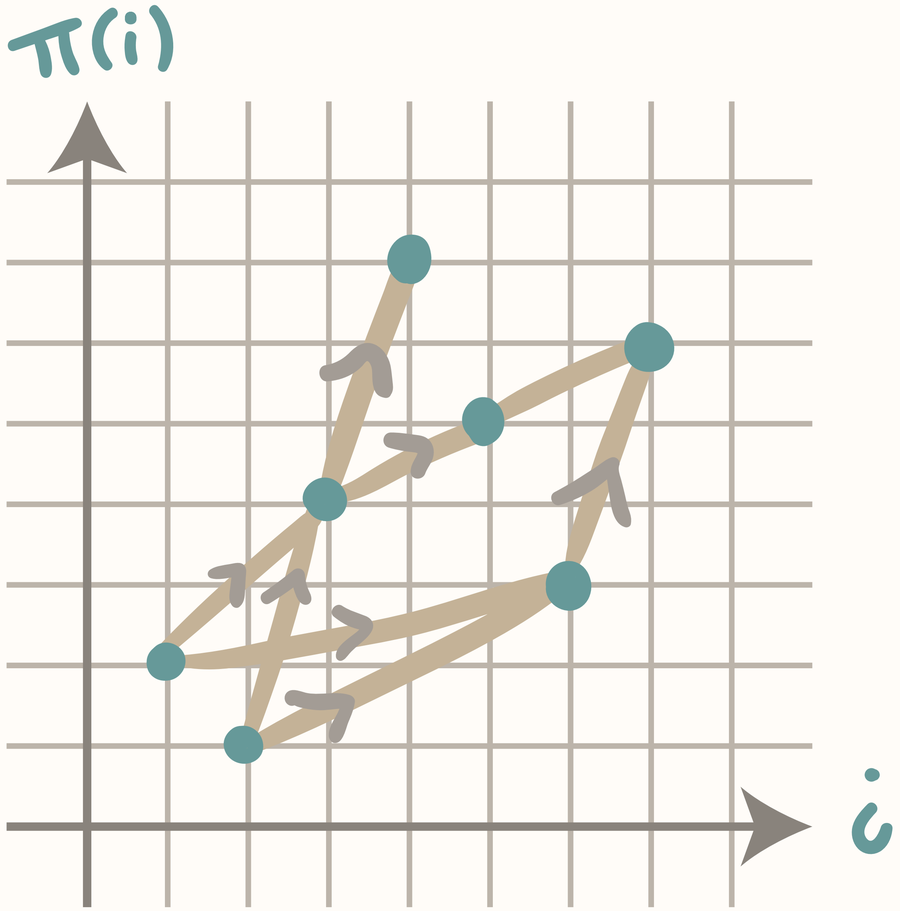
Plane permutations



Dominance diagram

= Dominance relation with no transitive edges

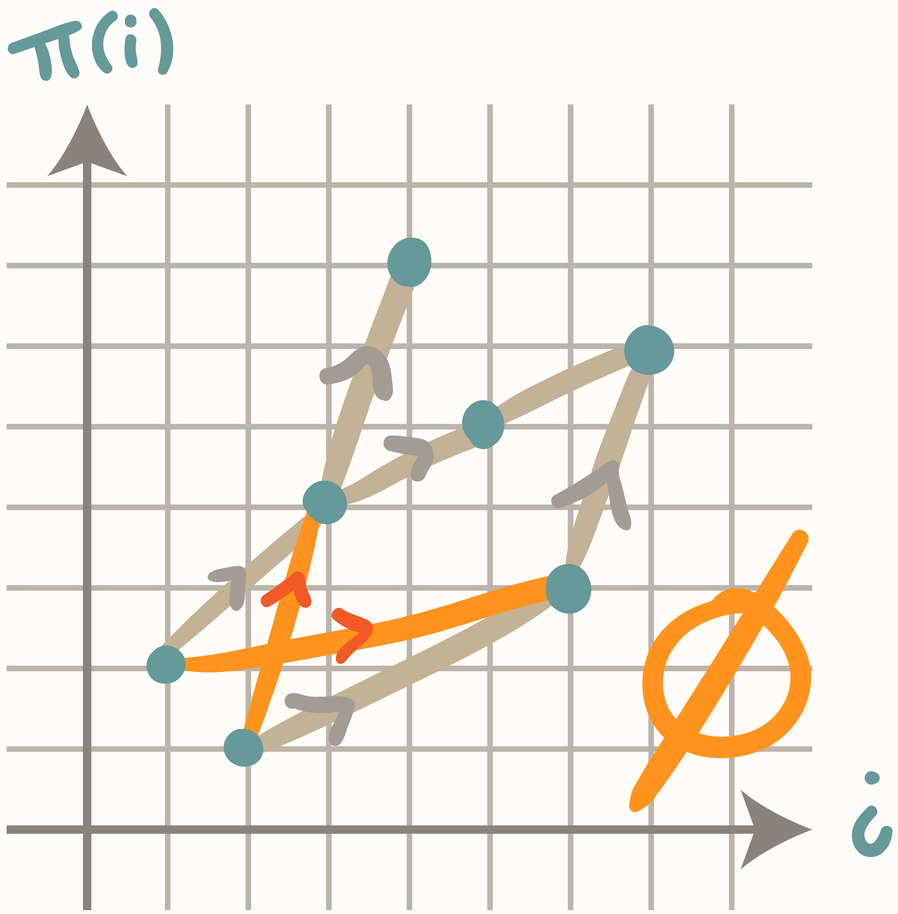
Plane permutations



Dominance diagram

= Dominance relation with no transitive edges

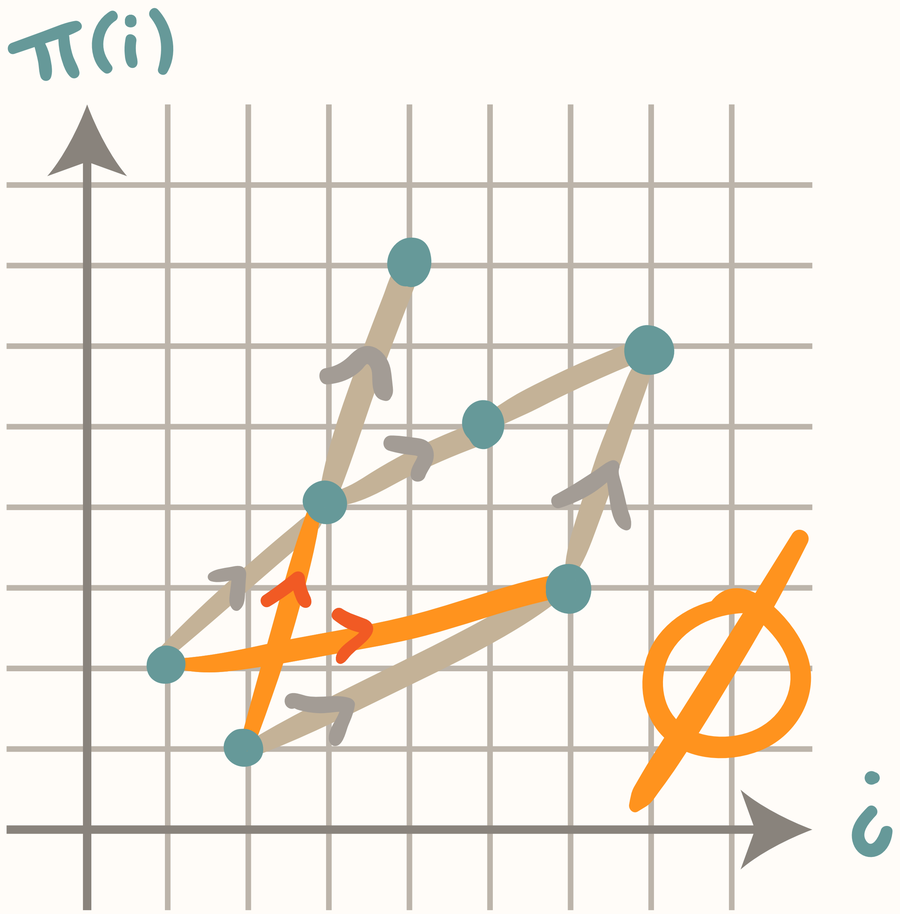
Plane permutations



Plane permutation

= No edge crossing in the dominance diagram

Plane permutations

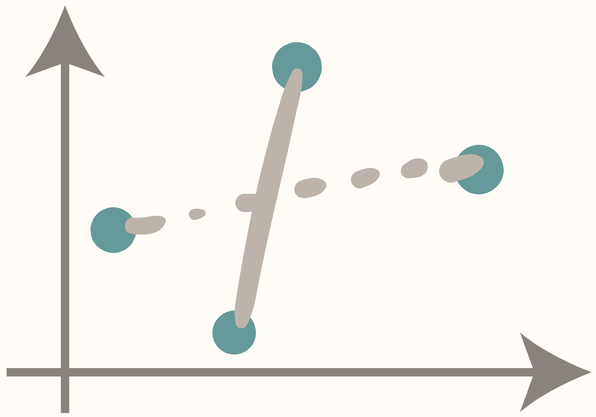


Plane permutation

= No edge crossing in the dominance diagram

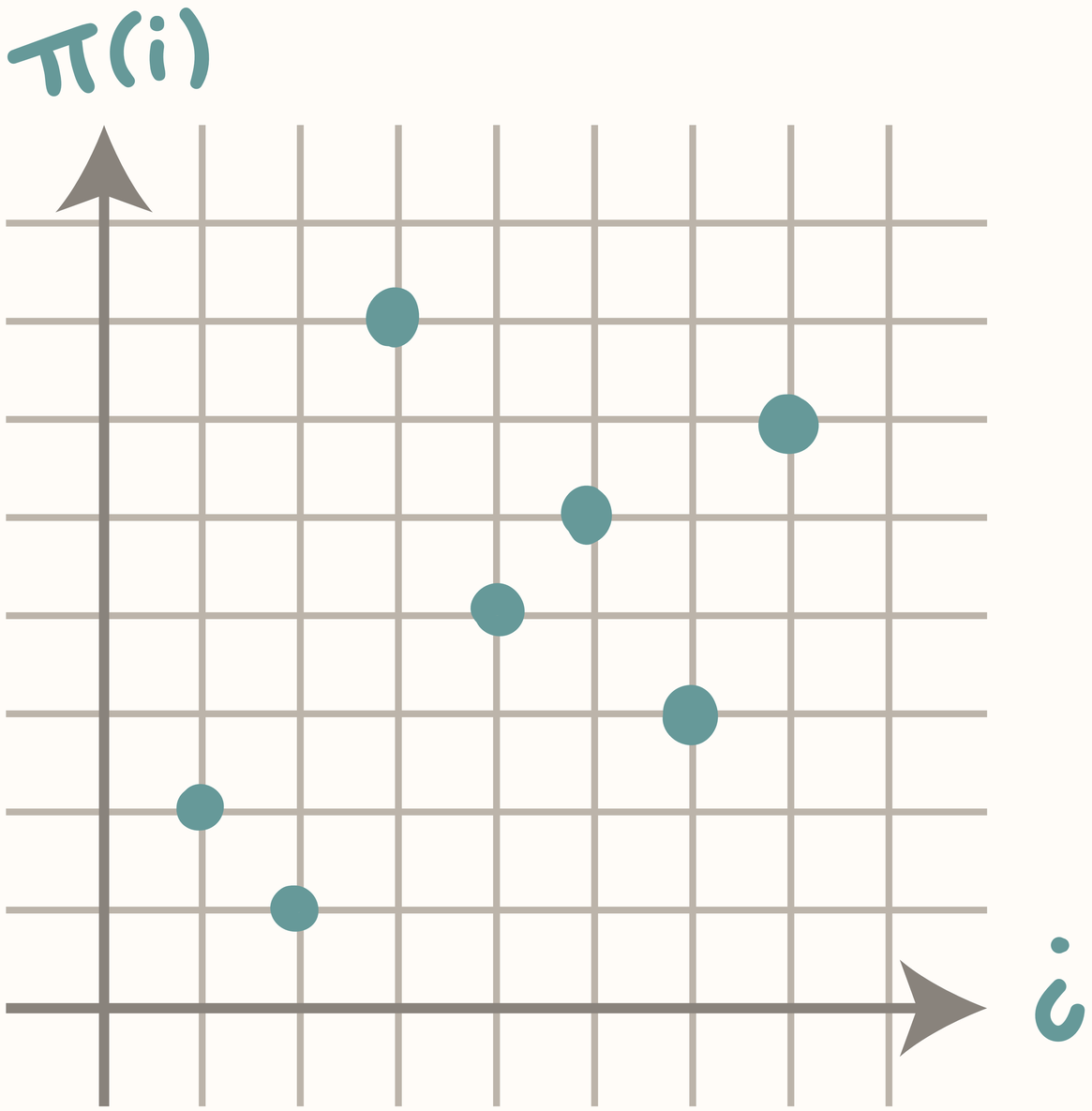
= Avoid the vincular pattern :

2 1 4 3



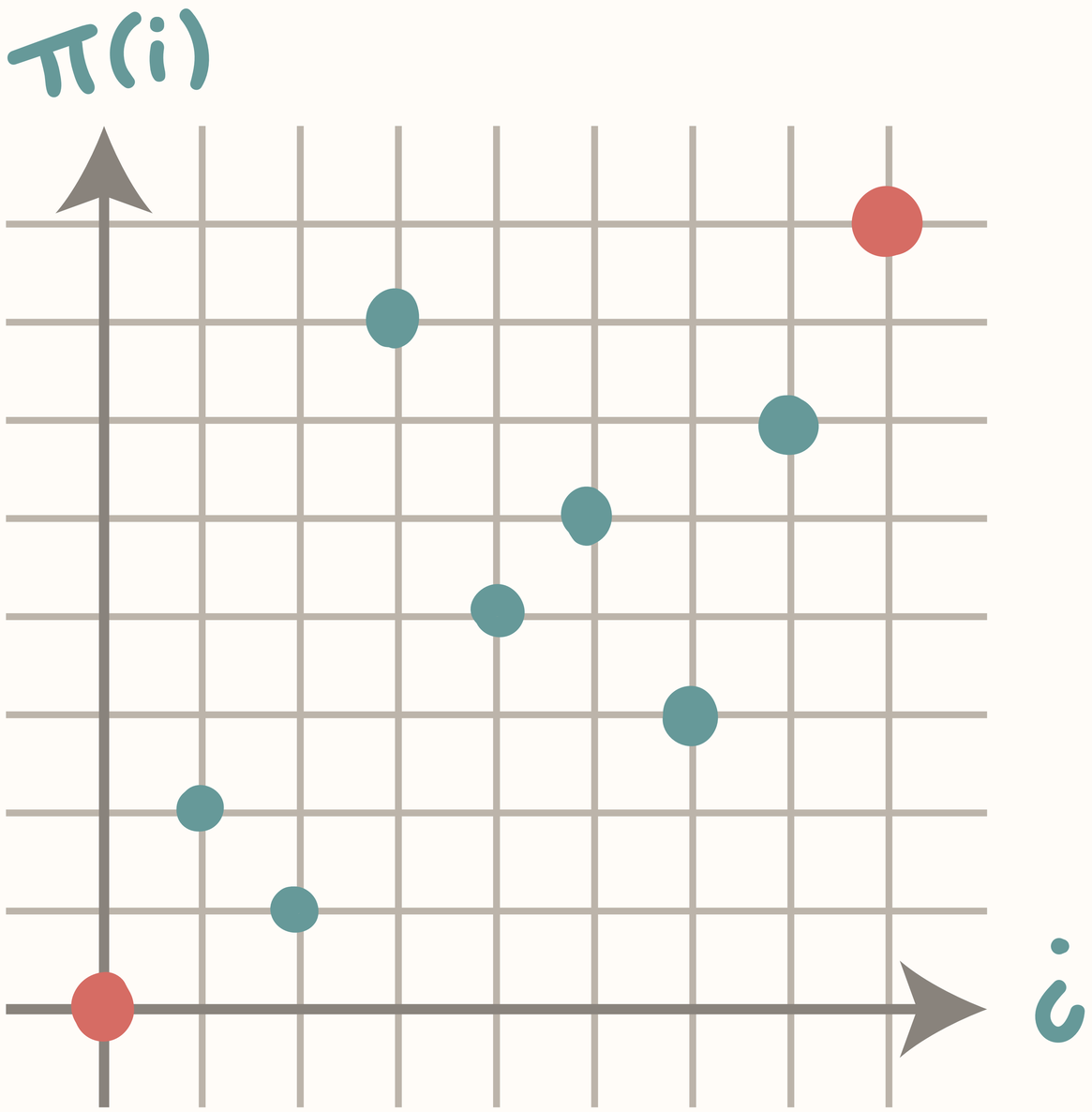
Link with plane permutations

Plane permutation $\longrightarrow$ *Poset*



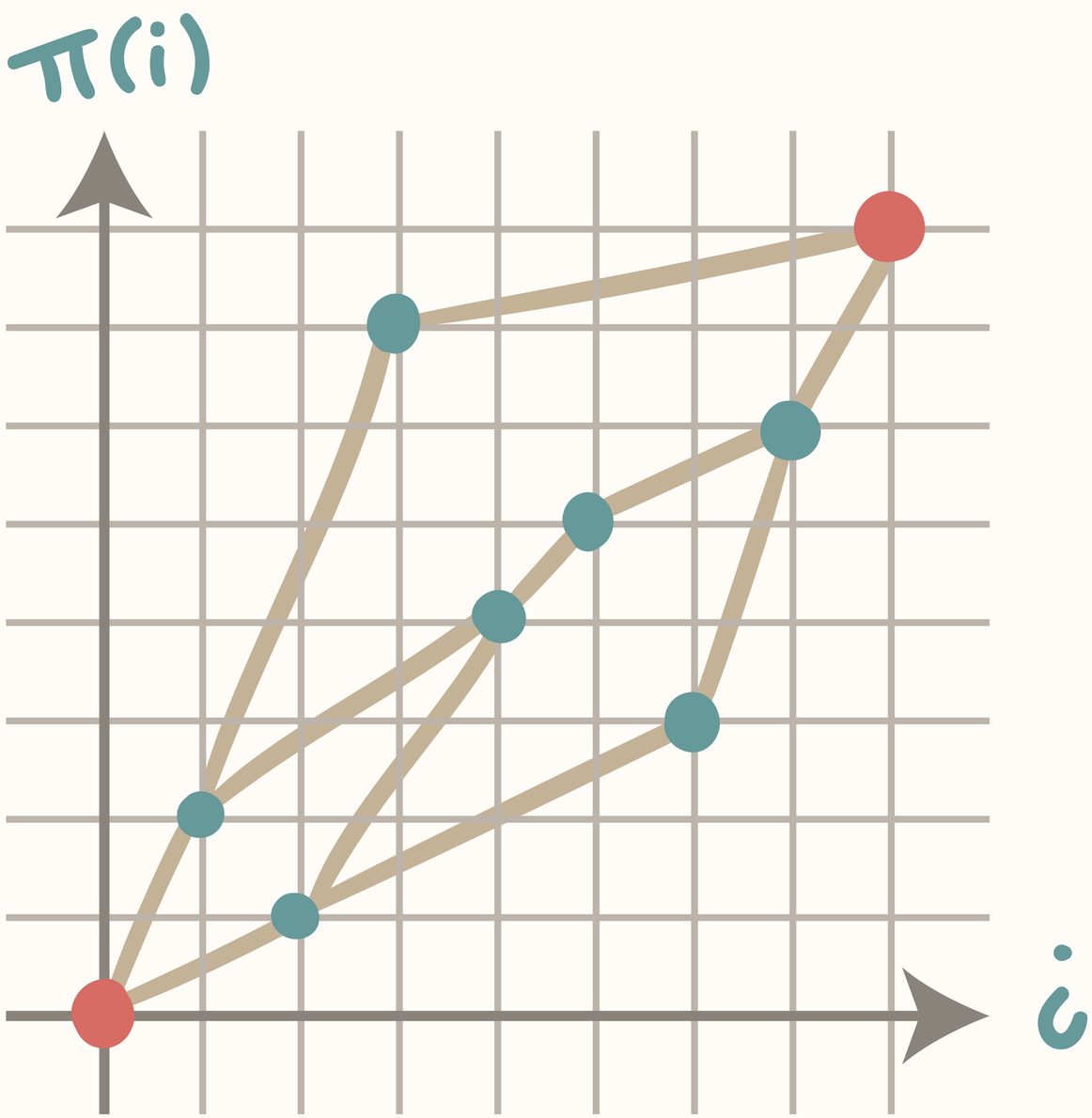
Link with plane permutations

Plane permutation $\longrightarrow$ *Poset*



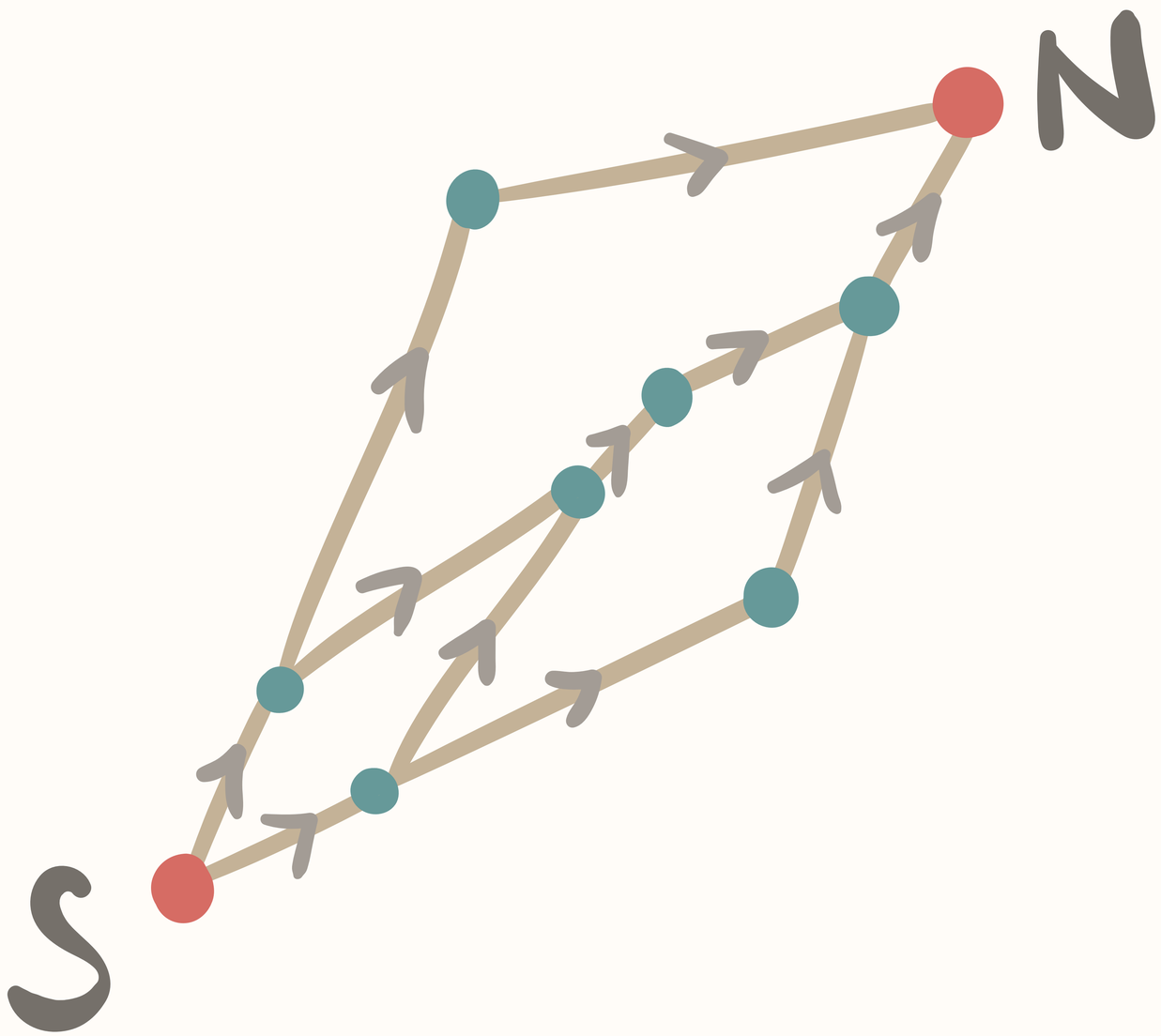
Link with plane permutations

Plane permutation $\longrightarrow$ *Poset*



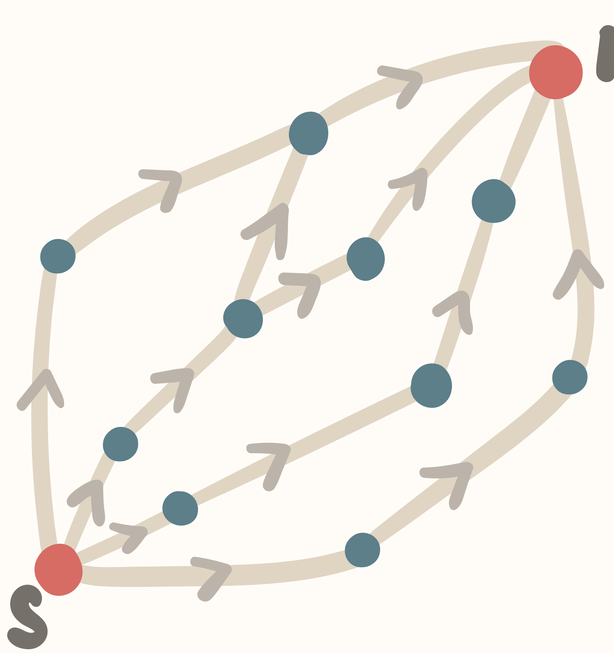
Link with plane permutations

Plane permutation $\longrightarrow$ *Poset*



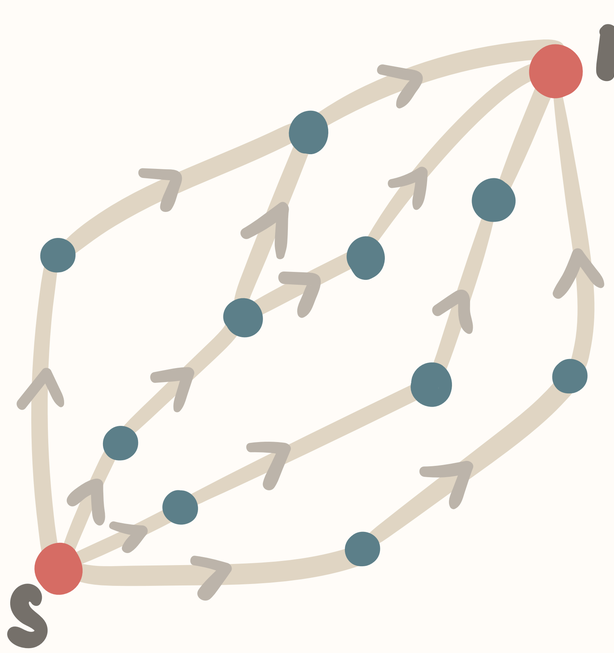
Link with plane permutations

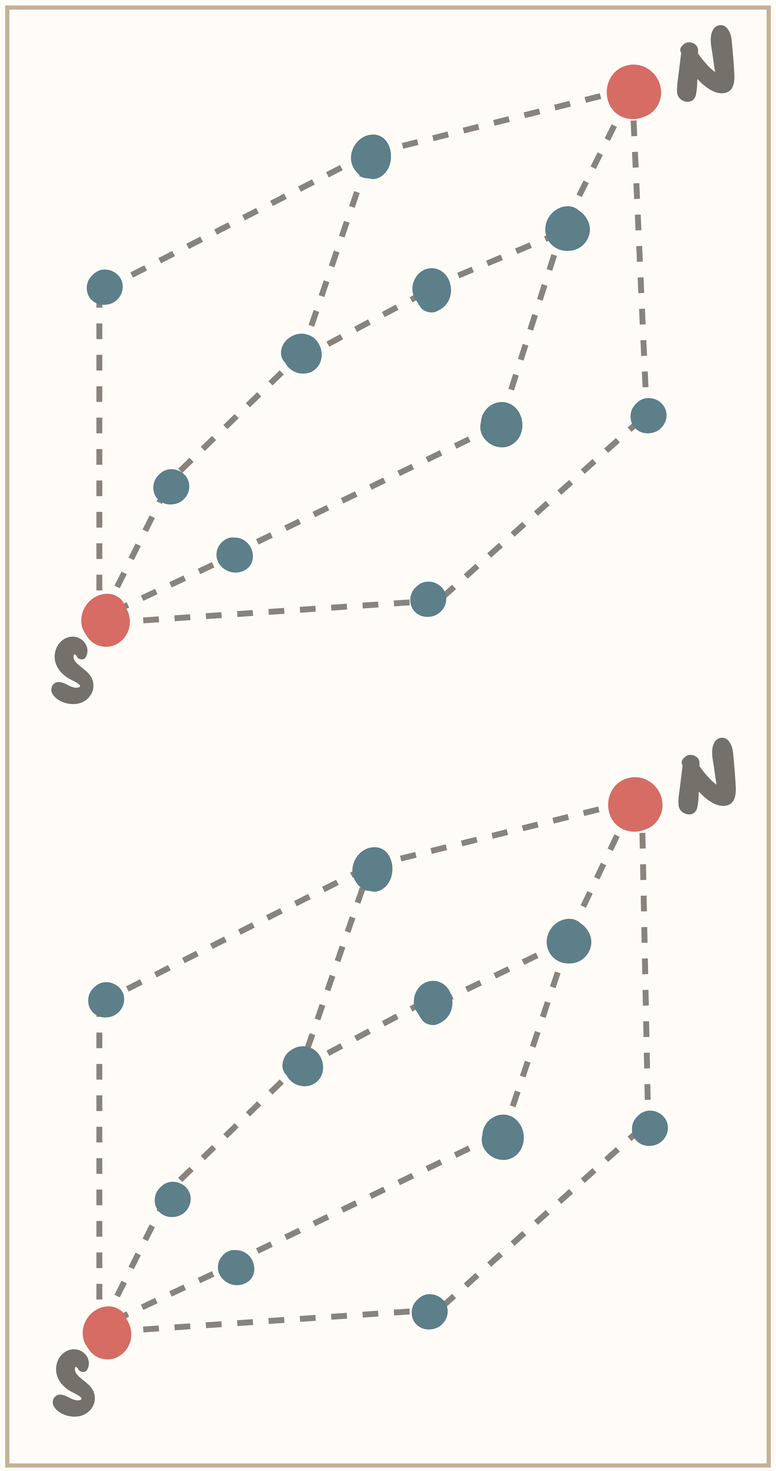
Poset $\longrightarrow$ *Plane permutation*



Link with plane permutations

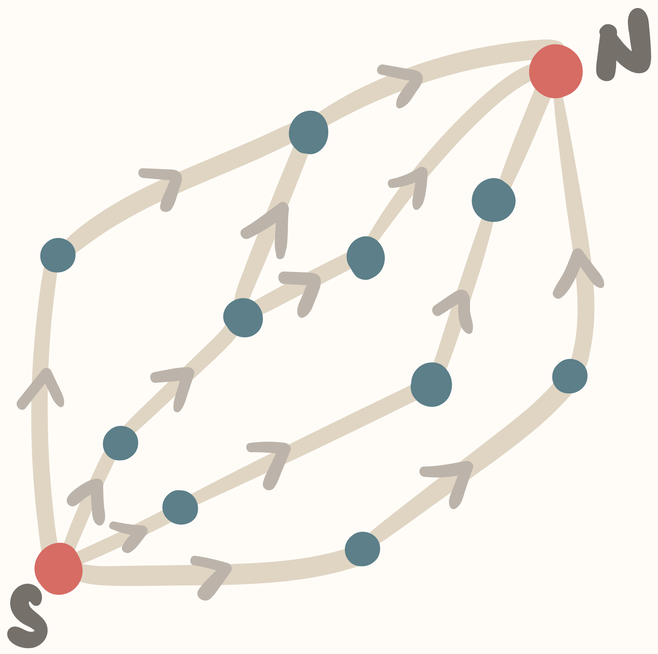
Poset ⟶ *Plane permutation*

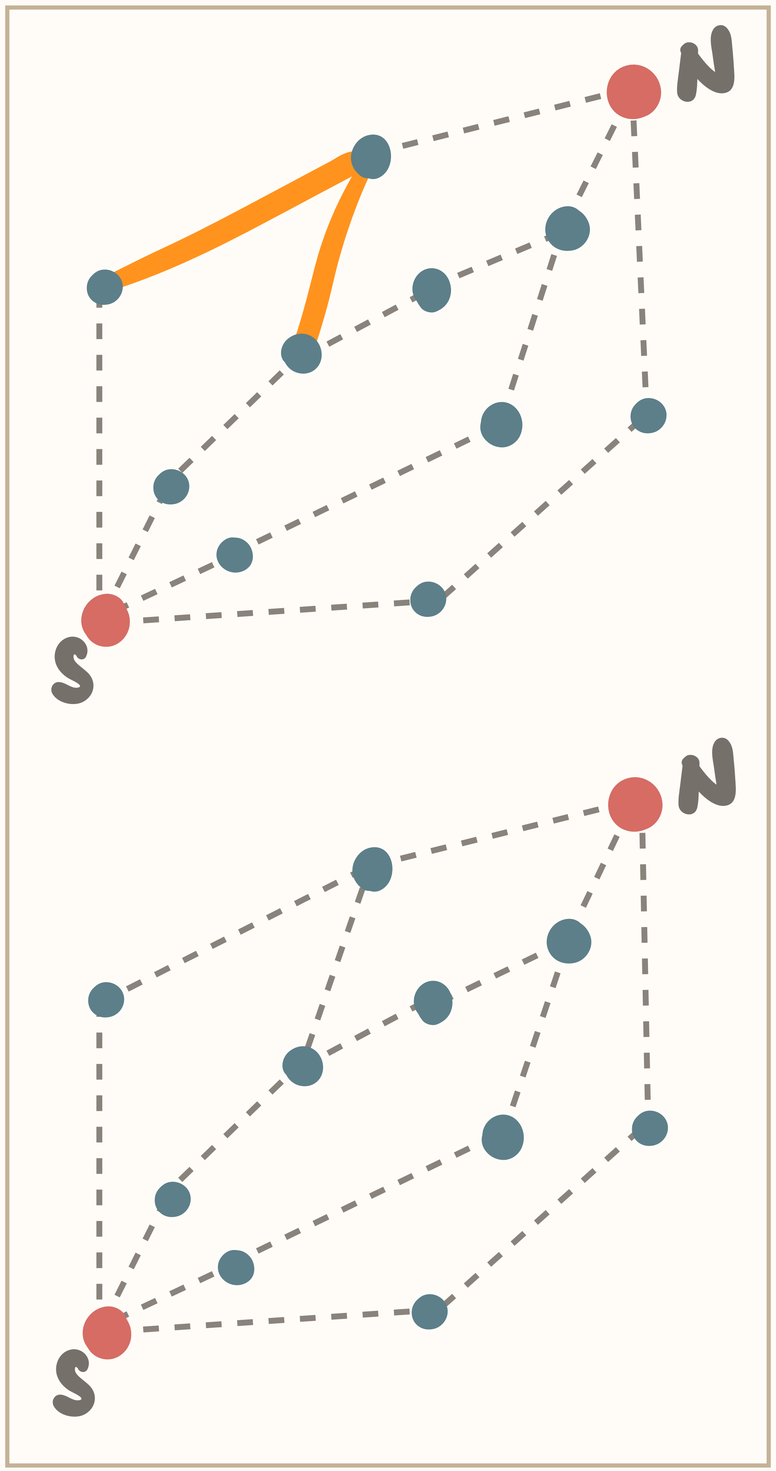




Link with plane permutations

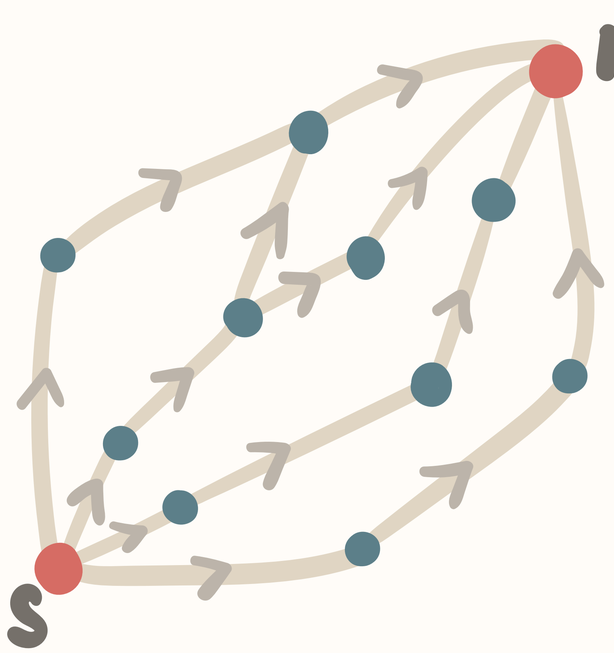
Poset ⟶ *Plane permutation*

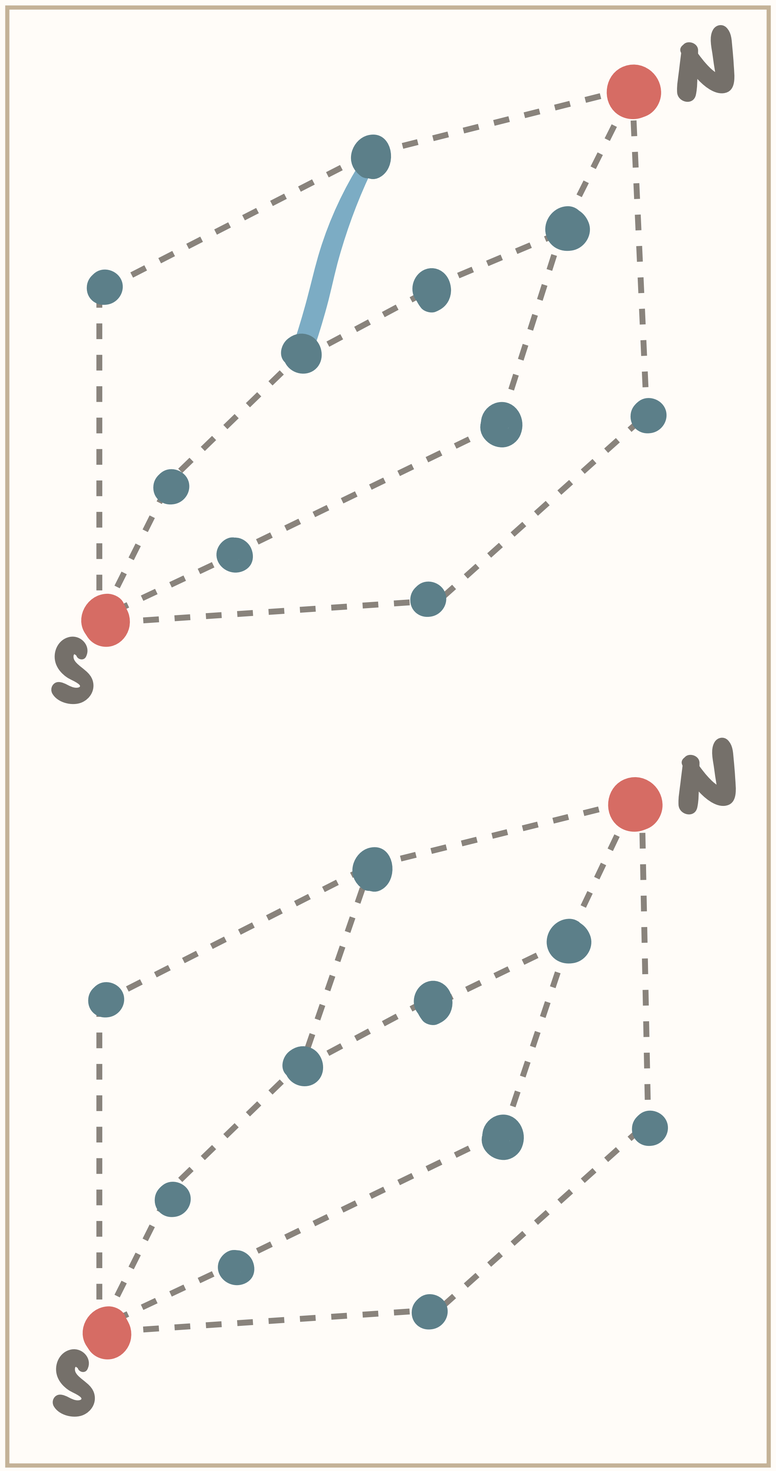




Link with plane permutations

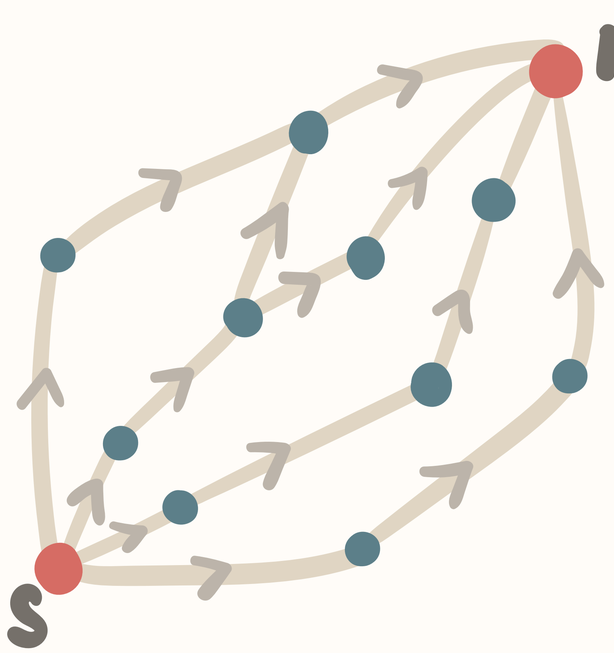
Poset ⟶ *Plane permutation*

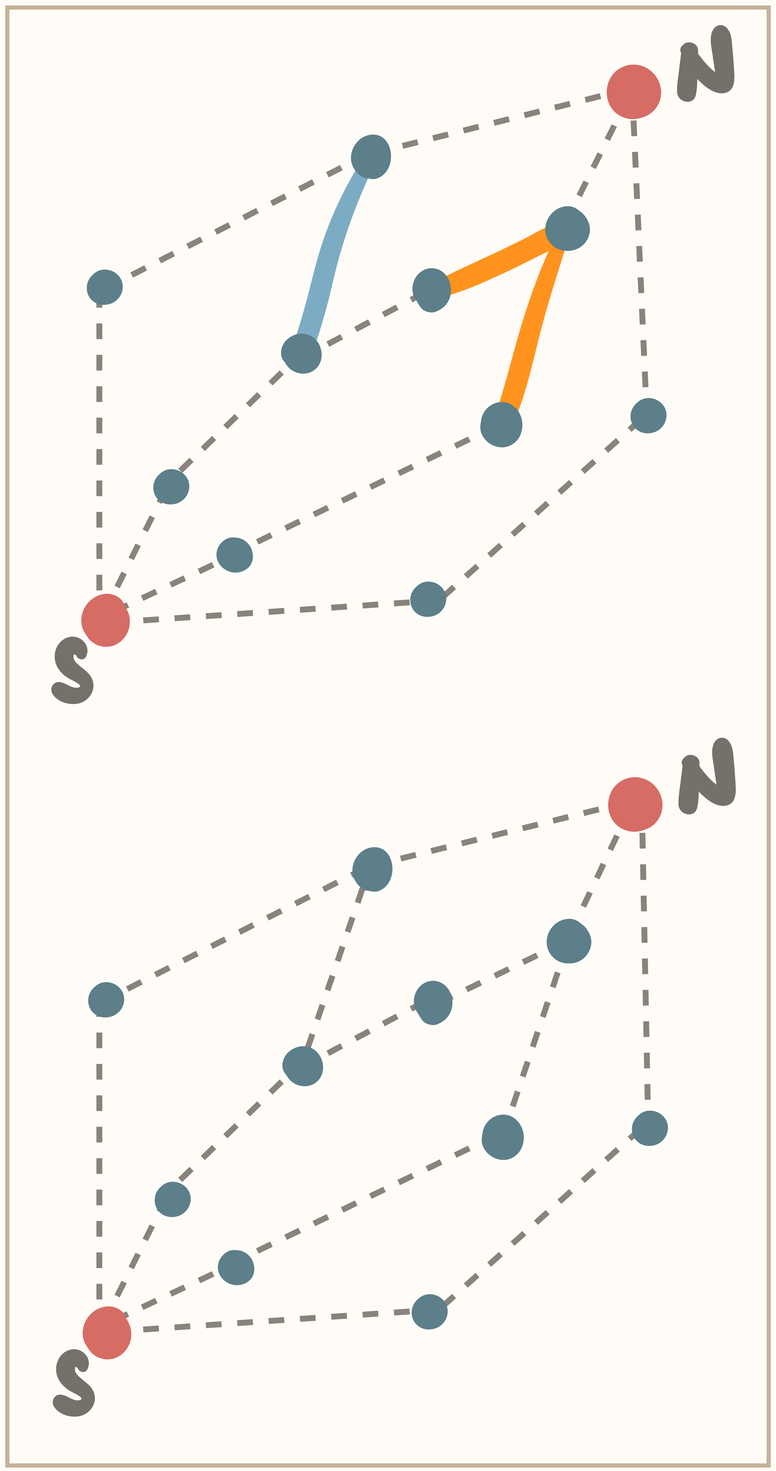




Link with plane permutations

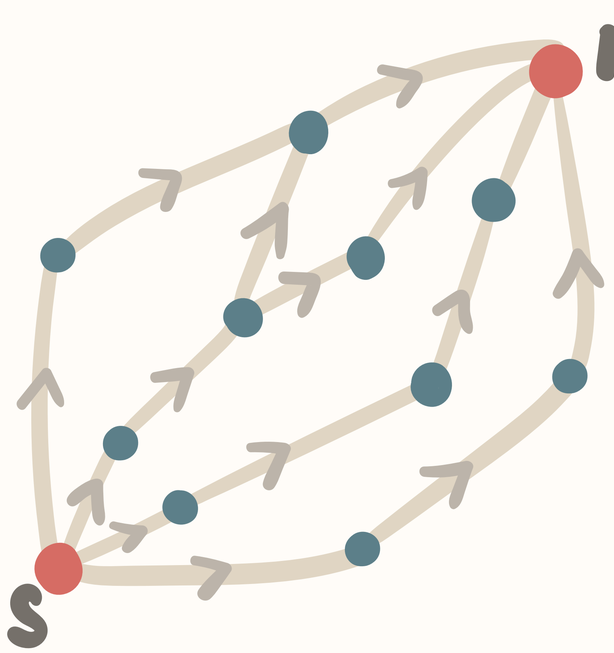
Poset ⟶ *Plane permutation*

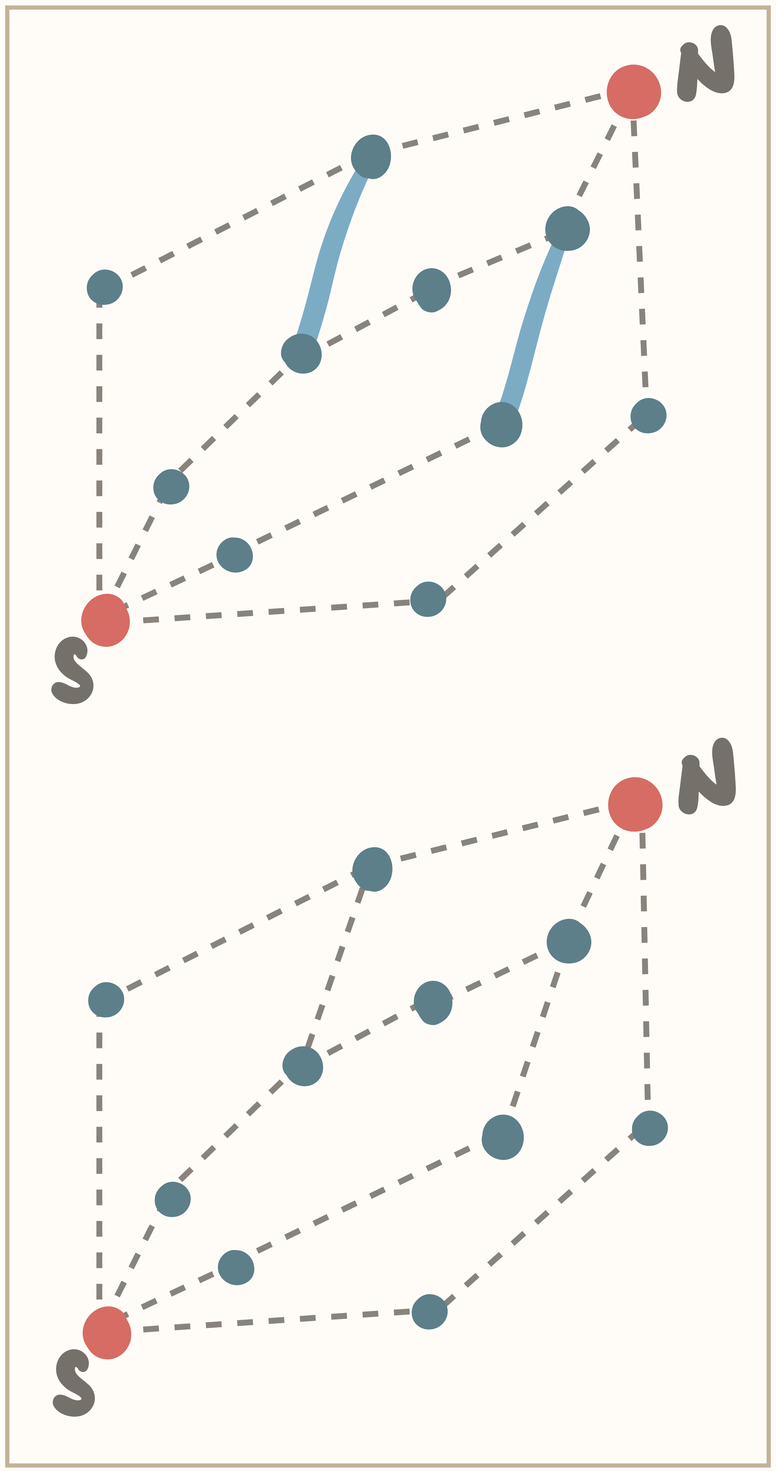




Link with plane permutations

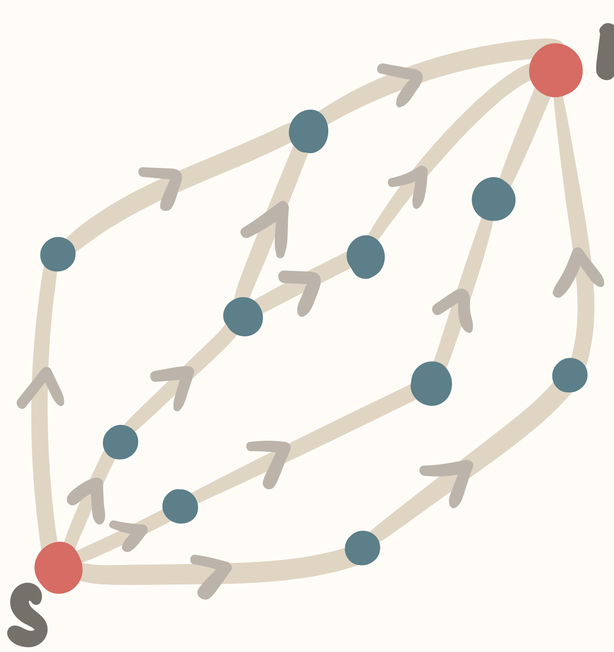
Poset ⟶ *Plane permutation*

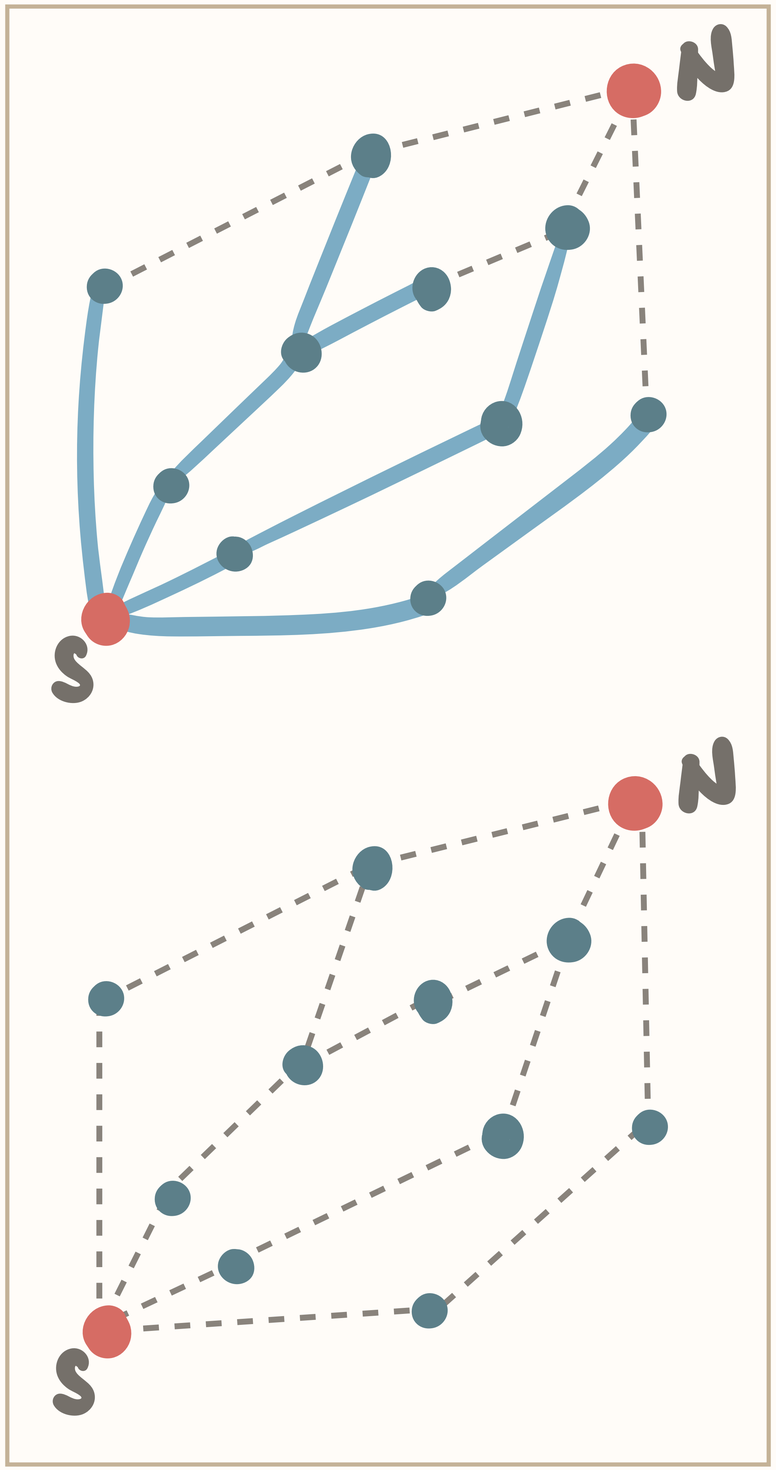




Link with plane permutations

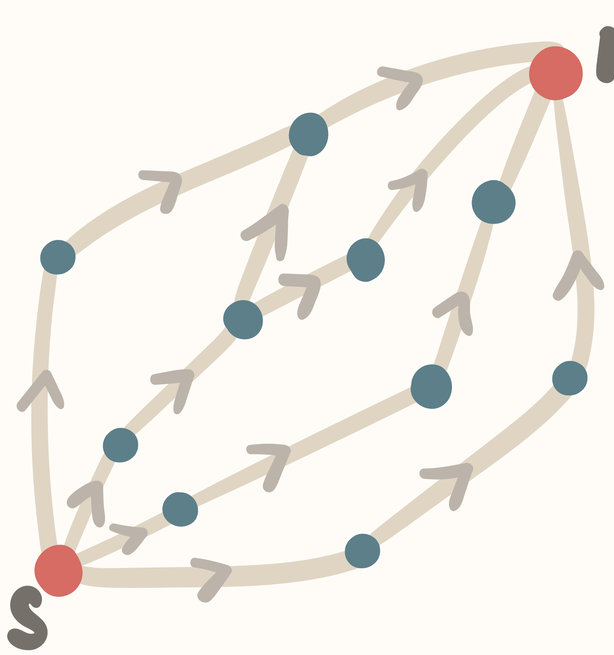
Poset $\longrightarrow$ *Plane permutation*

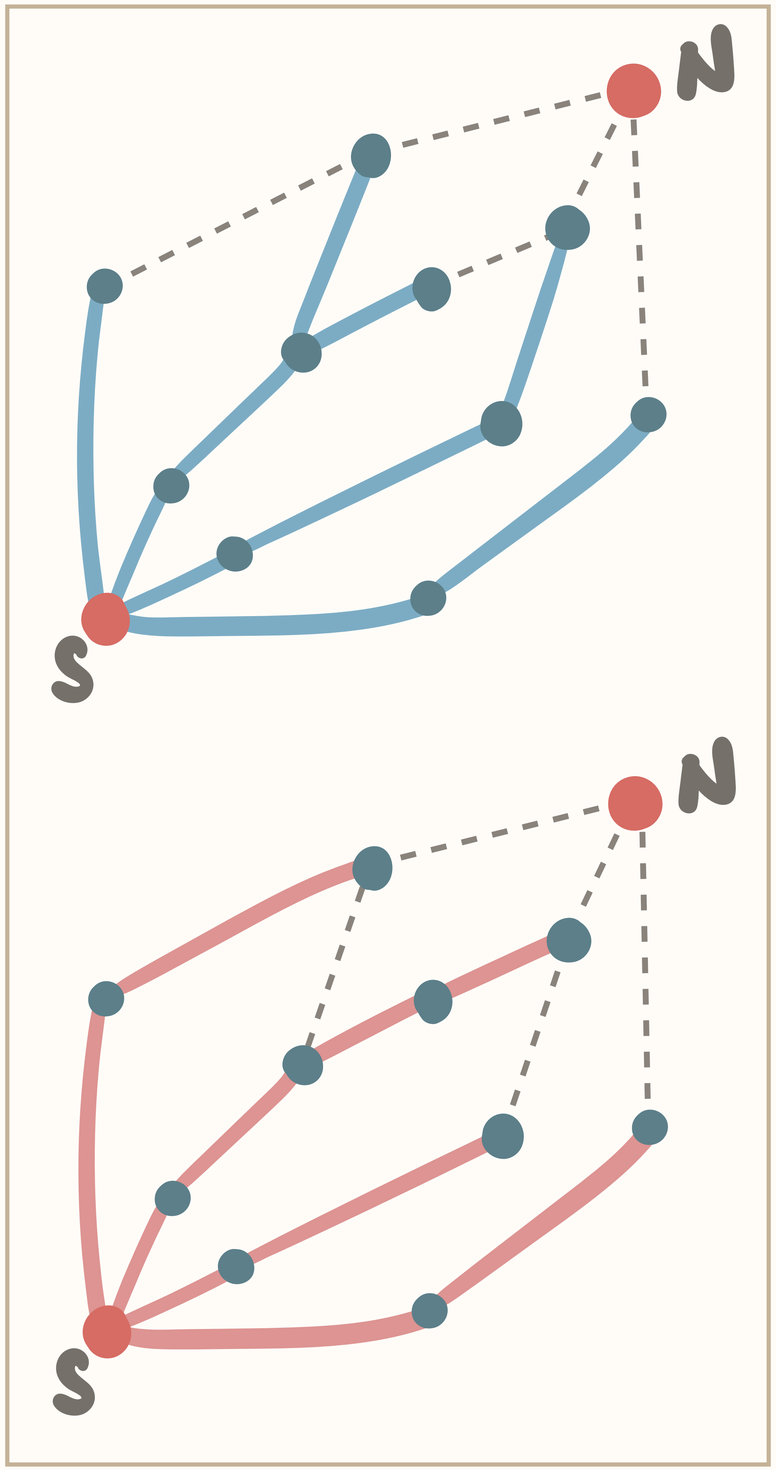




Link with plane permutations

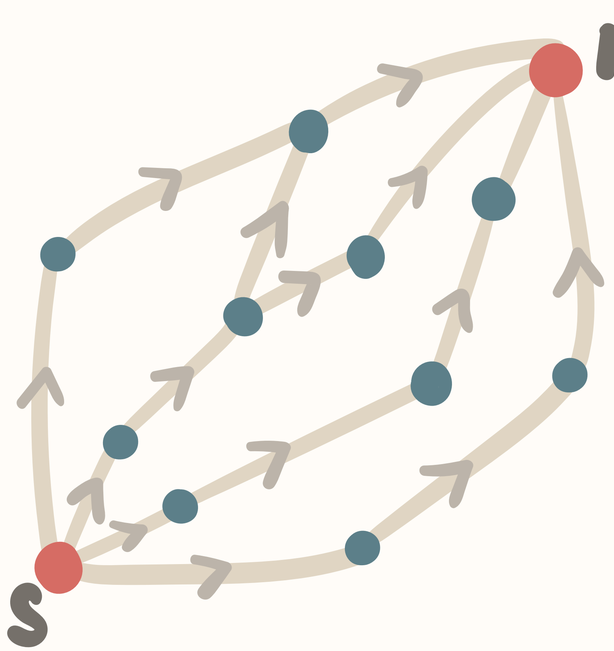
Poset $\longrightarrow$ *Plane permutation*

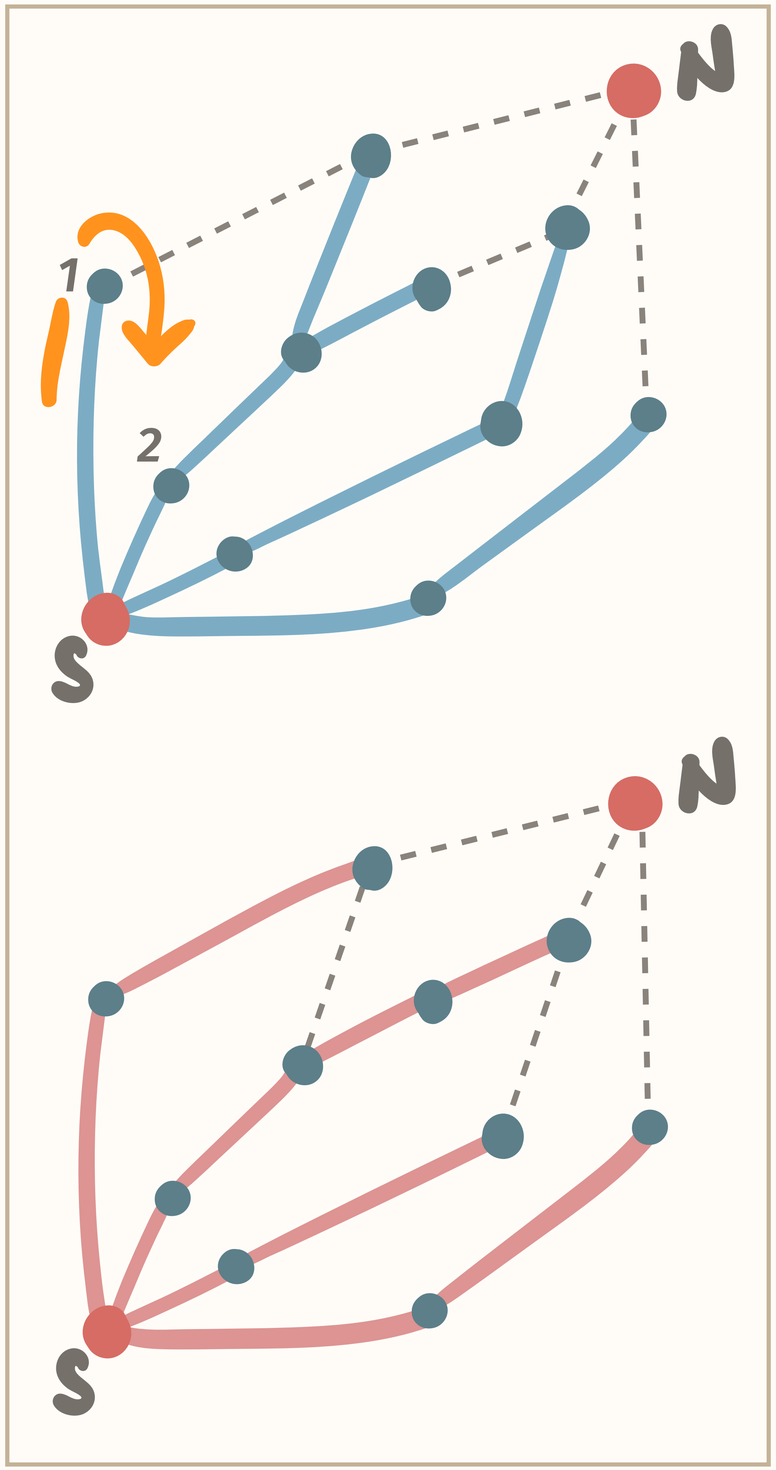




Link with plane permutations

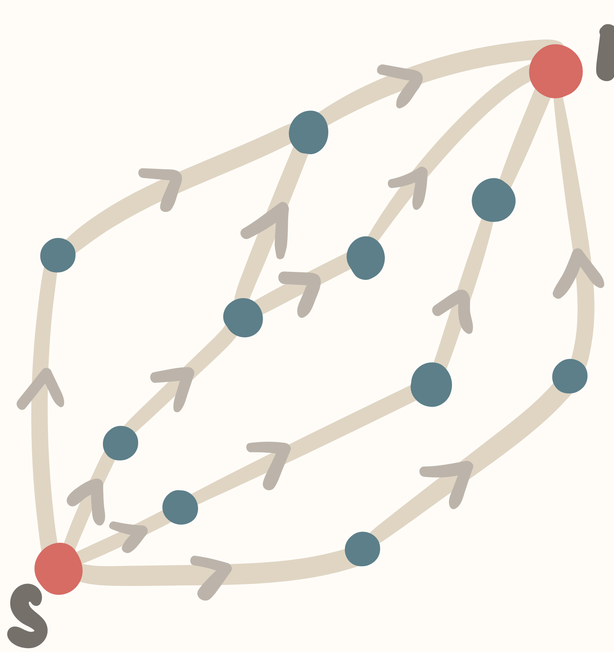
Poset ⟶ *Plane permutation*

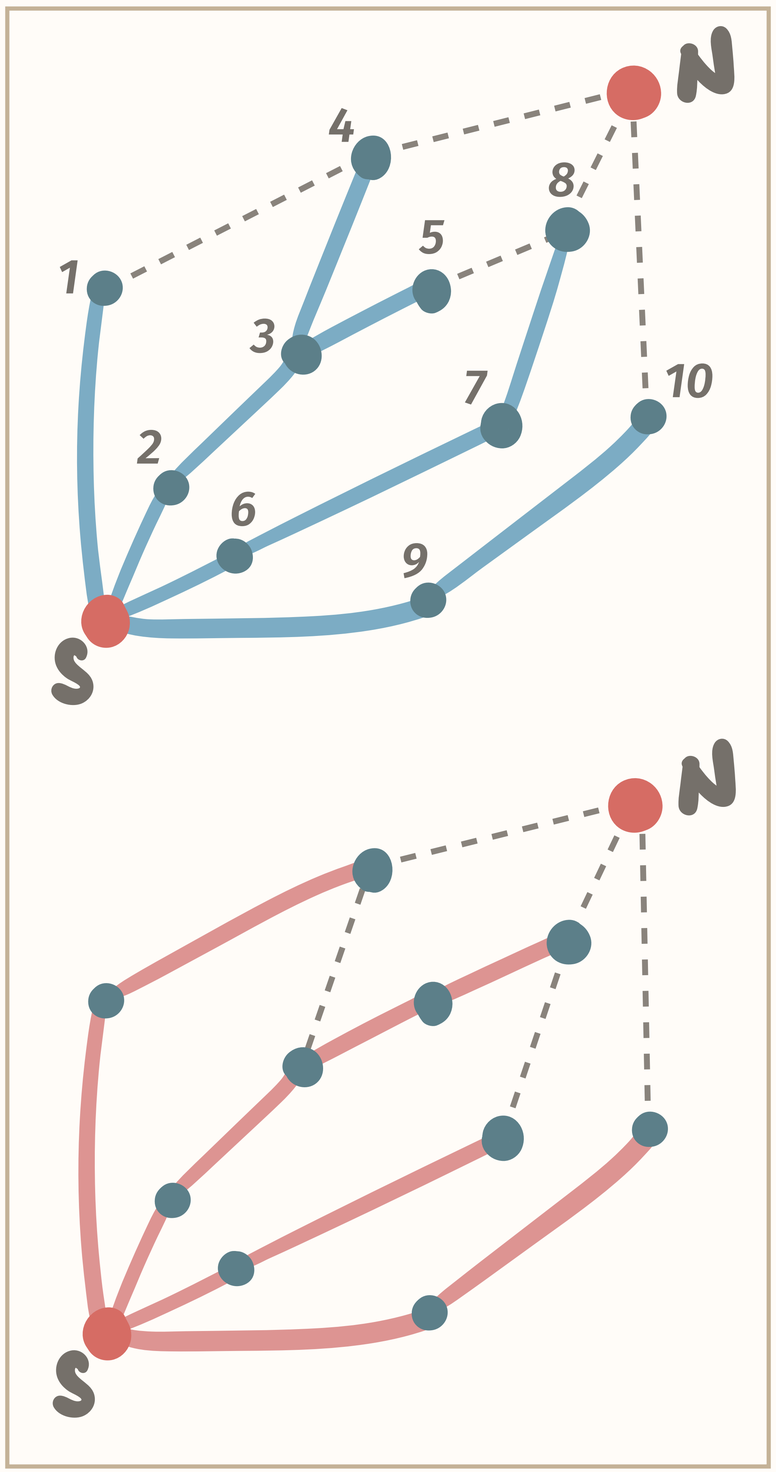




Link with plane permutations

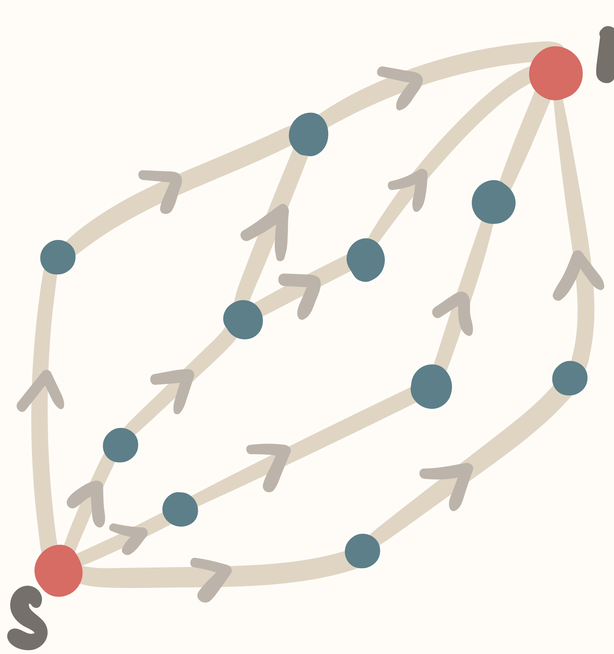
Poset ⟶ *Plane permutation*

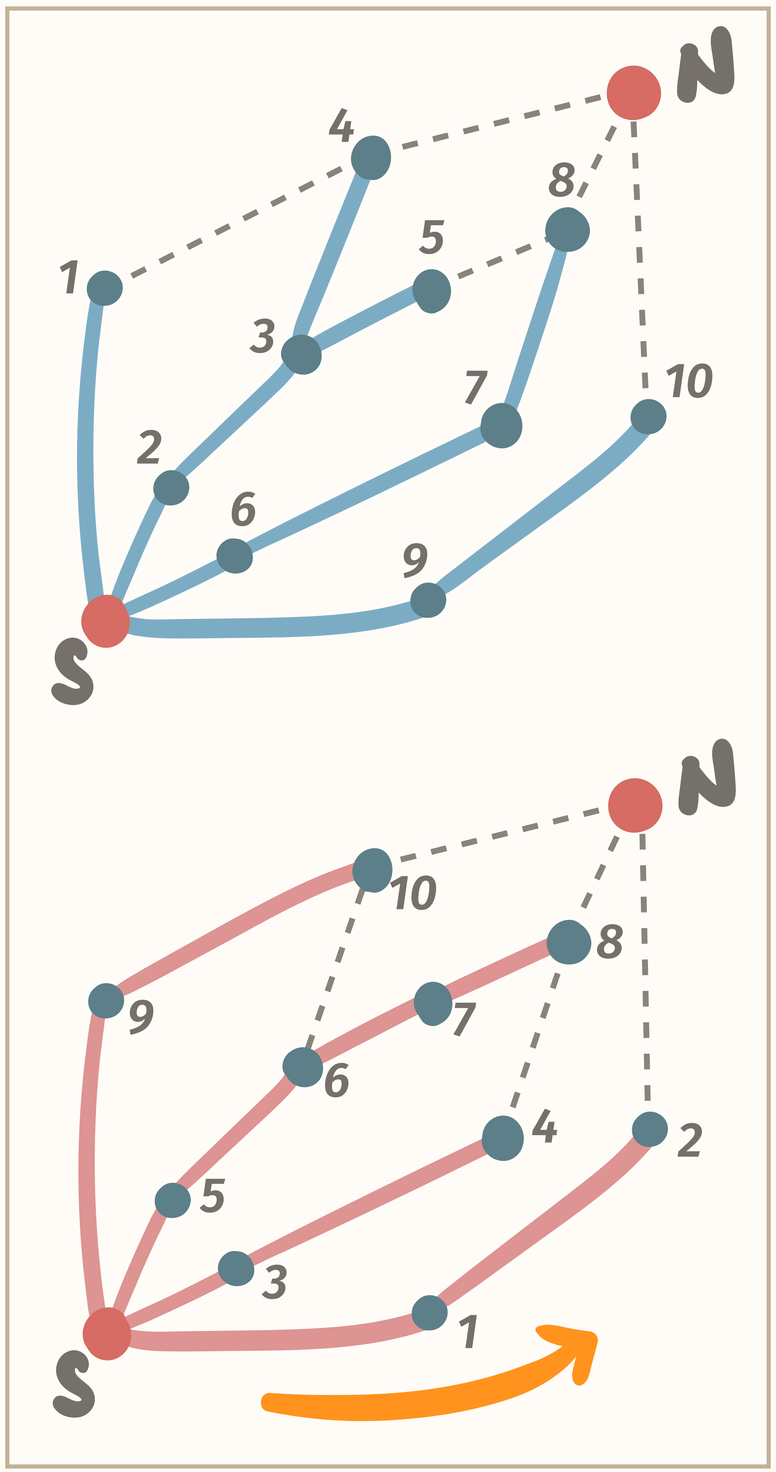




Link with plane permutations

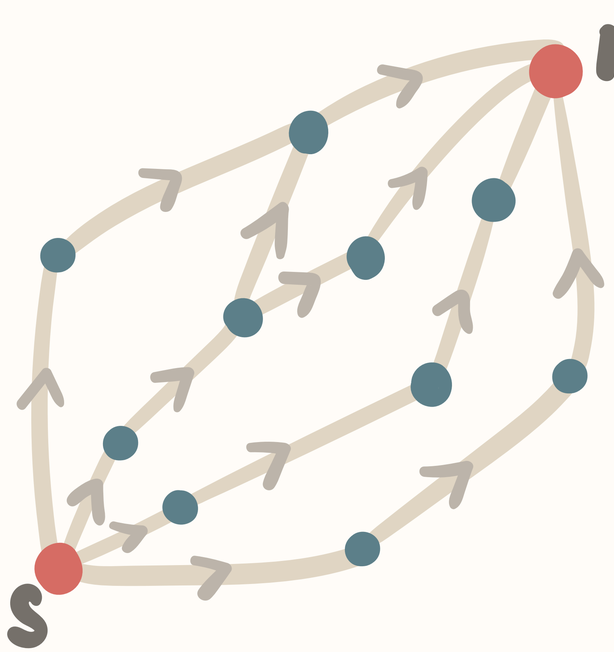
Poset ⟶ *Plane permutation*

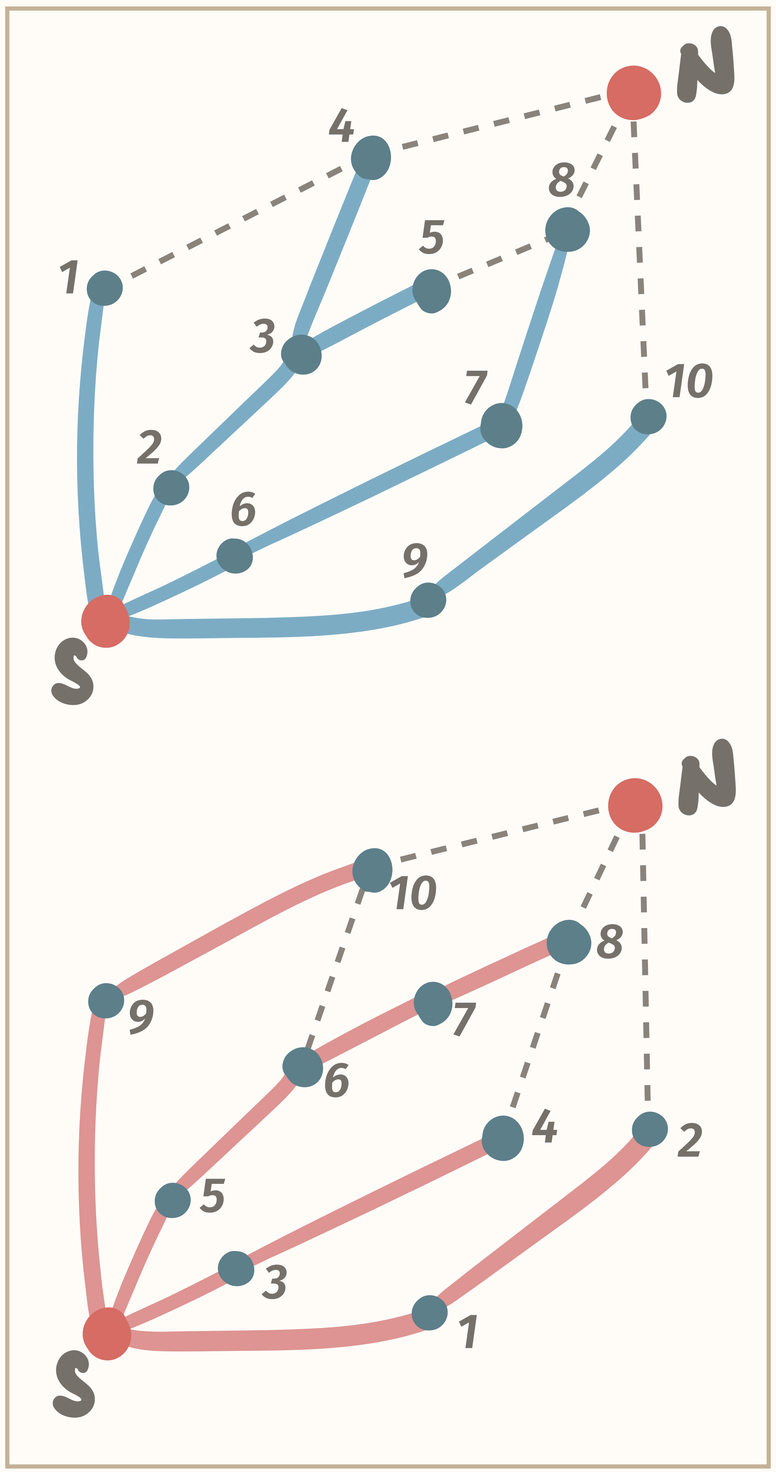


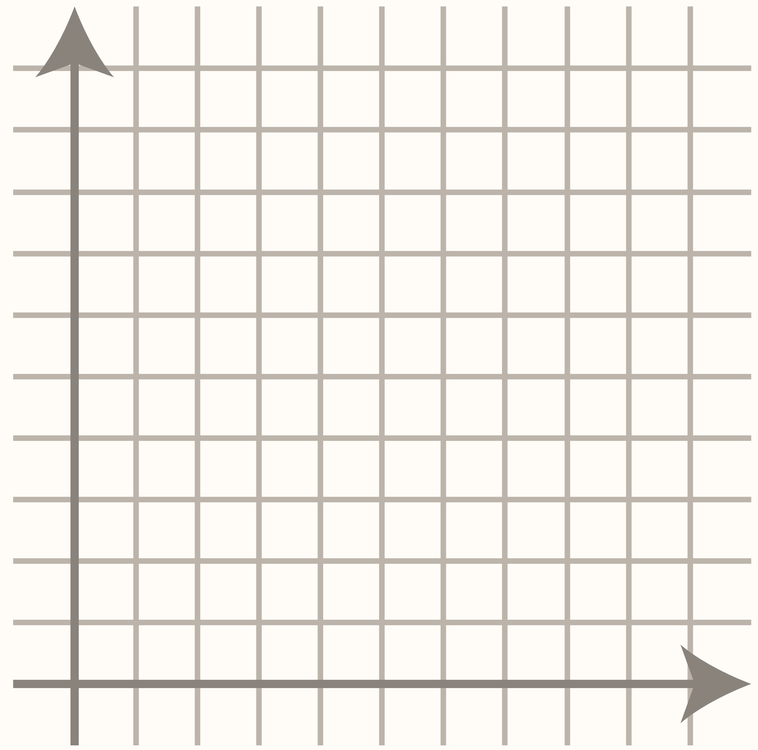


Link with plane permutations

Poset ⟶ *Plane permutation*

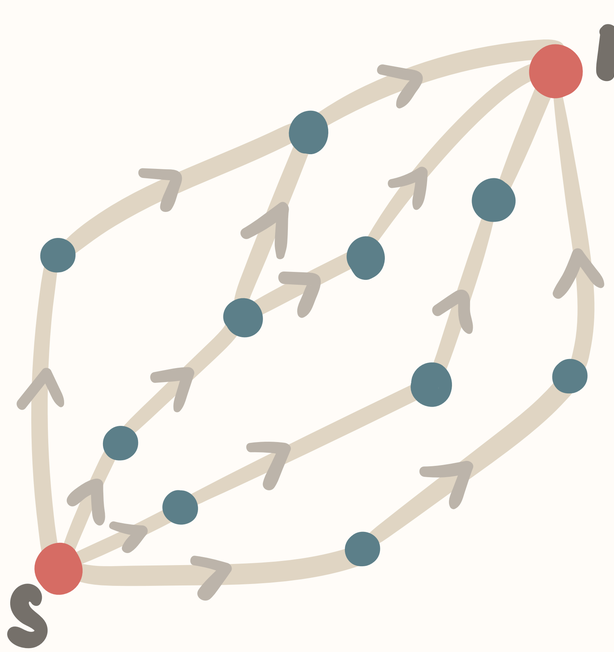


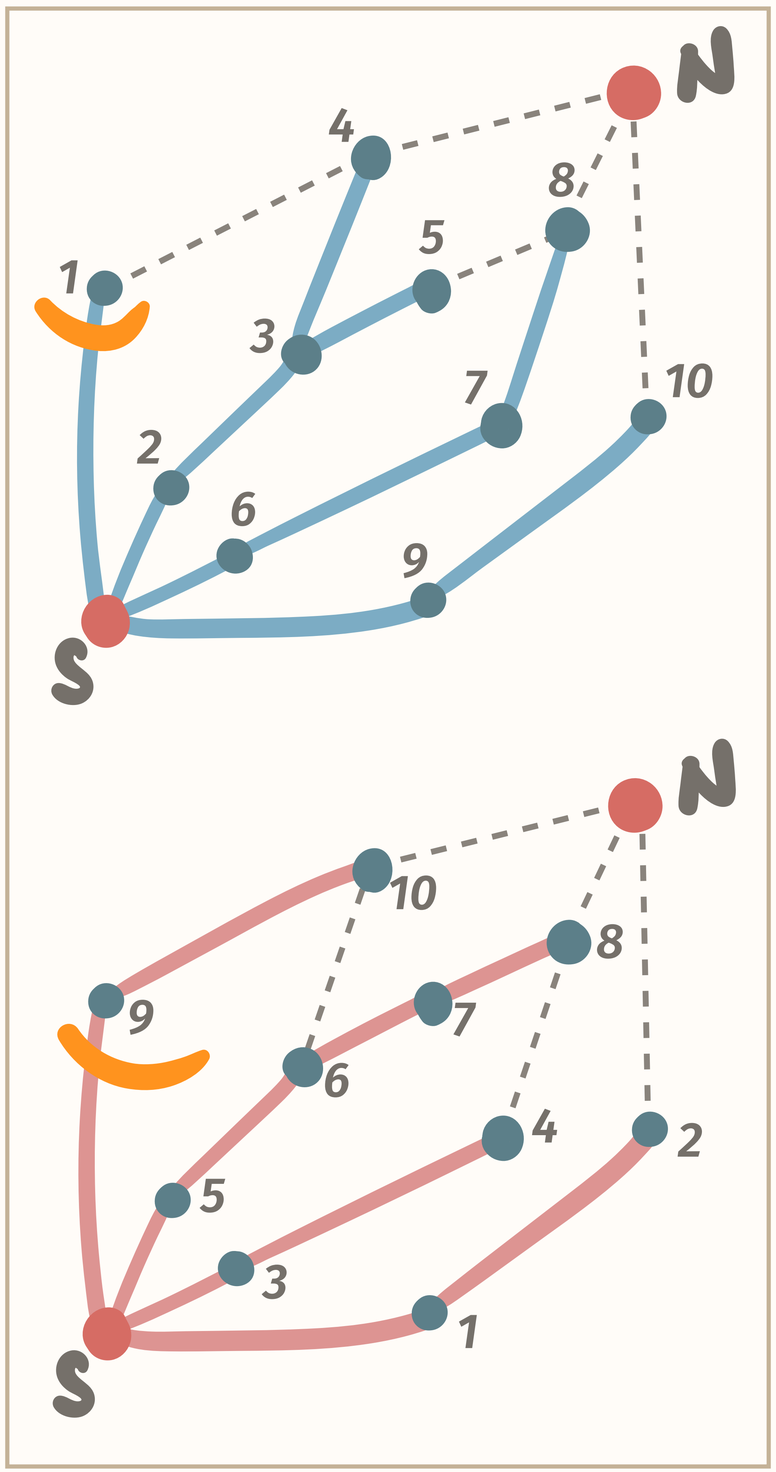


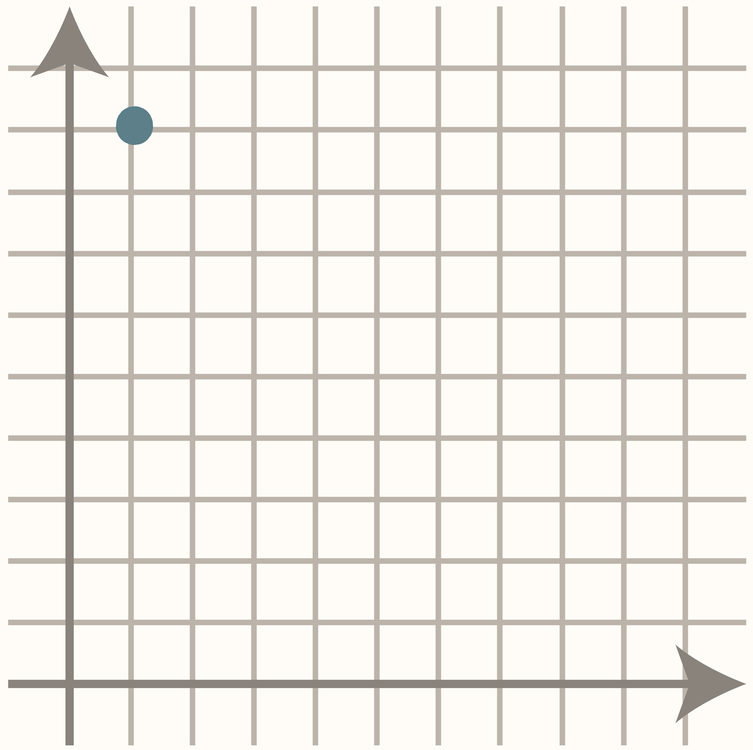


Link with plane permutations

Poset ⟶ *Plane permutation*



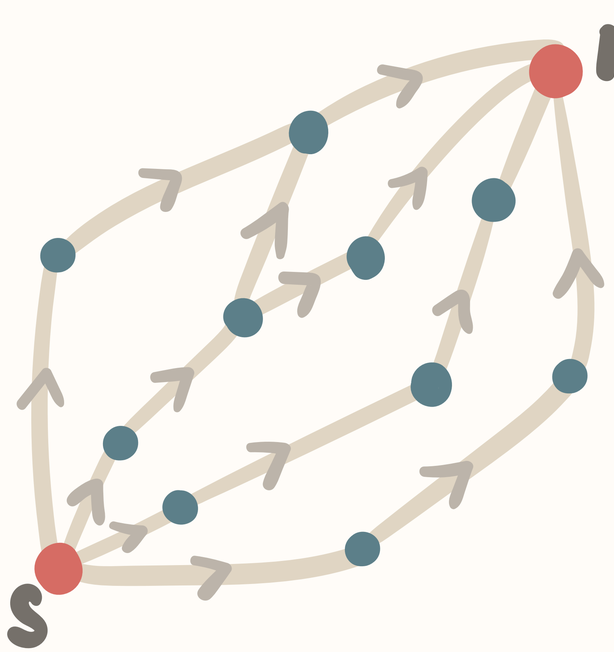


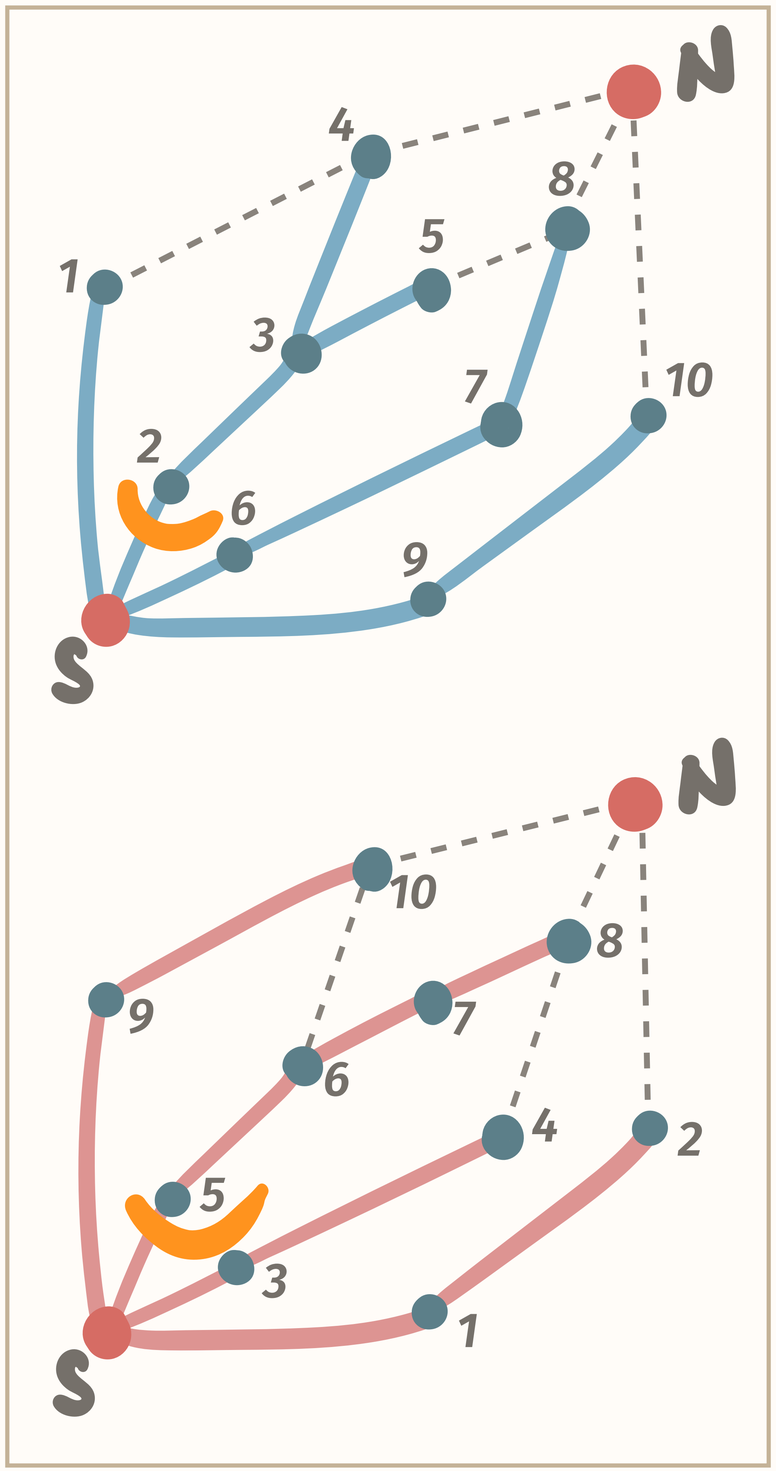


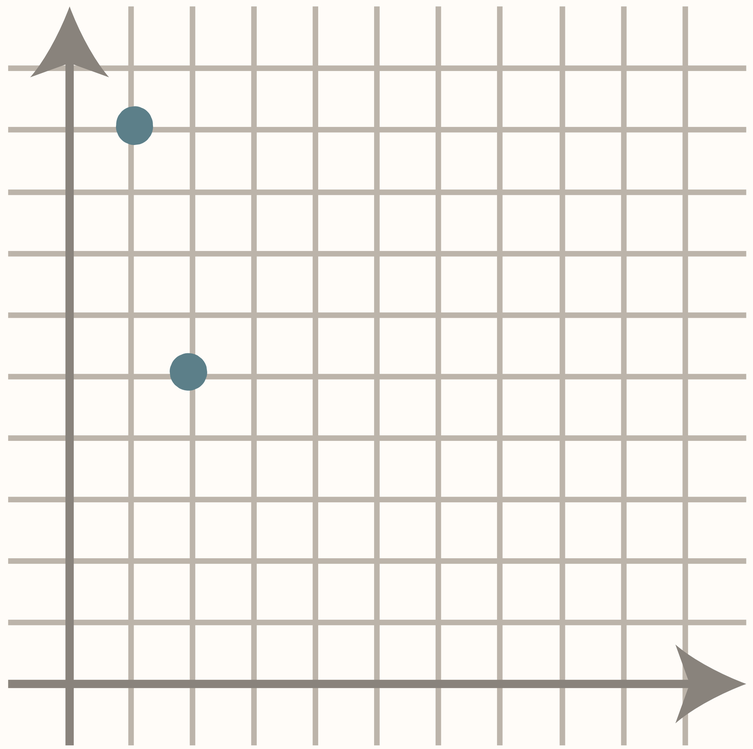
π : 1→9

Link with plane permutations

Poset ⟶ *Plane permutation*





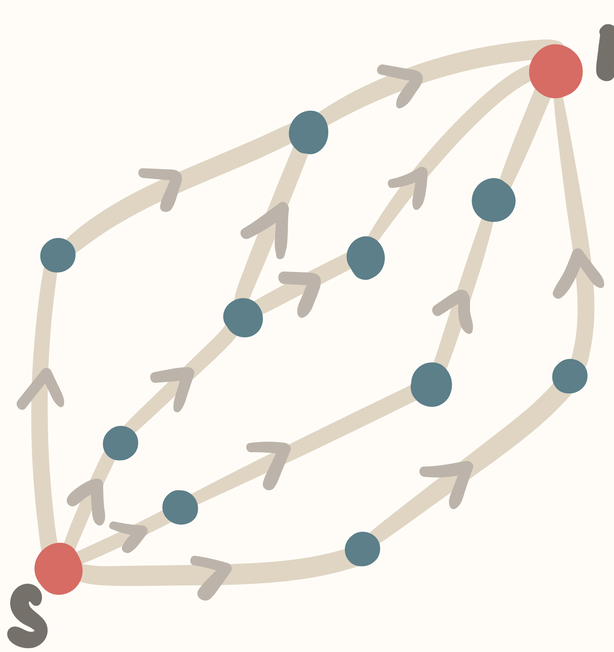


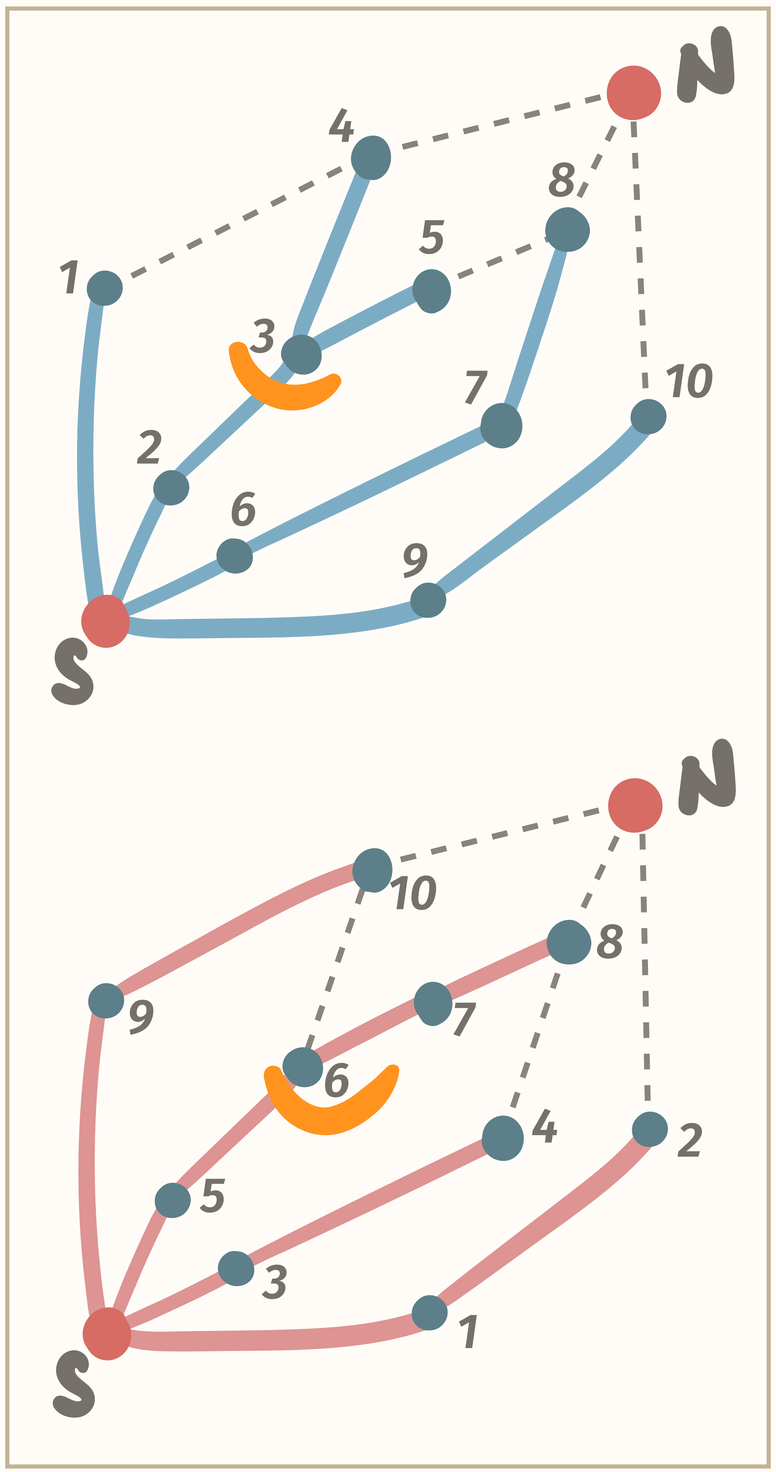
π : 1→9

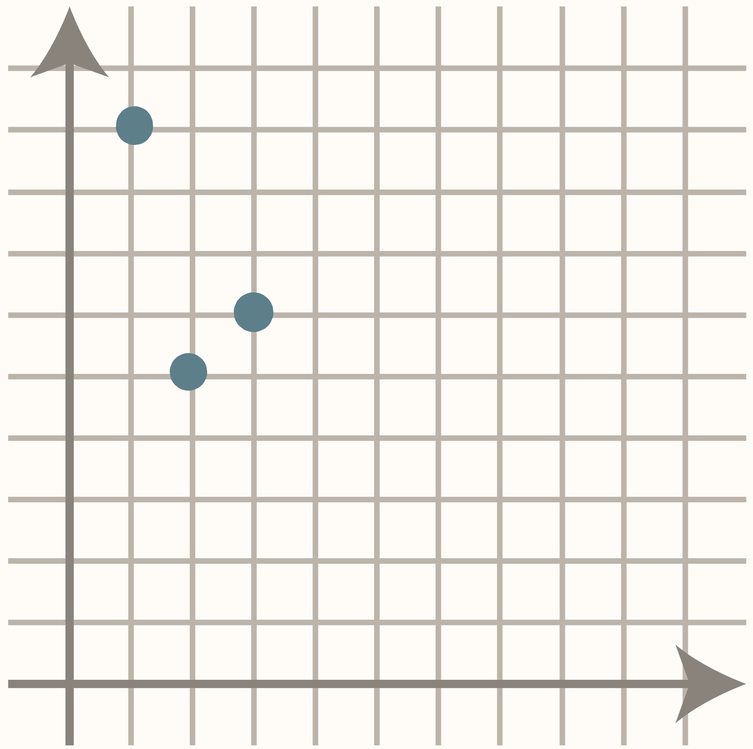
2→5

Link with plane permutations

Poset ⟶ *Plane permutation*



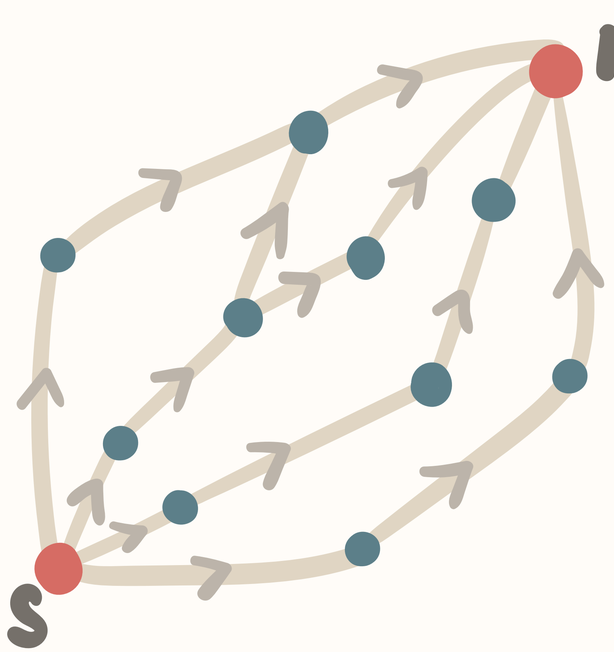


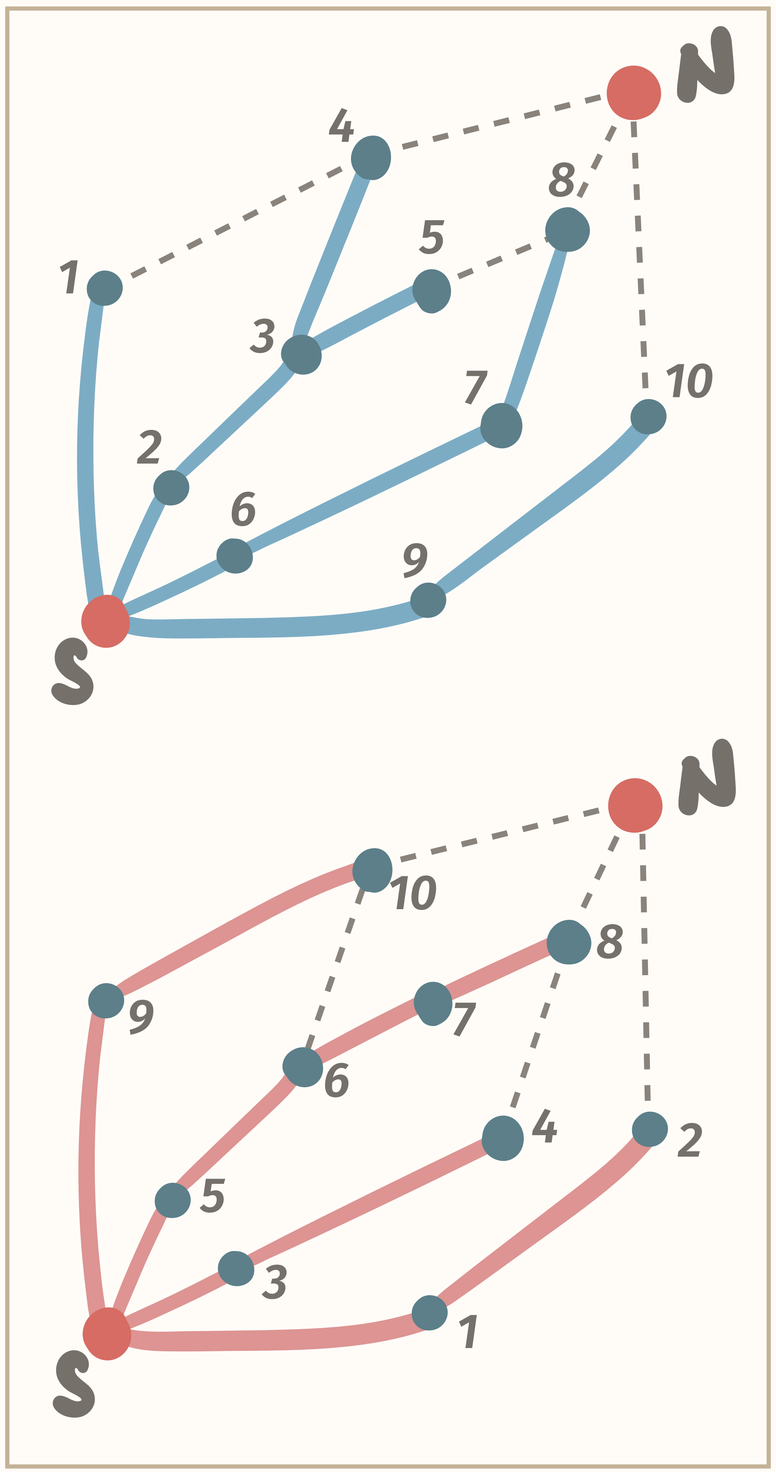


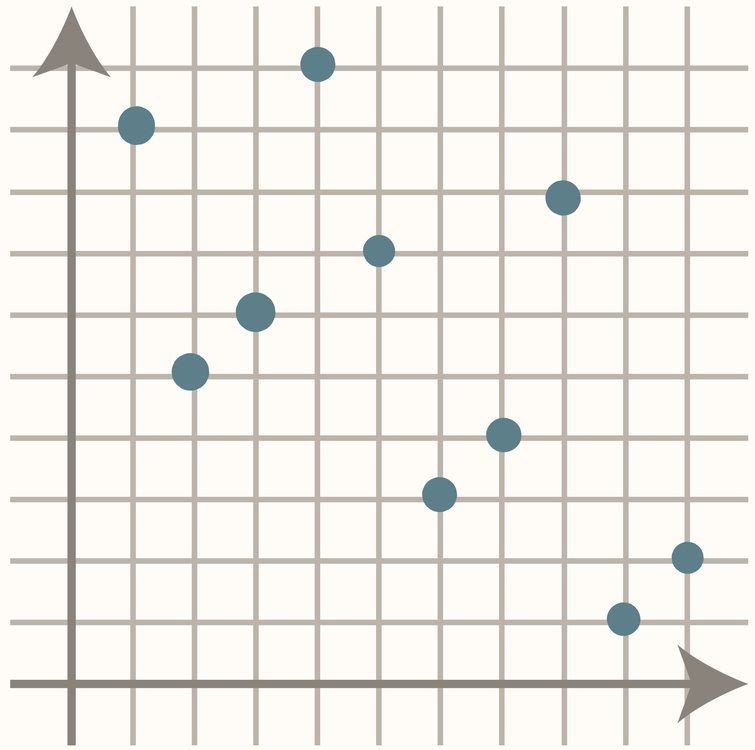
π: 1→9
2→5
3→6

Link with plane permutations

Poset ⟶ *Plane permutation*



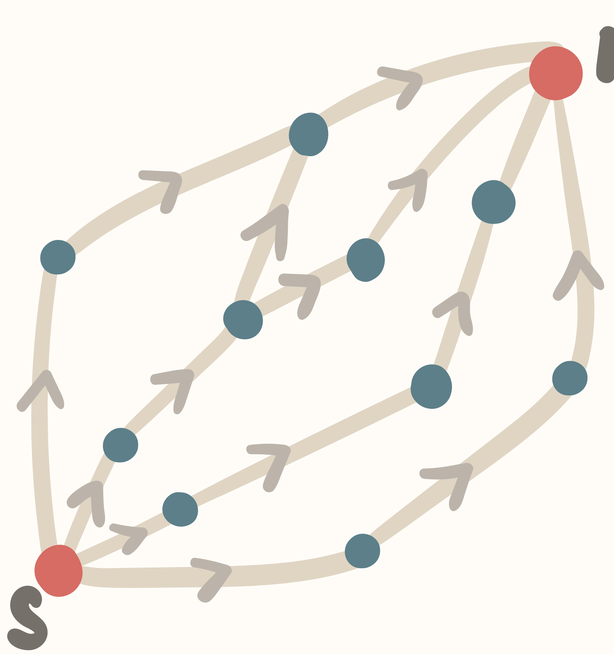


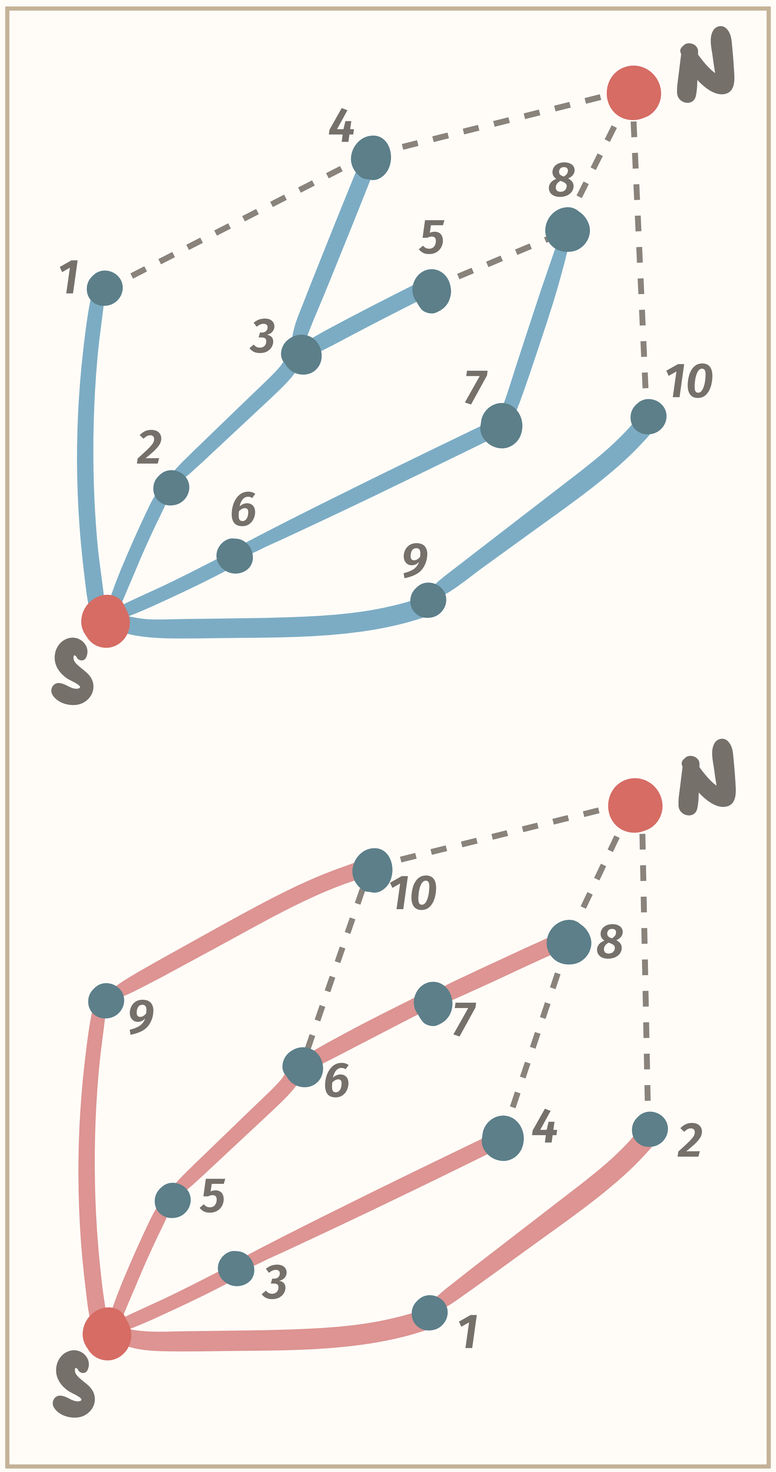


π: $1 \to 9$ $6 \to 3$
$2 \to 5$ $7 \to 4$
$3 \to 6$ $8 \to 8$
$4 \to 10$ $9 \to 1$
$5 \to 7$ $10 \to 2$

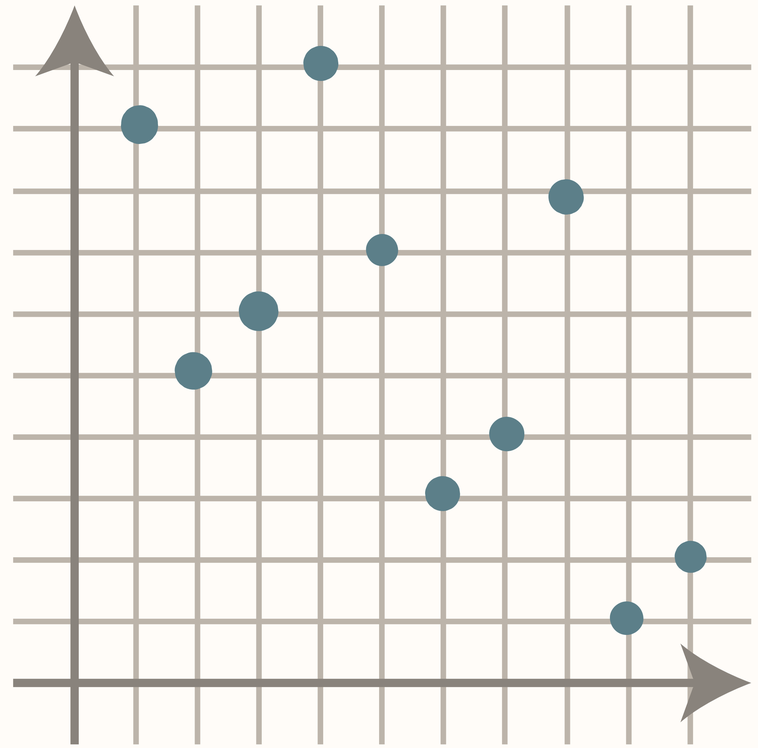
Link with plane permutations

Poset ⟶ *Plane permutation*





» *Area requirement and symmetry display of planar upward drawings,* G. Di Battista, R. Tamassia, and I. G. Tollis (1992)



π : 1→9 6→3
2→5 7→4
3→6 8→8
4→10 9→1
5→7 10→2